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# Neutrosophic Hypersoft Expert Set: Theory and Applications

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**Abstract.** Soft set-like models deal with single argument approximate functions while hypersoft set, an extension of the soft set, deals with multi-argument approximate functions. The soft set cannot handle situations when attributes are required to be further divided into disjoint attribute-valued sets. To overcome this situation, a hypersoft set has been developed. In different fields like decision making and medical diagnosis, many researchers developed models based on the soft set for the solution of many problems. But these models deal with only one expert who creates many problems for the users, primarily in designing questionnaires. To remove this discrepancy, we present a neutrosophic hypersoft expert set. This model not only solves the problem of dealing with one expert but also solves the problem of different parametric-valued sets parallel to different characteristics. In this study, we first introduce the concept of neutrosophic hypersoft expert sets, which is an amalgam of both structures i.e., neutrosophic set and hypersoft expert sets. Certain essential basic characteristics (i.e., subset, equal set, agree, disagree set, null set, whole relative set, and whole absolute set), aggregation operations (i.e., complement, restricted union, extended intersection, AND and OR), and results (i.e., idempotent, absorption, domination, identity, commutative, associative and distributive law) are discussed with examples. Some hybrid structures of the neutrosophic hypersoft expert set are developed with illustrated examples. In the end, a decision-making application is presented for the validity of the proposed theory.

**Keywords:** Soft Set; Soft Expert Set; Neutrosophic set; Hypersoft Set; Neutrosophic Hypersoft Expert Set.

## 1. Introduction

In some real-life issues in professional and information systems where we have a situation to deal with the truth-membership along with the falsity-membership for a correct description of an object in an uncertain and an ambiguous environment. Smarandache [1–3] characterized neutrosophic set as a generalization of classical sets, fuzzy set, intuitionistic fuzzy set. Membership functions are used to define fuzzy sets [4], while membership and non-membership

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functions both are used for intuitionistic fuzzy sets [5] and are used for solving problems having the data of imprecise, indeterminacy and inconsistent. Neutrosophic set (NS) has wide applications in different fields like decision making, medical diagnosis, data bases, control theory and topology etc. Wang et al [6] introduced single valued neutrosophic set (SVNS) and presented its set operations and different properties. The use NS and its hybridized structures in various fields has been continuing quickly [7]- [32].

Molodtsov [34] constructed soft set by taking the advantage of parameterization tool. Rahman et al. [35, 36] conceptualized  $m$ -convexity ( $m$ -concavity) and  $(m, n)$ -convexity ( $(m, n)$ -concavity) on soft sets with some properties. Maji et al. [37] made development by introducing fuzzy soft set to solve parametrization problems with uncertainty. Many researchers [38]- [44] advanced this theory and used in many fields. Rahman et al. [45] conceptualized  $(m-n)$ -convexity(concavity)on fuzzy soft set with applications in first and second senses. Alkhazaleh et al. [46, 47] made extensions in soft set by introducing soft and fuzzy soft expert sets. They used these structures for applications in decision-making problems(DMPs). Ihsan et al. [48, 49] conceptualized convexity on soft expert set and fuzzy soft expert set with certain properties. Broumi et al. [50] conceptualized intuitionistic fuzzy soft expert set and made its use in DMPs. Mehmet et al. [33] defined neutrosophic soft expert sets and applied it in DMPs.

In 2018, Smarandache [51] extended soft set to hypersoft set and used in daily life problems. In 2020, Saeed et al. [52] advanced this theory and explained its structures. In 2020, Rahman et al. [53], [54] worked on hypersoft set and introduced its some new structures like complex fuzzy hypersoft set. They also gave the concept of convexity (concavity) on it and proved its some basic properties. Ihsan et al. [58, 59] introduced the structures of hypersoft expert set and fuzzy hypersoft expert set with applications in DMPs. Kamaci and Saqlain [60] worked on  $n$ -ary fuzzy hypersoft expert set and applied in real life problem. Kamaci [61] gave hybrid structures of hypersoft set and rough set and applied in DMPs. He [62] introduced the structure of simplified neutrosophic multiplicative refined sets and their correlation coefficients with Application in medical pattern recognition. The neutrosophic soft set like structures have been investigated and applied in different fields like game theory and DMPs in [63–65].

Having motivation from [33]- [50], new notions of neutrosophic hypersoft expert set are developed and some hybrids of neutrosophic hypersoft expert set are established.

The remaining portion of the paper is constructed as: Section 2 describes the basic definitions of soft set, fuzzy set, intuitionistic fuzzy set, neutrosophic set, fuzzy soft expert set, hypersoft set and relevant definitions used in the proposed work. Section 3, presents notions of fuzzy hypersoft expert set, neutrosophic hypersoft expert set with properties. Section 4, describes the set theoretic operations of NHSES. Section 5, presents the basic properties and laws of

NHSES. Section 6 shows the hybrids of NHSES. Section 7, presents an application in decision making and section 8 contains the conclusions of the paper.

## 2. Preliminaries

In this portion, some elementary definitions are presented from the literature.

Suppose  $W$  be a set of experts and  $\mathbf{O}$  be a set of opinions,  $\mathbf{T} = \mathbf{F} \times W \times \mathbf{O}$ . Taking  $S \subseteq \mathbf{T}$  and  $\Delta$  as a set of universe with  $P(\Delta)$  is the power set of universe, while parameters set is  $\mathbf{F}$ .

**Definition 2.1.** [40] A set " $F_z$ " is called a *fuzzy set* written as  $F_z = \{(\hat{r}, B(\hat{r})) | \hat{r} \in \Delta\}$  with  $B : \Delta \rightarrow \mathbb{I}$  and  $B(\hat{r})$  represents the membership value of  $\hat{r} \in F_z$ .

**Definition 2.2.** [41] A set " $\mathcal{J}$ " is called an *intuitionistic fuzzy set* written as  $\mathcal{J} = \{(\check{a}, \langle Z_{\mathcal{J}}(\check{a}), X_{\mathcal{J}}(\check{a}) \rangle) | (\check{a} \in \Delta)\}$  with  $Z_{\mathcal{J}} : \mathbb{I} \rightarrow \Delta$ ,  $X_{\mathcal{J}} : \mathbb{I} \rightarrow \Delta$  and  $Z_{\mathcal{J}}(\check{a})$ ,  $X_{\mathcal{J}}(\check{a})$  represent the truth, falsity membership functions of  $\check{a} \in \Delta$  satisfying the inequality  $0 \leq Z_{\mathcal{J}}(\check{a}) + X_{\mathcal{J}}(\check{a}) \leq 1$ .

**Definition 2.3.** [39] A neutrosophic set  $\mathbf{N}$  in  $\Delta$  is defined by

$$N = \{ \langle y, (T_N(y), I_N(y), F_N(y)) \rangle : y \in \mathbf{F}, T_N, I_N, F_N \in ]^{-0}, 1^+[ \}$$

where  $T_N, I_N, F_N$  are truth, indeterminacy, and falsity membership functions and are real standard or nonstandard subsets of  $]^{-0}, 1^+[$ . Their sum does not have any restriction, that is,  $0^- \leq T_N(y), I_N(y), F_N(y) \leq 3^+$ . Here  $]^{-0}, 1^+[$  is named the nonstandard subset, which is the extension of real standard subsets  $[0, 1]$  where the nonstandard number  $1^+ = 1 + \epsilon$ , 1 is named the standard part, and  $\epsilon$  is named the nonstandard part.  $^{-0} = 0 - \epsilon$ , 0 is the standard part and  $\epsilon$  is named the nonstandard part, where  $\epsilon$  is closed to positive real number zero.

**Definition 2.4.** [34] A pair  $(\Psi_M, \mathbf{F})$  is named as *soft set* and  $\Psi_M$  is characterized by a mapping

$$\Psi_M : \mathbf{F} \rightarrow P(\Delta)$$

where  $P(\Delta)$  is the power set of universe of discourse.

**Definition 2.5.** [37] Let  $C \subseteq \mathbf{F}$ . A fuzzy soft set is a pair  $(R, C)$  and  $R$  is characterized as

$$R : C \rightarrow I^\Delta$$

where  $I^\Delta$  represents the collection of all fuzzy subsets of  $\Delta$ .

**Definition 2.6.** [46] A soft expert set is a pair  $(\Phi_H, S)$  with  $\Phi_H$  is characterized by a mapping

$$\Phi_H : S \rightarrow P(\Delta)$$

where  $S \subseteq \mathbf{F} \times W \times \mathbf{O}$ .

**Definition 2.7.** [47] A fuzzy soft expert set is a pair  $(\Psi_F, S)$  with  $\Psi_F$  is characterized by a mapping

$$\Psi_F : S \rightarrow I^\Delta$$

where  $S \subseteq \mathbf{F} \times W \times \mathbf{O}$  and  $I^\Delta$  represents the collection of all fuzzy subsets of  $\Delta$ .

**Definition 2.8.** [6] Let  $\mathcal{K}_{\bar{\delta}}, \mathcal{L}_{\bar{\delta}}$  and  $\mathcal{M}_{\bar{\delta}}$  represent truth, indeterminacy and falsity membership functions, then  $\bar{\delta}$  represents a single valued neutrosophic set such that  $0 \leq \mathcal{K}_{\bar{\delta}}(\beta) + \mathcal{L}_{\bar{\delta}}(\beta) + \mathcal{M}_{\bar{\delta}}(\beta) \leq 3$ . While  $\mathcal{K}_{\bar{\delta}}, \mathcal{L}_{\bar{\delta}}, \mathcal{M}_{\bar{\delta}} \in [0, 1]$  for all  $\beta$  in  $\Delta$ .

**Definition 2.9.** [51]

Let  $\times_1, \times_2, \times_3, \dots, \times_\epsilon$ , with  $\epsilon \geq 1$ , be  $\epsilon$  different characters having parallel characteristics values are the sets  $\bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_3, \dots, \bar{\lambda}_\epsilon$ , with  $\bar{\lambda}_p \cap \bar{\lambda}_q = \emptyset$ , for  $p \neq q$ , and  $p, q \in \{1, 2, 3, \dots, \epsilon\}$ . Then hypersoft expert set is a pair  $(\Upsilon, L)$  with  $\Upsilon$  is characterized by a mapping

$$\Upsilon : L \rightarrow P(\Delta)$$

where  $L = \bar{\lambda}_1 \times \bar{\lambda}_2 \times \bar{\lambda}_3 \times \dots \times \bar{\lambda}_\epsilon$ .

### 3. Neutrosophic Hypersoft Expert Set (NHSES)

In this portion, neutrosophic hypersoft expert set has been developed with the help of existing concept of neutrosophic soft expert set and some basic properties are presented.

**Definition 3.1.** [59] Fuzzy Hypersoft Expert Set (FHSES)

A pair  $(\mathcal{L}, \mathcal{F})$  represents a FHSES with  $\mathcal{L}$  is characterized by a mapping

$$\xi : \mathcal{F} \rightarrow I^\Delta$$

- $I^\Delta$  is being used a collection of all fuzzy subsets of  $\Delta$
- $\mathcal{F} \subseteq \mathcal{H} = \mathbb{A} \times \mathbb{B} \times \mathbb{C}$
- $\mathbb{A} = \mathbb{A}_1 \times \mathbb{A}_2 \times \mathbb{A}_3 \times \dots \times \mathbb{A}_p$  where  $\mathbb{A}_i$  are different characteristics-valued sets parallel to different characteristics  $\mathfrak{a}_i, i = 1, 2, 3, \dots, p$
- $\mathbb{B}$  represents an expert set
- $\mathbb{C}$  represents a conclusion set.

**Definition 3.2.** Neutrosophic Hypersoft Expert Set (NHSES)

A neutrosophic hypersoft expert set represents a pair  $(\bar{h}, \mathbb{G})$  if

$$\bar{h} : \mathbb{G} \rightarrow NF^\Delta$$

with  $NF^\Delta$  is being used as collection of all neutrosophic subsets of  $\Delta$  and  $\mathbb{A} \subseteq \mathcal{H} = \mathbb{A} \times \mathbb{B} \times \mathbb{C}$ .

**Example 3.3.** Assume that a worldwide organization expects to continue the assessment of specific experts about its sure items. Let  $\Delta = \{w_1, w_2, w_3, w_4\}$  be a set of products and  $\mathbb{A}_1 = \{p_{11}, p_{12}\}$ ,  $\mathbb{A}_2 = \{p_{21}, p_{22}\}$ ,  $\mathbb{A}_3 = \{p_{31}, p_{32}\}$ , be different characteristics sets for different characteristics  $p_1 =$  simple to use,  $p_2 =$  nature,  $p_3 =$  modest. Now  $\mathbb{A} = \mathbb{A}_1 \times \mathbb{A}_2 \times \mathbb{A}_3$

$$\mathbb{A} = \left\{ \begin{array}{l} v_1 = (p_{11}, p_{21}, p_{31}), v_2 = (p_{11}, p_{21}, p_{32}), v_3 = (p_{11}, p_{22}, p_{31}), v_4 = (p_{11}, p_{22}, p_{32}), \\ v_5 = (p_{12}, p_{21}, p_{31}), v_6 = (p_{12}, p_{21}, p_{32}), v_7 = (p_{12}, p_{22}, p_{31}), v_8 = (p_{12}, p_{22}, p_{32}) \end{array} \right\}$$

Now  $\mathcal{H} = \mathbb{A} \times \mathbb{C} \times \mathbb{A}$ .

$$\mathcal{H} = \left\{ \begin{array}{l} (v_1, c, 0), (v_1, c, 1), (v_1, d, 0), (v_1, d, 1), (v_1, e, 0), (v_1, e, 1), (v_2, c, 0), (v_2, c, 1), \\ (v_2, d, 0), (v_2, d, 1), (v_2, e, 0), (v_2, e, 1), (v_3, c, 0), (v_3, c, 1), (v_3, d, 0), (v_3, d, 1), \\ (v_3, e, 0), (v_3, e, 1), (v_4, c, 0), (v_4, c, 1), (v_4, d, 0), (v_4, d, 1), (v_4, e, 0), (v_4, e, 1), \\ (v_5, c, 0), (v_5, c, 1), (v_5, d, 0), (v_5, d, 1), (v_5, e, 0), (v_5, e, 1), (v_6, c, 0), (v_6, c, 1), \\ (v_6, d, 0), (v_6, d, 1), (v_6, e, 0), (v_6, e, 1), (v_7, c, 0), (v_7, c, 1), (v_7, d, 0), (v_7, d, 1), \\ (v_7, e, 0), (v_7, e, 1), (v_8, c, 0), (v_8, c, 1), (v_8, d, 0), (v_8, d, 1), (v_8, e, 0), (v_8, e, 1) \end{array} \right\}.$$

let

$$\mathbb{G} = \left\{ \begin{array}{l} (v_1, c, 0), (v_1, c, 1), (v_1, d, 0), (v_1, d, 1), (v_1, e, 0), (v_1, e, 1), \\ (v_2, c, 0), (v_2, c, 1), (v_2, d, 0), (v_2, d, 1), (v_2, e, 0), (v_2, e, 1), \\ (v_3, c, 0), (v_3, c, 1), (v_3, d, 0), (v_3, d, 1), (v_3, e, 0), (v_3, e, 1), \end{array} \right\}$$

be a subset of  $\mathcal{H}$  and  $\mathbb{C} = \{c, d, e\}$  be a set of specialists.

Following check relates the varieties of three specialists:

$$\begin{aligned} \hbar_1 &= \hbar(v_1, c, 1) = \left\{ \frac{w_1}{\langle 0.2, 0.5, 0.4 \rangle}, \frac{w_2}{\langle 0.7, 0.2, 0.5 \rangle}, \frac{w_3}{\langle 0.5, 0.4, 0.6 \rangle}, \frac{w_4}{\langle 0.1, 0.3, 0.6 \rangle} \right\}, \\ \hbar_2 &= \hbar(v_1, d, 1) = \left\{ \frac{w_1}{\langle 0.4, 0.2, 0.3 \rangle}, \frac{w_2}{\langle 0.8, 0.1, 0.5 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.6 \rangle}, \frac{w_4}{\langle 0.2, 0.5, 0.3 \rangle} \right\}, \\ \hbar_3 &= \hbar(v_1, e, 1) = \left\{ \frac{w_1}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_2}{\langle 0.5, 0.3, 0.6 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.7 \rangle}, \frac{w_4}{\langle 0.3, 0.5, 0.6 \rangle} \right\}, \\ \hbar_4 &= \hbar(v_2, c, 1) = \left\{ \frac{w_1}{\langle 0.9, 0.1, 0.3 \rangle}, \frac{w_2}{\langle 0.4, 0.5, 0.4 \rangle}, \frac{w_3}{\langle 0.7, 0.2, 0.6 \rangle}, \frac{w_4}{\langle 0.3, 0.4, 0.8 \rangle} \right\}, \\ \hbar_5 &= \hbar(v_2, d, 1) = \left\{ \frac{w_1}{\langle 0.4, 0.5, 0.6 \rangle}, \frac{w_2}{\langle 0.8, 0.1, 0.7 \rangle}, \frac{w_3}{\langle 0.3, 0.6, 0.5 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.7 \rangle} \right\}, \\ \hbar_6 &= \hbar(v_2, e, 1) = \left\{ \frac{w_1}{\langle 0.5, 0.4, 0.7 \rangle}, \frac{w_2}{\langle 0.3, 0.6, 0.4 \rangle}, \frac{w_3}{\langle 0.6, 0.2, 0.5 \rangle}, \frac{w_4}{\langle 0.8, 0.1, 0.6 \rangle} \right\}, \\ \hbar_7 &= \hbar(v_3, c, 1) = \left\{ \frac{w_1}{\langle 0.2, 0.7, 0.5 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.4 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.7 \rangle}, \frac{w_4}{\langle 0.5, 0.4, 0.8 \rangle} \right\}, \\ \hbar_8 &= \hbar(v_3, d, 1) = \left\{ \frac{w_1}{\langle 0.4, 0.3, 0.2 \rangle}, \frac{w_2}{\langle 0.6, 0.3, 0.1 \rangle}, \frac{w_3}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_4}{\langle 0.9, 0.1, 0.4 \rangle} \right\}, \\ \hbar_9 &= \hbar(v_3, e, 1) = \left\{ \frac{w_1}{\langle 0.7, 0.2, 0.6 \rangle}, \frac{w_2}{\langle 0.3, 0.5, 0.7 \rangle}, \frac{w_3}{\langle 0.5, 0.4, 0.5 \rangle}, \frac{w_4}{\langle 0.2, 0.7, 0.8 \rangle} \right\}, \\ \hbar_{10} &= \hbar(v_1, c, 0) = \left\{ \frac{w_1}{\langle 0.3, 0.2, 0.1 \rangle}, \frac{w_2}{\langle 0.2, 0.4, 0.5 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.8 \rangle}, \frac{w_4}{\langle 0.1, 0.8, 0.3 \rangle} \right\}, \\ \hbar_{11} &= \hbar(v_1, d, 0) = \left\{ \frac{w_1}{\langle 0.1, 0.8, 0.4 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.2 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.4 \rangle}, \frac{w_4}{\langle 0.2, 0.7, 0.5 \rangle} \right\}, \end{aligned}$$

$$\begin{aligned} \hbar_{12} &= \hbar(v_1, e, 0) = \left\{ \frac{w_1}{\langle 0.2, 0.7, 0.5 \rangle}, \frac{w_2}{\langle 0.1, 0.8, 0.6 \rangle}, \frac{w_3}{\langle 0.3, 0.5, 0.7 \rangle}, \frac{w_4}{\langle 0.5, 0.4, 0.6 \rangle} \right\}, \\ \hbar_{13} &= \hbar(v_2, c, 0) = \left\{ \frac{w_1}{\langle 0.8, 0.1, 0.6 \rangle}, \frac{w_2}{\langle 0.3, 0.6, 0.7 \rangle}, \frac{w_3}{\langle 0.5, 0.4, 0.8 \rangle}, \frac{w_4}{\langle 0.7, 0.2, 0.9 \rangle} \right\}, \\ \hbar_{14} &= \hbar(v_2, d, 0) = \left\{ \frac{w_1}{\langle 0.7, 0.2, 0.5 \rangle}, \frac{w_2}{\langle 0.2, 0.6, 0.4 \rangle}, \frac{w_3}{\langle 0.9, 0.1, 0.6 \rangle}, \frac{w_4}{\langle 0.4, 0.5, 0.7 \rangle} \right\}, \\ \hbar_{15} &= \hbar(v_2, e, 0) = \left\{ \frac{w_1}{\langle 0.6, 0.2, 0.5 \rangle}, \frac{w_2}{\langle 0.7, 0.2, 0.4 \rangle}, \frac{w_3}{\langle 0.3, 0.5, 0.4 \rangle}, \frac{w_4}{\langle 0.2, 0.7, 0.6 \rangle} \right\}, \\ \hbar_{16} &= \hbar(v_3, c, 0) = \left\{ \frac{w_1}{\langle 0.1, 0.7, 0.5 \rangle}, \frac{w_2}{\langle 0.4, 0.5, 0.7 \rangle}, \frac{w_3}{\langle 0.7, 0.2, 0.9 \rangle}, \frac{w_4}{\langle 0.8, 0.2, 0.4 \rangle} \right\}, \\ \hbar_{17} &= \hbar(v_3, d, 0) = \left\{ \frac{w_1}{\langle 0.2, 0.7, 0.4 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.6 \rangle}, \frac{w_3}{\langle 0.8, 0.2, 0.4 \rangle}, \frac{w_4}{\langle 0.3, 0.5, 0.7 \rangle} \right\}, \\ \hbar_{18} &= \hbar(v_3, e, 0) = \left\{ \frac{w_1}{\langle 0.5, 0.4, 0.2 \rangle}, \frac{w_2}{\langle 0.3, 0.6, 0.1 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.2 \rangle}, \frac{w_4}{\langle 0.1, 0.8, 0.3 \rangle} \right\}. \end{aligned}$$

