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A Decision-Making Approach Based on Correlation Coefficient For Generalized multi-Polar Neutrosophic Soft Set

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Abstract:

The paper proposes the generalized version of the multipolar neutrosophic soft set. The neutrosophic soft set (NSS) is an advanced extension of the neutrosophic set, which accommodates the alternatives' parametrized values. This paper extends the NSS to generalized multipolar NSS and introduces some fundamental operations for generalized multipolar NSS with their necessary properties. We define the correlation coefficient (CC) and weighted correlation coefficient (WCC) for the generalized multi-polar neutrosophic soft set. Several desirable properties for CC and WCC and their relationship are derived. Finally, based on established correlation measures, a decision-making algorithm under the neutrosophic environment is stated to tackle uncertain and vague information. The applicability of the proposed algorithm is demonstrated through a case study of the decision-making problem. A comparative analysis with several existing studies reveals the effectiveness of the approach.

Keywords: multipolar neutrosophic set; generalized multipolar neutrosophic soft set; CC; WCC; MCDM.

1. Introduction

Uncertainty plays a dynamic role in many areas of life (such as modeling, medicine, engineering, etc.). However, people have raised a common question: how do we express and use the concept of uncertainty in mathematical modeling. Many researchers planned and endorsed different methods to resolve those difficulties that contain hesitation. First of all, Zadeh presented the idea of fuzzy sets (FS) [1] to resolve uncertain complications. But in some cases, fuzzy sets are unable to handle the situation. To overcome such scenarios, the idea of interval-valued fuzzy sets (IVFS) was presented by Turksen [2]. In some cases, we must consider the object's nonmembership value, which cannot be dealt with by FS nor by IVFS. To conquer such issues, Atanassov planned the intuitionistic fuzzy set

(IFS) [3]. The idea proposed by Atanassov involves only under-considered data and membership and non-membership values. However, the IFS theory cannot handle the overall incompatibility and inaccurate information. To solve the problem of incompatibility and incorrect information, Smarandache [4] proposed the idea of NS. Molodtsov [5] presented a general mathematical tool for addressing uncertain environments known as soft set (SS). Maji et al. [6] extended the concept of SS and proposed fundamental operations with their desirable properties. Maji et al. [7] established a decision-making technique utilizing their developed operations and used it for decision making. Ali et al. [8] extended the notion of SS and developed some new operations with their properties. The authors [9] proved De Morgan's law by using different operators for the SS theory.

Maji [10] offered the idea of a neutrosophic soft set (NSS) with necessary operations and properties. The concept of the possibility NSS was developed by Karaaslan [11]. He also established a DM technique utilizing And-product based on the possibility NSS to solve DM issues. Broumi [12] created the generalized NSS with some operations and properties and used the proposed concept for DM. Deli and Subas [13] extended the notion of single-valued Neutrosophic numbers (SVNNs) and offered a DM approach to solving MCDM problems. They also developed the idea of cut sets for SVNNs. Wang et al. [14] presented the correlation coefficient (CC) for SVNSs and constructed a DM approach utilizing their developed correlation measure. Ye [15] offered the idea of simplified NSs and developed the aggregation operators (AOs) for simplified NSs, and established a DM methodology to solve MCDM problems utilizing his developed AOs. Masooma et al. [16] proposed a multipolar neutrosophic set by combining the multipolar fuzzy set and NS. They also established various characterization and operations with examples. Zulqarnain et al. [17] introduced some AOs and correlation coefficients for interval-valued IFSS. They also extended the TOPSIS technique using their developed correlation measures to solve the MADM problem. Zulqarnain et al. [18] introduced operational laws for Pythagorean fuzzy soft numbers (PFSNs). They developed AOs such as Pythagorean fuzzy soft weighted average and geometric using defined operational laws for PFSNs. They also planned a DM approach to solve MADM problems with the help of presented operators. Zulqarnain et al. [19] planned the TOPSIS methodology in the PFSS environment based on the correlation coefficient. They also established a DM methodology to resolve the MCGDM concerns and utilized the developed approach in green supply chain management.

Many mathematicians have developed various similarity measures, correlation coefficients, aggregation operators, and decision-making applications in the past few years. Zulqarnain et al. [20, 21] introduced some novel AOs for PFSS based on Einstein norms. Siddique et al. [22] proposed the score matrix for PFSS. Peng and Garg [23] proposed various PFS similarity measures with multiple parameters. Zulqarnain et al. [24, 25] presented the generalized neutrosophic TOPSIS and an integrated neutrosophic TOPSIS model and used their proposed techniques for supplier selection in the production industry. Saeed et al. [26] established the concept of mPNSS with properties and operators. They also developed a distance-based similarity measure and used the proposed similarity measure for decision-making and medical diagnosis. Zulqarnain et al. [27] developed some novel AOs for PFSS considering the interaction. Zulqarnain et al. [34] presented the generalized multipolar NSS and introduced some information measures to solve decision-making problems. They also extended the concept of multipolar NSS to multipolar interval-valued NSS with basic operations and their desirable properties [35].

In this era, professionals believe that real life is moving toward multi-polarization. Therefore, there is no doubt that the multi-polarization of information has played an essential role in the prosperity of many fields of science and technology. In neurobiology, multipolar neurons accumulate a lot of info from other neurons. The motivation for expanding and mixing this research work is gradually given in the whole manuscript. We prove that under any appropriate circumstances, different hybrid structures containing fuzzy sets will be converted into special privileges of GmPNSS. The concept of the neutrosophic environment of multipolar neutrosophic soft sets is novel. We discuss the effectiveness, flexibility, quality, and advantages of planned work and algorithms. This research will be the most versatile form that can be used to merge data in daily life complications. In the future, current work may be extended to different types of hybrid structures and decision-making techniques in numerous fields of life.

The following research is organized: In section 2, we recollected some basic definitions used in the subsequent sequel, such as NS, SS, NSS, and multipolar neutrosophic set. In section 3, we proposed the generalized version of mPNSS with its operations and introduced the idea of CC and WCC with their properties. Furthermore, a decision-making approach has been established based on developed CC. Finally, we use the developed method for decision-making in section 4. We also presented the comparative study of our proposed similarity measures and CC with existing techniques in section 5.

2. Preliminaries

In this section, we recollect some basic concepts such as the neutrosophic set, soft set, neutrosophic soft set, and m-polar neutrosophic soft set used in the following sequel.

Definition 2.1 [4]

Let $\boldsymbol{\mathcal{U}}$ be a universe, and $\boldsymbol{\mathcal{A}}$ be an NS on $\boldsymbol{\mathcal{U}}$ is defined as $\boldsymbol{\mathcal{A}} = \{\boldsymbol{u}, (\boldsymbol{u}_{\mathcal{A}}(\boldsymbol{u}), \boldsymbol{v}_{\mathcal{A}}(\boldsymbol{u}), \boldsymbol{w}_{\mathcal{A}}(\boldsymbol{u})): \boldsymbol{u} \in \boldsymbol{\mathcal{U}}\}$, where $\boldsymbol{u}$, $\boldsymbol{v}$, $\boldsymbol{w}$: $\boldsymbol{\mathcal{U}} \rightarrow]\mathbf{0}^-, \mathbf{1}^+[$ and $\mathbf{0}^- \leq \boldsymbol{u}_{\mathcal{A}}(\boldsymbol{u}) + \boldsymbol{v}_{\mathcal{A}}(\boldsymbol{u}) + \boldsymbol{w}_{\mathcal{A}}(\boldsymbol{u}) \leq \mathbf{3}^+$.

