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Abstract: Multi-objective assignment problems (MOAPs) emerge in a wide range of real-world scenarios, from everyday activities to large-scale industrial operations. In this study, a MOAP with fuzzy parameters is investigated, and the fuzziness is represented by a Type-2 fuzzy logic system. Because the T2FLS is more efficient in dealing with the uncertainty of a decision-making process, the current problem's many parameters are represented by Type-2 trapezoidal fuzzy numbers (T2TpFNs). T2TpFNs are first reduced to Type-1 fuzzy numbers, then to crisp numbers. Finally, the neutrosophic compromise programming technique (NCPT) is applied to produce a problem compromise solution. A numerical problem is used to demonstrate the validity and applicability of the NCPT for the current MOAP. Furthermore, a comparison of NCPT to other techniques such as FPT and IFPT shows its superiority.

Keywords: Multi-objective Optimization; Assignment Problem; Type-2 Fuzzy Logic; Neutrosophic Programming; Fuzzy Goal Programming; Intuitionistic Fuzzy Programming.

1. Introduction

An (AP) is a combinatorial discrete optimization decision making problem arising in operations research and project management. It is an indispensable part of human resource project management, one of the main project management areas. It includes selection, development and management/control of the project team. In literature, the assignment problem has also been called the maximum weight matching problem. It has a wide range of applications in many real-life projects related to, for instance, education [16], production planning in telecommunication [67], rail transport [70] and medicine [74]. A classical assignment problem deals with allocating n tasks to n agents so that each agent is assigned to a single task and only one agent performs each task to optimize a pre-defined objective. This may involve maximizing efficiency or minimizing assignment cost or execution time of the tasks.

Generally, a cost-minimizing assignment problem (CMAP) aims to find an assignment schedule that minimizes the total assignment cost. A time minimizing assignment problem (TMAP), also known as a bottleneck assignment problem, focuses on minimizing the overall execution time of all the tasks. The first polynomial-time algorithm, viz., Hungarian algorithm for solving a CMAP, was proposed by Kuhn [33] in 1955. Later, Ravindran and Ramaswamy [60] used the Hungarian approach to solve a single objective bottleneck assignment problem. Various researchers have discussed a number of variants of the CMAP as well as the TMAP [11,47,53,66,69,76]. Bogomolnaia and Moulin [11] discussed a random assignment problem with a unique solution in which probabilistic serial assignment has been characterized by efficiency in an ordinal sense and envy-freeness. Maxon and Bhadury [47] discussed a multi-period assignment problem with repetitive tasks and tried to integrate a human aspect into their analysis. Nuass [53] suggested an optimizing and heuristics approach for solving generalized assignment problems. Sasaki [66] discussed axiomatic characterizations like consistency and monotonicity of the core of assignment problems in his research. Sourd [69] addressed a persistent assignment problem to solve scheduling problems with periodic cost functions. Vatow and Orden [76] discussed a personnel assignment problem. A number of books are also available in the literature that discuss assignment problems and their variants thoroughly [12,22,51,72].

While making strategic planning decisions in many real-life situations related to economics, science and engineering, often there is a suggested need to optimize more than one objective simultaneously. It gives rise to multiobjective optimization problems (MOOPs). In MOOPs, the multiple objectives are mostly conflicting in nature, and therefore, a single optimal solution may or may not exist. One has to search for trade-off/compromising solution(s) that involves a loss in one of the objective values in return for the gains in the others. It is easy to determine the superiority of a solution over the others in a single objective optimization problem, but in a MOOP, compromising solutions' consistency is determined by the concept of dominance. Therefore, these compromising solutions form the so-called Pareto frontier of the problem and are called Pareto optimal solutions that give rise to non-dominated points of the problem in its criteria space. Likewise, depending upon various market segments in this competitive world, a business industry might choose a strategy to assign various jobs to various agents in such a way that some objectives are optimized simultaneously. These objectives may either involve minimizing total assignment cost or that of the overall execution time or both at a time. For instance, many business firms either follow low-cost strategies or follow better responsiveness and customer service rules. Assignment problems in which both these factors are taken into account become time-cost trade-off problems as the solution providing the lowest cost may not provide the least time as well. Such problems fall in the category of bi-objective/multiobjective assignment problems. These problems have been investigated intensively in literature by many researchers [1,6,7,19,23,30,48,55,57,75,77]. Adiche et al. [1] proposed a hybrid algorithm for solving MOAPs. Bao et al. [6] studied the 0-1 programming method to transform and solve a MOAP by transforming it to a single objective assignment problem (SOAP). Geetha et al. [19] discussed the cost-time trade-off in a multicriteria assignment problem, whereas Hammadi [23] solved a MOAP using a tabu search algorithm. Yadaiah et al. [77] discussed an assignment problem with multiple objectives viz., time-cost-quality using the Hungarian algorithm. Furthermore, in several real-world optimization issues, the decision-makers are not always able to assign precise values to the problem's many parameters.

Only a vague information may be available based on abrupt changes in the environmental conditions, sudden breakdown of machinery, changes in government policies like complete or partial lockdown in the concerned region (specifically, in the epidemic/pandemic scenario like Covid-19) that may result in sudden shortage of products with high demand or an increase in demand of the newly launched products etc. This vagueness may also be based on past experiences and knowledge about the related situations. Thus, there is uncertainty in the values of parameters which may be very large as well. The theory behind fuzzy techniques is based on the notion of relative graded membership, inspired by human perception and cognition processes. It can deal with information arising from cognition and computational perception that is partially true, imprecise or without sharp boundaries. In 1965, Lotfi A. Zadeh[80] published his first famous research paper on fuzzy sets. Since then, various computational optimization techniques based on fuzzy logic have been developed for pattern recognition and identifying, optimizing, controlling, and developing intelligent decision-making systems. It can also provide an effective means for conflict resolution of multiple criteria and assess the available options in a better way. Later, Zadeh [81] also discussed the concept of a linguistic variable and its application to approximate reasoning.

Assignment problems performed in turbulent times (e.g., economic crisis, pandemic, risks etc.) may also have complex parameter estimation that leads to the discussion of these problems in a fuzzy environment. Researchers have thoroughly discussed various SOAPs/MOAPs and their variants under fuzziness [9,13,14,17,25,26,33,37,38,39,40,41,42,43,44,49,59,61,65,71]. Biswas and Pramanik [9] discussed a MOAP in the context of military affairs with fuzzy costs as trapezoidal fuzzy numbers. To transform their problem into a crisp single objective assignment problem, they applied Yager's ranking method. Chen [13] proposed a fuzzy assignment paradigm that treated all individuals as having the same abilities. De and Yadav [14] proposed an algorithm to solve a MOAP with exponential (nonlinear) membership using an interactive fuzzy goal programming approach whereas Feng and Yang [17] discussed a bi-objective assignment problem and constructed a chance-constrained goal programming model for the problem. Huang et al. [25] discussed a fuzzy multicriteria decision-making approach for solving a bi-objective personnel assignment problem whereas Huang and Zhang [26] developed a mathematical model for a fuzzy assignment problem (FAP) with a set of qualification constraints. Then, they designed a tabu search algorithm based on fuzzy simulation to solve the problem. Kagade and Bajaj [31] solved a MOAP with cost coefficients of the objective functions as interval values. Li et al. [40] discussed FAPs and presented a metric uncertainty model of concentrated quantification value. The convergence of the solution algorithm developed by combining genetic algorithm and assignment problems has been analyzed using Markov chain theory. Lin and Wen [41] also considered an FAP with assignment costs as fuzzy numbers and proposed a methodology that reduces the problem, either to a linear fractional programming problem or to a bottleneck assignment problem. They used a labelling algorithm to solve the related linear fractional programming problem. Lin et al. [42] studied an FAP and performed advanced sensitivity analysis viz., Type II and Type III sensitivity analysis. Type II sensitivity analysis determined the range of perturbation so that the optimal solution remains optimal whereas Type III sensitivity analysis determined the range for which the rate at which the optimal value function changes remains unchanged. Liu and Gao [43] designed a genetic algorithm to solve the fuzzy weighted balance equilibrium multi-job assignment problem whereas Liu and Li [44] presented a fuzzy quadratic assignment problem with three penalty costs and developed a hybrid genetic algorithm to solve the problem. Mukherjee and Basu [49] proposed a fuzzy ranking method for solving assignment problems with fuzzy costs. Pramanik and Biswas [59] studied a MOAP in which time, costs and inefficiency were represented by generalized trapezoidal fuzzy numbers and developed a priority-based fuzzy goal programming method. A traffic assignment based on fuzzy choices has been discussed by Ridwan [61]. Sakawa et al. [65] used interactive fuzzy programming for the linear and linear fractional programming workforce and production assignment problems. Tada and Ishii [71] also discussed a bi-objective FAP. For some other fuzzy models of the assignment problem and its variants, one may refer to the works of Gupta and Mehlavat [21], Jose and Kuriakose [28], Majumdar and Bhunia[46], Mukherjee and Basu [50], Nirmala and Anju [54], Pandian and Kavitha [56]and Thorani and Shankar [73], Yang and Liu [78], Ye and Xu [79].

Generally, in fuzzy optimization theory, Type-1 fuzzy set (T1FS) is employed that represents the uncertainty of the parameters by the membership functions which are crisp numbers lying in the interval [0, 1]. From the beginning, one of the major issues with the T1FS is that it cannot handle the uncertainty of the parameters efficiently, specifically, in situations where there is further uncertainty associated with the membership functions of the parameters. There is a need to depict such uncertainties by fuzzy sets that have blur boundaries. Then, a Type-2 fuzzy set (T2FS) came into existence. Membership functions of T2FS are three dimensional that allow some additional degrees of freedom to manage these uncertainties in a better way. In recent years, researchers have discussed various decision-making problems using T2FS [15,20,27,29,34,35,36,45,52]. The problem studied in this paper is a MOAP with fuzzy parameters, represented by T2TpFNs. Firstly, a two-stage defuzzification process is used to convert these T2TpFNs to equivalent crisp values and then, the neutrosophic logic is applied to solve the problem. The definition of neutrosophic logic and the related literature review is provided in the next subsection.

1.1 Literature Review on Neutrosophic Logic

As mentioned in the previous section, the theory behind fuzzy techniques is based on the notion of relative graded membership, i.e., the degree of belongingness of a parameter in an interval or a fuzzy set. Nevertheless, sometimes it is important to discuss the non-belongingness or non-membership of that parameter to cater a more realistic scenario. Atanassov [5] proposed a generalization of fuzzy sets viz, intuitionistic fuzzy logic that incorporates both the aforementioned factors. In this approach, two different real numbers representing the degree of truth and degree of falsehood are associated with each parameter. However, a half-true expression in this logic is not always half false; there may be some hesitation degree as well. Many researchers have developed a number of intuitionistic fuzzy programming approaches which gained significant popularity among the existing multiobjective optimization techniques. Angelov [3,4] first discussed optimization in an intuitionistic fuzzy environment. Later on, various researchers discussed this technique to study assignment problems as well. Jose and Kuriakose [28] presented an algorithm for solving an assignment model in an intuitionistic fuzzy context. Mukherjee and Basu [50] solved an intuitionistic fuzzy assignment problem using similarity measures and score functions. Roy et al. [63] presented a new approach for solving intuitionistic fuzzy multiobjective transportation problems in which supply, demand and transportation costs are considered as intuitionistic fuzzy numbers. But certain real-world situations involve another factor called indeterminacy. In such problems, the indeterministic feature of ambiguous data plays an essential role in making a rational decision outside the reach of intuitionistic fuzzy set theory. Each membership function of the neutrosophic set is precisely quantified and independent. One obtains better and more refined results whenever the optimization is carried out in a neutrosophic or generalized neutrosophic setting. Many researchers have applied neutrosophic logic to solve various multiobjective optimization problems [2,18,32,58,62,64,82]. Aggarwal et al. [2] thoroughly discussed neutrosophic modelling and control. Freen et al. [18] discussed multiobjective nonlinear four-valued refined neutrosophic optimization. Kamal et al. [32] considered a multiobjective nonlinear selective maintenance allocation of system reliability and used a neutrosophic fuzzy goal programming approach to get the optimal solution.

Pintu and Tapan [58] presented a multiobjective nonlinear programming problem based on the neutrosophic optimization technique and discussed its application in the Riser Design problem. Rizk-Allah [62] also discussed a multiobjective transportation model under a neutrosophic environment. Şahin and Muhammed [64] studied a multicriteria neutrosophic group decision-making method based on TOPSIS for supplier selection. Zhang et al. [82] discussed neutrosophic interval sets and their applications in multicriteria decision-making problems. Next subsection discusses the motivation behind the present study.

