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Abstract: The purpose of this paper is to introduce the new concepts of neutrosophic orbit open set, neutrosophic orbit continuous, almost-neutrosophic orbit continuous, weakly-neutrosophic orbit continuous, neutrosophic orbit* continuous functions and analyze some of their interesting properties.

Keywords: Neutrosophic orbit set; Neutrosophic orbit open set; Neutrosophic orbit continuous.

1. Introduction

Fuzzy concept has invaded almost all branches of Mathematics since its introduction by Zadeh[23]. Fuzzy sets have applications in many fields such as information [21] and control [22]. The theory of fuzzy topological spaces was introduced and developed by Chang[7] and from then various notions in classical topology have been extended to fuzzy topological spaces[4, 5, 6]. Following this concept K.Atanassov[1,2,3] in 1983 devised the idea of intuitionistic fuzzy set on a universe X as a generalization of fuzzy set. Here besides the degree of membership a degree of non-membership for each element is also defined. The topological framework of intuitionistic fuzzy set was initiated by D.Coker[8].

As a generalization of intuitionistic fuzzy sets neutrosophic set was formulated by Smarandache. Smarandache[16,17,18] originally gave the definition of a neutrosophic set and neutrosophic logic. The neutrosophic logic is a formal frame trying to measure the truth, indeterminacy and falsehood. In 2012 Salama and Alblowi[19,21] introduced the concept of neutrosophic topological spaces. Prem Kumar Singh [14,15] introduced the concept of neutrosophic context analysis at distinct multi-granulation using single valued neutrosophic numbers and also graphical representation of lattices by applying interval valued neutrosophic numbers

The orbit in mathematics has an important role in the study of dynamical systems, an orbit is a collection of points associated by the evolution function of the dynamical system. One of the objectives of the modern theory of dynamical systems is using topological methods to understanding the properties of dynamical systems[12]. The concept of the fuzzy orbit set was introduced by R.Malathi and M.K.Uma[13] in 2017, as a generalization to the concept of the orbit point in general metric space[9]. Also, R.Malathi and M.K.Uma[13] introduced the concept of fuzzy orbit open sets and fuzzy orbit continuous mappings.

In this paper various novel concepts of neutrosophic orbit open set, almost-neutrosophic orbit continuous, weakly-neutrosophic orbit continuous, neutrosophic orbit* continuous are created which paves way to discuss. Some of its interesting properties and characterizations. Also neutrosophic orbit* continuous mappings are discussed with necessary examples and counterexamples.

2. Preliminaries

2.1 Definition [13] Let X be a non empty set. A neutrosophic set (NS for short) A is an object having the form $A = \langle x, A^T, A^I, A^F \rangle$ where A^T, A^I, A^F represent the degree of membership , the degree of indeterminacy and the degree of non-membership respectively of each element $x \in X$ to the set A.

2.2 Definition [13] Let X be a non empty set, $A = \langle x, A^T, A^I, A^F \rangle$ and $B = \langle x, B^T, B^I, B^F \rangle$ be neutrosophic sets on X, and let $\{A_i : i \in J\}$ be an arbitrary family of neutrosophic sets in X, where $A_i = \langle x, A^T, A^I, A^F \rangle$

(i) $A \subseteq B$ if and only if $A^T \leq B^T$, $A^I \geq B^I$ and $A^F \geq B^F$

(ii) $A = B$ if and only if $A \leq B$ and $B \leq A$.

(iii) $\overline{A} = \langle x, A^F, 1-A^I, A^T \rangle$

(iv) $A \cap B = \langle x, A^T \wedge B^T, A^I \vee B^I, A^F \vee B^F \rangle$

(v) $A \cup B = \langle x, A^T \vee B^T, A^I \wedge B^I, A^F \wedge B^F \rangle$

(vi) $\cup A_i = \langle x, \vee A_i^T, \wedge A_i^I, \wedge A_i^F \rangle$

(vii) $\cap A_i = \langle x, \wedge A_i^T, \vee A_i^I, \vee A_i^F \rangle$

(viii) $A - B = A \wedge \overline{B}$.

(ix) $0_N = \langle x,0,1,1 \rangle$; $1_N = \langle x,1,0,0 \rangle$.

2.3 Definition [18] A neutrosophic topology (NT for short) on a nonempty set X is a family τ of neutrosophic set in X satisfying the following axioms:

(i) $0_N, 1_N \in \tau$.

(ii) $G_1 \wedge G_2 \in \tau$ for any $G_1, G_2 \in \tau$.

(iii) $\vee G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called a neutrosophic topological space (NTS for short) and any neutrosophic set in τ is called a neutrosophic open set(NOS for short) in X. The complement A of a neutrosophic open set A is called a neutrosophic closed set (NCS for short) in X.

2.4 Definition [18] Let (X, τ) be a neutrosophic topological space and $A = \langle X, A^T, A^I, A^F \rangle$ be a set in X. Then the closure and interior of A are defined by

$Ncl(A) = \wedge\{K : K$ is a neutrosophic closed set in X and $A \leq K\}$,

$Nint(A) = \vee\{G : G$ is a neutrosophic open set in X and $G \leq A\}$.

It can be also shown that Ncl(A) is a neutrosophic closed set and Nint(A) is a neutrosophic open set in X, and A is a neutrosophic closed set in X iff Ncl(A) = A; and A is a neutrosophic open set in X iff Nint(A) = A.

2.5 Definition [9] Orbit of a point x in X under the mapping f is $O_f(x)=\{x, f(x), f^2(x),...\}$

2.6 Definition [10] A neutrosophic set $A=\langle x, A^T, A^I, A^F\rangle$ in a neutrosophic topological space (X, τ) is said to be a neutrosophic neighbourhood of a neutrosophic point $x_{r,t,s}$, $x\in X$, if there exists a neutrosophic open set $B=\langle x, B^T, B^I, B^F\rangle$ with $x_{r,t,s}\subseteq B \subseteq A$.

2.7 Corollary [11] Let A, $A_i(i\in J)$ be neutrosophic sets in X, B, $B_i(i\in K)$ be neutrosophic sets in Y and f: X → Y a function. Then

(a) $A_1\subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$,

(b) $B_1\subseteq B_2 \Rightarrow f^{-1}(A_1) \subseteq f^{-1}(A_2)$,

(c) $A\subseteq f^{-1}(f(A))$\{If f is injective, then $A= f^{-1}(f(A))$\},

(d) $f^{-1}(f(B))\subseteq B$\{If f is surjective, then $f^{-1}(f(B))= B$\},

(e) $\overline{f(A)} \subseteq f(\overline{A})$, if f is surjective,

(f) $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$.

3. Properties and characterization of neutrosophic orbit continuous Mappings

3.1 Definition A neutrosophic set A in a neutrosophic topological space (X, τ) is a neighbourhood of a neutrosophic set B, if there exists a neutrosophic open set O such that $B\subseteq O \subseteq A$.

3.2 Definition Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces. Let f: (X, τ) → (Y, σ) is said to be almost neutrosophic continuous, if for every neutrosophic set α and every neutrosophic open set μ with $f(\alpha) \leq \mu$, there exists a neutrosophic open set σ with $\alpha \leq \sigma$ such that $f(\sigma) \leq int(cl(\mu))$.

