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Abstract
In this article, we propose a MADM-strategy based on hyperbolic cosine similarity measures under the single valued neutrosophic set environment. Further, we also give some properties of the similarity measures by giving some suitable examples. We also solve a numerical example to validate our proposed MADM-model.

Keywords: MADM-Strategy; Neutrosophic Set; Similarity Measure; Distillation Unit.

1. Introduction

In the year 1998, Smarandache [18] grounded the concept of neutrosophic set (in short NS) as a generalization of the notion of fuzzy set [27] and intuitionistic fuzzy set (in short IFS) [1] theory to deal with incomplete and indeterminate information. In every NS, truth membership, indeterminacy membership, and falsity membership values of each element are independent of each other. Indeterminacy-membership plays a vital role in many real world multi attribute decision making (in short MADM) problems. In the year 2010, Wang et al. [21] presented the concept of single valued neutrosophic set (in short SVNS), which is the subclass of an NS. By using SVNS, we can represent incomplete, imprecise, and indeterminate information that helps in decision making in the real world. The notion of SVNS and the various extensions of SVNS have been used in the formation of MADM-model / MADM-algorithm in different branch (branches) of real world such as medical diagnosis, educational problem, social problems, decision making problems, conflict resolution, etc. In the year (In) 2014, Biswas et al. [2] proposed the entropy based grey relational analysis (in short GRA) method and developed a MADM-strategy under SVNS-environment. Afterwards, Dey et al. [4] proposed a MADM model for the select ion of weaver based on extended GRA method under the interval NS environment. Later on, Dey et al. [5] also proposed a MADM-strategy under the interval NS environment based on extended projection method. In the year 2016, Mondal et al. [11] studied the role of SVNS in data mining. In the year 2016, Pramanik et al. [13] proposed a MADM-strategy to choose the logistic center location. Later on, Mondal et al. [10] defined a similarity measure under the SVNS environment namely single valued neutrosophic hyperbolic sine similarity measure, and proposed a MADM-strategy based on it. In the year 2015, Pramanik and Mondal [15, 16] proposed two MADM-strategies under the rough neutrosophic set environment. Afterwards, several MADM-strategies has been developed by Ye [22-25], Ye and Zhang [26], etc. using different similarity measure under the SVNS environment.

In this study, we propose a MADM-strategy based using (on) the single valued weighted hyperbolic cosine similarity measure under the SVNS-environment. Further, we validate the proposed model by solving an illustrative numerical example entitled "Selection of the Most Suitable Distillation Unit under SVNS-Environment".

There is no study in the literature relating to MADM-strategy using single valued neutrosophic weighted hyperbolic cosine similarity measure under the SVNS-environment. To fill the research gap, we propose this MADM-strategy under SVNS-environment based on single valued neutrosophic weighted hyperbolic cosine similarity measure.

The rest of the paper has been split into the following sections:

In section-2, we recall SVNS and its different properties. In section-3, we introduce a new similarity measure namely single valued neutrosophic weighted hyperbolic cosine similarity measure of similarities between two single valued neutrosophic numbers. In section-4, we propose a MADM-strategy based on single valued neutrosophic weighted hyperbolic cosine similarity measure under the SVNS-environment. In section-5, we give a numerical example to show the applicability and effectiveness of the proposed MADM-strategy. In section-6, we conclude the work done in this paper by stating some future scope of research.

2. Preliminaries and Definitions

In this section, we give some basic definitions and results those are relevant for developing the main results of this article.

Definition 2.1. [18] A single valued neutrosophic set K over a fixed set L is defined by $K=\{(u, T_K(u), I_K(u), F_K(u)): u \in L\}$, where T_K, I_K, F_K are truth, indeterminacy and falsity membership mappings from L to [0, 1], and so $0 \le T_K(u) + I_K(u) + F_K(u) \le 3$.

