

10-5-2022

## Irresolute and its Contra Functions in Generalized Neutrosophic Topological Spaces

Santhi P

Yuvarani A

Vijaya S

Follow this and additional works at: [https://digitalrepository.unm.edu/nss\\_journal](https://digitalrepository.unm.edu/nss_journal)

---

### Recommended Citation

P, Santhi; Yuvarani A; and Vijaya S. "Irresolute and its Contra Functions in Generalized Neutrosophic Topological Spaces." *Neutrosophic Sets and Systems* 51, 1 (2022). [https://digitalrepository.unm.edu/nss\\_journal/vol51/iss1/8](https://digitalrepository.unm.edu/nss_journal/vol51/iss1/8)

This Article is brought to you for free and open access by UNM Digital Repository. It has been accepted for inclusion in Neutrosophic Sets and Systems by an authorized editor of UNM Digital Repository. For more information, please contact [disc@unm.edu](mailto:disc@unm.edu).



# Irresolute and its Contra Functions in Generalized Neutrosophic Topological Spaces

Santhi P <sup>1</sup>, Yuvarani A <sup>2</sup> and Vijaya S <sup>3\*</sup>

<sup>1</sup> PG & Research Department of Mathematics, The Standard Fireworks Rajaratnam College for Women, Madurai Kamaraj University, Sivakasi, Tamil Nadu, India, saayphd.11@gmail.com

<sup>2</sup> PG & Research Department of Mathematics, The American College, Madurai Kamaraj University, Madurai, Tamil Nadu, India, yuvamaths2003@gmail.com

<sup>3</sup> PG & Research Department of Mathematics, Thiagarajar College, Madurai Kamaraj University, Madurai, Tamil Nadu, India, viviphd.11@gmail.com

\* Correspondence: viviphd.11@gmail.com

**Abstract:** The intention to study the idea of Generalized Topological Spaces by means of Neutrosophic sets leads to develop this article. In this write up we launch new ideas on  $\lambda_N$ -Topological Spaces. We study some of its characteristics and behaviours of  $\lambda_N$ - $\alpha$ -irresolute function,  $\lambda_N$ -semi-irresolute function and  $\lambda_N$ -pre-irresolute function. Also we discuss the above for contra  $\lambda_N$ -irresolute functions and derived some relations between them.

**2010 Mathematics Subject Classification:** 54A05, 54B05, 54D10

**Keywords:**  $\lambda_N$ - $\alpha$ -irresolute function,  $\lambda_N$ -semi-irresolute function,  $\lambda_N$ -pre-irresolute function, contra  $\lambda_N$ - $\alpha$ -irresolute function, contra  $\lambda_N$ -semi-irresolute function, contra  $\lambda_N$ -pre-irresolute function.

---

## 1. Introduction

Zadeh [16] initiated fuzzy set theory in 1965 that deals with uncertainty in real life situations. Chang [2] designed fuzzy topology that gave a special note to the field of topology in 1968. Atanassov [1] in 1983, see the sights of intuitionistic fuzzy sets by considering both membership and non-membership of the elements. In 1997, Coker [4] worked on Intuitionistic fuzzy sets by extending the concepts of fuzziness and found a place for Intuitionistic fuzzy topological space.

Smarandache [5] to [7] & [14] introduced Neutrosophic set which is a generalization of fuzzy set and intuitionistic fuzzy set. This is a strong tool to discuss about the existence of incomplete, indeterminate and inconsistent information in the real life situation. Smarandache focused his observations en route for the degree of indeterminacy that directed into Neutrosophic Sets (NS). Soon after, Salama and Albawi [10] familiarized Neutrosophic Topological Spaces (NTS). Further, Salama, Smarandache and Valeri Kromov

[11] presented the continuous (Cts) functions in NTS. In [3], irresolute functions was introduced and analysed by Crossley and Hildebrand in Topological Spaces. Further, Vijaya [13] and Santhi [12] investigated the properties of  $\lambda$ - $\alpha$ -irresolute function and contra  $\lambda$ - $\alpha$ -irresolute function in Generalized Topological Spaces. In addition to that, properties of  $\alpha$ -irresolute function and contra  $\alpha$ -irresolute function in Nano Topological Spaces was look over by Yuvarani and et. al., [15]. By keeping all these works as a motivation, in 2020, Raksha Ben, Hari Siva Annam [8] & [9] contrived  $\lambda_N$ -Topological Space and deliberated its properties.

In this disquisition, we explore our perception of  $\lambda_N$ - $\alpha$ -irresolute function,  $\lambda_N$ -semi-irresolute function,  $\lambda_N$ -pre-irresolute function, contra  $\lambda_N$ - $\alpha$ -irresolute function, contra  $\lambda_N$ -semi-irresolute function, contra  $\lambda_N$ -pre-irresolute function and we have scrutinized about some of their basic properties. At every place the novel notions have been validated with apposite paradigms.

## 2. Prerequisites

### 2.1. Definition [10]

Let  $\Omega$  be a non-empty fixed set. A NS,  $E = \{ \langle \omega, M_E(\omega), I_E(\omega), N_E(\omega) \rangle : \omega \in \Omega \}$  where  $M_E(\omega)$ ,  $I_E(\omega)$  and  $N_E(\omega)$  represents the degree of membership, indeterminacy and non-membership functions respectively of every element  $\omega \in \Omega$ .

### 2.2. Remark [10]

A NS,  $E$  can be recognized as a structured triple  $E = \{ \langle \omega, M_E(\omega), I_E(\omega), N_E(\omega) \rangle : \omega \in \Omega \}$  in  $] -0, 1 +[$  on  $\Omega$ .

### 2.3. Remark [10]

The NS,  $0_N$  and  $1_N$  in  $\Omega$  is defined as

$$(P_1) \quad 0_N = \{ \langle \omega, 0, 0, 1 \rangle : \omega \in \Omega \}$$

$$(P_2) \quad 0_N = \{ \langle \omega, 0, 1, 1 \rangle : \omega \in \Omega \}$$

$$(P_3) \quad 0_N = \{ \langle \omega, 0, 1, 0 \rangle : \omega \in \Omega \}$$

$$(P_4) \quad 0_N = \{ \langle \omega, 0, 0, 0 \rangle : \omega \in \Omega \}$$

$$(P_5) \quad 1_N = \{ \langle \omega, 1, 0, 0 \rangle : \omega \in \Omega \}$$

$$(P_6) \quad 1_N = \{ \langle \omega, 1, 0, 1 \rangle : \omega \in \Omega \}$$

$$(P_7) \quad 1_N = \{ \langle \omega, 1, 1, 0 \rangle : \omega \in \Omega \}$$

$$(P_8) \quad 1_N = \{ \langle \omega, 1, 1, 1 \rangle : \omega \in \Omega \}$$

### 2.4. Definition [10]

If  $E = \{ \langle M_E(\omega), I_E(\omega), N_E(\omega) \rangle \}$ , then the complement of  $E$  on  $\Omega$  is

$$(P_9) \quad E' = \{ \langle \omega, 1 - M_E(\omega), 1 - I_E(\omega) \text{ and } 1 - N_E(\omega) \rangle : \omega \in \Omega \}$$

$$(P_{10}) \quad E' = \{ \langle \omega, N_E(\omega), I_E(\omega) \text{ and } M_E(\omega) \rangle : \omega \in \Omega \}$$

$$(P_{11}) \quad E' = \{ \langle \omega, N_E(\omega), 1 - I_E(\omega) \text{ and } M_E(\omega) \rangle : \omega \in \Omega \}$$

**2.5. Definition [10]**

Let  $\Omega$  be a non-empty set and let  $E = \{ \langle \omega, M_E(\omega), I_E(\omega), N_E(\omega) \rangle : \omega \in \Omega \}$  and  $F = \{ \langle \omega, M_F(\omega), I_F(\omega), N_F(\omega) \rangle : \omega \in \Omega \}$ . Then

- (i)  $E \subseteq F \Rightarrow M_E(\omega) \leq M_F(\omega), I_E(\omega) \leq I_F(\omega), N_E(\omega) \geq N_F(\omega), \forall \omega \in \Omega$
- (ii)  $E \subseteq F \Rightarrow M_E(\omega) \leq M_F(\omega), I_E(\omega) \geq I_F(\omega), N_E(\omega) \geq N_F(\omega), \forall \omega \in \Omega$