1.2 Study Motivation

This paper aims to present an efficient algorithmic solution procedure based on neutrosophic logic for a MOAP with conflicting objectives viz., assignment cost and execution time in which T2TpFNs are used to represent these parameters. Using the output processor of T2FS these T2TpFNs are initially reduced to Type-1 fuzzy numbers and then to crisp numbers. The proposed solution procedure is named as Neutrosophic compromise programming technique (NCPT). The selection of T2FS for the present study is due to the fact that its membership functions allow some additional degrees of freedom to manage the uncertainties/vagueness in the parameters (here, time and cost) in a better way. However, the advantage of neutrosophic logic, as mentioned in the previous subsection, is that it offers a neutral perspective to decide the best possible compromise solution(s) of a MOOP. It is shown that NCPT is the best solution technique for dealing for dealing with inaccurate, missing, and inconsistent information of the present MOAP when compared to the available solution techniques viz., fuzzy and Intuitionistic fuzzy programming techniques. This comparison has been done with the help of a numerical problem. LINGO software, created by LINDO Systems Inc., is used for all calculation-based frameworks.

The rest of the paper is structured as follows: In Section 2, mathematical statement of the present MOAP is given. It explains the basic as well as the fuzzy model of the problem viz., “Model 1” and “Model 2”, respectively. Section 3 discusses some basic mathematical preliminaries related to fuzzy, intuitionistic fuzzy and neutrosophic sets. Section 4 discusses the defuzzification process of T2TpFNs. In Section 5, three different solution techniques that are applied to the present MOAP have been discussed in detail. In Section 6, some real-world applications of the present MOAP are given. The efficacy of the proposed NCPT solution technique for a MOAP instance is addressed in Section 7. Section 8 discusses the performance and outcome of the proposed solution technique. It also provides its comparative study with the other two solution techniques. Advantage of using the NCPT solution technique instead of other commonly used techniques has been addressed in Section 9. Section 10 provides conclusion and the future aspects of the present study.

2. Mathematical Statement of MOAP

Nomenclature

Indices:

***i* -** Index for n workers, ($i=1, 2,\dots, n$)

***j* -** Index for n tasks, ($j=1,2,\dots, n$)

Decision Variable:

x_{ij} -Binary variable that takes the values 1 and 0 if j^{th} taskis assigned and not assigned to i^{th} worker, respectively. Equivalently,

$$x_{ij} = \begin{cases} 1 \text{ , } if \quad j^{th} \ task \ is assigned \ to \ i^{th} \ worker \\ 0 \text{ , } \quad otherwise \end{cases}$$

Parameters:

c_{ij} - Assignment cost of j^{th} task to the i^{th} worker

t_{ij} -execution time when i^{th} worker performs j^{th} task

Model 1:

The mathematical formulation of a MOAP with the above-mentioned parameters is as follows:

$$\left.\begin{aligned} & Min \ Z_1 = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij} \\ & Min \ Z_2 = \sum_{i=1}^{n}\sum_{j=1}^{n} t_{ij} x_{ij} \\ & \text{Subject to} \\ & \qquad \sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,2,\dots,n \\ & \qquad \sum_{i=1}^{m} x_{ij} = 1, \quad j = 1,2,\dots,n \\ & \qquad x_{ij} = 0 \text{ or } 1, \qquad i = 1,2,\dots,n, \ j = 1,2,\dots,n. \end{aligned}\right\}$$

In Model 1, time ($\tilde{\tilde{t}}_{ij}$) and cost ($\tilde{\tilde{c}}_{ij}$) parameters are assumed to be T2TpFNs.

Model 2:

$$\left.\begin{array}{l} Min\ Z_1=\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{\tilde{c}}_{ij}x_{ij} \\ Min\ Z_2=\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{\tilde{t}}_{ij}x_{ij} \\ \text{Subject to} \\ \text{Model}(1) \end{array}\right\}$$

3. Mathematical Preliminaries

Some basic definitions of fuzzy, intuitionistic fuzzy and neutrosophic set are discussed.

Definition 3.1 Fuzzy Set or Type I Fuzzy Set (T1FS) [10]

A fuzzy set $\tilde{C}$ is defined on the set Y of real numbers. Its membership function $\mu_{\tilde{C}}(y)$ can be characterized as:

$$\mu_{\tilde{C}}: Y \in [0,1];\ 0 \le \mu_{\tilde{C}}(y) \le 1,$$

Thus, a T1FS can be defined as: $\tilde{C}=\{(y,\mu_{\tilde{C}}(y): y \in Y\}$.

Definition 3.2 Defuzzification of T1FS [10]

Defuzzification is a process of transforming a fuzzy inference into a crisp output. For a Type-1 fuzzy number (T1FN) also, there exists an associated crisp quantity which is called defuzzified form of that T1FN. Let $\tilde{C}=(c_1,c_2,c_3,c_4)$ be a Type-1 Trapezoidal Fuzzy Number (T1TpFN). Using probability density function, defuzzified value of $\tilde{C}$ can be computed as:

$$V(\tilde{C})=\frac{1}{3}\left(c_1+c_2+c_3+c_4+\frac{(c_1c_2-c_3c_4)}{c_3+c_4-c_1-c_2}\right)$$

Definition 3.3 Type-2 Fuzzy Set (T2FS) [10]

Generalization of interval-valued fuzzy sets is known as T2FS, if the intervals are fuzzy. A T2FScan be expressed in four TIFS. . That means four membership functions of a T2FS are T1FSs, which depict the uncertainty of T2FS in a justified manner. Therefore, a membership function of T2FS is of the form $\mu_{\tilde{C}}: Y \to \chi([0,1])$ where $\chi([0,1])$ denotes the set of all T1FSs defined on the interval [0, 1].

Definition 3.4 Type-1 Trapezoidal Fuzzy Number (T1TpFN) [10]

A T1TpFN $\tilde{C}=(c_1,c_2,c_3,c_4)$ on Y with the membership function can be defined as:

$$\mu_{\tilde{C}}(y)=\begin{cases} \dfrac{y-c_1}{c_2-c_1} & \text{if } c_1 \le y \le c_2 \\ 1 & \text{if } c_2 \le y \le c_3 \\ \dfrac{c_4-y}{c_4-c_3} & \text{if } c_3 \le y \le c_4 \\ 0 & \text{if } y<c_1 \ or\ y>c_4 \end{cases}$$

Definition 3.5 Type-2 Trapezoidal Fuzzy Number (T2TpFN) [10]

A T2TpFN $\tilde{\tilde{C}}$ can be expressed in four T1TpFNs: $\tilde{\tilde{C}} = (\tilde{C}_1, \tilde{C}_2, \tilde{C}_3, \tilde{C}_4)$

Where, $\tilde{C}_1 = (c_1^L, c_0^L, c_0^R, c_1^R)$, $\tilde{C}_2 = (c_2^L, c_0^L, c_0^R, c_2^R)$, $\tilde{C}_3 = (c_3^L, c_0^L, c_0^R, c_3^R)$ and $\tilde{C}_4 = (c_4^L, c_0^L, c_0^R, c_4^R)$ represent T1TpFNs.

Now, primary membership functions $\mu_{\tilde{\tilde{C}}}(y)$ of $\tilde{\tilde{C}}$ can be defined as:

$$\mu_{\tilde{\tilde{\lambda}}}(y) = \begin{cases} (\ \mu_{1j}^L(y),\ \ \mu_{2j}^L(y),\ \ \mu_{3j}^L(y),\ \ \mu_{4j}^L(y)\ ) & \text{if } c_{j+1}^L \le y \le c_j^L \\ (\ \hat{\mu}_{1j}^R(y),\ \ \hat{\mu}_{2j}^R(y),\ \ \hat{\mu}_{3j}^R(y),\ \ \hat{\mu}_{4j}^R(y)\ ) & \text{if } c_j^R \le y \le c_{j+1}^R \\ 0 & \text{otherwise} \end{cases}$$

where

$$\hat{\mu}_{ij}^L(y) = \begin{cases} \dfrac{y - c_i^L}{c_0^L - c_i^L} & i > j \\ 0 & i \le j \end{cases} \quad ;\ (j = 0, 1, 2, 3;\ i = 1, 2, 3, 4)$$

$$\hat{\mu}_{ij}^R(y) = \begin{cases} \dfrac{c_i^R - y}{c_i^R - c_0^R} & i > j \\ 0 & i \le j \end{cases} \quad ;\ (j = 0, 1, 2, 3;\ i = 1, 2, 3, 4)$$

Secondary membership function $\mu_{\tilde{\tilde{C}}}(y)$ of $\tilde{\tilde{C}}$ can be defined as:

$$\begin{cases} \dfrac{\mu - \mu_{1j}^L(y)}{\mu_{2j}^L(y) - \mu_{1j}^L(y)} & \text{if } \mu_{1j}^L(y) \le \mu \le \mu_{2j}^L(y) \\ 1 & \text{if } \mu_{2j}^L(y) \le \mu \le \mu_{3j}^L(y) \\ \dfrac{\mu_{4j}^L(y) - \mu}{\mu_{4j}^L(y) - \mu_{3j}^L(y)} & \text{if } \mu_{3j}^L(y) \le \mu \le \mu_{4j}^L(y) \\ 0 & \text{otherwise} \end{cases} \quad ; \quad c_4^L \le y \le c_0^L$$

and

$$\begin{cases} \dfrac{\mu - \hat{\mu}_{1j}^R(y)}{\hat{\mu}_{2j}^R(y) - \hat{\mu}_{1j}^R(y)} & \text{if } \hat{\mu}_{1j}^R(y) \le \mu \le \hat{\mu}_{2j}^R(y) \\ 1 & \text{if } \hat{\mu}_{2j}^R(y) \le \mu \le \hat{\mu}_{3j}^R(y) \\ \dfrac{\hat{\mu}_{4j}^R(y) - \mu}{\hat{\mu}_{4j}^R(y) - \hat{\mu}_{3j}^R(y)} & \text{if } \hat{\mu}_{3j}^R(y) \le \mu \le \hat{\mu}_{4j}^R(y) \\ 0 & \text{otherwise} \end{cases} \quad ; \quad y_0^R \le y \le y_4^R$$

4. Defuzzification Technique of a T2TpFN

Since $\tilde{\tilde{c}}_{ij}$ and $\tilde{\tilde{t}}_{ij}$ in model 1 are assumed T2TpFN, therefore under this section, the defuzzification process of T2TpFNs is discussed. From definition 3.5, T2TpFN can be defined by four T1TpFNs and for each point of the universe of discourse of the T2TpFN, a T1TpFN corresponds as a secondary membership function. Therefore, a technique that defuzzifies a T1TpFN would be sufficient to provide a defuzzified value of the T2TpFN. The present defuzzification technique is divided into two stages. Stage-1 reduces T2TpFN into its equivalent T1TpFNs; however, Stage-2 defuzzifies these T1TpFNs to get the crisp values of the associated T2TpFN.

Stage 1.

Let $\tilde{\tilde{C}} = (\tilde{C}_1, \tilde{C}_2, \tilde{C}_3, \tilde{C}_4)$ be a T2TpFN where $\tilde{C}_i = (c_i^L, c_0^L, c_0^R, c_i^R);\ i = 1, 2, 3, 4$ denotes a T1TpFN. The membership function of a T1TpFN $\tilde{C}_i$ can be defined as:

$$\mu_{\tilde{C}_i}(y) = \begin{cases} \dfrac{y - c_i^L}{c_0^L - c_i^L} & \text{if } c_i^L \le y \le c_0^L \\ 1 & \text{if } c_0^L \le y \le c_0^R \\ \dfrac{c_i^R - y}{c_i^R - c_0^R} & \text{if } c_0^R \le y \le c_i^R \\ 0 & \text{otherwise} \end{cases}$$

Then, the probability density function $f_{\tilde{C}_i}(y)$ corresponding to the T1TpFN $\tilde{C}_i$ can be stated as:

$$f_{\tilde{C}_i}(y) = \begin{cases} \dfrac{2(y - c_i^L)}{(c_0^L - c_i^L)(c_0^R + c_i^R - c_0^L - c_i^L)} & \text{if } c_i^L \le y \le c_0^L \\ \dfrac{2}{(c_0^R + c_i^R - c_0^L - c_i^L)} & \text{if } c_0^L \le y \le c_0^R \\ \dfrac{2(c_i^R - y)}{(c_i^R - c_0^R)(c_0^R + c_i^R - c_0^L - c_i^L)} & \text{if } c_0^R \le y \le c_i^R \\ 0 & \text{otherwise} \end{cases} \qquad (i = 1, 2, 3, 4.)$$