The null SVNS (0_N) and the absolute SVNS (1_N) over a fixed set L are defined as follows:

(*i*) $0_N = \{(u, 0, 1, 1) : u \in L\}$,

(*ii*) $1_N = \{(u, 1, 0, 0) : u \in L\}$.

Example 2.1. Assume that $L=\{a, b\}$ be a fixed set. Then, $K=\{(a,0.3,0.2,0.6), (b,0.9,0.5,0.8)\}$ is a SVNS over L.

Definition 2.2. [18] Suppose that X={(u, $T_X(u)$, $I_X(u)$, $F_X(u)$): $u \in L$} and Y={(u, $T_Y(u)$, $I_Y(u)$, $F_Y(u)$): $u \in L$} be two SVNSs over L. Then, $X \subseteq Y$ if and only if $T_X(u) \leq T_Y(u)$, $I_X(u) \geq I_Y(u)$, $F_X(u) \geq F_Y(u)$, for all $u \in L$.

Example 2.2. Assume that L={a, b} be a fixed set. Let K={(a,0.3,0.5,0.6), (b,0.2,0.5,0.8)} and S={(a,0.4,0.3,0.6), (b,0.4,0.5,0.6)} be two SVNSs over L. Then, $K \subseteq S$.

Definition 2.3. [18] Assume that X = {(u, $T_X(u)$, $I_X(u)$, $F_X(u)$): $u \in L$} and Y = {(u, $T_Y(u)$, $I_Y(u)$, $F_Y(u)$): $u \in L$} be two SVN-Sets over L. Then, $X \cup Y$={(u, max {$T_X(u)$, $T_Y(u)$}, min {$I_X(u)$, $I_X(u)$}, min {$F_X(u)$, $F_X(u)$}): $u \in L$}.

Example 2.3. Suppose that K={(a,0.3,0.7,0.2), (b,0.9,0.4,0.8)} and S={(a,0.4,0.3,0.6), (b,0.4,0.5, 0.6)} be two SVNSs over a fixed set L={a, b}. Then, $K \cup S$={(a,0.4,0.3,0.2), (b,0.9,0.4,0.6)}.

Definition 2.4. [18] Suppose that X = {(u, $T_X(u)$, $I_X(u)$, $F_X(u)$): $u \in L$} and Y = {(u, $T_Y(u)$, $I_Y(u)$, $F_Y(u)$): $u \in L$} be two SVN-Sets over L. Then, $X \cap Y$ = {(u, min {$T_X(u)$, $T_Y(u)$}, max {$I_X(u)$, $I_X(u)$}, max {$F_X(u)$, $F_X(u)$}): $u \in L$}.

Example 2.4. Suppose that K and S be two SVNSs over a fixed set L={a, b} as shown in Example 2.3. Then, $K \cap S$={(a,0.3,0.7,0.6), (b,0.4,0.5,0.8)}.

Definition 2.5. [18] Suppose that X = {(u, $T_X(u)$, $I_X(u)$, $F_X(u)$): $u \in W$} and Y = {(u, $T_Y(u)$, $I_Y(u)$, $F_Y(u)$): $u \in L$} be two SVN-Sets over L. Then, X^c= {(u, 1-$T_X(u)$, 1-$I_X(u)$, 1-$F_X(u)$): $u \in L$}.

Example 2.5. Assume that K={(a,0.3,0.2,0.6), (b,0.9,0.5,0.8)} be a SVNS over L={a, b} as shown in Example 2.1. Then, K^c={(a,0.7,0.8,0.4), (b,0.1,0.5,0.2)}.

3. Single Valued Neutrosophic Hyperbolic Cosine Similarity Measure

In this section, we introduce a new similarity measure namely single valued neutrosophic weighted hyperbolic cosine similarity measure under the SVNS-environment. Then, we formulate some basic results based on it.