3.3 Definition Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces. Let f: (X, τ) → (Y, σ) is said to be weakly neutrosophic continuous, if for every neutrosophic set α and every neutrosophic open set μ with $f(\alpha) \leq \mu$, there exists a neutrosophic open set σ with $\alpha \leq \sigma$

such that $f(\sigma) \leq cl(\mu)$.

3.4 Definition Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces. Let f: (X, τ) → (Y, σ) is said to be slightly neutrosophic continuous, if for every neutrosophic set α and every neutrosophic open set μ with $f(\alpha) \leq \mu$, there exists a neutrosophic open set σ with $\alpha \leq \sigma$ such that $f(\sigma) \leq \mu$.

3.5 Definition Let X be a nonempty set and f : X → X be any mapping. Let α be any neutrosophic set in X. The neutrosophic orbit $O_f(\alpha)$ of α under the mapping f is defined as $O_{fT}(\alpha) = \{\alpha, f^1(\alpha), f^2(\alpha), \ldots f^n(\alpha)\}$, $O_{fI}(\alpha) = \{\alpha, f^1(\alpha), f^2(\alpha), \ldots f^n(\alpha)\}$, $O_{fF}(\alpha) = \{\alpha, f^1(\alpha), f^2(\alpha), \ldots f^n(\alpha)\}$ for $\alpha \in X$ and $n \in Z^+$.

3.6 Definition Let X be a nonempty set and let f : X→ X be any mapping. The neutrosophic orbit set of α under the mapping f is defined as $NO_f(\alpha) = \langle \alpha, O_{fT}(\alpha), O_{fI}(\alpha), O_{fF}(\alpha) \rangle$ for $\alpha \in X$, where $O_{fT}(\alpha) = \{\alpha \wedge f^1(\alpha) \wedge f^2(\alpha) \wedge \ldots \wedge f^n(\alpha)\}$, $O_{fI}(\alpha) = \{\alpha \vee f^1(\alpha) \vee f^2(\alpha) \vee \ldots \vee f^n(\alpha)\}$, $O_{fF}(\alpha) = \{\alpha \vee f^1(\alpha) \vee f^2(\alpha) \vee \ldots \vee f^n(\alpha)\}$.

3.7 Example Let X={a, b, c}. Define a neutrosophic set α where $\alpha^T : X \to \left]{}^-0, 1^+\right[$

$\alpha^I : X \to \left]{}^-0, 1^+\right[$

$\alpha^F : X \to \left]{}^-0, 1^+\right[$ as follows

$\alpha^T(a) = 0.5, \alpha^I(a) = 0.4, \alpha^F(a) = 0.5, \alpha^T(b) = 0.6, \alpha^I(b) = 0.5, \alpha^F(b) = 0.4, \alpha^T(c) = 0.7, \alpha^I(c) = 0.6, \alpha^F(c) = 0.3$

Define f : X→ X as f(a)=b, f(b)=c, f(c)=a. The neutrosophic orbit set of α under the mapping f is defined as $NO_f(\alpha) = \alpha \cap f^1(\alpha) \cap f^2(\alpha) \cap \ldots \cap f^n(\alpha)$

$NO_f(\alpha)(a) = \langle x, 0.7, 0.6, 0.3 \rangle$, $NO_f(\alpha)(b) = \langle x, 0.5, 0.4, 0.5 \rangle$, $NO_f(\alpha)(c) = \langle x, 0.6, 0.5, 0.4 \rangle$

3.8 Definition Let (X, τ) be a neutrosophic topological space. Let f : X → X be any mapping. The neutrosophic orbit set under the mapping f which is in neutrosophic topology τ is called neutrosophic orbit open set under the mapping f. Its complement is called a neutrosophic orbit closed set under the mapping f.

3.9 Example Let X={a, b, c}=Y. Define τ = {0_N, 1_N, α, γ }where $\lambda^T, \gamma^T : X \to \left]^-0, 1^+\right[$

$\alpha^I, \gamma^I : X \to \left]^-0, 1^+\right[$

$\alpha^F, \gamma^F : X \to \left]^-0, 1^+\right[$

are defined as

$\alpha^T(a) = 0.3, \alpha^I(a) = 0.5, \alpha^F(a) = 0.6, \alpha^T(b) = 0.4, \alpha^I(b) = 0.6,$

$\alpha(b) = 0.8, \alpha^T(c) = 0.1,$

$\alpha^I(c) = 0.3, \alpha^F(c) = 0.7$

$\gamma^T(a) = 0.3, \gamma^I(a) = 0.5, \gamma^F(a) = 0.6, \gamma^T(b) = 0, \gamma^I(b) = 0, \gamma^F(b) = 1,$

$\gamma^T(c) = 0, \gamma^I(c) = 0, \gamma^F(c) = 1.$

Define f : X→ X as f(a)=a, f(b)=a, f(c)=a. The neutrosophic orbit set of α under the mapping f is defined as $NO_f(\alpha)$ = $\alpha \cap f^1(\alpha) \cap f^2(\alpha) \cap \ldots \cap f^n(\alpha)$, $NO_f(\alpha) = \gamma$. Then γ is a neutrosophic orbit open set under the mapping f.

3.10 Definition Let (X, τ) be a neutrosophic topological space. Let f : X → X be any mapping. The neutrosophic orbit under the mapping f in a neutrosophic topological space (X, T) is said to be neutrosophic orbit clopen set under the mapping f, if it is both neutrosophic orbit open and neutrosophic orbit closed under the mapping f.

3.11 Definition A neutrosophic set α in a neutrosophic topological space (X, τ) is a neutrosophic orbit neighborhood, or NOnbhd for short, of a neutrosophic set μ, if there exists a neutrosophic orbit open set λ such that $\mu \subset \lambda \subset \alpha.$

3.12 Definition Let (X, τ) be a neutrosophic topological space and α = <X, α^1, α^2, α^3 > be a set in

X. Then the closure and interior of α are defined by

Ncl(α) = $\wedge\{\ \beta : \beta$ is a neutrosophic orbit closed set in X and $\alpha \leq \beta$ },

Nint(α) = $\vee\{\ \beta : \beta$ is a neutrosophic orbit open set in X and $\beta \leq \alpha$ }.

3.13 Definition Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces. Let f: X → X be a mapping. A mapping g : (X, τ) → (Y, σ) is said to be neutrosophic orbit continuous, if the inverse image of every neutrosophic open set in (Y, σ) is neutrosophic orbit open set under the mapping f in (X, τ).

3.14 Proposition Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces. Let g : (X, τ) → (Y,σ) and f_1 : X → X be any two mappings. Then the following are equivalent

(i) g is neutrosophic orbit continuous mapping

(ii) inverse image of every neutrosophic closed set in (Y, σ) is a neutrosophic orbit closed set under the mapping f_1 in (X, τ).

Proof: (i) $\Rightarrow$ (ii): Assume that g is a neutrosophic orbit continuous mapping. Let λ be any neutrosophic closed set in (Y, σ). Then $1-\lambda$ is a neutrosophic open set in in (Y, σ). Thus by assumption, $g^{-1}(1-\lambda)$ is a neutrosophic orbit open set under the mapping f_1 in (X, τ). Now, $g^{-1}(1-\lambda) = 1 - g^{-1}(\lambda)$. So, $g^{-1}(\lambda)$ is a neutrosophic orbit closed set under the mapping f_1 in (X, τ).