The NHSES can be described as  $(\hbar, \mathbb{G}) =$

$$\left\{ \left( \left( v_1, c, 1, \left\{ \frac{w_1}{\langle 0.2, 0.5, 0.4 \rangle}, \frac{w_2}{\langle 0.7, 0.2, 0.5 \rangle}, \frac{w_3}{\langle 0.5, 0.4, 0.6 \rangle}, \frac{w_4}{\langle 0.1, 0.3, 0.6 \rangle} \right\} \right), \left( v_1, d, 1, \left\{ \frac{w_1}{\langle 0.4, 0.2, 0.3 \rangle}, \frac{w_2}{\langle 0.8, 0.1, 0.5 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.6 \rangle}, \frac{w_4}{\langle 0.2, 0.5, 0.3 \rangle} \right\} \right) \right), \left( \left( v_1, e, 1, \left\{ \frac{w_1}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_2}{\langle 0.5, 0.3, 0.6 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.7 \rangle}, \frac{w_4}{\langle 0.3, 0.5, 0.6 \rangle} \right\} \right), \left( v_2, c, 1, \left\{ \frac{w_1}{\langle 0.9, 0.1, 0.3 \rangle}, \frac{w_2}{\langle 0.4, 0.5, 0.4 \rangle}, \frac{w_3}{\langle 0.7, 0.2, 0.6 \rangle}, \frac{w_4}{\langle 0.3, 0.4, 0.8 \rangle} \right\} \right) \right), \left( \left( v_2, d, 1, \left\{ \frac{w_1}{\langle 0.4, 0.5, 0.6 \rangle}, \frac{w_2}{\langle 0.8, 0.1, 0.7 \rangle}, \frac{w_3}{\langle 0.3, 0.6, 0.5 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.7 \rangle} \right\} \right), \left( v_2, e, 1, \left\{ \frac{w_1}{\langle 0.5, 0.4, 0.7 \rangle}, \frac{w_2}{\langle 0.3, 0.6, 0.4 \rangle}, \frac{w_3}{\langle 0.6, 0.2, 0.5 \rangle}, \frac{w_4}{\langle 0.8, 0.1, 0.6 \rangle} \right\} \right) \right), \left( \left( v_3, c, 1, \left\{ \frac{w_1}{\langle 0.2, 0.7, 0.5 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.4 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.7 \rangle}, \frac{w_4}{\langle 0.5, 0.4, 0.8 \rangle} \right\} \right), \left( v_3, d, 1, \left\{ \frac{w_1}{\langle 0.4, 0.3, 0.2 \rangle}, \frac{w_2}{\langle 0.6, 0.3, 0.1 \rangle}, \frac{w_3}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_4}{\langle 0.9, 0.1, 0.4 \rangle} \right\} \right) \right), \left( \left( v_3, e, 1, \left\{ \frac{w_1}{\langle 0.7, 0.2, 0.6 \rangle}, \frac{w_2}{\langle 0.3, 0.5, 0.7 \rangle}, \frac{w_3}{\langle 0.5, 0.4, 0.5 \rangle}, \frac{w_4}{\langle 0.2, 0.7, 0.8 \rangle} \right\} \right), \left( v_1, c, 0, \left\{ \frac{w_1}{\langle 0.3, 0.2, 0.1 \rangle}, \frac{w_2}{\langle 0.2, 0.4, 0.5 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.8 \rangle}, \frac{w_4}{\langle 0.1, 0.8, 0.3 \rangle} \right\} \right) \right), \left( \left( v_1, d, 0, \left\{ \frac{w_1}{\langle 0.1, 0.8, 0.4 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.2 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.4 \rangle}, \frac{w_4}{\langle 0.2, 0.7, 0.5 \rangle} \right\} \right), \left( v_1, e, 0, \left\{ \frac{w_1}{\langle 0.2, 0.7, 0.5 \rangle}, \frac{w_2}{\langle 0.1, 0.8, 0.6 \rangle}, \frac{w_3}{\langle 0.3, 0.5, 0.7 \rangle}, \frac{w_4}{\langle 0.5, 0.4, 0.6 \rangle} \right\} \right) \right), \left( \left( v_2, c, 0, \left\{ \frac{w_1}{\langle 0.8, 0.1, 0.6 \rangle}, \frac{w_2}{\langle 0.3, 0.6, 0.7 \rangle}, \frac{w_3}{\langle 0.5, 0.4, 0.8 \rangle}, \frac{w_4}{\langle 0.7, 0.2, 0.9 \rangle} \right\} \right), \left( v_2, d, 0, \left\{ \frac{w_1}{\langle 0.7, 0.2, 0.5 \rangle}, \frac{w_2}{\langle 0.2, 0.6, 0.4 \rangle}, \frac{w_3}{\langle 0.9, 0.1, 0.6 \rangle}, \frac{w_4}{\langle 0.4, 0.5, 0.7 \rangle} \right\} \right) \right), \left( \left( v_2, e, 0, \left\{ \frac{w_1}{\langle 0.6, 0.2, 0.5 \rangle}, \frac{w_2}{\langle 0.7, 0.2, 0.4 \rangle}, \frac{w_3}{\langle 0.3, 0.5, 0.4 \rangle}, \frac{w_4}{\langle 0.2, 0.7, 0.6 \rangle} \right\} \right), \left( v_3, c, 0, \left\{ \frac{w_1}{\langle 0.1, 0.7, 0.5 \rangle}, \frac{w_2}{\langle 0.4, 0.5, 0.7 \rangle}, \frac{w_3}{\langle 0.7, 0.2, 0.9 \rangle}, \frac{w_4}{\langle 0.8, 0.2, 0.4 \rangle} \right\} \right) \right), \left( \left( v_3, d, 0, \left\{ \frac{w_1}{\langle 0.2, 0.7, 0.4 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.6 \rangle}, \frac{w_3}{\langle 0.8, 0.2, 0.4 \rangle}, \frac{w_4}{\langle 0.3, 0.5, 0.7 \rangle} \right\} \right), \left( v_3, e, 0, \left\{ \frac{w_1}{\langle 0.5, 0.4, 0.2 \rangle}, \frac{w_2}{\langle 0.3, 0.6, 0.1 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.2 \rangle}, \frac{w_4}{\langle 0.1, 0.8, 0.3 \rangle} \right\} \right) \right) \right\}.$$

**Definition 3.4.** Neutrosophic Hypersoft Expert Subset

A NHSES  $(\hbar_1, \mathbb{G})$  is said to be NHSE subset of  $(\hbar_2, \mathbb{P})$ , if

- (i)  $\mathbb{G} \subseteq \mathbb{P}$ , (ii)  $\forall \gamma \in \mathbb{G}, \hbar_1(\gamma) \subseteq \hbar_2(\gamma)$  and denoted by  $(\hbar_1, \mathbb{G}) \subseteq (\hbar_2, \mathbb{P})$ .

**Example 3.5.** Considering Example 3.3, with two NHSESs

$$\mathbb{G}_1 = \left\{ (v_1, c, 1), (v_3, c, 0), (v_1, d, 1), (v_3, d, 1), (v_3, d, 0), (v_1, e, 0), (v_3, e, 1) \right\}$$

$$\mathbb{G}_2 = \left\{ (v_1, c, 1), (v_3, c, 0), (v_3, c, 1), (v_1, d, 1), (v_3, d, 1), (v_1, d, 0), (v_3, d, 0), (v_1, e, 0), (v_3, e, 1), (v_1, e, 1) \right\}.$$

It is clear that  $\mathbb{G}_1 \subset \mathbb{G}_2$ . Suppose  $(\tilde{h}_1, \mathbb{G}_1)$  and  $(\tilde{h}_2, \mathbb{G}_2)$  be defined as following

$$(\tilde{h}_1, \mathbb{G}_1) = \left\{ \begin{array}{l} \left( (v_1, c, 1), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.1, 0.6, 0.7 \rangle}, \frac{w_2}{\langle 0.6, 0.5, 0.8 \rangle}, \frac{w_3}{\langle 0.4, 0.6, 0.9 \rangle}, \frac{w_4}{\langle 0.1, 0.8, 0.6 \rangle} \end{array} \right\} \right), \\ \left( (v_1, d, 1), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.3, 0.4, 0.5 \rangle}, \frac{w_2}{\langle 0.6, 0.4, 0.6 \rangle}, \frac{w_3}{\langle 0.2, 0.5, 0.7 \rangle}, \frac{w_4}{\langle 0.1, 0.5, 0.6 \rangle} \end{array} \right\} \right), \\ \left( (v_3, d, 1), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.2, 0.6, 0.4 \rangle}, \frac{w_2}{\langle 0.5, 0.4, 0.7 \rangle}, \frac{w_3}{\langle 0.6, 0.5, 0.8 \rangle}, \frac{w_4}{\langle 0.8, 0.6, 0.4 \rangle} \end{array} \right\} \right), \\ \left( (v_3, e, 1), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.6, 0.4, 0.3 \rangle}, \frac{w_2}{\langle 0.2, 0.7, 0.6 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.3 \rangle}, \frac{w_4}{\langle 0.1, 0.7, 0.4 \rangle} \end{array} \right\} \right), \\ \left( (v_1, e, 0), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.1, 0.6, 0.3 \rangle}, \frac{w_2}{\langle 0.1, 0.7, 0.4 \rangle}, \frac{w_3}{\langle 0.2, 0.7, 0.6 \rangle}, \frac{w_4}{\langle 0.1, 0.6, 0.7 \rangle} \end{array} \right\} \right), \\ \left( (v_3, c, 0), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.1, 0.8, 0.6 \rangle}, \frac{w_2}{\langle 0.3, 0.6, 0.5 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.4 \rangle}, \frac{w_4}{\langle 0.7, 0.2, 0.6 \rangle} \end{array} \right\} \right), \\ \left( (v_3, d, 0), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.1, 0.7, 0.4 \rangle}, \frac{w_2}{\langle 0.6, 0.3, 0.6 \rangle}, \frac{w_3}{\langle 0.7, 0.2, 0.5 \rangle}, \frac{w_4}{\langle 0.2, 0.7, 0.4 \rangle} \end{array} \right\} \right) \end{array} \right\}$$

$$(\tilde{h}_2, \mathbb{G}_2) = \left\{ \begin{array}{l} \left( (v_1, c, 1), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.2, 0.3, 0.6 \rangle}, \frac{w_2}{\langle 0.7, 0.4, 0.7 \rangle}, \frac{w_3}{\langle 0.5, 0.4, 0.8 \rangle}, \frac{w_4}{\langle 0.2, 0.4, 0.5 \rangle} \end{array} \right\} \right), \\ \left( (v_1, d, 1), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.4, 0.3, 0.4 \rangle}, \frac{w_2}{\langle 0.8, 0.3, 0.5 \rangle}, \frac{w_3}{\langle 0.4, 0.3, 0.6 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.5 \rangle} \end{array} \right\} \right), \\ \left( (v_3, c, 1), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.1, 0.3, 0.4 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.3 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.4 \rangle}, \frac{w_4}{\langle 0.5, 0.3, 0.4 \rangle} \end{array} \right\} \right), \\ \left( (v_3, d, 1), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.4, 0.2, 0.3 \rangle}, \frac{w_2}{\langle 0.6, 0.3, 0.6 \rangle}, \frac{w_3}{\langle 0.7, 0.4, 0.5 \rangle}, \frac{w_4}{\langle 0.9, 0.5, 0.2 \rangle} \end{array} \right\} \right), \\ \left( (v_1, e, 1), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.7, 0.2, 0.4 \rangle}, \frac{w_2}{\langle 0.5, 0.2, 0.6 \rangle}, \frac{w_3}{\langle 0.6, 0.2, 0.7 \rangle}, \frac{w_4}{\langle 0.3, 0.5, 0.8 \rangle} \end{array} \right\} \right), \\ \left( (v_3, e, 1), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.7, 0.3, 0.1 \rangle}, \frac{w_2}{\langle 0.3, 0.5, 0.4 \rangle}, \frac{w_3}{\langle 0.5, 0.4, 0.2 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.3 \rangle} \end{array} \right\} \right), \\ \left( (v_1, e, 0), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.2, 0.5, 0.1 \rangle}, \frac{w_2}{\langle 0.2, 0.6, 0.3 \rangle}, \frac{w_3}{\langle 0.3, 0.5, 0.4 \rangle}, \frac{w_4}{\langle 0.5, 0.3, 0.5 \rangle} \end{array} \right\} \right), \\ \left( (v_1, d, 0), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.1, 0.6, 0.4 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.6 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.8 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.8 \rangle} \end{array} \right\} \right), \\ \left( (v_3, c, 0), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.2, 0.7, 0.4 \rangle}, \frac{w_2}{\langle 0.4, 0.5, 0.3 \rangle}, \frac{w_3}{\langle 0.7, 0.2, 0.1 \rangle}, \frac{w_4}{\langle 0.8, 0.1, 0.5 \rangle} \end{array} \right\} \right), \\ \left( (v_3, d, 0), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.2, 0.5, 0.1 \rangle}, \frac{w_2}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_3}{\langle 0.8, 0.2, 0.4 \rangle}, \frac{w_4}{\langle 0.3, 0.5, 0.2 \rangle} \end{array} \right\} \right) \end{array} \right\}$$

which shows that  $(\tilde{h}_1, \mathbb{G}_1) \subseteq (\tilde{h}_2, \mathbb{G}_2)$ .

**Definition 3.6.** Two NHSESs  $(\tilde{h}_1, \mathbb{G}_1)$  and  $(\tilde{h}_2, \mathbb{G}_2)$  over  $\Delta$  are said to be equal if  $(\tilde{h}_1, \mathbb{G}_1)$  is a NHSE subset of  $(\tilde{h}_2, \mathbb{G}_2)$  and  $(\tilde{h}_2, \mathbb{G}_2)$  is a neutrosophic hypersoft expert subset of  $(\tilde{h}_1, \mathbb{G}_1)$ .

**Definition 3.7.** The complement of a NHSES is characterized by as

$$(\tilde{h}, \mathbb{G})^c = \tilde{c}(\tilde{h}(\varsigma)) \quad \forall \varsigma \in \Delta \text{ while } \tilde{c} \text{ is a neutrosophic complement.}$$



**Example 3.8.** Finding complement of NHSES find in 3.3, we have

$$(\tilde{h}, \mathbb{G})^c = \left\{ \begin{array}{l} \left( (v_1, c, 1), \left\{ \frac{w_1}{\langle 0.4, 0.5, 0.2 \rangle}, \frac{w_2}{\langle 0.5, 0.8, 0.7 \rangle}, \frac{w_3}{\langle 0.6, 0.6, 0.5 \rangle}, \frac{w_4}{\langle 0.6, 0.7, 0.1 \rangle} \right\} \right), \\ \left( (v_1, d, 1), \left\{ \frac{w_1}{\langle 0.3, 0.8, 0.6 \rangle}, \frac{w_2}{\langle 0.5, 0.9, 0.8 \rangle}, \frac{w_3}{\langle 0.6, 0.5, 0.4 \rangle}, \frac{w_4}{\langle 0.3, 0.5, 0.2 \rangle} \right\} \right), \\ \left( (v_1, e, 1), \left\{ \frac{w_1}{\langle 0.3, 0.8, 0.7 \rangle}, \frac{w_2}{\langle 0.6, 0.7, 0.5 \rangle}, \frac{w_3}{\langle 0.7, 0.7, 0.6 \rangle}, \frac{w_4}{\langle 0.6, 0.5, 0.3 \rangle} \right\} \right), \\ \left( (v_2, c, 1), \left\{ \frac{w_1}{\langle 0.3, 0.9, 0.9 \rangle}, \frac{w_2}{\langle 0.4, 0.5, 0.4 \rangle}, \frac{w_3}{\langle 0.6, 0.8, 0.7 \rangle}, \frac{w_4}{\langle 0.8, 0.6, 0.3 \rangle} \right\} \right), \\ \left( (v_2, d, 1), \left\{ \frac{w_1}{\langle 0.6, 0.5, 0.4 \rangle}, \frac{w_2}{\langle 0.7, 0.9, 0.8 \rangle}, \frac{w_3}{\langle 0.5, 0.4, 0.3 \rangle}, \frac{w_4}{\langle 0.7, 0.4, 0.2 \rangle} \right\} \right), \\ \left( (v_2, e, 1), \left\{ \frac{w_1}{\langle 0.7, 0.6, 0.5 \rangle}, \frac{w_2}{\langle 0.4, 0.4, 0.3 \rangle}, \frac{w_3}{\langle 0.5, 0.8, 0.6 \rangle}, \frac{w_4}{\langle 0.6, 0.9, 0.8 \rangle} \right\} \right), \\ \left( (v_3, c, 1), \left\{ \frac{w_1}{\langle 0.5, 0.3, 0.2 \rangle}, \frac{w_2}{\langle 0.4, 0.9, 0.9 \rangle}, \frac{w_3}{\langle 0.7, 0.5, 0.4 \rangle}, \frac{w_4}{\langle 0.8, 0.6, 0.5 \rangle} \right\} \right), \\ \left( (v_3, d, 1), \left\{ \frac{w_1}{\langle 0.2, 0.7, 0.4 \rangle}, \frac{w_2}{\langle 0.1, 0.7, 0.6 \rangle}, \frac{w_3}{\langle 0.3, 0.8, 0.7 \rangle}, \frac{w_4}{\langle 0.4, 0.9, 0.9 \rangle} \right\} \right), \\ \left( (v_3, e, 1), \left\{ \frac{w_1}{\langle 0.6, 0.8, 0.7 \rangle}, \frac{w_2}{\langle 0.7, 0.5, 0.3 \rangle}, \frac{w_3}{\langle 0.5, 0.6, 0.5 \rangle}, \frac{w_4}{\langle 0.8, 0.3, 0.2 \rangle} \right\} \right), \\ \left( (v_1, c, 0), \left\{ \frac{w_1}{\langle 0.1, 0.8, 0.3 \rangle}, \frac{w_2}{\langle 0.5, 0.6, 0.2 \rangle}, \frac{w_3}{\langle 0.8, 0.5, 0.4 \rangle}, \frac{w_4}{\langle 0.3, 0.2, 0.1 \rangle} \right\} \right), \\ \left( (v_1, d, 0), \left\{ \frac{w_1}{\langle 0.4, 0.2, 0.1 \rangle}, \frac{w_2}{\langle 0.2, 0.9, 0.9 \rangle}, \frac{w_3}{\langle 0.4, 0.7, 0.6 \rangle}, \frac{w_4}{\langle 0.5, 0.3, 0.2 \rangle} \right\} \right), \\ \left( (v_1, e, 0), \left\{ \frac{w_1}{\langle 0.5, 0.3, 0.2 \rangle}, \frac{w_2}{\langle 0.6, 0.2, 0.1 \rangle}, \frac{w_3}{\langle 0.7, 0.5, 0.3 \rangle}, \frac{w_4}{\langle 0.6, 0.6, 0.5 \rangle} \right\} \right), \\ \left( (v_2, c, 0), \left\{ \frac{w_1}{\langle 0.6, 0.9, 0.8 \rangle}, \frac{w_2}{\langle 0.7, 0.4, 0.3 \rangle}, \frac{w_3}{\langle 0.8, 0.6, 0.5 \rangle}, \frac{w_4}{\langle 0.9, 0.8, 0.7 \rangle} \right\} \right), \\ \left( (v_2, d, 0), \left\{ \frac{w_1}{\langle 0.5, 0.8, 0.7 \rangle}, \frac{w_2}{\langle 0.4, 0.4, 0.2 \rangle}, \frac{w_3}{\langle 0.6, 0.9, 0.9 \rangle}, \frac{w_4}{\langle 0.7, 0.5, 0.4 \rangle} \right\} \right), \\ \left( (v_2, e, 0), \left\{ \frac{w_1}{\langle 0.5, 0.8, 0.6 \rangle}, \frac{w_2}{\langle 0.4, 0.8, 0.7 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.3 \rangle}, \frac{w_4}{\langle 0.6, 0.3, 0.2 \rangle} \right\} \right), \\ \left( (v_3, c, 0), \left\{ \frac{w_1}{\langle 0.5, 0.3, 0.1 \rangle}, \frac{w_2}{\langle 0.7, 0.5, 0.3 \rangle}, \frac{w_3}{\langle 0.9, 0.8, 0.8 \rangle}, \frac{w_4}{\langle 0.4, 0.8, 0.8 \rangle} \right\} \right), \\ \left( (v_3, d, 0), \left\{ \frac{w_1}{\langle 0.4, 0.3, 0.2 \rangle}, \frac{w_2}{\langle 0.6, 0.9, 0.9 \rangle}, \frac{w_3}{\langle 0.4, 0.8, 0.8 \rangle}, \frac{w_4}{\langle 0.7, 0.5, 0.3 \rangle} \right\} \right), \\ \left( (v_3, e, 0), \left\{ \frac{w_1}{\langle 0.2, 0.6, 0.5 \rangle}, \frac{w_2}{\langle 0.1, 0.4, 0.3 \rangle}, \frac{w_3}{\langle 0.2, 0.7, 0.6 \rangle}, \frac{w_4}{\langle 0.3, 0.2, 0.1 \rangle} \right\} \right) \end{array} \right\}.$$

**Definition 3.9.** An agree-NHSES is described by as  $(\tilde{h}, \mathbb{A})_{ag} = \{ \tilde{h}_{ag}(\varsigma) : \varsigma \in \mathbb{A} \times \mathbb{C} \times \{1\} \}$ .