Definition 3.1. Suppose that M = {(u_i, $T_M(u_i)$, $I_M(u_i)$, $F_M(u_i)$): $u_i \in L$, i=1, 2, …, n} and W = {(u_i, $T_W(u_i)$, $I_W(u_i)$, $F_W(u_i)$): $u_i \in L$, i=1, 2, …, n } be two SVNS over a non-empty set L. Then, the single valued neutrosophic hyperbolic cosine similarity measure (in short SVNHCSM) of the similarity between the SVNSs M and W is defined by:

$$\text{SVNHCSM}(M, W) = 1 - \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\cosh(|T_M(u_i)-T_W(u_i)| + |I_M(u_i)-I_W(u_i)| + |F_M(u_i)-F_W(u_i)|)}{11}\right) \quad (1)$$

Example 3.1. Let M = {(a,0.5,0.3,0.5), (b,0.3,0.5,0.4)} and W = {(a,0.6,0.4,0.3), (b,0.7,0.5, 0.4)} be two SVNSs over a fixed set L={a, b}. Then, SVNHCSM (M, W) = 0.9017206935.

Definition 3.2. Suppose that M = {(u_i, $T_M(u_i)$, $I_M(u_i)$, $F_M(u_i)$): $u_i \in L$, i=1, 2, …, n} and W = {(u_i, $T_W(u_i)$, $I_W(u_i)$, $F_W(u_i)$): $u_i \in L$, i=1, 2, …, n} be two SVNSs over a fixed set L. Then, the single valued neutrosophic weighted hyperbolic cosine similarity measure (in short SVNWHCSM) of the similarity between the SVNSs M and W is defined by:

$$\text{SVNWHCSM}(M, W) = 1 - \frac{1}{n}\sum_{i=1}^{n} w_i\left(\frac{\cosh(|T_M(u_i)-T_W(u_i)| + |I_M(u_i)-I_W(u_i)| + |F_M(u_i)-F_W(u_i)|)}{11}\right), \quad (2)$$

where $0 \leq w_i \leq 1$ and $\sum_{i=}^{n} w_i$=1.

Example 3.2. Let us consider two SVNSs M and W as shown in Example 3.1. Assume that w_1= 0.5 and w_2= 0.4 be the corresponding weights of M and W. Then, SVNWHCSM (M, W) = 0.9557743121.

Theorem 3.1. Let SVNHCSM (M, W) be the single valued neutrosophic hyperbolic cosine similarity measure between the SVNSs M and W. Then, 0 ≤ SVNHCSM (M, W) ≤ 1.

Proof. Suppose that $M = \{(u_i, T_M(u_i), I_M(u_i), F_M(u_i)): u\in L, i=1, 2, \ldots, n\}$ and $W = \{(u_i, T_W(u_i), I_W(u_i), F_W(u_i)): u_i\in L, i=1, 2, \ldots, n\}$ be two SVN-Sets over a fixed set L.

Now, $0 \le T_M(u_i), I_M(u_i), F_M(u_i), T_W(u_i), I_W(u_i), F_W(u_i) \le 1$, for each $i=1, 2, \ldots, n$

$\Rightarrow 0 \le |T_M(u_i) - T_W(u_i)| + |I_M(u_i) - I_W(u_i)| + |F_M(u_i) - F_W(u_i)| \le 3$, for each $i=1, 2, \ldots, n$

$\Rightarrow 0 \le \frac{cosh(|T_M(u_i)-T_W(u_i)| + |I_M(u_i)-I_W(u_i)| + |F_M(u_i)-F_W(u_i)|)}{11} \le 1$, for each $i=1, 2, \ldots, n$

$\Rightarrow 0 \le 1 - \frac{1}{n}\sum_{i=1}^{n}\frac{cosh(|T_M(u_i)-T_W(u_i)| + |I_M(u_i)-I_W(u_i)| + |F_M(u_i)-F_W(u_i)|)}{11} \le 1$

$\Rightarrow 0 \le$ SVNHCSM $(M, W) \le 1$.

Theorem 3.2. Assume that SVNHCSM (M, W) be the single valued neutrosophic hyperbolic cosine similarity measure of the similarities between two SVPNSs M and W. If $M = W$, then SVNHCSM $(M, W) = 1$.