Definition 2.2 [5]

Let $\mathcal{U}$ be the universal set and $\mathcal{E}$ be the set of attributes concerning $\mathcal{U}$. Let $\mathcal{P}(\mathcal{U})$ be the power set of $\mathcal{U}$ and $\mathcal{A} \subseteq \mathcal{E}$. A pair $(\mathcal{F}, \mathcal{A})$ is called a soft set over $\mathcal{U}$, and its mapping is given as

$$\mathcal{F}: \mathcal{A} \rightarrow \mathcal{P}(\mathcal{U})$$

It is also defined as:

$$(\mathcal{F}, \mathcal{A}) = \{\mathcal{F}(e) \in \mathcal{P}(\mathcal{U}): e \in \mathcal{E}, \mathcal{F}(e) = \emptyset \; if \; e \notin \mathcal{A}\}$$

Definition 2.3 [13]

Let $\mathcal{U}$ be the universal set and $\mathcal{E}$ be the set of attributes concerning $\mathcal{U}$. Let $\mathcal{P}(\mathcal{U})$ be the Neutrosophic values of $\mathcal{U}$ and $\mathcal{A} \subseteq \mathcal{E}$. A pair $(\mathcal{F}, \mathcal{A})$ is called a Neutrosophic soft set over $\mathcal{U}$ and its mapping is given as

$$\mathcal{F}: \mathcal{A} \rightarrow \mathcal{P}(\mathcal{U})$$

Definition 2.4 [19]

Let $\mathcal{U}$ be the universal set and $\mathcal{E}$ be the set of attributes concerning $\mathcal{U}$, then $\mathcal{F}_{\mathcal{E}}$ is said to multipolar neutrosophic set if

$\mathcal{F}_{\mathcal{E}} = \{u, (s_i \bullet \mathcal{u}_e(u),\ s_i \bullet \mathcal{v}_e(u), s_i \bullet \mathcal{w}_e(u)): u \in \mathcal{U},\ e \in \mathcal{E}, i = 1, 2, 3, \dots . m\}$, where $s_i \bullet \mathcal{u}_{\mathcal{E}}$, $s_i \bullet \mathcal{v}_{\mathcal{E}}, s_i \bullet \mathcal{w}_{\mathcal{E}}$: $\mathcal{U} \to$ [0, 1], and $0 \leq s_i \bullet \mathcal{u}_{\mathcal{E}}(u) + s_i \bullet \mathcal{v}_{\mathcal{E}}(u) + s_i \bullet \mathcal{w}_{\mathcal{E}}(u) \leq 3$; $i = 1, 2, 3, \dots . m$. $\mathcal{u}_e$, $\mathcal{v}_e$, and $\mathcal{w}_e$ represent the truth, indeterminacy, and falsity of the considered alternative.

3. Basic Operations and Correlation Coefficient for Generalized Multi-Polar Neutrosophic Soft Set

In this section, we develop the concept of GmPNSS and introduce aggregate operators on GmPNSS with their properties.

Definition 3.1

Let $\boldsymbol{\mathcal{U}}$ and E are universal and set of attributes respectively, and $\boldsymbol{\mathcal{A}} \subseteq$ E, if there exists a mapping Φ such as

$$\Phi\colon \mathcal{A} \to GmPNSS^{\mathcal{U}}$$

Then $(\Phi,\ \mathcal{A})$ is called GmPNSS over $\mathcal{U}$ defined as follows

$$\Upsilon_K = (\Phi,\ \mathcal{A}) = \left\{\left(e, \left(u, \Phi_{\mathcal{A}(e)}(u)\right)\right): e \in E, u \in \mathcal{U}\right\},$$

where $\Phi_{\mathcal{A}}(e) = \left\{u, \left(s_i \bullet \mathcal{u}_{\mathcal{A}(e)}(u), s_i \bullet \mathcal{v}_{\mathcal{A}(e)}(u),\ s_i \bullet \mathcal{w}_{\mathcal{A}(e)}(u)\right): u \in \mathcal{U}, e \in E;\ i \in 1, 2, 3, \dots, m\right\}$,

and

$0 \leq s_i \bullet \mathcal{u}_{\mathcal{A}(e)}(u) + s_i \bullet \mathcal{v}_{\mathcal{A}(e)}(u) + s_i \bullet \mathcal{w}_{\mathcal{A}(e)}(u) \leq 3$ for all $i \in 1, 2, 3,\dots, m$; $e \in E$ and $u \in \mathcal{U}$.

Definition 3.2

Let $\Upsilon_{\mathcal{A}}$ and Υ_B are two GmPNSS over $\mathcal{U}$, then $\Upsilon_{\mathcal{A}}$ is called a multi-polar neutrosophic soft subset of Υ_B. If

$s_i \bullet \mathcal{u}_{\mathcal{A}(e)}(u) \leq s_i \bullet \mathcal{u}_{B(e)}(u)$, $s_i \bullet \mathcal{v}_{\mathcal{A}(e)}(u) \leq s_i \bullet \mathcal{v}_{B(e)}(u)$ and $s_i \bullet \mathcal{w}_{\mathcal{A}(e)}(u) \geq s_i \bullet \mathcal{w}_{B(e)}(u)$ for all $i \in 1, 2, 3,\dots, m$; $e \in E$ and $u \in \mathcal{U}$.

Definition 3.3

Let $\Upsilon_{\mathcal{A}}$ and Υ_B are two GmPNSS over $\mathcal{U}$, then $\Upsilon_{\mathcal{A}} = \Upsilon_B$, if

$$s_i \bullet \mathcal{u}_{\mathcal{A}(e)}(u) \leq s_i \bullet \mathcal{u}_{B(e)}(u),\ s_i \bullet \mathcal{u}_{B(e)}(u) \leq s_i \bullet \mathcal{u}_{\mathcal{A}(e)}(u)$$
$$s_i \bullet \mathcal{v}_{\mathcal{A}(e)}(u) \leq s_i \bullet \mathcal{v}_{B(e)}(u),\ s_i \bullet \mathcal{v}_{B(e)}(u) \leq s_i \bullet \mathcal{v}_{\mathcal{A}(e)}(u)$$
$$s_i \bullet \mathcal{w}_{\mathcal{A}(e)}(u) \geq s_i \bullet \mathcal{w}_{B(e)}(u),\ s_i \bullet \mathcal{w}_{B(e)}(u) \geq s_i \bullet \mathcal{w}_{\mathcal{A}(e)}(u)$$

for all $i \in 1, 2, 3,\dots, m$; $e \in E$ and $u \in \mathcal{U}$.

Definition 3.4

Let $\boldsymbol{\mathcal{F}_{\breve{A}}} = \{\boldsymbol{u_k}, (\boldsymbol{s_i \bullet \mathcal{u}_{\breve{A}}(u_k)}, \boldsymbol{s_i \bullet \mathcal{v}_{\breve{A}}(u_k)}, \boldsymbol{s_i \bullet \mathcal{w}_{\breve{A}}(u_k)}): \boldsymbol{u_k \in \mathcal{U}};\ \boldsymbol{i \in 1, 2, 3, \dots, m}\}$ and $\mathcal{G}_{\breve{B}} = \{u_k, (s_i \bullet \mathcal{u}_{\breve{B}}(u_k), s_i \bullet \mathcal{v}_{\breve{B}}(u_k), s_i \bullet \mathcal{w}_{\breve{B}}(u_k)): u_k \in \mathcal{U};\ i \in 1, 2, 3, \dots, m\}$ are two GmPNSS over a set of parameters $E = \{x_1,\ x_2,\ x_3,\ \dots,\ x_n\}$. Then informational neutrosophic energies of two GmPNSS can be expressed as follows

$$\mathcal{E}_{GmPNSS}\ (\mathcal{F}_{\breve{A}}) = \sum_{j=1}^{z}\sum_{k=1}^{t}\left(\left(s_i \bullet \mathcal{u}_{\breve{A}_j}(u_k)\right)^2 + \left(s_i \bullet \mathcal{v}_{\breve{A}_j}(u_k)\right)^2 + \left(s_i \bullet \mathcal{w}_{\breve{A}_j}(u_k)\right)^2\right)$$
$$\mathcal{E}_{GmPNSS}\ (\mathcal{G}_{\breve{B}}) = \sum_{j=1}^{z}\sum_{k=1}^{t}\left(\left(s_i \bullet \mathcal{u}_{\breve{B}_j}(u_k)\right)^2 + \left(s_i \bullet \mathcal{v}_{\breve{B}_j}(u_k)\right)^2 + \left(s_i \bullet \mathcal{w}_{\breve{B}_j}(u_k)\right)^2\right)$$

Definition 3.5

The correlation of two GmPNSS can be presented as follows

$\varsigma_{GmPNSS}(\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}}) =$

$$\sum_{j=1}^{z}\sum_{k=1}^{t}\left\{\begin{pmatrix} s_i \bullet u_{\breve{A}_j}(u_k)s_i \bullet u_{\breve{B}_j}(u_k) + s_i \bullet v_{\breve{A}_j}(u_k)s_i \bullet v_{\breve{B}_j}(u_k) + \\ s_i \bullet w_{\breve{A}_j}(u_k)\, s_i \bullet w_{\breve{B}_j}(u_k) \end{pmatrix} : i \in 1,2,3,\ldots,m.\right\} \tag{3.1}$$