**2.6. Definition [10]**

Let  $\Omega$  be a non-empty set and  $E = \{ \langle \omega, M_E(\omega), I_E(\omega), N_E(\omega) \rangle : \omega \in \Omega \}$ ,  $F = \{ \langle \omega, M_F(\omega), I_F(\omega), N_F(\omega) \rangle : \omega \in \Omega \}$  are NSs. Then,

- (P12)  $E \cap F = \langle \omega, M_E(\omega) \wedge M_F(\omega), I_E(\omega) \vee I_F(\omega), N_E(\omega) \vee N_F(\omega) \rangle$
- (P13)  $E \cap F = \langle \omega, M_E(\omega) \wedge M_F(\omega), I_E(\omega) \wedge I_F(\omega), N_E(\omega) \vee N_F(\omega) \rangle$
- (P14)  $E \cup F = \langle \omega, M_E(\omega) \vee M_F(\omega), I_E(\omega) \wedge I_F(\omega), N_E(\omega) \wedge N_F(\omega) \rangle$
- (P15)  $E \cup F = \langle \omega, M_E(\omega) \vee M_F(\omega), I_E(\omega) \vee I_F(\omega), N_E(\omega) \wedge N_F(\omega) \rangle$

**2.7. Definition [9]**

Let  $\Omega \neq \emptyset$ . A family of Neutrosophic subsets of  $\Omega$  is  $\lambda_N$ -topology if it satisfies

- ( $\Delta_1$ )  $0_N \in \lambda_N$
- ( $\Delta_2$ )  $E_1 \cup E_2 \in \lambda_N$  for any  $E_1, E_2 \in \lambda_N$ .

**2.8. Remark [9]**

Members of  $\lambda_N$ -topology are  $\lambda_N$ -Open Sets ( $\lambda_N$ -OS) and their complements are  $\lambda_N$ -Closed Sets ( $\lambda_N$ -CS).

**2.9. Definition [9]**

Let  $(\Omega, \lambda_N)$  be a  $\lambda_N$ -TS and  $E = \{ \langle \omega, M_E(\omega), I_E(\omega), N_E(\omega) \rangle \}$  be a NS in  $\Omega$ . Then

$$\lambda_N\text{-Closure}(E) = \bigcap \{F: E \subseteq F, F \text{ is } \lambda_N\text{-CS}\}$$

$$\lambda_N\text{-Interior}(E) = \bigcup \{G: G \subseteq E, G \text{ is } \lambda_N\text{-OS}\}$$

**2.10. Definition [8]**

A NS,  $E$  in  $\lambda_N$ -TS is said to be

- (i)  $\lambda_N$ -Semi-Open Set ( $\lambda_N$ -SOS) if  $E \subseteq \lambda_N\text{-Cl}(\lambda_N\text{-Int}(E))$ ,
- (ii)  $\lambda_N$ -Pre-Open Set ( $\lambda_N$ -POS) if  $E \subseteq \lambda_N\text{-Int}(\lambda_N\text{-Cl}(E))$ ,
- (iii)  $\lambda_N$ - $\alpha$ -Open Set ( $\lambda_N$ - $\alpha$ OS) if  $E \subseteq \lambda_N\text{-Int}(\lambda_N\text{-Cl}(\lambda_N\text{-Int}(E)))$ .

**2.11. Lemma [8]**

Every  $\lambda_N$ - $\alpha$ OS is  $\lambda_N$ -SOS and  $\lambda_N$ -POS.

**2.12. Definition [8]**

Let the function  $h: (\Omega_1, \tau_1) \rightarrow (\Omega_2, \tau_2)$  is defined to be  $\lambda_N$ -Cts (resp.  $\lambda_N$ -SCts,  $\lambda_N$ -PCts,  $\lambda_N$ - $\alpha$ Cts) if the inverse image of  $\lambda_N$ -CS in  $(\Omega_2, \tau_2)$  is a  $\lambda_N$ -CS (resp.  $\lambda_N$ -SCS,  $\lambda_N$ -PCS,  $\lambda_N$ - $\alpha$ CS) in  $(\Omega_1, \tau_1)$ .

**3.  $\lambda_N$ -Irresolute Functions**

**3.1. Definition**

Let  $(\Omega_1, \tau_1)$  and  $(\Omega_2, \tau_2)$  be  $\lambda_N$ -TSs. Then  $h: \Omega_1 \rightarrow \Omega_2$  is said to be a  $\lambda_N$ - $\alpha$ -irresolute function (resp.  $\lambda_N$ -semi-irresolute,  $\lambda_N$ -pre-irresolute) if the inverse image of every  $\lambda_N$ - $\alpha$ OS (resp.  $\lambda_N$ -SOS,  $\lambda_N$ -POS) in  $(\Omega_2, \tau_2)$  is an  $\lambda_N$ - $\alpha$ OS (resp.  $\lambda_N$ -SOS,  $\lambda_N$ -POS) in  $(\Omega_1, \tau_1)$ .

### 3.2. Example

Let  $h: (\Omega_1, \tau_1) \rightarrow (\Omega_2, \tau_2)$  be defined as  $h(p) = s$  and  $h(q) = r$ , where  $\Omega_1 = \{p, q\}$  and  $\Omega_2 = \{r, s\}$ ,  $\tau_1 = \{0_N, A, B\}$ ,  $\tau_2 = \{0_N, C, D\}$ .

$$\begin{aligned} \text{(i)} \quad A &= \langle (0.2, 0.8, 0.9), (0.1, 0.7, 0.8) \rangle, & B &= \langle (0.3, 0.5, 0.6), (0.4, 0.6, 0.7) \rangle, \\ C &= \langle (0.1, 0.7, 0.8), (0.2, 0.8, 0.9) \rangle, & D &= \langle (0.4, 0.6, 0.7), (0.3, 0.5, 0.6) \rangle, \\ G &= \langle (0.3, 0.7, 0.8), (0.2, 0.6, 0.7) \rangle, & H &= \langle (0.2, 0.6, 0.7), (0.3, 0.7, 0.8) \rangle. \end{aligned}$$

Here  $\{0_N, A, B, G\}$  and  $\{0_N, C, D, H\}$  are  $\lambda_N$ - $\alpha$ OS of  $(\Omega_1, \tau_1)$  and  $(\Omega_2, \tau_2)$  respectively. Hence,  $h$  is a  $\lambda_N$ - $\alpha$ -irresolute function.

$$\begin{aligned} \text{(ii)} \quad A &= \langle (0.3, 0.7, 0.8), (0.2, 0.6, 0.8) \rangle, & B &= \langle (0.4, 0.6, 0.7), (0.5, 0.5, 0.6) \rangle, \\ C &= \langle (0.5, 0.5, 0.6), (0.4, 0.6, 0.7) \rangle, & D &= \langle (0.2, 0.6, 0.8), (0.3, 0.7, 0.8) \rangle, \\ G &= \langle (0.3, 0.7, 0.8), (0.4, 0.5, 0.7) \rangle, & H &= \langle (0.4, 0.5, 0.7), (0.3, 0.7, 0.8) \rangle. \end{aligned}$$

Here  $\{0_N, A, B, G\}$  and  $\{0_N, C, D, H\}$  are  $\lambda_N$ -SOS of  $(\Omega_1, \tau_1)$  and  $(\Omega_2, \tau_2)$  respectively. Therefore,  $h$  is a  $\lambda_N$ -semi-irresolute function.

$$\begin{aligned} \text{(iii)} \quad A &= \langle (0.3, 0.8, 0.9), (0.4, 0.7, 0.6) \rangle, & B &= \langle (0.4, 0.6, 0.7), (0.5, 0.6, 0.6) \rangle, \\ C &= \langle (0.5, 0.6, 0.6), (0.4, 0.6, 0.7) \rangle, & D &= \langle (0.4, 0.7, 0.6), (0.3, 0.8, 0.9) \rangle, \\ G &= \langle (0.2, 0.9, 0.9), (0.3, 0.8, 0.9) \rangle, & H &= \langle (0.3, 0.7, 0.8), (0.5, 0.5, 0.6) \rangle, \\ I &= \langle (0.3, 0.8, 0.9), (0.2, 0.9, 0.9) \rangle, & J &= \langle (0.5, 0.5, 0.6), (0.3, 0.7, 0.8) \rangle. \end{aligned}$$

Here  $\{0_N, A, B, G, H\}$  and  $\{0_N, C, D, I, J\}$  are  $\lambda_N$ -POS of  $(\Omega_1, \tau_1)$  and  $(\Omega_2, \tau_2)$  respectively and so  $h$  is a  $\lambda_N$ -pre-irresolute function.