(ii) $\Rightarrow$ (i): The proof is similar to (i) $\Rightarrow$ (ii).

3.15 Proposition Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces. Let g : (X, τ) → (Y,σ) and f_1 : X → X and f_2 : Y → Y be any two mappings. Then the following are equivalent

(i) g is neutrosophic orbit continuous mapping

(ii) for each neutrosophic set λ of X and every neutrosophic neighbourhood λ of

$g(\lambda), g^{-1}(\lambda)$ is a neutrosophic orbit neighbourhood of γ

(iii) for each neutrosophic set λ of X and every neutrosophic neighbourhood λ of $g(\lambda)$, there exists a neutrosophic orbit neighbourhood μ of γ such that $g(\mu) \leq \lambda$.

Proof:(i) $\Rightarrow$ (ii): Let γ be a neutrosophic set of X. Let λ be a neutrosophic neighbourhood of $g(\gamma)$. Then there exists a neutrosophic open set μ such that $g(\gamma) \leq \mu \leq \lambda$. Now $g^{-1}(g(\gamma)) \leq g^{-1}(\mu) \leq g^{-1}(\lambda)$. By hypothesis, $g^{-1}(\mu)$ is a neutrosophic orbit open set under the mapping f_1 in (X, τ). But, $\gamma \leq g^{-1}(g(\gamma))$. Thus $g^{-1}(\lambda)$ is a neutrosophic orbit neighbourhood of γ.

(ii) $\Rightarrow$ (iii): Let γ be a neutrosophic set of X. Let λ be a neutrosophic neighborhood of $g(\gamma)$. By hypothesis, $g^{-1}(\lambda)$ is a neutrosophic orbit neighbourhood of γ in (X, τ) such that $g^{-1}(g(\lambda)) \leq \lambda$.

(iii) $\Rightarrow$ (i): Let γ be a neutrosophic set of X such that $\gamma \leq g^{-1}(\lambda)$. Let λ be a neutrosophic orbit open set under the mapping f_2 in (Y, σ). Since every neutrosophic orbit open set is a neutrosophic neighborhood, λ is a neutrosophic neighbourhood of $g(\delta)$ in (Y, σ). Then by hypothesis, $g^{-1}(\lambda)$ is a neutrosophic orbit neighbourhood of γ in (X, τ). Since every neutrosophic orbit neighborhood set is a neutrosophic orbit open set, $g^{-1}(\lambda)$ is a neutrosophic orbit open set under the mapping f_1 in (X, τ). Thus g is neutrosophic orbit continuous.

3.16 Proposition Let (X, τ), (Y, σ) and (Z, η) be any three neutrosophic topological spaces. Let f_1 : X $\rightarrow$ X be any mappings. Let g : (X, τ) $\rightarrow$ (Y,σ) be neutrosophic orbit continuous and h : (Y, σ) $\rightarrow$ (Z, η) be neutrosophic continuous mappings, then their composition $h \circ g$ is neutrosophic orbit continuous.

Proof: Let λ be a open set of (Z, η). By Definition, $h^{-1}(\lambda)$ is a neutrosophic open set of (Y, σ). Since f is neutrosophic orbit continuous, $g^{-1}(h^{-1}(\lambda))$ is a neutrosophic orbit open set under the mapping f_1 of (X, τ). But $g^{-1}(h^{-1}(\lambda)) = (h \circ g)^{-1}(\lambda)$. Then $h \circ g$ is neutrosophic orbit continuous.

3.17 Definition Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces. Let $f_1 : X \rightarrow X$ and $f_2 : Y \rightarrow Y$ be any two mappings. A mapping g : (X, τ) → (Y, σ) is said to be neutrosophic orbit* continuous, if for every neutrosophic set α and every neutrosophic orbit open set μ under the mapping f_2 with g(α) $\leq$ μ, there exists a neutrosophic orbit open set λ under the mapping f_1 with $\alpha \leq \lambda$ such that g(λ) $\leq \mu$.

3.18 Example Let X={a, b, c}=Y. Define τ = {0_N, 1_N, λ, λ_1 }and σ = {0_N, 1_N, μ, μ_1 } where

$\lambda^T, {\lambda_1}^T, \mu^T, {\mu_1}^T : X \rightarrow \left]{}^-0, 1^+\right[$

$\lambda^I, {\lambda_1}^I, \mu^I, {\mu_1}^I : X \rightarrow \left]{}^-0, 1^+\right[$

$\lambda^F, {\lambda_1}^F, \mu^F, {\mu_1}^F : X \rightarrow \left]{}^-0, 1^+\right[$

are such that

$\lambda^T(a) = 0, \lambda^I(a) = 1, \lambda^F(a) = 1, \lambda^T(b) = 0, \lambda^I(b) = 1, \lambda^F(b) = 1, \lambda^T(c) = 0.6,$

$\lambda^I(c) = 0.5, \lambda^F(c) = 0.4$

${\lambda_1}^T(a) = 0.5, {\lambda_1}^I(a) = 0.4, {\lambda_1}^F(a) = 0.3, {\lambda_1}^T(b) = 0.7, {\lambda_1}^I(b) = 0.5, {\lambda_1}^F(b) = 0.6,$

${\lambda_1}^T(c) = 0.6, {\lambda_1}^I(c) = 0.5, {\lambda_1}^F(c) = 0.4$

$\mu^T(a) = 0.7, \mu^I(a) = 0.6, \mu^F(a) = 0.5, \mu^T(b) = 0.7, \mu^I(b) = 0.6, \mu^F(b) = 0.5,$

$\mu^T(c) = 0.7, \mu^I(c) = 0.6, \mu^F(c) = 0.5$

${\mu_1}^T(a) = 0.9, {\mu_1}^I(a) = 0.5, {\mu_1}^F(a) = 0.4, {\mu_1}^T(b) = 0.7, {\mu_1}^I(b) = 0.4, {\mu_1}^F(b) = 0.3,$

${\mu_1}^T(c) = 0.8, {\mu_1}^I(c) = 0.4, {\mu_1}^F(c) = 0.2$

Clearly (X, τ) and (Y, σ) are neutrosophic topological spaces.

Define g : (X, τ) → (Y, σ), f_1 : X → X and f_2 : Y→ Y as g(a) = b, g(b) = c, g(c) =a, f_1(a) = c, f_1(b) =c, f_1(c) = c and f_2(a) = b, f_2(b) =c, f_2(c) = a.

Let $\alpha^T : X \to \left]^-0, 1^+\right[$

$\alpha^I : X \to \left]^-0, 1^+\right[$

$\alpha^F : X \to \left]^-0, 1^+\right[$ be any neutrosophic set such that

$$\alpha^T(a) = 0, \alpha^I(a) = 0.8, \alpha^F(a) = 0.9, \alpha^T(b) = 0, \alpha^I(b) = 1, \alpha^F(b) = 1, \alpha^T(c) = 0, \alpha^I(c) = 1, \alpha^F(c) = 1$$

For the neutrosophic orbit open set μ under the mapping f_2 in (Y, σ), g(α) $\leq \mu$. Now, λ is a neutrosophic orbit open set under the mapping f_1 in (X, τ) with $\alpha \leq \lambda$ such that g(λ) $\leq \mu$. Hence g is neutrosophic orbit* continuous.

