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Rough Neutrosophic Ideals in a Ring

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Abstract. Aim of this paper is to introduce the notion of rough neutrosophic sets in rings also we discuss the sum and product of rough neutrosophic ideal in a ring. Also we prove the upper and lower approximation of neutrosophic subring is also a neutrosophic subring and some examples are discussed.

Keywords: Rough set; Neutrosophic set; Rough neutrosophic set; Neutrosophic ideal; Rough neutrosophic ideal.)

1. Introduction

Fuzzy set is introduced and described using membership functions by Zadeh in 1965 [14] in 1965. The notion of Rough sets was introduced by Pawlak [7] in his seminal paper of 1982. Crisp set and equivalence relation are the basic elements of Rough set theory. Rough set is based on result of approximating crisp sets known as the lower approximation and the upper approximation of a set introduced by Biswas and Nanda [2] in 1994. Approximation spaces are sets with multiple memberships but, fuzzy sets are with partial memberships. Many scholars Dubios et al [4], Gong et al [5], Leung et al [6], Sun et al [12] has developed many models upon different aspects. Rough sets and fuzzy sets, vague set and Intuitionistic fuzzy sets combine with various notions such as Generalized fuzzy rough sets, Intuitionistic fuzzy rough sets, Rough Intuitionistic fuzzy sets, and Rough vague sets were introduced.

Atanassov (1983) [1] introduced the notion of Intuitionistic fuzzy sets. They are the sets whose elements having degrees of membership and non-membership. Selvan, Senthil Kumar [8–10],

introduced the notion of rough intuitionistic fuzzy ideal(prime ideal) in rings in 2012. The generalizations of the theory of intuitionistic fuzzy sets is the theory of neutrosophic sets. The words neutrosophy and neutrosophic were introduced by Smarandache [11]. Neutrosophic concepts are very much useful in real life problem. For example, If a opinion is asked about a statement one may assign that the possibility that the statement true is 0.6 and the statement false is 0.8 and if it is not sure is 0.2. This idea is very much needful in various problems in real life situation. Neutrosophic sets are characterized by truth membership function , indeterminacy membership function and falsity membership function . Vildan cetkin and Halis Aygun [13] introduced an approach to single valued Neutrosophic ideals over a classical ring and Neutrosophic subring in 2018. Rough neutrosophic set is introduced by Broumi, Smarandache, and Dhar [3].

In this paper, we prove that any neutrosophic subring (ideal) of a ring is an upper and lower rough neutrosophic subring (ideal) of the ring.

2. Preliminaries

See [3], [7], [13] for basic concepts which are used in this work.

3. Operations on Rough Neutrosophic sets in a Ring

In this section we introduce the notion of *RNI* in a ring. Some basic properties of these ideals are proved and examples are given. Let C_R denote the congruence relation on R , throughout this section.

Theorem 3.1. *Let C_R and C_R' be the two congruence relations on R . If P and Q are any two NS of R , then the following properties are,*

$$(a) \underline{C_R}(P) \subseteq P \subseteq \overline{C_R}(P)$$

$$(b) \underline{C_R}(\underline{C_R}(P)) = \underline{C_R}(P)$$

$$(c) \overline{C_R}(\overline{C_R}(P)) = \overline{C_R}(P)$$

$$(d) \overline{C_R}(\underline{C_R}(P)) = \underline{C_R}(P)$$

$$(e) \underline{C_R}(\overline{C_R}(P)) = \overline{C_R}(P)$$

$$(f) (\overline{C_R}(P^c))^c = \underline{C_R}(P)$$

$$(g) (\underline{C_R}(P^c))^c = \overline{C_R}(P)$$

$$(h) \underline{C_R}(P \cap Q) = \underline{C_R}(P) \cap \underline{C_R}(Q)$$

$$(i) \overline{C_R}(P \cap Q) \subseteq \overline{C_R}(P) \cap \overline{C_R}(Q)$$

$$(j) \overline{C_R}(P \cup Q) = \overline{C_R}(P) \cup \overline{C_R}(Q)$$

$$(k) \underline{C_R}(P \cup Q) \supseteq \underline{C_R}(P) \cup \underline{C_R}(Q)$$