### 3.3. Theorem

Let  $(\Omega, \tau)$  be a  $\lambda_N$ -TS and  $E \subseteq \Omega$ . Then  $E$  is  $\lambda_N$ - $\alpha$ OS iff it is  $\lambda_N$ -SOS and  $\lambda_N$ -POS.

**Proof:**

If  $E$  is  $\lambda_N$ - $\alpha$ OS, then by Lemma 2.11,  $E$  is  $\lambda_N$ -SOS and  $\lambda_N$ -POS. Conversely if  $E$  is  $\lambda_N$ -SOS and  $\lambda_N$ -POS, then  $E \subseteq \lambda_N\text{-Cl}(\lambda_N\text{-Int}(E))$  and  $E \subseteq \lambda_N\text{-Int}(\lambda_N\text{-Cl}(E))$ . Therefore  $\lambda_N\text{-Int}(\lambda_N\text{-Cl}(E)) \subseteq \lambda_N\text{-Int}(\lambda_N\text{-Cl}(\lambda_N\text{-Cl}(\lambda_N\text{-Int}(E)))) = \lambda_N\text{-Int}(\lambda_N\text{-Cl}(\lambda_N\text{-Int}(E)))$ . That is  $\lambda_N\text{-Int}(\lambda_N\text{-Cl}(E)) \subseteq \lambda_N\text{-Int}(\lambda_N\text{-Cl}(\lambda_N\text{-Int}(E)))$ . Also  $E \subseteq \lambda_N\text{-Int}(\lambda_N\text{-Cl}(E)) \subseteq \lambda_N\text{-Int}(\lambda_N\text{-Cl}(\lambda_N\text{-Int}(E)))$  implies  $E \subseteq \lambda_N\text{-Int}(\lambda_N\text{-Cl}(\lambda_N\text{-Int}(E)))$ . Thus  $E$  is  $\lambda_N$ - $\alpha$ OS.

### 3.4. Theorem

Let  $h: \Omega_1 \rightarrow \Omega_2$  be a function, where  $(\Omega_1, \tau_1)$  and  $(\Omega_2, \tau_2)$  be  $\lambda_N$ -TSs. Then the succeeding are equivalent.

- (i)  $h$  is  $\lambda_N$ - $\alpha$ -irresolute.
- (ii)  $h^{-1}(E)$  is  $\lambda_N$ - $\alpha$ CS in  $(\Omega_1, \tau_1)$ , for every  $\lambda_N$ - $\alpha$ CS  $E$  in  $(\Omega_2, \tau_2)$ .
- (iii)  $h(\lambda_N\text{-}\alpha\text{Cl}(E)) \subseteq \lambda_N\text{-}\alpha\text{Cl}(h(E)) \quad \forall E \subseteq \Omega_1$ .
- (iv)  $\lambda_N\text{-}\alpha\text{Cl}(h^{-1}(E)) \subseteq h^{-1}(\lambda_N\text{-}\alpha\text{Cl}(E)) \quad \forall E \subseteq \Omega_2$ .
- (v)  $h^{-1}(\lambda_N\text{-}\alpha\text{Int}(E)) \subseteq \lambda_N\text{-}\alpha\text{Int}(h^{-1}(E)) \quad \forall E \subseteq \Omega_2$ .
- (vi)  $h$  is  $\lambda_N$ - $\alpha$ -irresolute for every  $\omega \in (\Omega_1, \tau_1)$ .

**Proof**

(i) implies (ii) It is obvious.

(ii) implies (iii) Let  $E \subseteq \Omega_1$ . In that case,  $\lambda_N\text{-}\alpha\text{Cl}(h(E))$  is a  $\lambda_N\text{-}\alpha\text{CS}$  of  $(\Omega_2, \tau_2)$ . By (ii),  $h^{-1}(\lambda_N\text{-}\alpha\text{Cl}(h(E)))$  is a  $\lambda_N\text{-}\alpha\text{CS}$  in  $(\Omega_1, \tau_1)$ , and  $\lambda_N\text{-}\alpha\text{Cl}(E) \subseteq \lambda_N\text{-}\alpha\text{Cl}(h^{-1}h(E)) \subseteq \lambda_N\text{-}\alpha\text{Cl}(h^{-1}(\lambda_N\text{-}\alpha\text{Cl}(h(E)))) = h^{-1}(\lambda_N\text{-}\alpha\text{Cl}(h(E)))$ . So  $h(\lambda_N\text{-}\alpha\text{Cl}(E)) \subseteq \lambda_N\text{-}\alpha\text{Cl}(h(E))$ .

(iii) implies (iv) Let  $E \subseteq \Omega_2$ . By (iii),  $h(\lambda_N\text{-}\alpha\text{Cl}(h^{-1}(E))) \subseteq \lambda_N\text{-}\alpha\text{Cl}(hh^{-1}(E)) \subseteq \lambda_N\text{-}\alpha\text{Cl}(E)$ . So  $\lambda_N\text{-}\alpha\text{Cl}(h^{-1}(E)) \subseteq h^{-1}(\lambda_N\text{-}\alpha\text{Cl}(E))$ .

(iv) implies (v) Let  $E \subseteq \Omega_2$ . By (iv),  $h^{-1}(\lambda_N\text{-}\alpha\text{Cl}(\Omega_2 - E)) \supseteq \lambda_N\text{-}\alpha\text{Cl}(h^{-1}(\Omega_2 - E)) = \lambda_N\text{-}\alpha\text{Cl}(\Omega_1 - h^{-1}(E))$ . Since  $\Omega_1 - \lambda_N\text{-}\alpha\text{Cl}(\Omega_1 - E) = \lambda_N\text{-}\alpha\text{Int}(E)$ , subsequently  $h^{-1}(\lambda_N\text{-}\alpha\text{Int}(E)) = h^{-1}(\Omega_2 - \lambda_N\text{-}\alpha\text{Cl}(\Omega_2 - E)) = \Omega_1 - h^{-1}(\lambda_N\text{-}\alpha\text{Cl}(\Omega_2 - E)) \subseteq \Omega_1 - \lambda_N\text{-}\alpha\text{Cl}(\Omega_1 - h^{-1}(E)) = \lambda_N\text{-}\alpha\text{Int}(h^{-1}(E))$ .

(v) implies (vi) Let  $E$  be any  $\lambda_N\text{-}\alpha\text{OS}$  of  $(\Omega_2, \tau_2)$ , subsequently  $E = \lambda_N\text{-}\alpha\text{Int}(E)$ . By (v),  $h^{-1}(E) = h^{-1}(\lambda_N\text{-}\alpha\text{Int}(E)) \subseteq \lambda_N\text{-}\alpha\text{Int}(h^{-1}(E)) \subseteq h^{-1}(E)$ . So,  $h^{-1}(E) = \lambda_N\text{-}\alpha\text{Int}(h^{-1}(E))$ . Thus,  $h^{-1}(E)$  is a  $\lambda_N\text{-}\alpha\text{OS}$  of  $(\Omega_1, \tau_1)$ . Therefore,  $h$  is  $\lambda_N\text{-}\alpha$ -irresolute.

(i) implies (vi) Let  $h$  be  $\lambda_N\text{-}\alpha$ -irresolute,  $\omega \in (\Omega_1, \tau_1)$  and any  $\lambda_N\text{-}\alpha\text{OS}$   $E$  of  $(\Omega_2, \tau_2)$ ,  $\exists h(\omega) \subseteq E$ . Then  $\omega \in h^{-1}(E) = \lambda_N\text{-}\alpha\text{Int}(h^{-1}(E))$ . Let  $F = h^{-1}(E)$  followed by  $F$  is a  $\lambda_N\text{-}\alpha\text{OS}$  of  $(\Omega_1, \tau_1)$  and so  $h(F) = hh^{-1}(E) \subseteq E$ . Thus,  $h$  is  $\lambda_N\text{-}\alpha$ -irresolute for each  $\omega \in (\Omega_1, \tau_1)$ .