Now, calculate the expected value $E(Y_{\tilde{C}_i})$ of $Y_{\tilde{C}_i}$ as $E(Y_{\tilde{C}_i}) = \int_{-\infty}^{\infty} y\, f_{\tilde{C}_i}(y)\,\mathrm{d}y$. This value is noted as the defuzzified value $V(\tilde{C}_i)$ of the T1TpFN $\tilde{C}_i$ for all $i \in \{1, 2, 3, 4\}$ i.e.,

$$\begin{aligned} V(\tilde{C}_i) = E(Y_{\tilde{C}_i}) &= \left[\int_{c_i^L}^{c_0^L} \frac{2(y - c_i^L)y}{(c_0^L - c_i^L)(c_0^R + c_i^R - c_0^L - c_i^L)}\mathrm{d}y + \int_{c_0^L}^{c_0^R} \frac{2}{(c_0^R + c_i^R - c_0^L - c_i^L)} y\mathrm{d}y + \int_{c_0^R}^{c_i^R} \frac{2(c_i^R - y)y}{(c_i^R - c_0^R)(c_0^R + c_i^R - c_0^L - c_i^L)}\mathrm{d}y \right] \\ &= \frac{1}{3(c_0^R + c_i^R - c_0^L - c_i^L)} \left[(c_0^R)^2 + (c_i^R)^2 - (c_0^L)^2 - (c_i^L)^2 + c_0^R c_i^R - c_0^L c_i^L \right] \\ &= \frac{1}{3}\left(c_0^L + c_i^L + c_0^R + c_i^R + \frac{c_0^L c_i^L - c_0^R c_i^R}{c_0^R + c_i^R - c_i^L - c_0^L}\right) \quad \text{for all } i = 1, 2, 3, 4. \end{aligned}$$

Thus, the T2TpFN based on the defuzzified values $V(\tilde{C}_i)$ of the T1TpFNs $\tilde{C}_i$ for all i's, can be defined as: $\tilde{\tilde{C}} = (V(\tilde{C}_1), V(\tilde{C}_2), V(\tilde{C}_3), V(\tilde{C}_4))$

$$=\begin{pmatrix}\frac{1}{3}\left(c_1^L+c_0^L+c_0^R+c_1^R+\frac{c_1^Lc_0^L-c_0^Rc_1^R}{c_0^R+c_1^R-c_1^L-c_0^L}\right),\frac{1}{3}\left(c_2^L+c_0^L+c_0^R+c_2^R+\frac{c_2^Lc_0^L-c_0^Rc_2^R}{c_0^R+c_2^R-c_2^L-c_0^L}\right),\\ \frac{1}{3}\left(c_3^L+c_0^L+c_0^R+c_3^R+\frac{c_3^Lc_0^L-c_0^Rc_3^R}{c_0^R+c_3^R-c_3^L-c_0^L}\right),\ \frac{1}{3}\left(c_4^L+c_0^L+c_0^R+c_4^R+\frac{c_4^Lc_0^L-c_0^Rc_4^R}{c_0^R+c_4^R-c_4^L-c_0^L}\right)\end{pmatrix}$$

Stage 2.

T1TpFNs are further defuzzified at this stage to generate the final defuzzified version of the T2TpFN as follows: $DV\left(\tilde{\tilde{C}}\right)=\frac{1}{3}\left(V(\tilde{C}_1)+V(\tilde{C}_2)+V(\tilde{C}_3)+V(\tilde{C}_4)+\frac{(V(\tilde{C}_1)V(\tilde{C}_2)-V(\tilde{C}_3)V(\tilde{C}_4))}{(V(\tilde{C}_3)+V(\tilde{C}_4)-V(\tilde{C}_1)\text{-}V(\tilde{C}_2))}\right)$

$$=\frac{1}{9}\left(\sum_{i=1}^{4}(c_i^L+c_i^R+c_0^L+c_0^R+\frac{c_i^Lc_0^L-c_0^Rc_i^R}{c_0^R+c_i^R-c_i^L-c_0^L})\right.$$

$$\left.+\frac{\prod_{i=1}^{2}\left(c_i^L+c_0^L+c_0^R+c_i^R+\frac{c_i^Lc_0^L-c_0^Rc_i^R}{c_0^R+c_i^R-c_i^L-c_0^L}\right)-\prod_{j=3}^{4}\left(c_j^L+c_0^L+c_0^R+c_j^R+\frac{c_j^Lc_0^L-c_0^Rc_j^R}{c_0^R+c_j^R-c_j^L-c_0^L}\right)}{\sum_{j=3}^{4}\left(c_j^L+\ c_j^R+\frac{c_j^Lc_0^L-c_0^Rc_j^R}{c_0^R+c_j^R-c_j^L-c_0^L}\right)-\sum_{i=1}^{2}\left(c_i^L+c_i^R+\frac{c_i^Lc_0^L-c_0^Rc_i^R}{c_0^R+c_i^R-c_i^L-c_0^L}\right)}\right)$$

The same procedure can be followed for T2TpFNs $\tilde{\tilde{t}}_{ij}$ to obtain their crisp values.

After the above defuzzification procedure, the resultant MOAP model finally takes the form

Model 3:

$$\left.\begin{array}{l}Min\ Z_1=\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{\tilde{c}}_{ij}x_{ij}\ =\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{3}\begin{pmatrix}V(\tilde{c}_1)+V(\tilde{c}_2)+\ V(\tilde{c}_3)+V(\tilde{c}_4)\\ +\frac{(V(\tilde{c}_1)V(\tilde{c}_2)-V(\tilde{c}_3)V(\tilde{c}_4))}{(V(\tilde{c}_3)+V(\tilde{c}_4)-V(\tilde{c}_1)-V(\tilde{c}_2))}\end{pmatrix}x_{ij}\\ Min\ Z_2=\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{\tilde{t}}_{ij}x_{ij}\ =\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{3}\begin{pmatrix}V(\tilde{t}_1)+V(\tilde{t}_2)+\ V(\tilde{t}_3)+V(\tilde{t}_4)\\ +\frac{(V(\tilde{t}_1)V(\tilde{t}_2)-V(\tilde{t}_3)V(\tilde{t}_4))}{(V(\tilde{t}_3)+V(\tilde{t}_3)-V(\tilde{t}_1)-V(\tilde{t}_2))}\end{pmatrix}x_{ij}\\ \qquad\text{Subject to;}\\ \qquad\qquad\sum_{j=1}^{n}x_{ij}=1,\quad i=1,2,...,n\\ \qquad\qquad\sum_{i=1}^{m}x_{ij}=1,\quad j=1,2,...,n\\ \qquad\qquad x_{ij}=0\text{ or}1,\quad i=1,2,...,n,\ j=1,2,...,n.\end{array}\right\}$$

5. Methodology

In this section, we discuss three different solution techniques viz.,

(i) Neutrosophic compromise programming technique
(ii) Fuzzy programming technique
(iii) Intuitionistic fuzzy programming.

The method of transforming a multiobjective optimization problem into a related single-objective optimization problem is also discussed for all the suggested approaches.

5.1 Neutrosophic Compromise Programming Technique (NCPT)

The extended version of the fuzzy and intuitionistic fuzzy sets has been classified as a neutrosophic set (NS) (defined below) with an additional membership function called indeterminacy. In some specific real-life decision-making problems, there are many cases in which decision-makers have indeterminacy or unbiased reasoning in decision-making. The principles of indeterminacy often lie between those of Truth and Lies. Literally, neutrosophic means neutral thought or awareness of indeterminacy, therefore, a NS has three distinct membership features viz., truth, indeterminacy and falsehood. On the other hand, in a fuzzy set, we maximize the degree of membership function which indicates that the element belongs to that set. In contrast, in an intuitionistic fuzzy set, two types of membership functions viz., the degree of membership (also known as the degree of truth) and the degree of non-membership (also known as the degree of falsehood) of an element, are considered. To be more specific, an NS maximizes the degree of truth and indeterminacy while decreasing the degree of falsehood. A NS represents a major touchstone in a decision-making process where the decision-maker can be entirely satisfied (with truth), partly satisfied (with indeterminacy) and dissatisfied (with falsehood). In any decision-making problem, these factors increase the strength of making the right decision or achieving an optimal solution. Since for MOOPs with conflicting objectives, the challenge of finding the best solution using classical approaches is a significantly complicated issue, the NCPT would be a useful technique for achieving the best compromise solution due to its aforementioned features.

Definition 5.1 Neutrosophic Set [63]

Let Y be the universe of discourse and $y \in Y$. A neutrosophic set (NS) P over Y is the set of triplets consisting of a truth membership function $T_P(y)$, indeterminacy membership function $I_P(y)$ and a false membership function $F_P(y)$, for $y \in Y$. Mathematically;

$$P = \{< y, T_P(y), I_P(y), F_P(y) > | \, y \in Y\}$$

Here, $T_P(y)$, $I_P(y)$ and $F_P(y)$ are real non-standard or standard functions with range $]0^-, 1^+[$, i.e., $T_p(y): Y \to]0^-, 1^+[$, $I_p(y): Y \to]0^-, 1^+[$ and $F_p(y): Y \to]0^-, 1^+[$. Assume that

$$0^- \le \sup T_p(y) + \sup I_p(y) + \sup F_p(y) \le 3^+$$

Now, the general formulation of a MOOP can be defined as:

$$\text{Minimize } \{Z_1(x), Z_2(x), ..., Z_L(x)\}$$
$$\text{Subject to}$$
$$g_m(x) \le b_m(x), \quad m = 1,2,..., M$$
$$x \ge 0, \; l = 1,2,..., L$$

where, $Z_l(x); l = 1,2,3,..., L$ denotes the *lth* objective function, $g_m(x); m = 1,2,3,..., M$ denotes the constraints and x denotes the decision variables. In 1970, Bellman and Zadeh [8] introduced the definitions of fuzzy decision (D), fuzzy goal (G) and fuzzy constraint (C) that are useful for solving any real-life optimization problems under uncertainty. Consequently, a fuzzy decision set is described as: $D = G \cap C$

On the same lines, a neutrosophic decision set D_N, with neutrosophic goal set G_L and neutrosophic constraints C_m can be defined as follows:

$$D_N = \{(\cap_{l=1}^{L} G_L) \cap (\cap_{m=1}^{M} C_m) = (x, T_D(x), I_D(x), F_D(x))\}$$

where

$$\begin{cases} T_D(x) = \min\begin{bmatrix} T_{G_1}(x), T_{G_2}(x), \ldots, T_{G_L}(x) \\ T_{C_1}(x), T_{C_2}(x), \ldots, T_{C_M}(x) \end{bmatrix} & \forall x \in X, \\ I_D(x) = \max\begin{bmatrix} I_{G_1}(x), I_{G_2}(x), \ldots, I_{G_L}(x) \\ I_{C_1}(x), I_{C_2}(x), \ldots, I_{C_M}(x) \end{bmatrix} & \forall x \in X, \\ F_D(x) = \max\begin{bmatrix} F_{G_1}(x), F_{G_2}(x), \ldots, F_{G_L}(x) \\ F_{C_1}(x), F_{C_2}(x), \ldots, F_{C_M}(x) \end{bmatrix} & \forall x \in X, \end{cases}$$

Here $T_D(x)$, $I_D(x)$ and $F_D(x)$ are the truth, indeterminacy and false membership functions, respectively, defined under the neutrosophic decision D_N.

To find the compromise solutions for a multiobjective decision making optimization issue, membership functions are created for each objective function and the lower and upper bounds are calculated as L_i and U_i respectively, by solving them individually under the stated constraints:

$$U_l = \max_l\{Z_l(X)\} \text{ and } L_l = \min_l\{Z_l(X)\} \text{ for all } l = 1,2,\ldots,L \tag{1}$$

Further, upper and lower bounds for l^{th} objectives under the NS can be determined as follows:

$$U_l^T = U_l, \qquad L_l^T = L_l \text{ for truth membership function (2)}$$

$$U_l^I = L_l^T + a_l, \qquad L_l^I = L_l^T \text{ for indeterminacy membership function} \tag{3}$$

$$U_l^F = U_l^T, \qquad L_l^F = L_l^T + b_l \text{ for false membership function} \tag{4}$$

where a_l and b_l are predetermined real values assigned by the decision-makers that lie in the interval (0, 1). Further, the linear membership function $T_l(Z_l(x))$ of truth, $I_l(Z_l(x))$ of indeterminacy and $F_l(Z_l(x))$ of falsity under the neutrosophic environment can be constructed as follows:

$$T_l(Z_l(x)) = \begin{cases} 1 & if\ Z_l(x) \le L_l^T \\ \dfrac{U_l^T - Z_l(x)}{U_j^T - L_j^T} & if\ L_l^T \le Z_l(x) \le U_l^T \\ 0 & if\ Z_l(x) \ge U_l^T \end{cases} \tag{5}$$

$$I_l((Z_l(x)) = \begin{cases} 1 & if\ Z_l(x) \le L_l^I \\ \dfrac{U_l^I - Z_l(x)}{U_j^I - L_j^I} & if\ L_l^I \le Z_l(x) \le U_l^I \\ 0 & if\ Z_l(x) \ge U_l^I \end{cases} \tag{6}$$

$$F_l((Z_l(x)) = \begin{cases} 0 & if\ Z_l(x) \le L_l^F \\ \dfrac{Z_l(x) - L_l^F}{U_j^F - L_j^F} & if\ L_l^F \le Z_l(x) \le U_l^F \\ 1 & if\ Z_l(x) \ge U_l^F \end{cases} \tag{7}$$

It should be noted here that $U_l^{(.)} \ne L_l^{(.)}$, $\forall\ l = 1,2,\ldots,L$.

If $U_l^{(.)} = L_l^{(.)}, \forall\ l = 1,2,..., L$, the membership value will be assumed to be 1.

