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Minimal Structures and Grill in Neutrosophic Topological Spaces

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Abstract: The main objective of this article is to introduce the notion of minimal structure (in short M-structure) and grill in neutrosophic topological space (in short N-T-space). Besides, we establish its relation with some existing notions on different types of open sets in N-T-space, and investigate some basic properties of the class of M-structure and grill in NT-space. Further, we furnish some suitable examples of M-structure and grill via NT-space.

Keywords: *Neutrosophic Set; M-structure; Neutrosophic Topology; Neutrosophic Grill Topology.*

1. Introduction: The concept of fuzzy set (in short F-set) theory was grounded by Zadeh [29] in 1965. Uncertainty plays an important role in our everyday life problems. Zadeh [29] associated the membership value with the elements to control the uncertainty. It was not sufficient to control uncertainty, so Atanaosv [3] added non-membership value along with the membership value, and introduced the notion of intuitionistic fuzzy set (in short IF-set). Still it was difficult to handle some real world problems under uncertainty, in particular for problems on decision making. In order to overcome this difficulty, Smarandache [24] considered the elements with truth-membership, indeterminacy-membership and false-membership values, and grounded the idea of neutrosophic set (in short N-set) theory in 1998. Till now, the concept of N-set has been applied in many branches of science and technology.

The notions of N-T-space was first grounded by Salama and Alblowi [22], followed by Salama and Alblowi [23], Iswaraya and Bageerathi [15], who studied the concept of neutrosophic semi-open set (in short NSO-set) and neutrosophic semi-closed set (in short NSC-set). Arokiarani et al. [2] studied some new notations and mappings via N-T-spaces. Iswarya and Bageerathi [15] established

the notion of neutrosophic semi-open sets via neutrosophic topological space. Afterwards, Rao and Srinivasa [21] introduced and studied neutrosophic pre-open set (in short NPO-set) and neutrosophic pre-closed set (in short NPC-set) via N-T-space. Das and Pramanik [7] grounded the notion of generalized neutrosophic b -open set via neutrosophic topological space. Later on, Das and Pramanik [8] introduced the notion of neutrosophic ϕ -open set and neutrosophic ϕ -continuous function via neutrosophic topological space. Recently, Das and Tripathy [11] presented the notion of neutrosophic simply b -open set via neutrosophic topological space. Thereafter, Das and Tripathy [13] introduced the idea of pairwise neutrosophic b -open sets via N-T-space. Parimala et al. [19] introduced the notion of neutrosophic nano ideal topological space. Later on Parimala et al. [20] grounded the idea of neutrosophic $\alpha\psi$ -homomorphism via neutrosophic topological spaces.

Makai et al. [16] introduced and studied the concept of minimal structure (in short M-structure) via topological space. It is found to have useful applications and the notion was investigated by Madok [17]. The notion of minimal structure in a fuzzy topological space was introduced by Alimohammady and Roohi [1] and further investigated by Tripathy and Debnath [27] and others.

In this article, we introduce the idea of minimal structure and grill via N-T-spaces. We establish its relation with some existing notions on several types of open sets via N-T-spaces. Besides, we investigate some basic properties of the class of minimal structures and grill via N-T-spaces. Further, we furnish some suitable examples on minimal structures and grill via N-T-spaces.

The rest of the paper is divided into following sections:

In section 2, we provide some definitions and results those are very useful for the preparation of the main results of this article. In section 3, we introduce the concept of M-structure and grill via NT-spaces, and proved some basic results on them. In section 4, we introduce an operator $()^*$ on M-structure via NT-spaces, and established several interesting results based on it. Finally, in section 5, we conclude the work done in this article.

2. Preliminaries & Definitions:

In this section, we provide some existing results on neutrosophic set and neutrosophic topology those are relevant to main results of this article.

Definition 2.1.[24] Suppose that \hat{G} be a fixed set. Then W , an N-set over \hat{G} is a set contains triplet having truth, indeterminacy and false membership values that can be characterized independently, denoted by T_w, I_w, F_w in the unit interval $[0, 1]$.

We denote the N-set W as follows:

$$W = \{(r, T_w(r), I_w(r), F_w(r)) : r \in \hat{G}\}, \text{ where } T_w(r), I_w(r), F_w(r) \in [0, 1], \text{ for all } r \in \hat{G}.$$

Since, no restriction on the values of $T_w(r), I_w(r)$ and $F_w(r)$ is imposed, so we have

$$0 \leq T_w(r) + I_w(r) + F_w(r) \leq 3, \text{ for all } r \in \hat{G}.$$

Example 2.1. Suppose that $\hat{G}=\{b, c\}$ be a fixed set. Clearly, $W=\{(b,0.4,0.8,0.7), (c,0.2,0.8,0.7)\}$ is an N-set over \hat{G} .

Definition 2.2.[24] Suppose that $W = \{(r, T_W(r), I_W(r), F_W(r)): r \in \hat{G}\}$ be an N-set over \hat{G} . Then, W^c i.e., the complement of W is defined by $W^c = \{(r, 1-T_W(r), 1-I_W(r), 1-F_W(r)): r \in \hat{G}\}$.