(vi) implies (i) Let  $E$  be a  $\lambda_N\text{-}\alpha\text{OS}$  of  $(\Omega_2, \tau_2)$ ,  $\omega \in h^{-1}(E)$ . Then  $h(\omega) \in E$ . By hypothesis there exists a  $\lambda_N\text{-}\alpha\text{OS}$   $F$  of  $(\Omega_1, \tau_1)$   $\exists \omega \in F$  and  $h(F) \subseteq E$ . Thus  $\omega \in F \subseteq h^{-1}(h(F)) \subseteq h^{-1}(E)$  and  $\omega \in F = \lambda_N\text{-}\alpha\text{Int}(F) \subseteq \lambda_N\text{-}\alpha\text{Int}(h^{-1}(E)) \Rightarrow h^{-1}(E) \subseteq \lambda_N\text{-}\alpha\text{Int}(h^{-1}(E))$ . Hence  $h^{-1}(E) = \lambda_N\text{-}\alpha\text{Int}(h^{-1}(E))$ . Thus,  $h$  is  $\lambda_N\text{-}\alpha$ -irresolute.

**3.5. Theorem**

Let  $h: \Omega_1 \rightarrow \Omega_2$  be a bijective function, where  $(\Omega_1, \tau_1)$  and  $(\Omega_2, \tau_2)$  be  $\lambda_N\text{-TSs}$ . Then  $h$  is  $\lambda_N\text{-}\alpha$ -irresolute iff  $\lambda_N\text{-}\alpha\text{Int}(h(E)) \subseteq h(\lambda_N\text{-}\alpha\text{Int}(E)) \quad \forall E \subseteq \Omega_1$ .

**Proof**

Let  $E \subseteq \Omega_1$ . By Theorem 3.4 and since  $h$  is bijective,  $h^{-1}(\lambda_N\text{-}\alpha\text{Int}(h(E))) \subseteq \lambda_N\text{-}\alpha\text{Int}(h^{-1}(h(E))) = \lambda_N\text{-}\alpha\text{Int}(E)$ . So,  $hh^{-1}(\lambda_N\text{-}\alpha\text{Int}(h(E))) \subseteq h(\lambda_N\text{-}\alpha\text{Int}(E))$ . Consequently  $\lambda_N\text{-}\alpha\text{Int}(h(E)) \subseteq h(\lambda_N\text{-}\alpha\text{Int}(E))$ .

Conversely, let  $E$  be a  $\lambda_N\text{-}\alpha\text{OS}$  of  $(\Omega_2, \tau_2)$ . Then  $E = \lambda_N\text{-}\alpha\text{Int}(E)$ . By hypothesis,  $h(\lambda_N\text{-}\alpha\text{Int}(h^{-1}(E))) \supseteq \lambda_N\text{-}\alpha\text{Int}(h(h^{-1}(E))) = \lambda_N\text{-}\alpha\text{Int}(E) = E$  implies  $h^{-1}h(\lambda_N\text{-}\alpha\text{Int}(h^{-1}(E))) \supseteq h^{-1}(E)$ . Since  $h$  is bijective,  $\lambda_N\text{-}\alpha\text{Int}(h^{-1}(E)) = h^{-1}h(\lambda_N\text{-}\alpha\text{Int}(h^{-1}(E))) \supseteq h^{-1}(E)$ .

Hence  $h^{-1}(E) = \lambda_N\text{-}\alpha\text{Int}(h^{-1}(E))$ . So  $h^{-1}(E)$  is  $\lambda_N\text{-}\alpha\text{OS}$  of  $(\Omega_1, \tau_1)$ . Thus,  $h$  is  $\lambda_N\text{-}\alpha$ -irresolute.

**3.6. Lemma**

Let  $(\Omega, \tau)$  be a  $\lambda_N\text{-TS}$  and  $E \subseteq \Omega$ . Then  $\lambda_N\text{-}\alpha\text{Int}(E) = E \cap \lambda_N\text{-Int}((\lambda_N\text{-Cl}(\lambda_N\text{-Int}(E)))$ ,  $\lambda_N\text{-}\alpha\text{Cl}(E) = E \cup \lambda_N\text{-Cl}(\lambda_N\text{-Int}(\lambda_N\text{-Cl}(E)))$ .

**3.7. Lemma**

Let  $(\Omega, \tau)$  be a  $\lambda_N\text{-TS}$ , then

(i)  $\lambda_N\text{-}\alpha\text{Cl}(E) \subseteq \lambda_N\text{-Cl}(E) \quad \forall E \subseteq \Omega$ .

(ii)  $\lambda_N\text{-Cl}(E) = \lambda_N\text{-}\alpha\text{Cl}(E) \quad \forall E \subseteq \Omega$  where  $E$  is  $\lambda_N\text{-}\alpha\text{OS}$ .

**Proof**

(i) Let  $E \subseteq \Omega$ . Since  $\lambda_N\text{-Int}(E) \subseteq \lambda_N\text{-}\alpha\text{Int}(E)$ ,  $U\text{-}\lambda_N\text{-Int}(E) \supseteq U\text{-}\lambda_N\text{-}\alpha\text{Int}(E)$ . Hence  $\lambda_N\text{-}\alpha\text{Cl}(E) \subseteq \lambda_N\text{-Cl}(E)$ .

(ii) Let  $E$  be any  $\lambda_N\text{-}\alpha\text{OS}$  of  $(\Omega, \tau)$ , then  $E \subseteq \lambda_N\text{-Int}(\lambda_N\text{-Cl}(\lambda_N\text{-Int}(E)))$ . Then  $\lambda_N\text{-Cl}(E) \subseteq \lambda_N\text{-Cl}(\lambda_N\text{-Int}(\lambda_N\text{-Cl}(\lambda_N\text{-Int}(E)))) = \lambda_N\text{-Cl}(\lambda_N\text{-Int}(E)) \subseteq \lambda_N\text{-Cl}(\lambda_N\text{-Int}(\lambda_N\text{-Cl}(E)))$ . So,  $\lambda_N\text{-Cl}(E) \subseteq E \cup \lambda_N\text{-Cl}(\lambda_N\text{-Int}(\lambda_N\text{-Cl}(E)))$ . By Lemma 3.6,  $\lambda_N\text{-Cl}(E) \subseteq \lambda_N\text{-}\alpha\text{Cl}(E)$ . By (i),  $\lambda_N\text{-}\alpha\text{Cl}(E) \subseteq \lambda_N\text{-Cl}(E)$ , therefore  $\lambda_N\text{-Cl}(E) = \lambda_N\text{-}\alpha\text{Cl}(E)$ .

**3.8. Theorem**

Let  $h: \Omega_1 \rightarrow \Omega_2$  be a  $\lambda_N\text{-}\alpha$ -irresolute function, where  $(\Omega_1, \tau_1)$  and  $(\Omega_2, \tau_2)$  be  $\lambda_N\text{-TSs}$ . Then  $\lambda_N\text{-Cl}(h^{-1}(E)) \subseteq h^{-1}(\lambda_N\text{-Cl}(E))$  for every  $\lambda_N\text{-OS}$   $E$  of  $\Omega_2$ .

**Proof**

Let  $E$  be any  $\lambda_N\text{-OS}$  of  $\Omega_2$ . Since  $h$  is  $\lambda_N\text{-}\alpha$ -irresolute and by Lemma 3.7,  $\lambda_N\text{-}\alpha\text{Cl}(h^{-1}(E)) = \lambda_N\text{-Cl}(h^{-1}(E))$ . By Theorem 3.4,  $\lambda_N\text{-}\alpha\text{Cl}(h^{-1}(E)) \subseteq h^{-1}(\lambda_N\text{-}\alpha\text{Cl}(E))$  and by Lemma 3.7,  $h^{-1}(\lambda_N\text{-}\alpha\text{Cl}(E)) \subseteq h^{-1}(\lambda_N\text{-Cl}(E))$ . Then  $\lambda_N\text{-}\alpha\text{Cl}(h^{-1}(E)) \subseteq h^{-1}(\lambda_N\text{-Cl}(E))$ . Therefore  $\lambda_N\text{-Cl}(h^{-1}(E)) \subseteq h^{-1}(\lambda_N\text{-Cl}(E))$ .

**3.9. Theorem**

Let  $(\Omega_1, \tau_1)$  and  $(\Omega_2, \tau_2)$  be  $\lambda_N\text{-TSs}$  and  $h: \Omega_1 \rightarrow \Omega_2$  is  $\lambda_N$ -semi-irresolute iff  $h^{-1}(E)$  is  $\lambda_N\text{-SCS}$  in  $\Omega_1$ ,  $\forall \lambda_N\text{-SCS}$   $E$  of  $\Omega_2$ .