Since the development of achievement functions helps to achieve the highest level or degree of satisfaction based on the priorities of the decision-makers, we also define a specific achievement variable for each membership function. The decision-maker may establish a target in a decision-making process to attain the maximum possible degree of satisfaction for the truth and indeterminacy membership functions while minimizing the degree of untruth as much as possible. After considering the linear membership of truth, indeterminacy and falsehood under neutrosophic nature, the mathematical expression of the neutrosophic compromise programming problem is given as

P1:

$$\left.\begin{array}{l} \text{Max } \min_{l=1,2,...,L}\ T_l(Z_l(x)) \\ \text{Max } \min_{l=1,2,...,L}\ I_l(Z_l(x)) \\ \text{Min } \max_{l=1,2,...,L}\ F_l(Z_l(x)) \\ \text{Subject to} \\ T_l(Z_l(x)) \ge I_l(Z_l(x)) \\ T_l(Z_l(x)) \ge F_l(Z_l(x)) \\ 0 \le T_l(Z_l(x)) + I_l(Z_l(x)) + F_l(Z_l(x)) \le 3 \end{array}\right\}$$

By using auxiliary parameters, the above problem **P1** can be transformed into a new problem, say, **P2** as follows

P2:

$$\left.\begin{array}{l} \text{Max } \alpha \\ \text{Max } \beta \\ \text{Min } \gamma \\ \text{Subject to} \\ T_l(Z_l(x)) \ge \alpha \\ I_l(Z_l(x)) \ge \beta \\ F_l(Z_l(x)) \le \gamma \\ \alpha \ge \beta,\ \ \alpha \ge \gamma,\ \ \ 0 \le \gamma + \beta + \gamma \le 3 \\ \gamma, \beta, \gamma \in [0,1] \end{array}\right\}$$

Here α , β and γ are the auxiliary variables for the truth, indeterminacy and false membership functions, respectively. Further, the above problem **P2** can be expressed in the purest form as the problem **P3** as follows

P3:

$$\left.\begin{array}{l}\text{Max}\ \phi(x)=\alpha+\beta-\gamma\\ \text{Subject to;}\\ Z_l(x)+(U_l^T-L_l^T)\alpha\le U_l^T\\ Z_l(x)+(U_l^I-L_l^I)\beta\le U_l^I\\ Z_l(x)-(U_l^F-L_l^F)\gamma\le L_l^F\\ \alpha\ge\beta,\ \alpha\ge\gamma,\ \ 0\le\gamma+\beta+\gamma\le 3\\ \gamma,\beta,\gamma\in[0,1]\end{array}\right\}$$

Based on the above formulations of a neutrosophic compromise programming technique, Model 2 of the present MOAP can be presented as a neutrosophic programming model in the following manner:

Model 4:

$$\text{Max}\ \phi(x)=\alpha+\beta-\gamma$$

Subject to;

$$\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{3}\begin{pmatrix}V(\tilde{c}_1)+V(\tilde{c}_2)+V(\tilde{c}_3)+V(\tilde{c}_4)\\ +\dfrac{(V(\tilde{c}_1)V(\tilde{c}_2)-V(\tilde{c}_3)V(\tilde{c}_4))}{(V(\tilde{c}_3)+V(\tilde{c}_4)-V(\tilde{c}_1)-V(\tilde{c}_2))}\end{pmatrix}x_{ij}\right\}+(U_1^T-L_1^T)\alpha\le U_1^T$$

$$\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{3}\begin{pmatrix}V(\tilde{c}_1)+V(\tilde{c}_2)+V(\tilde{c}_3)+V(\tilde{c}_4)\\ +\dfrac{(V(\tilde{c}_1)V(\tilde{c}_2)-V(\tilde{c}_3)V(\tilde{c}_4))}{(V(\tilde{c}_3)+V(\tilde{c}_4)-V(\tilde{c}_1)-V(\tilde{c}_2))}\end{pmatrix}x_{ij}\right\}+(U_1^I-L_1^I)\beta\le U_1^I$$

$$\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{3}\begin{pmatrix}V(\tilde{c}_1)+V(\tilde{c}_2)+V(\tilde{c}_3)+V(\tilde{c}_4)\\ +\dfrac{(V(\tilde{c}_1)V(\tilde{c}_2)-V(\tilde{c}_3)V(\tilde{c}_4))}{(V(\tilde{c}_3)+V(\tilde{c}_4)-V(\tilde{c}_1)-V(\tilde{c}_2))}\end{pmatrix}x_{ij}\right\}-(U_1^F-L_1^F)\gamma\le L_1^F$$

$$\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{3}\begin{pmatrix}V(\tilde{t}_1)+V(\tilde{t}_2)+V(\tilde{t}_3)+V(\tilde{t}_4)\\ +\dfrac{(V(\tilde{t}_1)V(\tilde{t}_2)-V(\tilde{t}_3)V(\tilde{t}_4))}{(V(\tilde{t}_3)+V(\tilde{t}_3)-V(\tilde{t}_1)-V(\tilde{t}_2))}\end{pmatrix}x_{ij}\right\}+(U_2^T-L_2^T)\alpha\le U_2^T$$

$$\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{3}\begin{pmatrix}V(\tilde{t}_1)+V(\tilde{t}_2)+V(\tilde{t}_3)+V(\tilde{t}_4)\\ +\dfrac{(V(\tilde{t}_1)V(\tilde{t}_2)-V(\tilde{t}_3)V(\tilde{t}_4))}{(V(\tilde{t}_3)+V(\tilde{t}_3)-V(\tilde{t}_1)-V(\tilde{t}_2))}\end{pmatrix}x_{ij}\right\}+(U_2^I-L_2^I)\beta\le U_2^I$$

$$\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{3}\begin{pmatrix}V(\tilde{t}_1)+V(\tilde{t}_2)+V(\tilde{t}_3)+V(\tilde{t}_4)\\ +\dfrac{(V(\tilde{t}_1)V(\tilde{t}_2)-V(\tilde{t}_3)V(\tilde{t}_4))}{(V(\tilde{t}_3)+V(\tilde{t}_3)-V(\tilde{t}_1)-V(\tilde{t}_2))}\end{pmatrix}x_{ij}\right\}-(U_2^F-L_2^F)\gamma\le L_2^F$$

$$\sum_{j=1}^{n}x_{ij}=1,\quad i=1,2,\dots,n,\ \sum_{i=1}^{m}x_{ij}=1,\quad j=1,2,\dots,n,$$

$$\alpha\ge\beta,\ \ \alpha\ge\gamma,\ \ 0\le\gamma+\beta+\gamma\le 3$$

$$\gamma,\beta,\gamma\in[0,1],\ x_{ij0}=0\,\text{or}\,1,$$

The following steps will be followed to discuss the present MOAP using NCPT.

Step 1. Formulate a MOAP under an uncertain environment as given by Model 2.
Step 2. Convert each fuzzy parameter of this problem into a crisp number using the defuzzification method discussed in Section 4.
Step 3. Calculate the best and worst solutions corresponding to each objective function under the given set of constraints using optimization software LINGO and create a payoff matrix (refer to Table 5).
Step 4. Determine the upper U_l and lower L_l bound, respectively, of each objectives using equation (1).
Step 5. With the help of these U_l and L_l values, find the upper and lower bound for all the membership functions (truth, indeterminacy and falsehood) using equations (2)-(4).
Step 6. Construct the linear membership function for the truth, indeterminacy and falsehood using equations (5)-(7).
Step 7. Construct the neutrosophic problem as problem P_2 and transform it into problem P_3.
Step 8. Solve the MOAP model as Model 4 and obtain the compromise solution using the Optimization Software Packages LINGO 16.0.

5.2 Fuzzy Programming Technique (FPT)

The problems involving undefined and imprecise parameters with multiple objectives are known to be typical mathematical problems. The fuzzy programming technique (FPT) is an effective and versatile solution technique for such a problem. Zimmermann [83] developed it in 1978, specifically to tackle MOOPs. A fuzzy programming model aims to optimize multiple objectives simultaneously, by reducing deviations from the goal features. Fuzzy programming needs the decision-makers to set a level of expectation for each target which is challenging as several uncertainties must also be considered in nature.
The general mathematical formulation of a fuzzy programming problem with l objectives and j constraints, with i decision variables, can be described as:

$$\left.\begin{array}{l}\text{Maximize } \lambda \\ \text{Subject to:} \\ \lambda \le \mu_l(x), \quad \forall\, l \\ g_j(x) \le 0, \quad j = 1,2,\ldots,n \\ x_i \ge 0, \quad i = 1,2,\ldots,m\end{array}\right\}$$

The following steps of the fuzzy programming technique can solve the MOAP given by Model 2.
Step 1. Find the optimal value of each objective function of the MOAP subject to the given set of constraints by ignoring all other objectives (use the optimization software LINGO).
Step 2. Calculate the best U_l and worst L_l values for each objective function separately and create a payoff matrix (Table 5).
Step 3. Define the membership function for each objective using equations (8) and (9) given below (refer [78]).
Membership function $\mu_l(Z_l(x))$ for lth objective function of minimization type

$$\mu_l(Z_l(x)) = \begin{cases} 1 & if \quad Z_l(x) \le L_l \\ \dfrac{U_l - Z_l(x)}{U_l - L_l} & if \quad L_l \le Z_l(x) \le U_l \\ 0 & if \quad Z_l(x) \ge U_l \end{cases} \tag{8}$$

Membership function for lth objective function of the maximization type

$$\mu_l(Z_l(x))=\begin{cases}1 & if \quad Z_l(x)\geq U_l\\ \dfrac{Z_l(x)-L_l}{U_l-L_l} & if \quad L_l\leq Z_l(x)\leq U_l\\ 0 & if \quad Z_l(x)\leq L_l\end{cases} \tag{9}$$

where L_l and U_l are the lower and upper bounds of the objective functions.

Finally, the MOAP can be defined as a fuzzy programming model as

Model 5:

$$\begin{aligned}
&\text{Maximize } \lambda\\
&\text{subject to:}\\
&\lambda\leq\mu_1\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{3}\begin{pmatrix}V(\tilde{c}_1)+V(\tilde{c}_2)+V(\tilde{c}_3)+V(\tilde{c}_4)\\ +\dfrac{(V(\tilde{c}_1)V(\tilde{c}_2)-V(\tilde{c}_3)V(\tilde{c}_4))}{(V(\tilde{c}_3)+V(\tilde{c}_4)-V(\tilde{c}_1)-V(\tilde{c}_2))}\end{pmatrix}x_{ij}\right\}\\
&\lambda\leq\mu_2\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{3}\begin{pmatrix}V(\tilde{t}_1)+V(\tilde{t}_2)+V(\tilde{t}_3)+V(\tilde{t}_4)\\ +\dfrac{(V(\tilde{t}_1)V(\tilde{t}_2)-V(\tilde{t}_3)V(\tilde{t}_4))}{(V(\tilde{t}_3)+V(\tilde{t}_3)-V(\tilde{t}_1)-V(\tilde{t}_2))}\end{pmatrix}x_{ij}\right\}\\
&\text{Model}(1)
\end{aligned}$$

Step 4. Solve this crisp MOAP above and obtain the compromise solution using the Optimization Software Package LINGO 16.0.

5.3 Intuitionistic Fuzzy Programming Technique (IFPT)

The intuitionistic fuzzy set theory is an alternative for defining a fuzzy set if the available knowledge is insufficient to describe an imprecise theory using a traditional fuzzy set. The degree of membership and non-membership for the objective functions and their limitations are concurrent and taken into account in such a way that the sum of both is either less than or equal to one.

The general mathematical formulation of a MOOP in the context of intuitionistic fuzzy programming is as follows:

$$\left.\begin{aligned}
&\text{Maximize } \alpha-\beta\\
&\text{Subject to}\\
&\mu_l(Z_l(x))\geq\alpha,\ \chi_l(Z_l(x))\leq\beta,\forall\, l\\
&\alpha+\beta\leq 1, \qquad \alpha\geq\beta,\ \beta\geq 0,\\
&g_k(x)\leq 0, \qquad k=1,2,...,K\\
&x_i\geq 0, \qquad i=1,2,...,m
\end{aligned}\right\} \tag{10}$$

where $\mu_l(Z_l(x))$ *and* $\chi_l(Z_l(x))$ are the membership and non-membership functions of the *lth* objective and α , β are their aspiration levels.

The following steps explain finding a compromise solution to the problem given by (10) using IFPT.

Step 1.Find the optimal value of each objective function of the MOOP subject to the given set of constraints by ignoring all other objectives, using the optimization software LINGO.

Step 2. Calculate the best U_l and worst L_l values for each objective function separately and create a payoff matrix (Table 5).

