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Abstract. Models of universe problems are brimming with complexities and uncertainties in almost every field of study, including engineering, mathematics, medical sciences, computer science, physics, management sciences, artificial intelligence, and operations research. To address these uncertainties, various theories have been developed, including probability, rough sets, fuzzy sets, soft ideals, and neutrosophic sets. Neutrosophic set theory is the focus of this paper. In this paper, we introduce the notions of neutrosophic ℵ-filters and neutrosophic ℵ-bi-filters in a semigroup and investigate several properties. Moreover, the relations of prime bi-ideal subset and prime neutrosophic ℵ-bi- ideal structure; neutrosophic ℵ-bi-filter and neutrosophic ℵ-bi-ideal structure; left (resp., right) filter and neutrosophic ℵ-left (resp., right) filter; neutrosophic ℵ-left(resp., right) filter and prime neutrosophic ℵ-left (resp., right) ideals in semigroups are discussed. Finally we prove that: let X be a semigroup and X_N be any neutrosophic structure. Then X_N is a neutrosophic ℵ-bi-filter of X if and only if X_{N^c} is a prime neutrosophic ℵ-bi-ideal of X.

Keywords: Semigroup; fuzzy sets; filter; bi-ideal; neutrosophic ℵ-bi-ideals.

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1. Introduction

In 1965, L.A. Zadeh [22] introduced the idea of Fuzzy sets which were represented using membership functions. Rather than a classic set, in the case of a fuzzy set A, x is an object that belong to this set with varying membership degrees in the range $[0, 1]$, where 0 and 1 denote, respectively, lack of membership and full membership. The investigation of algebraic structures has begun with the presentation of the idea of fuzzy subgroups in the spearheading paper of Rosenfeld [18]. Subsequently, many authors further studied fuzzy concept in semigroups(See [9–11, 19]). Of several higher-order fuzzy sets, the intuitionistic fuzzy set presented by Atanassov [3] has been seen as a profoundly useful idea in managing vagueness. Following the introduction of the intuitionistic fuzzy set concept, mathematicians published several papers extending classical and fuzzy mathematical concepts to the case of intuitionistic fuzzy mathematics.

In 1999, F. Smarandache [20] introduced the concept of neutrosophic set, which is the generalizations of fuzzy sets and intuitionistic fuzzy set. Neutrosophic set is a useful mathematical tool for dealing with incomplete, inconsistent and indeterminate information. The neutrosophic set theory is applied to algebraic structures, multiple attribute decision-making, and so on [1, 2, 6, 7, 12–17, 21].

For additional informations about neutrosophic set theory, we refer the readers to the below website *http://fs.unm.edu/neutrosophy.htm*.

In [12], M. Khan et al. introduced and investigated the concept of a neutrosophic ℵ-sub semigroup of a semigroup. The conditions for neutrosophic ℵ-structure to be neutrosophic ℵ-subsemigroup were given, and the characterization of neutrosophic ℵ-subsemigroup was discussed using neutrosophic ℵ-product. They also proved that the homomorphic preimage of a neutrosophic ℵ-subsemigroup is a neutrosophic ℵ-subsemigroup and that the onto homomorphic image of a neutrosophic ℵ-subsemigroup is a neutrosophic ℵ-subsemigroup. The notions of neutrosophic ℵ-ideals and neutrosophic ℵ- bi-ideals were defined to semigroups and obtained many useful results (See [5, 17]).

As a follow-up, in this paper we define the concept of neutrosophic ℵ-left (resp., bi-)filters in semigroup and describe the semigroup in terms of these notions. We also define prime neutrosophic ℵ-left ideals and prime neutrosophic ℵ-bi-ideal structures of semigroup and characterize the relations of neutrosophic ℵ-left filters and prime neutrosophic ℵ-left ideals in semigroups.

Throughout this paper, X denotes a semigroup and for $K, S \subseteq X$, we denote $KS := \{ks : k \in K, s \in S\}$.

Definition 1.1. [4] Let X be a semigroup and $\phi \neq K \subseteq X$. Then

(i) K is called a *subsemigroup* of X if $K^2 \subseteq K$.
(ii) K is called a *left (resp., right) ideal* of X if $XK \subseteq K$ (resp., $KX \subseteq K$).
(iii) If K is both a left and a right ideal of X, then it is called an *ideal* of X.
(iv) K is called a *bi-ideal subset* of X if $k \in K$ and $s \in X$ imply $ksk \in K$.

Definition 1.2. [10] Let X be a semigroup and K a subsemigroup of X. Then

(i) K is called *left (resp., right) filter* of X if $r, s \in X$, $rs \in K$ implies $s \in K$ (resp., $r \in K$).

(ii) K is called a *bi-filter* of X if $r, s \in X$, $rsr \in K$ implies $r \in K$.

Definition 1.3. [11] Let X be a semigroup and $\phi \neq K \subseteq X$. Then

(i) K is called a *prime subset* of X if $r, s \in X$, $rs \in K$ implies $r \in K$ or $s \in K$.
Equivalently, $S, T \subseteq X$, $ST \subseteq K$ implies $S \subseteq K$ or $T \subseteq K$.

(ii) K is called a *semiprime subset* of X if $r \in X$, $r^2 \in K$ implies $r \in K$.
Equivalently, $S \subseteq X$, $S^2 \subseteq K$ implies $S \subseteq K$.

2. Preliminary definitions and results of Neutrosophic ℵ- structure

In this section, we present the necessary fundamental concepts of neutrosophic ℵ-structures of X that we need in the sequel.

For a semigroup X, $\mathcal{F}(X, [-1, 0])$ is the collection of negative-valued functions from a set X to $[-1, 0]$. An element $g \in \mathcal{F}(X, [-1, 0])$ is called a ℵ-function on X and ℵ-structure means (X, g) of X.

Definition 2.1. [12] A *neutrosophic* ℵ- *structure* of X is defined to be the structure:

$$X_M := \frac{X}{(T_M, I_M, F_M)} = \left\{ \frac{l}{T_M(l), I_M(l), F_M(l)} \ : \ l \in X \right\}$$

where T_M is the negative truth membership function on X, I_M is the negative indeterminacy membership function on X and F_M is the negative falsity membership function on X.

Note that for any $k \in X, X_M$ fulfills the condition $-3 \leq T_M(k) + I_M(k) + F_M(k) \leq 0$.