**Example 3.10.** Finding agree-NHSES calculated in 3.3, we get

$$(\tilde{h}, \mathbb{G}) = \left\{ \begin{array}{l} \left( (v_1, c, 1), \left\{ \frac{w_1}{\langle 0.2, 0.5, 0.4 \rangle}, \frac{w_2}{\langle 0.7, 0.2, 0.5 \rangle}, \frac{w_3}{\langle 0.5, 0.4, 0.6 \rangle}, \frac{w_4}{\langle 0.1, 0.3, 0.6 \rangle} \right\} \right), \\ \left( (v_1, d, 1), \left\{ \frac{w_1}{\langle 0.4, 0.2, 0.3 \rangle}, \frac{w_2}{\langle 0.8, 0.1, 0.5 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.6 \rangle}, \frac{w_4}{\langle 0.2, 0.5, 0.3 \rangle} \right\} \right), \\ \left( (v_1, e, 1), \left\{ \frac{w_1}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_2}{\langle 0.5, 0.3, 0.6 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.7 \rangle}, \frac{w_4}{\langle 0.3, 0.5, 0.6 \rangle} \right\} \right), \\ \left( (v_2, c, 1), \left\{ \frac{w_1}{\langle 0.9, 0.1, 0.3 \rangle}, \frac{w_2}{\langle 0.4, 0.5, 0.4 \rangle}, \frac{w_3}{\langle 0.7, 0.2, 0.6 \rangle}, \frac{w_4}{\langle 0.3, 0.4, 0.8 \rangle} \right\} \right), \\ \left( (v_2, d, 1), \left\{ \frac{w_1}{\langle 0.4, 0.5, 0.6 \rangle}, \frac{w_2}{\langle 0.8, 0.1, 0.7 \rangle}, \frac{w_3}{\langle 0.3, 0.6, 0.5 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.7 \rangle} \right\} \right), \\ \left( (v_2, e, 1), \left\{ \frac{w_1}{\langle 0.5, 0.4, 0.7 \rangle}, \frac{w_2}{\langle 0.3, 0.6, 0.4 \rangle}, \frac{w_3}{\langle 0.6, 0.2, 0.5 \rangle}, \frac{w_4}{\langle 0.8, 0.1, 0.6 \rangle} \right\} \right), \\ \left( (v_3, c, 1), \left\{ \frac{w_1}{\langle 0.2, 0.7, 0.5 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.4 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.7 \rangle}, \frac{w_4}{\langle 0.5, 0.4, 0.8 \rangle} \right\} \right), \\ \left( (v_3, d, 1), \left\{ \frac{w_1}{\langle 0.4, 0.3, 0.2 \rangle}, \frac{w_2}{\langle 0.6, 0.3, 0.1 \rangle}, \frac{w_3}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_4}{\langle 0.9, 0.1, 0.4 \rangle} \right\} \right), \\ \left( (v_3, e, 1), \left\{ \frac{w_1}{\langle 0.7, 0.2, 0.6 \rangle}, \frac{w_2}{\langle 0.3, 0.5, 0.7 \rangle}, \frac{w_3}{\langle 0.5, 0.4, 0.5 \rangle}, \frac{w_4}{\langle 0.2, 0.7, 0.8 \rangle} \right\} \right) \end{array} \right\}.$$

**Definition 3.11.** A disagree-NHSES is described by as

$$(\tilde{h}, \mathbb{A})_{dag} = \{ \tilde{h}_{dag}(\varsigma) : \varsigma \in \mathbb{A} \times \mathbb{C} \times \{0\} \}.$$

**Example 3.12.** Getting disagree-NHSES calculated in 3.3,

$$(\tilde{h}, \mathbb{G}) = \left\{ \left( (v_1, c, 0), \left\{ \frac{w_1}{\langle 0.3, 0.2, 0.1 \rangle}, \frac{w_2}{\langle 0.2, 0.4, 0.5 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.8 \rangle}, \frac{w_4}{\langle 0.1, 0.8, 0.3 \rangle} \right\} \right), \right. \\ \left. \left( (v_1, d, 0), \left\{ \frac{w_1}{\langle 0.1, 0.8, 0.4 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.2 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.4 \rangle}, \frac{w_4}{\langle 0.2, 0.7, 0.5 \rangle} \right\} \right), \right. \\ \left. \left( (v_1, e, 0), \left\{ \frac{w_1}{\langle 0.2, 0.7, 0.5 \rangle}, \frac{w_2}{\langle 0.1, 0.8, 0.6 \rangle}, \frac{w_3}{\langle 0.3, 0.5, 0.7 \rangle}, \frac{w_4}{\langle 0.5, 0.4, 0.6 \rangle} \right\} \right), \right. \\ \left. \left( (v_2, c, 0), \left\{ \frac{w_1}{\langle 0.8, 0.1, 0.6 \rangle}, \frac{w_2}{\langle 0.3, 0.6, 0.7 \rangle}, \frac{w_3}{\langle 0.5, 0.4, 0.8 \rangle}, \frac{w_4}{\langle 0.7, 0.2, 0.9 \rangle} \right\} \right), \right. \\ \left. \left( (v_2, d, 0), \left\{ \frac{w_1}{\langle 0.7, 0.2, 0.5 \rangle}, \frac{w_2}{\langle 0.2, 0.6, 0.4 \rangle}, \frac{w_3}{\langle 0.9, 0.1, 0.6 \rangle}, \frac{w_4}{\langle 0.4, 0.5, 0.7 \rangle} \right\} \right), \right. \\ \left. \left( (v_2, e, 0), \left\{ \frac{w_1}{\langle 0.6, 0.2, 0.5 \rangle}, \frac{w_2}{\langle 0.7, 0.2, 0.4 \rangle}, \frac{w_3}{\langle 0.3, 0.5, 0.4 \rangle}, \frac{w_4}{\langle 0.2, 0.7, 0.6 \rangle} \right\} \right), \right. \\ \left. \left( (v_3, c, 0), \left\{ \frac{w_1}{\langle 0.1, 0.7, 0.5 \rangle}, \frac{w_2}{\langle 0.4, 0.5, 0.7 \rangle}, \frac{w_3}{\langle 0.7, 0.2, 0.9 \rangle}, \frac{w_4}{\langle 0.8, 0.2, 0.4 \rangle} \right\} \right), \right. \\ \left. \left( (v_3, d, 0), \left\{ \frac{w_1}{\langle 0.2, 0.7, 0.4 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.6 \rangle}, \frac{w_3}{\langle 0.8, 0.2, 0.4 \rangle}, \frac{w_4}{\langle 0.3, 0.5, 0.7 \rangle} \right\} \right), \right. \\ \left. \left( (v_3, e, 0), \left\{ \frac{w_1}{\langle 0.5, 0.4, 0.2 \rangle}, \frac{w_2}{\langle 0.3, 0.6, 0.1 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.2 \rangle}, \frac{w_4}{\langle 0.1, 0.8, 0.3 \rangle} \right\} \right) \right\}.$$

**Definition 3.13.** A NHSES  $(\tilde{h}_1, \mathbb{G}_1)$  is called a relative null NHSES w.r.t  $\mathbb{G}_1 \subset \mathbb{G}$ , denoted by  $(\tilde{h}_1, \mathbb{G}_1)$ , if  $\tilde{h}_1(g) = \emptyset, \forall g \in \mathbb{G}_1$ .

**Example 3.14.** Considering Example 3.3, we  
 $(\tilde{h}_1, \mathbb{G}_1) = \{((w_1, c, 1), \emptyset), ((w_2, d, 1), \emptyset), ((w_3, e, 1), \emptyset)\}.$

**Definition 3.15.** A NHSES  $(\tilde{h}_2, \mathbb{G}_2)$  is called a relative whole NHSES w.r.t  $\mathbb{G}_2 \subset \mathbb{G}$ , denoted by  $(\tilde{h}_2, \mathbb{G}_2)_\Delta$ , if  $\tilde{h}_2(g) = \Delta, \forall g \in \mathbb{G}_2$ .

**Example 3.16.** Considering Example 3.3, we have  
 $(\tilde{h}_2, \mathbb{G}_2)_\Delta = \{((w_1, c, 1), \Delta), ((w_2, d, 1), \Delta), ((w_3, e, 1), \Delta)\}$  where  $\mathbb{G}_2 \subseteq \mathbb{G}$ .

**Definition 3.17.** A NHSES  $(\tilde{h}, \mathbb{G})$  is called absolute whole NHSES denoted by  $(\tilde{h}, \mathbb{G})_\Delta$ , if  $\tilde{h}(g) = \Delta, \forall g \in \mathbb{G}$ .

**Example 3.18.** Considering Example 3.3, we have

$$(\Psi, \mathbb{S})_\Delta = \left\{ \begin{array}{l} ((w_1, c, 1), \Delta), ((w_1, d, 1), \Delta), ((w_1, e, 1), \Delta), ((w_3, c, 1), \Delta), \\ ((w_3, d, 1), \Delta), ((w_3, e, 1), \Delta), ((w_5, c, 1), \Delta), ((w_5, d, 1), \Delta), \\ ((w_5, e, 1), \Delta), ((w_1, c, 0), \Delta), ((w_1, d, 0), \Delta), ((w_1, e, 0), \Delta), \\ ((w_3, c, 0), \Delta), ((w_3, d, 0), \Delta), ((w_3, e, 0), \Delta), ((w_5, c, 0), \Delta), \\ ((w_5, d, 0), \Delta), ((w_5, e, 0), \Delta) \end{array} \right\}.$$

**Proposition 3.19.** Suppose  $(\tilde{h}_1, \mathbb{G}_1)_\Delta, (\tilde{h}_2, \mathbb{G}_2)_\Delta, (\tilde{h}_3, \mathbb{G}_3)_\Delta$ , be three NHSES-sets over  $\Delta$ , then

- $(\tilde{h}_1, \mathbb{G}_1) \subset (\tilde{h}_2, \mathbb{G}_2)_\Delta,$
- $(\tilde{h}_1, \mathbb{G}_1)_h \subset (\tilde{h}_1, \mathbb{G}_1),$
- $(\tilde{h}_1, \mathbb{G}_1) \subset (\tilde{h}_1, \mathbb{G}_1),$
- If  $(\tilde{h}_1, \mathbb{G}_1) \subset (\tilde{h}_2, \mathbb{G}_2)$ , and  $(\tilde{h}_2, \mathbb{G}_2) \subset (\tilde{h}_3, \mathbb{G}_3)$ , then  $(\tilde{h}_1, \mathbb{G}_1) \subset (\tilde{h}_3, \mathbb{S}_3).$

- If  $(\hbar_1, \mathbb{G}_1) = (\hbar_2, \mathbb{G}_2)$ , and  $(\hbar_2, \mathbb{G}_2) = (\hbar_3, \mathbb{G}_3)$ , then  $(\hbar_1, \mathbb{G}_1) = (\hbar_3, \mathbb{G}_3)$ .

**Proposition 3.20.** *If  $(\hbar, \mathbb{G})$  is a NHSES over  $\Delta$ , then*

- (1)  $((\hbar, \mathbb{G})^c)^c = (\hbar, \mathbb{G})$
- (2)  $(\hbar, \mathbb{G})_{ag}^c = (\hbar, \mathbb{G})_{dag}$
- (3)  $(\hbar, \mathbb{G})_{dag}^c = (\hbar, \mathbb{G})_{ag}$ .

#### 4. Set Theoretic Operations of NHSES

In this portion, some set theoretic operations are presented with detailed examples.

**Definition 4.1.** The union of  $(\hbar_1, \mathbb{G})$  and  $(\hbar_2, \mathbb{R})$  over  $\Delta$  is  $(\hbar_3, \mathbb{L})$  with  $\mathbb{L} = \mathbb{G} \cup \mathbb{R}$ , defined as

$$\hbar_3(\varsigma) = \begin{cases} \hbar_1(\varsigma) & ; \varsigma \in \mathbb{G} - \mathbb{R} \\ \hbar_2(\varsigma) & ; \varsigma \in \mathbb{R} - \mathbb{G} \\ \cup(\hbar_1(\varsigma), \hbar_2(\varsigma)) & ; \varsigma \in \mathbb{G} \cap \mathbb{R}. \end{cases}$$

where  $\cup(\hbar_1(\varsigma), \hbar_2(\varsigma)) = \{ \langle u, \max \{v_1(\varsigma), v_2(\varsigma)\}, 1/2\{v_1(\varsigma) + v_2(\varsigma)\}, \min \{\omega_1(\varsigma), \omega_2(\varsigma)\} \rangle : u \in \Delta \}$ .

**Example 4.2.** Considering Example 3.3, we see

$$\mathbb{G}_1 = \left\{ (v_1, c, 1), (v_3, c, 0), (v_1, d, 1), (v_3, d, 1), (v_3, d, 0), (v_1, e, 0), (v_3, e, 1) \right\}$$

$$\mathbb{G}_2 = \left\{ (v_1, c, 1), (v_3, c, 0), (v_3, c, 1), (v_1, d, 1), (v_3, d, 1), (v_1, e, 1), (v_3, d, 0), (v_1, e, 0), (v_3, e, 1), (v_1, d, 0) \right\}.$$

Suppose  $(\hbar_1, \mathbb{G}_1)$  and  $(\hbar_2, \mathbb{G}_2)$  over  $\Delta$  are two NHSESs such that

$$(\hbar_1, \mathbb{G}_1) = \left\{ \begin{array}{l} \left( (v_1, c, 1), \left\{ \frac{w_1}{\langle 0.1, 0.6, 0.4 \rangle}, \frac{w_2}{\langle 0.6, 0.3, 0.2 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.1 \rangle}, \frac{w_4}{\langle 0.1, 0.8, 0.5 \rangle} \right\} \right), \\ \left( (v_1, d, 1), \left\{ \frac{w_1}{\langle 0.3, 0.4, 0.5 \rangle}, \frac{w_2}{\langle 0.6, 0.2, 0.3 \rangle}, \frac{w_3}{\langle 0.2, 0.5, 0.6 \rangle}, \frac{w_4}{\langle 0.1, 0.5, 0.3 \rangle} \right\} \right), \\ \left( (v_3, d, 1), \left\{ \frac{w_1}{\langle 0.2, 0.6, 0.7 \rangle}, \frac{w_2}{\langle 0.5, 0.2, 0.3 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.5 \rangle}, \frac{w_4}{\langle 0.8, 0.1, 0.9 \rangle} \right\} \right), \\ \left( (v_3, e, 1), \left\{ \frac{w_1}{\langle 0.6, 0.2, 0.4 \rangle}, \frac{w_2}{\langle 0.2, 0.7, 0.6 \rangle}, \frac{w_3}{\langle 0.4, 0.3, 0.5 \rangle}, \frac{w_4}{\langle 0.1, 0.5, 0.4 \rangle} \right\} \right), \\ \left( (v_1, e, 0), \left\{ \frac{w_1}{\langle 0.1, 0.3, 0.5 \rangle}, \frac{w_2}{\langle 0.1, 0.7, 0.6 \rangle}, \frac{w_3}{\langle 0.2, 0.7, 0.4 \rangle}, \frac{w_4}{\langle 0.4, 0.6, 0.8 \rangle} \right\} \right), \\ \left( (v_3, c, 0), \left\{ \frac{w_1}{\langle 0.1, 0.6, 0.9 \rangle}, \frac{w_2}{\langle 0.3, 0.6, 0.7 \rangle}, \frac{w_3}{\langle 0.6, 0.1, 0.2 \rangle}, \frac{w_4}{\langle 0.7, 0.2, 0.3 \rangle} \right\} \right), \\ \left( (v_3, d, 0), \left\{ \frac{w_1}{\langle 0.1, 0.7, 0.3 \rangle}, \frac{w_2}{\langle 0.8, 0.1, 0.2 \rangle}, \frac{w_3}{\langle 0.7, 0.2, 0.4 \rangle}, \frac{w_4}{\langle 0.2, 0.7, 0.6 \rangle} \right\} \right) \end{array} \right\}$$

$$(\tilde{h}_2, \mathbb{G}_2) = \left\{ \begin{array}{l} \left( (v_1, c, 1), \left\{ \frac{w_1}{\langle 0.2, 0.3, 0.4 \rangle}, \frac{w_2}{\langle 0.7, 0.4, 0.5 \rangle}, \frac{w_3}{\langle 0.5, 0.4, 0.6 \rangle}, \frac{w_4}{\langle 0.2, 0.4, 0.7 \rangle} \right\} \right), \\ \left( (v_1, d, 1), \left\{ \frac{w_1}{\langle 0.4, 0.3, 0.8 \rangle}, \frac{w_2}{\langle 0.8, 0.3, 0.5 \rangle}, \frac{w_3}{\langle 0.4, 0.3, 0.5 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.7 \rangle} \right\} \right), \\ \left( (v_3, c, 1), \left\{ \frac{w_1}{\langle 0.1, 0.3, 0.6 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.7 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.8 \rangle}, \frac{w_4}{\langle 0.5, 0.3, 0.5 \rangle} \right\} \right), \\ \left( (v_3, d, 1), \left\{ \frac{w_1}{\langle 0.4, 0.2, 0.3 \rangle}, \frac{w_2}{\langle 0.6, 0.3, 0.5 \rangle}, \frac{w_3}{\langle 0.7, 0.4, 0.5 \rangle}, \frac{w_4}{\langle 0.9, 0.5, 0.7 \rangle} \right\} \right), \\ \left( (v_1, e, 1), \left\{ \frac{w_1}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_2}{\langle 0.5, 0.2, 0.4 \rangle}, \frac{w_3}{\langle 0.6, 0.2, 0.4 \rangle}, \frac{w_4}{\langle 0.3, 0.5, 0.6 \rangle} \right\} \right), \\ \left( (v_3, e, 1), \left\{ \frac{w_1}{\langle 0.7, 0.3, 0.7 \rangle}, \frac{w_2}{\langle 0.3, 0.5, 0.6 \rangle}, \frac{w_3}{\langle 0.5, 0.4, 0.3 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.4 \rangle} \right\} \right), \\ \left( (v_1, e, 0), \left\{ \frac{w_1}{\langle 0.2, 0.5, 0.4 \rangle}, \frac{w_2}{\langle 0.2, 0.6, 0.3 \rangle}, \frac{w_3}{\langle 0.3, 0.5, 0.6 \rangle}, \frac{w_4}{\langle 0.5, 0.3, 0.7 \rangle} \right\} \right), \\ \left( (v_1, d, 0), \left\{ \frac{w_1}{\langle 0.1, 0.6, 0.3 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.2 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.4 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.3 \rangle} \right\} \right), \\ \left( (v_3, c, 0), \left\{ \frac{w_1}{\langle 0.2, 0.7, 0.5 \rangle}, \frac{w_2}{\langle 0.4, 0.5, 0.6 \rangle}, \frac{w_3}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_4}{\langle 0.8, 0.1, 0.4 \rangle} \right\} \right), \\ \left( (v_3, d, 0), \left\{ \frac{w_1}{\langle 0.2, 0.5, 0.4 \rangle}, \frac{w_2}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_3}{\langle 0.8, 0.2, 0.6 \rangle}, \frac{w_4}{\langle 0.3, 0.5, 0.7 \rangle} \right\} \right) \end{array} \right\}.$$

Then  $(\tilde{h}_1, \mathbb{G}_1) \cup (\tilde{h}_2, \mathbb{G}_2) = (\tilde{h}_3, \mathbb{G}_3)$

$$(\tilde{h}_3, \mathbb{G}_3) = \left\{ \begin{array}{l} \left( (v_1, c, 1), \left\{ \frac{w_1}{\langle 0.2, 0.45, 0.4 \rangle}, \frac{w_2}{\langle 0.7, 0.35, 0.2 \rangle}, \frac{w_3}{\langle 0.5, 0.45, 0.1 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.5 \rangle} \right\} \right), \\ \left( (v_1, d, 1), \left\{ \frac{w_1}{\langle 0.4, 0.35, 0.5 \rangle}, \frac{w_2}{\langle 0.8, 0.25, 0.3 \rangle}, \frac{w_3}{\langle 0.4, 0.4, 0.5 \rangle}, \frac{w_4}{\langle 0.2, 0.55, 0.3 \rangle} \right\} \right), \\ \left( (v_3, c, 1), \left\{ \frac{w_1}{\langle 0.1, 0.3, 0.6 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.7 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.8 \rangle}, \frac{w_4}{\langle 0.5, 0.3, 0.5 \rangle} \right\} \right), \\ \left( (v_3, d, 1), \left\{ \frac{w_1}{\langle 0.4, 0.4, 0.3 \rangle}, \frac{w_2}{\langle 0.6, 0.25, 0.3 \rangle}, \frac{w_3}{\langle 0.7, 0.35, 0.5 \rangle}, \frac{w_4}{\langle 0.9, 0.3, 0.7 \rangle} \right\} \right), \\ \left( (v_1, e, 1), \left\{ \frac{w_1}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_2}{\langle 0.5, 0.2, 0.4 \rangle}, \frac{w_3}{\langle 0.6, 0.2, 0.4 \rangle}, \frac{w_4}{\langle 0.3, 0.5, 0.6 \rangle} \right\} \right), \\ \left( (v_3, e, 1), \left\{ \frac{w_1}{\langle 0.7, 0.2, 0.4 \rangle}, \frac{w_2}{\langle 0.3, 0.5, 0.6 \rangle}, \frac{w_3}{\langle 0.5, 0.3, 0.3 \rangle}, \frac{w_4}{\langle 0.2, 0.5, 0.4 \rangle} \right\} \right), \\ \left( (v_1, e, 0), \left\{ \frac{w_1}{\langle 0.2, 0.4, 0.4 \rangle}, \frac{w_2}{\langle 0.2, 0.65, 0.3 \rangle}, \frac{w_3}{\langle 0.3, 0.6, 0.4 \rangle}, \frac{w_4}{\langle 0.5, 0.45, 0.7 \rangle} \right\} \right), \\ \left( (v_1, d, 0), \left\{ \frac{w_1}{\langle 0.1, 0.6, 0.3 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.2 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.4 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.3 \rangle} \right\} \right), \\ \left( (v_3, c, 0), \left\{ \frac{w_1}{\langle 0.2, 0.65, 0.5 \rangle}, \frac{w_2}{\langle 0.4, 0.55, 0.6 \rangle}, \frac{w_3}{\langle 0.7, 0.15, 0.2 \rangle}, \frac{w_4}{\langle 0.8, 0.15, 0.3 \rangle} \right\} \right), \\ \left( (v_3, d, 0), \left\{ \frac{w_1}{\langle 0.2, 0.6, 0.3 \rangle}, \frac{w_2}{\langle 0.8, 0.15, 0.2 \rangle}, \frac{w_3}{\langle 0.8, 0.2, 0.4 \rangle}, \frac{w_4}{\langle 0.3, 0.6, 0.6 \rangle} \right\} \right) \end{array} \right\}.$$

**Definition 4.3.** Restricted Union of two NHSESs  $(\tilde{h}_1, \mathbb{G}_1)$  and  $(\tilde{h}_2, \mathbb{G}_2)$  over  $\Delta$  is  $(\tilde{h}_3, \mathbb{L})$  with  $\mathbb{L} = \mathbb{G}_1 \cap \mathbb{G}_2$ , defined as  $\tilde{h}_3(\varsigma) = \tilde{h}_1(\varsigma) \cup_{\mathbb{R}} \tilde{h}_2(\varsigma)$  for  $\varsigma \in \mathbb{G}_1 \cap \mathbb{G}_2$ .