Step 3. Construct the membership and non-membership functions $\mu_l(Z_l(x))$ *and* $\chi_l(Z_l(x))$, respectively, of l*th* objective function, for all values of l, using equations (11) and (12) given as

$$\mu_l(Z_l(x)) = \begin{cases} 1 & if \quad Z_l(x) \le L_l \\ \dfrac{U_l - Z_l(x)}{U_l - L_l} & if \quad L_l \le Z_l(x) \le U_l \\ 0 & if \quad Z_l(x) \ge U_l \end{cases} \tag{11}$$

and

$$\chi_l(Z_l(x)) = \begin{cases} 0 & if \quad Z_l(x) \le L_l \\ \dfrac{Z_l(x) - L_l}{U_l - L_l} & if \quad L_l \le Z_l(x) \le U_l \\ 1 & if \quad Z_l(x) \ge U_l \end{cases} \tag{12}$$

Now, Model 2 of the present MOAP can be defined using IFPT as follows:

Model 6:

$$\left.\begin{array}{l}
\text{Maximize } (\alpha - \beta) \\
\text{Subject to;} \\
\mu_1\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{3}\begin{pmatrix} V(\tilde{c}_1)+V(\tilde{c}_2)+V(\tilde{c}_3)+V(\tilde{c}_4) \\ +\dfrac{(V(\tilde{c}_1)V(\tilde{c}_2)-V(\tilde{c}_3)V(\tilde{c}_4))}{(V(\tilde{c}_3)+V(\tilde{c}_4)-V(\tilde{c}_1)-V(\tilde{c}_2))}\end{pmatrix}x_{ij}\right\} \ge \alpha, \\
\mu_2\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{3}\begin{pmatrix} V(\tilde{t}_1)+V(\tilde{t}_2)+V(\tilde{t}_3)+V(\tilde{t}_4) \\ +\dfrac{(V(\tilde{t}_1)V(\tilde{t}_2)-V(\tilde{t}_3)V(\tilde{t}_4))}{(V(\tilde{t}_3)+V(\tilde{t}_3)-V(\tilde{t}_1)-V(\tilde{t}_2))}\end{pmatrix}x_{ij}\right\} \ge \alpha \\
\chi_1\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{3}\begin{pmatrix} V(\tilde{c}_1)+V(\tilde{c}_2)+V(\tilde{c}_3)+V(\tilde{c}_4) \\ +\dfrac{(V(\tilde{c}_1)V(\tilde{c}_2)-V(\tilde{c}_3)V(\tilde{c}_4))}{(V(\tilde{c}_3)+V(\tilde{c}_4)-V(\tilde{c}_1)-V(\tilde{c}_2))}\end{pmatrix}x_{ij}\right\} \ge \beta \\
\chi_2\left\{\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{3}\begin{pmatrix} V(\tilde{t}_1)+V(\tilde{t}_2)+V(\tilde{t}_3)+V(\tilde{t}_4) \\ +\dfrac{(V(\tilde{t}_1)V(\tilde{t}_2)-V(\tilde{t}_3)V(\tilde{t}_4))}{(V(\tilde{t}_3)+V(\tilde{t}_3)-V(\tilde{t}_1)-V(\tilde{t}_2))}\end{pmatrix}x_{ij}\right\} \le \beta \\
\text{Model}(1), \\
\alpha + \beta \le 1, \quad \alpha \ge \beta, \; \beta \ge 0, \; x_{ij} = 0 \text{ or} 1
\end{array}\right\}$$

Step 4. Solve this crisp model of the present MOAP by using the Optimization Software Packages LINGO 16.0 and obtain a compromise solution.

A flow chart of the proposed optimization procedure using all the techniques mentioned above is given in Figure 1.

.

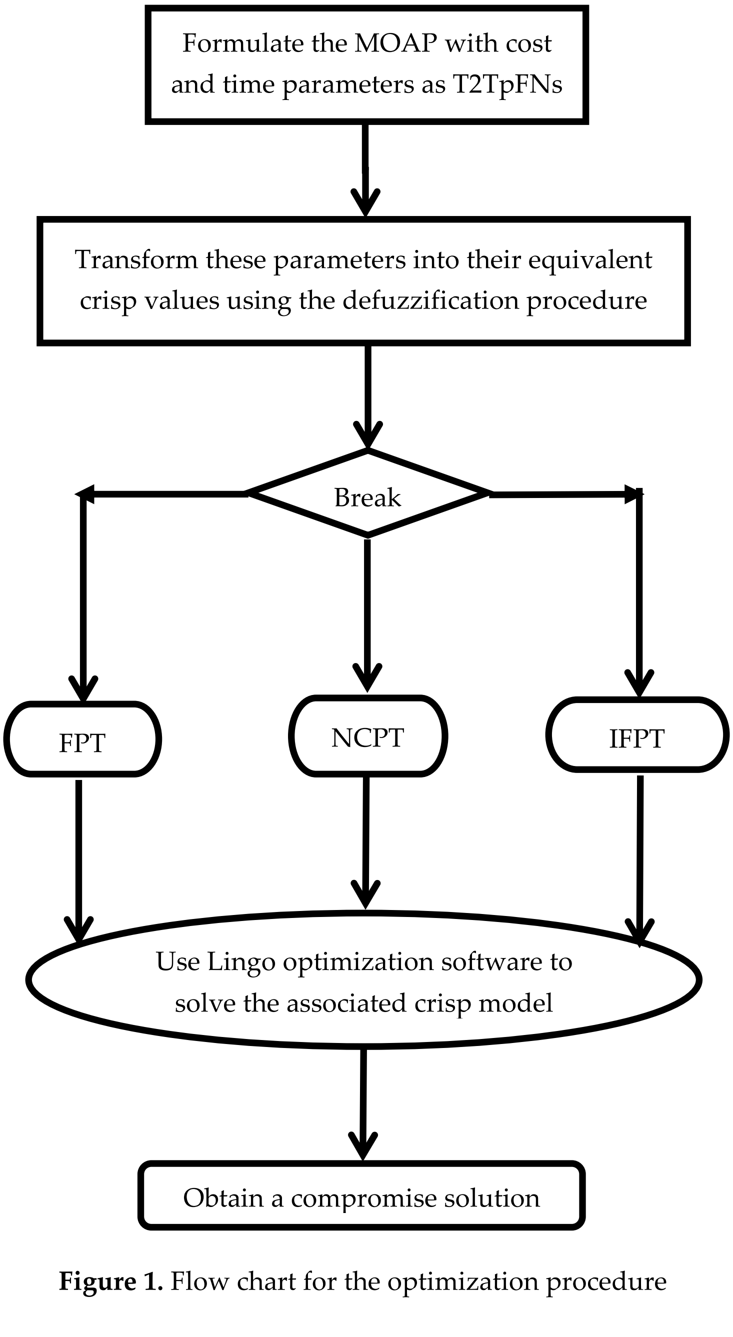

Figure 1. Flow chart for the optimization procedure

6. Real-World Applications

The present MOAP aims to minimize execution time and assignment cost, simultaneously. It finds its applications in many business scenarios where the quickest possible delivery of its product is as important as its financial budget. Generally, a quick mode of transportation may result in high transportation charges which mean that the objectives are conflicting in nature. So, the objective is to find such an assignment schedule that provides the best compromising solution to the problem. There may be many managerial implications of the present problem, but to quote some of them, consider the following real-life scenarios

(1) In an FMCG (fast-moving consumable goods) industry, due to the limited shelf life of the goods, it is important to deliver the products to the destinations as soon as possible. However, at the same time, the supply chain management team of the industry works to minimize the logistics cost. Therefore, it is important to find a way of transporting goods to minimize both objectives, simultaneously.

(2) In the commercial industry, road transportation is an extremely methodical way of hauling goods among various locations to improve the efficiency and growth of a business. Therefore, the use of heavy goods vehicles (HGVs) is an indispensable part of any business. Consider an industrial project of manufacturing some HGVs in minimum time and budget. For manufacturing various parts of an HGV in terms of both execution time and cost, quotations from various manufacturing units are taken. Then, an assignment schedule is looked for so that all the parts are produced in the minimum time and in the minimum budget so that a cost-efficient HGV is manufactured well in time. There are numerous other real-world situations of this kind that may give rise to the present MOAP.

7. Numerical Illustration

Consider an industrial manufacturing problem that uses third party operations. The product that the industry manufactures requires four major semi-finished parts. These semi-finished parts are finished and assembled to form the final product by the industry itself. All of these parts can be manufactured by any of the four different third party manufacturing units, which have imprecise values of the manufacturing time and cost corresponding to each part. The industry's objective is to assign the task of manufacturing four semi-finished parts to four third party manufacturing units so that all the parts are manufactured in the minimum time and with the least financial burden.

Here, the first objective Z_1 denotes the total manufacturing cost (in \$), and the second objective Z_2 denotes the total manufacturing time (in minutes) of all the four semi-finished parts. Table 1 shows the key attributes of the problem. The imprecise manufacturing costs and times quoted by all the third party manufacturing units for manufacturing each semi-finished part are given as T2TpFNs in Table 2 and Table 3, respectively. The two-phase defuzzification process (discussed in Section 4) is used to achieve a crisp value of each of these imprecise T2TpFNs. The crisp values corresponding to Stage 1 and Stage 2 of the defuzzification process are summarized in Table 4. Table 5 provides the best and worst values of both the objective functions, achieved by solving each of them individually under a given set of constraints.

Table 1. Main attributes of the problem

Number of third party manufacturing units (i)	*4*
Number of tasks (j)	*4*

Table 2. Imprecise manufacturing costs as T2TpFNs

Z_1	Task 1	Task 2	Task 3	Task 4
Manufacturing unit 1	[(38,40,42,46); (35,40,42,48); (32,40,42,48); (31,40,42,55)]	[(43,45,46,49); (41,45,46,54); (38,45,46,56); (36,45,46,59)]	[(51,53,55,58); (49,53,55,60); (46,53,55,64); (44,53,55,67)]	[(65,67,69,72); (62,67,69,74); (60,67,69,78); (59,67,69,80)]
Manufacturing unit 2	[(35,37,39,43); (32,37,39,45);	[(69,71,73,76); (67,71,73,80);	[(66,68,70,74); (62,68,70,77);	[(77,79,82,86); (74,79,82,89);

	(29,37,39,49); (28,37,39,54)]	(65,71,73,83); (62,71,73,85)]	(60,68,70,81); (57,68,70,85)]	(72,79,82,94); (68,79,82,97)]
Manufacturing unit 3	[(89,91,94,98); (87,91,94,102); (85,91,94,106); (83,91,94,109)]	[(83,85,86,88); (82,85,86,91); (80,85,86,94); (77,85,86,98)]	[(96,98,100,104); (94,98,100,107); (91,98,100,110); (88,98,100,114)]	[(61,63,64,67); (58,63,64,71); (56,63,64,75); (53,63,64,79)]
Manufacturing unit 4	[(58,60,63,67); (56,60,63,71); (53,60,63,74); (51,60,63,78)]	[(35,38,40,43); (33,38,40,44); (32,38,40,45); (30,38,40,49)]	[(56,58,60,64); (54,58,60,68); (51,58,60,70); (49,58,60,74)]	[(73,75,77,81); (70,75,77,84); (68,75,77,87); (65,75,77,89)]

Table 3. Imprecise manufacturing times asT2TpFNs

Z_2	Task 1	Task 2	Task 3	Task 4
Manufacturing unit 1	[(218,220,222,225); (216,220,222,227); (213,220,222,231); (210,220,222,234)]	[(242,245,246,249); (240,245,246,252); (237,245,246,255); (234,245,246,259)]	[(211,209,215,218); (209,213,215,220); (206,213,215,224); (203,213,215,227)]	[(225,227,229,233); (224,227,229,235); (221,227,229,239); (217,227,229,244)]
Manufacturing unit 2	[(262,264,266,270); (260,264,266,273); (257,264,266,275); (254,264,266,276)]	[(250,252,254,257); (248,252,254,260); (245,252,254,264); (241,252,254,267)]	[(231,233,234,237); (228,233,234,240); 226,233,234,244); 223,233,234,247)]	[(255,257,259,262); (252,257,259,264); (249,257,259,267); (247,257,259,270)]
Manufacturing unit 3	[(278,280,281,284); (275,280,281,286); (273,280,281,289); (270,280,281,293)]	[(283,285,287,290); (280,285,287,292); (277,285,287,294); (274,285,287,298)]	[(295,297,299,303); (292,297,299,306); (290,297,299,309); (287,297,299,314)]	[(288,290,292,295); (285,290,292,298); (283,290,292,301); (280,290,292,303)]
Manufacturing unit 4	[(242,244,246,249); (240,244,246,253); (238,244,246,257); (236,244,246,261)]	[(285,287,289,292); (283,287,289,295); (281,287,289,297); (279,287,289,303)]	[(257,259,261,265); (255,259,261,268); (253,259,261,270); (251,259,261,274)]	[(273,275,277,282); (271,275,277,285); (269,275,277,288); (267,275,277,303)]

Table 4. Crisp values of the manufacturing costs and times obtained by the two-stage defuzzification process