Definition 2.2. For a subset K of X, consider the neutrosophic ℵ-structure

$$\chi_K(X_N) = \frac{X}{(\chi_K(T)_N, \chi_K(I)_N, \chi_K(F)_N)}$$

where

$$\chi_K(T)_N : X \to [-1, 0], \ x \to \begin{cases} -1 & if \ x \in K \\ 0 & if \ x \notin K, \end{cases}$$

$$\chi_K(I)_N : X \to [-1, 0], \ x \to \begin{cases} 0 & if \ x \in K \\ -1 & if \ x \notin K, \end{cases}$$

$$\chi_K(F)_N : X \to [-1, 0], \ x \to \begin{cases} -1 & if \ x \in K \\ 0 & if \ x \notin K, \end{cases}$$

which is called the *characteristic neutrosophic* ℵ*-structure* of K over X.

Definition 2.3. [12] Let X be a semigroup. Then for any $X_N := \frac{X}{(T_N, I_N, F_N)}$ and $X_M := \frac{X}{(T_M, I_M, F_M)}$.

(i) X_M is called a *neutrosophic* $\aleph$*-substructure* of X_N, denoted by $X_N \subseteq X_M$, if it satisfies the below condition for any $l \in X$,

$$T_N(l) \geq T_M(l), I_N(l) \leq I_M(l), F_N(l) \geq F_M(l).$$

If $X_N \subseteq X_M$ and $X_M \subseteq X_N$, then we say that $X_N = X_M$.

(ii) The union of X_N and X_M is a neutrosophic $\aleph$-structure over X is defined as

$$X_N \cup X_M = X_{N\cup M} = (X; T_{N\cup M}, I_{N\cup M}, F_{N\cup M}),$$

where

$$(T_N \cup T_M)(k) = T_{N\cup M}(k) = T_N(k) \wedge T_M(k),$$
$$(I_N \cup I_M)(k) = I_{N\cup M}(k) = I_N(k) \vee I_M(k),$$
$$(F_N \cup F_M)(k) = F_{N\cup M}(k) = F_N(k) \wedge F_M(k) \ for\ any\ k \in X.$$

(iii) The intersection of X_N and X_M is a neutrosophic $\aleph$-structure over X is defined as

$$X_N \cap X_M = X_{N\cap M} = (X; T_{N\cap M}, I_{N\cap M}, F_{N\cap M}),$$

where

$$(T_N \cap T_M)(k) = T_{N\cap M}(k) = T_N(k) \vee T_M(k),$$
$$(I_N \cap I_M)(k) = I_{N\cap M}(k) = I_N(k) \wedge I_M(k),$$
$$(F_N \cap F_M)(k) = F_{N\cap M}(k) = F_N(k) \vee F_M(k) \ for\ any\ k \in X.$$

Definition 2.4. [12] Let $X_N = \frac{X}{(T_N, I_N, F_N)}$. Then the *complement of* X_N, denoted by X_{N^c} over U, is defined to be a neutrosophic $\aleph$-structure

$$X_{N^c} := \frac{X}{(T_{N^c}, I_{N^c}, F_{N^c})},$$

over X, where $T_{N^c}(l) = -1 - T_N(l); I_{N^c}(l) = -1 - I_N(l)$ and $F_{N^c}(l) = -1 - F_N(l)\ \forall l \in X$.

Definition 2.5. [12] Let $X_N = \frac{X}{(T_N, I_N, F_N)}$ and $\mu, \lambda, \nu \in [-1, 0]$ with $-3 \leq \mu + \lambda + \nu \leq 0$. Consider the following sets:

$$T_N^{\mu} = \{k \in X \mid T_N(k) \leq \mu\},$$
$$I_N^{\lambda} = \{k \in X \mid I_N(k) \geq \lambda\},$$
$$I_N^{\nu} = \{k \in X | F_N(k) \leq \nu\}.$$

Then the set $X_N(\mu, \lambda, \nu) = \{k \in X | T_N(k) \leq \mu, I_N(k) \geq \lambda, F_N(k) \leq \nu\}$ is called a (μ, λ, ν)-*level set* of X_N. Note that $X_N(\mu, \lambda, \nu) = T_N^{\mu} \cap I_N^{\lambda} \cap F_N^{\nu}$.

Definition 2.6. [12] A neutrosophic $\aleph$-structure X_M of X is called a *neutrosophic* $\aleph$-*subsemigroup* if it satisfies:

$$(\forall\ k, s \in X) \begin{pmatrix} T_M(ks) \leq T_M(k) \vee T_M(s) \\ I_M(ks) \geq I_M(k) \wedge I_M(s) \\ F_M(ks) \leq F_M(k) \vee F_M(s) \end{pmatrix}.$$

Definition 2.7. [5] A neutrosophic ℵ-structure X_M of X is called a *neutrosophic ℵ-left (resp., right) ideal* if it satisfies the below condition: for any $k, s \in X$

$$\begin{pmatrix} T_M(ks) \leq T_M(s)(resp., T_M(ks) \leq T_M(k)) \\ I_M(ks) \geq I_M(s)(resp., I_M(ks) \geq I_M(k)) \\ F_M(ks) \leq F_M(s)(resp., F_M(ks) \leq F_M(k)) \end{pmatrix}.$$

If X_M is both a neutrosophic ℵ-left and a neutrosophic ℵ-right ideal of X, then it is called a *neutrosophic ℵ-ideal* of X.

Definition 2.8. A neutrosophic ℵ-subsemigroup X_M is called a *neutrosophic ℵ-left(resp., right) filter* of X if it satisfies the below condition: for any $k, s \in X$

$$\begin{pmatrix} T_M(ks) \geq T_M(s)(resp., T_M(ks) \geq T_M(k)) \\ I_M(ks) \leq I_M(s)(resp., I_M(ks) \leq I_M(k)) \\ F_M(ks) \geq F_M(s)(resp., F_M(ks) \geq F_M(k)) \end{pmatrix}.$$

Definition 2.9. A neutrosophic ℵ-subsemigroup X_M is called a *neutrosophic ℵ-filter* if it both a neutrosophic ℵ-left filter and a neutrosophic ℵ-right filter of X.

Equivalently, a neutrosophic ℵ-subsemigroup X_M over X is called a *neutrosophic ℵ-filter* of X if it satisfies:

$$(\forall k, s \in X) \begin{pmatrix} T_M(ks) = T_M(k) \vee T_M(s) \\ I_M(ks) = I_M(k) \wedge I_M(s) \\ F_M(ks) = F_M(k) \vee F_M(s) \end{pmatrix}.$$

The following example shows that there are some neutrosophic ℵ-subsemigroups in X, which are neither neutrosophic ℵ-left filters nor neutrosophic ℵ-right filters of X.