3.19 Proposition Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces. Let g : (X, τ) → (Y,σ) be a mappings. Then the following are equivalent

(i) g is neutrosophic orbit* continuous.

(ii) inverse image of every neutrosophic orbit open set of (Y, σ) is neutrosophic orbit open set of (X, τ).

(iii) inverse image of every neutrosophic orbit clopen set of (Y, σ) is neutrosophic orbit open set of (X, τ).

Proof:(i) ⇒ (ii): Let f_1 : X → X and f_2 : Y → Y be any two mappings. Let μ be a neutrosophic orbit open set under the mapping f_2 of (Y, σ) and any neutrosophic set α with g(α) $\leq \mu$. Since g is neutrosophic orbit* continuous, there exists a neutrosophic orbit open set λ under the mapping f_1 of (X, τ) with $\alpha \leq \lambda$ such that g(λ) $\leq \mu$. Hence $g^{-1}(\mu)$ is a neutrosophic orbit open set.

(ii) $\Rightarrow$ (iii): Let μ be a neutrosophic orbit open set under the mapping f_2 of (Y, σ). By (ii) $g^{-1}(\mu)$ is a neutrosophic orbit open set under the mapping f_1 of (X, τ). Now $1-\mu$ is also neutrosophic orbit clopen set. By (ii) $g^{-1}(1-\mu)$ is neutrosophic orbit open set under the mapping f_1 in (X, τ). So $1-g^{-1}(1-\mu)$ is neutrosophic orbit closed set under the mapping f_1 in (X, τ). This implies that $g^{-1}(\mu)$ is neutrosophic orbit closed. Therefore, $g^{-1}(\mu)$ is a neutrosophic orbit open set clopen set in (X, τ).

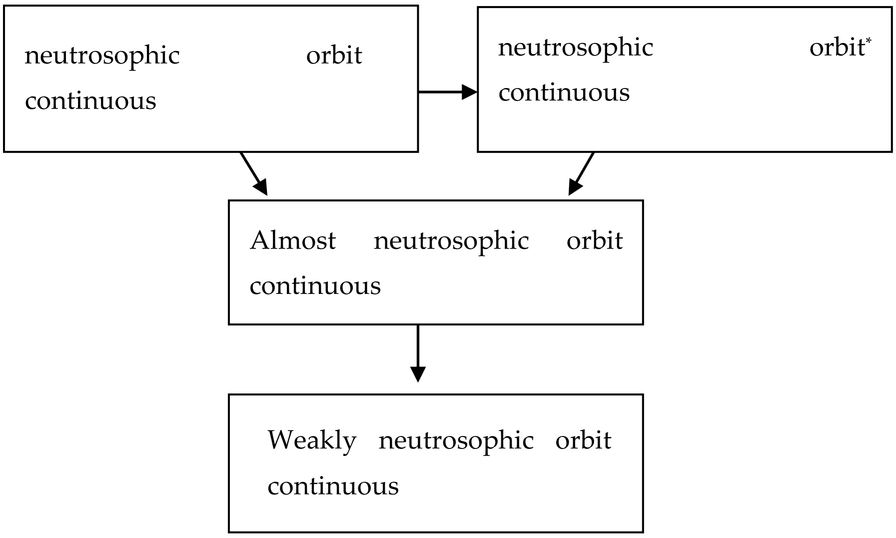
(iii) $\Rightarrow$ (i): Let μ be a neutrosophic orbit clopen set under the mapping f_2 and any fuzzy set α with g(α) $\leq$ μ. Now $g^{-1}(\mu)$ is neutrosophic orbit open set under the mapping f_1 of (X, τ) and $g(g^{-1}(\mu)) \leq \mu$. Hence, g is neutrosophic orbit* continuous.

3.20 Definition Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces. Let $f_1 : X \rightarrow X$ and $f_2 : Y \rightarrow Y$ be an y two mappings. A mapping g : (X, τ) $\rightarrow$ (Y, σ) is said to be almost-neutrosophic orbit continuous, if for every neutrosophic set α and every neutrosophic orbit open set μ under the mapping f_2 with g(α) $\leq$ μ, there exists a neutrosophic orbit open set λ under the mapping f_1 with $\alpha \leq \lambda$ such that g(λ) $\leq$ int (cl(μ)).

3.21 Definition Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces. Let $f_1 : X \rightarrow X$ and $f_2 : Y \rightarrow Y$ be any two mappings. A mapping g : (X, τ) $\rightarrow$ (Y, σ) is said to be weakly-neutrosophic orbit continuous, if for every neutrosophic set α and every neutrosophic orbit open set μ under the mapping f_2 with g(α) $\leq$ μ, there exists a neutrosophic orbit open set λ under the mapping f_1 with $\alpha \leq \lambda$ such that g(λ) $\leq$ cl(μ).

3.22 Remark

Figure 1.



3.23 Proposition Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces. If g : (X, τ) → (Y,σ) is neutrosophic orbit continuous, then g is almost neutrosophic orbit continuous.

Proof. Let $f_1 : X \to X$ and $f_2 : Y \to Y$ be any two mappings. Let α be any neutrosophic set and μ be any neutrosophic orbit open set under the mapping f_2 with $g(\alpha) \leq \mu$. By Corollary 2.7, $\alpha \leq g^{-1}(g(\alpha)) \leq g^{-1}(\mu)$. Then $\leq g^{-1}(\mu)$. Since g is neutrosophic orbit continuous, $\alpha \leq g^{-1}(\mu) = \lambda, \lambda$ is a neutrosophic orbit open set under the mapping f_1. By Corollary 2.7, $gg^{-1}(\mu) \leq \mu$. Thus g(λ) = $gg^{-1}(\mu) \leq \mu$. Since μ is neutrosophic orbit open, μ is neutrosophic open and hence $\mu \leq$ int(cl(μ)) which implies that g(λ) $\leq$ int(cl(μ)). So g is almost-neutrosophic orbit continuous.