Definition 3.6

Let $\mathcal{F}_{\breve{A}}$ and $\mathcal{G}_{\breve{B}}$ are two GmPNSS, then the CC between them can be defined as follows

$$\mathcal{R}_{GmPNSS}\ (\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}}) = \frac{\varsigma_{GmPNSS}(\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}})}{\sqrt{\varepsilon_{GmPNSS}(\mathcal{F}_{\breve{A}},\mathcal{F}_{\breve{A}})\cdot \varepsilon_{GmPNSS}(\mathcal{G}_{\breve{B}},\mathcal{G}_{\breve{B}})}} \tag{3.2}$$

$$\mathcal{R}_{GmPNSS}\ (\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}}) = \frac{\sum_{j=1}^{z}\sum_{k=1}^{t}\left(s_i \bullet u_{\breve{A}_j}(u_k)s_i \bullet u_{\breve{B}_j}(u_k) + s_i \bullet v_{\breve{A}_j}(u_k)s_i \bullet v_{\breve{B}_j}(u_k) + s_i \bullet w_{\breve{A}_j}(u_k)\, s_i \bullet w_{\breve{B}_j}(u_k)\right)}{\sqrt{\sum_{j=1}^{z}\sum_{k=1}^{t}\left(\left(s_i \bullet u_{\breve{A}_j}(u_k)\right)^2 + \left(s_i \bullet v_{\breve{A}_j}(u_k)\right)^2 + \left(s_i \bullet w_{\breve{A}_j}(u_k)\right)^2\right)}\sqrt{\sum_{j=1}^{z}\sum_{k=1}^{t}\left(\left(s_i \bullet u_{\breve{B}_j}(u_k)\right)^2 + \left(s_i \bullet v_{\breve{B}_j}(u_k)\right)^2 + \left(s_i \bullet w_{\breve{B}_j}(u_k)\right)^2\right)}}$$

Proposition 3.7

Let $\mathcal{F}_{\breve{A}}$ and $\mathcal{G}_{\breve{B}}$ are two GmPNSS, then the CC $\mathcal{R}_{GmPNSS}\ (\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}})$ between them satisfied the following properties

1. $0 \leq \mathcal{R}_{GmPNSS}\ (\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}}) \leq 1$
2. $\mathcal{R}_{GmPNSS}\ (\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}}) = \mathcal{R}_{GmPNSS}\ (\mathcal{G}_{\breve{B}},\ \mathcal{F}_{\breve{A}})$
3. If $\mathcal{F}_{\breve{A}} = \mathcal{G}_{\breve{B}}$ i. e; $s_i \bullet u_{\breve{A}_j}(u_k) = s_i \bullet u_{\breve{B}_j}(u_k)$, $s_i \bullet v_{\breve{A}_j}(u_k) = s_i \bullet v_{\breve{B}_j}(u_k)$, and $s_i \bullet w_{\breve{A}_j}(u_k) = s_i \bullet w_{\breve{B}_j}(u_k)$ for all j, k, where $i \in 1,2,3,\ldots,m$, then $\mathcal{R}_{GmPNSS}\ (\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}}) = 1$.

Proof 1

$\mathcal{R}_{GmPNSS}\ (\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}}) \geq 0$ is trivial, so we just need to prove that $\mathcal{R}_{GmPNSS}\ (\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}}) \leq 1$.

As we know that

$$\varsigma_{GmPNSS}\ (\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}}) = \sum_{j=1}^{z}\sum_{k=1}^{t}\begin{pmatrix} s_i \bullet u_{\breve{A}_j}(u_k)s_i \bullet u_{\breve{B}_j}(u_k) + s_i \bullet v_{\breve{A}_j}(u_k)s_i \bullet v_{\breve{B}_j}(u_k) + \\ s_i \bullet w_{\breve{A}_j}(u_k)\, s_i \bullet w_{\breve{B}_j}(u_k) \end{pmatrix}$$

$$= \sum_{j=1}^{z}\left(s_i \bullet u_{\breve{A}_j}(u_1)s_i \bullet u_{\breve{B}_j}(u_1) + s_i \bullet v_{\breve{A}_j}(u_1)s_i \bullet v_{\breve{B}_j}(u_1) + s_i \bullet w_{\breve{A}_j}(u_1)\, s_i \bullet w_{\breve{B}_j}(u_1)\right)$$

$$+ \sum_{j=1}^{z}\left(s_i \bullet u_{\breve{A}_j}(u_2)s_i \bullet u_{\breve{B}_j}(u_2) + s_i \bullet v_{\breve{A}_j}(u_2)s_i \bullet v_{\breve{B}_j}(u_2) + s_i \bullet w_{\breve{A}_j}(u_2)\, s_i \bullet w_{\breve{B}_j}(u_2)\right)$$

$$+$$

$$\vdots$$

$$+ \sum_{j=1}^{z}\left(s_i \bullet u_{\breve{A}_j}(u_t)s_i \bullet u_{\breve{B}_j}(u_t) + s_i \bullet v_{\breve{A}_j}(u_t)s_i \bullet v_{\breve{B}_j}(u_t) + s_i \bullet w_{\breve{A}_j}(u_t)\, s_i \bullet w_{\breve{B}_j}(u_t)\right)$$

$$=$$

$$\left\{\begin{matrix} \left(s_i \bullet u_{\breve{A}_1}(u_1)s_i \bullet u_{\breve{B}_1}(u_1) + s_i \bullet v_{\breve{A}_1}(u_1)s_i \bullet v_{\breve{B}_1}(u_1) + s_i \bullet w_{\breve{A}_1}(u_1)\, s_i \bullet w_{\breve{B}_1}(u_1)\right) + \\ \left(s_i \bullet u_{\breve{A}_2}(u_1)s_i \bullet u_{\breve{B}_2}(u_1) + s_i \bullet v_{\breve{A}_2}(u_1)s_i \bullet v_{\breve{B}_2}(u_1) + s_i \bullet w_{\breve{A}_2}(u_1)\, s_i \bullet w_{\breve{B}_2}(u_1)\right) + \cdots + \\ \left(s_i \bullet u_{\breve{A}_z}(u_1)s_i \bullet u_{\breve{B}_z}(u_1) + s_i \bullet v_{\breve{A}_z}(u_1)s_i \bullet v_{\breve{B}_z}(u_1) + s_i \bullet w_{\breve{A}_z}(u_1)\, s_i \bullet w_{\breve{B}_z}(u_1)\right) \end{matrix}\right\} +$$

$$\left\{\begin{matrix} \left(s_i \bullet u_{\breve{A}_1}(u_2)s_i \bullet u_{\breve{B}_1}(u_2) + s_i \bullet v_{\breve{A}_1}(u_2)s_i \bullet v_{\breve{B}_1}(u_2) + s_i \bullet w_{\breve{A}_1}(u_2)\, s_i \bullet w_{\breve{B}_1}(u_2)\right) + \\ \left(s_i \bullet u_{\breve{A}_2}(u_2)s_i \bullet u_{\breve{B}_2}(u_2) + s_i \bullet v_{\breve{A}_2}(u_2)s_i \bullet v_{\breve{B}_2}(u_2) + s_i \bullet w_{\breve{A}_2}(u_2)\, s_i \bullet w_{\breve{B}_2}(u_2)\right) + \cdots + \\ \left(s_i \bullet u_{\breve{A}_z}(u_2)s_i \bullet u_{\breve{B}_z}(u_2) + s_i \bullet v_{\breve{A}_z}(u_2)s_i \bullet v_{\breve{B}_z}(u_2) + s_i \bullet w_{\breve{A}_z}(u_2)\, s_i \bullet w_{\breve{B}_z}(u_2)\right) \end{matrix}\right\} + \cdots +$$

$$\left\{\begin{matrix} \left(s_i \bullet u_{\breve{A}_1}(u_k)s_i \bullet u_{\breve{B}_1}(u_k) + s_i \bullet v_{\breve{A}_1}(u_k)s_i \bullet v_{\breve{B}_1}(u_k) + s_i \bullet w_{\breve{A}_1}(u_k)\, s_i \bullet w_{\breve{B}_1}(u_k)\right) + \\ \left(s_i \bullet u_{\breve{A}_2}(u_k)s_i \bullet u_{\breve{B}_2}(u_k) + s_i \bullet v_{\breve{A}_2}(u_k)s_i \bullet v_{\breve{B}_2}(u_k) + s_i \bullet w_{\breve{A}_2}(u_k)\, s_i \bullet w_{\breve{B}_2}(u_k)\right) + \cdots + \\ \left(s_i \bullet u_{\breve{A}_z}(u_k)s_i \bullet u_{\breve{B}_z}(u_k) + s_i \bullet v_{\breve{A}_z}(u_k)s_i \bullet v_{\breve{B}_z}(u_k) + s_i \bullet w_{\breve{A}_z}(u_k)\, s_i \bullet w_{\breve{B}_z}(u_k)\right) \end{matrix}\right\}$$

$$= \sum_{j=1}^{z}\left(s_i \bullet u_{\breve{A}_j}(u_1)s_i \bullet u_{\breve{B}_j}(u_1) + s_i \bullet u_{\breve{A}_j}(u_2)s_i \bullet u_{\breve{B}_j}(u_2) + \cdots + s_i \bullet u_{\breve{A}_j}(u_t)s_i \bullet u_{\breve{B}_j}(u_t)\right)$$

$$+ \sum_{j=1}^{z}\left(s_i \bullet v_{\breve{A}_j}(u_1)s_i \bullet v_{\breve{B}_j}(u_1) + s_i \bullet v_{\breve{A}_j}(u_2)s_i \bullet v_{\breve{B}_j}(u_2) + \cdots + s_i \bullet v_{\breve{A}_j}(u_t)s_i \bullet v_{\breve{B}_j}(u_t)\right)$$

$$+ \sum_{j=1}^{z}\left(s_i \bullet w_{\breve{A}_j}(u_1)s_i \bullet w_{\breve{B}_j}(u_1) + s_i \bullet w_{\breve{A}_j}(u_2)s_i \bullet w_{\breve{B}_j}(u_2) + \cdots + s_i \bullet w_{\breve{A}_j}(u_t)s_i \bullet w_{\breve{B}_j}(u_t)\right)$$