Example 2.10. Consider the semigroup X, the set of all positive integers, with respect to multiplication. Then $X_N = \left\{ \frac{k}{(-\frac{1}{k}, 0, -\frac{1}{k})} : k \in X \right\}$ is a neutrosophic ℵ-subsemigroup of X, but not a neutrosophic ℵ-left filter as well as not a neutrosophic ℵ-right filter of X. □

Example 2.11. Let $X = \{1, 2, 3, 4, 5\}$ be a finite semigroup with the below multiplication table:

.	1	2	3	4	5
1	1	1	1	1	1
2	1	2	3	1	1
3	1	1	1	2	3
4	1	4	5	1	1
5	1	1	1	4	5

Then $X_N = \left\{ \frac{1}{(-0.5,-0.7,-0.4)}, \frac{2}{(-0.4,-0.8,-0.3)}, \frac{3}{(-0.4,-0.8,-0.3)}, \frac{4}{(-0.4,-0.7,-0.3)}, \frac{5}{(-0.4,-0.7,-0.3)} \right\}$ is a neutrosophic ℵ-subsemigroup of X. Here $I_N(3.3) \nleq I_N(3)$. So X_N is neither a neutrosophic ℵ-left filter nor a neutrosophic ℵ-right filter of X. □

Example 2.12. Let $X = \{k, r, s\}$ be a semigroup with the below multiplication table:

.	k	r	s
k	k	k	k
r	r	r	r
s	s	s	s

Then $X_N = \left\{ \frac{k}{(-0.5,-0.5,-0.7)}, \frac{r}{(-0.4,-0.6,-0.6)}, \frac{s}{(-0.3,-0.7,-0.5)} \right\}$ is a neutrosophic $\aleph$-right filter, but not a neutrosophic $\aleph$-left filter of X as $T_N(kr) \ngeq T_N(r), I_N(kr) \nleq I_N(r)$ and $F_N(kr) \ngeq F_N(r)$. □

Definition 2.13. A neutrosophic structure X_N of X is a *neutrosophic* $\aleph$-*bi-ideal structure* if it satisfies:

$$(\forall k, s \in X) \begin{pmatrix} T_N(ksk) \leq T_N(k) \\ I_N(ksk) \geq I_N(k) \\ F_N(ksk) \leq F_N(k) \end{pmatrix}.$$

Definition 2.14. A neutrosophic $\aleph$-subsemigroup X_N of X is called a *neutrosophic* $\aleph$-*bi-filter* if it satisfies:

$$(\forall k, s \in X) \begin{pmatrix} T_N(ksk) \geq T_N(k) \\ I_N(ksk) \leq I_N(k) \\ F_N(ksk) \geq F_N(k) \end{pmatrix}.$$

Example 2.15. Let X be the set of all non-negative integers except one. Then X is a semigroup with usual multiplication.

Consider $X_M = \left\{ \begin{array}{c} \frac{0}{(-0.1,-0.8,-0.1)}, \frac{2}{(-0.6,-0.5,-0.6)}, \frac{3}{(-0.7,-0.4,-0.8)}, \frac{6}{(-0.8,-0.3,-0.9)}, \\ \frac{otherwise}{(-0.2,-0.6,-0.3)} \end{array} \right\}$.

Then X_M is a neutrosophic $\aleph$-bi-filter of X, but not a filter as $T_N(2.3) = T_N(6) = -0.8 \ngeq T_N(3)$. □

Definition 2.16. Let $X_N = \frac{X}{(T_N, I_N, F_N)}$. Then X_N is called *prime neutrosophic* $\aleph$-*structure* of X if it satisfies:

$$(\forall k, s \in X) \begin{pmatrix} T_N(ks) \geq T_N(k) \wedge T_N(s) \\ I_N(ks) \leq I_N(k) \vee I_N(s) \\ F_N(ks) \geq F_N(k) \wedge F_N(s) \end{pmatrix}.$$

Definition 2.17. Let $X_N = \frac{X}{(T_N, I_N, F_N)}$. Then X_N is called *semiprime neutrosophic* $\aleph$-*structure* of X if it satisfies:

$$(\forall k \in X) \begin{pmatrix} T_N(k^2) \geq T_N(k) \\ I_N(k^2) \leq I_N(k) \\ F_N(k^2) \geq F_N(k) \end{pmatrix}.$$

Note 2.18. *Clearly every prime neutrosophic* $\aleph$-*structure of* X *is a semi prime neutrosophic* $\aleph$-*structure of* X, *but converse is not true.*

Example 2.19. Let $X = \{0, k, r, s\}$ be a semigroup with the following multiplication table:

.	0	k	r	s
0	0	0	0	0
k	0	0	s	r
r	0	s	0	k
s	0	r	k	0

Then $X_N = \left\{ \frac{0}{(-0.1,-0.9,-0.2}, \frac{k}{(-0.4,-0.5,-0.6)}, \frac{r}{(-0.5,-0.6,-0.7)}, \frac{s}{(-0.6,-0.4,-0.8)} \right\}$ is a semi-prime neutrosophic ℵ- structure of X, but it is not a prime neutrosophic ℵ-structure of X since $T_N(kr) \ngeq T_N(k) \wedge T_N(r); I_N(kr) \nleq I_N(k) \vee I_N(r)$ and $F_N(kr) \ngeq F_N(k) \wedge F_N(r)$. □

3. **Neutrosophic ℵ-filters and Neutrosophic ℵ-bi-filters**

Lemma 3.1. *Let* $X_N = \frac{X}{(T_N, I_N, F_N)}$; $X_M = \frac{X}{(T_M, I_M, F_M)}$ *and* $X_O = \frac{X}{(T_O, I_O, F_O)}$. *Then*

(i) $X_N \subseteq X_M$ *if and only if* $X_{N^c} \supseteq X_{M^c}$.
(ii) $X_O \subseteq X_N \cup X_M$ *if and only if* $X_{O^c} \supseteq X_{N^c} \cap X_{M^c}$.
(iii) $X_O \subseteq X_N \cap X_M$ *if and only if* $X_{O^c} \supseteq X_{N^c} \cup X_{M^c}$.