Proof. Suppose that $M = \{(u_i, T_M(u_i), I_M(u_i), F_M(u_i)): u_i\in L, i=1, 2, \ldots, n\}$ and $W = \{(u_i, T_W(u_i), I_W(u_i), F_W(u_i)): u_i \in L, i=1, 2, \ldots, n\}$ be two SVN-Sets over a fixed set L such that $M = W$.

So, $T_M(u_i) = T_W(u_i), I_M(u_i) = I_W(u_i), F_M(u_i) = F_W(u_i)$, for each $u_i\in L$ $(i=1, 2, \ldots, n)$

$\Rightarrow |T_M(u_i) - T_W(u_i)| = 0, |I_M(u_i) - I_W(u_i)| = 0, |F_M(u_i) - F_W(u_i)| = 0$, for each $u_i\in L$ $(i=1, 2, \ldots, n)$

$\Rightarrow cosh(|T_M(u_i) - T_W(u_i)| + |I_M(u_i) - I_W(u_i)| + |F_M(u_i) - F_W(u_i)|) = 0$, for each $u_i\in L$ $(i=1, 2, \ldots, n)$

$\Rightarrow \frac{1}{n}\sum_{i=1}^{n}\frac{cosh(|T_M(u_i)-T_W(u_i)| + |I_M(u_i)-I_W(u_i)| + |F_M(u_i)-F_W(u_i)|)}{11} = 0$

$\Rightarrow 1 - \frac{1}{n}\sum_{i=1}^{n}\frac{cosh(|T_M(u_i)-T_W(u_i)| + |I_M(u_i)-I_W(u_i)| + |F_M(u_i)-F_W(u_i)|)}{11} = 1$

$\Rightarrow$ SVNHCSM $(M, W) = 1$.

Theorem 3.3. Assume that SVNHCSM (M, W) be the single valued neutrosophic hyperbolic cosine similarity measure of the similarities between two SVN-Sets M and W. Then, SVNHCSM (M, W) = SVNHCSM (W, M).

Proof. Suppose that $M = \{(u_i, T_M(u_i), I_M(u_i), F_M(u_i)): u\in L, i=1, 2, \ldots, n\}$ and $W = \{(u_i, T_W(u_i), I_W(u_i), F_W(u_i)): u_i\in L, i=1, 2, \ldots, n\}$ be two SVN-Sets over L.

Now, SVNHCSM (M, W)

$= 1- \frac{1}{n}\sum_{i=1}^{n}\left(\frac{cosh(|T_M(u_i)-T_W(u_i)| + |I_M(u_i)-I_W(u_i)| + |F_M(u_i)-F_W(u_i)|)}{11}\right)$

$= 1- \frac{1}{n}\sum_{i=1}^{n}\left(\frac{cosh(|T_W(u_i)-T_M(u_i)| + |I_W(u_i)-I_M(u_i)| + |F_W(u_i)-F_M(u_i)|)}{11}\right)$

= SVNHCSM (W, M).

Therefore, SVNHCSM (M, W) = SVNHCSM (M, W).

Theorem 3.4. Suppose that SVNHCSM (M, W) be the single valued neutrosophic hyperbolic cosine similarity measure of the similarity between the SVN-Sets M and W. If Q be a SVN-Set over L such that $M\subseteq W\subseteq Q$, then SVNHCSM $(M, W) \ge$ SVNHCSM (M, Q) and SVNHCSM $(W, Q) \ge$ SVNHCSM (M, Q).