By using Cauchy-Schwarz inequality, we get

$$(\varsigma_{GmPNSS}\ (\mathcal{F}_{\breve{A}}, \mathcal{G}_{\breve{B}}))^2 \leq \left\{\begin{array}{c} \sum_{j=1}^{z}\left(\left(s_i \bullet u_{\breve{A}_j}(u_1)\right)^2 + \left(s_i \bullet u_{\breve{A}_j}(u_2)\right)^2 + \cdots + \left(s_i \bullet u_{\breve{A}_j}(u_t)\right)^2\right) + \\ \sum_{j=1}^{z}\left(\left(s_i \bullet v_{\breve{A}_j}(u_1)\right)^2 + \left(s_i \bullet v_{\breve{A}_j}(u_2)\right)^2 + \cdots + \left(s_i \bullet v_{\breve{A}_j}(u_t)\right)^2\right) + \cdots + \\ \sum_{j=1}^{z}\left(\left(s_i \bullet w_{\breve{A}_j}(u_1)\right)^2 + \left(s_i \bullet w_{\breve{A}_j}(u_2)\right)^2 + \cdots + \left(s_i \bullet w_{\breve{A}_j}(u_t)\right)^2\right) \end{array}\right\} \times$$

$$\left\{\begin{array}{c} \sum_{j=1}^{z}\left(\left(s_i \bullet u_{\breve{B}_j}(u_1)\right)^2 + \left(s_i \bullet u_{\breve{B}_j}(u_2)\right)^2 + \cdots + \left(s_i \bullet u_{\breve{B}_j}(u_t)\right)^2\right) + \\ \sum_{j=1}^{z}\left(\left(s_i \bullet v_{\breve{B}_j}(u_1)\right)^2 + \left(s_i \bullet v_{\breve{B}_j}(u_2)\right)^2 + \cdots + \left(s_i \bullet v_{\breve{B}_j}(u_t)\right)^2\right) + \cdots + \\ \sum_{j=1}^{z}\left(\left(s_i \bullet w_{\breve{B}_j}(u_1)\right)^2 + \left(s_i \bullet w_{\breve{B}_j}(u_2)\right)^2 + \cdots + \left(s_i \bullet w_{\breve{B}_j}(u_t)\right)^2\right) \end{array}\right\}$$

$$= \left\{\sum_{j=1}^{z}\sum_{k=1}^{t}\left(\left(s_i \bullet u_{\breve{A}_j}(u_t)\right)^2 + \left(s_i \bullet v_{\breve{A}_j}(u_t)\right)^2 + \left(s_i \bullet w_{\breve{A}_j}(u_t)\right)^2\right)\right\} \times \left\{\sum_{j=1}^{z}\sum_{k=1}^{t}\left(\left(s_i \bullet u_{\breve{B}_j}(u_t)\right)^2 + \left(s_i \bullet v_{\breve{B}_j}(u_t)\right)^2 + \left(s_i \bullet w_{\breve{B}_j}(u_t)\right)^2\right)\right\}$$

$$= \mathcal{E}_{GmPNSS}\ (\mathcal{F}_{\breve{A}}) \cdot \mathcal{E}_{GmPNSS}\ (\mathcal{G}_{\breve{B}})$$

Therefore, $(\varsigma_{GmPNSS}\ (\mathcal{F}_{\breve{A}}, \mathcal{G}_{\breve{B}}))^2 \leq \mathcal{E}_{GmPNSS}\ (\mathcal{F}_{\breve{A}}) \cdot \mathcal{E}_{GmPNSS}\ (\mathcal{G}_{\breve{B}})$. Hence, by using Definition 3.6, we get

$\mathcal{R}_{GmPNSS}\ (\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}}) \leq 1$, so $0 \leq \mathcal{R}_{GmPNSS}\ (\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}}) \leq 1$.

Proof 2 The proof is obvious.

Proof 3

As we know that

$$\mathcal{R}_{GmPNSS}\ (\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}}) = \frac{\sum_{j=1}^{z}\sum_{k=1}^{t}\left(s_i \bullet u_{\breve{B}_j}(u_k) s_i \bullet u_{\breve{B}_j}(u_k) + s_i \bullet v_{\breve{B}_j}(u_k) s_i \bullet v_{\breve{B}_j}(u_k) + s_i \bullet w_{\breve{B}_j}(u_k)\ s_i \bullet w_{\breve{B}_j}(u_k)\right)}{\sqrt{\sum_{j=1}^{z}\sum_{k=1}^{t}\left(\left(s_i \bullet u_{\breve{B}_j}(u_k)\right)^2 + \left(s_i \bullet v_{\breve{B}_j}(u_k)\right)^2 + \left(s_i \bullet w_{\breve{B}_j}(u_k)\right)^2\right)}\sqrt{\sum_{j=1}^{z}\sum_{k=1}^{t}\left(\left(s_i \bullet u_{\breve{B}_j}(u_k)\right)^2 + \left(s_i \bullet v_{\breve{B}_j}(u_k)\right)^2 + \left(s_i \bullet w_{\breve{B}_j}(u_k)\right)^2\right)}}$$

As we know that $s_i \bullet u_{\breve{A}_j}(u_k) = s_i \bullet u_{\breve{B}_j}(u_k)$, $s_i \bullet v_{\breve{A}_j}(u_k) = s_i \bullet v_{\breve{B}_j}(u_k)$, and $s_i \bullet w_{\breve{A}_j}(u_k) = s_i \bullet w_{\breve{B}_j}(u_k)$, for all j, k, so by using Definition 3.6, we have

$$\mathcal{R}_{GmPNSS}\ (\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}}) = \frac{\sum_{j=1}^{z}\sum_{k=1}^{t}\left(\left(s_i \bullet u_{\breve{B}_j}(u_k)\right)^2 + \left(s_i \bullet v_{\breve{B}_j}(u_k)\right)^2 + \left(s_i \bullet w_{\breve{B}_j}(u_k)\right)^2\right)}{\sqrt{\sum_{j=1}^{z}\sum_{k=1}^{t}\left(\left(s_i \bullet u_{\breve{B}_j}(u_k)\right)^2 + \left(s_i \bullet v_{\breve{B}_j}(u_k)\right)^2 + \left(s_i \bullet w_{\breve{B}_j}(u_k)\right)^2\right)}\sqrt{\sum_{j=1}^{z}\sum_{k=1}^{t}\left(\left(s_i \bullet u_{\breve{B}_j}(u_k)\right)^2 + \left(s_i \bullet v_{\breve{B}_j}(u_k)\right)^2 + \left(s_i \bullet w_{\breve{B}_j}(u_k)\right)^2\right)}}$$

Hence

$\mathcal{R}_{GmPNSS}\ (\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}}) = 1$.

Definition 3.8

Let $\mathcal{F}_{\breve{A}}$ and $\mathcal{G}_{\breve{B}}$ are two GmPNSS, then the CC between them also can be defined as follows

$$\mathcal{R}^1_{GmPNSS}\ (\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}}) = \frac{\varsigma_{GmPNSS}(\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}})}{max\{\mathcal{E}_{GmPNSS}(\mathcal{F}_{\breve{A}},\mathcal{F}_{\breve{A}}), \mathcal{E}_{GmPNSS}(\mathcal{G}_{\breve{B}},\mathcal{G}_{\breve{B}})\}} \tag{3.3}$$

$$\mathcal{R}^1_{GmPNSS}(\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}}) = \frac{\sum_{j=1}^{z}\sum_{k=1}^{t}\left(s_i \bullet u_{\breve{A}_j}(u_k) s_i \bullet u_{\breve{B}_j}(u_k) + s_i \bullet v_{\breve{A}_j}(u_k) s_i \bullet v_{\breve{B}_j}(u_k) + s_i \bullet w_{\breve{A}_j}(u_k)\ s_i \bullet w_{\breve{B}_j}(u_k)\right)}{max\left\{\begin{array}{c}\sum_{j=1}^{z}\sum_{k=1}^{t}\left(\left(s_i \bullet u_{\breve{A}_j}(u_k)\right)^2 + \left(s_i \bullet v_{\breve{A}_j}(u_k)\right)^2 + \left(s_i \bullet w_{\breve{A}_j}(u_k)\right)^2\right), \\ \sum_{j=1}^{s}\sum_{k=1}^{t}\left(\left(s_i \bullet u_{\breve{B}_j}(u_k)\right)^2 + \left(s_i \bullet v_{\breve{B}_j}(u_k)\right)^2 + \left(s_i \bullet w_{\breve{B}_j}(u_k)\right)^2\right)\end{array}\right\}}$$

Proposition 3.9

Let $\mathcal{F}_{\breve{A}}$ and $\mathcal{G}_{\breve{B}}$ are two GmPNSS, then the CC $\mathcal{R}^1_{GmPNSS}\ (\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}})$ between them satisfied the following properties.