Example 2.2. Suppose that $\hat{G}=\{b, c\}$ be a fixed set. Let $W=\{(b,0.5,0.5,0.7), (c,0.5,0.7,0.8)\}$ be an N-set over \hat{G} . Then, the complement of W is $W^c=\{(b,0.5,0.5,0.3), (c,0.5,0.3,0.2)\}$.

Definition 2.3.[24] An N-set $W = \{(r, T_W(r), I_W(r), F_W(r)): r \in \hat{G}\}$ is called a subset of an N-set $L = \{(r, T_L(r), I_L(r), F_L(r)): r \in \hat{G}\}$ (i.e., $W \subseteq L$) if and only if $T_W(r) \leq T_L(r), I_W(r) \geq I_L(r), F_W(r) \geq F_L(r)$, for each $r \in \hat{G}$.

Example 2.3. Suppose that $\hat{G}=\{b, c\}$ be a fixed set. Let $M=\{(b,0.5,0.5,0.7), (c,0.5,0.7,0.8)\}$ and $W=\{(b,0.7,0.2,0.5), (c,0.9,0.5,0.3)\}$ be two N-sets over \hat{G} . Clearly, $M \subseteq W$.

Definition 2.4.[24] Assume that $W = \{(r, T_W(r), I_W(r), F_W(r)): r \in \hat{G}\}$ and $L = \{(r, T_L(r), I_L(r), F_L(r)): r \in \hat{G}\}$ be any two N-sets over a fixed set \hat{G} . Then, their intersection and union are defined as follows:

$$W \cap L = \{(r, T_N(r) \wedge T_L(r), I_N(r) \vee I_L(r), F_N(r) \vee F_L(r)): r \in \hat{G}\},$$

$$W \cup L = \{(r, T_N(r) \vee T_L(r), I_N(r) \wedge I_L(r), F_N(r) \wedge F_L(r)): r \in \hat{G}\}.$$

Example 2.4. Suppose that $\hat{G}=\{b, c\}$ be a fixed set. Let $W=\{(b,0.5,0.5,0.7), (c,0.5,0.7,0.8)\}$ and $M=\{(b,0.7,0.2,0.5), (c,0.9,0.5,0.3)\}$ be two N-sets over \hat{G} . Then, $W \cup M=\{(b,0.7,0.2,0.5), (c,0.9,0.5,0.3)\}$ and $W \cap M=\{(b,0.5,0.5,0.7), (c,0.5,0.7,0.8)\}$.

Definition 2.5.[22] The null N-set (0_N) and absolute N-set (1_N) over \hat{G} are represented as follows:

(i) $0_N = \{(r, 0, 0, 1) : r \in \hat{G}\};$

(ii) $0_N = \{(r, 0, 1, 0) : r \in \hat{G}\};$

(iii) $0_N = \{(r, 0, 1, 1) : r \in \hat{G}\};$

(iv) $0_N = \{(r, 0, 0, 0) : r \in \hat{G}\};$

(v) $1_N = \{(r, 1, 0, 1) : r \in \hat{G}\};$

(vi) $1_N = \{(r, 1, 1, 0) : r \in \hat{G}\};$

(vii) $1_N = \{(r, 1, 0, 0) : r \in \hat{G}\};$

(viii) $1_N = \{(r, 1, 1, 1) : r \in \hat{G}\}.$

Clearly, $0_N \subseteq 1_N$. We have, for any N-set W , $0_N \subseteq W \subseteq 1_N$.

Throughout the article, we will use $0_N = \{(r, 0, 1, 1) : r \in \hat{G}\}$ and $1_N = \{(r, 1, 0, 0) : r \in \hat{G}\}$.

Definition 2.6.[22] Assume that \hat{G} be a fixed set. Then τ , a family of N-sets over \hat{G} is called an N-T on \hat{G} if the following condition holds:

- (i) $0_N, 1_N \in \tau$;
- (ii) $W_1, W_2 \in \tau \Rightarrow W_1 \cap W_2 \in \tau$;
- (iii) $\{W_i : i \in \Delta\} \subseteq \tau \Rightarrow \cup_{i \in \Delta} W_i \in \tau$.

Then, the pair (\hat{G}, τ) is called an N-T-space. If $W \in \tau$, then W is called an neutrosophic open set (in short N-O-set) in (\hat{G}, τ) , and the complement of W is called an neutrosophic closed set (in short N-C-set) in (\hat{G}, τ) .

Example 2.5. Suppose that W, E and Z be three N-sets over a fixed set $\hat{G} = \{p, q\}$ such that:

$$W = \{(p, 0.7, 0.5, 0.7), (q, 0.5, 0.1, 0.5) : p, q \in \hat{G}\};$$

$$E = \{(p, 0.6, 0.9, 0.8), (q, 0.5, 0.3, 0.8) : p, q \in \hat{G}\};$$

$$Z = \{(p, 0.5, 1.0, 0.8), (q, 0.4, 0.4, 0.9) : p, q \in \hat{G}\}.$$

Then, the collection $\tau = \{0_N, 1_N, W, E, Z\}$ forms a neutrosophic topology on \hat{G} . Here, $0_N, 1_N, W, E, Z$ are NOSs in (\hat{G}, τ) , and their complements $1_N, 0_N, W^c = \{(p, 0.3, 0.5, 0.3), (q, 0.5, 0.9, 0.5) : p, q \in \hat{G}\}$, $E^c = \{(p, 0.4, 0.1, 0.2), (q, 0.5, 0.7, 0.2) : p, q \in \hat{G}\}$ and $Z^c = \{(p, 0.5, 0.0, 0.2), (q, 0.6, 0.6, 0.1) : p, q \in \hat{G}\}$ are NCSs in (\hat{G}, τ) .