Proof: (i) For any $a \in X$, we have

$$X_N \subseteq X_M \Leftrightarrow \begin{pmatrix} T_N(a) \geq T_M(a) \\ I_N(a) \leq I_M(a) \\ F_N(a) \geq F_M(a) \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} -T_N(a) \leq -T_M(a) \\ -I_N(a) \geq -I_M(a) \\ -F_N(a) \leq -F_M(a) \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} -1-T_N(a) \leq -1 - T_M(a) \\ -1-I_N(a) \geq -1-I_M(a) \\ -1 - F_N(a) \leq -1 - F_M(a) \end{pmatrix}$$

$$\Leftrightarrow X_{N^c} \supseteq X_{M^c}.$$

(ii) For any $a \in X$, we have

$$X_O \subseteq X_M \cup X_N$$
$$\Leftrightarrow \begin{pmatrix} T_O(a) \geq T_M(a) \wedge T_N(a) \\ I_O(a) \leq I_M(a) \vee I_N(a) \\ F_O(a) \geq F_M(a) \wedge F_N(a) \end{pmatrix}$$
$$\Leftrightarrow \begin{pmatrix} -T_O(a) \leq -(T_M(a) \wedge T_N(a)) \\ -I_O(a) \geq -(I_M(a) \vee I_N(a)) \\ -F_O(a) \leq -(F_M(a) \wedge F_N(a)) \end{pmatrix}$$
$$\Leftrightarrow \begin{pmatrix} -T_O(a) \leq -T_M(a) \vee -T_N(a)) \\ -I_O(a) \geq -I_M(a) \wedge -I_N(a)) \\ -F_O(a) \leq -F_M(a) \vee -F_N(a)) \end{pmatrix}$$
$$\Leftrightarrow \begin{pmatrix} -1 - T_O(a) \leq (-1 - T_M(a)) \vee (-1 - T_N(a)) \\ -1 - I_O(a) \geq (-1 - I_M(a)) \wedge (-1 - I_N(a)) \\ -1 - F_O(a) \leq (-1 - F_M(a)) \vee (-1 - F_N(a)) \end{pmatrix}$$
$$\Leftrightarrow X_{O^c} \supseteq X_{M^c} \cap X_{N^c}.$$

(iii) For any $a \in X$, we have

$$X_O \subseteq X_M \cap X_N$$
$$\Leftrightarrow \begin{pmatrix} T_O(a) \geq T_M(a) \ \vee \ T_N(a) \\ I_O(a) \leq I_M(a) \ \wedge \ I_N(a) \\ F_O(a) \geq F_M(a) \ \vee \ F_N(a) \end{pmatrix}$$
$$\Leftrightarrow \begin{pmatrix} -T_O(a) \leq -(T_M(a) \ \vee \ T_N(a)) \\ -I_O(a) \geq -(I_M(a) \ \wedge \ I_N(a)) \\ -F_O(a) \leq -(F_M(a) \ \vee \ F_N(a)) \end{pmatrix}$$
$$\Leftrightarrow \begin{pmatrix} -T_O(a) \leq -T_M(a) \ \wedge -T_N(a)) \\ -I_O(a) \geq -I_M(a) \ \vee - I_N(a)) \\ -F_O(a) \leq -F_M(a) \ \wedge \ -F_N(a)) \end{pmatrix}$$
$$\Leftrightarrow \begin{pmatrix} -1 - T_O(a) \leq (-1 - T_M(a)) \ \wedge (-1 - T_N(a)) \\ -1 - I_O(a) \geq (-1 - I_M(a)) \vee \ (-1 - I_N(a)) \\ -1 - F_O(a) \leq (-1 - F_M(a)) \ \wedge (-1 - F_N(a)) \end{pmatrix}$$
$$\Leftrightarrow X_{O^c} \supseteq X_{M^c} \cup X_{N^c}.$$

So $X_O \subseteq X_N \cup X_M$ if and only if $X_{O^c} \supseteq X_{N^c} \cap X_{M^c}$. □

Theorem 3.2. *For $\Phi \neq K \subseteq X$ and $X_N = \frac{X}{(T_N, I_N, F_N)}$, the below assertions are equivalent:*

(i) $\chi_K(X_N)$ of X is a neutrosophic $\aleph$-subsemigroup,

(ii) K of X is a subsemigroup.

Proof: Suppose $\chi_K(X_N)$ is a neutrosophic $\aleph$-subsemigroup of X. Let $k, s \in K$. Then

$$\begin{aligned} \chi_K(T)_N(ks) \leq &\quad \chi_K(T)_N(k) \vee \chi_K(T)_N(s) = -1, \\ \chi_K(I)_N(ks) \geq &\quad \chi_K(I)_N(k) \wedge \chi_K(I)_N(s) = 0, \\ \chi_K(F)_N(ks) \leq &\quad \chi_K(F)_N(k) \vee \chi_K(F)_N(s) = -1. \end{aligned}$$

Thus $ks \in K$ and hence K is a subsemigroup of X.

Conversely, suppose that K is a subsemigroup of X and let $k, s \in X$.

If $k, s \in K$, then $ks \in K$. Now

$$\begin{aligned} \chi_K(T)_N(ks) = &\quad -1 = \chi_K(T)_N(k) \vee \chi_K(T)_N(s), \\ \chi_K(I)_N(ks) = &\quad 0 = \chi_K(I)_N(k) \wedge \chi_K(I)_N(k), \\ \chi_K(F)_N(ks) = &\quad -1 = \chi_K(F)_N(k) \vee \chi_K(F)_N(k). \end{aligned}$$

If $k \notin K$ or $s \notin K$, then

$$\begin{aligned} \chi_K(T)_N(ks) \leq &\quad 0 = \chi_K(T)_N(k) \vee \chi_K(T)_N(s), \\ \chi_K(I)_N(ks) \geq &\quad -1 = \chi_K(I)_N(k) \wedge \chi_K(I)_N(s), \\ \chi_K(F)_N(ks) \leq &\quad 0 = \chi_K(F)_N(k) \vee \chi_K(F)_N(s). \end{aligned}$$

So $\chi_K(X_N)$ of X is a neutrosophic $\aleph$-subsemigroup. □

Theorem 3.3. *For $\Phi \neq K \subseteq X$ and $X_N = \frac{X}{(T_N, I_N, F_N)}$, the below assertions are equivalent:*

(i) $\chi_K(X_N)$ of X is a neutrosophic $\aleph$-bi-ideal structure,

(ii) K is a bi-ideal subset of X.

Proof: Suppose $\chi_K(X_N)$ is a neutrosophic $\aleph-$bi-ideal structure of X. Let $k \in K$ and $s \in X$. Then

$$\begin{aligned} \chi_K(T)_N(ksk) \leq &\quad \chi_K(T)_N(k) = -1, \\ \chi_K(I)_N(ksk) \geq &\quad \chi_K(I)_N(k) = 0, \\ \chi_K(F)_N(ksk) \leq &\quad \chi_K(F)_N(k) = -1. \end{aligned}$$

Thus $ksk \in K$ and hence K is a bi-ideal subset of X.

Conversely, suppose K is a bi-ideal subset of X. Let $k, s \in X$.