3.24 Remark The converse of the Proposition 3.15 need not be true as shown in the following example.

3.25 Example Let X={a, b, c}=Y. Define τ = {0_N, 1_N, λ, λ_1 }and σ= {0_N, 1_N, μ, μ_1, μ_2, μ_3} where $\lambda^T, {\lambda_1}^T, \mu^T, {\mu_1}^T, {\mu_2}^T, {\mu_3}^T : X \to \left]^-0, 1^+\right[$

$\lambda^I, {\lambda_1}^I, \mu^I, {\mu_1}^I, {\mu_2}^I, {\mu_3}^I : X \to \left] {}^-0, 1^+ \right[$

$\lambda^F, {\lambda_1}^F, \mu^F, {\mu_1}^F, {\mu_2}^F, {\mu_3}^F : X \to \left] {}^-0, 1^+ \right[$

are such that

$\lambda^T(a) = 0, \lambda^I(a) = 1, \lambda^F(a) = 1, \lambda^T(b) = 0, \lambda^I(b) = 1, \lambda^F(b) = 1, \lambda^T(c) = 0.3,$

$\lambda^I(c) = 0.4, \lambda^F(c) = 0.4$

${\lambda_1}^T(a) = 0.7, {\lambda_1}^I(a) = 0.4, {\lambda_1}^F(a) = 0.5, {\lambda_1}^T(b) = 0.6, {\lambda_1}^I(b) = 0.3, {\lambda_1}^F(b) = 0.5,$

${\lambda_1}^T(c) = 0.3, {\lambda_1}^I(c) = 0.4, {\lambda_1}^F(c) = 0.4$

$\mu^T(a) = 0.6, \mu^I(a) = 0.5, \mu^F(a) = 0.4, \mu^T(b) = 0.6, \mu^I(b) = 0.5, \mu^F(b) = 0.4,$

$\mu^T(c) = 0.6, \mu^I(c) = 0.5, \mu^F(c) = 0.4$

${\mu_1}^T(a) = 0.6, {\mu_1}^I(a) = 0.5, {\mu_1}^F(a) = 0.3, {\mu_1}^T(b) = 0.7, {\mu_1}^I(b) = 0.2, {\mu_1}^F(b) = 0.3,$

${\mu_1}^T(c) = 0.8, {\mu_1}^I(c) = 0.2, {\mu_1}^F(c) = 0.1$

${\mu_2}^T(a) = 0.3, {\mu_2}^I(a) = 0.5, {\mu_2}^F(a) = 0.4, {\mu_2}^T(b) = 0.3, {\mu_2}^I(b) = 0.5, {\mu_2}^F(b) = 0.4,$

${\mu_2}^T(c) = 0.3, {\mu_2}^I(c) = 0.5, {\mu_2}^F(c) = 0.4$

${\mu_3}^T(a) = 0.3, {\mu_3}^I(a) = 0.5, {\mu_3}^F(a) = 0.4, {\mu_3}^T(b) = 0.4, {\mu_3}^I(b) = 0.5, {\mu_3}^F(b) = 0.4,$

${\mu_3}^T(c) = 0.5, {\mu_3}^I(c) = 0.5, {\mu_3}^F(c) = 0.4$

Clearly (X, τ) and (Y, σ) are neutrosophic topological spaces.

Define g : (X, τ) → (Y, σ), f_1 : X → X and f_2 : Y→ Y as g(a) = b, g(b) = c, g(c) =a, f_1(a) = c, f_1(b) =c, f_1(c) = c and f_2(a) = b, f_2(b) =c, f_2(c) = a.

Let $\alpha^T : X \to \left] {}^-0, 1^+ \right[$

$\alpha^I : X \to \left] {}^-0, 1^+ \right[$

$\alpha^F : X \to \left]{}^{-}0, 1^{+}\right[$ be any neutrosophic set such that

$\alpha^T(a) = 0, \alpha^I(a) = 1, \alpha^F(a) = 1, \alpha^T(b) = 0, \alpha^I(b) = 1, \alpha^F(b) = 1, \alpha^T(c) = 0.2, \alpha^I(c) = 0.5, \alpha^F(c) = 0.8$

For the neutrosophic orbit open set μ under the mapping f_2 in (Y, σ) with g(α) $\leq$ μ.

Now the neutrosophic orbit open set λ under the mapping f_1 in (X, τ) with $\alpha \leq \lambda$ such that g(λ) $\leq$ int (cl(μ)). Then g is almost neutrosophic orbit continuous.

Now the neutrosophic open sets μ_1, μ_2 and μ_3 in (Y, σ), but $g^{-1}(\mu_1), g^{-1}(\mu_2)$ and $g^{-1}(\mu_3)$ are not neutrosophic orbit open under the mapping f_1 in (X, τ). Thus g is not neutrosophic orbit continuous.

3.26 Proposition Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces. If g : (X, τ) → (Y,σ) is neutrosophic orbit continuous, then g is weakly neutrosophic orbit continuous.

Proof: Let f_1 : X → X and f_2 : Y→ Y be any two mappings. Let α be any neutrosophic set and μ be any neutrosophic orbit open set under the mapping f_2 with $g(\alpha) \leq \mu$. By Corollary 2.7, $\alpha \leq g^{-1}(g(\alpha)) \leq g^{-1}(\mu)$. Then $\alpha \leq g^{-1}(\mu)$. Since g is neutrosophic orbit continuous, $\alpha \leq g^{-1}(\mu) = \lambda, \lambda$ is a neutrosophic orbit open set under the mapping f_1. By Corollary 2.7, $gg^{-1}(\mu) \leq \mu$. Thus g(λ) = $gg^{-1}(\mu) \leq cl(\mu)$. So g is weakly neutrosophic orbit continuous.

3.27 Remark The converse of the Proposition 3.18 need not be true as shown in the following example.

3.28 Example Let X={a, b, c}=Y. Define τ = {0_N, 1_N, λ, λ_1 }and σ= {0_N, 1_N, μ, μ_1, μ_2, μ_3} where

$\lambda^T, {\lambda_1}^T, \mu^T, {\mu_1}^T, {\mu_2}^T, {\mu_3}^T : X \to \left]{}^{-}0, 1^{+}\right[$

$\lambda^I, {\lambda_1}^I, \mu^I, {\mu_1}^I, {\mu_2}^I, {\mu_3}^I : X \to \left]{}^{-}0, 1^{+}\right[$

$\lambda^F, {\lambda_1}^F, \mu^F, {\mu_1}^F, {\mu_2}^F, {\mu_3}^F : X \to \left]{}^{-}0, 1^{+}\right[$

are such that

$\lambda^T(a) = 0, \lambda^I(a) = 1, \lambda^F(a) = 1, \lambda^T(b) = 0.5, \lambda^I(b) = 0.4, \lambda^F(b) = 0.6, \lambda^T(c) = 0,$

$\lambda^I(c) = 1, \lambda^F(c) = 1$

${\lambda_1}^T(a) = 0.7, {\lambda_1}^I(a) = 0.4, {\lambda_1}^F(a) = 0.3, {\lambda_1}^T(b) = 0.5, {\lambda_1}^I(b) = 0.4, {\lambda_1}^F(b) = 0.6,$

${\lambda_1}^T(c) = 0.4, {\lambda_1}^I(c) = 0.2, {\lambda_1}^F(c) = 0.3$

$\mu^T(a) = 0.3, \mu^I(a) = 0.4, \mu^F(a) = 0.5, \mu^T(b) = 0.3, \mu^I(b) = 0.4, \mu^F(b) = 0.5,$

$\mu^T(c) = 0.3, \mu^I(c) = 0.4, \mu^F(c) = 0.5$

${\mu_1}^T(a) = 0.6, {\mu_1}^I(a) = 0.3, {\mu_1}^F(a) = 0.4, {\mu_1}^T(b) = 0.8, {\mu_1}^I(b) = 0.2, {\mu_1}^F(b) = 0.4,$

${\mu_1}^T(c) = 0.9, {\mu_1}^I(c) = 0.4, {\mu_1}^F(c) = 0.3$

${\mu_2}^T(a) = 0.6, {\mu_2}^I(a) = 0.3, {\mu_2}^F(a) = 0.5, {\mu_2}^T(b) = 0.6, {\mu_2}^I(b) = 0.3, {\mu_2}^F(b) = 0.4,$

${\mu_2}^T(c) = 0.6, {\mu_2}^I(c) = 0.4, {\mu_2}^F(c) = 0.3$

${\mu_3}^T(a) = 0.3, {\mu_3}^I(a) = 0.4, {\mu_3}^F(a) = 0.5, {\mu_3}^T(b) = 0.4, {\mu_3}^I(b) = 0.3, {\mu_3}^F(b) = 0.4,$

${\mu_3}^T(c) = 0.5, {\mu_3}^I(c) = 0.4, {\mu_3}^F(c) = 0.5$

Clearly (X, τ) and (Y, σ) are neutrosophic topological spaces.