1. $0 \leq \mathcal{R}^1_{GmPNSS}\ (\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}}) \leq 1$
2. $\mathcal{R}^1_{GmPNSS}\ (\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}}) = \mathcal{R}^1_{GmPNSS}\ (\mathcal{G}_{\breve{B}},\ \mathcal{F}_{\breve{A}})$

3. If $\mathcal{F}_{\breve{A}} = \mathcal{G}_{\breve{B}}$ i. e; $s_i \bullet u_{\breve{A}_j}(u_k) = s_i \bullet u_{\breve{B}_j}(u_k)$, $s_i \bullet v_{\breve{A}_j}(u_k) = s_i \bullet v_{\breve{B}_j}(u_k)$, and $s_i \bullet w_{\breve{A}_j}(u_k) = s_i \bullet w_{\breve{B}_j}(u_k)$ for all j, k, where $i \in 1, 2, 3, \ldots, m$, then $\mathcal{R}^1_{GmPNSS}(\mathcal{F}_{\breve{A}}, \mathcal{G}_{\breve{B}}) = 1$.

Proof

We can prove easily according to Definition 3.7.

It is important to anticipate the weight of IVNSS for functional determinations. When a decision-maker alleviates a distinct weight for each object in the universe of discourse, the result of the purpose may be distinctive. So, it is necessary to consider the weights before making a decision. Let $\acute{\omega} = \{\dot{\omega}_1, \dot{\omega}_2, \dot{\omega}_3, \ldots, \dot{\omega}_m\}$ be a weight vector for experts such as $\dot{\omega}_k > 0$, $\sum_{k=1}^{m} \dot{\omega}_k = 1$ and $\delta = \{\delta_1, \delta_2, \delta_3, \ldots, \delta_n\}$ be a weight vector for parameters such as $\delta_i > 0$, $\sum_{i=1}^{n} \delta_i = 1$. By extending definitions 3.6, 3.8, the notion of WCC has been developed in the following.

Definition 3.10

For two GmPNSS $\mathcal{F}_{\breve{A}}$ and $\mathcal{G}_{\breve{B}}$, the WCC between them can be defined as follows

$$\mathcal{R}_{GWmPNSS}(\mathcal{F}_{\breve{A}}, \mathcal{G}_{\breve{B}}) = \frac{\varsigma_{GmPNSS}(\mathcal{F}_{\breve{A}}, \mathcal{G}_{\breve{B}})}{\sqrt{\varepsilon_{GmPNSS}(\mathcal{F}_{\breve{A}}, \mathcal{F}_{\breve{A}})\, \varepsilon_{GmPNSS}(\mathcal{G}_{\breve{B}}, \mathcal{G}_{\breve{B}})}} \tag{3.4}$$

$$\mathcal{R}_{GWmPNSS}(\mathcal{F}_{\breve{A}}, \mathcal{G}_{\breve{B}}) = \frac{\sum_{j=1}^{z} \delta_j \left(\sum_{k=1}^{t} \dot{\omega}_k \left(s_i \bullet u_{\breve{A}_j}(u_k) s_i \bullet u_{\breve{B}_j}(u_k) + s_i \bullet v_{\breve{A}_j}(u_k) s_i \bullet v_{\breve{B}_j}(u_k) + s_i \bullet w_{\breve{A}_j}(u_k)\, s_i \bullet w_{\breve{B}_j}(u_k) \right) \right)}{\left(\begin{array}{c} \sqrt{\sum_{j=1}^{z} \delta_j \left(\sum_{k=1}^{t} \dot{\omega}_k \left(\left(s_i \bullet u_{\breve{A}_j}(u_k) \right)^2 + \left(s_i \bullet v_{\breve{A}_j}(u_k) \right)^2 + \left(s_i \bullet w_{\breve{A}_j}(u_k) \right)^2 \right) \right)} \\ \sqrt{\sum_{j=1}^{z} \delta_j \left(\sum_{k=1}^{t} \dot{\omega}_k \left(\left(s_i \bullet u_{\breve{B}_j}(u_k) \right)^2 + \left(s_i \bullet v_{\breve{B}_j}(u_k) \right)^2 + \left(s_i \bullet w_{\breve{B}_j}(u_k) \right)^2 \right) \right)} \end{array} \right)}$$

Definition 3.11

Let $\mathcal{F}_{\breve{A}}$ and $\mathcal{G}_{\breve{B}}$ are two GmPNSS, then the WCC between them can be defined as follows

$$\mathcal{R}^1_{GWmPNSS}(\mathcal{F}_{\breve{A}}, \mathcal{G}_{\breve{B}}) = \frac{\varsigma_{GmPNSS}(\mathcal{F}_{\breve{A}}, \mathcal{G}_{\breve{B}})}{max\{\varepsilon_{GmPNSS}(\mathcal{F}_{\breve{A}}, \mathcal{F}_{\breve{A}}), \varepsilon_{GmPNSS}(\mathcal{G}_{\breve{B}}, \mathcal{G}_{\breve{B}})\}} \tag{3.5}$$

$$\mathcal{R}^1_{GWmPNSS}(\mathcal{F}_{\breve{A}}, \mathcal{G}_{\breve{B}}) = \frac{\sum_{j=1}^{z} \delta_j \left(\sum_{k=1}^{t} \dot{\omega}_k \left(s_i \bullet u_{\breve{A}_j}(u_k) s_i \bullet u_{\breve{B}_j}(u_k) + s_i \bullet v_{\breve{A}_j}(u_k) s_i \bullet v_{\breve{B}_j}(u_k) + s_i \bullet w_{\breve{A}_j}(u_k)\, s_i \bullet w_{\breve{B}_j}(u_k) \right) \right)}{max \left\{ \begin{array}{c} \sum_{j=1}^{z} \delta_j \left(\sum_{k=1}^{t} \dot{\omega}_k \left(\left(s_i \bullet u_{\breve{A}_j}(u_k) \right)^2 + \left(s_i \bullet v_{\breve{A}_j}(u_k) \right)^2 + \left(s_i \bullet w_{\breve{A}_j}(u_k) \right)^2 \right) \right), \\ \sum_{j=1}^{z} \delta_j \left(\sum_{k=1}^{t} \dot{\omega}_k \left(\left(s_i \bullet u_{\breve{B}_j}(u_k) \right)^2 + \left(s_i \bullet v_{\breve{B}_j}(u_k) \right)^2 + \left(s_i \bullet w_{\breve{B}_j}(u_k) \right)^2 \right) \right) \end{array} \right\}}$$

If we consider $\acute{\omega} = \{\frac{1}{t}, \frac{1}{t}, \ldots, \frac{1}{t}\}$ and $\delta = \{\frac{1}{z}, \frac{1}{z}, \ldots, \frac{1}{z}\}$, then $\mathcal{R}_{GWmPNSS}(\mathcal{F}_{\breve{A}}, \mathcal{G}_{\breve{B}})$ and $\mathcal{R}^1_{GWmPNSS}(\mathcal{F}_{\breve{A}}, \mathcal{G}_{\breve{B}})$ are reduced to $\mathcal{R}_{GmPNSS}(\mathcal{F}_{\breve{A}}, \mathcal{G}_{\breve{B}})$ and $\mathcal{R}^1_{GmPNSS}(\mathcal{F}_{\breve{A}}, \mathcal{G}_{\breve{B}})$ respectively defined in 3.6 and 3.8.