$$(l) P \subseteq Q \Rightarrow \overline{C_R}(P) \subseteq \overline{C_R}(Q)$$

$$(m) P \subseteq Q \Rightarrow \underline{C_R}(P) \subseteq \underline{C_R}(Q)$$

$$\begin{aligned} (n)C_R \subseteq C_R' &\Rightarrow \underline{C}_R(P) \supseteq \underline{C}_R'(P) \\ (o)C_R \subseteq C_R' &\Rightarrow \overline{C}_R(P) \subseteq \overline{C}_R'(P). \end{aligned}$$

Proof. Proof is obvious. \square

Theorem 3.2. *If P and Q are any two NS of R , then $\overline{C}_R(P) + \overline{C}_R(Q) \subseteq \overline{C}_R(P + Q)$.*

Proof. Since P and Q be any two NS of R . Then

$$\overline{C}_R(P) + \overline{C}_R(Q) = \{[\overline{C}_R(P(n_t)) + \overline{C}_R(Q(n_t))], [\overline{C}_R(P(n_i)) + \overline{C}_R(Q(n_i))], [\overline{C}_R(P(n_f)) + \overline{C}_R(Q(n_f))]\}.$$

$$\overline{C}_R(P + Q) = \{[\overline{C}_R(P(n_t) + Q(n_t))], [\overline{C}_R(P(n_i) + Q(n_i))], [\overline{C}_R(P(n_f) + Q(n_f))]\}.$$

we've to prove, $\overline{C}_R(P) + \overline{C}_R(Q) \subseteq \overline{C}_R(P + Q)$.

For this we want to prove, $\forall \alpha \in R$

$$(\overline{C}_R(P(n_t)) + \overline{C}_R(Q(n_t)))(\alpha) \leq \overline{C}_R(P(n_t) + Q(n_t))(\alpha)$$

$$(\overline{C}_R(P(n_i)) + \overline{C}_R(Q(n_i)))(\alpha) \geq \overline{C}_R(P(n_i) + Q(n_i))(\alpha)$$

$$(\overline{C}_R(P(n_f)) + \overline{C}_R(Q(n_f)))(\alpha) \geq \overline{C}_R(P(n_f) + Q(n_f))(\alpha)$$

Consider,

$$\begin{aligned} (\overline{C}_R(P(n_t)) + \overline{C}_R(Q(n_t)))(\alpha) &= \bigvee_{\alpha=\beta+\gamma} [\overline{C}_R(P(n_t))(\beta) \wedge \overline{C}_R(Q(n_t))(\gamma)] \\ &= \bigvee_{\alpha=\beta+\gamma} [(\bigvee_{x \in [\beta]_{C_R}} (P(n_t)(x)) \wedge (\bigvee_{y \in [\gamma]_{C_R}} (Q(n_t)(y)))] \\ &= \bigvee_{\alpha=\beta+\gamma} [(\bigvee_{\substack{x \in [\beta]_{C_R} \\ y \in [\gamma]_{C_R}}} (P(n_t)(x) \wedge Q(n_t)(y)))] \\ &\leq \bigvee_{\alpha=\beta+\gamma} [(\bigvee_{x+y \in [\beta+\gamma]_{C_R}} (P(n_t)(x) \wedge Q(n_t)(y)))] \\ &= \bigvee_{x+y \in [\alpha]_{C_R}} (P(n_t)(x) \wedge Q(n_t)(y)) \\ &= \bigvee_{\substack{z \in [\alpha]_{C_R} \\ z=x+y}} (P(n_t)(x) \wedge Q(n_t)(y)) \\ &= \bigvee_{z \in [\alpha]_{C_R}} \bigvee_{z=x+y} (P(n_t)(x) \wedge Q(n_t)(y)) \\ &= \bigvee_{z \in [\alpha]_{C_R}} [P(n_t) + Q(n_t)](z) \\ &= \overline{C}_R(P(n_t) + Q(n_t))(z) \end{aligned}$$