**Example 4.4.** Considering Example 3.3, we see

$$\mathbb{G}_1 = \left\{ (v_1, c, 1), (v_3, c, 0), (v_1, d, 1), (v_3, d, 1), (v_3, d, 0), (v_1, e, 0), (v_3, e, 1) \right\}, \quad \mathbb{G}_2 = \left\{ (v_1, c, 1), (v_3, c, 0), (v_3, c, 1), (v_1, d, 1), (v_3, d, 1), (v_1, d, 0), (v_3, d, 0), (v_1, e, 0), (v_3, e, 1), (v_1, e, 1) \right\}.$$

Suppose  $(\hbar_1, \mathbb{G}_1)$  and  $(\hbar_2, \mathbb{G}_2)$  over  $\Delta$  are two NHSEs such that

$$(\hbar_1, \mathbb{G}_1) = \left\{ \begin{array}{l} \left( (v_1, c, 1), \left\{ \frac{w_1}{\langle 0.1, 0.6, 0.4 \rangle}, \frac{w_2}{\langle 0.6, 0.3, 0.2 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.1 \rangle}, \frac{w_4}{\langle 0.1, 0.8, 0.5 \rangle} \right\} \right), \\ \left( (v_1, d, 1), \left\{ \frac{w_1}{\langle 0.3, 0.4, 0.5 \rangle}, \frac{w_2}{\langle 0.6, 0.2, 0.3 \rangle}, \frac{w_3}{\langle 0.2, 0.5, 0.6 \rangle}, \frac{w_4}{\langle 0.1, 0.5, 0.3 \rangle} \right\} \right), \\ \left( (v_3, d, 1), \left\{ \frac{w_1}{\langle 0.2, 0.6, 0.7 \rangle}, \frac{w_2}{\langle 0.5, 0.2, 0.3 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.5 \rangle}, \frac{w_4}{\langle 0.8, 0.1, 0.9 \rangle} \right\} \right), \\ \left( (v_3, e, 1), \left\{ \frac{w_1}{\langle 0.6, 0.2, 0.4 \rangle}, \frac{w_2}{\langle 0.2, 0.7, 0.6 \rangle}, \frac{w_3}{\langle 0.4, 0.3, 0.5 \rangle}, \frac{w_4}{\langle 0.1, 0.5, 0.4 \rangle} \right\} \right), \\ \left( (v_1, e, 0), \left\{ \frac{w_1}{\langle 0.1, 0.3, 0.5 \rangle}, \frac{w_2}{\langle 0.1, 0.7, 0.6 \rangle}, \frac{w_3}{\langle 0.2, 0.7, 0.4 \rangle}, \frac{w_4}{\langle 0.4, 0.6, 0.8 \rangle} \right\} \right), \\ \left( (v_3, c, 0), \left\{ \frac{w_1}{\langle 0.1, 0.6, 0.9 \rangle}, \frac{w_2}{\langle 0.3, 0.6, 0.7 \rangle}, \frac{w_3}{\langle 0.6, 0.1, 0.2 \rangle}, \frac{w_4}{\langle 0.7, 0.2, 0.3 \rangle} \right\} \right), \\ \left( (v_3, d, 0), \left\{ \frac{w_1}{\langle 0.1, 0.7, 0.3 \rangle}, \frac{w_2}{\langle 0.8, 0.1, 0.2 \rangle}, \frac{w_3}{\langle 0.7, 0.2, 0.4 \rangle}, \frac{w_4}{\langle 0.2, 0.7, 0.6 \rangle} \right\} \right) \end{array} \right\},$$

$$(\hbar_2, \mathbb{G}_2) = \left\{ \begin{array}{l} \left( (v_1, c, 1), \left\{ \frac{w_1}{\langle 0.2, 0.3, 0.4 \rangle}, \frac{w_2}{\langle 0.7, 0.4, 0.5 \rangle}, \frac{w_3}{\langle 0.5, 0.4, 0.6 \rangle}, \frac{w_4}{\langle 0.2, 0.4, 0.7 \rangle} \right\} \right), \\ \left( (v_1, d, 1), \left\{ \frac{w_1}{\langle 0.4, 0.3, 0.8 \rangle}, \frac{w_2}{\langle 0.8, 0.3, 0.5 \rangle}, \frac{w_3}{\langle 0.4, 0.3, 0.5 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.7 \rangle} \right\} \right), \\ \left( (v_3, c, 1), \left\{ \frac{w_1}{\langle 0.1, 0.3, 0.6 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.7 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.8 \rangle}, \frac{w_4}{\langle 0.5, 0.3, 0.5 \rangle} \right\} \right), \\ \left( (v_3, d, 1), \left\{ \frac{w_1}{\langle 0.4, 0.2, 0.3 \rangle}, \frac{w_2}{\langle 0.6, 0.3, 0.5 \rangle}, \frac{w_3}{\langle 0.7, 0.4, 0.5 \rangle}, \frac{w_4}{\langle 0.9, 0.5, 0.7 \rangle} \right\} \right), \\ \left( (v_1, e, 1), \left\{ \frac{w_1}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_2}{\langle 0.5, 0.2, 0.4 \rangle}, \frac{w_3}{\langle 0.6, 0.2, 0.4 \rangle}, \frac{w_4}{\langle 0.3, 0.5, 0.6 \rangle} \right\} \right), \\ \left( (v_3, e, 1), \left\{ \frac{w_1}{\langle 0.7, 0.3, 0.7 \rangle}, \frac{w_2}{\langle 0.3, 0.5, 0.6 \rangle}, \frac{w_3}{\langle 0.5, 0.4, 0.3 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.4 \rangle} \right\} \right), \\ \left( (v_1, e, 0), \left\{ \frac{w_1}{\langle 0.2, 0.5, 0.4 \rangle}, \frac{w_2}{\langle 0.2, 0.6, 0.3 \rangle}, \frac{w_3}{\langle 0.3, 0.5, 0.6 \rangle}, \frac{w_4}{\langle 0.5, 0.3, 0.7 \rangle} \right\} \right), \\ \left( (v_1, d, 0), \left\{ \frac{w_1}{\langle 0.1, 0.6, 0.3 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.2 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.4 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.3 \rangle} \right\} \right), \\ \left( (v_3, c, 0), \left\{ \frac{w_1}{\langle 0.2, 0.7, 0.5 \rangle}, \frac{w_2}{\langle 0.4, 0.5, 0.6 \rangle}, \frac{w_3}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_4}{\langle 0.8, 0.1, 0.4 \rangle} \right\} \right), \\ \left( (v_3, d, 0), \left\{ \frac{w_1}{\langle 0.2, 0.5, 0.4 \rangle}, \frac{w_2}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_3}{\langle 0.8, 0.2, 0.6 \rangle}, \frac{w_4}{\langle 0.3, 0.5, 0.7 \rangle} \right\} \right) \end{array} \right\}.$$

Then  $(\hbar_1, \mathbb{G}_1) \cup_{\mathbb{R}} (\hbar_2, \mathbb{G}_2) = (\hbar_3, \mathbb{L})$

$$(\hbar_3, \mathbb{L}) = \left\{ \begin{array}{l} \left( (v_1, c, 1), \left\{ \frac{w_1}{\langle 0.2, 0.45, 0.4 \rangle}, \frac{w_2}{\langle 0.7, 0.35, 0.2 \rangle}, \frac{w_3}{\langle 0.5, 0.45, 0.1 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.5 \rangle} \right\} \right), \\ \left( (v_1, d, 1), \left\{ \frac{w_1}{\langle 0.4, 0.35, 0.5 \rangle}, \frac{w_2}{\langle 0.8, 0.25, 0.3 \rangle}, \frac{w_3}{\langle 0.4, 0.4, 0.5 \rangle}, \frac{w_4}{\langle 0.2, 0.55, 0.3 \rangle} \right\} \right), \\ \left( (v_3, d, 1), \left\{ \frac{w_1}{\langle 0.4, 0.4, 0.3 \rangle}, \frac{w_2}{\langle 0.6, 0.25, 0.3 \rangle}, \frac{w_3}{\langle 0.7, 0.35, 0.5 \rangle}, \frac{w_4}{\langle 0.9, 0.3, 0.7 \rangle} \right\} \right), \\ \left( (v_1, e, 1), \left\{ \frac{w_1}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_2}{\langle 0.5, 0.2, 0.4 \rangle}, \frac{w_3}{\langle 0.6, 0.2, 0.4 \rangle}, \frac{w_4}{\langle 0.3, 0.5, 0.6 \rangle} \right\} \right), \\ \left( (v_3, e, 1), \left\{ \frac{w_1}{\langle 0.7, 0.2, 0.4 \rangle}, \frac{w_2}{\langle 0.3, 0.5, 0.6 \rangle}, \frac{w_3}{\langle 0.5, 0.3, 0.3 \rangle}, \frac{w_4}{\langle 0.2, 0.5, 0.4 \rangle} \right\} \right), \\ \left( (v_1, e, 0), \left\{ \frac{w_1}{\langle 0.2, 0.6, 0.4 \rangle}, \frac{w_2}{\langle 0.2, 0.1, 0.3 \rangle}, \frac{w_3}{\langle 0.3, 0.3, 0.4 \rangle}, \frac{w_4}{\langle 0.5, 0.6, 0.7 \rangle} \right\} \right), \\ \left( (v_3, c, 0), \left\{ \frac{w_1}{\langle 0.2, 0.65, 0.5 \rangle}, \frac{w_2}{\langle 0.4, 0.55, 0.6 \rangle}, \frac{w_3}{\langle 0.7, 0.15, 0.2 \rangle}, \frac{w_4}{\langle 0.8, 0.15, 0.3 \rangle} \right\} \right), \\ \left( (v_3, d, 0), \left\{ \frac{w_1}{\langle 0.2, 0.6, 0.3 \rangle}, \frac{w_2}{\langle 0.8, 0.15, 0.2 \rangle}, \frac{w_3}{\langle 0.8, 0.2, 0.4 \rangle}, \frac{w_4}{\langle 0.3, 0.6, 0.6 \rangle} \right\} \right) \end{array} \right\}.$$

**Proposition 4.5.** *If  $(\hbar_1, \mathbb{G}_1), (\hbar_2, \mathbb{G}_2)$  and  $(\hbar_3, \mathbb{G}_3)$  are three NHSEs, then*

- (1)  $(\hbar_1, \mathbb{G}_1) \cup (\hbar_2, \mathbb{G}_2) = (\hbar_2, \mathbb{G}_2) \cup (\hbar_1, \mathbb{G}_1)$
- (2)  $((\hbar_1, \mathbb{G}_1) \cup (\hbar_2, \mathbb{G}_2)) \cup (\hbar_3, \mathbb{G}_3) = (\hbar_1, \mathbb{G}_1) \cup ((\hbar_2, \mathbb{G}_2) \cup (\hbar_3, \mathbb{G}_3))$
- (3)  $(\hbar, \mathbb{G}) \cup \Phi = (\hbar, \mathbb{G})$ .

**Definition 4.6.** The intersection of two NHSESs  $(\hbar_1, \mathbb{G})$  and  $(\hbar_2, \mathbb{R})$  over  $\Delta$  is  $(\hbar_3, \mathbb{L})$  with  $\mathbb{L} = \mathbb{G} \cap \mathbb{R}$ , defined as

$$\hbar_3(\varsigma) = \begin{cases} \hbar_1(\varsigma) & ; \varsigma \in \mathbb{G} - \mathbb{R} \\ \hbar_2(\varsigma) & ; \varsigma \in \mathbb{R} - \mathbb{G} \\ \cap(\hbar_1(\varsigma), \hbar_2(\varsigma)) & ; \varsigma \in \mathbb{G} \cap \mathbb{R} \end{cases}$$

where  $\cap(\hbar_1(\varsigma), \hbar_2(\varsigma)) = \{ \langle u, \min \{ \nu_1(\varsigma), \nu_2(\varsigma) \}, 1/2\{ \nu_1(\varsigma) + \nu_2(\varsigma) \}, \max \{ \omega_1(\varsigma), \omega_2(\varsigma) \} \rangle : u \in \Delta \}$ .

**Example 4.7.** Reconsidering Example 3.3, we have

$$\mathbb{G}_1 = \{ (v_1, c, 1), (v_3, c, 0), (v_1, d, 1), (v_3, d, 1), (v_3, d, 0), (v_1, e, 0), (v_3, e, 1) \}$$

$$\mathbb{G}_2 = \{ (v_1, c, 1), (v_3, c, 0), (v_3, c, 1), (v_1, d, 1), (v_3, d, 1), (v_1, d, 0), (v_3, d, 0), (v_1, e, 0), (v_3, e, 1), (v_1, e, 1) \}$$

Suppose  $(\hbar_1, \mathbb{G}_1)$  and  $(\hbar_2, \mathbb{G}_2)$  are two NHSESs over  $\Delta$  such that

$$(\hbar_1, \mathbb{G}_1) = \left\{ \begin{array}{l} \left( (v_1, c, 1), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.1, 0.6, 0.4 \rangle}, \frac{w_2}{\langle 0.6, 0.3, 0.2 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.1 \rangle}, \frac{w_4}{\langle 0.1, 0.8, 0.5 \rangle} \end{array} \right\} \right), \\ \left( (v_1, d, 1), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.3, 0.4, 0.5 \rangle}, \frac{w_2}{\langle 0.6, 0.2, 0.3 \rangle}, \frac{w_3}{\langle 0.2, 0.5, 0.6 \rangle}, \frac{w_4}{\langle 0.1, 0.5, 0.3 \rangle} \end{array} \right\} \right), \\ \left( (v_3, d, 1), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.2, 0.6, 0.7 \rangle}, \frac{w_2}{\langle 0.5, 0.2, 0.3 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.5 \rangle}, \frac{w_4}{\langle 0.8, 0.1, 0.9 \rangle} \end{array} \right\} \right), \\ \left( (v_3, e, 1), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.6, 0.2, 0.4 \rangle}, \frac{w_2}{\langle 0.2, 0.7, 0.6 \rangle}, \frac{w_3}{\langle 0.4, 0.3, 0.5 \rangle}, \frac{w_4}{\langle 0.1, 0.5, 0.4 \rangle} \end{array} \right\} \right), \\ \left( (v_1, e, 0), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.1, 0.3, 0.5 \rangle}, \frac{w_2}{\langle 0.1, 0.7, 0.6 \rangle}, \frac{w_3}{\langle 0.2, 0.7, 0.4 \rangle}, \frac{w_4}{\langle 0.4, 0.6, 0.8 \rangle} \end{array} \right\} \right), \\ \left( (v_3, c, 0), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.1, 0.6, 0.9 \rangle}, \frac{w_2}{\langle 0.3, 0.6, 0.7 \rangle}, \frac{w_3}{\langle 0.6, 0.1, 0.2 \rangle}, \frac{w_4}{\langle 0.7, 0.2, 0.3 \rangle} \end{array} \right\} \right), \\ \left( (v_3, d, 0), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.1, 0.7, 0.3 \rangle}, \frac{w_2}{\langle 0.8, 0.1, 0.2 \rangle}, \frac{w_3}{\langle 0.7, 0.2, 0.4 \rangle}, \frac{w_4}{\langle 0.2, 0.7, 0.6 \rangle} \end{array} \right\} \right) \end{array} \right\}$$

$$(\hbar_2, \mathbb{G}_2) = \left\{ \begin{array}{l} \left( (v_1, c, 1), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.2, 0.3, 0.4 \rangle}, \frac{w_2}{\langle 0.7, 0.4, 0.5 \rangle}, \frac{w_3}{\langle 0.5, 0.4, 0.6 \rangle}, \frac{w_4}{\langle 0.2, 0.4, 0.7 \rangle} \end{array} \right\} \right), \\ \left( (v_1, d, 1), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.4, 0.3, 0.8 \rangle}, \frac{w_2}{\langle 0.8, 0.3, 0.5 \rangle}, \frac{w_3}{\langle 0.4, 0.3, 0.5 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.7 \rangle} \end{array} \right\} \right), \\ \left( (v_3, c, 1), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.1, 0.3, 0.6 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.7 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.8 \rangle}, \frac{w_4}{\langle 0.5, 0.3, 0.5 \rangle} \end{array} \right\} \right), \\ \left( (v_3, d, 1), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.4, 0.2, 0.3 \rangle}, \frac{w_2}{\langle 0.6, 0.3, 0.5 \rangle}, \frac{w_3}{\langle 0.7, 0.4, 0.5 \rangle}, \frac{w_4}{\langle 0.9, 0.5, 0.7 \rangle} \end{array} \right\} \right), \\ \left( (v_1, e, 1), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_2}{\langle 0.5, 0.2, 0.4 \rangle}, \frac{w_3}{\langle 0.6, 0.2, 0.4 \rangle}, \frac{w_4}{\langle 0.3, 0.5, 0.6 \rangle} \end{array} \right\} \right), \\ \left( (v_3, e, 1), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.7, 0.3, 0.7 \rangle}, \frac{w_2}{\langle 0.3, 0.5, 0.6 \rangle}, \frac{w_3}{\langle 0.5, 0.4, 0.3 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.4 \rangle} \end{array} \right\} \right), \\ \left( (v_1, e, 0), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.2, 0.5, 0.4 \rangle}, \frac{w_2}{\langle 0.2, 0.6, 0.3 \rangle}, \frac{w_3}{\langle 0.3, 0.5, 0.6 \rangle}, \frac{w_4}{\langle 0.5, 0.3, 0.7 \rangle} \end{array} \right\} \right), \\ \left( (v_1, d, 0), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.1, 0.6, 0.3 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.2 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.4 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.3 \rangle} \end{array} \right\} \right), \\ \left( (v_3, c, 0), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.2, 0.7, 0.5 \rangle}, \frac{w_2}{\langle 0.4, 0.5, 0.6 \rangle}, \frac{w_3}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_4}{\langle 0.8, 0.1, 0.4 \rangle} \end{array} \right\} \right), \\ \left( (v_3, d, 0), \left\{ \begin{array}{l} \frac{w_1}{\langle 0.2, 0.5, 0.4 \rangle}, \frac{w_2}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_3}{\langle 0.8, 0.2, 0.6 \rangle}, \frac{w_4}{\langle 0.3, 0.5, 0.7 \rangle} \end{array} \right\} \right) \end{array} \right\}$$

Then  $(\hbar_1, \mathbb{G}_1) \cap (\hbar_2, \mathbb{G}_2) = (\hbar_3, \mathbb{G}_3)$

$$(\hbar_3, \mathbb{G}_3) = \left\{ \begin{array}{l} \left( (v_1, c, 1), \left\{ \frac{w_1}{\langle 0.1, 0.45, 0.4 \rangle}, \frac{w_2}{\langle 0.6, 0.35, 0.5 \rangle}, \frac{w_3}{\langle 0.4, 0.45, 0.6 \rangle}, \frac{w_4}{\langle 0.1, 0.6, 0.7 \rangle} \right\} \right), \\ \left( (v_1, d, 1), \left\{ \frac{w_1}{\langle 0.3, 0.35, 0.8 \rangle}, \frac{w_2}{\langle 0.6, 0.25, 0.5 \rangle}, \frac{w_3}{\langle 0.2, 0.4, 0.6 \rangle}, \frac{w_4}{\langle 0.1, 0.55, 0.7 \rangle} \right\} \right), \\ \left( (v_3, d, 1), \left\{ \frac{w_1}{\langle 0.2, 0.4, 0.7 \rangle}, \frac{w_2}{\langle 0.5, 0.25, 0.5 \rangle}, \frac{w_3}{\langle 0.6, 0.35, 0.5 \rangle}, \frac{w_4}{\langle 0.8, 0.30, 0.7 \rangle} \right\} \right), \\ \left( (v_3, e, 1), \left\{ \frac{w_1}{\langle 0.6, 0.25, 0.7 \rangle}, \frac{w_2}{\langle 0.2, 0.60, 0.6 \rangle}, \frac{w_3}{\langle 0.4, 0.35, 0.5 \rangle}, \frac{w_4}{\langle 0.1, 0.55, 0.4 \rangle} \right\} \right), \\ \left( (v_1, e, 0), \left\{ \frac{w_1}{\langle 0.1, 0.40, 0.5 \rangle}, \frac{w_2}{\langle 0.1, 0.65, 0.6 \rangle}, \frac{w_3}{\langle 0.2, 0.60, 0.6 \rangle}, \frac{w_4}{\langle 0.4, 0.45, 0.8 \rangle} \right\} \right), \\ \left( (v_3, c, 0), \left\{ \frac{w_1}{\langle 0.1, 0.65, 0.9 \rangle}, \frac{w_2}{\langle 0.3, 0.55, 0.7 \rangle}, \frac{w_3}{\langle 0.6, 0.15, 0.3 \rangle}, \frac{w_4}{\langle 0.7, 0.15, 0.4 \rangle} \right\} \right), \\ \left( (v_3, d, 0), \left\{ \frac{w_1}{\langle 0.1, 0.60, 0.4 \rangle}, \frac{w_2}{\langle 0.8, 0.15, 0.3 \rangle}, \frac{w_3}{\langle 0.7, 0.20, 0.6 \rangle}, \frac{w_4}{\langle 0.2, 0.60, 0.7 \rangle} \right\} \right), \end{array} \right\}.$$