The neutrosophic interior and neutrosophic closure of an N-set are defined as follows:

Definition 2.7.[22] Assume that τ be an N-T on \hat{G} . Suppose that W be an N-set over \hat{G} . Then,

- (i) neutrosophic interior (in short N_{int}) of W is the union of all N-O-sets in (\hat{G}, τ) those are contained in W , i.e., $N_{int}(W) = \cup \{E : E \text{ is an N-O-set in } \hat{G} \text{ such that } E \subseteq W\}$;
- (ii) neutrosophic closure (in short N_{cl}) of W is the intersection of all N-C-sets in (\hat{G}, τ) those containing W , i.e., $N_{cl}(W) = \cap \{F : F \text{ is an N-C-set in } \hat{G} \text{ such that } W \subseteq F\}$.

Clearly $N_{int}(W)$ is the largest N-O-set contained in W , and $N_{cl}(W)$ is the smallest N-C-set containing W .

Example 2.6. Suppose that (\hat{G}, τ) be an NT-space as shown in **Example 2.5**. Suppose that $U = \{(p, 0.5, 0.5, 0.7), (q, 0.5, 0.7, 0.8)\}$ be an N-set over \hat{G} . Then, $N_{int}(U) = 0_N$ and $N_{cl}(U) = \{(p, 0.5, 0.0, 0.2), (q, 0.6, 0.6, 0.1)\}$.

Proposition 2.1.[22] For any N-set B in (\hat{G}, τ) , we have the following:

- (i) $N_{int}(B^c) = (N_{cl}(B))^c$;
- (ii) $N_{cl}(B^c) = (N_{int}(B))^c$.

Definition 2.2.[21] Suppose that (\hat{G}, τ) be an N-T-space, and W be an N-set over \hat{G} . Then, W is called

- (i) NSO-set if and only if $W \subseteq N_{cl}(N_{int}(W))$;

(ii) NPO-set if and only if $W \subseteq N_{int}(N_{cl}(W))$.

The collection of all NSO sets and NPO sets in (\hat{G}, τ) are denoted by $NSO(\tau)$ and $NPO(\tau)$.

Example 2.7. Suppose that $\hat{G}=\{a, b\}$ be a fixed set. Clearly, (\hat{G}, τ) is an NT-space, where $\tau=\{0_N, 1_N, \{(a, 0.3, 0.3, 0.4), (b, 0.4, 0.4, 0.3): a, b \in \hat{G}\}, \{(a, 0.4, 0.1, 0.4), (b, 0.5, 0.3, 0.1): a, b \in \hat{G}\}\}$. Then, the N-set $Q=\{(a, 0.6, 0.1, 0.4), (b, 0.9, 0.2, 0.1): a, b \in \hat{G}\}$ is an NSO set, and $P=\{(a, 0.3, 0.2, 0.9), (b, 0.3, 0.3, 0.4): a, b \in \hat{G}\}$ is an NPO set in (\hat{G}, τ) .

Definition 2.8.[2] Assume that (\hat{G}, τ) be an N-T-space. Then W , an N-set over \hat{G} is called an neutrosophic b -open set (in short N- b -O-set) in (\hat{G}, τ) if and only if $W \subseteq N_{int}(N_{cl}(W)) \cup N_{cl}(N_{int}(W))$.

An N-set G is called an neutrosophic b -closed set (in short N- b -C-set) in (\hat{G}, τ) if and only if its complement is an N- b -O-set in (\hat{G}, τ) . The collection of all neutrosophic b -open sets in (\hat{G}, τ) is denoted by N- b -O(τ).

Example 2.8. Suppose that (\hat{G}, τ) be an NT-space as shown in **Example 2.7**. Then, the neutrosophic set $P=\{(a, 0.3, 0.2, 0.9), (b, 0.3, 0.3, 0.4): a, b \in \hat{G}\}$ is an neutrosophic b -open set in (\hat{G}, τ) .

3. Minimal Structure in Neutrosophic Topological Space

In this section, we procure the notions of M-structure and grill in N-T-space.

Definition 3.1. A family M of neutrosophic subsets of \hat{G} i.e., $M \subset P(\hat{G})$, where $P(\hat{G})$ is the collection of all N-sets defined over \hat{G} is said to be a M-structure on \hat{G} if 0_N and 1_N belong to M . By (\hat{G}, M) , we denote the neutrosophic minimal space (in short N-M-space). The members of M are called neutrosophic minimal-open (in short N-m-O) subset of \hat{G} .

