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Abstract. A number of types of semi-continuous linguistic neutrosophic functions are introduced in this paper. Furthermore, these types of functions are demonstrated with appropriate examples. Theorems and properties are discussed in great detail.

Keywords: Linguistic neutrosophic semi continuous function; Linguistic neutrosophic quasi semi continuous function; Linguistic neutrosophic perfectly semi continuous function; Linguistic neutrosophic totally continuous function; Linguistic neutrosophic strongly semi continuous function; Linguistic neutrosophic slightly semi continuous function; Linguistic neutrosophic semi totally continuous function;

1. **Introduction**

In many system oriented implementations in which typical and standard logic was not suitable due to contradictory circumstances or unpredictability, the Fuzzy logic was utilized primarily, which was discovered by Zadeh(1965) [7]. This idea concerned with the membership or truth value of every elements of the fuzzy set. Along with truth value, the false value or non-membership was adjoined in intuitionistic fuzzy sets which was found by Atanassov [1]. Furthermore, in a new class of sets called neutrosophic sets which was given by Smarandache(1999) [5], possesses an additional membership called indeterminate membership. Neutrosophic sets have a wide range of applications over many real life fields.

Linguistic sets were invented by Fang [3], which has a variety of applications in day to day life. Gayathri and Helen(2021) [4] have found a topological space merging linguistic neutrosophic sets and topological spaces, termed as linguistic neutrosophic topology. The concept of linguistic neutrosophic semi continuous function is recasted into many forms of continuity by using linguistic neutrosophic semi open sets. Interrelations are analyzed among these types of linguistic neutrosophic continuity and counter examples are established to vindicate that the reverse implication of the result is not holds true.

2. Preambles

Definition 2.1. [7] Let S be a space of points (objects), with a generic element in x denoted by S. A neutrosophic set A in S is characterized by a truth-membership function T_A, an indeterminacy membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is
$T_A : S \to]0^-, 1^+[, I_A : S \to]0^-, 1^+[, F_A : S \to]0^-, 1^+[$
There is no restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2.2. [7] Let S be a space of points (objects), with a generic element in x denoted by S. A single valued neutrosophic set (SVNS) A in S is characterized by truth-membership function T_A, indeterminacy-membership function I_A and falsity-membership function F_A. For each point S in S, $T_A(x), I_A(x), F_A(x) \in [0, 1]$.
When S is continuous, a SVNS A can be written as $A = \int \langle T(x), I(x), F(x)\rangle / x \in S$.
When S is discrete, a SVNS A can be written as $A = \sum \langle T(x_i), I(x_i), F(x_i)\rangle / x_i \in S$.

Definition 2.3. [3] Let $S = \{s_\theta | \theta = 0, 1, 2, ..., \tau\}$ be a finite and totally ordered discrete term set, where τ is the even value and s_θ represents a possible value for a linguistic variable.

Su [8] extended the discrete linguistic term set S into a continuous term set $S = \{s_\theta | \theta \in [0, q]\}$, where, if $s_\theta \in S$, then w call s_θ the original term, otherwise it is called as a virtual term.

Definition 2.4. [3] Let $Q = \{s_0, s_1, s_2, ..., s_t\}$ be a linguistic term set (LTS) with odd cardinality $t+1$ and $\overline{Q} = \{s_h / s_0 \leq s_h \leq s_t, h \in [0, t]\}$. Then, a linguistic single valued neutrosophic set A is defined by,
$A = \{\langle x, s_\theta(x), s_\psi(x), s_\sigma(x)\rangle | x \in S\}$, where $s_\theta(x), s_\psi(x), s_\sigma(x) \in \overline{Q}$ represent the linguistic truth, linguistic indeterminacy and linguistic falsity degrees of S to A, respectively, with condition $0 \leq \theta + \psi + \sigma \leq 3t$. This triplet $(s_\theta, s_\psi, s_\sigma)$ is called a linguistic single valued neutrosophic number.

Definition 2.5. [3] Let $\alpha = (s_\theta, s_\psi, s_\sigma), \alpha_1 = (s_{\theta 1}, s_{\psi_1}, s_{\sigma 1}), \alpha_2 = (s_{\theta 2}, s_{\psi_2}, s_{\sigma 2})$ be three LSVNNs, then

(1) $\alpha^c = (s_\sigma, s_\psi, s_\theta)$;
(2) $\alpha_1 \cup \alpha_2 = (max(\theta_1, \theta_2), max(\psi_1, \psi_2), min(\sigma_1, \sigma_2))$;
(3) $\alpha_1 \cap \alpha_2 = (min(\theta_1, \theta_2), min(\psi_1, \psi_2), max(\sigma_1, \sigma_2))$;

(4) $\alpha_1 = \alpha_2$ iff $\theta_1 = \theta_2, \psi_1 = \psi_2, \sigma_1 = \sigma_2$;

Definition 2.6. [4] For a linguistic neutrosophic topology τ, the collection of linguistic neutrosophic sets should obey,

(1) $0_{LN}, 1_{LN} \in \tau$
(2) $K_1 \bigcap K_2 \in \tau$ for any $K_1, K_2 \in \tau$
(3) $\bigcup K_i \in \tau, \forall \{K_i : i \in J\} \subseteq \tau$

We call, the pair (S_{LN}, τ_{LN}), a linguistic neutrosophic topological space.