And

$$\begin{aligned}
(\overline{C_R}(P(n_i)) + \overline{C_R}(Q(n_i)))(\alpha) &= \bigwedge_{\alpha=\beta+\gamma} [\overline{C_R}(P(n_i))(\beta) \vee \overline{C_R}(Q(n_i))(\gamma)] \\
&= \bigwedge_{\alpha=\beta+\gamma} [(\bigwedge_{x \in [\beta]_{C_R}} (P(n_i)(x)) \vee (\bigwedge_{y \in [\gamma]_{C_R}} (Q(n_i)(y)))] \\
&= \bigwedge_{\alpha=\beta+\gamma} [(\bigwedge_{\substack{x \in [\beta]_{C_R} \\ y \in [\gamma]_{C_R}}} (P(n_i)(x) \vee Q(n_i)(y)))] \\
&\geq \bigwedge_{\alpha=\beta+\gamma} [(\bigwedge_{x+y \in [\beta+\gamma]_{C_R}} (P(n_i)(x) \vee Q(n_i)(y)))] \\
&= \bigwedge_{x+y \in [\alpha]_{C_R}} (P(n_i)(x) \vee Q(n_i)(y)) \\
&= \bigwedge_{\substack{z \in [\alpha]_{C_R} \\ z=x+y}} (P(n_i)(x) \vee Q(n_i)(y)) \\
&= \bigwedge_{z \in [\alpha]_{C_R}} \bigwedge_{z=x+y} (P(n_i)(x) \vee Q(n_i)(y)) \\
&= \bigwedge_{z \in [\alpha]_{C_R}} [P(n_i) + Q(n_i)](z) \\
&= \overline{C_R}(P(n_i) + Q(n_i))(z)
\end{aligned}$$

Also,

$$\begin{aligned}
(\overline{C_R}(P(n_f)) + \overline{C_R}(Q(n_f)))(\alpha) &= \bigwedge_{\alpha=\beta+\gamma} [\overline{C_R}(P(n_f))(\beta) \vee \overline{C_R}(Q(n_f))(\gamma)] \\
&= \bigwedge_{\alpha=\beta+\gamma} [(\bigwedge_{x \in [\beta]_{C_R}} (P(n_f)(x)) \vee (\bigwedge_{y \in [\gamma]_{C_R}} (Q(n_f)(y)))] \\
&= \bigwedge_{\alpha=\beta+\gamma} [(\bigwedge_{\substack{x \in [\beta]_{C_R} \\ y \in [\gamma]_{C_R}}} (P(n_f)(x) \vee Q(n_f)(y)))] \\
&\geq \bigwedge_{\alpha=\beta+\gamma} [(\bigwedge_{x+y \in [\beta+\gamma]_{C_R}} (P(n_f)(x) \vee Q(n_f)(y)))] \\
&= \bigwedge_{x+y \in [\alpha]_{C_R}} (P(n_f)(x) \vee Q(n_f)(y)) \\
&= \bigwedge_{\substack{z \in [\alpha]_{C_R} \\ z=x+y}} (P(n_f)(x) \vee Q(n_f)(y)) \\
&= \bigwedge_{z \in [\alpha]_{C_R}} \bigwedge_{z=x+y} (P(n_f)(x) \vee Q(n_f)(y)) \\
&= \bigwedge_{z \in [\alpha]_{C_R}} [P(n_f) + Q(n_f)](z) \\
&= \overline{C_R}(P(n_f) + Q(n_f))(z)
\end{aligned}$$

Hence Proved.

□

Theorem 3.3. *If P and Q are any two NS of R , then $\underline{C}_R(P) + \underline{C}_R(Q) \subseteq \underline{C}_R(P + Q)$.*

Proof. This proof is similar to Theorem 3.2. \square

Theorem 3.4. *Let P and Q are any two NS of R , then $\overline{C}_R(P).\overline{C}_R(Q) \subseteq \overline{C}_R(P.Q)$.*

Proof. Since P and Q be any two NS of R . Then,

$$\overline{C}_R(P).\overline{C}_R(Q) = \{[\overline{C}_R(P(n_t)).\overline{C}_R(Q(n_t))], [\overline{C}_R(P(n_i)).\overline{C}_R(Q(n_i))], [\overline{C}_R(P(n_f)).\overline{C}_R(Q(n_f))]\}$$

$$\overline{C}_R(P.Q) = \{[\overline{C}_R(P(n_t).Q(n_t))], [\overline{C}_R(P(n_i).Q(n_i))], [\overline{C}_R(P(n_f).Q(n_f))]\}$$

To prove, $\overline{C}_R(P).\overline{C}_R(Q) \subseteq \overline{C}_R(P.Q)$.