Example 3.1. Let W, E and Z be three neutrosophic sets over a fixed set $\hat{G}=\{b, c\}$ such that:

$$W=\{(b, 0.7, 0.5, 0.7), (c, 0.5, 0.1, 0.5): b, c \in \hat{G}\};$$

$$E=\{(b, 0.6, 0.9, 0.8), (c, 0.5, 0.3, 0.8): b, c \in \hat{G}\};$$

$$Z=\{(b, 0.5, 1.0, 0.8), (c, 0.4, 0.4, 0.9): b, c \in \hat{G}\}.$$

Clearly, the collection $M=\{0_N, 1_N, W, E, Z\}$ forms a neutrosophic minimal structure on \hat{G} , and the pair (\hat{G}, M) is a neutrosophic minimal structure space.

Definition 3.2. The complement of N-m-O set W is an neutrosophic m-closed set (in short N-m-C set) in (\hat{G}, M) .

Example 3.2. Let us consider a neutrosophic minimal structure space as shown in Example 3.1. Here, $0_N, 1_N, W, E, Z$ are neutrosophic minimal open sets in (\hat{G}, M) , and $(0_N)^c = 1_N, (1_N)^c = 0_N, W^c =$

$\{(b,0.3,0.5,0.3), (c,0.5,0.9,0.5)\}$, $E^c=\{(b,0.4,0.1,0.2), (c, 0.5,0.7,0.2)\}$, $Z^c = \{(b,0.5,0.0,0.2), (c,0.6,0.6,0.1)\}$ are neutrosophic minimal closed sets in (\hat{G}, M) .

Example 3.3. From the above definitions, it is clear that NPO-sets, NSO-sets, N- α -O-sets, N- b -O-sets are N-m-O sets.

Example 3.4. Let W, E and Z be three neutrosophic sets over a non-empty set $\hat{G}=\{b, c\}$ such that:

$$W=\{(b,0.7,0.5,0.7), (c,0.5,0.5,0.1) : b, c \in X\};$$

$$E=\{(b,0.6,0.8,0.9), (c,0.5,0.8,0.3) : b, c \in X\};$$

$$Z=\{(b,0.5,0.8,1.0), (c,0.4,0.9,0.4) : b, c \in X\}.$$

Here, the family $\tau=\{0_N, 1_N, W, E, Z\}$ forms a neutrosophic topology on X , and so (\hat{G}, τ) is a neutrosophic topological space. Suppose $M = \tau \cup \text{NPO}(\tau) \cup \text{NSO}(\tau) \cup \text{N-}b\text{-O}(\tau)$, then (\hat{G}, M) is a neutrosophic minimal structure. Now, from the above it is clear that, every neutrosophic pre-open sets, neutrosophic semi-open sets, neutrosophic b -open sets in (\hat{G}, τ) are neutrosophic m -open sets in (\hat{G}, M) . Further, it is also seen that, every neutrosophic m -open set in (\hat{G}, M) is also a neutrosophic pre-open set, neutrosophic semi-open set, neutrosophic b -open set in (\hat{G}, τ) .

Remark 3.1. We can define neutrosophic minimal interior (in short N_{mint}), neutrosophic minimal closure (in short N_{mcl}) etc. in an N-M-space as we have define in the previous section.

Example 3.5. Suppose that (\hat{G}, M) be a neutrosophic minimal structure space as shown in Example 3.1. Then, the neutrosophic minimal interior of $D=\{(b,0.2,0.6,0.4), (c,0.4,0.9,0.7)\}$ is $N_{m-int}(D)=\{(b,0,1,1), (c,0,1,1)\}$, and the neutrosophic minimal closure of $D=\{(b,0.2,0.6,0.4), (c,0.4,0.9,0.7)\}$ is $N_{m-cl}(D)=\{(b,0.3, 0.5,0.3), (c,0.5,0.9,0.5)\}$ respectively.

In view of the definitions given in this article, we state the following result without proof.

Theorem 3.1. Suppose that (\hat{G}, M) be an N-M-space. Then, for any N-sets S and R over \hat{G} , the following holds:

- (i) $(N_{mcl}(S))^c = N_{mcl}(S^c)$ and $(N_{mint}(S))^c = N_{mcl}(S^c)$.
- (ii) $N_{mcl}(S)=S$ if and only if S is an N-m-C set in (\hat{G}, M) .
- (iii) $N_{mint}(S)=S$ if and only if S is an N-m-O set in (\hat{G}, M) .
- (iv) $S \subseteq R \Rightarrow N_{mcl}(S) \subseteq N_{mcl}(R)$ and $N_{mint}(S) \subseteq N_{mint}(R)$.
- (v) $S \subseteq N_{mcl}(S)$ and $N_{mint}(S) \subseteq S$.
- (vi) $N_{mcl}(N_{mcl}(S))=N_{mcl}(S)$ and $N_{mint}(N_{mint}(S))=N_{mint}(S)$.

Theorem 3.2. Assume that (\hat{G}, M) be an N-M-space. Suppose that M satisfies the property B . Then, for an neutrosophic subset S of \hat{G} , the followings hold:

- (i) $S \in M$ if and only if $N_{\min}(S) = A$.
- (ii) S is N-m-C set if and only if $N_{\text{mcl}}(S) = S$.
- (iii) $N_{\min}(S) \in M$ and $N_{\text{mcl}}(S)$ is an N-m-C set.