Definition 2.7. [4] Let (S_{LN}, τ_{LN}) be a linguistic neutrosophic topological space (LNTS). Then,

- $(S_{LN}, \tau_{LN})^c$ is the dual linguistic neutrosophic topology, whose elements are ${K^C}_{LN}$ for $K_{LN} \in (S_{LN}, \tau_{LN})$.
- Any open set in τ_{LN} is known as linguistic neutrosophic open set(LNOS).
- Any closed set in τ_{LN} is known as linguistic neutrosophic closed set(LNCS) if and only if it's complement is linguistic neutrosophic open set.

3. Types of Linguistic Neutrosophic Semi continuous Functions

Definition 3.1. A mapping from $f : S_{LN} \to T_{LN}$ is a linguistic neutrosophic quasi semi continuous if the inverse image $f^{-1}(K_{LN})$ of every linguistic neutrosophic semi open set K_{LN} of T_{LN} is a linguistic neutrosophic open set in S_{LN}.

Theorem 3.2. *A mapping from $f : S_{LN} \to T_{LN}$ is a linguistic neutrosophic quasi semi continuous if and only if the inverse image $f^{-1}(K_{LN})$ of every linguistic neutrosophic semi closed set K_{LN} of T_{LN} is a linguistic neutrosophic closed set in S_{LN}.*

Proof: Necessity Part: Let $f : S_{LN} \to T_{LN}$ be linguistic neutrosophic quasi semi continuous and V_{LN} be any linguistic neutrosophic semi closed set in T_{LN}. Then $T_{LN} \backslash V_{LN}$ is linguistic neutrosophic semi open set in T_{LN}. As f is linguistic neutrosophic quasi semi continuous, $f^{-1}(T_{LN} \backslash V_{LN}) = S_{LN} \backslash f^{-1}(V_{LN})$ is linguistic neutrosophic semi open set in S_{LN}. Hence, the set $f^{-1}(V_{LN})$ is linguistic neutrosophic semi closed and thus the function f is linguistic neutrosophic quasi semi continuous.
Sufficiency Part: Let the set $f^{-1}(V_{LN})$ be linguistic neutrosophic semi closed in S_{LN} for each linguistic neutrosophic closed set in T_{LN}. Let V_{LN} be any linguistic neutrosophic open set in T_{LN}, then $T_{LN} \backslash V_{LN}$ is linguistic neutrosophic closed set in T_{LN}.
By assumption, the set $f^{-1}(T_{LN} \backslash V_{LN}) = S_{LN} \backslash f^{-1}(V_{LN})$ is linguistic neutrosophic semi closed in S_{LN}, which implies $f^{-1}(V_{LN})$ is linguistic neutrosophic semi open in S_{LN}. So, the mapping f is linguistic neutrosophic quasi semi continuous.

Remark 3.3. The above theorem is established by the following example.

Example 3.4. Let the universe of discourse be $U = \{p, q, r, s, t\}$ and let $S_{LN} = \{q, r\} = T_{LN}$. The set of all linguistic term set be L= {never familiar(l_0), almost never familiar(l_1), slightly familiar(l_2), some what familiar(l_3),occasionally familiar(l_4),moderately familiar(l_5), almost every time familiar(l_6), frequently familiar(l_7), extremely familiar(l_8)}. Let $f : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ be the linguistic neutrosophic identity mapping, where $\tau_{LN} = \{0_{LN}, 1_{LN}, K_{LN}\}$ and $\eta_{LN} = \{0_{LN}, 1_{LN}, H_{LN}\}$. The linguistic neutrosophic sets K_{LN} and H_{LN} are given by $K_{LN} = \{\langle, q,(l_3, l_6, l_3)\rangle, \langle r,(l_4, l_3, l_2)\rangle\}$ and $H_{LN} = \{\langle q,(l_3, l_6, l_3)\rangle, \langle r,(l_2, l_3, l_4)\rangle\}$ respectively. Here the inverse image $f^{-1}(H_{LN})$ is linguistic neutrosophic closed in S_{LN}.

Theorem 3.5. *If the mapping $f : S_{LN} \rightarrow T_{LN}$ is linguistic neutrosophic strongly semi continuous, then it is linguistic neutrosophic quasi semi continuous.*

Proof: Let V_{LN} be a linguistic neutrosophic semi open set in T_{LN}. Since f is linguistic neutrosophic strongly semi continuous, $f^{-1}(V_{LN})$ is linguistic neutrosophic semi cl-open in S_{LN}. Thus, f is linguistic neutrosophic quasi semi continuous.

Remark 3.6. The converse part of the above theorem need not be true in general, which is given by a counter example.