If $k \in K$, then $ksk \in K$. Now

$$\begin{aligned} \chi_K(T)_N(ksk) = &\quad -1 = \chi_K(T)_N(k), \\ \chi_K(I)_N(ksk) = &\quad 0 = \chi_K(I)_N(k), \\ \chi_K(F)_N(ksk) = &\quad -1 = \chi_K(F)_N(k). \end{aligned}$$

If $k \notin K$, then

$$\begin{aligned} \chi_K(T)_N(ksk) \leq &\quad 0 = \chi_K(T)_N(k), \\ \chi_K(I)_N(ksk) \geq &\quad -1 = \chi_K(I)_N(k) \\ \chi_K(F)_N(ksk) \leq &\quad 0 = \chi_K(F)_N(k). \end{aligned}$$

Therefore $\chi_K(X_N)$ of X is a neutrosophic $\aleph-$bi-ideal structure. □

Theorem 3.4. *For $\Phi \neq K \subseteq X$ and $X_N = \frac{X}{(T_N, I_N, F_N)}$, the below assertions are equivalent:*

(i) $\chi_K(X_N)$ *of* X *is a neutrosophic* $\aleph$*-bi-filter,*

(ii) K *of* X *is a bi-filter.*

Proof: Suppose $\chi_K(X_N)$ of X is a neutrosophic $\aleph$-bi-ideal. Then by Theorem 3.2, K is a subsemigroup of X. Let $k \in K$ and $s \in X$ with $ksk \in K$. Then

$$-1 = \chi_K(T)_N(ksk) \leq \chi_K(T)_N(k) = -1,$$
$$0 = \chi_K(I)_N(ksk) \geq \chi_K(I)_N(k) = 0,$$
$$-1 = \chi_K(F)_N(ksk\ ) \leq \chi_K(F)_N(k) = -1.$$

Thus $k \in K$ and hence K is a bi-filter of X,

Conversely, suppose K of X is a bi-filter. Then by Theorem 3.2, we have $\chi_K(X_N)$ of X is a neutrosophic $\aleph$-subsemigroup.

Let $s, k \in X$.

If k$\in K$, then $ksk \in K$. Now

$$\chi_K(T)_N(ksk) = \ \ -1 = \chi_K(T)_N(k),$$
$$\chi_K(I)_N(ksk) = \ \ 0 = \chi_K(I)_N(k),$$
$$\chi_K(F)_N(ksk) = \ \ -1 = \chi_K(F)_N(k).$$

If $k \notin K$, then

$$\chi_K(T)_N(ksk) \leq \ \ 0 = \chi_K(T)_N(k),$$
$$\chi_K(I)_N(ksk) \geq \ \ -1 = \chi_K(I)_N(k),$$
$$\chi_K(F)_N(ksk) \leq \ \ 0 = \chi_K(F)_N(k).$$

So $\chi_K(X_N)$ of X is a neutrosophic $\aleph$-bi-filter. □

Theorem 3.5. *For* $X_N = \frac{X}{(T_N, I_N, F_N)}$*, the below assertions are equivalent:*

(i) X_N *of* X *is a neutrosophic* $\aleph-$ *left (resp., right) ideal,*

(ii) The non-empty sets T_N^α*,* I_N^β *and* F_N^γ *are left (resp., right) ideals of* X $\forall \alpha, \beta, \gamma \in [-1, 0]$.

Proof: Suppose X_N is a neutrosophic $\aleph$-left ideal of X and $\alpha, \beta, \gamma \in [-1, 0]$.

Let $k \in T_N^\alpha \cap I_N^\beta \cap F_N^\gamma; s \in X$. Then

$$T_N(sk) \leq T_N(k) \leq \alpha,$$
$$I_N(sk) \geq I_N(k) \geq \beta,$$
$$F_N(sk) \leq F_N(k) \leq \gamma$$

which imply $sk \in T_N^\alpha \cap I_N^\beta \cap F_N^\gamma$. So T_N^α, I_N^β and F_N^γ are left ideals of X.

Conversely, assume that T_N^α, I_N^β and F_N^γ are left ideals of X for any $\alpha, \beta, \gamma \in [-1, 0]$. Then by Theorem 3.2 of [5], X_N of X is a neutrosophic $\aleph$-left ideal. □

Theorem 3.6. *For* $\Phi \neq K \subseteq X$ *and* $X_N = \frac{X}{(T_N, I_N, F_N)}$*, the below statements are equivalent:*

(i) K *is a prime left (resp., right) ideal of* X*,*

(ii) $\chi_K(X_N)$ *is a prime neutrosophic* $\aleph$*-left (resp., right) ideal of* X*.*

Proof: Suppose that K is a prime left ideal of X. Then by Theorem 3.2 of [5], $\chi_K(X_N)$ of X is a neutrosophic $\aleph$-left ideal. Let $k, s \in X$.

If $ks \notin K$, then

$$\begin{aligned} \chi_K(T)_N(ks) &= 0 \geq \chi_K(T)_N(k) \wedge \chi_K(T)_N(s), \\ \chi_K(I)_N(ks) &= -1 \leq \chi_K(I)_N(k) \vee \chi_K(I)_N(s), \\ \chi_K(F)_N(ks) &= 0 \geq \chi_K(F)_N(k) \wedge \chi_K(F)_N(s). \end{aligned}$$

If $ks \in K$, then $k \in K$ or $s \in K$. So

$$\begin{aligned} \chi_K(T)_N(ks) = & \quad -1 = \chi_K(T)_N(k) \wedge \chi_K(T)_N(s), \\ \chi_K(I)_N(ks) = & \quad 0 = \chi_K(I)_N(k) \vee \chi_K(I)_N(s), \\ \chi_K(F)_N(ks) = & \quad -1 = \chi_K(F)_N(k) \wedge \chi_K(F)_N(s). \end{aligned}$$

Hence $\chi_K(X_N)$ is a prime neutrosophic $\aleph-$ left ideal of X.

Conversely, suppose $\chi_K(X_N)$ of X is a prime neutrosophic $\aleph-$ left (resp., right) ideal. Then by Theorem 3.2 of [5], K of X is a left ideal.

Let $k, s \in S$ with $ks \in K$. Suppose that $k \notin K$ and s$\notin K$. Then

$$\begin{aligned} -1 =& \chi_K(T)_N(ks) \geq \chi_K(T)_N(k) \wedge \chi_K(T)_N(s) = 0, \\ 0 =& \chi_K(I)_N(ks) \leq \chi_K(I)_N(k) \vee \chi_K(T)_N(s) = -1, \\ -1 =& \chi_K(F)_N(ks) \geq \chi_K(F)_N(k) \wedge \chi_K(T)_N(s) = 0 \end{aligned}$$

which are not possible. Thus $k \in K$ or $s \in K$, and hence K of X is a prime left ideal. □

Theorem 3.7. *Let $X_N = \frac{X}{(T_N, I_N, F_N)}$. Then the below assertions are equivalent:*

(i) X_N of X is a prime neutrosophic $\aleph$- left (resp., right) ideal,

(ii) The non-empty sets T_N^{α}, I_N^{β} and F_N^{γ} are prime left (resp., right) ideals of X for all $\alpha, \beta, \gamma \in [-1, 0]$.