Proposition 3.12

Let $\mathcal{F}_{\breve{A}}$ and $\mathcal{G}_{\breve{B}}$ are two GmPNSS, then the CC $\mathcal{R}_{GWmPNSS}(\mathcal{F}_{\breve{A}}, \mathcal{G}_{\breve{B}})$ between them satisfied the following properties

1. $0 \leq \mathcal{R}_{GWmPNSS}(\mathcal{F}_{\breve{A}}, \mathcal{G}_{\breve{B}}) \leq 1$
2. $\mathcal{R}_{GWmPNSS}(\mathcal{F}_{\breve{A}}, \mathcal{G}_{\breve{B}}) = \mathcal{R}_{GWmPNSS}(\mathcal{G}_{\breve{B}}, \mathcal{F}_{\breve{A}})$
3. If $\mathcal{F}_{\breve{A}} = \mathcal{G}_{\breve{B}}$ i. e; $s_i \bullet u_{\breve{A}_j}(u_k) = s_i \bullet u_{\breve{B}_j}(u_k)$, $s_i \bullet v_{\breve{A}_j}(u_k) = s_i \bullet v_{\breve{B}_j}(u_k)$, and $s_i \bullet w_{\breve{A}_j}(u_k) = s_i \bullet w_{\breve{B}_j}(u_k)$ for all j, k, where $i \in 1, 2, 3, \ldots, m$, then $\mathcal{R}_{GWmPNSS}(\mathcal{F}_{\breve{A}}, \mathcal{G}_{\breve{B}}) = 1$.

Proof

Similar to Proposition 3.7.

4. Application of Correlation Coefficient of GmPNSS for Decision Making

In this section, we proposed the algorithm for GmPNSS by using developed CC. We also used the proposed method for decision-making in real-life problems.

4.1. Algorithm for Correlation Coefficient of GmPNSS

Step 1. Pick out the set containing parameters.

Step 2. Construct the GmPNSS according to experts.

Step 3. Find the informational neutrosophic energies of any two GmPNSS.

Step 4. Calculate the correlation between two GmPNSS by using the following formula

$$\varsigma_{GmPNSS}(\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}}) = \sum_{j=1}^{z}\sum_{k=1}^{t}\begin{pmatrix} s_i \bullet u_{\breve{A}_j}(u_k) s_i \bullet u_{\breve{B}_j}(u_k) + s_i \bullet v_{\breve{A}_j}(u_k) s_i \bullet v_{\breve{B}_j}(u_k) + \\ s_i \bullet w_{\breve{A}_j}(u_k)\ s_i \bullet w_{\breve{B}_j}(u_k) : i \in 1, 2, 3, \ldots, m. \end{pmatrix}$$

Step 5. Calculate the CC between any two GmPNSS by using the following formula

$$\mathcal{R}_{GmPNSS}\ (\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}}) = \frac{\varsigma_{GmPNSS}(\mathcal{F}_{\breve{A}},\ \mathcal{G}_{\breve{B}})}{\sqrt{\varepsilon_{GmPNSS}(\mathcal{F}_{\breve{A}},\mathcal{F}_{\breve{A}}) \cdot \varepsilon_{GmPNSS}(\mathcal{G}_{\breve{B}},\mathcal{G}_{\breve{B}})}}$$

Step 6. Pick the most suitable alternate with a supreme correlation value

Step 7. Analyze the results.

The graphical representation of the proposed model is given in the following Figure 1.

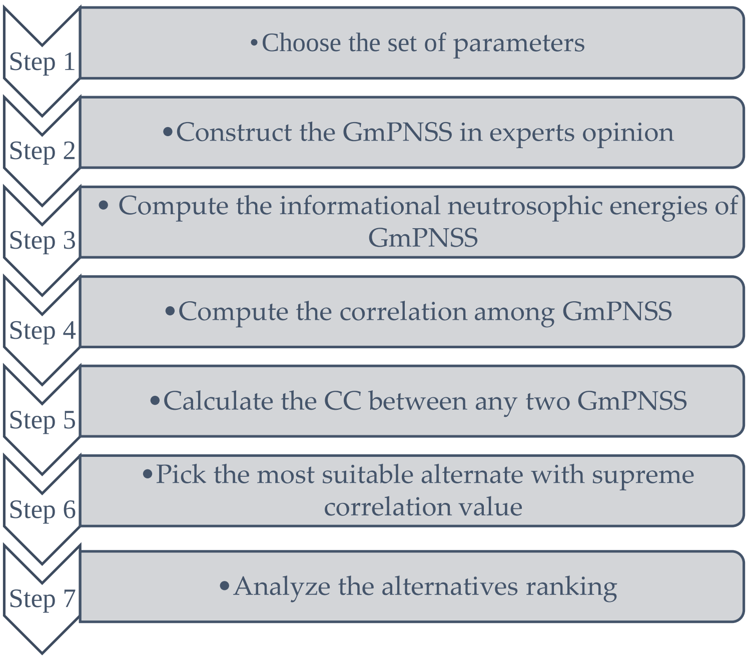

Figure 1: Flowchart of the proposed model

4.2. Problem Formulation and Application of CC for GmPNSS in Decision Making

Department of the scientific discipline of some universities U will have one scholarship for the post-doctorate position. Several scholars apply to get a scholarship but referable probabilistic along with CGPA (cumulative grade points average), simply four scholars call for enrolled for undervaluation such as $S = \{S_1,\ S_2,\ S_3,\ S_4\ \}$ be a set of selected scholars call for the interview. The president of the university hires a committee of four experts $X = \{X_1,\ X_2,\ X_3,\ X_4\}$ for the selection post-doctoral scholar. First of all, the committee decides the set of parameters such as $E = \{x_1,\ x_2,\ x_3\}$, where x_1, x_2, and x_3 represents the research papers, research quality, and communication skills for selecting post-doctoral scholars. The experts evaluate the scholars under defined parameters and forward the evaluation performa to the university's president. Finally, the university president scrutinizes the one best scholar based on the expert’s evaluation for the post-doctoral scholarship.

4.3. Application of GmPNSS For Decision Making

Assume $S = \{S_1, S_2, S_3, S_4\}$ be a set of scholars who are shortlisted for interview and $E = \{x_1 =$ research paper, x_2 = research quality, x_3 = interview$\}$ be a set of parameters for the selection of scholarship. Let $\mathcal{F}$ and $\mathcal{G} \subseteq E$. Then we construct the G3-PNSS $\Phi_{\mathcal{F}}(x)$ according to the requirement of the scientific discipline department.

Table 1. Construction of G3-PNSS of all Scholars According to Department Requirement

$\Phi_{\mathcal{F}}(x)$	x_1	x_2	x_3
$\mathbf{X_1}$	(.82,.55,.63),(.55,.46,.28),(.43,.38,.60)	(.43,.68,.86),(.47,.67,.56),(.42,.51,.33)	(.73,.48,.53),(.87,.43,.77),(.76,.53,.62)
$\mathbf{X_2}$	(.50,.62,.52),(.93,.57,.80),(.66,.48,.52)	(.77,.54,.81),(.75,.54,.72),(.53,.54,.69)	(.64,.48,.59),(.32,.58,.22),(.94,.64,.62)
$\mathbf{X_3}$	(.29,.25,.41),(.73,.34,.32),(.64,.44,.56)	(.36,.45,.27),(.47,.65,.21),(.61,.37,.39)	(.57,.25,.41),(.72,.55,.29),(.64,.31,.34)
$\mathbf{X_4}$	(.91,.50,.16),(.30,.24,.63),(.16,.55,.20)	(.69,.52,.61),(.37,.44,.23),(.46,.37,.29)	(.39,.35,.67),(.47,.24,.32),(.40,.71,.56)

Now we will construct the G3-PNSS $\varphi_{\mathcal{G}}^{t}$ according to four experts, where t = 1, 2, 3, 4.

Table 2. G3-PNSS Evaluation Report According to Experts of S_1

$\boldsymbol{\varphi_{\mathcal{G}}^{1}}$	x_1	x_2	x_3
$\mathbf{X_1}$	(.13,.15,.22),(.89,.78,.83),(.77,.82,.91)	(.91,.50,.16),(.30,.24,.63),(.16,.55,.20)	(.69,.52,.61),(.37,.44,.23),(.46,.37,.29)
$\mathbf{X_2}$	(.79,.84,.93),(.36,.18,.26),(.21,.24,.16)	(.39,.35,.67),(.47,.24,.32),(.40,.71,.56)	(.76,.62,.41),(.36,.49,.79),(.53,.59,.91)
$\mathbf{X_3}$	(.07,.23,.32),(.12,.18,.20),(.74,.79,.88)	(.70,.22,.11),(.67,.43,.53),(.41,.57,.49)	(.87,.58,.66),(.77,.22,.56),(.57,.33,.29)
$\mathbf{X_4}$	(.23,.12,.17),(.25,.16,.22),(.14,.16,.18)	(.74,.62,.66),(.67,.41,.93),(.85,.67,.99)	(.27,.29,.61),(.71,.43,.21),(.47,.70,.89)