Example 3.7. Let the universe of discourse be as in example (3.4). And let $f : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic mapping defined by $f(a) = c, f(c) = a$, where $\tau_{LN} = \{0_{LN}, 1_{LN}, K_{LN}, H_{LN}\}$ and $\eta_{LN} = \{0_{LN}, 1_{LN}, M_{LN}\}$. The linguistic neutrosophic sets K_{LN}, H_{LN} and M_{LN} are given by $K_{LN} = \{\langle q,(l_4, l_5, l_2)\rangle, \langle r,(l_3, l_2, l_4)\rangle\}, H_{LN} = \{\langle q,(l_4, l_6, l_4)\rangle, \langle r,(l_4, l_3, l_8)\rangle\}$ and $M_{LN} = \{\langle q,(l_4, l_6, l_4)\rangle, \langle r,(l_8, l_3, l_4)\rangle\}$ respectively. Now, the mapping f is linguistic neutrosophic quasi continuous but not linguistic neutrosophic strongly semi continuous.

Definition 3.8. A mapping from $f : S_{LN} \rightarrow T_{LN}$ is said to be a linguistic neutrosophic perfectly semi continuous mapping if the inverse image $f(E_{LN})$ of every linguistic neutrosophic semi open set E_{LN} of T_{LN} is linguistic neutrosophic cl-open set in S_{LN}.

Example 3.9. Let the universe of discourse be as in example (3.4). And let $f : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic mapping defined by $f(a) = c, f(c) = a$, where $\tau_{LN} = \{0_{LN}, 1_{LN}, E_{LN}, F_{LN}\}$ and $\eta_{LN} = \{0_{LN}, 1_{LN}, G_{LN}\}$. The linguistic neutrosophic sets E_{LN}, F_{LN} and G_{LN} are given by $E_{LN} = \{\langle q,(l_3, l_6, l_3)\rangle, \langle r,(l_2, l_5, l_2)\rangle\}, F_{LN} = \{\langle q,(l_3, l_5, l_3)\rangle, \langle r,(l_4, l_5, l_8)\rangle\}$ and $G_{LN} = \{\langle q,(l_3, l_6, l_3)\rangle, \langle r,(l_2, l_5, l_2)\rangle\}$ respectively. Now, the mapping f is linguistic neutrosophic perfectly semi continuous.

Theorem 3.10. *A mapping $f : S_{LN} \to T_{LN}$ is linguistic neutrosophic perfectly semi continuous if and only if the inverse image $f^{-1}(E_{LN})$ of every linguistic neutrosophic semi closed set E_{LN} of T_{LN} is linguistic neutrosophic cl-open set in S_{LN}.*

Proof:

Necessity Part: Let $f : S_{LN} \to T_{LN}$ be linguistic neutrosophic perfectly semi continuous and V_{LN} be any linguistic neutrosophic semi closed set in T_{LN}. Then $T_{LN} \backslash V_{LN}$ is linguistic neutrosophic semi open set in T_{LN}. As f is linguistic neutrosophic perfectly semi continuous, $f^{-1}(T_{LN} \backslash V_{LN}) = S_{LN} \backslash f^{-1}(V_{LN})$ is linguistic neutrosophic cl-open set in S_{LN}. Hence, the set $f^{-1}(V_{LN})$ is linguistic neutrosophic cl-open and thus the function f is linguistic neutrosophic perfectly semi continuous.

Sufficiency Part: Let the set $f^{-1}(V_{LN})$ be linguistic neutrosophic cl-open in S_{LN} for each linguistic neutrosophic semi closed set in T_{LN}. Let V_{LN} be any linguistic neutrosophic semi open set in T_{LN}, then $T_{LN} \backslash V_{LN}$ is linguistic neutrosophic semi closed set in T_{LN}. By assumption, the set $f^{-1}(T_{LN} \backslash V_{LN}) = S_{LN} \backslash f^{-1}(V_{LN})$ is linguistic neutrosophic cl-open in S_{LN}, which implies $f^{-1}(V_{LN})$ is linguistic neutrosophic cl-open in S_{LN}. So, the mapping f is linguistic neutrosophic perfectly semi continuous.

Theorem 3.11. *If the mapping $f : S_{LN} \to T_{LN}$ is linguistic neutrosophic perfectly semi continuous, then it is linguistic neutrosophic quasi semi continuous.*

Proof: Let V_{LN} be a linguistic neutrosophic semi open set in T_{LN}. Since f is linguistic neutrosophic perfectly semi continuous, $f^{-1}(V_{LN})$ is linguistic neutrosophic cl-open in S_{LN}. Thus, f is linguistic neutrosophic quasi semi continuous.

Remark 3.12. The converse part of the above theorem need not be true in general, which is given by a counter example.

Example 3.13. Let the linguistic term set be as in example (3.5) and $f : S_{LN} \to T_{LN}$ be any linguistic neutrosophic mapping. Now, the mapping f is linguistic neutrosophic quasi continuous but not linguistic neutrosophic perfectly semi continuous.