**Proof**

If  $h$  is  $\lambda_N$ -semi-irresolute, then for every  $\lambda_N\text{-SOS}$   $F$  of  $\Omega_2$ ,  $h^{-1}(F)$  is  $\lambda_N\text{-SOS}$  in  $\Omega_1$ . If  $E$  is any  $\lambda_N\text{-SCS}$  of  $\Omega_2$ , then  $\Omega_2 - E$  is  $\lambda_N\text{-SOS}$ . As a consequence,  $h^{-1}(\Omega_2 - E)$  is  $\lambda_N\text{-SOS}$  but  $h^{-1}(\Omega_2 - E) = \Omega_1 - h^{-1}(E)$  so that  $h^{-1}(E)$  is  $\lambda_N\text{-SCS}$  in  $\Omega_1$ .

Conversely, if, for all  $\lambda_N\text{-SCS}$   $E$  of  $\Omega_2$ ,  $h^{-1}(E)$  is  $\lambda_N\text{-SCS}$  in  $\Omega_1$  and if  $F$  is any  $\lambda_N\text{-SOS}$  of  $\Omega_2$ , then  $\Omega_2 - F$  is  $\lambda_N\text{-SCS}$ . Also  $h^{-1}(\Omega_2 - F) = \Omega_1 - h^{-1}(F)$  is  $\lambda_N\text{-SCS}$  in  $\Omega_1$ . Accordingly  $h^{-1}(F)$  is  $\lambda_N\text{-SOS}$  in  $\Omega_1$ . As a result,  $h$  is  $\lambda_N$ -semi-irresolute.

**3.10. Theorem**

If  $h_1: (\Omega_1, \tau_1) \rightarrow (\Omega_2, \tau_2)$  is  $\lambda_N$ -semi-irresolute and  $h_2: (\Omega_2, \tau_2) \rightarrow (\Omega_3, \tau_3)$  is  $\lambda_N$ -semi-irresolute, then  $h_2 \circ h_1 : (\Omega_1, \tau_1) \rightarrow (\Omega_3, \tau_3)$  is  $\lambda_N$ -semi-irresolute.

**Proof**

If  $E \subseteq \Omega_3$  is  $\lambda_N\text{-SOS}$ , then  $h_2^{-1}(E)$  is  $\lambda_N\text{-SOS}$  in  $\Omega_2$  because  $h_2$  is  $\lambda_N$ -semi-irresolute. Consequently since  $h_1$  is  $\lambda_N$ -semi-irresolute,  $h_1^{-1}(h_2^{-1}(E)) = (h_2 \circ h_1)^{-1}(E)$  is  $\lambda_N\text{-SOS}$  in  $\Omega_1$ . Hence  $h_2 \circ h_1$  is  $\lambda_N$ -semi-irresolute.

**3.11. Example ( $h_2 \circ h_1$  is  $\lambda_N$ -semi-irresolute  $\not\Rightarrow h_1$  &  $h_2$  is  $\lambda_N$ -semi-irresolute)**

Let  $h_1: (\Omega_1, \tau_1) \rightarrow (\Omega_2, \tau_2)$  be defined by  $h_1(p) = s$ ,  $h_1(q) = r$  and  $h_2: (\Omega_2, \tau_2) \rightarrow (\Omega_3, \tau_3)$  be defined by  $h_2(r) = u$  and  $h_2(s) = v$  where  $\Omega_1 = \{p, q\}$ ,  $\Omega_2 = \{r, s\}$  and  $\Omega_3 = \{u, v\}$ . Let  $\tau_1 = \{0_N, A, B\}$ ,

$\tau_2 = \{0_N, C, D\}$  and  $\tau_3 = \{0_N, E, F\}$ . Now,  $\{0_N, A, B, G\}$ ,  $\{0_N, C, D, H\}$  and  $\{0_N, E, F, I\}$  are  $\lambda_N$ -SOS of  $(\Omega_1, \tau_1)$ ,  $(\Omega_2, \tau_2)$  and  $(\Omega_3, \tau_3)$  respectively, where

$$\begin{aligned} A &= \langle (0.3, 0.7, 0.8), (0.2, 0.6, 0.8) \rangle, & B &= \langle (0.4, 0.6, 0.7), (0.5, 0.5, 0.6) \rangle, \\ C &= \langle (0.8, 0.4, 0.2), (0.8, 0.3, 0.3) \rangle, & D &= \langle (0.6, 0.5, 0.5), (0.7, 0.4, 0.4) \rangle, \\ E &= \langle (0.2, 0.6, 0.8), (0.3, 0.7, 0.8) \rangle, & F &= \langle (0.5, 0.5, 0.6), (0.4, 0.6, 0.7) \rangle, \\ G &= \langle (0.3, 0.7, 0.8), (0.4, 0.5, 0.7) \rangle, & H &= \langle (0.7, 0.5, 0.4), (0.8, 0.3, 0.3) \rangle, \\ I &= \langle (0.4, 0.5, 0.7), (0.3, 0.7, 0.8) \rangle. \end{aligned}$$

Here,  $h_2 \circ h_1: \Omega_1 \rightarrow \Omega_3$  defined by  $h_2 \circ h_1(p) = v$  and  $h_2 \circ h_1(q) = u$  is  $\lambda_N$ -semi-irresolute, but  $h_1$  and  $h_2$  are not  $\lambda_N$ -semi-irresolute.

### 3.12. Corollary

Let  $(\Omega_1, \tau_1)$ ,  $(\Omega_2, \tau_2)$  and  $(\Omega_3, \tau_3)$  be  $\lambda_N$ -TSs. If  $h_1: \Omega_1 \rightarrow \Omega_2$  and  $h_2: \Omega_2 \rightarrow \Omega_3$  are  $\lambda_N$ - $\alpha$ -irresolute then  $h_2 \circ h_1: \Omega_1 \rightarrow \Omega_3$  is  $\lambda_N$ - $\alpha$ -irresolute.

#### Proof

Let  $E$  is  $\lambda_N$ - $\alpha$ OS in  $(\Omega_3, \tau_3)$ . Since  $h_2$  is  $\lambda_N$ - $\alpha$ -irresolute,  $h_2^{-1}(E)$  is  $\lambda_N$ - $\alpha$ OS in  $(\Omega_2, \tau_2)$ . Also since  $h_1$  is  $\lambda_N$ - $\alpha$ -irresolute,  $h_1^{-1}(h_2^{-1}(E)) = (h_2 \circ h_1)^{-1}(E)$  is  $\lambda_N$ - $\alpha$ OS in  $(\Omega_1, \tau_1)$ . Therefore  $h_2 \circ h_1$  is  $\lambda_N$ - $\alpha$ -irresolute.

### 3.13. Corollary

If  $h_1: (\Omega_1, \tau_1) \rightarrow (\Omega_2, \tau_2)$  is  $\lambda_N$ - $\alpha$ -irresolute (resp.  $\lambda_N$ -semi-irresolute,  $\lambda_N$ -pre-irresolute) and  $h_2: (\Omega_2, \tau_2) \rightarrow (\Omega_3, \tau_3)$  is  $\lambda_N$ - $\alpha$ Cts (resp.  $\lambda_N$ -SCts,  $\lambda_N$ -PCts) then  $h_2 \circ h_1: (\Omega_1, \tau_1) \rightarrow (\Omega_3, \tau_3)$  is  $\lambda_N$ - $\alpha$ Cts (resp.  $\lambda_N$ -SCts,  $\lambda_N$ -PCts).

#### Proof

Let  $E$  is  $\lambda_N$ -OS in  $(\Omega_3, \tau_3)$ . Since  $h_2$  is  $\lambda_N$ - $\alpha$ Cts (resp.  $\lambda_N$ -SCts,  $\lambda_N$ -PCts),  $h_2^{-1}(E)$  is  $\lambda_N$ - $\alpha$ OS (resp.  $\lambda_N$ -SOS,  $\lambda_N$ -POS) in  $(\Omega_2, \tau_2)$ . Also since  $h_1$  is  $\lambda_N$ - $\alpha$ -irresolute (resp.  $\lambda_N$ -semi-irresolute,  $\lambda_N$ -pre-irresolute),  $h_1^{-1}(h_2^{-1}(E)) = (h_2 \circ h_1)^{-1}(E)$  is  $\lambda_N$ - $\alpha$ OS (resp.  $\lambda_N$ -SOS,  $\lambda_N$ -POS) in  $(\Omega_1, \tau_1)$ . Therefore  $h_2 \circ h_1$  is  $\lambda_N$ - $\alpha$ Cts (resp.  $\lambda_N$ -SCts,  $\lambda_N$ -PCts).