Definition 3.3. An N-M-structure M on a non-empty set \hat{G} is said to be have property B if the union of only family of neutrosophic subsets belonging M belongs to M .

Example 3.6. Suppose that R, E and Y be three neutrosophic sets over a fixed set $\hat{G} = \{b, c\}$ such that:

$$R = \{(b, 0.8, 0.5, 0.8), (c, 0.6, 0.1, 0.6) : b, c \in \hat{G}\};$$

$$E = \{(b, 0.7, 0.9, 0.9), (c, 0.6, 0.3, 0.9) : b, c \in \hat{G}\};$$

$$Y = \{(b, 0.6, 1.0, 0.9), (c, 0.5, 0.4, 1.0) : b, c \in \hat{G}\}.$$

Here, the collection $M = \{0_N, 1_N, R, E, Y\}$ forms a N-M-structure on \hat{G} , and so the pair (\hat{G}, M) is a neutrosophic minimal structure space. Clearly, the N-M-structure on \hat{G} satisfied the property B which was stated in Definition 3.3.

Definition 3.4. An N-M-structure (\hat{G}, M) satisfies the property J if the finite intersection of N-m-O sets is an N-m-O set.

Example 3.7. Let us consider a N-M-structure M on \hat{G} as shown in Example 3.6. Clearly, the N-M-structure M on \hat{G} satisfied the property j which was stated in Definition 3.4.

Remark 3.2. If a N-M-structure M on \hat{G} is a neutrosophic topology on \hat{G} , then M satisfied the property j which was stated in Definition 3.4.

Definition 3.5. A family G ($0_N \notin G$) of N-sets over \hat{G} is called a grill on \hat{G} if G satisfies the following condition:

- (i) $S \in G$ and $S \subseteq R \Rightarrow R \in G$;
- (ii) $S, R \subseteq \hat{G}$ and $S \cup R \in G \Rightarrow S \in G$ or $R \in G$.

Example 3.8. Suppose that $\hat{G} = \{b, c\}$ be a fixed set. Then, the collection $M = \{1_N, \{(b, 0.9, 0.0, 0.0), (c, 0.9, 0.0, 0.0)\}, \{(b, 0.8, 0.0, 0.0), (c, 0.8, 0.0, 0.0)\}\}$ forms a grill on \hat{G} .

Remark 3.3. Since $0_N \notin G$, so G is not a N-M-structure on \hat{G} . An N-M-structure with a grill is called as a neutrosophic grill minimal space (in short N-G-M-space), denoted by (\hat{G}, M, G) .

4. (*)-Operator on Neutrosophic Minimal Structure:

Definition 4.1. Suppose that (\hat{G}, M, G) be an N-G-M-space. A function $(\)^{*m}: P(\hat{G}) \rightarrow P(\hat{G})$ called as neutrosophic minimal local function (N-M-L-function) is defined by

$$(\)^{*m} = \{x \in \hat{G}: S \cap U \in M, \text{ for all } U \in M(x)\}.$$

Now we discuss about some properties of the neutrosophic minimal local function $(\)^{*m}$ in (\hat{G}, M, G) .

Definition 4.2. Assume that (\hat{G}, M, G) be a N-G-M-space. Then, the boundary of a N-set S over \hat{G} is defined by $(\partial S)^{*m} = (S)^{*m} \cap (S^c)^{*m}$.

We state the following two results without prove in view of the above definitions.

Proposition 4.1. Suppose that (\hat{G}, M, G) be a N-G-M-space. Then, the following holds:

- (i) $(0_N)^{*m} = 0_N$;
- (ii) $(S)^{*m} = 0_N$, if $S \notin G$;
- (iii) $(S)^{*mP} \subseteq (S)^{*mQ}$, where P and Q are neutrosophic grill on \hat{G} with $P \subseteq Q$.

Proposition 4.2. Suppose that $P(\hat{G})$ be the collection of all neutrosophic sets defined over \hat{G} . Assume that (\hat{G}, M, G) be a N-G-M-space. Then, for $S \in P(\hat{G})$,

- (i) $(S)^{*m} = S \cup (\partial S)^{*m}$;
- (ii) $(S)^{*m} = \bigcap_{n \in \Delta} F_n$, where $\{F_n\}_{n \in \Delta}$ is the collection of all $(\)^{*m}$ -closed sets in (\hat{G}, M, G) .

Theorem 4.1. Assume that (\hat{G}, M, G) be a N-G-M-space. Then, the following holds:

- (i) $S, R \in P(\hat{G})$ and $S \subseteq R \Rightarrow (S)^{*m} \subseteq (R)^{*m}$;
- (ii) For $S \subseteq \hat{G}$, $N_{mcl}(S)^{*m} \subseteq (S)^{*m}$;
- (iii) For $S \subseteq \hat{G}$, $(S)^{*m}$ is a N-m-C set;
- (iv) For $S \subseteq \hat{G}$, $(S)^{*m} \subseteq N_{mcl}(S)$;
- (v) For $S \subseteq \hat{G}$, $[(S)^{*m}]^{*m} \subseteq (S)^{*m}$.