Theorem 3.14. *If the mapping $f : S_{LN} \to T_{LN}$ is linguistic neutrosophic strongly continuous, then it is linguistic neutrosophic perfectly semi continuous.*

Proof: Let V_{LN} be a linguistic neutrosophic semi open set in T_{LN}. Since f is linguistic neutrosophic strongly semi continuous, $f^{-1}(V_{LN})$ is linguistic neutrosophic cl-open in S_{LN}. Thus, f is linguistic neutrosophic perfectly semi continuous.

Remark 3.15. The reverse part of the above theorem need not be true in general, which is given by a counter example.

Example 3.16. Let the universe of discourse be $U = \{p, q, r, s, t\}$ and let $S_{LN} = \{p, q, r\} = T_{LN}$ and the linguistic term set be as in example (3.5). Let $f : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic identity mapping, where $\tau_{LN} = \{0_{LN}, 1_{LN}, E_{LN}\}$ and $\eta_{LN} = \{0_{LN}, 1_{LN}, A_{LN}, B_{LN}\}$. The linguistic neutrosophic sets E_{LN}, A_{LN} and B_{LN} are given by $E_{LN} = \{\langle p, (l_3, l_5, l_3)\rangle, \langle q, (l_2, l_1, l_2)\rangle, \langle r, (l_8, l_5, l_8)\rangle\}$, $A_{LN} = \{\langle p, (l_3, l_5, l_3)\rangle, \langle q, (l_2, l_1, l_2)\rangle, \langle r, (l_8, l_5, l_8)\rangle\}$ and $B_{LN} = \{\langle p, (l_7, l_5, l_2)\rangle, \langle q, (l_6, l_6, l_3)\rangle, \langle r, (l_3, l_2, l_3)\rangle\}$ respectively. The inverse image of H_{LN} in T_{LN}, is E_{LN} in S_{LN}, which is a linguistic neutrosophic cl-open set. Then the mapping f is linguistic neutrosophic perfectly semi continuous but not linguistic neutrosophic strongly continuous.

Theorem 3.17. *Let (S_{LN}, τ_{LN}) be a discrete linguistic neutrosophic topological space and (T_{LN}, η_{LN}) be any linguistic neutrosophic topological space such that $f : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ is a mapping. Then the following are equivalent.*

(1) *f is linguistic neutrosophic perfectly semi continuous*
(2) *f is linguistic neutrosophic quasi semi continuous*

Proof: (1) $\Rightarrow$ (2): Let U_{LN} be linguistic neutrosophic semi open set in (T_{LN}, η_{LN}) and the function f be linguistic neutrosophic perfectly semi continuous, (i.e) the inverse image $f^{-1}(U_{LN})$ of any linguistic neutrosophic semi open set in (T_{LN}, η_{LN}), is linguistic neutrosophic cl-open in (S_{LN}, τ_{LN}). This implies the function f is linguistic neutrosophic quasi semi continuous.
(2) $\Rightarrow$ (1): Let U_{LN} be linguistic neutrosophic semi open set in (T_{LN}, η_{LN}), then the set $f^{-1}(U_{LN})$ is linguistic neutrosophic open in (S_{LN}, τ_{LN}), since the function f is linguistic neutrosophic quasi semi continuous. Thus, $f^{-1}(U_{LN})$ is linguistic neutrosophic closed as (S_{LN}, τ_{LN}) is a discrete linguistic neutrosophic topological space, (i.e) $f^{-1}(U_{LN})$ is linguistic neutrosophic cl-open which implies the mapping f is linguistic neutrosophic perfectly semi continuous.

Theorem 3.18. *Let $f : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ and $g : (T_{LN}, \eta_{LN}) \rightarrow (P_{LN}, \mu_{LN})$ be any two mappings. Then their composition $g \circ f$ is*

(1) *linguistic neutrosophic semi continuous if g is linguistic neutrosophic strongly continuous and f is linguistic neutrosophic semi continuous.*
(2) *linguistic neutrosophic perfectly semi continuous if g is linguistic neutrosophic perfectly semi continuous and f is linguistic neutrosophic continuous.*

Proof: (1): Let $g : (T_{LN}, \eta_{LN}) \rightarrow (P_{LN}, \mu_{LN})$ be linguistic neutrosophic strongly continuous and f is linguistic neutrosophic semi continuous. Let U_{LN} be any linguistic neutrosophic closed set in (P_{LN}, μ_{LN}). Then, $g^{-1}(U_{LN})$ is linguistic neutrosophic cl-open set in (T_{LN}, η_{LN}) as g