**Definition 4.8.** Extended intersection of two NHSEs  $(\hbar_1, \mathbb{S})$  and  $(\hbar_2, \mathbb{R})$  over  $\Delta$  is  $(\hbar_3, \mathbb{L})$  with  $\mathbb{L} = \mathbb{S} \cup \mathbb{R}$ , defined as

$$\hbar_3(\varsigma) = \begin{cases} \hbar_1(\varsigma) & ; \varsigma \in \mathbb{S} - \mathbb{R} \\ \hbar_2(\varsigma) & ; \varsigma \in \mathbb{R} - \mathbb{S} \\ \hbar_1(\varsigma) \cap \hbar_2(\varsigma) & ; \varsigma \in \mathbb{S} \cap \mathbb{R}. \end{cases}$$

**Example 4.9.** Considering Example 3.3, we have

$$\mathbb{G}_1 = \left\{ (v_1, c, 1), (v_3, c, 0), (v_1, d, 1), (v_3, d, 1), (v_3, d, 0), (v_1, e, 0), (v_3, e, 1) \right\}$$

$$\mathbb{G}_2 = \left\{ (v_1, c, 1), (v_3, c, 0), (v_3, c, 1), (v_1, d, 1), (v_3, d, 1), (v_1, d, 0), (v_3, d, 0), (v_1, e, 0), (v_3, e, 1), (v_1, e, 1) \right\}.$$

Suppose  $(\hbar_1, \mathbb{G}_1)$  and  $(\hbar_2, \mathbb{G}_2)$  are two NHSEs over  $\Delta$  such that

$$(\hbar_1, \mathbb{G}_1) = \left\{ \begin{array}{l} \left( (v_1, c, 1), \left\{ \frac{w_1}{\langle 0.1, 0.6, 0.4 \rangle}, \frac{w_2}{\langle 0.6, 0.3, 0.2 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.1 \rangle}, \frac{w_4}{\langle 0.1, 0.8, 0.5 \rangle} \right\} \right), \\ \left( (v_1, d, 1), \left\{ \frac{w_1}{\langle 0.3, 0.4, 0.5 \rangle}, \frac{w_2}{\langle 0.6, 0.2, 0.3 \rangle}, \frac{w_3}{\langle 0.2, 0.5, 0.6 \rangle}, \frac{w_4}{\langle 0.1, 0.5, 0.3 \rangle} \right\} \right), \\ \left( (v_3, d, 1), \left\{ \frac{w_1}{\langle 0.2, 0.6, 0.7 \rangle}, \frac{w_2}{\langle 0.5, 0.2, 0.3 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.5 \rangle}, \frac{w_4}{\langle 0.8, 0.1, 0.9 \rangle} \right\} \right), \\ \left( (v_3, e, 1), \left\{ \frac{w_1}{\langle 0.6, 0.2, 0.4 \rangle}, \frac{w_2}{\langle 0.2, 0.7, 0.6 \rangle}, \frac{w_3}{\langle 0.4, 0.3, 0.5 \rangle}, \frac{w_4}{\langle 0.1, 0.5, 0.4 \rangle} \right\} \right), \\ \left( (v_1, e, 0), \left\{ \frac{w_1}{\langle 0.1, 0.3, 0.5 \rangle}, \frac{w_2}{\langle 0.1, 0.7, 0.6 \rangle}, \frac{w_3}{\langle 0.2, 0.7, 0.4 \rangle}, \frac{w_4}{\langle 0.4, 0.6, 0.8 \rangle} \right\} \right), \\ \left( (v_3, c, 0), \left\{ \frac{w_1}{\langle 0.1, 0.6, 0.9 \rangle}, \frac{w_2}{\langle 0.3, 0.6, 0.7 \rangle}, \frac{w_3}{\langle 0.6, 0.1, 0.2 \rangle}, \frac{w_4}{\langle 0.7, 0.2, 0.3 \rangle} \right\} \right), \\ \left( (v_3, d, 0), \left\{ \frac{w_1}{\langle 0.1, 0.7, 0.3 \rangle}, \frac{w_2}{\langle 0.8, 0.1, 0.2 \rangle}, \frac{w_3}{\langle 0.7, 0.2, 0.4 \rangle}, \frac{w_4}{\langle 0.2, 0.7, 0.6 \rangle} \right\} \right), \end{array} \right\}$$

$$(\hbar_2, \mathbb{G}_2) = \left\{ \left( (v_1, c, 1), \left\{ \frac{w_1}{\langle 0.2, 0.3, 0.4 \rangle}, \frac{w_2}{\langle 0.7, 0.4, 0.5 \rangle}, \frac{w_3}{\langle 0.5, 0.4, 0.6 \rangle}, \frac{w_4}{\langle 0.2, 0.4, 0.7 \rangle} \right\} \right), \right. \\ \left. \left( (v_1, d, 1), \left\{ \frac{w_1}{\langle 0.4, 0.3, 0.8 \rangle}, \frac{w_2}{\langle 0.8, 0.3, 0.5 \rangle}, \frac{w_3}{\langle 0.4, 0.3, 0.5 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.7 \rangle} \right\} \right), \right. \\ \left. \left( (v_3, c, 1), \left\{ \frac{w_1}{\langle 0.1, 0.3, 0.6 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.7 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.8 \rangle}, \frac{w_4}{\langle 0.5, 0.3, 0.5 \rangle} \right\} \right), \right. \\ \left. \left( (v_3, d, 1), \left\{ \frac{w_1}{\langle 0.4, 0.2, 0.3 \rangle}, \frac{w_2}{\langle 0.6, 0.3, 0.5 \rangle}, \frac{w_3}{\langle 0.7, 0.4, 0.5 \rangle}, \frac{w_4}{\langle 0.9, 0.5, 0.7 \rangle} \right\} \right), \right. \\ \left. \left( (v_1, e, 1), \left\{ \frac{w_1}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_2}{\langle 0.5, 0.2, 0.4 \rangle}, \frac{w_3}{\langle 0.6, 0.2, 0.4 \rangle}, \frac{w_4}{\langle 0.3, 0.5, 0.6 \rangle} \right\} \right), \right. \\ \left. \left( (v_3, e, 1), \left\{ \frac{w_1}{\langle 0.7, 0.3, 0.7 \rangle}, \frac{w_2}{\langle 0.3, 0.5, 0.6 \rangle}, \frac{w_3}{\langle 0.5, 0.4, 0.3 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.4 \rangle} \right\} \right), \right. \\ \left. \left( (v_1, e, 0), \left\{ \frac{w_1}{\langle 0.2, 0.5, 0.4 \rangle}, \frac{w_2}{\langle 0.2, 0.6, 0.3 \rangle}, \frac{w_3}{\langle 0.3, 0.5, 0.6 \rangle}, \frac{w_4}{\langle 0.5, 0.3, 0.7 \rangle} \right\} \right), \right. \\ \left. \left( (v_1, d, 0), \left\{ \frac{w_1}{\langle 0.1, 0.6, 0.3 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.2 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.4 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.3 \rangle} \right\} \right), \right. \\ \left. \left( (v_3, c, 0), \left\{ \frac{w_1}{\langle 0.2, 0.7, 0.5 \rangle}, \frac{w_2}{\langle 0.4, 0.5, 0.6 \rangle}, \frac{w_3}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_4}{\langle 0.8, 0.1, 0.4 \rangle} \right\} \right), \right. \\ \left. \left( (v_3, d, 0), \left\{ \frac{w_1}{\langle 0.2, 0.5, 0.4 \rangle}, \frac{w_2}{\langle 0.7, 0.2, 0.3 \rangle}, \frac{w_3}{\langle 0.8, 0.2, 0.6 \rangle}, \frac{w_4}{\langle 0.3, 0.5, 0.7 \rangle} \right\} \right) \right\}.$$

Then  $(\hbar_1, \mathbb{G}_1) \cap_E (\hbar_2, \mathbb{G}_2) = (\hbar_3, \mathbb{L})$

$$(\hbar_3, \mathbb{L}) = \left\{ \left( (v_1, c, 1), \left\{ \frac{w_1}{\langle 0.1, 0.6, 0.4 \rangle}, \frac{w_2}{\langle 0.6, 0.4, 0.5 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.6 \rangle}, \frac{w_4}{\langle 0.1, 0.8, 0.7 \rangle} \right\} \right), \right. \\ \left. \left( (v_1, d, 1), \left\{ \frac{w_1}{\langle 0.3, 0.4, 0.8 \rangle}, \frac{w_2}{\langle 0.6, 0.3, 0.5 \rangle}, \frac{w_3}{\langle 0.2, 0.5, 0.6 \rangle}, \frac{w_4}{\langle 0.1, 0.6, 0.7 \rangle} \right\} \right), \right. \\ \left. \left( (v_3, d, 1), \left\{ \frac{w_1}{\langle 0.2, 0.6, 0.7 \rangle}, \frac{w_2}{\langle 0.5, 0.4, 0.5 \rangle}, \frac{w_3}{\langle 0.6, 0.4, 0.5 \rangle}, \frac{w_4}{\langle 0.8, 0.1, 0.7 \rangle} \right\} \right), \right. \\ \left. \left( (v_3, e, 1), \left\{ \frac{w_1}{\langle 0.6, 0.3, 0.7 \rangle}, \frac{w_2}{\langle 0.2, 0.7, 0.6 \rangle}, \frac{w_3}{\langle 0.4, 0.4, 0.5 \rangle}, \frac{w_4}{\langle 0.1, 0.6, 0.4 \rangle} \right\} \right), \right. \\ \left. \left( (v_1, e, 0), \left\{ \frac{w_1}{\langle 0.1, 0.5, 0.5 \rangle}, \frac{w_2}{\langle 0.1, 0.6, 0.6 \rangle}, \frac{w_3}{\langle 0.2, 0.7, 0.6 \rangle}, \frac{w_4}{\langle 0.4, 0.6, 0.8 \rangle} \right\} \right), \right. \\ \left. \left( (v_3, c, 0), \left\{ \frac{w_1}{\langle 0.1, 0.7, 0.9 \rangle}, \frac{w_2}{\langle 0.3, 0.6, 0.7 \rangle}, \frac{w_3}{\langle 0.6, 0.2, 0.3 \rangle}, \frac{w_4}{\langle 0.7, 0.2, 0.4 \rangle} \right\} \right), \right. \\ \left. \left( (v_3, d, 0), \left\{ \frac{w_1}{\langle 0.1, 0.7, 0.4 \rangle}, \frac{w_2}{\langle 0.8, 0.2, 0.3 \rangle}, \frac{w_3}{\langle 0.7, 0.2, 0.6 \rangle}, \frac{w_4}{\langle 0.2, 0.7, 0.7 \rangle} \right\} \right), \right. \\ \left. \left( (v_1, d, 0), \left\{ \frac{w_1}{\langle 0.1, 0.6, 0.3 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.2 \rangle}, \frac{w_3}{\langle 0.6, 0.3, 0.4 \rangle}, \frac{w_4}{\langle 0.2, 0.6, 0.3 \rangle} \right\} \right), \right. \\ \left. \left( (v_1, e, 0), \left\{ \frac{w_1}{\langle 0.2, 0.5, 0.4 \rangle}, \frac{w_2}{\langle 0.2, 0.6, 0.3 \rangle}, \frac{w_3}{\langle 0.3, 0.5, 0.6 \rangle}, \frac{w_4}{\langle 0.5, 0.3, 0.7 \rangle} \right\} \right), \right. \\ \left. \left( (v_3, c, 1), \left\{ \frac{w_1}{\langle 0.1, 0.3, 0.6 \rangle}, \frac{w_2}{\langle 0.9, 0.1, 0.7 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.8 \rangle}, \frac{w_4}{\langle 0.5, 0.3, 0.5 \rangle} \right\} \right) \right\}.$$

**Proposition 4.10.** *If  $(\hbar_1, \mathbb{G}_1), (\hbar_2, \mathbb{G}_2)$  and  $(\hbar_3, \mathbb{G}_3)$  are three NHSEs then*

- (1)  $(\hbar_1, \mathbb{G}_1) \cap (\hbar_2, \mathbb{G}_2) = (\hbar_2, \mathbb{G}_2) \cap (\hbar_1, \mathbb{G}_1)$
- (2)  $((\hbar_1, \mathbb{G}_1) \cap (\hbar_2, \mathbb{G}_2)) \cap (\hbar_3, \mathbb{G}_3) = (\hbar_1, \mathbb{G}_1) \cap ((\hbar_2, \mathbb{G}_2) \cap (\hbar_3, \mathbb{G}_3))$
- (3)  $(\hbar, \mathbb{G}) \cap \phi = \phi$ .

**Proposition 4.11.** *If  $(\hbar_1, \mathbb{G}_1), (\hbar_2, \mathbb{G}_2)$  and  $(\hbar_3, \mathbb{G}_3)$  are three NHSEs, then*

- (1)  $(\hbar_1, \mathbb{G}_1) \cup ((\hbar_2, \mathbb{G}_2) \cap (\hbar_3, \mathbb{G}_3)) =$   
 $((\hbar_1, \mathbb{G}_1) \cup ((\hbar_2, \mathbb{G}_2)) \cap ((\hbar_1, \mathbb{G}_1) \cup (\hbar_3, \mathbb{G}_3))$
- (2)  $(\hbar_1, \mathbb{G}_1) \cap ((\hbar_2, \mathbb{G}_2) \cup (\hbar_3, \mathbb{G}_3)) = ((\hbar_1, \mathbb{G}_1) \cap ((\hbar_2, \mathbb{G}_2)) \cup ((\hbar_1, \mathbb{G}_1) \cap (\hbar_3, \mathbb{G}_3)).$

**Definition 4.12.** *If  $(\hbar_1, \mathbb{G}_1)$  and  $(\hbar_2, \mathbb{G}_2)$  are two NHSEs over  $\Delta$  then  $(\hbar_1, \mathbb{G}_1)$  AND  $(\hbar_2, \mathbb{G}_2)$  denoted by  $(\hbar_1, \mathbb{G}_1) \wedge (\hbar_2, \mathbb{G}_2)$  is defined by*

$$(\hbar_1, \mathbb{G}_1) \wedge (\hbar_2, \mathbb{G}_2) = (\hbar_3, \mathbb{G}_1 \times \mathbb{G}_2), \text{ while } \hbar_3(\varsigma, \gamma) = \hbar_1(\varsigma) \cap \hbar_2(\gamma), \forall (\varsigma, \gamma) \in \mathbb{G}_1 \times \mathbb{G}_2.$$



**Example 4.13.** Considering Example 3.3, we have

$$\mathbb{G}_1 = \left\{ (v_1, c, 1), (v_1, d, 1), (v_3, c, 0) \right\}, \mathbb{G}_2 = \left\{ (v_1, c, 0), (v_3, c, 1) \right\}.$$

Suppose  $(\tilde{h}_1, \mathbb{G}_1)$  and  $(\tilde{h}_2, \mathbb{G}_2)$  over  $\Delta$  are two NHSESs such that

$$(\tilde{h}_1, \mathbb{G}_1) = \left\{ \left( (v_1, c, 1), \left\{ \frac{w_1}{\langle 0.1, 0.6, 0.4 \rangle}, \frac{w_2}{\langle 0.6, 0.4, 0.5 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.6 \rangle}, \frac{w_4}{\langle 0.1, 0.8, 0.7 \rangle} \right\} \right), \right. \\ \left. \left( (v_1, d, 1), \left\{ \frac{w_1}{\langle 0.3, 0.4, 0.8 \rangle}, \frac{w_2}{\langle 0.6, 0.3, 0.5 \rangle}, \frac{w_3}{\langle 0.2, 0.5, 0.6 \rangle}, \frac{w_4}{\langle 0.1, 0.6, 0.7 \rangle} \right\} \right), \right. \\ \left. \left( (v_3, c, 0), \left\{ \frac{w_1}{\langle 0.1, 0.6, 0.9 \rangle}, \frac{w_2}{\langle 0.3, 0.6, 0.7 \rangle}, \frac{w_3}{\langle 0.6, 0.1, 0.2 \rangle}, \frac{w_4}{\langle 0.7, 0.2, 0.3 \rangle} \right\} \right) \right\}.$$

$$(\tilde{h}_2, \mathbb{G}_2) = \left\{ \left( (v_1, c, 0), \left\{ \frac{w_1}{\langle 0.2, 0.1, 0.3 \rangle}, \frac{w_2}{\langle 0.7, 0.2, 0.4 \rangle}, \frac{w_3}{\langle 0.5, 0.2, 0.5 \rangle}, \frac{w_4}{\langle 0.2, 0.3, 0.6 \rangle} \right\} \right), \right. \\ \left. \left( (v_3, c, 1), \left\{ \frac{w_1}{\langle 0.1, 0.5, 0.6 \rangle}, \frac{w_2}{\langle 0.4, 0.2, 0.5 \rangle}, \frac{w_3}{\langle 0.7, 0.1, 0.2 \rangle}, \frac{w_4}{\langle 0.8, 0.1, 0.4 \rangle} \right\} \right) \right\}.$$

Then  $(\tilde{h}_1, \mathbb{G}_1) \wedge (\tilde{h}_2, \mathbb{G}_2) = (\tilde{h}_3, \mathbb{G}_1 \times \mathbb{G}_2)$ ,

$$(\tilde{h}_3, \mathbb{G}_1 \times \mathbb{G}_2) = \left\{ \left( ((v_1, c, 1), (v_1, c, 0)), \left\{ \frac{w_1}{\langle 0.1, 0.35, 0.4 \rangle}, \frac{w_2}{\langle 0.6, 0.30, 0.5 \rangle}, \frac{w_3}{\langle 0.4, 0.35, 0.6 \rangle}, \frac{w_4}{\langle 0.1, 0.55, 0.7 \rangle} \right\} \right), \right. \\ \left( ((v_1, d, 1), (v_1, c, 0)), \left\{ \frac{w_1}{\langle 0.2, 0.25, 0.8 \rangle}, \frac{w_2}{\langle 0.6, 0.25, 0.5 \rangle}, \frac{w_3}{\langle 0.2, 0.35, 0.6 \rangle}, \frac{w_4}{\langle 0.1, 0.45, 0.7 \rangle} \right\} \right), \\ \left( ((v_1, d, 1), (v_3, c, 1)), \left\{ \frac{w_1}{\langle 0.1, 0.45, 0.8 \rangle}, \frac{w_2}{\langle 0.4, 0.25, 0.5 \rangle}, \frac{w_3}{\langle 0.2, 0.30, 0.6 \rangle}, \frac{w_4}{\langle 0.1, 0.35, 0.7 \rangle} \right\} \right), \\ \left( ((v_1, c, 1), (v_3, c, 1)), \left\{ \frac{w_1}{\langle 0.1, 0.55, 0.6 \rangle}, \frac{w_2}{\langle 0.4, 0.30, 0.5 \rangle}, \frac{w_3}{\langle 0.4, 0.30, 0.6 \rangle}, \frac{w_4}{\langle 0.1, 0.45, 0.7 \rangle} \right\} \right), \\ \left( ((v_3, c, 0), (v_1, c, 0)), \left\{ \frac{w_1}{\langle 0.1, 0.35, 0.9 \rangle}, \frac{w_2}{\langle 0.3, 0.40, 0.7 \rangle}, \frac{w_3}{\langle 0.5, 0.15, 0.5 \rangle}, \frac{w_4}{\langle 0.2, 0.25, 0.6 \rangle} \right\} \right), \\ \left. \left( ((v_3, c, 0), (v_3, c, 1)), \left\{ \frac{w_1}{\langle 0.1, 0.55, 0.9 \rangle}, \frac{w_2}{\langle 0.3, 0.40, 0.7 \rangle}, \frac{w_3}{\langle 0.6, 0.10, 0.2 \rangle}, \frac{w_4}{\langle 0.7, 0.15, 0.4 \rangle} \right\} \right) \right\}.$$

**Definition 4.14.** If  $(\tilde{h}_1, \mathbb{G}_1)$  and  $(\tilde{h}_2, \mathbb{G}_2)$  are two NHSESs over  $\Delta$ , then  $(\tilde{h}_1, \mathbb{G}_1)$  OR  $(\tilde{h}_2, \mathbb{G}_2)$  denoted by  $(\tilde{h}_1, \mathbb{G}_1) \vee (\tilde{h}_2, \mathbb{G}_2)$  is defined by  $(\tilde{h}_1, \mathbb{G}_1) \vee (\tilde{h}_2, \mathbb{G}_2) = (\tilde{h}_3, \mathbb{G}_1 \times \mathbb{G}_2)$ , while  $\tilde{h}_3(\delta, \gamma) = \tilde{h}_1(\delta) \cup \tilde{h}_2(\gamma), \forall (\delta, \gamma) \in \mathbb{G}_1 \times \mathbb{G}_2$ .