It is enough to prove that, $\forall \alpha \in R$

$$(\overline{C}_R(P(n_t)).\overline{C}_R(Q(n_t)))(\alpha) \leq \overline{C}_R(P(n_t).Q(n_t))(\alpha)$$

$$(\overline{C}_R(P(n_i)).\overline{C}_R(Q(n_i)))(\alpha) \geq \overline{C}_R(P(n_i).Q(n_i))(\alpha)$$

$$(\overline{C}_R(P(n_f)).\overline{C}_R(Q(n_f)))(\alpha) \geq \overline{C}_R(P(n_f).Q(n_f))(\alpha)$$

Consider,

$$\begin{aligned} (\overline{C}_R(P(n_t)).\overline{C}_R(Q(n_t)))(\alpha) &= \bigvee_{\alpha=\beta\gamma} [\overline{C}_R(P(n_t))(\beta) \wedge \overline{C}_R(Q(n_t))(\gamma)] \\ &= \bigvee_{\alpha=\beta\gamma} [(\bigvee_{x \in [\beta]_{C_R}} (P(n_t)(x)) \wedge (\bigvee_{y \in [\gamma]_{C_R}} (Q(n_t)(y)))] \\ &= \bigvee_{\alpha=\beta\gamma} [(\bigvee_{\substack{x \in [\beta]_{C_R} \\ y \in [\gamma]_{C_R}}} (P(n_t)(x) \wedge Q(n_t)(y)))] \\ &\leq \bigvee_{\alpha=\beta\gamma} [(\bigvee_{xy \in [\beta\gamma]_{C_R}} (P(n_t)(x) \wedge Q(n_t)(y)))] \\ &= \bigvee_{xy \in [\alpha]_{C_R}} (P(n_t)(x) \wedge Q(n_t)(y)) \\ &= \bigvee_{\substack{z \in [\alpha]_{C_R} \\ z=xy}} (P(n_t)(x) \wedge Q(n_t)(y)) \\ &= \bigvee_{z \in [\alpha]_{C_R}} \bigvee_{z=xy} (P(n_t)(x) \wedge Q(n_t)(y)) \\ &= \bigvee_{z \in [\alpha]_{C_R}} [P(n_t).Q(n_t)](z) \\ &= \overline{C}_R(P(n_t).Q(n_t))(z) \end{aligned}$$

$$\begin{aligned}
(\overline{C_R}(P(n_i)).\overline{C_R}(Q(n_i)))(\alpha) &= \bigwedge_{\alpha=\beta\gamma} [\overline{C_R}(P(n_i))(\beta) \vee \overline{C_R}(Q(n_i))(\gamma)] \\
&= \bigwedge_{\alpha=\beta\gamma} [(\bigwedge_{x \in [\beta]_{C_R}} (P(n_i)(x)) \vee (\bigwedge_{y \in [\gamma]_{C_R}} (Q(n_i)(y)))] \\
&= \bigwedge_{\alpha=\beta\gamma} [(\bigwedge_{\substack{x \in [\beta]_{C_R} \\ y \in [\gamma]_{C_R}}} (P(n_i)(x) \vee Q(n_i)(y)))] \\
&\geq \bigwedge_{\alpha=\beta\gamma} [(\bigwedge_{xy \in [\beta\gamma]_{C_R}} (P(n_i)(x) \vee Q(n_i)(y)))] \\
&= \bigwedge_{xy \in [\alpha]_{C_R}} (P(n_i)(x) \vee Q(n_i)(y)) \\
&= \bigwedge_{\substack{z \in [\alpha]_{C_R} \\ z=xy}} (P(n_i)(x) \vee Q(n_i)(y)) \\
&= \bigwedge_{z \in [\alpha]_{C_R}} \bigwedge_{z=xy} (P(n_i)(x) \vee Q(n_i)(y)) \\
&= \bigwedge_{z \in [\alpha]_{C_R}} [P(n_i).Q(n_i)](z) \\
&= \overline{C_R}(P(n_i).Q(n_i))(z)
\end{aligned}$$