is linguistic neutrosophic strongly semi continuous. Now, $f^{-1}(g^{-1}(U_{LN})) = (g \circ f)^{-1}(U_{LN})$ is linguistic neutrosophic semi closed set in (S_{LN}, τ_{LN}), since f is linguistic neutrosophic semi continuous. Thus, $g \circ f : (S_{LN}, \tau_{LN}) \to (P_{LN}, \mu_{LN})$ is linguistic neutrosophic semi continuous.
(2): Let f be linguistic neutrosophic continuous and g be linguistic neutrosophic perfectly semi continuous. Let U_{LN} be any linguistic neutrosophic semi closed set in (P_{LN}, μ_{LN}). Then, $g^{-1}(U_{LN})$ is linguistic neutrosophic cl-open set in (T_{LN}, η_{LN}) as g is linguistic neutrosophic perfectly semi continuous. Since f is linguistic neutrosophic continuous, $f^{-1}(g^{-1}(U_{LN})) = (g \circ f)^{-1}(U_{LN})$ is linguistic neutrosophic semi closed set in (S_{LN}, τ_{LN}), which implies $g \circ f : (S_{LN}, \tau_{LN}) \to (P_{LN}, \mu_{LN})$ is linguistic neutrosophic perfectly semi continuous.

Definition 3.19. A function $f : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ is called as linguistic neutrosophic totally semi continuous if the inverse image of every linguistic neutrosophic open subset of (T_{LN}, η_{LN}) is a linguistic neutrosophic semi cl-open subset of (S_{LN}, τ_{LN}).

Remark 3.20. It is clear that every linguistic neutrosophic totally continuous function is linguistic neutrosophic totally semi continuous but the reverse implication is not true which can be seen from the counter example.

Example 3.21. Let the universe of discourse be $U = \{x, y, z, w\}$. The set of all linguistic terms be L= { very strongly disagree(l_0), strongly disagree(l_1), disagree(l_2), mostly disagree(l_3), slightly disagree(l_4), neither disagree nor agree(l_5), slightly agree(l_6), mostly agree(l_7), agree(l_8), strongly agree(l_9), very strongly agree(l_{10})}.
And $S_{LN} = \{y, z\} = T_{LN}, \tau_{LN} = \{0_{LN}, 1_{LN}, E_{LN}, F_{LN}\}$ and $\eta_{LN} = \{0_{LN}, 1_{LN}, K_{LN}\}$, defines linguistic neutrosophic topology where, $E_{LN} = \{\langle y, (l_3, l_4, l_2)\rangle, \langle z, (l_3, l_5, l_3)\rangle\}, F_{LN} = \{\langle y, (l_2, l_4, l_3)\rangle, \langle z, (l_3, l_5, l_3)\rangle), K_{LN} = (\langle y, l_4, l_2, l_3\rangle, \langle z, l_5, l_3, l_3\rangle)$. The mapping A function $f : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN}$ is defined by $f(a) = c, f(b) = a, f(c) = b$. Then f is a linguistic neutrosophic totally semi continuous function but not linguistic neutrosophic totally continuous.

Definition 3.22. A function $f : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ is called as linguistic neutrosophic strongly semi continuous if the inverse image of every linguistic neutrosophic subset of (T_{LN}, η_{LN}) is a linguistic neutrosophic semi cl-open subset of (S_{LN}, τ_{LN}).

Remark 3.23. Obviously, LN strong semi continuity $\Rightarrow$ LN totally semi continuity $\Rightarrow$ LN semi continuity.
Given below is an example of a linguistic neutrosophic function which is linguistic neutrosophic totally semi continuous but not linguistic neutrosophic strongly semi continuous.

Example 3.24. In example (3.5), the mapping f is linguistic neutrosophic semi continuous. Clearly, the inverse image $f^{-1}(H_{LN})$ is not linguistic neutrosophic closed and hence it is not linguistic neutrosophic semi cl-open. Thus, f is not linguistic neutrosophic totally semi continuous and linguistic neutrosophic strongly semi continuous.

Theorem 3.25. *Every linguistic neutrosophic totally semi continuous function into T_1 space is linguistic neutrosophic strongly semi continuous.*

Proof:

In a T_1 space all linguistic neutrosophic singleton sets are closed. Hence, $f^{-1}(A_{LN})$ is linguistic neutrosophic semi cl-open in S_{LN} for every linguistic neutrosophic subset A_{LN} of T_{LN}.

Definition 3.26. A function $f : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ is called as linguistic neutrosophic slightly semi continuous if for every $s \in S_{LN}$ and for each cl-open subset V_{LN} of T_{LN} containing $f(s)$, there exists a linguistic neutrosophic semi open subset U_{LN} of S_{LN} such that $s \in U_{LN}$ and $f(U_{LN}) \subseteq V_{LN}$.

Theorem 3.27. *Every linguistic neutrosophic slightly semi continuous function into a linguistic neutrosophic discrete space is linguistic neutrosophic strongly semi continuous.*

Proof:

Let $f : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic slightly semi continuous function from a linguistic neutrosophic space S_{LN} into a linguistic neutrosophic discrete space T_{LN}. Let A_{LN} be any linguistic neutrosophic subset of T_{LN}, then A_{LN} is a linguistic neutrosophic cl-open subset of T_{LN}. Hence $f^{-1}(A_{LN})$ is linguistic neutrosophic cl-open set of S_{LN}. Thus, f is linguistic neutrosophic strongly semi continuous.