### 3.14. Theorem

Let  $(\Omega_1, \tau_1)$  and  $(\Omega_2, \tau_2)$  be  $\lambda_N$ -TSs. If  $h: \Omega_1 \rightarrow \Omega_2$  is  $\lambda_N$ -semi-irresolute and  $\lambda_N$ -pre-irresolute then  $h$  is  $\lambda_N$ - $\alpha$ -irresolute.

#### Proof

Let  $E$  is  $\lambda_N$ - $\alpha$ OS in  $(\Omega_2, \tau_2)$ , then by Theorem 3.3,  $E$  is  $\lambda_N$ -SOS and  $\lambda_N$ -POS. Since  $h$  is  $\lambda_N$ -semi-irresolute and  $\lambda_N$ -pre-irresolute,  $h^{-1}(E)$  is  $\lambda_N$ -SOS and  $\lambda_N$ -POS. Therefore  $h^{-1}(E)$  is  $\lambda_N$ - $\alpha$ OS. Hence  $h$  is  $\lambda_N$ - $\alpha$ -irresolute.

### 3.15. Theorem

Let  $(\Omega_1, \tau_1)$  and  $(\Omega_2, \tau_2)$  be  $\lambda_N$ -TSs. A function  $h: \Omega_1 \rightarrow \Omega_2$  is  $\lambda_N$ - $\alpha$ Cts iff it is  $\lambda_N$ -SCts and  $\lambda_N$ -PCts.

#### Proof

It is clear from Theorem 3.3.



### 4. Contra $\lambda_N$ -Irresolute Functions

#### 4.1. Definition

Let  $(\Omega_1, \tau_1)$  and  $(\Omega_2, \tau_2)$  be  $\lambda_N$ -TSs. Then  $h: \Omega_1 \rightarrow \Omega_2$  is said to be contra  $\lambda_N$ - $\alpha$ -irresolute (resp. contra  $\lambda_N$ -semi-irresolute, contra  $\lambda_N$ -pre-irresolute) if the inverse image of every  $\lambda_N$ - $\alpha$ OS (resp.  $\lambda_N$ -SOS,  $\lambda_N$ -POS) in  $(\Omega_2, \tau_2)$  is a  $\lambda_N$ - $\alpha$ CS (resp.  $\lambda_N$ -SCS,  $\lambda_N$ -PCS) in  $(\Omega_1, \tau_1)$ .

#### 4.2. Example

(i) Let  $h: (\Omega_1, \tau_1) \rightarrow (\Omega_2, \tau_2)$  be defined as  $h(s) = u$  and  $h(t) = v$ , where  $\Omega_1 = \{s, t\}$  and  $\Omega_2 = \{u, v\}$ ,  $\tau_1 = \{0_N, A, B\}$ ,  $\tau_2 = \{0_N, C, D\}$ .

$$\begin{aligned} A &= \langle (0.2, 0.8, 0.9), (0.1, 0.7, 0.8) \rangle, & B &= \langle (0.3, 0.5, 0.6), (0.4, 0.6, 0.7) \rangle, \\ C &= \langle (0.8, 0.3, 0.1), (0.9, 0.2, 0.2) \rangle, & D &= \langle (0.7, 0.4, 0.4), (0.6, 0.5, 0.3) \rangle, \\ G &= \langle (0.3, 0.7, 0.8), (0.2, 0.6, 0.7) \rangle, & H &= \langle (0.7, 0.4, 0.2), (0.8, 0.3, 0.3) \rangle. \end{aligned}$$

Here,  $\{A', B', G', 1_N\}$  are  $\lambda_N$ - $\alpha$ CS of  $(\Omega_1, \tau_1)$  and  $\{0_N, C, D, H\}$  are  $\lambda_N$ - $\alpha$ OS of  $(\Omega_2, \tau_2)$ . Consequently,  $h$  is contra  $\lambda_N$ - $\alpha$ -irresolute function.

(ii) Let  $h: (\Omega_1, \tau_1) \rightarrow (\Omega_2, \tau_2)$  be defined as  $h(p) = v$ ,  $h(q) = w$  and  $h(r) = u$ , where  $\Omega_1 = \{p, q, r\}$  and  $\Omega_2 = \{u, v, w\}$ ,  $\tau_1 = \{0_N, A, B\}$ ,  $\tau_2 = \{0_N, C, D\}$ .

$$\begin{aligned} A &= \langle (0.2, 0.6, 0.8), (0.1, 0.7, 0.9), (0.2, 0.8, 0.9) \rangle, & B &= \langle (0.3, 0.4, 0.7), (0.2, 0.5, 0.8), (0.4, 0.6, 0.7) \rangle, \\ C &= \langle (0.9, 0.3, 0.1), (0.9, 0.2, 0.2), (0.8, 0.4, 0.2) \rangle, & D &= \langle (0.8, 0.5, 0.2), (0.7, 0.4, 0.4), (0.7, 0.6, 0.3) \rangle, \\ G &= \langle (0.3, 0.5, 0.7), (0.2, 0.6, 0.9), (0.3, 0.7, 0.8) \rangle, & H &= \langle (0.9, 0.4, 0.2), (0.8, 0.3, 0.3), (0.7, 0.5, 0.3) \rangle. \end{aligned}$$

Here,  $\{A', B', G', 1_N\}$  are  $\lambda_N$ -SCS of  $(\Omega_1, \tau_1)$  and  $\{0_N, C, D, H\}$  are  $\lambda_N$ -SOS of  $(\Omega_2, \tau_2)$ . Hence  $h$  is contra  $\lambda_N$ -semi-irresolute function.

(iii) Let  $h: (\Omega_1, \tau_1) \rightarrow (\Omega_2, \tau_2)$  be defined as  $h(p) = w$ ,  $h(q) = u$  and  $h(r) = v$ , where  $\Omega_1 = \{p, q, r\}$  and  $\Omega_2 = \{u, v, w\}$ ,  $\tau_1 = \{0_N, A, B\}$ ,  $\tau_2 = \{0_N, C, D\}$ .

$$\begin{aligned} A &= \langle (0.2, 0.7, 0.7), (0.3, 0.7, 0.8), (0.1, 0.8, 0.8) \rangle, & B &= \langle (0.3, 0.7, 0.6), (0.4, 0.6, 0.7), (0.2, 0.7, 0.8) \rangle, \\ C &= \langle (0.9, 0.1, 0.1), (0.8, 0.2, 0.2), (0.8, 0.3, 0.2) \rangle, & D &= \langle (0.8, 0.3, 0.2), (0.6, 0.3, 0.3), (0.7, 0.4, 0.4) \rangle, \\ G &= \langle (0.2, 0.8, 0.8), (0.2, 0.7, 0.8), (0.1, 0.9, 0.9) \rangle, & H &= \langle (0.8, 0.2, 0.1), (0.7, 0.3, 0.2), (0.8, 0.3, 0.3) \rangle. \end{aligned}$$

Here,  $\{A', B', G', 1_N\}$  are  $\lambda_N$ -PCS of  $(\Omega_1, \tau_1)$  and  $\{0_N, C, D, H\}$  are  $\lambda_N$ -POS of  $(\Omega_2, \tau_2)$ . That's why  $h$  is contra  $\lambda_N$ -pre-irresolute function.

#### 4.3. Theorem

Let  $(\Omega_1, \tau_1)$  and  $(\Omega_2, \tau_2)$  be  $\lambda_N$ -TSs. Then  $h: \Omega_1 \rightarrow \Omega_2$  is contra  $\lambda_N$ - $\alpha$ -irresolute iff for every  $\lambda_N$ - $\alpha$ CS  $E$  of  $\Omega_2$ ,  $h^{-1}(E)$  is  $\lambda_N$ - $\alpha$ OS in  $\Omega_1$ .