$$\begin{aligned}
(\overline{C_R}(P(n_f)).\overline{C_R}(Q(n_f)))(\alpha) &= \bigwedge_{\alpha=\beta+\gamma} [\overline{C_R}(P(n_f))(\beta) \vee \overline{C_R}(Q(n_f))(\gamma)] \\
&= \bigwedge_{\alpha=\beta\gamma} [(\bigwedge_{x \in [\beta]_{C_R}} (P(n_f)(x)) \vee (\bigwedge_{y \in [\gamma]_{C_R}} (Q(n_f)(y)))] \\
&= \bigwedge_{\alpha=\beta\gamma} [(\bigwedge_{\substack{x \in [\beta]_{C_R} \\ y \in [\gamma]_{C_R}}} (P(n_f)(x) \vee Q(n_f)(y)))] \\
&\geq \bigwedge_{\alpha=\beta\gamma} [(\bigwedge_{xy \in [\beta\gamma]_{C_R}} (P(n_f)(x) \vee Q(n_f)(y)))] \\
&= \bigwedge_{xy \in [\alpha]_{C_R}} (P(n_f)(x) \vee Q(n_f)(y)) \\
&= \bigwedge_{\substack{z \in [\alpha]_{C_R} \\ z=xy}} (P(n_f)(x) \vee Q(n_f)(y)) \\
&= \bigwedge_{z \in [\alpha]_{C_R}} \bigwedge_{z=xy} (P(n_f)(x) \vee Q(n_f)(y)) \\
&= \bigwedge_{z \in [\alpha]_{C_R}} [P(n_f).Q(n_f)](z) \\
&= \overline{C_R}(P(n_f).Q(n_f))(z)
\end{aligned}$$

Hence proved. \square

Theorem 3.5. Let P and Q are any two NS of R , then

$$\underline{C_R}(P).\underline{C_R}(Q) \subseteq \underline{C_R}(P.Q).$$

Proof. This proof is similar to Theorem 3.4. \square

4. Rough Neutrosophic Subring (RNSR) of Ring

Definition 4.1. A *NSR* is called an *RNSR* if it is both upper *RNSR* and lower *RNSR* of R .

Definition 4.2. A *NSR* is said to be an lower (upper) *RNSR* of R if its lower(upper) approximation is also a *NSR* of R .

Theorem 4.3. If K be a *NSR* of R , then K is an upper *RNSR* of R .

Proof. Since K is a *NSR* of R . Now, $\forall a, b \in R$

$$\begin{aligned} \overline{C}_R(K(n_t))(a - b) &= \bigvee_{c \in [a-b]_{C_R}} K(n_t)(c) \\ &= \bigvee_{x-y \in [a]_{C_R} - [b]_{C_R}} K(n_t)(x - y) \\ &\geq \bigvee_{\substack{x \in [a]_{C_R} \\ y \in [b]_{C_R}}} [K(n_t)(x) \wedge K(n_t)(y)] \\ &= [\bigvee_{x \in [a]_{C_R}} K(n_t)(x)] \wedge [\bigvee_{y \in [b]_{C_R}} K(n_t)(y)] \\ &= \overline{C}_R(K(n_t))(x) \wedge \overline{C}_R(K(n_t))(y) \end{aligned}$$