Theorem 3.28. *If $f : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ is linguistic neutrosophic slightly semi continuous function and $g : (T_{LN}, \eta_{LN}) \to (R_{LN}, \mu_{LN})$ is linguistic neutrosophic totally continuous, then $g \circ f$ is linguistic neutrosophic totally semi continuous.*

Proof:

Let A_{LN} be linguistic neutrosophic open subset of (R_{LN}, μ_{LN}). Then $g^{-1}(A_{LN})$ is a linguistic neutrosophic semi cl-open subset of (T_{LN}, η_{LN}). As f is linguistic neutrosophic slightly semi continuous, we have, $f^{-1}(g^{-1}(A_{LN})) = (g \circ f)^{-1}(A_{LN})$ is linguistic neutrosophic semi cl-open subset of (S_{LN}, τ_{LN}). Hence $g \circ f$ is linguistic neutrosophic totally semi continuous.

Definition 3.29. A function $f : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ is called as linguistic neutrosophic totally continuous if the inverse image of every linguistic neutrosophic open subset of (T_{LN}, η_{LN}) is a linguistic neutrosophic cl-open subset of (S_{LN}, τ_{LN}).

Definition 3.30. A function $f : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ is called as linguistic neutrosophic semi totally continuous if the inverse image of every linguistic neutrosophic semi open subset of (T_{LN}, η_{LN}) is a linguistic neutrosophic cl-open subset of (S_{LN}, τ_{LN}).

Example 3.31. Let the universe of discourse be $U = \{a, b, c, d, e\}$ and let $S_{LN} = \{b\} = T_{LN}$. The set of all linguistic term set be L= {no healing(l_0), deterioting(l_1), chronic(l_2), some what chronic(l_3), extremely chronic(l_4), very ill(l_5), ill(l_6), no healing(l_7), healing(l_8), slowly healing(l_9), fastly healing(l_{10})}. Let $f : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be the linguistic neutrosophic mapping defined by $f(b) = c, f(c) = b$. And $\tau_{LN} = \{0_{LN}, 1_{LN}, \langle b, (l_3, l_1, l_3)\rangle\}$ and $\eta_{LN} = \{0_{LN}, 1_{LN}, \langle b, (l_3, l_2, l_1)\rangle, \langle b, (l_1, l_1, l_2)\rangle\}$ be linguistic neutrosophic topologies. The set $E_{LN} = \langle b, (l_3, l_2, l_1)\rangle$ is linguistic neutrosophic semi open subset of (T_{LN}, η_{LN}). The inverse image $f^{-1}(E_{LN})$ in (S_{LN}, τ_{LN}) is both linguistic neutrosophic semi closed and linguistic neutrosophic semi open. Thus, the map f is linguistic neutrosophic semi totally continuous.

Theorem 3.32. *A function $f : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ is linguistic neutrosophic semi totally continuous if and only if the inverse image of each linguistic neutrosophic semi closed subset of (T_{LN}, η_{LN}) is a linguistic neutrosophic cl-open subset of (S_{LN}, τ_{LN}).*

Proof:

Let K_{LN} be any linguistic neutrosophic semi closed subset in (T_{LN}, η_{LN}), then $T_{LN} \backslash K_{LN}$ is linguistic neutrosophic semi open subset of T_{LN}. Then $f^{-1}(T_{LN} \backslash K_{LN})$ is linguistic neutrosophic cl-open in S_{LN}, (i.e) $S_{LN} \backslash f^{-1}(K_{LN})$ is linguistic neutrosophic cl-open in S_{LN}. Therefore, $f^{-1}(K_{LN})$ is linguistic neutrosophic cl-open in S_{LN}.

Conversely, if H_{LN} is linguistic neutrosophic semi open subset of (T_{LN}, η_{LN}), then $T_{LN} \backslash H_{LN}$ is linguistic neutrosophic semi closed subset of T_{LN}. Then, $f^{-1}(T_{LN} \backslash H_{LN}) = S_{LN} \backslash f^{-1}(H_{LN})$ is linguistic neutrosophic cl-open in S_{LN} and hence $f^{-1}(H_{LN})$ is linguistic neutrosophic cl-open in S_{LN}. Therefore, the inverse image of every linguistic neutrosophic semi open subset of T_{LN} is a linguistic neutrosophic cl-open subset of S_{LN}. Thus, f is linguistic neutrosophic semi totally continuous.

Theorem 3.33. *Every linguistic neutrosophic semi totally continuous mapping is a linguistic neutrosophic totally continuous mapping.*

Proof:

Let H_{LN} be any linguistic neutrosophic open subset of T_{LN}, where $f : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ is linguistic neutrosophic semi totally continuous mapping. As each linguistic neutrosophic open set is linguistic neutrosophic semi open, H_{LN} is linguistic neutrosophic semi open in T_{LN} and $f^{-1}(H_{LN})$ is linguistic neutrosophic cl-open subset of S_{LN}. Thus, the inverse image of every open subset of T_{LN} is linguistic neutrosophic cl-open in S_{LN} which implies, f is totally continuous. The converse part is not holds true which is given by a counter example.