Define g : (X, τ) → (Y, σ), $f_1 : X \to X$ and $f_2 : Y \to Y$ as g(a) = b, g(b) = c, g(c) =a, $f_1(a) = b$, $f_1(b)$ =b, $f_1(c)$ = b and $f_2(a) = b$, $f_2(b)$ =c, $f_2(c)$ = a.

Let $\alpha^T : X \to \left]^-0, 1^+\right[$

$\alpha^I : X \to \left]^-0, 1^+\right[$

$\alpha^F : X \to \left]^-0, 1^+\right[$ be any neutrosophic set such that

$\alpha^T(a) = 0, \alpha^I(a) = 1, \alpha^F(a) = 1, \alpha^T(b) = 0, \alpha^I(b) = 0.6, \alpha^F(b) = 0.8, \alpha^T(c) = 0, \alpha^I(c) = 1, \alpha^F(c) = 1$

For the neutrosophic orbit open set μ under the mapping f_2 in (Y, σ) with g(α) $\leq$ μ.

Now the neutrosophic orbit open set λ under the mapping f_1 in (X, τ) with $\alpha \leq \lambda$ such that g(λ) $\leq$ cl(μ). Then g is weakly neutrosophic orbit continuous.

Now the neutrosophic open sets μ_1, μ_2 and μ_3 in (Y, σ), but $g^{-1}(\mu_1), g^{-1}(\mu_2)$ and $g^{-1}(\mu_3)$ are not neutrosophic orbit open under the mapping f_1 in (X, τ). Thus g is not neutrosophic orbit continuous.

3.29 Proposition Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces. If g : (X, τ) → (Y,σ) is almost neutrosophic orbit continuous, then g is weakly neutrosophic orbit continuous.

Proof. Let f_1 : X → X and f_2 : Y→ Y be any two mappings. Let α be any neutrosophic set and μ be any neutrosophic orbit open set under the mapping f_2 with $g(\alpha) \leq \mu$. Since g is almost neutrosophic orbit continuous, there exists a neutrosophic orbit open set λ under the mapping f_1 with $\alpha \leq \lambda$ such that g(λ) $\leq$ int(cl(μ)), which implies that g(λ) $\leq$ cl(μ). Then g is weakly neutrosophic orbit continuous.

3.30 Remark The converse of the Proposition 3.21 need not be true as shown in the following example.

3.31 Example Let X={a, b, c}=Y. Define τ = {0_N, 1_N, λ, λ_1 }and σ= {0_N, 1_N, μ, μ_1, μ_2, μ_3} where

$\lambda^T, {\lambda_1}^T, \mu^T, {\mu_1}^T, {\mu_2}^T, {\mu_3}^T : X \to \left]{}^{-}0, 1^{+}\right[$

$\lambda^I, {\lambda_1}^I, \mu^I, {\mu_1}^I, {\mu_2}^I, {\mu_3}^I : X \to \left]{}^{-}0, 1^{+}\right[$

$\lambda^F, {\lambda_1}^F, \mu^F, {\mu_1}^F, {\mu_2}^F, {\mu_3}^F : X \to \left]{}^{-}0, 1^{+}\right[$

are such that

$\lambda^T(a) = 0, \lambda^I(a) = 1, \lambda^F(a) = 1, \lambda^T(b) = 0.4, \lambda^I(b) = 0.4, \lambda^F(b) = 0.6, \lambda^T(c) = 0,$

$\lambda^I(c) = 1, \lambda^F(c) = 1$

${\lambda_1}^T(a) = 0.6, {\lambda_1}^I(a) = 0.4, {\lambda_1}^F(a) = 0.3, {\lambda_1}^T(b) = 0.4, {\lambda_1}^I(b) = 0.4, {\lambda_1}^F(b) = 0.6,$

${\lambda_1}^T = 0.4, {\lambda_1}^I(c) = 0.6, {\lambda_1}^F(c) = 0.6$

$\mu^T(a) = 0.3, \mu^I(a) = 0.6, \mu^F(a) = 0.4, \mu^T(b) = 0.3, \mu^I(b) = 0.6, \mu^F(b) = 0.4,$

$\mu^T(c) = 0.3, \mu^I(c) = 0.6, \mu^F(c) = 0.4$

${\mu_1}^T(a) = 0.3, {\mu_1}^I(a) = 0.5, {\mu_1}^F(a) = 0.4, {\mu_1}^T(b) = 0.4, {\mu_1}^I(b) = 0.4, {\mu_1}^F(b) = 0.3,$

${\mu_1}^T(c) = 0.5, {\mu_1}^I(c) = 0.4, {\mu_1}^F(c) = 0.2$

${\mu_2}^T(a) = 0.6, {\mu_2}^I(a) = 0.5, {\mu_2}^F(a) = 0.3, {\mu_2}^T(b) = 0.6, {\mu_2}^I(b) = 0.3, {\mu_2}^F(b) = 0.2,$

${\mu_2}^T(c) = 0.6, {\mu_2}^I(c) = 0.4, {\mu_2}^F(c) = 0.1$

${\mu_3}^T(a) = 0.6, {\mu_3}^I(a) = 0.4, {\mu_3}^F(a) = 0.2, {\mu_3}^T(b) = 0.8, {\mu_3}^I(b) = 0.3, {\mu_3}^F(b) = 0.1,$

${\mu_3}^T(c) = 0.9, {\mu_3}^I(c) = 0.4, {\mu_3}^F(c) = 0.1$

Clearly (X, τ) and (Y, σ) are neutrosophic topological spaces.

Define g : (X, τ) → (Y, σ), $f_1 : X \to X$ and $f_2 : Y \to Y$ as g(a) = b, g(b) = c, g(c) =a, f_1(a) = b, f_1(b) =b, f_1(c) = b and f_2(a) = b, f_2(b) =c, f_2(c) = a.

Let $\alpha^T : X \to \left]^-0, 1^+\right[$

$\alpha^I : X \to \left]^-0, 1^+\right[$

$\alpha^F : X \to \left]^-0, 1^+\right[$ be any neutrosophic set such that

$\alpha^T(a) = 0, \alpha^I(a) = 1, \alpha^F(a) = 1, \alpha^T(b) = 0.3, \alpha^I(b) = 0.7, \alpha^F(b) = 0.8, \alpha^T(c) = 0, \alpha^I(c) = 1, \alpha^F(c) = 1$

For the neutrosophic orbit open set μ under the mapping f_2 in (Y, σ) with g(α) $\leq$ μ.