Z_1 (Cost)			Z_2 (Time)		
c_{ij}	$V(\tilde{\tilde{c}}_{ij})$	$DV(\tilde{\tilde{c}}_{ij})$	t_{ij}	$V(\tilde{\tilde{t}}_{ij})$	$DV(\tilde{\tilde{t}}_{ij})$
c_{11}	(41.60,41.31,41.63,42.28)	41.73	t_{11}	(221.29,221.30,221.63,221.64)	221.46
c_{12}	(45.80,46.78,46.47,46.47)	46.35	t_{12}	(245.50,245.80,245.82,246.15)	245.82
c_{13}	(54.29,54.30,54.63,54.96)	54.55	t_{13}	(214.29,214.30,214.63,214.64)	214.46
c_{14}	(68.29,68.00,68.63,68.95)	68.46	t_{14}	(228.60,228.92,229.26,229.60)	229.09
c_{21}	(38.60,38.11,38.63,39.92)	38.87	t_{21}	(265.60,265.93,265.63,265.00)	265.51
c_{22}	(72.29,72.93,73.26,72.96)	72.83	t_{22}	(253.29,253.61,253.95,253.64)	253.64
c_{23}	(69.60,69.31,69.95,70.28)	69.78	t_{23}	(233.80,233.82,233.47,234.48)	235.95
c_{24}	(81.08,81.11,82.06,81.77)	81.50	t_{24}	(258.29,258.00,258.01,258.32)	258.20

c_{31}	(93.08,93.60,94.37,94.71)	93.93	t_{31}	(280.80,280.50,280.82,281.15)	280.81
c_{32}	(86.31,86.56,85.79,85.81)	86.12	t_{32}	(286.29,286.00,285.68,286.09)	286.31
c_{33}	(99.60,99.43,99.95,100.28)	99.80	t_{32}	(298.60,298.00,298.62,299.60)	298.94
c_{34}	(63.80,64.14,64.80,65.32)	64.52	t_{34}	(291.29,291.31,291.63,291.32)	291.41
c_{41}	(62.08,62.72,62.75,63.40)	62.73	t_{41}	(244.07,244.91,243.31,243.45)	243.96
c_{42}	(39.00,38.88,38.68,39.20)	38.79	t_{42}	(287.62,287.59,287.09,289.01)	288.11
c_{43}	(59.60,60.25,59.63,59.92)	59.94	t_{43}	(259.97,260.32,260.74,261.01)	260.50
c_{44}	(76.60,76.62,76.95,76.64)	76.72	t_{44}	(275.98,276.08,276.58,276.96)	276.40

The T2TpFN defuzzification process is divided into two stages. In stage I, the defuzzification technique transforms T2TpFN to T1TpFN, and in stage II, the T1TpFNs were again used to obtain the defuzzified value of T2TpFN.

Now, using the above available data in Table 2 and 3, the MOAP (Model 2) with Type 2 fuzzy parameters can be described as follows:

Stage I.

$Min\ Z_1 =$

$$\begin{pmatrix} \frac{1}{3}\left(38+40+42+46+\frac{38\times40-42\times46}{38+40-42-46}\right), & \frac{1}{3}\left(35+40+42+48+\frac{35\times40-42\times48}{35+40-42-48}\right) \\ \frac{1}{3}\left(32+40+42+46+\frac{32\times40-42\times46}{32+40-42-46}\right), & \frac{1}{3}\left(31+40+42+55+\frac{31\times40-42\times55}{31+40-42-55}\right) \end{pmatrix} x_{11}$$

$$+\begin{pmatrix} \frac{1}{3}\left(43+45+46+49+\frac{43\times45-46\times49}{43+45-46-49}\right), & \frac{1}{3}\left(41+45+46+54+\frac{41\times45-46\times54}{41+45-46-54}\right) \\ \frac{1}{3}\left(38+45+46+56+\frac{38\times45-46\times56}{38+45-46-56}\right), & \frac{1}{3}\left(36+45+46+59+\frac{36\times45-46\times59}{36+45-46-59}\right) \end{pmatrix} x_{12}$$

$$+\begin{pmatrix} \frac{1}{3}\left(51+53+55+58+\frac{51\times53-55\times58}{51+53-55-58}\right), & \frac{1}{3}\left(49+53+55+60+\frac{49\times53-55\times60}{49+53-55-60}\right) \\ \frac{1}{3}\left(46+53+55+64+\frac{46\times53-55\times64}{46+53-55-64}\right), & \frac{1}{3}\left(44+53+55+67+\frac{44\times53-55\times67}{44+53-55-67}\right) \end{pmatrix} x_{13}$$

$$+\begin{pmatrix} \frac{1}{3}\left(65+67+69+72+\frac{65\times67-69\times72}{65+67-69-72}\right), & \frac{1}{3}\left(62+67+69+74+\frac{62\times67-69\times74}{62+67-69-74}\right) \\ \frac{1}{3}\left(60+67+69+78+\frac{60\times67-69\times78}{60+67-69-78}\right), & \frac{1}{3}\left(59+67+69+80+\frac{59\times67-69\times80}{59+67-69-80}\right) \end{pmatrix} x_{14}$$

$$+\begin{pmatrix} \frac{1}{3}\left(35+37+39+43+\frac{35\times37-39\times43}{35+37-39-43}\right), & \frac{1}{3}\left(32+37+39+45+\frac{32\times37-39\times45}{32+37-39-45}\right) \\ \frac{1}{3}\left(29+37+39+49+\frac{29\times37-39\times49}{29+37-39-49}\right), & \frac{1}{3}\left(28+37+39+49+\frac{28\times37-39\times54}{28+37-39-54}\right) \end{pmatrix} x_{21}$$

$$+\begin{pmatrix} \frac{1}{3}\left(69+71+73+76+\frac{69\times71-73\times76}{69+71-73-76}\right), & \frac{1}{3}\left(67+71+73+80+\frac{67\times71-73\times80}{67+71-73-80}\right) \\ \frac{1}{3}\left(65+71+73+83+\frac{65\times71-73\times83}{65+71-73-83}\right), & \frac{1}{3}\left(62+71+73+85+\frac{62\times71-73\times85}{62+71-73-85}\right) \end{pmatrix} x_{22}$$

$$+\begin{pmatrix} \frac{1}{3}\left(66+68+70+74+\frac{66\times68-70\times74}{66+68-70-74}\right), & \frac{1}{3}\left(62+68+70+77+\frac{62\times68-70\times77}{62+68-70-77}\right) \\ \frac{1}{3}\left(60+68+70+81+\frac{60\times68-70\times81}{60+68-70-81}\right), & \frac{1}{3}\left(57+68+70+85+\frac{57\times68-70\times85}{57+68-70-85}\right) \end{pmatrix} x_{23}$$

$$+\begin{pmatrix} \frac{1}{3}\left(77+79+82+86+\frac{77\times79-82\times86}{77+79-82-86}\right), & \frac{1}{3}\left(74+79+82+89+\frac{74\times79-82\times89}{74+79-82-89}\right) \\ \frac{1}{3}\left(72+79+82+94+\frac{72\times79-82\times94}{72+79-82-94}\right), & \frac{1}{3}\left(68+79+82+97+\frac{68\times79-82\times97}{68+79-82-97}\right) \end{pmatrix} x_{24}$$

$$+\begin{pmatrix} \frac{1}{3}\left(89+91+94+98+\frac{89\times91-94\times98}{89+91-94-98}\right), & \frac{1}{3}\left(87+91+94+102+\frac{87\times91-94\times102}{87+91-94-102}\right) \\ \frac{1}{3}\left(85+91+94+106+\frac{85\times91-94\times106}{85+91-94-106}\right), & \frac{1}{3}\left(83+91+94+109+\frac{83\times91-94\times109}{83+91-94-109}\right) \end{pmatrix} x_{31}$$

$$+\begin{pmatrix} \frac{1}{3}\left(83+85+86+88+\frac{83\times85-86\times88}{83+85-86-88}\right), & \frac{1}{3}\left(82+85+86+91+\frac{82\times85-86\times91}{82+85-86-91}\right) \\ \frac{1}{3}\left(80+85+86+94+\frac{80\times85-86\times94}{80+85-86-94}\right), & \frac{1}{3}\left(77+85+86+98+\frac{77\times85-86\times98}{77+85-86-98}\right) \end{pmatrix} x_{32}$$

$$+\left(\begin{array}{l}\frac{1}{3}\left(96+98+100+104+\frac{96\times 98-100\times 104}{96+98-100-104}\right),\ \frac{1}{3}\left(94+98+100+107+\frac{94\times 98-100\times 107}{94+98-100-107}\right)\\ \frac{1}{3}\left(91+98+100+110+\frac{91\times 98-100\times 110}{91+98-100-110}\right),\ \frac{1}{3}\left(88+98+100+114+\frac{88\times 98-100\times 114}{88+98-100-114}\right)\end{array}\right)x_{33}$$

$$+\left(\begin{array}{l}\frac{1}{3}\left(61+63+64+67+\frac{61\times 63-64\times 67}{61+63-64-67}\right),\ \frac{1}{3}\left(58+63+64+71+\frac{58\times 63-64\times 71}{58+63-64-71}\right)\\ \frac{1}{3}\left(56+63+64+75+\frac{56\times 63-64\times 75}{56+63-64-75}\right),\ \frac{1}{3}\left(53+63+64+79+\frac{53\times 63-64\times 79}{53+63-64-79}\right)\end{array}\right)x_{34}$$

$$+\left(\begin{array}{l}\frac{1}{3}\left(58+60+63+67+\frac{58\times 60-63\times 67}{58+60-63-67}\right),\ \frac{1}{3}\left(56+60+63+71+\frac{56\times 60-63\times 71}{56+60-63-71}\right)\\ \frac{1}{3}\left(53+60+63+74+\frac{53\times 60-63\times 74}{51+60-63-74}\right),\ \frac{1}{3}\left(51+60+63+78+\frac{51\times 60-63\times 78}{51+60-63-78}\right)\end{array}\right)x_{41}$$

$$+\left(\begin{array}{l}\frac{1}{3}\left(35+38+40+43+\frac{35\times 38-40\times 43}{35+38-40-43}\right),\ \frac{1}{3}\left(33+38+40+44+\frac{33\times 38-40\times 44}{33+38-40-44}\right)\\ \frac{1}{3}\left(32+38+40+45+\frac{32\times 38-40\times 45}{32+38-40-45}\right),\ \frac{1}{3}\left(30+38+40+49+\frac{30\times 38-40\times 49}{30+38-40-49}\right)\end{array}\right)x_{42}$$

$$+\left(\begin{array}{l}\frac{1}{3}\left(56+58+60+64+\frac{56\times 58-60\times 64}{56+58-60-64}\right),\ \frac{1}{3}\left(54+58+60+68+\frac{54\times 58-60\times 68}{54+58-60-68}\right)\\ \frac{1}{3}\left(51+58+60+70+\frac{51\times 58-60\times 70}{51+58-60-70}\right),\ \frac{1}{3}\left(49+58+60+74+\frac{49\times 58-60\times 74}{49+58-60-74}\right)\end{array}\right)x_{43}$$

$$+\left(\begin{array}{l}\frac{1}{3}\left(73+75+77+81+\frac{73\times 75-77\times 81}{73+75-77-81}\right),\ \frac{1}{3}\left(70+75+77+84+\frac{70\times 75-77\times 84}{70+75-77-84}\right)\\ \frac{1}{3}\left(68+75+77+87+\frac{68\times 75-77\times 87}{68+75-77-87}\right),\ \frac{1}{3}\left(65+75+77+89+\frac{65\times 75-77\times 89}{65+75-77-89}\right)\end{array}\right)x_{44}$$