Example 3.34. Let the universe of discourse be $U = \{x, y, z\}$ and $S_{LN} = \{y, z\} = T_{LN}$. The set of all linguistic term set be L= {no healing(l_0), deterioting(l_1), chronic(l_2), some what chronic(l_3), extremely chronic(l_4), very ill(l_5), ill(l_6), no healing(l_7), healing(l_8), slowly healing(l_9), fastly healing(l_{10})}.

Let $f : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be a linguistic neutrosophic mapping,defined by $f(b) = c, f(c) = b$ where $\tau_{LN} = \{0_{LN}, 1_{LN}, K_{LN}\}$ and $\eta_{LN} = \{0_{LN}, 1_{LN}, H_{LN}\}$. The linguistic neutrosophic sets K_{LN} and H_{LN} are given by $K_{LN} = \{\langle y, (l_4, l_5, l_4)\rangle, \langle z, (l_9, l_9, l_9)\rangle\}$ and $H_{LN} = \{\langle y, (l_5, l_4, l_4)\rangle, \langle z, (l_9, l_9, l_9)\rangle\}$ respectively. The inverse image $f^{-1}(H_{LN})$ is linguistic neutrosophic cl-open in S_{LN}. Thus, the map f is linguistic neutrosophic totally continuous.

Let $D_{LN} = \langle y, (l_2, l_4, l_5)\rangle, \langle z, (l_6, l_5, l_9)\rangle$ be any linguistic neutrosophic set in T_{LN}. Then D_{LN} is linguistic neutrosophic semi open but the inverse image is not linguistic neutrosophic cl-open subset of S_{LN}. Thus, the map f is not linguistic neutrosophic semi totally continuous.

Theorem 3.35. *Every linguistic neutrosophic semi totally continuous mapping is a linguistic neutrosophic totally semi continuous mapping.*

Proof:

Let H_{LN} be any linguistic neutrosophic open subset of T_{LN}, where $f : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ is linguistic neutrosophic semi totally continuous mapping. As each linguistic neutrosophic open set is linguistic neutrosophic semi open, H_{LN} is linguistic neutrosophic semi open in T_{LN} and $f^{-1}(H_{LN})$ is linguistic neutrosophic semi cl-open subset of S_{LN}, as f is linguistic neutrosophic semi totally continuous mapping. Thus, the inverse image of every open subset of T_{LN} is linguistic neutrosophic semi cl-open in S_{LN} which implies, f is totally semi continuous. The converse part is not holds true which is given by a counter example.

Example 3.36. In example (3.5), let $S_{LN} = \{q, t\} = T_{LN}$. Let $f : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ be the linguistic neutrosophic mapping defined by $f(b) = c, f(c) = b$. And $\tau_{LN} = \{0_{LN}, 1_{LN}, K_{LN}\}$ and $\eta_{LN} = \{0_{LN}, 1_{LN}, H_{LN}\}$ be linguistic neutrosophic topologies.The linguistic neutrosophic sets K_{LN} and H_{LN} are given by $K_{LN} = \{\langle q, (l_3, l_4, l_2)\rangle, \langle t, (l_5, l_5, l_2)\rangle\}$ and $H_{LN} = \{\langle q, (l_3, l_3, l_2)\rangle, \langle ts, (l_4, l_4, l_5)\rangle\}$ respectively. Linguistic neutrosophic semi open sets in S_{LN} are $\{0_{LN}, 1_{LN}, \langle q, (l_3, l_2, l_3)\rangle, \langle t, (l_4, l_5, l_4)\rangle\}$ and in T_{LN} are $\{0_{LN}, 1_{LN}, \langle q, (l_4, l_5, l_0)\rangle, \langle t, (l_4, l_6, l_2)\rangle\}$.

Then the inverse image of the linguistic neutrosophic open set in (T_{LN}, η_{LN}) is linguistic neutrosophic semi cl-open in (S_{LN}, τ_{LN}) whereas the inverse image of the linguistic neutrosophic semi open set in T_{LN} is not linguistic neutrosophic semi cl-open. Thus, the map f is linguistic neutrosophic totally semi continuous but not linguistic neutrosophic semi totally continuous.

Theorem 3.37. *Every linguistic neutrosophic strongly continuous mapping is a linguistic neutrosophic semi totally continuous mapping.*

Proof:

Let $f : (S_{LN}, \tau_{LN}) \to (T_{LN}, \eta_{LN})$ is linguistic neutrosophic strongly continuous mapping and

M_{LN} be any linguistic neutrosophic semi open subset of T_{LN}. Now, $f^{-1}(M_{LN})$ is linguistic neutrosophic semi cl-open subset of S_{LN}, by definition. Thus, the inverse image of every semi open subset of T_{LN} is linguistic neutrosophic cl-open in S_{LN} which implies,f is semi totally continuous. The converse part is not holds true which is given by a counter example.