#### Proof

If  $h$  is contra  $\lambda_N$ - $\alpha$ -irresolute, then for each  $\lambda_N$ - $\alpha$ OS  $F$  of  $\Omega_2$ ,  $h^{-1}(F)$  is  $\lambda_N$ - $\alpha$ CS in  $\Omega_1$ . If  $E$  is any  $\lambda_N$ - $\alpha$ CS of  $\Omega_2$ , then  $\Omega_2 - E$  is  $\lambda_N$ - $\alpha$ OS. Thus  $h^{-1}(\Omega_2 - E)$  is  $\lambda_N$ - $\alpha$ CS but  $h^{-1}(\Omega_2 - E) = \Omega_1 - h^{-1}(E)$  so that  $h^{-1}(E)$  is  $\lambda_N$ - $\alpha$ OS in  $\Omega_1$ .

Conversely, if, for all  $\lambda_N$ - $\alpha$ CS  $E$  of  $\Omega_2$ ,  $h^{-1}(E)$  is  $\lambda_N$ - $\alpha$ OS in  $\Omega_1$  and if  $F$  is any  $\lambda_N$ - $\alpha$ OS of  $\Omega_2$ , then  $\Omega_2 - F$  is  $\lambda_N$ - $\alpha$ CS. Also,  $h^{-1}(\Omega_2 - F) = \Omega_1 - h^{-1}(F)$  is  $\lambda_N$ - $\alpha$ OS. Thus  $h^{-1}(F)$  is  $\lambda_N$ - $\alpha$ CS in  $\Omega_1$ . Hence  $h$  is contra  $\lambda_N$ - $\alpha$ -irresolute.

**4.4. Corollary**

Let  $(\Omega_1, \tau_1)$  and  $(\Omega_2, \tau_2)$  be  $\lambda_N$ -TSSs. Then  $h: \Omega_1 \rightarrow \Omega_2$  is contra  $\lambda_N$ -semi-irresolute (contra  $\lambda_N$ -pre-irresolute) iff for every  $\lambda_N$ -SCS ( $\lambda_N$ -PCS)  $E$  of  $\Omega_2$ ,  $h^{-1}(E)$  is  $\lambda_N$ -SOS ( $\lambda_N$ -POS) in  $\Omega_1$ .

**Proof**

If  $h$  is contra  $\lambda_N$ -semi-irresolute (contra  $\lambda_N$ -pre-irresolute), then for each  $\lambda_N$ -SOS ( $\lambda_N$ -POS)  $F$  of  $\Omega_2$ ,  $h^{-1}(F)$  is  $\lambda_N$ -SCS ( $\lambda_N$ -PCS) in  $\Omega_1$ . If  $E$  is any  $\lambda_N$ -SCS ( $\lambda_N$ -PCS) of  $\Omega_2$ , then  $\Omega_2 - E$  is  $\lambda_N$ -SOS ( $\lambda_N$ -POS). Thus  $h^{-1}(\Omega_2 - E)$  is  $\lambda_N$ -SCS ( $\lambda_N$ -PCS) but  $h^{-1}(\Omega_2 - E) = \Omega_1 - h^{-1}(E)$  so that  $h^{-1}(E)$  is  $\lambda_N$ -SOS ( $\lambda_N$ -POS) in  $\Omega_1$ .

Conversely, if, for all  $\lambda_N$ -SCS ( $\lambda_N$ -PCS)  $E$  of  $\Omega_2$ ,  $h^{-1}(E)$  is  $\lambda_N$ -SOS ( $\lambda_N$ -POS) in  $\Omega_1$  and if  $F$  is any  $\lambda_N$ -SOS ( $\lambda_N$ -POS) of  $\Omega_2$ , then  $\Omega_2 - F$  is  $\lambda_N$ -SCS ( $\lambda_N$ -PCS). Also,  $h^{-1}(\Omega_2 - F) = \Omega_1 - h^{-1}(F)$  is  $\lambda_N$ -SCS ( $\lambda_N$ -PCS). Thus  $h^{-1}(F)$  is  $\lambda_N$ -SOS ( $\lambda_N$ -POS) in  $\Omega_1$ . Hence  $h$  is contra  $\lambda_N$ -semi-irresolute (contra  $\lambda_N$ -pre-irresolute).

**4.5. Theorem**

Let  $(\Omega_1, \tau_1)$ ,  $(\Omega_2, \tau_2)$  and  $(\Omega_3, \tau_3)$  be  $\lambda_N$ -TSSs. If  $h_1: \Omega_1 \rightarrow \Omega_2$  and  $h_2: \Omega_2 \rightarrow \Omega_3$  are contra  $\lambda_N$ -semi-irresolute functions, then  $h_2 \circ h_1: \Omega_1 \rightarrow \Omega_3$  is  $\lambda_N$ -semi-irresolute.

**Proof**

If  $E \subseteq Z$  is  $\lambda_N$ -SOS, then  $h_2^{-1}(E)$  is  $\lambda_N$ -SCS in  $\Omega_2$  because  $h_2$  is contra  $\lambda_N$ -semi-irresolute. Consequently, since  $h_1$  is contra  $\lambda_N$ -semi-irresolute,  $h_1^{-1}(h_2^{-1}(E)) = (h_2 \circ h_1)^{-1}(E)$  is  $\lambda_N$ -SOS in  $\Omega_1$ . Hence  $h_2 \circ h_1$  is  $\lambda_N$ -semi-irresolute.

**4.6. Example ( $h_2 \circ h_1$  is  $\lambda_N$ -semi-irresolute  $\nRightarrow$   $h_1$  &  $h_2$  is contra  $\lambda_N$ -semi-irresolute)**

Let  $h_1: (\Omega_1, \tau_1) \rightarrow (\Omega_2, \tau_2)$  be defined by  $h_1(l) = q$ ,  $h_1(m) = r$ ,  $h_1(n) = p$  and  $h_2: (\Omega_2, \tau_2) \rightarrow (\Omega_3, \tau_3)$  be defined by  $h_2(p) = v$ ,  $h_2(q) = w$  and  $h_2(r) = u$  where  $\Omega_1 = \{l, m, n\}$ ,  $\Omega_2 = \{p, q, r\}$  and  $\Omega_3 = \{u, v, w\}$ . Let  $\tau_1 = \{0_N, A, B\}$ ,  $\tau_2 = \{0_N, C, D\}$  and  $\tau_3 = \{0_N, E, F, I\}$ . Here,  $\{0_N, A, B, G\}$ ,  $\{0_N, C, D, H\}$  and  $\{0_N, E, F, I\}$  are  $\lambda_N$ -SOS of  $(\Omega_1, \tau_1)$ ,  $(\Omega_2, \tau_2)$  and  $(\Omega_3, \tau_3)$  where

$$\begin{aligned}
 A &= \langle (0.2, 0.6, 0.8), (0.1, 0.7, 0.9), (0.2, 0.8, 0.9) \rangle, & B &= \langle (0.3, 0.4, 0.7), (0.2, 0.5, 0.8), (0.4, 0.6, 0.7) \rangle, \\
 C &= \langle 0.2, 0.8, 0.9, (0.2, 0.6, 0.8), (0.1, 0.7, 0.9) \rangle, & D &= \langle 0.4, 0.6, 0.7, (0.3, 0.4, 0.7), (0.2, 0.5, 0.8) \rangle, \\
 E &= \langle (0.1, 0.7, 0.9), (0.2, 0.8, 0.9), (0.2, 0.6, 0.8) \rangle, & F &= \langle (0.2, 0.5, 0.8), (0.4, 0.6, 0.7), (0.3, 0.4, 0.7) \rangle, \\
 G &= \langle (0.3, 0.5, 0.7), (0.2, 0.6, 0.9), (0.3, 0.7, 0.8) \rangle, & H &= \langle (0.3, 0.7, 0.8), (0.3, 0.5, 0.7), (0.2, 0.6, 0.9) \rangle, \\
 I &= \langle (0.2, 0.6, 0.9), (0.3, 0.7, 0.8), (0.3, 0.5, 0.7) \rangle.
 \end{aligned}$$

Here,  $h_2 \circ h_1: \Omega_1 \rightarrow \Omega_3$  which is defined by  $h_2 \circ h_1(l) = w$ ,  $h_2 \circ h_1(m) = u$  and  $h_2 \circ h_1(n) = v$  is  $\lambda_N$ -semi-irresolute, but  $h_1$  and  $h_2$  are not contra  $\lambda_N$ -semi-irresolute.