Proof. Suppose that $M = \{(u_i, T_M(u_i), I_M(u_i), F_M(u_i)): u_i\in L, i=1, 2, \ldots, n\}$ and $W = \{(u_i, T_W(u_i), I_W(u_i), F_W(u_i)): u_i\in L, i=1, 2, \ldots, n\}$ be two SVN-Sets over L. Let Q be a SVN-Set over L such that $M\subseteq W\subseteq Q$. Since $M\subseteq W\subseteq Q$, so $|T_M(u_i)-T_W(u_i)| \le |T_M(u_i)-T_Q(u_i)|$, $|I_M(u_i)-I_W(u_i)| \le |I_M(u_i)-I_Q(u_i)|$, $|F_M(u_i)-F_W(u_i)| \le |F_M(u_i)-F_Q(u_i)|$, $\forall u_i\in L$, $i=1, 2, \ldots, n$.

Now, we have

SVNHCSM (M, W)

$$= 1- \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\cosh(|T_M(u_i)-T_W(u_i)| + |I_M(u_i)-I_W(u_i)| + |F_M(u_i)-F_W(u_i)|)}{11}\right)$$

$$\geq 1- \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\cosh(|T_M(u_i)-T_Q(u_i)| + |I_M(u_i)-I_Q(u_i)| + |F_M(u_i)-F_Q(u_i)|)}{11}\right)$$

= SVNHCSM (M, Q).

Therefore, SVNHCSM (M, W) ≥ SVNHCSM (M, Q).

Again, from $M \subseteq W \subseteq Q$ it can be say that $|T_W(u_i)-T_Q(u_i)| \leq |T_M(u_i)-T_Q(u_i)|$, $|I_W(u_i)-I_Q(u_i)| \leq |I_M(u_i)-I_Q(u_i)|$, $|F_M(u_i)-F_W(u_i)| \leq |F_M(u_i)-F_Q(u_i)|$, $\forall u_i \in L$, i=1, 2, …, n.

Now, we have

SVNHCSM (W, Q)

$$= 1- \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\cosh(|T_W(u_i)-T_Q(u_i)| + |I_W(u_i)-I_Q(u_i)| + |F_W(u_i)-F_Q(u_i)|)}{11}\right)$$

$$\geq 1- \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\cosh(|T_M(u_i)-T_Q(u_i)| + |I_M(u_i)-I_Q(u_i)| + |F_M(u_i)-F_Q(u_i)|)}{11}\right)$$

= SVNHCSM (M, Q).

Therefore, SVNHCSM (M, W) ≥ SVNHCSM (M, Q).

4. SVNWHCSM Based MADM Strategy

Let Q = {Q_1, Q_2, ..., Q_n} be the fixed set of possible alternatives and P = {P_1, P_2, ..., P_m} be the collection of attributes for a multi attribute decision making (in short MADM) problem. Then, a decision maker can provide their evaluation information of each alternative Q_i (i = 1, 2, ..., n) against the attributes P_j (j = 1, 2, ..., m) in terms of SVNS. Then, the whole evaluation information of all alternatives can be expressed by a decision matrix.

The following are the steps of the proposed MADM-technique:

Step-1: Construct the Decision Matrix Using the SVNS

The whole evaluation information of each alternative Q_i (i = 1, 2,..., n) based on the attributes P_j (j = 1, 2, ..., m) is expressed in terms of SVN-Set E_{Q_i}= {(P_j, $T_{ij}(Q_i, P_j)$, $I_{ij}(Q_i, P_j)$, $F_{ij}(Q_i, P_j)$): $P_j \in P$}, where ($T_{ij}(Q_i, P_j)$, $I_{ij}(Q_i, P_j)$, $F_{ij}(Q_i, P_j)$) denotes the evaluation assessment of Q_i (i = 1, 2,..., n) against P_j (j = 1, 2, ..., m).