Proof: Suppose X_N of X is a prime neutrosophic $\aleph$-left ideal. Then by Theorem 3.5, T_N^{α}, I_N^{β} and F_N^{γ} are left ideals of X for $\alpha, \beta, \gamma \in [-1, 0]$.

Let $k, s \in X$ with $ks \in T_N^{\alpha} \cap I_N^{\beta} \cap F_N^{\gamma}$. Then $\alpha \geq T_N(ks) \geq T_N(k) \wedge T_N(s)$ implies $\alpha \geq T_N(k)$ or $\alpha \geq T_N(s)$. So $k \in T_N^{\alpha}$ or $s \in T_N^{\alpha}$. Also $\beta \leq I_N(ks) \leq I_N(k) \vee I_N(s)$ gives $\beta \leq I_N(k)$ or $\beta \leq I_N(s)$. So $k \in I_N^{\beta}$ or $s \in I_N^{\beta}$. Also $\gamma \geq F_N(ks) \geq F_N(k) \wedge F_N(s)$ implies $\gamma \geq F_N(k)$ or $\gamma \geq F_N(s)$. So $k \in F_N^{\gamma}$ or $s \in F_N^{\gamma}$.

Therefore T_N^{α}, I_N^{β} and F_N^{γ} are prime left ideals of X.

Conversely, suppose T_N^{α}, I_N^{β} and F_N^{γ} are prime left ideals of X for all $\alpha, \beta, \gamma \in [-1, 0]$. Then by Theorem 3.5, X_N of X is a neutrosophic $\aleph$-left ideal.

Let $k, s \in X$. Then $T_N(ks) = \alpha_1$; $I_N(ks) = \beta_1$ and $F_N(ks) = \gamma_1$ for some $\alpha_1, \beta_1, \gamma_1 \in [-1, 0]$ which imply $s \in T_N^{\alpha_1} \cap I_N^{\beta_1} \cap F_N^{\gamma_1}$. Since $T_N^{\alpha_1}$ is prime, we have $k \in T_N^{\alpha_1}$ or $s \in T_N^{\alpha_1}$ which implies $T_N(k) \leq \alpha_1$ or $T_N(s) \leq \alpha_1$. Since $I_N^{\beta_1}$ is prime, we have $k \in I_N^{\beta_1}$ or $s \in I_N^{\beta_1}$ which implies

$I_N(k) \geq \beta_1$ or $I_N(s) \geq \beta_1$. Since $F_N^{\gamma_1}$ is prime, we have $k \in F_N^{\gamma_1}$ or $s \in F_N^{\gamma_1}$ which implies $F_N(k) \leq \gamma_1$ or $F_N(s) \leq \gamma_1$. Now

$$\begin{aligned} T_N(ks) = &\ \alpha_1 \geq T_N(k) \wedge T_N(s), \\ I_N(ks) = &\ \beta_1 \leq I_N(k) \vee I_N(s), \\ F_N(ks) = &\ \gamma_1 \geq F_N(k) \wedge F_N(s). \end{aligned}$$

So X_N of X is a prime neutrosophic ℵ- left ideal. □

Theorem 3.8. *For $X_N = \frac{X}{(T_N, I_N, F_N)}$, the below assertions are equivalent:*

(i) X_N of X is a semiprime neutrosophic ℵ- left (resp., right) ideal,

(ii) The non-empty sets T_N^{α}, I_N^{β} and I_N^{γ} are semiprime left (resp., right) ideals of X for any $\alpha, \beta, \gamma \in [-1, 0]$.

Proof: Suppose X_N of X is a semiprime neutrosophic ℵ- left ideal. Then by Theorem 3.5, T_N^{α}, I_N^{β} and F_N^{γ} are left ideals of X for $\alpha, \beta, \gamma \in [-1, 0]$.

Let $r \in X$ with $r^2 \in T_N^{\alpha} \cap I_N^{\beta} \cap F_N^{\gamma}$. Then $\alpha \geq T_N(r^2) \geq T_N(r)$ implies $\alpha \geq T_N(r)$. So $r \in T_N^{\alpha}$. Also $\beta \leq I_N(r^2) \leq I_N(r)$ implies $\beta \leq I_N(r)$. So $r \in I_N^{\beta}$. Also $\gamma \geq F_N(r^2) \geq F_N(r)$ implies $\gamma \geq F_N(r)$. So $r \in F_N^{\gamma}$. Hence T_N^{α}, I_N^{β} and F_N^{γ} are semiprime left ideals of X.

Conversely, suppose T_N^{α}, I_N^{β} and F_N^{γ} are semiprime left ideals of X $\forall \alpha, \beta, \gamma \in [-1,\ 0]$. Then by Theorem 3.5, X_N of X is a neutrosophic ℵ- left ideal. Let $r \in X$. Then $T_N(r^2) = \alpha_1$; $I_N(r^2) = \beta_1$ and $F_N(r^2) = \gamma_1$ for some $\alpha_1, \beta_1, \gamma_1 \in [-1, 0]$ which imply $r^2 \in T_N^{\alpha_1} \cap I_N^{\beta_1} \cap F_N^{\gamma_1}$. Since $T_N^{\alpha_1}$, $I_N^{\beta_1}$ and $F_N^{\gamma_1}$ are semiprime, we have $r \in T_N^{\alpha_1}$ gives $T_N(r) \leq \alpha_1$; $r \in I_N^{\beta_1}$ gives $I_N(r) \geq \beta_1$ and $r \in F_N^{\gamma_1}$ gives $F_N(r) \leq \gamma_1$.

Now

$$\begin{aligned} T_N(r^2) = &\ \alpha_1 \geq T_N(r), \\ I_N(r^2) = &\ \beta_1 \leq I_N(r), \\ F_N(r^2) = &\ \gamma_1 \geq F_N(r). \end{aligned}$$

So X_N is semiprime neutrosophic ℵ-left ideal. □

Theorem 3.9. *Let $X_N = \frac{X}{(T_N, I_N, F_N)}$. Then the below assertions are equivalent:*

(i) X_N of X is a neutrosophic ℵ-bi-ideal structure,

(ii) The non-empty sets T_N^{α}, I_N^{β} and F_N^{γ} are bi-ideal subsets of X for all $\alpha, \beta, \gamma \in [-1, 0]$.

Proof: Suppose X_N of X is a neutrosophic ℵ-bi-ideal structure and $\alpha, \beta, \gamma \in [-1, 0]$.

Let $k \in T_N^{\alpha} \cap I_N^{\beta} \cap F_N^{\gamma}; s \in X$. Then

$$\begin{aligned} T_N(ksk) \leq &\ T_N(k) \leq \alpha, \\ I_N(ksk) \geq &\ I_N(k) \geq \beta, \\ F_N(ksk) \leq &\ F_N(k) \leq \gamma \end{aligned}$$

which imply $ksk \in T_N^{\alpha} \cap I_N^{\beta} \cap F_N^{\gamma}$. So T_N^{α}, I_N^{β} and F_N^{γ} are bi-ideal subsets of X.