**Example 4.15.** Considering Example 3.3, we see

$$\mathbb{G}_1 = \left\{ (v_1, c, 1), (v_1, d, 1), (v_3, c, 0) \right\}, \mathbb{G}_2 = \left\{ (v_1, c, 0), (v_3, c, 1) \right\}.$$

Suppose  $(\tilde{h}_1, \mathbb{G}_1)$  and  $(\tilde{h}_2, \mathbb{G}_2)$  over  $\Delta$  are two NHSESs such that

$$(\tilde{h}_1, \mathbb{G}_1) = \left\{ \left( (v_1, c, 1), \left\{ \frac{w_1}{\langle 0.1, 0.6, 0.4 \rangle}, \frac{w_2}{\langle 0.6, 0.4, 0.5 \rangle}, \frac{w_3}{\langle 0.4, 0.5, 0.6 \rangle}, \frac{w_4}{\langle 0.1, 0.8, 0.7 \rangle} \right\} \right), \right. \\ \left( (v_1, d, 1), \left\{ \frac{w_1}{\langle 0.3, 0.4, 0.8 \rangle}, \frac{w_2}{\langle 0.6, 0.3, 0.5 \rangle}, \frac{w_3}{\langle 0.2, 0.5, 0.6 \rangle}, \frac{w_4}{\langle 0.1, 0.6, 0.7 \rangle} \right\} \right), \\ \left. \left( (v_3, c, 0), \left\{ \frac{w_1}{\langle 0.1, 0.6, 0.9 \rangle}, \frac{w_2}{\langle 0.3, 0.6, 0.7 \rangle}, \frac{w_3}{\langle 0.6, 0.1, 0.2 \rangle}, \frac{w_4}{\langle 0.7, 0.2, 0.3 \rangle} \right\} \right) \right\}$$

$$(\tilde{h}_2, \mathbb{G}_2) = \left\{ \left( (v_1, c, 0), \left\{ \frac{w_1}{\langle 0.2, 0.1, 0.3 \rangle}, \frac{w_2}{\langle 0.7, 0.2, 0.4 \rangle}, \frac{w_3}{\langle 0.5, 0.2, 0.5 \rangle}, \frac{w_4}{\langle 0.2, 0.3, 0.6 \rangle} \right\} \right), \right. \\ \left. \left( (v_3, c, 1), \left\{ \frac{w_1}{\langle 0.1, 0.5, 0.6 \rangle}, \frac{w_2}{\langle 0.4, 0.2, 0.5 \rangle}, \frac{w_3}{\langle 0.7, 0.1, 0.2 \rangle}, \frac{w_4}{\langle 0.8, 0.1, 0.4 \rangle} \right\} \right) \right\}.$$

Then  $(\hbar_3, \mathbb{G}_3) \vee (\hbar_2, \mathbb{G}_2) = (\hbar_3, \mathbb{G}_1 \times \mathbb{G}_2)$ ,

$$(\hbar_3, \mathbb{G}_1 \times \mathbb{G}_2) = \left\{ \begin{array}{l} \left( ((v_1, c, 1), (v_1, c, 0)), \left\{ \frac{w_1}{\langle 0.2, 0.35, 0.3 \rangle}, \frac{w_2}{\langle 0.7, 0.30, 0.4 \rangle}, \frac{w_3}{\langle 0.5, 0.35, 0.5 \rangle}, \frac{w_4}{\langle 0.2, 0.55, 0.6 \rangle} \right\} \right), \\ \left( ((v_1, d, 1), (v_1, c, 0)), \left\{ \frac{w_1}{\langle 0.3, 0.25, 0.3 \rangle}, \frac{w_2}{\langle 0.7, 0.25, 0.4 \rangle}, \frac{w_3}{\langle 0.5, 0.35, 0.5 \rangle}, \frac{w_4}{\langle 0.2, 0.45, 0.6 \rangle} \right\} \right), \\ \left( ((v_1, d, 1), (v_3, c, 1)), \left\{ \frac{w_1}{\langle 0.3, 0.45, 0.6 \rangle}, \frac{w_2}{\langle 0.6, 0.25, 0.5 \rangle}, \frac{w_3}{\langle 0.7, 0.30, 0.2 \rangle}, \frac{w_4}{\langle 0.8, 0.35, 0.4 \rangle} \right\} \right), \\ \left( ((v_1, c, 1), (v_3, c, 1)), \left\{ \frac{w_1}{\langle 0.1, 0.55, 0.4 \rangle}, \frac{w_2}{\langle 0.6, 0.30, 0.5 \rangle}, \frac{w_3}{\langle 0.7, 0.30, 0.2 \rangle}, \frac{w_4}{\langle 0.8, 0.45, 0.4 \rangle} \right\} \right), \\ \left( ((v_3, c, 0), (v_1, c, 0)), \left\{ \frac{w_1}{\langle 0.2, 0.35, 0.3 \rangle}, \frac{w_2}{\langle 0.7, 0.40, 0.4 \rangle}, \frac{w_3}{\langle 0.6, 0.15, 0.2 \rangle}, \frac{w_4}{\langle 0.7, 0.25, 0.3 \rangle} \right\} \right), \\ \left( ((v_3, c, 0), (v_3, c, 1)), \left\{ \frac{w_1}{\langle 0.1, 0.55, 0.6 \rangle}, \frac{w_2}{\langle 0.4, 0.40, 0.5 \rangle}, \frac{w_3}{\langle 0.7, 0.10, 0.2 \rangle}, \frac{w_4}{\langle 0.8, 0.15, 0.4 \rangle} \right\} \right), \end{array} \right\}.$$

**Proposition 4.16.** *If  $(\hbar_1, \mathbb{G}_1), (\hbar_2, \mathbb{G}_2)$  and  $(\hbar_3, \mathbb{G}_3)$  are three NHSESs over  $\Delta$ , then*

- (1)  $((\hbar_1, \mathbb{G}_1) \wedge (\hbar_2, \mathbb{G}_2))^c = ((\hbar_1, \mathbb{G}_1))^c \vee ((\hbar_2, \mathbb{G}_2))^c$
- (2)  $((\hbar_1, \mathbb{G}_1) \vee (\hbar_2, \mathbb{G}_2))^c = ((\hbar_1, \mathbb{G}_1))^c \wedge ((\hbar_2, \mathbb{G}_2))^c$ .

**Proposition 4.17.** *If  $(\hbar_1, \mathbb{G}_1), (\hbar_2, \mathbb{G}_2)$  and  $(\hbar_3, \mathbb{G}_3)$  are three NHSESs over  $\Delta$ , then*

- (1)  $((\hbar_1, \mathbb{G}_1) \wedge (\hbar_2, \mathbb{G}_2)) \wedge (\hbar_3, \mathbb{G}_3) = (\hbar_1, \mathbb{G}_1) \wedge ((\hbar_2, \mathbb{G}_2) \wedge (\hbar_3, \mathbb{G}_3))$
- (2)  $((\hbar_1, \mathbb{G}_1) \vee (\hbar_2, \mathbb{G}_2)) \vee (\hbar_3, \mathbb{G}_3) = (\hbar_1, \mathbb{G}_1) \vee ((\hbar_2, \mathbb{G}_2) \vee (\hbar_3, \mathbb{G}_3))$
- (3)  $(\hbar_1, \mathbb{G}_1) \vee ((\hbar_2, \mathbb{G}_2) \wedge (\hbar_3, \mathbb{G}_3)) = ((\hbar_1, \mathbb{G}_1) \vee ((\hbar_2, \mathbb{G}_2)) \wedge ((\hbar_1, \mathbb{G}_1) \vee (\hbar_3, \mathbb{G}_3))$
- (4)  $(\hbar_1, \mathbb{G}_1) \wedge ((\hbar_2, \mathbb{G}_2) \vee (\hbar_3, \mathbb{G}_3)) = ((\hbar_1, \mathbb{G}_1) \wedge ((\hbar_2, \mathbb{G}_2)) \vee ((\hbar_1, \mathbb{G}_1) \wedge (\hbar_3, \mathbb{G}_3))$ .

### 5. Basic Properties and Laws of Neutrosophic Hypersoft Expert Set Operations

In this important part of the paper, certain important characteristics and laws are explained for NHSES.

Here  $(\hbar, \mathbb{G}), (\hbar, \mathbb{G}_1), (\hbar, \mathbb{G}_2), (\hbar, \mathbb{G}_3)$  and  $(\hbar_1, \mathbb{G})$  are NHSESs over  $\Delta$

- Idempotent Laws
  - (a)  $(\hbar, \mathbb{G}) \cup (\hbar, \mathbb{G}) = (\hbar, \mathbb{G}) = (\hbar, \mathbb{G}) \cup_R (\hbar, \mathbb{G})$
  - (b)  $(\hbar, \mathbb{G}) \cap (\hbar, \mathbb{G}) = (\hbar, \mathbb{G}) = (\hbar, \mathbb{G}) \cap_\varepsilon (\hbar, \mathbb{G})$
- Identity Laws
  - (a)  $(\hbar, \mathbb{G}) \cup (\hbar, \mathbb{G})_\Phi = (\hbar, \mathbb{G}) = (\hbar, \mathbb{G}) \cup_R (\hbar, \mathbb{G})_\Phi$
  - (b)  $(\hbar, \mathbb{G}) \cap (\hbar, \mathbb{G})_\Delta = (\hbar, \mathbb{G}) = (\hbar, \mathbb{G}) \cap_\varepsilon (\hbar, \mathbb{G})_\Delta$ .
- Domination Laws
  - (a)  $(\hbar, \mathbb{G}) \cup (\hbar, \mathbb{G})_\Delta = (\hbar, \mathbb{G})_\Delta = (\hbar, \mathbb{G}) \cup_R (\hbar, \mathbb{G})_\Delta$
  - (b)  $(\hbar, \mathbb{G}) \cap (\hbar, \mathbb{G})_\Phi = (\hbar, \mathbb{G})_\Phi = (\hbar, \mathbb{G}) \cap_\varepsilon (\hbar, \mathbb{G})_\Phi$ .
- Characteristic of Exclusion
  - (a)  $(\hbar, \mathbb{G}) \cup (\hbar, \mathbb{G})^c = (\hbar, \mathbb{G})_\Delta = (\hbar, \mathbb{G}) \cup_R (\hbar, \mathbb{G})^c$ .
- Characteristic of Contradiction
  - (a)  $(\hbar, \mathbb{G}) \cap (\hbar, \mathbb{G})^c = (\hbar, \mathbb{G})_\Phi = (\hbar, \mathbb{G}) \cap_\varepsilon (\hbar, \mathbb{G})^c$ .

- Absorption Laws

- (a)  $(\bar{h}, \mathbb{G}_1) \cup ((\bar{h}, \mathbb{G}_1) \cap (\bar{h}, \mathbb{G}_1)) = (\bar{h}, \mathbb{G}_1)$
- (b)  $(\bar{h}, \mathbb{G}_1) \cap ((\bar{h}, \mathbb{G}_1) \cup (\bar{h}, \mathbb{G}_1)) = (\bar{h}, \mathbb{G}_1)$
- (c)  $(\bar{h}, \mathbb{G}_1) \cup_R ((\bar{h}, \mathbb{G}_1) \cap_\varepsilon (\bar{h}, \mathbb{G}_1)) = (\bar{h}, \mathbb{G}_1)$
- (d)  $(\bar{h}, \mathbb{G}_1) \cap_\varepsilon ((\bar{h}, \mathbb{G}_1) \cup_R (\bar{h}, \mathbb{G}_1)) = (\bar{h}, \mathbb{G}_1)$ .

- Absorption Laws

- (a)  $((\bar{h}, \mathbb{G}_1) \cup (\bar{h}, \mathbb{G}_2)) = ((\bar{h}, \mathbb{G}_1) \cup (\bar{h}, \mathbb{G}_2))$
- (b)  $((\bar{h}, \mathbb{G}_1) \cup_R (\bar{h}, \mathbb{G}_2)) = ((\bar{h}, \mathbb{G}_1) \cup_R (\bar{h}, \mathbb{G}_2))$
- (c)  $((\bar{h}, \mathbb{G}_1) \cap (\bar{h}, \mathbb{G}_2)) = ((\bar{h}, \mathbb{G}_1) \cap (\bar{h}, \mathbb{G}_2))$
- (d)  $((\bar{h}, \mathbb{G}_1) \cap_\varepsilon (\bar{h}, \mathbb{G}_2)) = ((\bar{h}, \mathbb{G}_1) \cap_\varepsilon (\bar{h}, \mathbb{G}_2))$ .

- Associative Laws

- (a)  $(\bar{h}, \mathbb{G}_1) \cup ((\bar{h}, \mathbb{G}_2) \cup (\bar{h}_1, \mathbb{G}_3)) = ((\bar{h}, \mathbb{G}_1) \cup (\bar{h}, \mathbb{G}_2)) \cup (\bar{h}_1, \mathbb{G}_3)$
- (b)  $(\bar{h}, \mathbb{G}_1) \cup_R ((\bar{h}, \mathbb{G}_2) \cup_R (\bar{h}_1, \mathbb{G}_3)) = ((\bar{h}, \mathbb{G}_1) \cup_R (\bar{h}, \mathbb{G}_2)) \cup_R (\bar{h}_1, \mathbb{G}_3)$
- (c)  $(\bar{h}, \mathbb{G}_1) \cap ((\bar{h}, \mathbb{G}_2) \cap (\bar{h}_1, \mathbb{G}_3)) = ((\bar{h}, \mathbb{G}_1) \cap (\bar{h}, \mathbb{G}_2)) \cap (\bar{h}_1, \mathbb{G}_3)$
- (d)  $(\bar{h}, \mathbb{G}_1) \cap_\varepsilon ((\bar{h}, \mathbb{G}_2) \cap_\varepsilon (\bar{h}_1, \mathbb{G}_3)) = ((\bar{h}, \mathbb{G}_1) \cap_\varepsilon (\bar{h}, \mathbb{G}_2)) \cap_\varepsilon (\bar{h}_1, \mathbb{G}_3)$
- (e)  $(\bar{h}, \mathbb{G}_1) \vee ((\bar{h}, \mathbb{G}_2) \vee (\bar{h}_1, \mathbb{G}_3)) = ((\bar{h}, \mathbb{G}_1) \vee (\bar{h}, \mathbb{G}_2)) \vee (\bar{h}_1, \mathbb{G}_3)$
- (e)  $(\bar{h}, \mathbb{G}_1) \wedge ((\bar{h}, \mathbb{G}_2) \wedge (\bar{h}_1, \mathbb{G}_3)) = ((\bar{h}, \mathbb{G}_1) \wedge (\bar{h}, \mathbb{G}_2)) \wedge (\bar{h}_1, \mathbb{G}_3)$ .

- De Morgan's Laws

- (a)  $((\bar{h}, \mathbb{G}_1) \cup (\bar{h}, \mathbb{G}_2))^c = (\bar{h}, \mathbb{G}_1)^c \cap_\varepsilon (\bar{h}, \mathbb{G}_2)^c$
- (b)  $((\bar{h}, \mathbb{G}_1) \cap_\varepsilon (\bar{h}, \mathbb{G}_2))^c = (\bar{h}, \mathbb{G}_1)^c \cup (\bar{h}, \mathbb{G}_2)^c$
- (c)  $((\bar{h}, \mathbb{G}_1) \vee (\bar{h}, \mathbb{G}_2))^c = (\bar{h}, \mathbb{G}_1)^c \wedge (\bar{h}, \mathbb{G}_2)^c$
- (d)  $((\bar{h}, \mathbb{G}_1) \wedge (\bar{h}, \mathbb{G}_2))^c = (\bar{h}, \mathbb{G}_1)^c \vee (\bar{h}, \mathbb{G}_2)^c$ .

- Distributive Laws

- (a)  $(\bar{h}, \mathbb{G}_1) \cup ((\bar{h}, \mathbb{G}_2) \cap (\bar{h}_1, \mathbb{G}_3)) = ((\bar{h}, \mathbb{G}_1) \cup (\bar{h}, \mathbb{G}_2)) \cap ((\bar{h}, \mathbb{G}_1) \cup (\bar{h}_1, \mathbb{G}_3))$
- (b)  $(\bar{h}, \mathbb{G}_1) \cap ((\bar{h}, \mathbb{G}_2) \cup (\bar{h}_1, \mathbb{G}_3)) = ((\bar{h}, \mathbb{G}_1) \cap (\bar{h}, \mathbb{G}_2)) \cup ((\bar{h}, \mathbb{G}_1) \cap (\bar{h}_1, \mathbb{G}_3))$
- (c)  $(\bar{h}, \mathbb{G}_1) \cup_R ((\bar{h}, \mathbb{G}_2) \cap_\varepsilon (\bar{h}_1, \mathbb{G}_3)) = ((\bar{h}, \mathbb{G}_1) \cup_R (\bar{h}, \mathbb{G}_2)) \cap_\varepsilon ((\bar{h}, \mathbb{G}_1) \cup_R (\bar{h}_1, \mathbb{G}_3))$
- (d)  $(\bar{h}, \mathbb{G}_1) \cap_\varepsilon ((\bar{h}, \mathbb{G}_2) \cup_R (\bar{h}_1, \mathbb{G}_3)) = ((\bar{h}, \mathbb{G}_1) \cap_\varepsilon (\bar{h}, \mathbb{G}_2)) \cup_R ((\bar{h}, \mathbb{G}_1) \cap_\varepsilon (\bar{h}_1, \mathbb{G}_3))$
- (c)  $(\bar{h}, \mathbb{G}_1) \cup_R ((\bar{h}, \mathbb{G}_2) \cap (\bar{h}_1, \mathbb{G}_3)) = ((\bar{h}, \mathbb{G}_1) \cup_R (\bar{h}, \mathbb{G}_2)) \cap ((\bar{h}, \mathbb{G}_1) \cup_R (\bar{h}_1, \mathbb{G}_3))$
- (e)  $(\bar{h}, \mathbb{G}_1) \cap ((\bar{h}, \mathbb{G}_2) \cup_R (\bar{h}_1, \mathbb{G}_3)) = ((\bar{h}, \mathbb{G}_1) \cap (\bar{h}, \mathbb{G}_2)) \cup_R ((\bar{h}, \mathbb{G}_1) \cap (\bar{h}_1, \mathbb{G}_3))$ .

## 6. Hybrids of Neutrosophic Hypersoft Expert Set

In this study, some hybridized structures of NHSES are presented. Suppose  $\mathbf{Y}$  denotes the set of expert and  $\mathbf{O}$  be a set of opinions,  $T = \mathbf{F} \times \mathbf{Y} \times \mathbf{O}$ . Taking  $A \subseteq T$  and  $\Delta$  denotes the universe, while  $\mathbf{F}$  used for parameters.

**Definition 6.1.** A bipolar neutrosophic hypersoft expert set is a pair  $(\mathbb{B}, A)$  and is characterized by a mapping

$$\mathbb{B} : A \rightarrow P(\Delta)$$

where

$$(\mathbb{B}, A) = \{ \langle x, v_{B(e)}^+(x), \nu_{B(e)}^+(x), \omega_{B(e)}^+(x), v_{B(e)}^-(x), \nu_{B(e)}^-(x), \omega_{B(e)}^-(x) \rangle : \forall e \in A, x \in \Delta \}$$

, where  $v_{B(e)}^+, \nu_{B(e)}^+, \omega_{B(e)}^+ : \Delta \rightarrow [0, 1], v_{B(e)}^-, \nu_{B(e)}^-, \omega_{B(e)}^- : \Delta \rightarrow [0, 1]$ .