Then, we can build the decision matrix (DM[Q|P]) as follows:

	P_1	P_2	….	P_m
Q_1	[$T_{11}(Q_1, P_1)$, $I_{11}(Q_1, P_1)$, $F_{11}(Q_1, P_1)$]	[$T_{12}(Q_1, P_2)$, $I_{12}(Q_1, P_2)$, $F_{12}(Q_1, P_2)$]	….	[$T_{1m}(Q_1, P_m)$, $I_{1m}(Q_1, P_m)$, $F_{1m}(Q_1, P_m)$]
Q_2	[$T_{21}(Q_2, P_1)$, $I_{21}(Q_2, P_1)$, $F_{21}(Q_2, P_1)$]	[$T_{22}(Q_2, P_2)$, $I_{22}(Q_2, P_2)$, $F_{22}(Q_2, P_2)$]	….	[$T_{2m}(Q_2, P_m)$, $I_{2m}(Q_2, P_m)$, $F_{2m}(Q_2, P_m)$]
….	……………………..	………………………..	….	……………………..

Q_n	$[T_{n1}(Q_n, P_1), I_{n1}(Q_n, P_1), F_{n1}(Q_n, P_1)]$	$[T_{n2}(Q_n, P_2), I_{n2}(Q_n, P_2), F_{n2}(Q_n, P_2)]$	….	$[T_{nm}(Q_n, P_m), I_{nm}(Q_n, P_m), F_{nm}(Q_n, P_m)]$

Step-2: Determination of the Attributes Weight

In an MADM-strategy, the weights of the attributes play an important role in taking decision. When the weights of the attributes are totally unknown to the decision makers, then the attribute weights can be determined by using the compromise function defined in equation (3).

Compromise Function: The compromise function of Q is defined by:

$$\Omega_j=\sum_{i=1}^{n} (2+T_{ij}(Q_i, P_j)-I_{ij}(Q_i, P_j)-F_{ij}(Q_i, P_j))/3 \tag{3}$$

Then the desired weight of the j^{th} attribute is defined by $w_j= \frac{\Omega_j}{\sum_{j=1}^{m}\Omega_j}$ (4)

Here, $\sum_{j=1}^{m} w_j=1$.

Step-3: Determination of ideal solution

In any similarity measure based MADM-strategy, the selection of ideal solution is the key factor to find the most suitable alternative. In our proposed MADM-strategy, we take the absolute SVNS 1_N as an ideal solution to find the suitable alternative.

Step-4: Determination of single valued neutrosophic weighted hyperbolic cosine similarity value

After the formation of ideal solution in step-3, by using eq. (1), we calculate the SVNWHCSM values for every alternative between the ideal solution and the corresponding SVNS from decision matrix DM[$Q|P$] that formed in step-1.

Step-5: Ranking Order of the Alternatives

In this step, we arrange the all the single valued neutrosophic weighted hyperbolic cosine similarity value in ascending order. The alternative with the lowest single valued neutrosophic weighted hyperbolic cosine similarity value with the ideal solution is the most suitable alternative for selection.

Step-6: End.

5. Validation of the Proposed MADM-strategy

In this section, we demonstrate a numerical example to show the real life applicability of the proposed MADM-strategy.

Example 5.1. "Selection of the Most Suitable Distillation Unit under SVNS-Environment"

Distillation units are one of the essential laboratory equipment in modern day science. A solvent distillation unit or distilled machine comes in various designs, capacities and lab grade solvent purity level. The distillation process removes minerals and microbiological contaminants and can reduce levels of chemical contaminants through boiling the target solvent. The distillation apparatus structurally consists of flask with heating elements embedded in glass and a fused spiral coil tapered round glass, joints at the top double walled condenser with ground glass joints.

Successful distillation depends on several factors, including the difference in boiling points of the materials in the mixture, and therefore the difference in their vapor pressures, the type of apparatus used, and the care exercised by the experimentalist. In heating, the lowest boiling distills first (most volatile), having a maximum boiling point distills last, and others subsequently or not at all. Distillation is a simple apparatus with entirely satisfactory for the purification of a solvent containing nonvolatile material and is reasonably adequate for separating liquids of wide-ranged boiling points. Industrially, distillation is the basis for the separation of crude oil into the various, more useful hydrocarbon fractions. Chemically, distillation is the principal method for purifying liquids (e.g. samples, or solvents for performing reactions).