$Min\ Z_2 =$

$$\left(\begin{array}{l}\frac{1}{3}\left(218+220+222+225+\frac{218\times 220-222\times 225}{218+220-222-225}\right),\ \frac{1}{3}\left(216+220+222+227+\frac{216\times 220-222\times 227}{216+220-222-227}\right)\\ \frac{1}{3}\left(213+220+222+231+\frac{213\times 220-222\times 231}{213+220-222-231}\right),\ \frac{1}{3}\left(210+220+222+234+\frac{210\times 220-222\times 234}{210+220-222-234}\right)\end{array}\right)x_{11}$$

$$+\left(\begin{array}{l}\frac{1}{3}\left(242+245+246+249+\frac{242\times 245-246\times 249}{242+245-246-249}\right),\ \frac{1}{3}\left(240+245+246+252+\frac{240\times 245-246\times 252}{240+245-246-252}\right)\\ \frac{1}{3}\left(237+245+246+255+\frac{237\times 245-246\times 255}{237+245-246-255}\right),\ \frac{1}{3}\left(234+245+246+259+\frac{234\times 245-246\times 259}{234+245-246-259}\right)\end{array}\right)x_{12}$$

$$+\left(\begin{array}{l}\frac{1}{3}\left(211+209+215+218+\frac{211\times 209-215\times 218}{211+209-215-218}\right),\ \frac{1}{3}\left(209+213+215+220+\frac{209\times 213-215\times 220}{209+213-215-220}\right)\\ \frac{1}{3}\left(206+213+215+224+\frac{206\times 213-215\times 224}{206+213-215-224}\right),\ \frac{1}{3}\left(203+213+215+227+\frac{203\times 213-215\times 227}{203+213-215-227}\right)\end{array}\right)x_{13}$$

$$+\left(\begin{array}{l}\frac{1}{3}\left(225+227+229+233+\frac{225\times 227-229\times 233}{225+227-229-233}\right),\ \frac{1}{3}\left(224+227+229+235+\frac{224\times 227-229\times 235}{224+227-229-235}\right)\\ \frac{1}{3}\left(221+227+229+239+\frac{221\times 227-229\times 239}{221+227-229-239}\right),\ \frac{1}{3}\left(217+227+229+244+\frac{217\times 227-229\times 244}{217+227-229-244}\right)\end{array}\right)x_{14}$$

$$+\begin{pmatrix} \frac{1}{3}\left(262+264+266+270+\frac{262\times264-266\times270}{262+264-266-270}\right), & \frac{1}{3}\left(260+264+266+273+\frac{260\times264-266\times273}{260+264-266-273}\right) \\ \frac{1}{3}\left(257+264+266+275+\frac{257\times264-266\times275}{257+264-266-275}\right), & \frac{1}{3}\left(254+264+266+276+\frac{254\times264-266\times276}{254+264-266-276}\right) \end{pmatrix} x_{21}$$

$$+\begin{pmatrix} \frac{1}{3}\left(250+252+254+257+\frac{250\times252-254\times257}{250+252-254-257}\right), & \frac{1}{3}\left(248+252+254+260+\frac{248\times252-254\times260}{248+252-254-260}\right) \\ \frac{1}{3}\left(245+252+254+264+\frac{245\times252-254\times264}{245+252-254-264}\right), & \frac{1}{3}\left(241+252+254+267+\frac{241\times252-254\times267}{241+252-254-267}\right) \end{pmatrix} x_{22}$$

$$+\begin{pmatrix} \frac{1}{3}\left(231+233+234+237+\frac{231\times233-234\times237}{231+233-234-237}\right), & \frac{1}{3}\left(228+233+234+240+\frac{228\times233-234\times240}{228+233-234-240}\right) \\ \frac{1}{3}\left(226+233+234+244+\frac{226\times233-234\times244}{226+233-234-244}\right), & \frac{1}{3}\left(223+233+234+247+\frac{223\times233-234\times247}{223+233-234-247}\right) \end{pmatrix} x_{23}$$

$$+\begin{pmatrix} \frac{1}{3}\left(255+257+259+262+\frac{255\times257-259\times262}{255+257-259-262}\right), & \frac{1}{3}\left(252+257+259+264+\frac{252\times257-259\times264}{252+257-259-264}\right) \\ \frac{1}{3}\left(249+257+259+267+\frac{249\times257-259\times267}{249+257-259-267}\right), & \frac{1}{3}\left(247+257+259+270+\frac{247\times257-259\times270}{247+257-259-270}\right) \end{pmatrix} x_{24}$$

$$+\begin{pmatrix} \frac{1}{3}\left(278+280+281+284+\frac{278\times280-281\times284}{278+280-281-284}\right), & \frac{1}{3}\left(275+280+281+286+\frac{275\times280-281\times286}{275+280-281-286}\right) \\ \frac{1}{3}\left(273+280+281+289+\frac{273\times280-281\times289}{273+280-281-289}\right), & \frac{1}{3}\left(270+280+281+293+\frac{270\times280-281\times293}{270+280-281-293}\right) \end{pmatrix} x_{31}$$

$$+\begin{pmatrix} \frac{1}{3}\left(283+285+287+290+\frac{283\times285-287\times290}{283+285-287-290}\right), & \frac{1}{3}\left(280+285+287+292+\frac{280\times285-287\times292}{280+285-287-292}\right) \\ \frac{1}{3}\left(277+285+287+294+\frac{277\times285-287\times294}{277+285-287-294}\right), & \frac{1}{3}\left(274+285+287+298+\frac{274\times285-287\times298}{274+285-287-298}\right) \end{pmatrix} x_{32}$$

$$+\begin{pmatrix} \frac{1}{3}\left(295+297+299+303+\frac{295\times297-299\times303}{295+297-299-303}\right), & \frac{1}{3}\left(292+297+299+306+\frac{292\times297-299\times306}{292+297-299-306}\right) \\ \frac{1}{3}\left(290+297+299+309+\frac{290\times297-299\times309}{295+297-299-309}\right), & \frac{1}{3}\left(287+297+299+314+\frac{287\times297-299\times314}{287+297-299-314}\right) \end{pmatrix} x_{33}$$

$$+\begin{pmatrix} \frac{1}{3}\left(288+290+292+295+\frac{288\times290-292\times295}{288+290-292-295}\right), & \frac{1}{3}\left(285+290+292+298+\frac{285\times290-292\times298}{285+290-292-298}\right) \\ \frac{1}{3}\left(283+290+292+301+\frac{283\times290-292\times301}{283+290-292-301}\right), & \frac{1}{3}\left(280+290+292+303+\frac{280\times290-292\times303}{280+290-292-303}\right) \end{pmatrix} x_{34}$$

$$+\begin{pmatrix} \frac{1}{3}\left(242+244+246+249+\frac{242\times244-246\times249}{242+244-246-249}\right), & \frac{1}{3}\left(240+244+246+253+\frac{240\times244-246\times253}{240+244-246-253}\right) \\ \frac{1}{3}\left(238+244+246+257+\frac{238\times244-246\times257}{238+244-246-257}\right), & \frac{1}{3}\left(236+244+246+261+\frac{236\times244-246\times261}{236+244-246-261}\right) \end{pmatrix} x_{41}$$

$$+\begin{pmatrix} \frac{1}{3}\left(285+287+289+292+\frac{285\times287-289\times292}{285+287-289-292}\right), & \frac{1}{3}\left(283+287+289+295+\frac{283\times287-289\times295}{283+287-289-295}\right) \\ \frac{1}{3}\left(281+287+289+297+\frac{281\times287-289\times297}{281+287-289-297}\right), & \frac{1}{3}\left(279+287+289+303+\frac{279\times287-289\times303}{279+287-289-303}\right) \end{pmatrix} x_{42}$$

$$+\begin{pmatrix} \frac{1}{3}\left(257+259+261+265+\frac{257\times259-261\times265}{257+259-261-265}\right), & \frac{1}{3}\left(255+259+261+268+\frac{255\times259-261\times268}{255+259-261-268}\right) \\ \frac{1}{3}\left(253+259+261+270+\frac{253\times259-261\times270}{253+259-261-270}\right), & \frac{1}{3}\left(251+259+261+274+\frac{251\times259-261\times274}{251+259-261-274}\right) \end{pmatrix} x_{43}$$

$$+\begin{pmatrix} \frac{1}{3}\left(273+275+277+282+\frac{273\times 275-277\times 282}{273+275-277-282}\right), \frac{1}{3}\left(271+275+277+288+\frac{271\times 275-277\times 285}{271+275-277-285}\right) \\ \frac{1}{3}\left(269+275+277+288+\frac{269\times 275-277\times 288}{269+275-277-288}\right), \frac{1}{3}\left(267+275+277+303+\frac{267\times 275-277\times 303}{267+275-277-303}\right) \end{pmatrix} x_{44}$$

Subjec to;

$$x_{11}+x_{12}+x_{13}+x_{14}=1$$
$$x_{21}+x_{22}+x_{23}+x_{24}=1$$
$$x_{31}+x_{32}+x_{33}+x_{34}=1$$
$$x_{41}+x_{42}+x_{43}+x_{44}=1$$
$$x_{11}+x_{21}+x_{31}+x_{41}=1$$
$$x_{12}+x_{222}+x_{32}+x_{42}=1$$
$$x_{13}+x_{23}+x_{33}+x_{43}=1$$
$$x_{14}+x_{24}+x_{34}+x_{44}=1$$

Stage II.

$Min\ Z_1 =$

$$\frac{1}{3}\left(41.60+41.31+41.63+42.28+\frac{41.60\times 41.31-41.63\times 42.28}{41.60+41.31-41.63-42.28}\right)x_{11}$$
$$+\frac{1}{3}\left(45.80+46.78+46.47+46.49+\frac{45.80\times 46.78-46.47\times 46.49}{45.80+46.78-46.47-46.49}\right)x_{12}$$
$$+\frac{1}{3}\left(54.29+54.30+54.63+54.96+\frac{54.29\times 54.30-54.63\times 54.96}{54.29+54.30-54.63-54.96}\right)x_{13}$$
$$+\frac{1}{3}\left(68.29+68.00+68.63+68.95+\frac{68.29\times 68.00-68.63\times 68.95}{68.29+68.00-68.63-68.95}\right)x_{14}$$
$$+\frac{1}{3}\left(38.60+38.11+38.63+39.92+\frac{38.60\times 38.11-38.63\times 39.92}{38.60+38.11-38.63-39.92}\right)x_{21}$$
$$+\frac{1}{3}\left(72.29+72.93+73.26+72.96+\frac{72.29\times 72.93-73.26\times 72.96}{72.29+72.93-73.26-72.96}\right)x_{22}$$
$$+\frac{1}{3}\left(69.60+69.31+69.95+70.28+\frac{69.60\times 69.31-69.95\times 70.28}{69.60+69.31-69.95-70.28}\right)x_{23}$$
$$+\frac{1}{3}\left(81.08+81.11+82.06+81.77+\frac{81.08\times 81.11-82.06\times 81.77}{81.08+81.11-82.06-81.77}\right)x_{24}$$
$$+\frac{1}{3}\left(93.08+93.60+94.37+94.71+\frac{893.08\times 93.60-94.37\times 94.71}{93.08+93.60-94.37-94.71}\right)x_{31}$$
$$+\frac{1}{3}\left(86.31+86.56+85.79+85.81+\frac{86.31\times 86.56-85.79\times 85.81}{86.31+86.56-85.79-85.81}\right)x_{32}$$
$$+\frac{1}{3}\left(99.60+99.43+99.95+100.28+\frac{99.60\times 99.43-99.95\times 100.28}{99.60+99.43-99.95-100.28}\right)x_{33}$$
$$+\frac{1}{3}\left(63.80+64.14+64.80+65.32+\frac{63.80\times 64.14+64.80\times 65.32}{63.80+64.14-64.80-65.32}\right)x_{34}$$
$$+\frac{1}{3}\left(62.08+62.72+62.75+63.40+\frac{62.08\times 62.72-62.75\times 63.40}{62.08+62.72-62.75-63.40}\right)x_{41}$$
$$+\frac{1}{3}\left(39.00+38.88+38.68+39.20+\frac{39.00\times 38.88-38.68\times 39.20}{39.00+38.88-38.68-39.20}\right)x_{42}$$
$$+\frac{1}{3}\left(59.60+60.28+59.63+59.92+\frac{59.60\times 60.28-59.63\times 59.92}{59.60+60.28-59.63-59.92}\right)x_{43}$$

$$+\frac{1}{3}\left(76.60+76.62+76.95+76.64+\frac{76.60\times 76.62-76.95\times 76.64}{76.60+76.62-76.95-76.64}\right)x_{44}$$

$$Min\ Z_2 =$$

$$\frac{1}{3}\left(221.29+221.30+221.63+221.64+\frac{221.29\times 221.30-221.63\times 221.64}{221.29+221.30-221.63-221.64}\right)x_{11}$$
$$+\frac{1}{3}\left(245.50+245.80+245.82+246.15+\frac{245.50\times 245.80-245.82\times 246.15}{245.50+245.80-245.82-246.15}\right)x_{12}$$
$$+\frac{1}{3}\left(214.29+214.30+214.63+214.64+\frac{214.29\times 214.30-214.63\times 214.64}{214.29+214.30-214.63214.64}\right)x_{13}$$
$$+\frac{1}{3}\left(228.60+228.92+229.26+229.60+\frac{228.60\times 228.92-229.26\times 229.60}{228.60+228.92-229.26-229.60}\right)x_{14}$$
$$+\frac{1}{3}\left(265.60+265.93+265.63+265.00+\frac{265.60\times 265.93-265.63\times 265.00}{265.60+265.93-265.63-265.00}\right)x_{21}$$
$$+\frac{1}{3}\left(253.29+253.61+253.95+253.64+\frac{253.29\times 253.61-253.95\times 253.64}{253.29+253.61-253.95-253.64}\right)x_{22}$$
$$+\frac{1}{3}\left(233.80+233.82+233.47+234.48+\frac{233.80\times 233.82-233.47\times 234.48}{233.80+233.82-233.47-234.48}\right)x_{23}$$
$$+\frac{1}{3}\left(258.29+258.00+258.01+258.32+\frac{258.29\times 258.00-258.01\times 258.32}{258.29+258.00-258.01-258.32}\right)x_{24}$$
$$+\frac{1}{3}\left(280.80+280.50+280.82+281.15+\frac{280.80\times 280.50-280.82\times 281.15}{280.80+280.50-280.82-281.15}\right)x_{31}$$
$$+\frac{1}{3}\left(286.29+286.00+285.68+286.09+\frac{286.29\times 286.00-285.68\times 286.09}{286.29+286.00-285.68-286.09}\right)x_{32}$$
$$+\frac{1}{3}\left(298.60+298.00+298.62+299.60+\frac{298.60\times 298.00-298.62\times 299.60}{298.60+298.00-298.62-299.60}\right)x_{33}$$
$$+\frac{1}{3}\left(291.29+291.31+291.63+291.60+\frac{291.29\times 291.31-291.63\times 291.60}{291.29+291.31-291.63-291.60}\right)x_{34}$$
$$+\frac{1}{3}\left(244.07+244.91+243.31+243.45+\frac{244.07\times 244.91-243.31\times 243.45}{244.07+244.91-243.31-243.45}\right)x_{41}$$
$$+\frac{1}{3}\left(287.62+287.59+287.09+289.01+\frac{287.62\times 287.59-287.09\times 289.01}{287.62+287.59-287.09-289.01}\right)x_{42}$$
$$+\frac{1}{3}\left(259.97+260.32+260.74+260.01+\frac{259.97\times 260.32+260.74\times 260.01}{259.97+260.32-260.74-260.01}\right)x_{43}$$
$$+\frac{1}{3}\left(275.98+276.08+276.58+276.96+\frac{275.98\times 276.08-276.58\times 276.96}{275.98+276.08-276.58-276.96}\right)x_{44}$$