Conversely, suppose T_N^{α}, I_N^{β} and F_N^{γ} are bi-ideal subsets of X for all $\alpha, \beta, \gamma \in [-1, 0]$.

If there are $r, s \in X$ such that $T_N(rsr) > T_N(r)$, then $T_N(rsr) > t_\alpha \geq T_N(r)$ for some $t_\alpha \in [-1, 0)$ which implies $r \in T_N^{t_\alpha}(r)$ and $rsr \notin T_N^{t_\alpha}(r)$, a contradiction. So $T_N(rsr) \leq T_N(r)$.

If there are $r, s \in X$ such that $I_N(rsr) < I_N(r)$, then $I_N(rsr) < t_\beta \leq I_N(r)$ for some $t_\beta \in (-1, 0]$ which implies $r \in I_N^{t_\beta}(r)$ and $rsr \notin I_N^{t_\beta}(r)$, a contradiction. So $I_N(rsr) \geq I_N(r)$.

If there are $r, s \in X$ such that $F_N(rsr) > F_N(r)$, then $F_N(rsr) > t_\gamma \geq F_N(r)$ for some $t_\gamma \in [-1, 0)$ which implies $r \in F_N^{t_\gamma}(r)$ and $rsr \notin F_N^{t_\gamma}(r)$, a contradiction. So $F_N(rsr) \leq F_N(r)$.

Therefore X_N is a neutrosophic ℵ-bi-ideal structure. □

Theorem 3.10. *Let $X_N = \frac{X}{(T_N, I_N, F_N)}$. Then the below assertions are equivalent:*

(i) X_N of X is a prime neutrosophic ℵ-bi-ideal structure,

(ii) The non-empty sets T_N^α, I_N^β and F_N^γ are prime bi-ideal subsets of X for any $\alpha, \beta, \gamma \in [-1, 0]$.

Proof: Suppose X_N of X is a prime neutrosophic ℵ- bi-ideal structure and $\alpha, \beta, \gamma \in [-1, 0]$. Let $k, s \in X$ with $ks \in T_N^\alpha \cap I_N^\beta \cap F_N^\gamma$. Then $\alpha \geq T_N(ks) \geq T_N(k) \wedge T_N(s)$ implies $\alpha \geq T_N(k)$ or $\alpha \geq T_N(s)$. So $k \in T_N^\alpha$ or $s \in T_N^\alpha$. Also $\beta \leq I_N(ks) \leq I_N(k) \vee I_N(s)$ implies $\beta \leq I_N(k)$ or $\beta \leq I_N(s)$. So $k \in I_N^\beta$ or $s \in I_N^\beta$. Also $\gamma \geq F_N(ks) \geq F_N(k) \wedge F_N(s)$ implies $\gamma \geq F_N(k)$ or $\gamma \geq F_N(s)$. So $k \in F_N^\gamma$ or $s \in F_N^\gamma$. Hence T_N^α, I_N^β and F_N^γ are prime left ideals of X.

Conversely, suppose T_N^α, I_N^β and F_N^γ are prime bi-ideal subsets of X $\forall \alpha, \beta, \gamma \in [-1, 0]$. Then by Theorem 3.9, X_N of X is a neutrosophic ℵ-bi-ideal. Let $k, s \in X$. Then $T_N(ks) = \alpha_1$;$I_N(ks) = \beta_1$ and $F_N(ks) = \gamma_1$ for some $\alpha_1, \beta_1, \gamma_1 \in [-1, 0]$ which imply $ks \in T_N^{\alpha_1} \cap I_N^{\beta_1} \cap F_N^{\gamma_1}$. Since $T_N^{\alpha_1}$ is prime bi-ideal, $k \in T_N^{\alpha_1}$ or $s \in T_N^{\alpha_1}$ which implies $T_N(k) \leq \alpha_1$ or $T_N(s) \leq \alpha_1$.

Since $I_N^{\beta_1}$ is prime bi-ideal, $k \in I_N^{\beta_1}$ or $s \in I_N^{\beta_1}$ which implies $I_N(k) \geq \beta_1$ or $I_N(s) \geq \beta_1$. Also $F_N^{\gamma_1}$ is prime bi-ideal, $k \in F_N^{\gamma_1}$ or $s \in F_N^{\gamma_1}$ which implies $F_N(k) \leq \gamma_1$ or $F_N(s) \leq \gamma_1$. Now

$$\begin{aligned} T_N(ks) &= \alpha_1 \geq T_N(k) \wedge T_N(s), \\ I_N(ks) &= \beta_1 \leq I_N(k) \vee I_N(s), \\ F_N(ks) &= \gamma_1 \geq F_N(k) \wedge F_N(s). \end{aligned}$$

Therefore X_N is a prime neutrosophic ℵ-bi-ideal structure of X. □

Theorem 3.11. *For $X_N = \frac{X}{(T_N, I_N, F_N)}$, the below assertions are equivalent:*

(i) X_N is a semiprime neutrosophic ℵ-bi- ideal structure of X,

(ii) The non-empty sets T_N^α, I_N^β and F_N^γ are semiprime bi-ideal subsets of X for all $\alpha, \beta, \gamma \in [-1, 0]$.

Proof: It is similar to the proof of Theorem 3.10. □

Theorem 3.12. *For $X_N = \frac{X}{(T_N, I_N, F_N)}$ and $\Phi \neq K \subseteq X$, the below statements are equivalent:*

(i) K is a prime bi-ideal subset of X,

(ii) $\chi_K(X_N)$ of X is a prime neutrosophic ℵ-bi-ideal structure.

Proof: It is similar to the proof of Theorem 3.6. □

Theorem 3.13. *For $X_N = \frac{X}{(T_N, I_N, F_N)}$, the below assertions are equivalent:*

(i) X_N of X is a neutrosophic ℵ-bi-filter,

(ii) X_{N^c} of X is a neutrosophic ℵ-bi-ideal structure.

Proof: It is trivial as for $k, s \in X$, we have

$$\begin{pmatrix} T_N(ksk) \geq T_N(k) \\ I_N(ksk) \leq I_N(k) \\ F_N(ksk) \geq F_N(k) \end{pmatrix} \Leftrightarrow \begin{pmatrix} T_{N^c}(ksk) \leq T_{N^c}(k) \\ I_{N^c}(ksk) \geq I_{N^c}(k) \\ F_{N^c}(ksk) \leq F_{N^c}(k) \end{pmatrix}.$$

□

Theorem 3.14. *For $\Phi \neq K \subseteq X$ and $X_N = \frac{X}{(T_N, I_N, F_N)}$, the below assertions are equivalent:*

(i) K is a left (resp., right) filter of X,

(ii) $\chi_K(X_N)$ is a neutrosophic ℵ-left (resp., right) filter of X.