Structure:

A distilling flask, a source of heat or a hot bath, condenser, receiving flask to collect the condensed vapors or distillate are the basic structural units of an ideal distillation apparatus. For laboratory use, the apparatus is commonly made of glass and connected with corks, rubber bungs, or ground-glass joints, wherein in industrial applications, larger equipment of metal or ceramic is used. The underlying mechanism of distillation is the differences in volatility between individual components. With sufficient heat applied, a gas phase (vapor) is formed from the liquid solution. The liquid product is subsequently condensed from the gas phase by the removal of the heat.

Process:

There are many types of distillation units used in modern laboratories and industries based on their application. Some are simple distillation, fractional distillation, steam distillation, and vacuum distillation.

(i) **Simple distillation:** In simple distillation heating of the liquid mixture at the boiling point and immediately condensing the resulting vapors. This method is only effective for mixtures wherein the boiling points of the liquids are considerably different (~ 25^{o}C).

(ii) **Fractional distillation:** Simple distillation is not efficient for separating liquids whose boiling points lie close to one another. Fractional distillation is often used to separate mixtures of liquids that have similar boiling points. It involves several vaporization-condensation steps (which take place in a fractioning column). This process is also known as rectification.

(iii) **Steam distillation:** Steam distillation is often used to separate components from a mixture of heat-sensitive components. The process is processed by passing steam through the mixture (which is slightly heated) to vaporize it. It establishes a high heat transfer rate without the need for a source of high temperatures. The resultant vapor is condensed to afford the required distillate liquid. The process of steam distillation is used to obtain essential oil constituents and herbal distillates from several aromatic flowers/herbs.

(iv) **Vacuum distillation:** Vacuum distillation is ideal for separating mixtures of liquids with very high boiling points. To boil these compounds, heating to high temperatures is an inefficient method. Therefore, the pressure of the surroundings is lowered instead. The lowering of the pressure enables the component to boil at lower temperatures. Once the vapor pressure of the component is equal to the surrounding pressure, it is converted into vapor. These vapors are then condensed and collected as the distillate. The vacuum distillation method is also used to obtain high-purity samples of compounds that decompose at high temperatures.

Suppose that, a bio-science department of an institution needs a distillation unit for their laboratory research. In market there are several types of distillation units. But it is difficult to choose the most suitable distillation unit among the possible distillation units those are available in the market. For this the decision maker (institution) can choose the some attributes on which basis customers /users/ institutions are being interested to buy distillation unit for their laboratory purpose or other needs.

- **Capacity or productivity (E_1):** Production of required solvent as per hour is one of the most important criteria that buyers looking for. On average a laboratory distillation unit can produce 2.0-2.5 liters of distillate per hour where an industrial unit can produce much more than a laboratory distillation unit.
- **The material used in the vessel (E_2):** The distillation flask should preferably be round-bottomed rather than a flat-bottomed one for smoothness of boiling. The material used in the vessel should be very heat resistant and light-weighted. There are two major glass materials used maximum glass distillation units *i.e*. borosilicate glass and quartz glass material.
- **Automation Grade of machine (E_3):** Machine-operating systems are the most advanced technology for all of us where it works automatically and without human involvement. So the criteria should be either a semi-automatic or automatic process.
- **Usage/Application of machine (E_4):** The application of any machine defines the existence of that machine. This is one of the price-dependent criteria among all.
- **Temperature Control Range (E_5):** Temperature measurement is a common control parameter in distillation cooling and heating processes. Depending upon the application and process fluid, temperature control may be used for cooling distillate to condense high volatility products into liquid phase, or heating of process fluid to vaporize the high volatility components for easier separation. The lower limit of the range is the temperature indicated by the thermometer when the first drop of condensate leaves the tip of the condenser, and the upper limit is the temperature at which the last drop evaporates from the lowest point in the distillation flask.
- **Price (E_6):** Generally, there are two types of cost named fixed cost and variable cost, which are used along with numbers of units for determining the selling price of the product. Cost of materials plays a very significant role in their selection. The application and material of glass are charged with the cost of distillation units issued to them.