Table 3. G3-PNSS Evaluation Report According to Experts of S_2

$\boldsymbol{\varphi_{\mathcal{G}}^{2}}$	x_1	x_2	x_3
$\mathbf{X_1}$	(.16,.20,.27),(.83,.87,.89),(70,.75,.86)	(.91,.50,.16),(.30,.24,.63),(.16,.55,.20)	(.69,.52,.61),(.37,.44,.23),(.46,.37,.29)
$\mathbf{X_2}$	(.13,.21,.24),(.18,.20,.20),(.70,.84,.90)	(.39,.35,.67),(.47,.24,.32),(.40,.71,.56)	(.76,.62,.41),(.36,.49,.79),(.53,.59,.91)
$\mathbf{X_3}$	(.20,.16,.27),(.29,.17,.26),(.14,.15,.12)	(.70,.22,.11),(.67,.43,.53),(.41,.57,.49)	(.87,.58,.66),(.77,.22,.56),(.57,.33,.29)
$\mathbf{X_4}$	(.88,.81,.90),(.40,.20,.26),(.22,.27,.17)	(.74,.62,.66),(.67,.41,.93),(.85,.67,.99)	(.27,.29,.61),(.71,.43,.21),(.47,.70,.89)

Table 4. G3-PNSS Evaluation Report According to Experts of S_3

$\boldsymbol{\varphi_{\mathcal{G}}^{3}}$	x_1	x_2	x_3
$\mathbf{X_1}$	(.77,.49,.61),(.71,.43,.21),(.47,.40,.69)	(.47,.59,.76),(.67,.62,.56),(.57,.43,.29)	(.70,.54,.61),(.79,.44,.63),(.61,.41,.51)
$\mathbf{X_2}$	(.60,.32,.32),(.77,.49,.83),(.76,.32,.59)	(.76,.62,.61),(.56,.49,.79),(.53,.59,.81)	(.69,.62,.67),(.57,.74,.43),(.86,.47,.79)
$\mathbf{X_3}$	(.60,.22,.21),(.67,.43,.53),(.49,.57,.49)	(.29,.72,.41),(.30,.66,.29),(.56,.32,.39)	(.74,.52,.66),(.67,.41,.93),(.85,.47,.59)
$\mathbf{X_4}$	(.74,.26,.37),(.49,.41,.63),(.44,.35,.32)	(.41,.66,.51),(.39,.27,.36),(.41,.51,.21)	(.60,.16,.47),(.31,.17,.24),(.54,.35,.24)

Table 5. G3-PNSS Evaluation Report According to Experts of S_4

$\boldsymbol{\varphi_{\mathcal{G}}^{4}}$	x_1	x_2	x_3

$\mathbf{X_1}$	(.23,.13,.22),(.31,.25,.43),(.19,.22,.27)	(.43,.68,.86),(.47,.67,.56),(.42,.51,.33)	(.82,.55,.63),(.55,.46,.28),(.43,.38,.60)
$\mathbf{X_2}$	(.10,.13,.11),(.91,.84,.69),(.31,.30,.28)	(.27,.29,.61),(.71,.43,.21),(.47,.70,.89)	(.50,.62,.52),(.93,.57,.80),(.66,.48,.52)
$\mathbf{X_3}$	(.70,.22,.11),(.67,.43,.53),(.41,.57,.49)	(.70,.22,.11),(.67,.43,.53),(.41,.57,.49)	(.36,.45,.27),(.47,.65,.21),(.61,.37,.39)
$\mathbf{X_4}$	(.45,.16,.27),(.91,.67,.23),(.64,.88,.10)	(.67,.81,.17),(.21,.54,.71),(.41,.54,.21)	(.20,.76,.47),(.39,.17,.46),(.41,.53,.22)

Solution by Using Developed Algorithm

Now, by using Tables 1, 2, 3, 4, and 5, we can find the correlation coefficient for each alternative by using equation 3.2 given as $\mathcal{R}_{GmPNSS}$ $(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}}^{1})$ = .8374, $\mathcal{R}_{GmPNSS}$ $(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}}^{2})$ = .7821, $\mathcal{R}_{GmPNSS}$ $(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}}^{3})$ = .9462, and $\mathcal{R}_{GmPNSS}$ $(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}}^{4})$ = .9422. This shows that $\mathcal{R}_{GmPNSS}$ $(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}}^{3}) >$ $\mathcal{R}_{GmPNSS}$ $(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}}^{4}) >$ $\mathcal{R}_{GmPNSS}$ $(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}}^{1}) >$ $\mathcal{R}_{GmPNSS}$ $(\Phi_{\mathcal{F}}, \varphi_{\mathcal{G}}^{2})$. Hence S_3 is the best scholar for a postdoctoral position.

5. Result Discussion and Comparative Analysis

In the subsequent section, we will debate the suggested method's effectiveness, simplicity, flexibility, and good position. A brief comparative analysis has been presented among our proposed and prevailing approaches.

5.1 Advantages and Superiority of Proposed Approach

This manuscript has developed a novel DM technique based on CC utilizing GmPNSS. The developed approach is more operative and delivers the appropriate results in MCDM problems comparative to existing techniques. Through this scientific research and comparison, we have realized that the suggested approach's outcomes are more general than conventional methods. However, compared to the current DM method, the DM process contains more information to deal with uncertain data. In addition to the fact that the hybrid structure of multiple FSs becomes a particular case of GmPNSS, add some appropriate conditions. Among them, the information associated with the object may be displayed precisely and analytically, so GmPNSS is an effective power tool to cope with imprecise and uncertain information in the DM process. Hence, our approach is more suitable, pliable, and better than FS's distinctive, accessible hybrid structures.

Table 6: Comparative analysis between some existing techniques and the proposed approach

	Set	Truthiness	Indeterminacy	Falsity	Multi-polarity	Loss of information
Chen et al. [28]	mPFS	✓	×	×	✓	×
Xu et al. [29]	IFS	✓	×	✓	×	×
Zhang et al. [30]	IFS	✓	×	✓	×	✓
Ali et al. [31]	BPNSS	✓	✓	✓	×	×
Proposed approach	GmPNSS	✓	✓	✓	✓	×

It turns out to be a contemporary problem. Why do we particularize novel algorithms according to the present novel structure? Several indications indicate that the recommended methodology can be exceptional compared to other existing methods. We remember that IFS, picture fuzzy set, FS, hesitant fuzzy set, NS, and different fuzzy sets have been restricted by the hybrid structure and cannot provide complete information regarding the situation. But our m-polarity model GmPNSS can be most suitable for MCDM because it can deal with truthiness, indeterminacy, and falsity. Due to the exaggerated multipolar neutrosophy, those three degrees have been independent of each other and furnish many information regarding alternative specifications. The similarity measures of other available hybrid structures are converted into a particular case of mPIVNSS. The comparative analysis with some prevailing techniques is listening above Table 6. Therefore, the model is more versatile and can quickly solve complications comparative to intuitionistic, neutrosophy, hesitation, image, and ambiguity substitution. The similarity measure established for GmPNSS becomes better than the existing similarity measure for MCDM.

5.2 Discussion

Chen et al. [28] multi-polar information of fuzzy sets deals with the membership value of the objects; the multi-polar fuzzy set cannot handle the circumstances when the objects have indeterminacy and falsity information. Xu et al. [29] and Zhang et al. [30] IFS only deal with the membership and non-membership values of the alternatives. These techniques cannot deal with the multi-polar information and indeterminacy of the alternative. Ali et al. [24] dealt with the truthiness, indeterminacy, and falsity grades for substitutes, but these techniques cannot manage multiple data. Instead, our established technique is an innovative method that can handle various information alternatives. But, on the other hand, the strategy we have progressed is about truth, indeterminacy, and the falsity of other options. So, the methodology we have offered is very efficient and will provide better outcomes for experts through additional information.

5.3 Comparative analysis

In this article, we propose a novel algorithm. First, an algorithm is proposed based on the correlation coefficient for GmPNSS. Next, utilize the developed algorithm to solve practical problems in real-life, that is, to select a postdoctoral position. The obtained results show that the proposed technique is effective and beneficial. Finally, the ranking of all alternatives using the existing methodologies gives the same final decision, that is, the position of "postdoctoral" is selected as S_3. All rankings are also calculated by applying existing methods with the same case study. The proposed method is also compared with other existing methods, Saeed et al. [26], Riaz et al. [16, 32], and Mohd Kamal et al. [33]. But these techniques cannot manage multiple data. Instead, our established technique is an innovative method that can handle various information alternatives. But, on the other hand, the strategy we have progressed is about truth, indeterminacy, and the falsity of alternatives. So, the methodology we have offered is very efficient and will provide better outcomes for experts through additional information.