**Example 6.2.** Considering Example 3.3 with  $\Delta = \{w_1, w_2\}$ , we have bipolar neutrosophic hypersoft expert set as

$$(\mathbb{B}, A) = \left\{ \left( \begin{array}{l} (v_1, c, 1), \left\{ \begin{array}{l} \overline{\langle 0.2, 0.5, 0.4, -0.1, -0.2, -0.3 \rangle}^{w_1}, \overline{\langle 0.1, 0.3, 0.6, -0.2, -0.3, -0.2 \rangle}^{w_2} \\ \overline{\langle 0.4, 0.2, 0.3, -0.1, -0.1, -0.2 \rangle}^{w_1}, \overline{\langle 0.2, 0.5, 0.3, -0.1, -0.2, -0.5 \rangle}^{w_2} \end{array} \right\} \\ (v_1, d, 1), \left\{ \begin{array}{l} \overline{\langle 0.7, 0.2, 0.3, -0.3, -0.1, -0.2 \rangle}^{w_1}, \overline{\langle 0.3, 0.5, 0.6, -0.2, -0.3, -0.4 \rangle}^{w_2} \\ \overline{\langle 0.9, 0.1, 0.3, -0.3, -0.2, -0.1 \rangle}^{w_1}, \overline{\langle 0.3, 0.4, 0.8, -0.1, -0.7, -0.4 \rangle}^{w_2} \end{array} \right\} \\ (v_2, c, 1), \left\{ \begin{array}{l} \overline{\langle 0.4, 0.5, 0.6, -0.2, -0.3, -0.4 \rangle}^{w_1}, \overline{\langle 0.2, 0.6, 0.7, -0.1, -0.3, -0.4 \rangle}^{w_2} \\ \overline{\langle 0.5, 0.4, 0.7, -0.1, -0.2, -0.3 \rangle}^{w_1}, \overline{\langle 0.8, 0.1, 0.6, -0.2, -0.2, -0.3 \rangle}^{w_2} \end{array} \right\} \\ (v_2, d, 1), \left\{ \begin{array}{l} \overline{\langle 0.3, 0.2, 0.1, -0.1, -0.2, -0.3 \rangle}^{w_1}, \overline{\langle 0.1, 0.8, 0.3, -0.1, -0.7, -0.2 \rangle}^{w_2} \\ \overline{\langle 0.1, 0.8, 0.4, -0.1, -0.2, -0.3 \rangle}^{w_1}, \overline{\langle 0.2, 0.7, 0.5, -0.1, -0.3, -0.4 \rangle}^{w_2} \end{array} \right\} \\ (v_1, c, 0), \left\{ \begin{array}{l} \overline{\langle 0.3, 0.2, 0.1, -0.1, -0.2, -0.3 \rangle}^{w_1}, \overline{\langle 0.1, 0.8, 0.3, -0.1, -0.7, -0.2 \rangle}^{w_2} \\ \overline{\langle 0.1, 0.8, 0.4, -0.1, -0.2, -0.3 \rangle}^{w_1}, \overline{\langle 0.2, 0.7, 0.5, -0.1, -0.3, -0.4 \rangle}^{w_2} \end{array} \right\} \\ (v_1, d, 0), \left\{ \begin{array}{l} \overline{\langle 0.3, 0.2, 0.1, -0.1, -0.2, -0.3 \rangle}^{w_1}, \overline{\langle 0.1, 0.8, 0.3, -0.1, -0.7, -0.2 \rangle}^{w_2} \\ \overline{\langle 0.1, 0.8, 0.4, -0.1, -0.2, -0.3 \rangle}^{w_1}, \overline{\langle 0.2, 0.7, 0.5, -0.1, -0.3, -0.4 \rangle}^{w_2} \end{array} \right\} \\ (v_1, e, 0), \left\{ \begin{array}{l} \overline{\langle 0.2, 0.7, 0.5, -0.1, -0.3, -0.4 \rangle}^{w_1}, \overline{\langle 0.5, 0.4, 0.6, -0.1, -0.3, -0.4 \rangle}^{w_2} \\ \overline{\langle 0.2, 0.7, 0.5, -0.1, -0.3, -0.4 \rangle}^{w_1}, \overline{\langle 0.5, 0.4, 0.6, -0.1, -0.3, -0.4 \rangle}^{w_2} \end{array} \right\} \end{array} \right\}.$$

**Definition 6.3.** A complex neutrosophic hypersoft expert set  $(\mathcal{C}, A)$  is characterized by a mapping

$$\mathcal{C} : A \rightarrow \mathcal{CN}^\Delta$$

where  $\mathcal{CN}^\Delta$  denotes the collection of all complex neutrosophic subsets of  $\Delta$  and

$$(\mathcal{C}, A) = \{ \langle x, v_{C(e)}(x), \nu_{C(e)}(x), \omega_{C(e)}(x) \rangle : \forall e \in A, x \in \Delta \}, \text{ where}$$

$$v_{C(e)}(x) = aC(e)(x).e^{jC(e)(x)}, \nu_{C(e)}(x) = bC(e)(x).e^{jC(e)(x)}, \omega_{C(e)}(x) = cC(e)(x).e^{jC(e)(x)}$$

for all  $u \in \Delta$  while  $v_{C(e)}, \nu_{C(e)}, \omega_{C(e)}$  are complex-valued truth, indeterminacy and falsity membership functions and these values lie within the unit circle in the complex plane and both the amplitude terms  $aC(e)(x), bC(e)(x), cC(e)(x)$  and the phase terms  $v_{C(e)}(x), \nu_{C(e)}(x), \omega_{C(e)}(x)$  are real valued such that  $0 \leq aC(e)(x) + bC(e)(x) + cC(e)(x) \leq 3$  while  $aC(e)(x), bC(e)(x), cC(e)(x) \in [0, 1]$ .

**Example 6.4.** Considering Example 3.3, we have complex neutrosophic hypersoft expert as

$$(\mathbb{C}, A) = \left\{ \left( (v_1, c, 1), \left\{ \left\langle \frac{w_1}{\langle 0.1e^{j2\pi(0.3)}, 0.6e^{j2\pi(0.3)}, 0.7e^{j2\pi(0.3)} \rangle}, \frac{w_2}{\langle 0.9e^{j2\pi(0.3)}, 0.7e^{j2\pi(0.3)}, 0.8e^{j2\pi(0.3)} \rangle} \right\rangle \right), \right. \\ \left. \left( (v_1, d, 1), \left\{ \left\langle \frac{w_1}{\langle 0.2e^{j2\pi(0.3)}, 0.6e^{j2\pi(0.3)}, 0.3e^{j2\pi(0.3)} \rangle}, \frac{w_2}{\langle 0.1e^{j2\pi(0.3)}, 0.8e^{j2\pi(0.3)}, 0.2e^{j2\pi(0.3)} \rangle} \right\rangle \right), \right. \\ \left. \left( (v_1, e, 1), \left\{ \left\langle \frac{w_1}{\langle 0.6e^{j2\pi(0.3)}, 0.7e^{j2\pi(0.3)}, 0.8e^{j2\pi(0.3)} \rangle}, \frac{w_2}{\langle 0.6e^{j2\pi(0.3)}, 0.9e^{j2\pi(0.3)}, 0.2e^{j2\pi(0.3)} \rangle} \right\rangle \right), \right. \\ \left. \left( (v_2, c, 1), \left\{ \left\langle \frac{w_1}{\langle 0.4e^{j2\pi(0.3)}, 0.5e^{j2\pi(0.3)}, 0.4e^{j2\pi(0.3)} \rangle}, \frac{w_2}{\langle 0.1e^{j2\pi(0.3)}, 0.8e^{j2\pi(0.3)}, 0.5e^{j2\pi(0.3)} \rangle} \right\rangle \right), \right. \\ \left. \left( (v_2, d, 1), \left\{ \left\langle \frac{w_1}{\langle 0.4e^{j2\pi(0.3)}, 0.7e^{j2\pi(0.3)}, 0.4e^{j2\pi(0.3)} \rangle}, \frac{w_2}{\langle 0.6e^{j2\pi(0.3)}, 0.8e^{j2\pi(0.3)}, 0.1e^{j2\pi(0.3)} \rangle} \right\rangle \right), \right. \\ \left. \left( (v_2, e, 1), \left\{ \left\langle \frac{w_1}{\langle 0.4e^{j2\pi(0.3)}, 0.7e^{j2\pi(0.3)}, 0.2e^{j2\pi(0.3)} \rangle}, \frac{w_2}{\langle 0.4e^{j2\pi(0.3)}, 0.6e^{j2\pi(0.3)}, 0.6e^{j2\pi(0.3)} \rangle} \right\rangle \right), \right. \\ \left. \left( (v_1, c, 0), \left\{ \left\langle \frac{w_1}{\langle 0.3e^{j2\pi(0.3)}, 0.3e^{j2\pi(0.3)}, 0.2e^{j2\pi(0.3)} \rangle}, \frac{w_2}{\langle 0.1e^{j2\pi(0.3)}, 0.6e^{j2\pi(0.3)}, 0.1e^{j2\pi(0.3)} \rangle} \right\rangle \right), \right. \\ \left. \left( (v_1, d, 0), \left\{ \left\langle \frac{w_1}{\langle 0.7e^{j2\pi(0.3)}, 0.4e^{j2\pi(0.3)}, 0.2e^{j2\pi(0.3)} \rangle}, \frac{w_2}{\langle 0.1e^{j2\pi(0.3)}, 0.6e^{j2\pi(0.3)}, 0.2e^{j2\pi(0.3)} \rangle} \right\rangle \right), \right. \\ \left. \left( (v_1, e, 0), \left\{ \left\langle \frac{w_1}{\langle 0.6e^{j2\pi(0.3)}, 0.7e^{j2\pi(0.3)}, 0.4e^{j2\pi(0.3)} \rangle}, \frac{w_2}{\langle 0.5e^{j2\pi(0.3)}, 0.9e^{j2\pi(0.3)}, 0.4e^{j2\pi(0.3)} \rangle} \right\rangle \right), \right. \\ \left. \left( (v_2, c, 0), \left\{ \left\langle \frac{w_1}{\langle 0.6e^{j2\pi(0.3)}, 0.7e^{j2\pi(0.3)}, 0.5e^{j2\pi(0.3)} \rangle}, \frac{w_2}{\langle 0.7e^{j2\pi(0.3)}, 0.9e^{j2\pi(0.3)}, 0.3e^{j2\pi(0.3)} \rangle} \right\rangle \right) \right\}.$$

**Definition 6.5.** A pair  $(F, H)$  is called a fuzzy parameterized complex neutrosophic hypersoft expert set(FP-CNHSES) over  $\Delta$ , where  $F$  is a mapping given by

$$F : H \rightarrow CN^\Delta$$

where  $CN^\Delta$  is the collection of all complex neutrosophic subsets of  $\Delta$ .

It can also be written as  $(F, H) = \left\{ \left( t, \left\{ \frac{w}{F(t)(x)} : x \in \Delta \right\} \right) : t \in H \right\}$

where  $H \subseteq \mathcal{G} \times \mathcal{D} \times \mathbb{C} = \left\{ \left( \frac{\alpha}{\mathfrak{S}(\alpha)} : \beta, \gamma \in \Delta \right) : \alpha \in \mathcal{G}, \beta \in \mathcal{D}, \gamma \in \mathcal{C} \right\}$  with  $\mathfrak{S}$  is a corresponding membership function of fuzzy set and

$$(F, H) = \langle x, v_{C(e)}(x), \nu_{C(e)}(x), \omega_{C(e)}(x) \rangle : \forall e \in H, x \in \Delta,$$

where

$$v_{C(e)}(x) = aC(e)(x).e^{jC(e)(x)}, \nu_{C(e)}(x) = bC(e)(x).e^{jC(e)(x)}, \omega_{C(e)}(x) = cC(e)(x).e^{jC(e)(x)}$$

for all  $x \in \Delta$  while  $v_{C(e)}, \nu_{C(e)}, \omega_{C(e)}$  are complex-valued truth, indeterminacy and falsity membership functions for or the FP-CNHSES and these values lie within the unit circle in the complex plane and both the amplitude terms  $aC(e)(x), bC(e)(x), cC(e)(x)$  and the phase terms  $vC(e)(x), \nu C(e)(x), \omega C(e)(x)$  are real valued such that  $0 \leq aC(e)(x)+bC(e)(x)+cC(e)(x) \leq 3$ .

**Example 6.6.** Considering Example 3.3 with  $\mathfrak{S} = \left\{ \frac{v_1}{0.2}, \frac{v_2}{0.3}, \frac{v_3}{0.5} \right\}$  as a fuzzy subset of  $FZ(E)$ . We can define FP-CNHSES as

$$(F, H) = \left\{ \begin{array}{l} \left( \left( \frac{v_1}{0.2}, c, 1 \right), \left\{ \frac{w_1}{\langle 0.1e^{2\pi(0.2)}, 0.6e^{2\pi(0.2)}, 0.7e^{2\pi(0.2)} \rangle}, \frac{w_2}{\langle 0.9e^{2\pi(0.2)}, 0.7e^{2\pi(0.2)}, 0.8e^{2\pi(0.2)} \rangle} \right\} \right), \\ \left( \left( \frac{v_2}{0.3}, d, 1 \right), \left\{ \frac{w_1}{\langle 0.2e^{2\pi(0.2)}, 0.6e^{2\pi(0.2)}, 0.3e^{2\pi(0.2)} \rangle}, \frac{w_2}{\langle 0.1e^{2\pi(0.2)}, 0.8e^{2\pi(0.2)}, 0.2e^{2\pi(0.2)} \rangle} \right\} \right), \\ \left( \left( \frac{v_3}{0.5}, e, 1 \right), \left\{ \frac{w_1}{\langle 0.6e^{2\pi(0.2)}, 0.7e^{2\pi(0.2)}, 0.8e^{2\pi(0.2)} \rangle}, \frac{w_2}{\langle 0.6e^{2\pi(0.2)}, 0.9e^{2\pi(0.2)}, 0.2e^{2\pi(0.2)} \rangle} \right\} \right), \\ \left( \left( \frac{v_1}{0.2}, c, 1 \right), \left\{ \frac{w_1}{\langle 0.4e^{2\pi(0.2)}, 0.5e^{2\pi(0.2)}, 0.4e^{2\pi(0.2)} \rangle}, \frac{w_2}{\langle 0.1e^{2\pi(0.3)}, 0.8e^{2\pi(0.2)}, 0.5e^{2\pi(0.2)} \rangle} \right\} \right), \\ \left( \left( \frac{v_2}{0.3}, d, 1 \right), \left\{ \frac{w_1}{\langle 0.4e^{2\pi(0.2)}, 0.7e^{2\pi(0.2)}, 0.4e^{2\pi(0.2)} \rangle}, \frac{w_2}{\langle 0.6e^{2\pi(0.2)}, 0.8e^{2\pi(0.2)}, 0.1e^{2\pi(0.2)} \rangle} \right\} \right), \\ \left( \left( \frac{v_3}{0.5}, e, 1 \right), \left\{ \frac{w_1}{\langle 0.4e^{2\pi(0.2)}, 0.7e^{2\pi(0.2)}, 0.2e^{2\pi(0.2)} \rangle}, \frac{w_2}{\langle 0.4e^{2\pi(0.2)}, 0.6e^{2\pi(0.2)}, 0.6e^{2\pi(0.2)} \rangle} \right\} \right), \\ \left( \left( \frac{v_1}{0.2}, c, 0 \right), \left\{ \frac{w_1}{\langle 0.3e^{2\pi(0.3)}, 0.3e^{2\pi(0.2)}, 0.2e^{2\pi(0.2)} \rangle}, \frac{w_2}{\langle 0.1e^{2\pi(0.3)}, 0.6e^{2\pi(0.2)}, 0.1e^{2\pi(0.2)} \rangle} \right\} \right), \\ \left( \left( \frac{v_2}{0.3}, d, 0 \right), \left\{ \frac{w_1}{\langle 0.7e^{2\pi(0.2)}, 0.4e^{2\pi(0.2)}, 0.2e^{2\pi(0.2)} \rangle}, \frac{w_2}{\langle 0.1e^{2\pi(0.2)}, 0.6e^{2\pi(0.2)}, 0.2e^{2\pi(0.2)} \rangle} \right\} \right), \\ \left( \left( \frac{v_3}{0.5}, e, 0 \right), \left\{ \frac{w_1}{\langle 0.6e^{2\pi(0.2)}, 0.7e^{2\pi(0.2)}, 0.4e^{2\pi(0.2)} \rangle}, \frac{w_2}{\langle 0.5e^{2\pi(0.2)}, 0.9e^{2\pi(0.2)}, 0.4e^{2\pi(0.2)} \rangle} \right\} \right), \\ \left( \left( \frac{v_1}{0.2}, c, 0 \right), \left\{ \frac{w_1}{\langle 0.6e^{2\pi(0.2)}, 0.7e^{2\pi(0.2)}, 0.5e^{2\pi(0.2)} \rangle}, \frac{w_2}{\langle 0.7e^{2\pi(0.3)}, 0.9e^{2\pi(0.2)}, 0.3e^{2\pi(0.2)} \rangle} \right\} \right) \end{array} \right\}.$$

**Definition 6.7.** A pair  $(V, A)$  is called a neutrosophic vague hypersoft expert set is a pair  $(V, A)$ , with  $V$  representing a mapping  $V : A \rightarrow NV^\Delta$ , and  $NV^\Delta$  is being used for the power neutrosophic vague set of  $\Delta$ . Let mapping  $V$  is defined by as  $V(t) = V(t)(x)$ ,  $x \in \Delta$ . For each  $t_i \in A$ ,  $V(t_i) = V(t_i)(x)$ , where  $V(t_i)$  represents the truth, indeterminacy and falsity membership functions of  $\Delta$  in  $V(t_i)$ . Hence  $V(t_i)$  can be written as

$$V(t_i) = \left\{ \frac{x_i}{V(t_i)x_i} \right\}, \text{ for } i = 1, 2, 3, \dots$$

where  $V(t_i)(x_i) = [v - \omega(t_i)(x_i), v + \omega(t_i)(x_i)]$ ,  $[I - \omega(t_i)(x_i), I + \omega(t_i)(x_i)]$ ,  $[w - \omega(t_i)(x_i), w + \omega(t_i)(x_i)]$  and  $v + \omega(t_i)(x_i) = 1 - \omega(t_i)(x_i)$ ,  $\omega + \omega(t_i)(x_i) = 1 - v - \omega(t_i)(x_i)$  with  $[v\omega(t_i)(x_i), v + \omega(t_i)(x_i)]$ ,  $[I - \omega(t_i)(x_i), I + \omega(t_i)(x_i)]$  representing the truth, indeterminacy and falsity-membership functions of each of the elements  $x_i \in \Delta$ , respectively.

**Example 6.8.** Considering Example 3.3 with  $\Delta = \{w_1, w_2\}$ , we have neutrosophic vague hypersoft expert set as

$$(\mathbb{B}, A) = \left\{ \begin{array}{l} \left( (v_1, c, 1), \left\{ \frac{w_1}{\langle [0.2,0.5],[0.4,0.1],[0.2,0.3] \rangle}, \frac{w_2}{\langle [0.1,0.3],[0.6,0.2],[0.3,0.2] \rangle} \right\} \right), \\ \left( (v_1, d, 1), \left\{ \frac{w_1}{\langle [0.4,0.2],[0.3,0.1],[0.1,0.2] \rangle}, \frac{w_2}{\langle [0.2,0.5],[0.3,0.1],[0.2,0.5] \rangle} \right\} \right), \\ \left( (v_1, e, 1), \left\{ \frac{w_1}{\langle [0.7,0.2],[0.3,0.8],[0.1,0.6] \rangle}, \frac{w_2}{\langle [0.3,0.5],[0.6,0.2],[0.3,0.4] \rangle} \right\} \right), \\ \left( (v_2, c, 1), \left\{ \frac{w_1}{\langle [0.9,0.1],[0.3,0.3],[0.2,0.1] \rangle}, \frac{w_2}{\langle [0.3,0.4],[0.8,0.1],[0.7,0.4] \rangle} \right\} \right), \\ \left( (v_2, d, 1), \left\{ \frac{w_1}{\langle [0.4,0.5],[0.6,0.2],[0.3,0.4] \rangle}, \frac{w_2}{\langle [0.2,0.6],[0.7,0.1],[0.3,0.4] \rangle} \right\} \right), \\ \left( (v_2, e, 1), \left\{ \frac{w_1}{\langle [0.5,0.4],[0.7,0.1],[0.2,0.3] \rangle}, \frac{w_2}{\langle [0.8,0.1],[0.6,0.2],[0.2,0.3] \rangle} \right\} \right), \\ \left( (v_1, c, 0), \left\{ \frac{w_1}{\langle [0.3,0.2],[0.1,0.1],[0.2,0.3] \rangle}, \frac{w_2}{\langle [0.1,0.8],[0.3,0.1],[0.7,0.2] \rangle} \right\} \right), \\ \left( (v_1, d, 0), \left\{ \frac{w_1}{\langle [0.1,0.8],[0.4,0.1],[0.2,0.3] \rangle}, \frac{w_2}{\langle [0.2,0.7],[0.5,0.1],[0.3,0.4] \rangle} \right\} \right), \\ \left( (v_1, e, 0), \left\{ \frac{w_1}{\langle [0.2,0.7],[0.5,0.1],[0.3,0.4] \rangle}, \frac{w_2}{\langle [0.5,0.4],[0.6,0.1],[0.3,0.4] \rangle} \right\} \right) \end{array} \right\}.$$

## 7. An Application to Neutrosophic Hypersoft Expert Set

An application of NHSES theory related to the decision-making problem is presented while using an algorithmic technique..

### Statement of the problem

Mr Jay needs to buy a mask from a business opportunity for his own wellbeing. He takes help from his a few companions (Henry, John and Watson) who have skill in mask buying.

### Proposed Algorithm For Selection Of Mask

The accompanying calculation is embraced for this choice (purchase).

- (1) Construct NHSES  $(\bar{h}, \mathbb{G})$ ,
- (2) Determine an Agree and Disagree-NHSES,
- (3) Compute  $d_i = \sum_i t_{ij}$  for Agree-NHSES,
- (4) Determine  $q_i = \sum_i t_{ij}$  for Disagree-NHSES,
- (5) Determine  $g_j = d_j - q_j$  for Agree and Disagree-NHSES,
- (6) Compute  $n$ , for which  $p_n = \max p_j$  for best solution of the product.