$$\begin{aligned} \overline{C}_R(K(n_i))(a - b) &= \bigwedge_{c \in [a-b]_{C_R}} K(n_i)(c) \\ &= \bigwedge_{x-y \in [a]_{C_R} - [b]_{C_R}} K(n_i)(x - y) \\ &\leq \bigwedge_{\substack{x \in [a]_{C_R} \\ y \in [b]_{C_R}}} [P(n_i)(x) \vee K(n_i)(y)] \\ &= [\bigwedge_{x \in [a]_{C_R}} K(n_i)(x)] \vee [\bigwedge_{y \in [b]_{C_R}} K(n_i)(y)] \\ &= \overline{C}_R(K(n_i))(x) \vee \overline{C}_R(K(n_i))(y) \end{aligned}$$

$$\begin{aligned} \overline{C}_R(K(n_f))(a - b) &= \bigwedge_{c \in [a-b]_{C_R}} K(n_f)(c) \\ &= \bigwedge_{x-y \in [a]_{C_R} - [b]_{C_R}} K(n_f)(x - y) \\ &\leq \bigwedge_{\substack{x \in [a]_{C_R} \\ y \in [b]_{C_R}}} [K(n_f)(x) \vee K(n_f)(y)] \\ &= [\bigwedge_{x \in [a]_{C_R}} K(n_f)(x)] \vee [\bigwedge_{y \in [b]_{C_R}} K(n_f)(y)] \\ &= \overline{C}_R(K(n_f))(x) \vee \overline{C}_R(K(n_f))(y) \end{aligned}$$

$\forall a, b \in R$

$$\begin{aligned}
 \overline{C_R}(K(n_t))(ab) &= \bigvee_{c \in [ab]_{C_R}} K(n_t)(c) \\
 &= \bigvee_{xy \in [a]_{C_R} [b]_{C_R}} K(n_t)(xy) \\
 &\geq \bigvee_{x \in [a]_{C_R}, y \in [b]_{C_R}} [K(n_t)(x) \wedge K(n_t)(y)] \\
 &= \left[\bigvee_{x \in [a]_{C_R}} K(n_t)(x) \right] \wedge \left[\bigvee_{y \in [b]_{C_R}} K(n_t)(y) \right] \\
 &= \overline{C_R}(K(n_t))(x) \wedge \overline{C_R}(K(n_t))(y)
 \end{aligned}$$

$$\begin{aligned}
 \overline{C_R}(K(n_i))(ab) &= \bigwedge_{c \in [ab]_{C_R}} K(n_i)(c) \\
 &= \bigwedge_{xy \in [a]_{C_R} [b]_{C_R}} K(n_i)(xy) \\
 &\leq \bigwedge_{x \in [a]_{C_R}, y \in [b]_{C_R}} [K(n_i)(x) \vee K(n_i)(y)] \\
 &= \left[\bigwedge_{x \in [a]_{C_R}} K(n_i)(x) \right] \vee \left[\bigwedge_{y \in [b]_{C_R}} K(n_i)(y) \right] \\
 &= \overline{C_R}(K(n_i))(x) \vee \overline{C_R}(K(n_i))(y)
 \end{aligned}$$

$$\begin{aligned}
 \overline{C_R}(K(n_f))(ab) &= \bigwedge_{c \in [ab]_{C_R}} K(n_f)(c) \\
 &= \bigwedge_{xy \in [a]_{C_R} [b]_{C_R}} K(n_f)(xy) \\
 &\leq \bigwedge_{x \in [a]_{C_R}, y \in [b]_{C_R}} [K(n_f)(x) \vee K(n_f)(y)] \\
 &= \left[\bigwedge_{x \in [a]_{C_R}} K(n_f)(x) \right] \vee \left[\bigwedge_{y \in [b]_{C_R}} K(n_f)(y) \right] \\
 &= \overline{C_R}(K(n_f))(x) \vee \overline{C_R}(K(n_f))(y)
 \end{aligned}$$

Hence, $\overline{C_R}(K)$ is a NSR of R . Thus K is an upper RNSR of R . \square

Theorem 4.4. *If K be a NSR of R , then K is a lower RNSR of R .*

Proof. This proof is similar to Theorem 4.3 \square

Corollary 4.5. *Let K be the NSR of R . Then K is a rough RNSR of R .*

Proof. By applying Theorem 4.3 and 4.4 we get the result. \square

Definition 4.6. A NI is called an RNI if it is both upper RNI and lower RNI of R .