Example 3.38. In example (3.5), let $S_{LN} = \{q, t\} = T_{LN}$. Let $f : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ be the linguistic neutrosophic identity mapping. And $\tau_{LN} = \{0_{LN}, 1_{LN}, K_{LN}\}$ and $\eta_{LN} = \{0_{LN}, 1_{LN}, H_{LN}\}$ be linguistic neutrosophic topologies.The linguistic neutrosophic sets K_{LN} and H_{LN} are given by $K_{LN} = \{\langle q, (l_3, l_2, l_3)\rangle, \langle t, (l_4, l_5, l_4)\rangle\}$ and $H_{LN} = \{\langle q, (l_3, l_4, l_2)\rangle, \langle t, (l_5, l_5, l_3)\rangle\}$ respectively. Linguistic neutrosophic semi open sets in T_{LN} are $\{0_{LN}, 1_{LN}, \langle q, (l_3, l_2, l_3)\rangle, \langle t, (l_4, l_5, l_4)\rangle\}$.
Now, the inverse image of the linguistic neutrosophic semi open set in T_{LN} is linguistic neutrosophic open in S_{LN}. Let $\{\langle q, (l_3, l_6, l_0)\rangle, \langle t, (l_3, l_3, l_1)\rangle\}$ be any linguistic neutrosophic set whose inverse image is neither linguistic neutrosophic open nor linguistic neutrosophic closed in S_{LN}. Therefore, the map f is linguistic neutrosophic semi totally continuous but not linguistic neutrosophic strongly continuous mapping.

Theorem 3.39. *Let $f : (S_{LN}, \tau_{LN}) \rightarrow (T_{LN}, \eta_{LN})$ be a mapping, from a linguistic neutrosophic topological space (S_{LN}, τ_{LN}) into a linguistic neutrosophic topological space (T_{LN}, η_{LN}). Then the following statements are equivalent.*

(1) *f is linguistic neutrosophic semi totally continuous mapping.*
(2) *for every $s \in S_{LN}$ and for each linguistic neutrosophic semi open set M_{LN} in (T_{LN}, η_{LN}) with $f(s) \in T_{LN}$, there exists a linguistic neutrosophic cl-open set K_{LN} in S_{LN} such that $s \in K_{LN}$ and $f(K_{LN}) \subset M_{LN}$.*

Proof:
(1) $\Rightarrow$ (2): If f is linguistic neutrosophic semi totally continuous and M_{LN} be any linguistic neutrosophic semi open set in (T_{LN}, η_{LN}) containing $f(s)$ so that $s \in f^{-1}(M_{LN})$. Since f is linguistic neutrosophic semi totally continuous, $f^{-1}(M_{LN})$ is linguistic neutrosophic cl-open in S_{LN} and $s \in M_{LN}$. Also, let $K_{LN} = f^{-1}(M_{LN})$, then $f(K_{LN}) = f(f^{-1}(M_{LN})) \subset M_{LN}$ which implies $f(K_{LN}) \subset M_{LN}$.
(2) $\Rightarrow$ (1): Let M_{LN} be linguistic neutrosophic semi open set in T_{LN} and $s \in f^{-1}(M_{LN})$ be any arbitrary linguistic neutrosophic point, then $f(s) \in M_{LN}$. Thus, from the assumption, there exists a linguistic neutrosophic cl-open set $f(G_{LN}) \in S_{LN}$ containing s such that $f(G_{LN}) \subset M_{LN}$, which implies $s \in G_{LN} \subset f^{-1}(M_{LN})$.
Now, $f^{-1}(M_{LN})$ is linguistic neutrosophic cl-open neighborhood of s. As s is arbitrary, $f^{-1}(M_{LN})$ is linguistic neutrosophic cl-open neighborhood of each of its points. Thus,

$f^{-1}(M_{LN})$ is linguistic neutrosophic cl-open set in S_{LN} and hence f is linguistic neutrosophic semi totally continuous.

Remark 3.40. The implications of all linguistic neutrosophic continuous functions are given below.

$$\begin{bmatrix} & A & B & C & D & E & F & G \\ A & - & 1 & 0 & 1 & 0 & 0 & 1 \\ B & 0 & - & 0 & 0 & 0 & 0 & 0 \\ C & 0 & 1 & - & 1 & 0 & 0 & 0 \\ D & 0 & 0 & 0 & - & 0 & 0 & 0 \\ E & 1(indiscrete) & 0 & 0 & 0 & - & 0 & 0 \\ F & 0 & 0 & 0 & 0 & 0 & - & 0 \\ G & 1(T1) & 0 & 0 & 0 & 0 & 0 & - \end{bmatrix}$$

where, A - LN strongly semi continuous, B - LN quasi semi continuous, C - LN perfectly semi continuous, D - LN contra semi continuous, E - LN slightly semi continuous, F - LN semi totally continuous, G - LN totally semi continuous, respectively.

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