Now the neutrosophic orbit open set λ under the mapping f_1 in (X, τ) with $\alpha \leq \lambda$ such that g(λ) $\leq$ cl(μ). Then g is weakly neutrosophic orbit continuous.

Now, g(α) $\leq$ μ. But there is no neutrosophic orbit open set λ under the mapping f_1 with $\alpha \leq \lambda$ such that g(λ) $\nleq$ int(cl(μ)).Thus g is not almost neutrosophic orbit continuous.

3.32 Proposition Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces. If g : (X, τ) → (Y,σ) is neutrosophic orbit* continuous, then g is almost neutrosophic orbit continuous.

Proof. Let $f_1 : X \to X$ and $f_2 : Y \to Y$ be any two mappings. Let α be any neutrosophic set and μ be any neutrosophic orbit open set under the mapping f_2 with $g(\alpha) \leq \mu$. Since g is neutrosophic orbit* continuous, there exists a neutrosophic orbit open set λ under the mapping f_1 with $\alpha \leq \lambda$ such that g(λ) $\leq \mu$. Since μ is neutrosophic open, which implies that g(λ) $\leq$ int(cl(μ)). Then g is almost neutrosophic orbit continuous.

3.33 Remark The converse of the Proposition 3.24 need not be true as shown in the following example.

3.34 Example Let X={a, b, c}=Y. Define τ = {0_N, 1_N, λ, λ_1 }and σ= {0_N, 1_N, μ, μ_1} where

$$\lambda^T, {\lambda_1}^T, \mu^T, {\mu_1}^T : X \to \left]^-0, 1^+\right[$$

$$\lambda^I, {\lambda_1}^I, \mu^I, {\mu_1}^I : X \to \left]^-0, 1^+\right[$$

$$\lambda^F, {\lambda_1}^F, \mu^F, {\mu_1}^F : X \to \left]^-0, 1^+\right[$$

are such that

$$\lambda^T(a) = 0.4, \lambda^I(a) = 0.5, \lambda^F(a) = 0.5, \lambda^T(b) = 0.4, \lambda^I(b) = 0.5, \lambda^F(b) = 0.5, \lambda^T(c) = 0.4,$$

$$\lambda^I(c) = 0.5, \lambda^F(c) = 0.5$$

$${\lambda_1}^T(a) = 0.4, {\lambda_1}^I(a) = 0.4, {\lambda_1}^F(a) = 0.5, {\lambda_1}^T(b) = 0.5, {\lambda_1}^I(b) = 0.5, {\lambda_1}^F(b) = 0.4,$$

$${\lambda_1}^T = 0.6, {\lambda_1}^I(c) = 0.5, {\lambda_1}^F(c) = 0.4$$

$$\mu^T(a) = 0.3, \mu^I(a) = 0.5, \mu^F(a) = 0.6, \mu^T(b) = 0.3, \mu^I(b) = 0.5, \mu^F(b) = 0.6,$$

$$\mu^T(c) = 0.3, \mu^I(c) = 0.5, \mu^F(c) = 0.6$$

$${\mu_1}^T(a) = 0.4, {\mu_1}^I(a) = 0.5, {\mu_1}^F(a) = 0.5, {\mu_1}^T(b) = 0.4, {\mu_1}^I(b) = 0.5, {\mu_1}^F(b) = 0.5,$$

$${\mu_1}^T(c) = 0.5, {\mu_1}^I(c) = 0.5, {\mu_1}^F(c) = 0.5$$

Clearly (X, τ) and (Y, σ) are neutrosophic topological spaces.

Define g : (X, τ) → (Y, σ), f_1 : X → X and f_2 : Y→ Y as g(a) = b, g(b) = c, g(c) =a, f_1(a) = b, f_1(b) =c, f_1(c) = a and f_2(a) = b, f_2(b) =c, f_2(c) = a.

Let $\alpha^T : X \to \left]{}^{-}0, 1^{+}\right[$

$\alpha^I : X \to \left]{}^{-}0, 1^{+}\right[$

$\alpha^F : X \to \left]{}^{-}0, 1^{+}\right[$ be any neutrosophic set such that

$\alpha^T(a) = 0.2, \alpha^I(a) = 0.6, \alpha^F(a) = 0.8, \alpha^T(b) = 0.2, \alpha^I(b) = 0.6, \alpha^F(b) = 0.6, \alpha^T(c) = 0.2, \alpha^I(c) = 0.6, \alpha^F(c) = 0.7$

For the neutrosophic orbit open set μ under the mapping f_2 in (Y, σ) with g(α) $\leq$ μ.

Now the neutrosophic orbit open set λ under the mapping f_1 in (X, τ) with $\alpha \leq \lambda$ such that g(λ) $\leq$ int(cl(μ)). Then g is almost neutrosophic orbit continuous.

Now, g(α) $\leq$ μ. But there is no neutrosophic orbit open set λ under the mapping f_1 with $\alpha \leq \lambda$ such that g(λ) $\nleq$ μ.Thus g is not neutrosophic orbit* continuous.

3.35 Proposition Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces. If g : (X, τ) → (Y,σ) is neutrosophic orbit* continuous, then g is weakly neutrosophic orbit continuous.

Proof. Let f_1 : X → X and f_2 : Y→ Y be any two mappings. Let α be any neutrosophic set and μ be any neutrosophic orbit open set under the mapping f_2 with $g(\alpha) \leq \mu$. Since g is neutrosophic orbit* continuous, there exists a neutrosophic orbit open set λ under the mapping f_1 with $\alpha \leq \lambda$ such that g(λ) $\leq$ μ, which implies that g(λ) $\leq$ cl(μ). Then g is weakly neutrosophic orbit continuous.

3.36 Remark The converse of the Proposition 3.27 need not be true as shown in the following example.

3.37 Example Let X={a, b, c}=Y. Define τ = {0_N, 1_N, λ, λ_1 }and σ = {0_N, 1_N, μ, μ_1 } where

$\lambda^T, {\lambda_1}^T, \mu^T, {\mu_1}^T : X \to \left]^-0, 1^+\right[$

$\lambda^I, {\lambda_1}^I, \mu^I, {\mu_1}^I : X \to \left]^-0, 1^+\right[$

$\lambda^F, {\lambda_1}^F, \mu^F, {\mu_1}^F : X \to \left]^-0, 1^+\right[$

are such that

$\lambda^T(a) = 0, \lambda^I(a) = 1, \lambda^F(a) = 1, \lambda^T(b) = 0.7, \lambda^I(b) = 0.7, \lambda^F(b) = 0.7, \lambda^T(c) = 0,$

$\lambda^I(c) = 1, \lambda^F(c) = 1$

${\lambda_1}^T(a) = 0.6, {\lambda_1}^I(a) = 0.3, {\lambda_1}^F(a) = 0.3, {\lambda_1}^T(b) = 0.7, {\lambda_1}^I(b) = 0.7, {\lambda_1}^F(b) = 0.7,$

${\lambda_1}^T = 0.4, {\lambda_1}^I(c) = 0.1, {\lambda_1}^F(c) = 0.4$

$\mu^T(a) = 0.6, \mu^I(a) = 0.5, \mu^F(a) = 0.4, \mu^T(b) = 0.6, \mu^I(b) = 0.5, \mu^F(b) = 0.4,$

$\mu^T(c) = 0.6, \mu^I(c) = 0.5, \mu^F(c) = 0.4$

${\mu_1}^T(a) = 0.6, {\mu_1}^I(a) = 0.4, {\mu_1}^F(a) = 0.3, {\mu_1}^T(b) = 0.7, {\mu_1}^I(b) = 0.5, {\mu_1}^F(b) = 0.2,$

${\mu_1}^T(c) = 0.8, {\mu_1}^I(c) = 0.4, {\mu_1}^F(c) = 0.4$

Clearly (X, τ) and (Y, σ) are neutrosophic topological spaces.