Definition 4.7. A NI is said to be an lower (upper) RNI of R if its lower(upper) approximation is also an NI of R .

Theorem 4.8. If K be a NI of R , then K is an upper RNI of R .

Proof. Since K is a NI of R . We've to prove that, $\forall a, b \in R$

$$\begin{aligned}\overline{C}_R(K(n_t))(ab) &\geq \overline{C}_R(K(n_t))(a) \vee \overline{C}_R(K(n_t))(b) \\ \overline{C}_R(K(n_i))(ab) &\leq \overline{C}_R(K(n_i))(a) \wedge \overline{C}_R(K(n_i))(b) \\ \overline{C}_R(K(n_f))(ab) &\leq \overline{C}_R(K(n_f))(a) \wedge \overline{C}_R(K(n_f))(b)\end{aligned}$$

Now,

$$\begin{aligned}\overline{C}_R(K(n_i))(ab) &= \bigvee_{c \in [ab]_{C_R}} K(n_i)(c) \\ &\geq \bigvee_{c \in [a]_{C_R} [b]_{C_R}} K(n_i)(c) \\ &= \bigvee_{xy \in [a]_{C_R} [b]_{C_R}} K(n_i)(xy) \\ &\geq \bigvee_{x \in [a]_{C_R}, y \in [b]_{C_R}} [K(n_i)(x) \vee K(n_i)(y)] \\ &= [\bigvee_{x \in [a]_{C_R}} K(n_i)(x)] \vee [\bigvee_{y \in [b]_{C_R}} K(n_i)(y)] \\ &= \overline{C}_R(K(n_i))(x) \vee \overline{C}_R(K(n_i))(y)\end{aligned}$$

$$\begin{aligned}\overline{C}_R(K(n_i))(ab) &= \bigwedge_{c \in [ab]_{C_R}} K(n_i)(c) \\ &\leq \bigwedge_{c \in [a]_{C_R} [b]_{C_R}} K(n_i)(c) \\ &= \bigwedge_{xy \in [a]_{C_R} [b]_{C_R}} K(n_i)(xy) \\ &\leq \bigwedge_{x \in [a]_{C_R}, y \in [b]_{C_R}} [K(n_i)(x) \wedge K(n_i)(y)] \\ &= [\bigwedge_{x \in [a]_{C_R}} K(n_i)(x)] \wedge [\bigwedge_{y \in [b]_{C_R}} K(n_i)(y)] \\ &= \overline{C}_R(K(n_i))(x) \wedge \overline{C}_R(K(n_i))(y)\end{aligned}$$

$$\begin{aligned}
\overline{C_R}(K(n_f))(ab) &= \bigwedge_{c \in [ab]_{C_R}} K(n_f)(c) \\
&\leq \bigwedge_{c \in [a]_{C_R} [b]_{C_R}} K(n_f)(c) \\
&= \bigwedge_{xy \in [a]_{C_R} [b]_{C_R}} K(n_f)(xy) \\
&\leq \bigwedge_{x \in [a]_{C_R}, y \in [b]_{C_R}} [K(n_f)(x) \wedge K(n_f)(y)] \\
&= \left[\bigwedge_{x \in [a]_{C_R}} K(n_f)(x) \right] \wedge \left[\bigwedge_{y \in [b]_{C_R}} K(n_f)(y) \right] \\
&= \overline{C_R}(K(n_f))(x) \wedge \overline{C_R}(K(n_f))(y)
\end{aligned}$$

Hence, $\overline{C_R}(K)$ is a NI of R . Thus K is an upper RNI of R . \square

Theorem 4.9. *If K be a NI of R , then K is a lower RNI of R .*

Proof. This proof is similar to Theorem 4.8. \square

Corollary 4.10. *If K be the NI of R . then K is a RNI of R .*

Proof. By applying Theorem 4.8 and 4.9 we get the result. \square

5. Conclusion

In this paper, we discussed the notion of rough neutrosophic set in a ring and their properties. Also, we proved that any neutrosophic ideal of a ring is an rough neutrosophic ideal of a ring. For further research one can extend this to other algebraic systems.

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