### Step-1

Let eight categories of mask which are being used for the universe of discourse  $\Omega = \{b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8\}$  and  $X = \{\rho_1 = Henry, \rho_2 = John, \rho_3 = Watson\}$  be a set of experts. The prescribed attributes for the attribute-valued sets are :

$$O_1 = Brand = \{o_1 = new, o_2 = old\}$$

$$O_2 = Price = \{o_3 = 100dollar, o_4 = 50dollar\}$$

$$O_3 = Colour = \{o_5 = black, o_6 = blue\}$$

$$O_4 = Quality = \{o_7 = good, o_8 = better\}$$

$$O_5 = Shape = \{o_9 = circular, o_{10} = square\}$$

and then  $O = O_1 \times O_2 \times O_3 \times O_4 \times O_5$

$$O = \left\{ \begin{array}{l} (o_1, o_3, o_5, o_7, o_9), (o_1, o_3, o_5, o_7, o_{10}), (o_1, o_3, o_5, o_8, o_9), (o_1, o_3, o_5, o_8, o_{10}), (o_1, o_3, o_6, o_7, o_9), \\ (o_1, o_3, o_6, o_7, o_{10}), (o_1, o_3, o_6, o_8, o_9), (o_1, o_3, o_6, o_8, o_{10}), (o_1, o_4, o_5, o_7, o_9), (o_1, o_4, o_5, o_7, o_{10}), \\ (o_1, o_4, o_5, o_8, o_9), (o_1, o_4, o_5, o_8, o_{10}), (o_1, o_4, o_6, o_7, o_9), (o_1, o_4, o_6, o_7, o_{10}), (o_1, o_4, o_6, o_8, o_9), \\ (o_1, o_4, o_6, o_8, o_{10}), (o_2, o_3, o_5, o_7, o_9), (o_2, o_3, o_5, o_7, o_{10}), (o_2, o_3, o_5, o_8, o_9), (o_2, o_3, o_5, o_8, o_{10}), \\ (o_2, o_3, o_6, o_7, o_9), (o_2, o_3, o_6, o_7, o_{10}), (o_2, o_3, o_6, o_8, o_9), (o_2, o_3, o_6, o_8, o_{10}), (o_2, o_4, o_5, o_7, o_9), \\ (o_2, o_4, o_5, o_7, o_{10}), (o_2, o_4, o_5, o_8, o_9), (o_2, o_4, o_5, o_8, o_{10}), (o_2, o_4, o_6, o_7, o_9), (o_2, o_4, o_6, o_7, o_{10}), \\ (o_2, o_4, o_6, o_8, o_9), (o_2, o_4, o_6, o_8, o_{10}) \end{array} \right\}$$

Now take  $Q \subseteq O$  as

$$Q = \{q_1 = (o_1, o_3, o_5, o_7, o_9), q_2 = (o_1, o_3, o_6, o_7, o_{10}), q_3 = (o_1, o_4, o_6, o_8, o_9), q_4 =$$

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$$(o_2, o_3, o_6, o_8, o_9), q_5 = (o_2, o_4, o_6, o_7, o_{10})\}$$

$$(\tilde{h}, \mathbb{G}) = \left\{ \begin{array}{l} ((q_1, \rho_1, 1) = \{o_2, o_3, o_4, o_5, o_6, o_8\}), ((q_1, \rho_2, 1) = \{o_1, o_2, o_3, o_7\}), ((q_2, \rho_1, 1) = \{o_5, o_8\}), \\ ((q_2, \rho_2, 1) = \{o_1, o_2, o_3, o_4, o_5, o_6, o_8\}), ((q_3, \rho_1, 1) = \{o_4, o_7\}), ((q_3, \rho_2, 1) = \{o_1, o_2, o_4, o_5, o_8\}), \\ ((q_3, \rho_3, 1) = \{o_1, o_5, o_7, o_8\}), \\ ((q_4, \rho_1, 1) = \{o_1, o_7, o_8\}), ((q_4, \rho_2, 1) = \{o_1, o_4, o_8\}), ((q_4, \rho_3, 1) = \{o_1, o_6, o_7, o_8\}), \\ ((q_5, \rho_1, 1) = \{o_3, o_7, o_8\}), ((q_5, \rho_2, 1) = \{o_1, o_2, o_3, o_4, o_5, o_8\}), ((q_5, \rho_3, 0) = \{o_1, o_3, o_6\}) \\ ((q_5, \rho_3, 1) = \{o_2, o_3, o_5, o_7, o_8\}), ((q_1, \rho_1, 0) = \{o_3, o_5, o_6\}), ((q_1, \rho_2, 0) = \{o_2, o_3, o_6, o_7\}), \\ ((q_1, \rho_3, 0) = \{o_3, o_4\}), ((q_2, \rho_1, 0) = \{o_1, o_2, o_4, o_5, o_6, o_7\}), ((q_2, \rho_2, 0) = \{o_2, o_7\}), \\ ((q_2, \rho_3, 0) = \{o_2, o_3, o_4, o_5, o_6\}), ((q_3, \rho_1, 0) = \{o_1, o_2, o_6, o_8\}), ((q_3, \rho_2, 0) = \{o_3, o_4, o_6, o_7\}), \\ ((q_3, \rho_3, 0) = \{o_2, o_3, o_4, o_5, o_7\}), ((q_5, \rho_1, 0) = \{o_4, o_6, o_7\}), ((q_4, \rho_1, 0) = \{o_2, o_3, o_3, o_4, o_5, o_7\}), \\ ((q_4, \rho_3, 0) = \{o_2, o_3, o_4, o_5\}), ((q_5, \rho_2, 0) = \{o_2, o_3, o_6, o_7\}), \\ ((q_1, \rho_3, 1) = \{o_1, o_3, o_4, o_6, o_7, o_8\}), ((q_4, \rho_2, 0) = \{o_2, o_3, o_6, o_7\}), \\ ((q_2, \rho_3, 1) = \{o_1, o_2, o_4, o_7, o_8\}), \end{array} \right\}$$

is a NHSES.

**Step-2**

The Agree and Disagree-NHSES are represented by Table 1 and Table 2 respectively, also when  $o_i \in F_1(\beta)$  then  $o_{ij} = \surd = 1$  diversely  $o_{ij} = \times = 0$ , and if

$$o_i \in F_0(\beta)$$

then  $o_{ij} = \surd = 1$  diversely  $o_{ij} = \times = 0$  while  $o_{ij}$  are being used as members of Tables 1 and 2.

**Step-(3-5)**

presents The  $d_i = \sum_i o_{ij}$  for Agree-NHSES,  $q_i = \sum_i o_{ij}$  for Disagree-NHSES are presented in Table 3 and  $g_j = d_j - q_j$  have been shown and to choose product  $p_n = \max p_j$  for solution.

**Step-6-Decision**

Since  $g_8$  is maximum in above Table 3, so category  $b_8$  is preferred to be selected for purchase.

**8. Conclusions**

In this paper,

- The fundamentals of neutrosophic hypersoft expert set are established and some necessary properties like subset, equal set, agree and disagree set, relative whole and relative null set, absolute whole set are explained with detailed examples.
- Some theoretic operations like union, restricted union, intersection, extended intersection, complement, AND and OR are generalized.
- Some basic laws such as idempotent, absorption, domination, identity, associative and distributive are discussed with examples.



TABLE 1. Agree-NHSES

$B$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
$(p_1, \rho_1)$	×	✓	✓	✓	✓	✓	×	×
$(p_2, \rho_1)$	×	✓	×	×	×	×	✓	×
$(p_3, \rho_1)$	×	×	×	✓	×	×	✓	×
$(p_4, \rho_1)$	✓	×	×	×	×	×	✓	✓
$(p_5, \rho_1)$	✓	✓	✓	✓	×	×	×	✓
$(p_1, \rho_2)$	✓	×	✓	×	×	×	✓	×
$(p_2, \rho_2)$	✓	✓	✓	×	✓	✓	×	✓
$(p_3, \rho_2)$	×	✓	✓	×	×	×	✓	×
$(p_4, \rho_2)$	✓	×	×	×	×	×	×	✓
$(p_5, \rho_2)$	×	×	×	×	✓	×	×	×
$(p_1, \rho_3)$	✓	×	✓	✓	×	✓	✓	×
$(p_2, \rho_3)$	✓	✓	×	✓	✓	×	✓	×
$(p_3, \rho_3)$	✓	×	✓	×	✓	×	✓	×
$(p_4, \rho_3)$	✓	✓	✓	✓	×	✓	✓	×
$(p_5, \rho_3)$	×	✓	✓	✓	✓	×	✓	×
$d_j = \sum_i n_{ij}$	$d_1 = 09$	$d_2 = 08$	$d_3 = 9$	$d_4 = 7$	$d_5 = 06$	$d_6 = 4$	$d_7 = 10$	$d_8 = 11$

TABLE 2. Disagree-NHSES

$B$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$
$(p_1, \rho_1)$	×	×	✓	×	×	✓	×	×
$(p_2, \rho_1)$	✓	✓	×	✓	×	✓	✓	×
$(p_3, \rho_1)$	✓	✓	×	×	×	✓	×	✓
$(p_4, \rho_1)$	×	✓	✓	✓	✓	✓	×	×
$(p_5, \rho_1)$	×	×	×	✓	×	✓	✓	×
$(p_1, \rho_2)$	×	✓	✓	×	×	✓	✓	×
$(p_2, \rho_2)$	×	✓	×	×	×	×	✓	×
$(p_3, \rho_2)$	✓	✓	×	×	×	✓	×	✓
$(p_4, \rho_2)$	×	✓	✓	×	×	✓	✓	×
$(p_5, \rho_2)$	×	✓	✓	×	×	✓	✓	×
$(p_1, \rho_3)$	×	×	✓	✓	×	×	×	×
$(p_2, \rho_3)$	×	×	✓	×	✓	✓	×	×
$(p_3, \rho_3)$	×	✓	✓	✓	✓	✓	×	×
$(p_4, \rho_3)$	×	✓	✓	✓	✓	×	×	×
$(p_5, \rho_3)$	×	✓	×	✓	×	✓	×	×
$p_i = \sum_j n_{ij}$	$p_1 = 3$	$p_2 = 11$	$p_3 = 9$	$p_4 = 7$	$p_5 = 4$	$p_6 = 12$	$p_7 = 6$	$p_8 = 2$

- Some hybridized structures of neutrosophic hypersoft expert set are established with illustrative examples.

TABLE 3. Optimal

$d_i = \sum_i n_{ij}$	$q_i = \sum_i n_{ij}$	$g_j = d_j - q_j$
$d_1 = 09$	$q_1 = 3$	$g_1 = 6$
$d_2 = 8$	$q_2 = 11$	$g_2 = -3$
$d_3 = 9$	$q_3 = 9$	$g_3 = 0$
$d_4 = 7$	$q_4 = 7$	$g_4 = 0$
$d_5 = 06$	$q_5 = 4$	$g_5 = 2$
$d_6 = 4$	$q_6 = 12$	$g_6 = -8$
$d_7 = 10$	$q_7 = 6$	$g_7 = 4$
$d_8 = 11$	$q_8 = 2$	$g_8 = 9$

- An algorithm is developed to explain the procedure of decision making problem.
- An application related to the mask purchasing is described with the help of proposed algorithm.
- Future task may include the extension of the existing work for other neutrosophic hypersoft expert-like hybrids i.e., generalized neutrosophic, generalized interval valued neutrosophic, neutrosophic vague, interval-valued neutrosophic, etc. This new work will give an outstanding extension to existing theories for dealing with truthness, indeterminacy and falsity membership functions.

## References

1. Smarandache. F, A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic. Rehoboth: American Research Press, **2005**.
2. Smarandache. F, Neutrosophic set, a generalization of the intuitionistic fuzzy sets, *International Journal of Pure and Applied Mathematics*, **2005**, 24(3), 287-297.
3. Smarandache. F, Neutrosophic seta generalization of the intuitionistic fuzzy set, *Journal of Defense Resources Management (JoDRM)*, **2010**, 1(1), 107-116.
4. Zadeh, L. A., Fuzzy sets, *Information and Control*, **1965**, 8, 338-353.
5. Atanassov. K. T, Intuitionistic Fuzzy Sets, *Fuzzy Sets and Systems*, **1986**, 20(1), 87-96.
6. Wang. H, Smarandache. F, Zhang. Y, and Sunderraman. R, Single valued Neutrosophic Sets, *Multispace and Multistructure*, **2010**, 4, 410-413.
7. Kharal, A. A., Neutrosophic multi-criteria decision making method, *New Mathematics and Natural Computation*, **2010**, 10(02), 143-162.
8. Ansaria. A. Q., Biswas. R, and Aggarwal. S, Neutrosophic classifier, *An extension of fuzzy classifier, Applied Soft Computing*, **2013**, 13, 563-573.
9. Salama. A. A., Alblowi. S. A, Neutrosophic Set and Neutrosophic Topological Spaces, *ISOR Journal of Mathematics*, **2012**, 3(3), 31-35.
10. Salama. A. A., Neutrosophic Crisp Point and Neutrosophic Crisp Ideals, *Neutrosophic Sets and Systems*, **2013**, 1(1), 50-54.
11. Salama. A. A., and Smarandache. F, Filters via Neutrosophic Crisp Sets, *Neutrosophic Sets and Systems*, **2013**, 1(1), 34-38.

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12. Rabounski. D., Rabounski, D., Smarandache, F., and Borissova, L. *Neutrosophic methods in general relativity*, **2013**, (Vol. 10). Infinite Study.
13. Ye. J. Similarity measure between interval neutrosophic sets and their applications in multicriteria decision making, *Journal of intelligent and fuzzy systems*, **2014**, 26, 165-172.
14. Ye, J. Multiple attribute group decision-making method with completely unknown weights based on similarity measures under single valued neutrosophic environment, *Journal of Intelligent and Fuzzy Systems*, **2014**, 27(6), 2927-2935.
15. Ye. J. Single valued neutrosophic cross-entropy for multicriteria decision making problems, *Applied Mathematical Modelling*, **2014**, 38, 1170-1175.
16. Ye. J. Single valued neutrosophic minimum spanning tree and its clustering method, *Journal of Intelligent Systems*, **2014**, 23(3), 311-324.
17. Ye, J. Multicriteria decision-making method using the correlation coefficient under single-valued neutrosophic environment, *International Journal of General Systems*, **2014**, 42(4), 386-394.
18. Ye, J. Some aggregation operators of interval neutrosophic linguistic numbers for multiple attribute decision making, *Journal of Intelligent and Fuzzy Systems*, **2014**, 27(5), 2231-2241.
19. Ye. J. Vector similarity measure of simplified neutrosophic sets and their application in multicriteria decision making, *International Journal of Fuzzy Systems*, **2014**, 16(2), 204-215.
20. Ye. J. A Multicriteria decision-making method using aggregation operators for simplified neutrosophic sets, *Journal of Intelligent and Fuzzy System*, **2014**, 26, 24592466.
21. Broumi. S, and Smarandache. F, Intuitionistic Neutrosophic Soft Set, *Journal of Information and Computing Science*, **2013**, 8(2), 130-140.
22. Broumi. S, Generalized Neutrosophic Soft Set, *International Journal of Computer Science, Engineering and Information Technology*, **2013**, 3(2), 17-30.
23. Broumi. S, and Smarandache. F, More on Intuitionistic Neutrosophic Soft Sets, *Computer Science and Information Technology*, **2013**, 1(4), 257-268.
24. Broumi. S, Generalized Neutrosophic Soft Set, *International Journal of Computer Science, Engineering and Information Technology*, **2013**, 3(2), 17-30.
25. Broumi. S, Smarandache. F., Several Similarity Measures of Neutrosophic Sets, *Neutrosophic Sets and Systems*, **2013**, 1, 54-62.
26. Broumi. S, Deli. I, and Smarandache. F., Relations on Interval Valued Neutrosophic Soft Sets, *Journal of New Results in Science*, **2014**, 5, 1-20.
27. Broumi. S, Deli. I, Smarandache. F., Neutrosophic Parameterized Soft Set theory and its decision making problem, *Italian Journal of Pure and Applied Mathematics*, **2014**, 32, 1 -12.
28. Broumi. S, Smarandache. F., On Neutrosophic Implications, *Neutrosophic Sets and Systems*, **2014**, Vol. 2, 9-17.
29. Broumi. S, Smarandache. F., Rough neutrosophic sets, *Italian Journal of Pure and applied Mathematics*, **2014**, 32, 493-502.
30. Broumi. S, Sahin. R and Smarandache. F., Generalized Interval Neutrosophic Soft Set and its Decision Making Problem, *Journal of New Results in Science*, **2014**, 7, 29-47.
31. Broumi. S, Smarandache. F., and Maji, P. K., Intuitionistic Neutrosophic Soft Set over Rings, *Mathematics and Statistics*, **2014**, 2(3), 120-126.
32. Broumi. S, Smarandache. F., Single valued neutrosophic trapezoid linguistic aggregation operators based multi-attribute decision making, *Bulletin of Pure and Applied Sciences- Mathematics and Statistics*, **2014**, 33(2), 135-155.
33. Broumi. S, Smarandache. F., Single valued neutrosophic soft expert sets and their application in decision making, *Journal of New Theory*, **2015**, 3, 67-88.

34. Molodtsov, D, Soft set theory first results, *Computers and Mathematics with Applications*, **1999**, 37(4-5), 19-31.
35. Rahman, A. U. Saeed, M. Ihsan, M. Arshad, M. & Ayaz, S., A Conceptual Framework of m-Convex and m-Concave Sets Under Soft Set Environment With Properties, *Transactions in Mathematical and Computational Sciences*, **2021**,1(1), pp. 5060.
36. Rahman, A. U. Saeed, M. Arshad, M. Ihsan, M. & Ahmad, M. R., (m, n)-convexity-concavity on Fuzzy soft set with applications in First and Second Sense, *Punjab University Journal Of Mathematics*, **2021**, 1(53), pp19-33.
37. Maji, P. K., Roy, A. R. and Biswas, R., Fuzzy Soft Sets, *Journal of Fuzzy Mathematics*, **2001**, 9, 589-602.
38. Chen, D, Tsang, E. C. C, D., Yeung, S, and Wang, X, The parameterization reduction of soft sets and its applications, *Computers and Mathematics with Applications*, **2005**, 49(5-6), 757-763.
39. Maji, P. K., Biswas, R, and Roy, A. R., Soft set theory, *Computers and Mathematics with Applications*, **2003** 45(4-5), 555-562.
40. Maji, P. K., Roy, A. R, and Biswas, R, An application of soft sets in a decision making problem, *Computers and Mathematics with Applications*, **2002**, 44(8-9), 1077-1083.
41. Alkhazaleh, S., Salleh, A. R., and Hassan. N, Soft multisets theory, *Applied Mathematical Sciences*, **2011** 5(72), 3561-3573.
42. Alkhazaleh. S., Salleh, A. R., and Hassan. N., Possibility fuzzy soft set, *Advances in Decision Sciences*, **2011**.
43. Alkhazaleh. S., Salleh, A. R., and Hassan. N., Fuzzy parameterized interval-valued fuzzy soft set, *Applied Mathematical Sciences*, **2011**, 5(67), 3335-3346.
44. Ali. M.I., Feng, F, Liu, X, Min, W. K, and Shabir, M., On some new operations in soft set theory, *Computers and Mathematics with Applications*, **2009**, 57(9), 1547-1553.
45. A. U. Rahman, M. Saeed, M. Arshad, M. Ihsan, & M. R. Ahmad, (m, n)-convexity-concavity on Fuzzy soft set with applications in First and Second Sense, *Punjab University Journal Of Mathematics*, **2021**, (53)1, pp19-33.
46. Alkhazaleh, S., Salleh, A.R., Soft Expert Sets, *Advances in Decision Sciences*, **2011**, 757868-1.
47. Alkhazaleh, S., and Salleh, A.R., Fuzzy soft expert set and its application, *Applied Mathematics*, **2014**, 5, 1349-1368.
48. Ihsan, M., Saeed, M., & Rahman, A.U., A Rudimentary Approach to Develop Context for Convexity cum Concavity on Soft Expert Set with Some Generalized Results, *Punjab University Journal of Mathematics*, **2021**, 9(53), 621-629.
49. Ihsan, M. Rahman, A. U., Saeed, M., & Khalifa, H. A. E. W., Convexity-Cum-Concavity on Fuzzy Soft Expert Set with Certain Properties, *International Journal of Fuzzy Logic and Intelligent Systems*, **2021**, 3(21), 233-242.
50. Broumi, S., and Smarandache, F., Intuitionistic fuzzy soft expert sets and its application in decision making, *Journal of new theory*, **2015**, 1, 89-105.
51. Smarandache, F., Extension of Soft Set to Hypersoft Set and then to Plithogenic Hypersoft Set, *Neutrosophic Sets Syst*, **2018**, 22, 168-170.
52. Saeed, M., Ahsan, M., Siddique, M. K., and Ahmad, M. R., A Study of The Fundamentals of Hypersoft Set Theory, *International Journal of Scientific and Engineering Research*, **2020**, 11.
53. Rahman, A.U., Saeed, M., Smarandache, F., and Ahmad, M.R., Development of Hybrids of Hypersoft Set with Complex Fuzzy Set, Complex Intuitionistic Fuzzy set and Complex Neutrosophic Set, *Neutrosophic Sets and Syst*, **2020**, 38, 335-354.
54. Rahman, A.U., Saeed, M., and Smarandache, F., Convex and Concave Hypersoft Sets with Some Properties, *Neutrosophic Sets Syst*, **2020**, 38, 497-508.

55. Al-Quran, A., Hassan, N., Neutrosophic vague soft expert set theory, *Journal of Intelligent Fuzzy Systems*, **2016**, 30(6), 3691-3702
56. Bashir, M., Salleh, A. R., Fuzzy parameterized soft expert set, *In Abstract and Applied Analysis*, **2012**. Hindawi.
57. Bashir, M., Salleh, A. R., Possibility fuzzy soft expert set, *Open Journal of Applied Sciences*, **2012**, 12, 208-211.
58. Ihsan, M. Rahman, A.U. M. Saeed, M., Hypersoft Expert Set With Application in Decision Making for Recruitment Process, *Neutrosophic Sets and Systems* **2021**, 42, 191-207.
59. Ihsan, M. Rahman, A.U. M. Saeed, M., Fuzzy Hypersoft Expert Set with Application in Decision Making for the Best Selection of Product, *Neutrosophic Sets and Systems*, **2021**, 46, 318-336.
60. Kamac, H. and Saqlain, M., n-ary Fuzzy hypersoft expert sets, *Neutrosophic Sets and Systems* **2021**, 1(43), 15.
61. Kamac, H., On Hybrid Structures of Hypersoft Sets and Rough Sets. **2021**.
62. Kamac, H., Simplified Neutrosophic Multiplicative Refined Sets and Their Correlation Coefficients with Application in Medical Pattern Recognition. *Neutrosophic Sets and Systems*, **2021**, 41, 270-285.
63. Kamac, H., Rough Approximations of Complex Quadripartitioned Single Valued Neutrosophic Sets, *Journal of New Theory*, (34), 45-63.
64. Kamac, H., Linguistic single-valued neutrosophic soft sets with applications in game theory, *International Journal of Intelligent Systems*, **2021**.
65. Kamac, H., Garg, H., & Petchimuthu, S., Bipolar trapezoidal neutrosophic sets and their Dombi operators with applications in multicriteria decision making, *Soft Computing*, (**2021**), 1-24.

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