Proof. (i) Assume that $S \subseteq R$ and $x \in (S)^{*m}$. Then, for all $U \in M$, we have by definition, that $U \cap S \in G$. Thus by definition of neutrosophic grill we have $U \cap R \in G$. Hence, $x \in (R)^{*m}$. Therefore, $(S)^{*m} \subseteq (R)^{*m}$.

(ii) Assume that $x \notin N_{mcl}(S)$, for some $S \in \hat{G}$. Then by a known result there exists an $U_x \in M$ such that $U_x \cap S = 0_N \notin G$. Therefore, $x \notin (S)^{*m}$. Hence, we have $(S)^{*m} \subseteq N_{mcl}(S)$.

(iii) Assume that $x \in N_{mcl}(S)^{*m}$ and $U \in M(x)$, then $U \cap (S)^{*m} \neq 0_N$. Next, let $y \in U \cap (S)^{*m}$. Then, we have $y \in U$ and $y \in (S)^{*m}$. Therefore, $U \cap S \in G$, which implies $x \in (S)^{*m}$.

Thus, we have $N_{mcl}(S)^{*m} \subseteq (S)^{*m}$.

(iv) Suppose that $S \in P(\hat{G})$. Then, we have $(S)^{*m} \subseteq N_{mcl}(S)^{*m}$ and $N_{mcl}(S)^{*m} = (S)^{*m}$. Thus, we have for any $(S)^{*m} = N_{mcl}(S)^{*m}$, since $N_{mcl}(S)^{*m}$ is an N-m-C set, so $(S)^{*m}$ is an N-m-C set.

(v) Suppose that $S \in P(\hat{G})$. Then from (iv) we have $N_{mcl}(S)^{*m} \subseteq (S)^{*m}$. Further on considering $(S)^{*m}$ is place of S , from (vi) we have $((S)^{*m})^m \subseteq N_{mcl}(S)^{*m}$. Hence, from these two inclusion we have $((S)^{*m})^m \subseteq (S)^{*m}$.

Theorem 4.2. Assume that (\hat{G}, M, G) be a N-G-M-space. Suppose that (\hat{G}, M) satisfies the property J. Then, the following holds:

(i) $(S \cup R)^{*m} = (S)^{*m} \cup (R)^{*m}$, for $S, R \subseteq M$;

(ii) $(S \cap R)^{*m} \subseteq (S)^{*m} \cap (R)^{*m}$, for $S, R \subseteq M$.

Proof. (i) We have $S \subseteq S \cup R, R \subseteq S \cup R$. Thus, we have $(S)^{*m} \subseteq (S \cup R)^{*m}$ and $(R)^{*m} \subseteq (S \cup R)^{*m}$. This implies, $(S)^{*m} \cup (R)^{*m} \subseteq (S \cup R)^{*m}$ (1)

Suppose that $x \notin (S)^{*m} \cup (R)^{*m}$. Therefore there exists $U_1, U_2 \in M(x)$ such that $U_1 \cap S \notin G, U_2 \cap R \notin G$. This implies, $(U_1 \cap S) \cup (U_2 \cap R) \notin G$.

Now, $U_1, U_2 \in M(x) \Rightarrow U_1 \cap U_2 \in M(x)$ and $(S \cup R) \cap (U_1 \cap U_2) \subseteq (U_1 \cap S) \cup (U_2 \cap R) \notin G$.

Therefore, $x \notin (S \cup R)^{*m}$. Thus, we have $(S \cup R)^{*m} \subseteq (S)^{*m} \cup (R)^{*m}$ (2)

From (1) and (2) we have, $(S \cup R)^{*m} = (S)^{*m} \cup (R)^{*m}$.

(ii) We have $S \cap R \subseteq S$ and $S \cap R \subseteq R$. This implies, $(S \cap R)^{*m} \subseteq (S)^{*m}$ and $(S \cap R)^{*m} \subseteq (R)^{*m}$. Therefore, $(S \cap R)^{*m} \subseteq (S)^{*m} \cap (R)^{*m}$.

Theorem 4.3. Suppose that (\hat{G}, M, G) be a N-G-M-space. Assume that (\hat{G}, M) satisfies the property J. Then, the following holds:

(i) For $W \subseteq M$ and $S \subseteq \hat{G}$, $W \cap (S)^{*m} = W \cap (W \cap S)^{*m}$;

(ii) For $S, R \subseteq \hat{G}$, $[(S)^{*m} \setminus (R)^{*m}] = [(S \setminus R)^{*m} \setminus (R)^{*m}]$;

(iii) For $S, R \subseteq \hat{G}$, with $R \notin G$. $(S \cup R)^{*m} = (S)^{*m} = (S \setminus R)^{*m}$.

Proof. (i) It is known that $(W \cap S) \subseteq S$.

Now, $(W \cap S) \subseteq S$

$\Rightarrow (W \cap S)^{*m} \subseteq (S)^{*m}$

$\Rightarrow W \cap (W \cap S)^{*m} \subseteq W \cap (S)^{*m}$ (3)

Assume that $x \in W \cap (S)^{*m}$ and $V \in M(x)$.