Hence, the selection of a suitable distillation unit for biological laboratory can be considered as a multi-attribute-decision-making problem.

Assume that, the decision maker select four alternatives after the initial screening. Let $\H{U}$ = $\{P_1, P_2, P_3, P_4\}$ be the universal set of available distillation units from which the decision maker will buy a suitable distillation unit. Let $E = \{E_1, E_2, E_3, E_4, E_5, E_6\}$ be the set of attributes based on which the decision maker will select the most suitable distillation units. Then, the tabular representation of the information of distillation units P_1, P_2, P_3, P_4 against the attributes $E_1, E_2, E_3, E_4, E_5, E_6$ are given in Table-1.

Table-1

	E_1	E_2	E_3	E_4	E_5	E_6
P_1	(0.8,0.1,0.2)	(0.9,0.2,0.3)	(0.7,0.1,0.1)	(0.9,0.0,0.1)	(1.0,0.3,0.1)	(0.9,0.2,0.1)
P_2	(0.2,0.6,0.5)	(0.1,0.4,0.3)	(0.0,0.2,0.3)	(0.1,0.2,0.0)	(0.5,0.1,0.1)	(0.0,0.1,0.2)
P_3	(0.8,0.4,0.2)	(0.6,0.5,0.5)	(0.5,0.2,0.1)	(0.4,0.5,0.4)	(0.6,0.4,0.2)	(0.2,0.3,0.1)
P_4	(0.8,0.2,0.2)	(0.7,0.3,0.2)	(0.9,0.0,0.1)	(1.0,0.2,0.1)	(0.9,0.2,0.1)	(0.8,0.1,0.1)

Now, by using the eq. (3) and eq. (4), we have w_1= 0.1607378, w_2 = 0.1528327, w_3 = 0.1712780, w_4 = 0.1699605, w_5 = 0.1778656, and w_6 = 0.1673254.

The ideal solution is $P^*= 1_N$ = {(E_1, 1, 0, 0), (E_2, 1, 0, 0), (E_3, 1, 0, 0), (E_4, 1, 0, 0), (E_5, 1, 0, 0), (E_6, 1, 0, 0)}. The single valued neutrosophic weighted hyperbolic cosine similarity measure of similarities between the possible alternatives (distillation units) and the ideal solution (ideal distillation unit) are:
SVNWHCSM (P_1, P^*) = 0.9833013,
SVNWHCSM (P_2, P^*) = 0.9669271,
SVNWHCSM (P_3, P^*) = 0.9734845,
and SVNWHCSM (P_4, P^*) = 0.9830226.

Here, SVNWHCSM (P_2, P^*) < SVNWHCSM (P_3, P^*) < SVNWHCSM (P_4, P^*) < SVNWHCSM (P_1, P^*). Therefore, the alternative P_2 is the most suitable alternative among the set of possible alternative. Hence, the institution can buy the distillation unit P_2 for their laboratory related work.

6. Conclusions

In the study, we have proposed a new similarity measure namely single valued neutrosophic weighted hyperbolic cosine similarity measures of similarities between two SVNSs and proved some of their basic properties. Further, we have developed a novel MADM-strategy based on the proposed single valued neutrosophic weighted hyperbolic cosine similarity measure under the SVNS environment. Then, we validate our proposed MADM-strategy by solving an illustrative MADM-problem namely "Selection of the Most Suitable Distillation Unit for Biological Laboratory under SVNS-environment" to demonstrate the applicability and effectiveness of our proposed MADM-strategy.

The proposed MADM-strategy also can be used to deal with other real life problems in real world such as decision making [3-4, 6-8, 13], data mining [11], medical diagnosis [15-16].

The data used in this paper has not taken from any source. We have considered these numbers for the verification of our algorithm. However, this algorithm can apply for any real source data.

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