Define g : (X, τ) → (Y, σ), f_1 : X → X and f_2 : Y→ Y as g(a) = b, g(b) = c, g(c) =a, f_1(a) = b, f_1(b) =b, f_1(c) = b and f_2(a) = b, f_2(b) =c, f_2(c) = a.

Let $\alpha^T : X \to \left]^-0, 1^+\right[$

$\alpha^I : X \to \left]^-0, 1^+\right[$

$\alpha^F : X \to \left]^-0, 1^+\right[$ be any neutrosophic set such that

$\alpha^T(a) = 0, \alpha^I(a) = 1, \alpha^F(a) = 1, \alpha^T(b) = 0.3, \alpha^I(b) = 0.8, \alpha^F(b) = 0.8, \alpha^T(c) = 0, \alpha^I(c) = 1, \alpha^F(c) = 1$

For the neutrosophic orbit open set μ under the mapping f_2 in (Y, σ) with g(α) $\leq$ μ.

Now the neutrosophic orbit open set λ under the mapping f_1 in (X, τ) with $\alpha \leq \lambda$ such that g(λ) $\leq$ cl(μ). Then g is weakly neutrosophic orbit continuous.

Now, g(α) $\leq$ μ. But there is no neutrosophic orbit open set λ under the mapping f_1 with $\alpha \leq \lambda$ such that g(λ) $\nleq$ μ.Thus g is not neutrosophic orbit* continuous.

3.38 Proposition Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces. If g : (X, τ) → (Y,σ) is neutrosophic orbit continuous, then g is neutrosophic orbit* continuous.

Proof: Let $f_1 : X \to X$ and $f_2 : Y \to Y$ be any two mappings. Let α be any neutrosophic set and μ be any neutrosophic orbit open set under the mapping f_2 with $g(\alpha) \leq \mu$. By Corollary 2.7, $\alpha \leq g^{-1}(g(\alpha)) \leq g^{-1}(\mu)$. Then $\alpha \leq g^{-1}(\mu)$. Since g is neutrosophic orbit continuous, $\alpha \leq g^{-1}(\mu) = \lambda, \lambda$ is a neutrosophic orbit open set under the mapping f_1. By Corollary 2.7, $gg^{-1}(\mu) \leq \mu$. Therefore g(λ) = $gg^{-1}(\mu) \leq \mu$ which implies that g(λ) $\leq$ μ. Then g is neutrosophic orbit* continuous.

3.39 Remark The converse of the Proposition 3.30 need not be true as shown in the following example.

3.40 Example Let X={a, b, c}=Y. Define τ = {0_N, 1_N, λ, λ_1 }and σ = {0_N, 1_N, μ, μ_1 } where

$\lambda^T, {\lambda_1}^T, \mu^T, {\mu_1}^T : X \to \left]^-0, 1^+\right[$

$\lambda^I, {\lambda_1}^I, \mu^I, {\mu_1}^I : X \to \left]^-0, 1^+\right[$

$\lambda^F, {\lambda_1}^F, \mu^F, {\mu_1}^F : X \to \left]^-0, 1^+\right[$

are such that

$\lambda^T(a) = 0, \lambda^I(a) = 1, \lambda^F(a) = 1, \lambda^T(b) = 0, \lambda^I(b) = 1, \lambda^F(b) = 1, \lambda^T(c) = 0.5,$

$\lambda^I(c) = 0.6, \lambda^F(c) = 0.8$

${\lambda_1}^T(a) = 0.4, {\lambda_1}^I(a) = 0.5, {\lambda_1}^F(a) = 0.7, {\lambda_1}^T(b) = 0.6, {\lambda_1}^I(b) = 0.4, {\lambda_1}^F(b) = 0.3,$

${\lambda_1}^T = 0.5, {\lambda_1}^I(c) = 0.6, {\lambda_1}^F(c) = 0.4$

$\mu^T(a) = 0.6, \mu^I(a) = 0.5, \mu^F(a) = 0.4, \mu^T(b) = 0.6, \mu^I(b) = 0.5, \mu^F(b) = 0.4,$

$\mu^T(c) = 0.6, \mu^I(c) = 0.5, \mu^F(c) = 0.4$

${\mu_1}^T(a) = 0.6, {\mu_1}^I(a) = 0.4, {\mu_1}^F(a) = 0.3, {\mu_1}^T(b) = 0.7, {\mu_1}^I(b) = 0.5, {\mu_1}^F(b) = 0.2,$

${\mu_1}^T(c) = 0.8, {\mu_1}^I(c) = 0.4, {\mu_1}^F(c) = 0.4$

Clearly (X, τ) and (Y, σ) are neutrosophic topological spaces.

Define g : (X, τ) → (Y, σ), f_1 : X → X and f_2 : Y→ Y as g(a) = b, g(b) = c, g(c) =a, f_1(a) = c, f_1(b) =c, f_1(c) = c and f_2(a) = b, f_2(b) =c, f_2(c) = a.

Let $\alpha^T : X \to \left]{}^-0, 1^+\right[$

$\alpha^I : X \to \left]{}^-0, 1^+\right[$

$\alpha^F : X \to \left]{}^-0, 1^+\right[$ be any neutrosophic set such that

$\alpha^T(a) = 0, \alpha^I(a) = 1, \alpha^F(a) = 1, \alpha^T(b) = 0, \alpha^I(b) = 1, \alpha^F(b) = 1, \alpha^T(c) = 0.2, \alpha^I(c) = 0.8, \alpha^F(c) = 0.9$

For the neutrosophic orbit open set μ under the mapping f_2 in (Y, σ), g(α) $\leq \mu$.

Now the neutrosophic orbit open set λ under the mapping f_1 in (X, τ) with $\alpha \leq \lambda$ such that g(λ) $\leq \mu$. Then g is neutrosophic orbit* continuous.

Now the neutrosophic open sets μ_1 in (Y, σ), but $g^{-1}(\mu_1)$ is not neutrosophic orbit open under the mapping f_1 in (X, τ). Thus g is not neutrosophic orbit continuous.

4. Conclusions

In this paper, we study the collection of neutrosophic orbit open sets under the mapping f : X → X. The characterization of neutrosophic orbit continuous functions are studied. Some interrelations are discussed with suitable examples. This paper paves way in future to introduce and study the family of all neutrosophic orbit open sets constructs a neutrosophic topological space.

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