Then, we have $W \cap V \in M(x)$ and $x \in (S)^{*m}$ implies $(W \cap V) \cap S \in G$.

Thus, $(W \cap S) \cap V \in G$. Thus, we have $x \in (W \cap S)^{*m}$, which implies $x \in W \cap (W \cap S)^{*m}$.

Hence, $(W \cap S)^{*m} \subseteq W \cap (W \cap S)^{*m}$ (4)

From (3) and (4), we have $W \cap (S)^{*m} = W \cap (W \cap S)^{*m}$.

(ii) We have,

$$\begin{aligned} (S)^{*m} &= [(S \setminus R) \cup (S \cap R)]^m \\ &= (S \setminus R)^m \cup (S \cap R)^m \quad [\text{by part (i)}] \\ &\subseteq [(S \setminus R)^m \cup (R)^m]. \end{aligned}$$

Thus, we have $[(S)^{*m} \setminus (R)^{*m}] \subseteq [(S \setminus R)^{*m} \cup (R)^{*m}]$.

We have, $S \setminus R \subseteq S \Rightarrow (S \setminus R)^{*m} \subseteq (S)^{*m}$. This implies, $[(S \setminus R)^{*m} \setminus (R)^{*m}] \subseteq [(S)^{*m} \setminus (R)^{*m}]$.

Hence, we have $[(S)^{*m} \setminus (R)^{*m}] = [(S \setminus R)^{*m} \setminus (R)^{*m}]$.

(iii) By Theorem 4.4 (i), we have for $S, R \subseteq \hat{G}$, $(S \cup R)^{*m} = (S)^{*m} \cup (R)^{*m}$.

Further the earlier result we have $R \notin G$ implies $(R)^{*m} = 0_N$, so $(S)^{*m} \cup (R)^{*m} = (S)^{*m}$.

We have, $S \setminus R \subseteq S \Rightarrow (S \setminus R)^{*m} \subseteq (S)^{*m}$, by part (iii) we have,

$$[(S)^{*m} \setminus (R)^{*m}] \subseteq (S \setminus R)^{*m}, \text{ since } (R)^{*m} = 0_N \text{ implies } (S)^{*m} \subseteq (S \setminus R)^{*m}.$$

Thus, we have $(S)^{*m} = (S \setminus R)^{*m}$.

Definition 4.3. Suppose that (\hat{G}, M, G) be an N-G-M-space. Then, the mapping $N_{clmG}: P(\hat{G}) \rightarrow P(\hat{G})$ is define by $N_{clmG}(S) = S \cup (S)^{*m}$, for all $S \in P(\hat{G})$.

Theorem 4.4. The mapping $N_{clmG}: P(\hat{G}) \rightarrow P(\hat{G})$ satisfies the Kuratowski closure axioms.

Proof. We have, $N_{clmG}(0_N) = 0_N \cup (0_N)^{*m} = 0_N \cup 0_N = 0_N$. By the definition of N_{clmG} , we have for all $S \in P(\hat{G})$, $N_{clmG}(S) = S \cup (S)^{*m} \supseteq S$.

Further, we have

$$\begin{aligned} N_{clmG}(S \cup R) &= (S \cup R) \cup (S \cup R)^{*m} \\ &= ((S \cup R) \cup ((S)^{*m} \cup (R)^{*m})) && \text{[by Theorem 4.3]} \\ &= (S \cup (S)^{*m}) \cup (R \cup (R)^{*m}) = N_{clmG}(S) \cup N_{clmG}(R). \end{aligned}$$

Suppose that $S \in P(\hat{G})$. Then, we have

$$\begin{aligned} N_{clmG}(N_{clmG}(S)) &= N_{clmG}(S \cup (S)^{*m}) \\ &= [(S \cup (S)^{*m})] \cup [(S \cup (S)^{*m})^*m] \\ &= [(S \cup (S)^{*m})] \cup [(S)^{*m} \cup ((S)^{*m})^*m] && \text{[by Theorem 4.3]} \\ &= S \cup (S)^{*m} \end{aligned}$$

Hence, the mapping N_{clmG} satisfies the Kuratowski closure axioms.

5. Conclusions: In this article, we have introduced the notion of minimal structure and grill via N-T-spaces. Besides, we have established its relation with some existing notions on several types of open sets via N-T-spaces, and investigated some basic properties of the class of minimal structures and grill via N-T-spaces. Further, we have furnished some suitable examples on minimal structures and grill via N-T-spaces. In the future, it is hoped that the notion of minimal structures and grill on N-T-spaces can also be applied in neutrosophic supra topological space [5], quadripartitioned neutrosophic topological space [6], pentapartitioned neutrosophic topological space [12], neutrosophic bitopological space [18], neutrosophic tri-topological space [10], neutrosophic soft topological space [9], neutrosophic multiset topological space [14], multiset mixed topological space [4], etc.