**4.7. Corollary**

Let  $(\Omega_1, \tau_1)$ ,  $(\Omega_2, \tau_2)$  and  $(\Omega_3, \tau_3)$  be  $\lambda_N$ -TSSs. If  $h_1: \Omega_1 \rightarrow \Omega_2$  and  $h_2: \Omega_2 \rightarrow \Omega_3$  are contra  $\lambda_N$ - $\alpha$ -irresolute (contra  $\lambda_N$ -pre-irresolute) functions, then  $h_2 \circ h_1: \Omega_1 \rightarrow \Omega_3$  is a  $\lambda_N$ - $\alpha$ -irresolute ( $\lambda_N$ -pre-irresolute) function.

**4.8. Theorem**

Let  $(\Omega_1, \tau_1)$  and  $(\Omega_2, \tau_2)$  be  $\lambda_N$ -TSs. If  $h: \Omega_1 \rightarrow \Omega_2$  is contra  $\lambda_N$ - $\alpha$ -irresolute, then it is contra  $\lambda_N$ - $\alpha$ Cts.

**Proof**

Let  $E$  be any  $\lambda_N$ -OS in  $\Omega_2$ . Then  $E$  is  $\lambda_N$ - $\alpha$ OS in  $\Omega_2$ . Since  $h$  is contra  $\lambda_N$ - $\alpha$ -irresolute,  $h^{-1}(E)$  is a  $\lambda_N$ - $\alpha$ CS in  $\Omega_1$ . It shows that  $h$  is contra  $\lambda_N$ - $\alpha$ Cts function.

**4.9. Theorem**

Let  $(\Omega_1, \tau_1)$ ,  $(\Omega_2, \tau_2)$  and  $(\Omega_3, \tau_3)$  be  $\lambda_N$ -TSs. If  $h_1: \Omega_1 \rightarrow \Omega_2$  is contra  $\lambda_N$ - $\alpha$ -irresolute and  $h_2: \Omega_2 \rightarrow \Omega_3$  is contra  $\lambda_N$ - $\alpha$ Cts, then  $h_2 \circ h_1: \Omega_1 \rightarrow \Omega_3$  is  $\lambda_N$ - $\alpha$ Cts.

**Proof**

Let  $E \subseteq \Omega_3$  is  $\lambda_N$ -OS. Since  $h_2$  is contra  $\lambda_N$ - $\alpha$ Cts,  $h_2^{-1}(E)$  is  $\lambda_N$ - $\alpha$ CS in  $\Omega_2$ . Consequently, since  $h_1$  is contra  $\lambda_N$ - $\alpha$ -irresolute,  $h_1^{-1}(h_2^{-1}(E)) = (h_2 \circ h_1)^{-1}(E)$  is  $\lambda_N$ - $\alpha$ OS in  $\Omega_1$ , by Theorem 4.3. Hence  $h_2 \circ h_1$  is  $\lambda_N$ - $\alpha$ Cts.

**4.10. Corollary**

Let  $(\Omega_1, \tau_1)$ ,  $(\Omega_2, \tau_2)$  and  $(\Omega_3, \tau_3)$  be  $\lambda_N$ -TSs, and  $h_1: \Omega_1 \rightarrow \Omega_2$  and  $h_2: \Omega_2 \rightarrow \Omega_3$  be two functions. Then if  $h_1$  is contra  $\lambda_N$ -semi-irresolute (contra  $\lambda_N$ -pre-irresolute) and  $h_2$  is contra  $\lambda_N$ -SCts (contra  $\lambda_N$ -PCts), then  $h_2 \circ h_1: \Omega_1 \rightarrow \Omega_3$  is  $\lambda_N$ -SCts ( $\lambda_N$ -PCts).

**4.11. Theorem**

Let  $(\Omega_1, \tau_1)$  and  $(\Omega_2, \tau_2)$  be  $\lambda_N$ -TSs. If  $h: \Omega_1 \rightarrow \Omega_2$  is contra  $\lambda_N$ -semi-irresolute and contra  $\lambda_N$ -pre-irresolute, then  $h$  is contra  $\lambda_N$ - $\alpha$ -irresolute.

**Proof**

Let  $E$  is  $\lambda_N$ - $\alpha$ OS in  $(\Omega_2, \tau_2)$ , then by Theorem 3.3,  $E$  is  $\lambda_N$ -SOS and  $\lambda_N$ -POS. Since  $h$  is contra  $\lambda_N$ -semi-irresolute and contra  $\lambda_N$ -pre-irresolute,  $h^{-1}(E)$  is  $\lambda_N$ -SCS and  $\lambda_N$ -PCS. Therefore  $h^{-1}(E)$  is  $\lambda_N$ - $\alpha$ CS. Hence  $h$  is contra  $\lambda_N$ - $\alpha$ -irresolute.

**5. Conclusion**

In this confab, we instigated  $\lambda_N$ - $\alpha$ -irresolute function,  $\lambda_N$ -semi-irresolute function and  $\lambda_N$ -pre-irresolute function on  $\lambda_N$ -TS. Subsequently, we have analyzed its various properties. Followed by this, the new postulations of contra  $\lambda_N$ - $\alpha$ -irresolute function, contra  $\lambda_N$ -semi-irresolute function and contra  $\lambda_N$ -pre-irresolute function were put forth on  $\lambda_N$ -TS and their features were probed along with illustrations.

$\lambda_N$ -TS idea can be further developed and extended in the actual life applications such as medical field, robotics, machine learning, neural networks, natural image sensing, speech recognition, and so on.

In future, it provokes to apply these perceptions in further extensions of  $\lambda_N$ -TS such as almost continuity and its unique characteristics in  $G_N$ -TSs along with some separation axioms related to  $G_N$ -TSs. Also, this concept may be extended to Intuitionistic Fuzzy and Neutrosophic Fixed Point Theory.

**References**

1. Atanassov K.T: Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 20(1986), 87–96.
2. Chang C.L: Fuzzy topological spaces, *Journal of Mathematical Analysis and Application*, 24(1968), 183–190.
3. Crossley S.G and Hildebrand S.K: Semi topological properties, *Fund. Math.*, 74(1972), 233-254.
4. Dogan Coker: An introduction to intuitionistic fuzzy topological spaces, *Fuzzy Sets and Systems*, 88(1997), 81–89.
5. Floretin Smarandache: Neutrosophy and Neutrosophic Logic, *First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA, 2002.*
6. Floretin Smarandache: Neutrosophic Set: A Generalization of Intuitionistic Fuzzy set, *Journal of Defense Resources Management*, 1(2010), 107–116.
7. Floretin Smarandache: A Unifying Field in Logic: Neutrosophic Logic. Neutrosophy, Neutrosophic set, Neutrosophic Probability, *American Research Press, Rehoboth, NM, 1999.*
8. Raksha Ben N, Hari Siva Annam G: Some new open sets in  $\mu_N$  topological space, *Malaya Journal of Matematik*, 9(1)(2021), 89-94.
9. Raksha Ben N, Hari Siva Annam G: Generalized Topological Spaces via Neutrosophic Sets, *J. Math. Comput. Sci.*, 11(2021), 716-734.
10. Salama A.A and Alblowi S.A: Neutrosophic set and Neutrosophic topological space, *ISOR J. Mathematics*, 3(4)(2012), 31–35.
11. Salama A.A, Florentin Smarandache and Valeri Kroumov: Neutrosophic Closed set and Neutrosophic Continuous Function, *Neutrosophic Sets and Systems*, 4(2014), 4–8.
12. Santhi P and Poovazhaki R: On Generalized Topological Contra Quotient Functions, *Indian Journal of Mathematics Research*, 1(1)(2013), 123-136.
13. Vijaya S and Poovazhaki R: On Generalized Topological Quotient Functions, *Journal of Advanced Research in Scientific Computing*, 6(2)(2014), 1-10, Online ISSN: 1943-2364.
14. Wadel Faris Al-Omeri and Florentin Smarandache: New Neutrosophic Sets via Neutrosophic Topological Spaces, *New Trends in Neutrosophic Theory and Applications*, (2)(2016), 1-10.
15. Yuvarani A, Vijaya S and Santhi P: Weaker forms of Nano Irresolute and its Contra Functions, *Ratio Mathematica*, (43)(2022), doi: <http://dx.doi.org/10.23755/rm.v43i0.764>, ISSN:1592-7415, eISSN:2282-8214.
16. Zadeh L.A: Fuzzy set, *Inform and Control*, 8(1965), 338–353.

Received: July 25, 2022. Accepted: September 22, 2022