Subject to;

$$x_{11}+x_{12}+x_{13}+x_{14}=1$$
$$x_{21}+x_{22}+x_{23}+x_{24}=1$$
$$x_{31}+x_{32}+x_{33}+x_{34}=1$$
$$x_{41}+x_{42}+x_{43}+x_{44}=1$$
$$x_{11}+x_{21}+x_{31}+x_{41}=1$$
$$x_{12}+x_{222}+x_{32}+x_{42}=1$$
$$x_{13}+x_{23}+x_{33}+x_{43}=1$$
$$x_{14}+x_{24}+x_{34}+x_{44}=1$$

All parameters are in T2TpFNs and are translated to a crisp value using the procedure described above. The crisp value is presented in Table 4 for each objective function, repetitively. After using the crisp value the equivalent crisp MOAP can be defined as follows:

Using these crisp values of the manufacturing costs and manufacturing times which are obtained by using the two-stage defuzzification process, the present MOAP can be expressed as Model 7:
Model 7

$$
\begin{aligned}
Min\, Z_1 &= 41.73x_{11} + 46.35x_{12} + 54.55x_{13} + 68.46x_{14} + 38.87x_{21} + 72.83x_{22} + 69.78x_{23} + 81.50x_{24} \\
&\quad + 93.93x_{31} + 86.12x_{32} + 99.82x_{33} + 64.52x_{34} + 62.73x_{41} + 38.79x_{42} + 59.94x_{43} + 76.72x_{44} \\
Min\, Z_2 &= 221.46x_{11} + 245.82x_{12} + 214.46x_{13} + 229.09x_{14} + 265.51x_{21} + 253.64x_{22} + 235.95x_{23} \\
&\quad + 258.20x_{24} + 280.81x_{31} + 286.31x_{32} + 298.94x_{33} + 291.41x_{34} + 243.96x_{41} + 288.11x_{42} \\
&\quad + 260.51x_{43} + 276.40x_{44}
\end{aligned}
$$

$$
\begin{aligned}
&Subject\ \ to; \\
&\quad x_{11} + x_{12} + x_{13} + x_{14} = 1; x_{21} + x_{22} + x_{23} + x_{24} = 1; x_{31} + x_{32} + x_{33} + x_{34} = 1 \\
&\quad x_{41} + x_{42} + x_{43} + x_{44} = 1; x_{11} + x_{21} + x_{31} + x_{41} = 1; x_{12} + x_{222} + x_{32} + x_{42} = 1 \\
&\quad x_{13} + x_{23} + x_{33} + x_{43} = 1; x_{14} + x_{24} + x_{34} + x_{44} = 1
\end{aligned}
$$

Now, each objective function is minimized subject to the given set of constraints by ignoring the other objective function. This provides the minimum value of each objective function and the corresponding value (written as Max) of the other one. These values are depicted in Table 5, which is called the Payoff matrix.

Table 5. Payoff matrix

	Z_1 (Total manufacturing cost)	Z_2 (Total manufacturing time)
Max	287.07	1059.49
Min	196	995.31

Thus, the following inequalities hold for each objective function

$$196 \le Z_1 \le 287.07, \qquad 995.31 \le Z_2 \le 1059.49$$

8. Results and Discussion

The above MOAP is solved using three solution techniques viz., NCPT, FPT and IFPT. The best compromise solution obtained by each of these methods is given in Table 6.
1. While solving Model 7 using NCPT, we find each objective function's upper and lower bounds by solving them separately, subject to the given constraints. Then, we designed the linear membership functions for truth, indeterminacy and falsehood, respectively and maximized the truth and indeterminacy value and minimized the false value. Using Model 4 and LINGO 16.0 optimization software, we obtained the optimal solution of Model 7 as

$$
\begin{aligned}
&x_{11} = 0, x_{12} = 0,\ x_{13} = 1, x_{14} = 0\ \ x_{21} = 0,\ x_{22} = 1, x_{23} = 0, x_{24} = 0\ \ x_{31} = 0,\ x_{32} = 0,\ x_{33} = 0, x_{34} = 1, \\
&x_{41} = 0, x_{42} = 0, x_{43} = 0, x_{44} = 1,\ \gamma = 0.98546,\ Z_1 = 227.04,\quad Z_2 = 1003.05.
\end{aligned}
$$

2. While solving Model 7 using FPT, we designed the linear membership functions of both the objectives and maximized them. Using Model 5 and LINGO 16.0 optimization software, we obtained the optimal solutions of Model 7 as

$$x_{11}=1,\ x_{12}=0,\ x_{13}=0, x_{14}=0\ x_{21}=0,\ x_{22}=1, x_{23}=0, x_{24}=0\ x_{31}=0,\ x_{32}=0,\ x_{33}=0, x_{34}=1,$$
$$x_{41}=0, x_{42}=0, x_{43}=1, x_{44}=0.\ \gamma=0.5, \qquad Z_1=239.02, \quad Z_2=1027.02$$

3. While solving Model 7 using IFPT, we first designed the linear membership and non-membership functions and then maximized the membership function and minimized the non-membership function. Using Model 6 and LINGO 16.0 optimization software, we obtained the optimal solution of Model 7 as

$$x_{11}=1,\ x_{12}=0,\ x_{13}=0, x_{14}=0\ x_{21}=0,\ x_{22}=1, x_{23}=0, x_{24}=0\ x_{31}=0,\ x_{32}=0,\ x_{33}=0, x_{34}=1,$$
$$x_{41}=0, x_{42}=0, x_{43}=1, x_{44}=0.\ \gamma=0.5, \qquad Z_1=239.02, \quad Z_2=1027.02$$

From Table 6, we can easily conclude that the optimal solution of the present MOAP derived from the technique NCPT is more desirable and therefore, NCPT is a more suitable technique than the FPT and IFPT. This is due to the same reason that the fuzzy and the intuitionistic fuzzy logics are based on the truth function only, however, in real-world decision-making problems, the decision may result in the form of agreement, disagreement or the state of being unsure. Since the concept of neutrosophy allows the decision-makers to consider all these aspects together, NCPT performed better than the other techniques for the present MOAPs. Thus, the main advantage of the present study on MOAPs over existing literature is to solve the problem by considering degrees of truthness, falsehood, and indeterminacy altogether which may help the decision-maker make a better and more realistic decision. From Table 6, it is concluded that the best compromise solution of the present MOAP given by NCPT, provides the total manufacturing cost as 224.04 $ and the total manufacturing time of all the semi-finished parts as 1003.05 mins. To be more precise, a graphical representation of the compromise optimal solutions of the present MOAP, extracted from different solution approaches is given in Figure 2.

Table 6. Optimal solutions obtained by NCPT, FPT and IFPT

Objective functions / **Decision variables**	**NCPT**	**FPT**	**IFPT**
Min Z_1	227.04	239.02	239.02
Min Z_2	1003.05	1027.02	1027.02
x_{11}	0	1	1
x_{12}	0	0	0
x_{13}	1	0	0
x_{14}	0	0	0
x_{21}	0	0	0
x_{22}	1	1	1
x_{23}	0	0	0
x_{24}	0	0	0

x_{31}	0	0	0
x_{32}	0	0	0
x_{33}	0	0	0
x_{34}	1	1	1
x_{41}	0	0	0
x_{42}	0	0	0
x_{43}	0	1	1
x_{44}	1	0	0

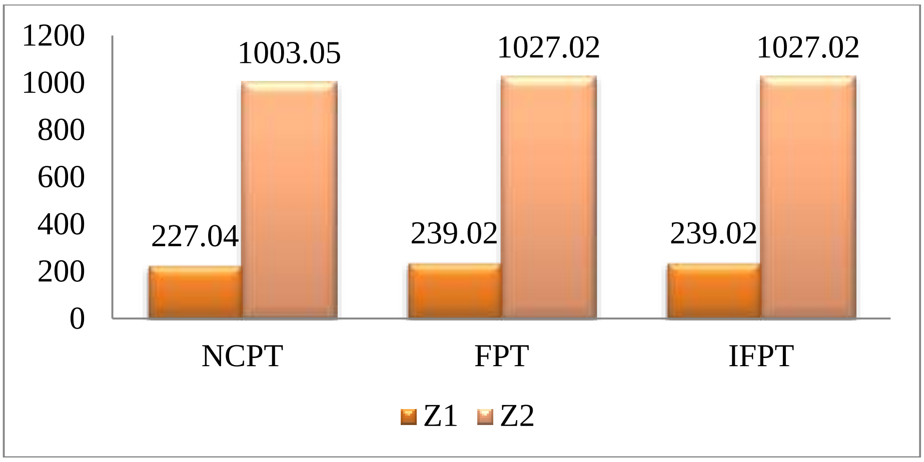

Figure 2. Comparison of objective values obtained from NCPT, FPT & IFPT

9. Advantage and Comparison of the Proposed Work with Some Existing Ones in Literature

The present problem is a MOAP with conflicting objectives which is discussed under fuzziness. This formulation of an assignment problem caters to a more realistic scenario arising in various commercial situations with vague information.

Further, in the study of MOAP under uncertainty, most of the authors like Biswas and Pramanik [9], Huang and Zhang [25], Jose and Kuriakose [27], Lin and Wen [36], Liu and Goa [38], Majumdar and Bhunia[41] and Thorani and Shankar [68] have used the concept of Type 1 fuzzy set (TIFS) whose membership functions are expressed as absolute numbers. The T1FS, in general, cannot handle the vagueness of the parameters efficiently as its membership functions are crisp. In contrast to this, Type 2 fuzzy sets (T2FS) can model the uncertainties/vagueness of optimization problems more appropriately as its membership functions are also presented as fuzzy numbers. To be more precise, the membership functions of T1FS are two-dimensional whereas the membership functions of T2FS are three-dimensional. This additional degree of freedom makes it possible to model the vagueness/uncertainties of an optimization problem more efficiently. So, the formulation of the present problem with T2TpF parameters is another advantage of the present study.

Furthermore, De and Yadav [14], Mukherjee and Basu [44], Pramanik and Biswas [54] and Sakawa et al. [60] are some of the authors who discussed assignment problems in an uncertain environment and either used fuzzy programming techniques or used the intuitionistic fuzzy programming techniques. The disadvantage of these techniques is that they can only handle information in the context of membership and/or non-membership function of a parameter but not the information related to indeterminacy or inconsistency in the parameter values. The neutrosophic approach discussed in this paper overcomes this limitation. In its theory, indeterminacy is quantified directly while the truth, indeterminacy and falsehood membership functions are independent. Since the present MOAP under uncertainty with T2TpF parameters is discussed using neutrosophic logic, this may be considered as another advantage of the present problem over existing literature. The efficiency of this technique over the existing ones reflects in Table 6.

10. Conclusion and Future Aspects

The current paper uses neutrosophic logic to solve MOAP in an uncertain environment. T2TpFNs are used to represent all of the uncertain parameters of the MOAP. The model is then crisped using a two-stage defuzzification procedure that finds the crisp values of these T2TpFNs. This crisp model is solved by using three solution techniques viz., FPT, IFPT and NCPT. The primary goal of this work is to solve the MOAP utilising NCPT and demonstrate its superiority over the others techniques described above. A numerical demonstration is shown that clearly shows that the NCPT outperforms the other two solution strategies that are also capable of dealing with uncertainty.

The concept of neutrosophic may be included into a multiobjective transportation model in future study. A MOAP's stochastic model may also be explored and solved using NCPT. Fuzzy-random or fuzzy-stochastic variations of a multiobjective assignment or transportation issue are also possibilities. Furthermore, the NCPT may be used in a variety of domains such as management science, financial management, and decision-making science, among others.

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