References:

1. Alimohammady, S., & Roohi, M. (2006). Fuzzy minimal structure and fuzzy minimal vector space. *Chaos, Solotons & Fractals*, 27, 599-605.
2. Arokiarani, I., Dhavaseelan, R., Jafari S., & Parimala M. (2017). On some new notations and functions in neutrosophic topological spaces. *Neutrosophic Sets Systems*, 16, 16-19.
3. Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, 20, 87-96.
4. Tripathy, B.C., & Das, R. (In Press). Multiset Mixed Topological Space. *Transactions of A. Razmadze Mathematical Institute*.
5. Das, S. (2021). Neutrosophic Supra Simply Open Set and Neutrosophic Supra Simply Compact Space. *Neutrosophic Sets and Systems*, 43, 105-113. DOI: 10.5281/zenodo.4914821
6. Das, S., Das, R., & Granados, C. (2021). Topology on Quadripartitioned Neutrosophic Sets. *Neutrosophic Sets and Systems*, 45, 54-61.
7. Das, S., & Pramanik, S. (2020). Generalized neutrosophic b -open sets in neutrosophic topological space. *Neutrosophic Sets and Systems*, 35, 522-530.
8. Das, S., & Pramanik, S. (2020). Neutrosophic Φ -open sets and neutrosophic Φ -continuous functions. *Neutrosophic Sets and Systems*, 38, 355-367.
9. Das, S., & Pramanik, S. (2020). Neutrosophic simply soft open set in neutrosophic soft topological space. *Neutrosophic Sets and Systems*, 38, 235-243.
10. Das, S., & Pramanik, S. (2021). Neutrosophic Tri-Topological Space. *Neutrosophic Sets and Systems*, 45, 366-377.
11. Das, S., & Tripathy, B.C. (In Press). Neutrosophic simply b -open set in neutrosophic topological spaces. *Iraqi Journal of Science*.
12. Das, S., & Tripathy, B.C. (2021). Pentapartitioned Neutrosophic Topological Space. *Neutrosophic Sets and Systems*, 45, 121-132.
13. Das, S., & Tripathy, B.C. (2020). Pairwise neutrosophic- b -open set in neutrosophic bitopological spaces. *Neutrosophic Sets and Systems*, 38, 135-144.
14. Das R., & Tripathy, B.C. (2020). Neutrosophic multiset topological space. *Neutrosophic Sets and Systems*, 35, 142-152.
15. Iswarya, P., & Bageerathi, K. (2016). On Neutrosophic semi-open sets in Neutrosophic topological spaces. *Internat. Jour. Math Trends Tech.*, 37(3), 214-223.
16. Makai, H., Umehara, J., & Noiri, T. (1996). Every J topological space is pre $T_{1/2}$. *Mem Fac Sci Kochi Univ Ser Math.*, 12, 33-42.
17. Modak, S. (2017). Minimal spaces with a mathematical structure. *Jour Assoc Arab Univ Basic Appl Sc.*, 22, 98-101.

18. Ozturk, T.Y., & Ozkan, A. (2019). Neutrosophic Bitopological Spaces. *Neutrosophic Sets and Systems*, 30, 88-97.
19. Parimala, M., Karthika, M., Jafari, S., Smarandache, F., & Udhayakumar, R. (2019). Neutrosophic Nano ideal topological structure. *Neutrosophic Sets and Systems*, 24, 70-76.
20. Parimala, M., Jeevitha, R., Jafari, S., Smarandache, F., & Udhayakumar, R. (2018). Neutrosophic $\alpha\psi$ -Homeomorphism in Neutrosophic Topological Spaces. *Information*, 9, 187; doi:10.3390/info9080187
21. Rao, V.V., & Srinivasa R. (2017). Neutrosophic pre-open sets and pre-closed sets in Neutrosophic topology. *Internat Jour Chem Tech Research*, 10(10), 449-458.
22. Salama, A.A., & Alblowi, S.A. (2012). Neutrosophic set and neutrosophic topological space. *ISOR Journal of Mathematics*, 3(4), 31-35.
23. Salama, A.A., & Alblowi, S.A. (2012). Generalized neutrosophic set and generalized neutrosophic topological space. *Comp. Sci. Engg.*, 21, 29-132.
24. Smarandache, F. (2005). Neutrosophic set: a generalization of the intuitionistic fuzzy sets. *Inter J Pure Appl Math.*, 24, 287-297.
25. Tripathy, B.C., & Das, S. (2021). Pairwise Neutrosophic b -Continuous Function in Neutrosophic Bitopological Spaces. *Neutrosophic Sets and Systems*, 43, 82-92.
26. Tripathy, B.C., & Debnath, S. (2013). γ -open sets and γ -continuous mappings in fuzzy bitopological spaces. *Journal of Intelligent and Fuzzy Systems*, 24(3), 631-635.
27. Tripathy, B.C., & Debnath, S. (2019). Fuzzy m -structures, m -open multifunctions and bitopological spaces. *Boletim da Sociedade Paranaense de Matemática*, 37(4), 119-128.
28. Tripathy, B.C., & Sarma, D.J. (2011). On b -locally open sets in bitopological spaces. *Kyungpook Math J.*, 51(4), 429-433.
29. Zadeh, L.A. (1965). Fuzzy sets. *Information and Control*, 8, 338-353.

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