Table 7. Comparison Between mPNSS and GmPNSS Techniques

Method	Alternative final Ranking	Optimal Choice
Riaz et al. [16]	$S_3 > S_2 > S_1 > S_4$	S_3
Saeed et al. [26]	$S_3 > S_4 > S_2 > S_1$	S_3
Riaz et al. [32]	$S_3 > S_2 > S_1 > S_4$	S_3
Mohd Kamal et al. [33]	$S_3 > S_4 > S_2 > S_1$	S_3
Proposed Approach	$S_3 > S_4 > S_1 > S_2$	S_3

6. Conclusion

In this manuscript, a novel hybrid structure has been established by GmPNSS by extending the mPNSS. We have developed the CC and WCC with their properties in the content of GmPNSS. A novel algorithm for GmPNSS utilizing our developed measure has been constructed to solve MCDM problems. A comparative analysis was also performed to demonstrate the proposed method.

Through comparative analysis, it is observed that the proposed technique exhibited higher steadiness and pragmatism for decision-makers in the DM process. Based on the results obtained, it is concluded that this method is most suitable for solving the MCDM problem in today's life. We will apply these techniques to other fields in future work, such as mathematical programming, cluster analysis, etc. This research article has pragmatic boundaries and can be immensely helpful in real-life dimensions: including the medical profession, pattern recognition, economics, etc.

References

1. L. A. Zadeh, Fuzzy Sets, Information and Control, 8(1965) 338–353.
2. I. B. Turksen, Interval Valued Fuzzy Sets Based on Normal Forms, Fuzzy Sets and Systems, 20(1986) 191–210.
3. K. Atanassov, Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems, 20(1986) 87–96.
4. F. Smarandache, Neutrosophic set – a generalization of intuitionistic fuzzy sets, International Journal of Pure and Applied Mathematics, 24(3)(2005) 287–297.
5. D. Molodtsov, Soft Set Theory First Results, Computers & Mathematics with Applications, 37(1999) 19–31.
6. P. K. Maji, R. Biswas, A. R. Roy, Soft set theory, Computers and Mathematics with Applications, 45(4–5)(2003) 555–562.
7. P. K. Maji, A. R. Roy, R. Biswas, An Application of Soft Sets in A Decision Making Problem, Computers and Mathematics with Applications, 44(2002) 1077–1083.
8. M. I. Ali, F. Feng, X. Liu, W. Keun, M. Shabir, On some new operations in soft set theory, Computers and Mathematics with Applications, 57(9)(2009) 1547–1553.
9. A. Sezgin, A. O. Atagun, On operations of soft sets, Computers and Mathematics with Applications, 61(5)(2011) 1457–1467.
10. P. K. Maji, *Neutrosophic soft set*, Annals of Fuzzy Mathematics and Informatics., **5(1)** (2013), 157–168.
11. F. Karaaslan, *Possibility neutrosophic soft sets and PNS-decision making method,* Applied Soft Computing Journal., **54** (2016), 403–414.
12. S. Broumi, *Generalized Neutrosophic Soft Set,* International Journal of Computer Science, Engineering and Information Technology., **3(2)** (2013), 17–30.
13. I. Deli, Y. Şubaş, Y. *A ranking method of single valued neutrosophic numbers and its applications to multi-attribute decision making problems*, Int. J. Mach. Learn. & Cyber., **8** (2017), 1309–1322.
14. H. Wang, F. Smarandache, Y. Zhang, *Single valued neutrosophic sets*, Int. J. Gen. Syst., **42** (2013), 386–394.
15. J. Ye, *A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets*, Journal of Intelligent and Fuzzy Systems., **26** (2014), 2459–2466.
16. M. R. Hashmi, M. Riaz, F. Smarandache, *m-Polar Neutrosophic Topology with Applications to Multi criteria Decision-Making in Medical Diagnosis and Clustering Analysis*, Int. J. Fuzzy Syst., **2019**, Dec (2019).
17. R. M. Zulqarnain, X. L. Xin, M. Saqlain, W. A. Khan, TOPSIS Method Based on the Correlation Coefficient of Interval-Valued Intuitionistic Fuzzy Soft Sets and Aggregation Operators with Their Application in Decision-Making, Journal of Mathematics, 2021, 1-16. https://doi.org/10.1155/2021/6656858.
18. R.M. Zulqarnain, X.L. Xin, H. Garg, W.A. Khan, Aggregation Operators of Pythagorean Fuzzy Soft Sets with Their Application for Green Supplier Chain Management, Journal of Intelligent & Fuzzy Systems (2021) 40 (3), 5545-5563. DOI: 10.3233/JIFS-202781.
19. R. M. Zulqarnain, X.L. Xin, I. Siddique, W. A. Khan, M. A. Yousif, TOPSIS Method Based on Correlation Coefficient under Pythagorean Fuzzy Soft Environment and Its Application towards Green Supply Chain Management. Sustainability 13 (2021), 1642. https://doi.org/10.3390/su13041642.

20. R. M. Zulqarnain, I. Siddique, Salwa A. El Morsy, Einstein Ordered Weighted Geometric Operator for Pythagorean Fuzzy Soft Set with its Application to Solve MAGDM Problem, Mathematical Problems in Engineering, 2021.
21. R. M. Zulqarnain, I. Siddique, A. Iampan, S. Ahmad, G. Jovanov, Đ. Vranješ, J. Vasiljević, Pythagorean Fuzzy Soft Einstein Ordered Weighted Average Operator in Sustainable Supplier Selection Problem, Mathematical Problems in Engineering, 2021.
22. I. Siddique, Rana Muhammad Zulqarnain, R. Ali, A. Alburaikan, A. Iampan, H. A. El-Wahed Khalifa, A Decision-Making Approach Based on Score Matrix for Pythagorean Fuzzy Soft Set, Computational Intelligence and Neuroscience, 2021.
23. X. Peng, H. Garg, *Multiparametric similarity measures on Pythagorean fuzzy sets with applications to pattern recognition,* Applied Intelligence., (2019), https://doi.org/10.1007/s10489-019-01445-0.
24. R. M. Zulqarnain, X. L. Xin, M. Saeed, F. Smarandache, N. Ahmad, Generalized Neutrosophic TOPSIS to Solve Multi-Criteria Decision-Making Problems, Neutrosophic Sets and Systems, 38, 276-292, (2020).
25. R. M. Zulqarnain, X. L. Xin, M. Saqlain, F. Smarandache, M. I. Ahamad, An integrated model of Neutrosophic TOPSIS with application in Multi-Criteria Decision-Making Problem. Neutrosophic Sets, and Systems, 40,118-133, (2021).
26. M. Saeed, M. Saqlain, A. Mehmood, K. Naseer, S. Yaqoob, *Multi-Polar Neutrosophic Soft Sets with Application in Medical Diagnosis and Decision-Making*, Neutrosophic Sets and Systems., **33** (2020) 183-207.
27. R. M. Zulqarnain, X. L. Xin, H.Garg, R. Ali, Interaction Aggregation Operators to Solve Multi Criteria Decision Making Problem Under Pythagorean Fuzzy Soft Environment, Journal of Intelligent and Fuzzy Systems, 41 (1), 1151-1171, 2021 ,DOI: 10.3233/JIFS-210098.
28. Chen, J., Li, S., Ma, S., Wang, X.: m-Polar fuzzy sets: an extension of bipolar fuzzy sets. Sci World J 2014, 416530 (2014).
29. Xu, Z.; Chen, J.; Wu, J. Clustering algorithm for intuitionistic fuzzy sets, Information Sciences. 2008, 178, 3775–3790.
30. Zhang, H. M.; Xu, Z. S.; Chen, Q. On clustering approach to intuitionstic fuzzy sets. Contr Decis 22, 882–888 (2007).
31. Mumtaz Ali, Le Hoang Son, Irfan Deli and Nguyen Dang Tien, Bipolar neutrosophic soft sets and applications in decision making, Journal of Intelligent & Fuzzy Systems 33 (2017) 4077–4087.
32. M. Riaz, N. Khalid, I. Zareef, D. Afzal, Neutrosophic N-Soft Sets with TOPSIS method for Multiple Attribute Decision Making, 32 (2020) 1-24. 10.5281/zenodo.3723131.
33. M. Kamal, N. Liyana, L. Abdullah, Multi-Valued Neutrosophic Soft Set, Malaysian Journal of Mathematical Sciences, 13 (2019) 153-168.
34. R. M. Zulqarnain, H. Garg, I. Siddique, A. Alsubie, N. Hamadneh, I. Khan, Algorithms for a generalized multi-polar neutrosophic soft set with information measures to solve medical diagnoses and decision-making problems, Journal of Mathematics, Volume 2021, Article ID 6654657, 1-30, https://doi.org/10.1155/2021/6654657.
35. R. M. Zulqarnain, I. Siddique, A. Iampan, E. Bonyah, Algorithms For Multi-Polar Interval-Valued Neutrosophic Soft Set With Information Measures to Solve Multi-Criteria Decision-Making Problem, Computational Intelligence and Neuroscience, 2021.

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