Proof: Suppose K of X is a left filter. Then by Theorem 3.12, $\chi_K(X_N)$ of X is a neutrosophic ℵ-subsemigroup. Let k, $t \in X$.

If $kt \notin K$, then

$$\begin{aligned} \chi_K(T)_N(kt) = &\quad 0 \geq \chi_K(T)_N(t), \\ \chi_K(I)_N(kt) = &\quad -1 \leq \chi_K(I)_N(t), \\ \chi_K(F)_N(kt) = &\quad 0 \geq \chi_K(F)_N(t). \end{aligned}$$

If $kt \in K$, then $k \in K$. So

$$\begin{aligned} \chi_K(T)_N(kt) = &\quad -1 = \chi_K(T)_N(t), \\ \chi_K(I)_N(kt) = &\quad 0 = \chi_K(I)_N(t), \\ \chi_K(F)_N(kt) = &\quad -1 = \chi_K(F)_N(t). \end{aligned}$$

Hence $\chi_K(X_N)$ of X is a neutrosophic ℵ-left filter.

Conversely, suppose $\chi_K(X_N)$ of X is a neutrosophic ℵ-left (resp., right) filter. Then by Theorem 3.12, K is a subsemigroup of X.

Let $r, s \in S$ such that $rs \in K$. Suppose that $s \notin K$. Then

$$\begin{aligned} -1 &= \chi_K(T)_N(rs) \geq \chi_K(T)_N(s) = 0, \\ 0 &= \chi_K(I)_N(rs) \leq \chi_K(I)_N(s) = -1, \\ -1 &= \chi_K(F)_N(xy) \geq \chi_K(F)_N(s) = 0, \end{aligned}$$

which are not possible.

Thus $s \in K$ and hence K of X is a left filter. □

Theorem 3.15. *Let $X_N = \frac{X}{(T_N,\ I_N,\ F_N)}$. Then the below statements are equivalent:*

(i) X_N is a neutrosophic ℵ− left (resp., right) filter of X,

(ii) X_{N^c} is a prime neutrosophic ℵ− left (resp., right) ideal of X.

Proof: Suppose X_N of X is a neutrosophic $\aleph-$ left filter. Then X_N of X is a neutrosophic $\aleph-$ subsemigroup. For k, $s \in X$, we have

$$\begin{pmatrix} T_N(ks) \geq T_N(s) \\ I_N(ks) \leq I_N(s) \\ F_N(ks) \geq F_N(s) \end{pmatrix} \Leftrightarrow \begin{pmatrix} T_{N^c}(ks) \leq T_{N^c}(s) \\ I_{N^c}(ks) \geq I_{N^c}(s) \\ F_{N^c}(ks) \leq F_{N^c}(s) \end{pmatrix} \quad (a)$$

So X_{N^c} of X is a neutrosophic $\aleph-$ left ideal.

Since X_N is neutrosophic $\aleph$- subsemigroup, we have

$$\begin{pmatrix} T_N(ks) \leq T_N(k) \vee T_N(s) \\ I_N(ks) \geq I_N(k) \wedge I_N(s) \\ F_N(ks) \leq F_N(k) \vee F_N(s) \end{pmatrix} \Leftrightarrow \begin{pmatrix} T_{N^c}(ks) \geq T_{N^c}(k) \wedge T_{N^c}(s) \\ I_{N^c}(ks) \leq I_{N^c}(k) \vee I_{N^c}(s) \\ F_{N^c}(ks) \geq F_{N^c}(k) \wedge F_{N^c}(s) \end{pmatrix}.$$

Therefore X_{N^c} is a prime neutrosophic $\aleph$-left ideal of X.

Conversely, suppose X_{N^c} of X is a prime neutrosophic $\aleph$- left ideal. Then X_{N^c} of X is a neutrosophic $\aleph$- left ideal. Then by (a), we have X_N of X is a neutrosophic $\aleph$-left filter. □

Theorem 3.16. *Let $X_N = \frac{X}{(T_N, I_N, F_N)}$. Then the below statements are equivalent:*

(i) X_N is a neutrosophic $\aleph$-bi-filter of X,

(ii) X_{N^c} is a prime neutrosophic $\aleph$-bi-ideal structure of X.

Proof: Suppose X_N is a neutrosophic $\aleph$-bi-filter of X. Then X_N is a neutrosophic $\aleph$-subsemigroup of X. For any $k, s \in X$, we have

$$\begin{pmatrix} T_N(ksk) \leq T_N(k) \\ I_N(ksk) \geq I_N(k) \\ F_N(ksk) \leq F_N(k) \end{pmatrix} \Leftrightarrow \begin{pmatrix} T_{N^c}(ksk) \geq T_{N^c}(k) \\ I_{N^c}(ksk) \leq I_{N^c}(k) \\ F_{N^c}(ksk) \geq F_{N^c}(k) \end{pmatrix} \quad (1)$$

So X_{N^c} is a neutrosophic $\aleph$-bi-ideal structure of X.

Since X_N is a neutrosophic $\aleph$-subsemigroup of X, we have

$$\begin{pmatrix} T_N(ks) \leq T_N(k) \vee T_N(s) \\ I_N(ks) \geq I_N(k) \wedge I_N(s) \\ F_N(ks) \leq F_N(k) \vee F_N(s) \end{pmatrix} \Leftrightarrow \begin{pmatrix} T_{N^c}(ks) \geq T_{N^c}(k) \wedge T_{N^c}(s) \\ I_{N^c}(ks) \leq I_{N^c}(k) \vee I_{N^c}(s) \\ F_{N^c}(ks) \geq F_{N^c}(k) \wedge F_{N^c}(s) \end{pmatrix}$$

Therefore X_{N^c} is a prime neutrosophic $\aleph$-bi-ideal structure of X.

Conversely, suppose X_{N^c} of X is a prime neutrosophic $\aleph$-bi-ideal structure. Then X_{N^c} of X is a neutrosophic $\aleph$-bi-ideal structure. Then by (1), we have X_N of X is a neutrosophic $\aleph$-bi-filter. □

4. **Conclusion**

In this paper, we have characterized the concept neutrosophic $\aleph$-bi-filter of X and described semigroup as far as neutrosophic $\aleph$-bi-ideal and neutrosophic $\aleph$-bi-filter of X. We likewise characterized the notions neutrosophic ℵ-left filters and prime neutrosophic ℵ-left ideals of X and portrayed semigroup in terms of these notions.

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