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Abstract. In this paper, first of all, considering the notions of single-valued neutrosophic and general fuzzy automata we present the concept of single-valued neutrosophic general machine, to simplicity, SVNGM. Also, for a given SVNGM $\mathcal{M}$, we give the concept of single-valued neutrosophic sub-general machine (SVNSGM) of $\mathcal{M}$. Moreover, we show that if there exists a strong homomorphism between two SVNGM, then there is a connection between the SVNSGM of them. Further, we give the notion of single-valued neutrosophic strong sub-general machine (SVNSSGM). In addition, we show that for a given SVNGM $\mathcal{M}$ if $\mathcal{M}'$ is a SVNSSGM of $\mathcal{M}$, then $\mathcal{M}'$ is a SVNSGM of $\mathcal{M}$, but the converse does not hold.

Keywords: Neutrosophic set; Automata; Intuitionistic set; Submachine; General fuzzy automata

1. Introduction

The idea of ‘fuzzy‘ and a number of other notions in mathematics and other fields were fuzzified by Zadeh [18] in 1965. The concept of fuzzy automaton suggested by Wee [17] and Santos [10]. Doostfatemeh and Kremer [3] introduced the concept of general fuzzy automata.

An intuitionistic fuzzy set may be considered an alternative approach when the available information is not sufficient to define the vagueness of the conventional fuzzy set. In fuzzy sets the degree of acceptance is taken into account solely however intuitionistic fuzzy set is characterized by a membership function and a non-membership function, the only need is that the sum of both values is less and equal to one. Intuitionistic fuzzy set will solely contend with incomplete information but not the indeterminate information and inconsistent information that commonly exists within the certainty system.

In intuitionistic fuzzy sets, indeterminacy is its hesitation part by default. Neutrosophy is one of the helpful tools for managing uncertainty in concrete problems. Neutrosophy is a branch of philosophy that was introduced by Florentin Smarandache [4–6]. Afterwards, to generalise, Shamsizadeh and Zahedi introduced and studied the concept of intuitionistic general fuzzy automata [12]. For further details see recent literature such as [1, 2, 7, 11, 13, 14].

Neutrosophy deals with the origin, nature and scope of neutralities, as well as their interactions with various ideational spectra. Neutrosophy is the foundation of neutrosophic sets (derivative of neutrosophy). Wang et al. [16] introduced single valued neutrosophic sets that is a neutrosophic set defined in the range [0, 1]. Wang et.al: [15] presented the concepts of interval-valued neutrosophic sets. Tahir Mahmood presented to the idea of interval neutrosophic finite state machine [9]. In 2019 [8] the idea of neutrosophic general fuzzy automata was presented by Kavikumar. The basic advantage of incorporating neutrosophic sets into general fuzzy automata is the ability to bring indeterminacy membership and nonmembership in every transition and active states that help us to overcome the uncertain situation at the time of predicting the next active state.

The present paper is organized as follows: Section 2 encompasses preliminary information pertaining to the content of the paper. In Section 3 by considering the notions of single-valued neutrosophic and general fuzzy automata we focus on the study of the concepts of single-valued neutrosophic general machine (SVNGM). Also, for a given SVNGM $\mathcal{M}$, we confer the concept of single-valued neutrosophic sub-general machine (SVNSGM) of $\mathcal{M}$. Moreover, we show that if there exists a strong homomorphism between two SVNGM, then there is a connection between SVNSGM of them. Section 4 is towards the study the notion of single-valued neutrosophic strong sub-general machine (SVNSSGM). Also, we show that for a given SVNGM $\mathcal{M}$ if $\mathcal{M}'$ is a SVNSSGM of $\mathcal{M}$, then $\mathcal{M}'$ is a SVNSGM of $\mathcal{M}$, but the converse is not true.

2. **Preliminaries**

In this section, some concepts and definitions related to single-valued neutrosophy and automata are introduced.

Definition 2.1. [3] A general fuzzy automaton (GFA) is considered as:

$$\tilde{F} = (Q, \Sigma, \tilde{R}, Z, \tilde{\delta}, \omega, F_1, F_2),$$

where (i) Q is a finite set of states, (ii) Σ is a finite set of input symbols, (iii) $\tilde{R}$ is the set of fuzzy start states, $\tilde{R} \subseteq \tilde{P}(Q)$, (iv) Z is a finite set of output symbols, $Z = \{b_1, b_2, \ldots, b_k\}$, (v) $\omega : Q \to Z$ is the output function, (vi) $\tilde{\delta} : (Q \times [0,1]) \times \Sigma \times Q \to [0,1]$ is the augmented transition function. (vii) Function $F_1 : [0,1] \times [0,1] \to [0,1]$ is called membership assignment function. Function $F_1(\mu, \delta)$, as is seen, is motivated by two parameters μ and δ, wherever μ is that the membership value of a predecessor and δ is that the value of a transition.

With this definition, the process that happens upon the transition from state q_i to q_j an input a_k is characterized by:

$$\mu^{t+1}(q_j) = \tilde{\delta}((q_i, \mu^t(q_i)), a_k, q_j) = F_1(\mu^t(q_i), \delta(q_i, a_k, q_j)).$$

It denote that membership value (mv) of the state q_j at time $t+1$ is calculated by function F_1 utilizing both the membership value of q_i at time t and the value of the transition.

(viii) $F_2 : [0,1]^* \to [0,1]$, is called multi-membership resolution function. The multi-membership resolution function determines the multi-membership active states and assigns them a unique membership value.

Definition 2.2. Let Σ be a space of points, with a generic element in Σ denoted by x. A neutrosophic set A in Σ is characterised by a truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is $T_A : \Sigma \to]0^-, 1^+[, I_A : \Sigma \to]0^-, 1^+[, F_A : \Sigma \to]0^-, 1^+[$. There is no restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

Definition 2.3. Single-valued neutrosophic set is the immediate results of neutrosophic set if it is defined over standard unit interval $[0,1]$ instead of the non-standard unit interval $]0^-, 1^+[$. A single-valued neutrosophic subset (SVNS) A of Q is defined by $SVNS(A) = \{(x, T_A(x), I_A(x), F_A(x)) \big| x \in \Sigma\}$, where $T_A(x), I_A(x), F_A(x) : \Sigma \to [0,1]$ such that $0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3$.

3. Single-valued neutrosophic general machine

Definition 3.1. A single-valued neutrosophy general machine (SVNGM) $\mathcal{M}$ is a six-tuple machine denoted by $\mathcal{M} = (Q, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$, where

1. Q is a finite set of states,
2. Σ is a finite set of input symbols,
3. $\tilde{R} \subseteq \tilde{P}(Q)$ is the set of single-valued neutrosophic initial states,
4. $\tilde{\delta} : (Q \times [0,1] \times [0,1] \times [0,1]) \times \Sigma \times Q \to [0,1] \times [0,1] \times [0,1]$ is the single-valued neutrosophic augmented transition function,
5. $E_1 = (E_1^T, E_1^I, E_1^F)$, where $E_1^T : [0,1] \times [0,1] \to [0,1]$ is a t-norm and it is called the truth-membership assignment function. $E_1^T(T, T_\delta)$ is motivated by two parameters T and T_δ, where T is the truth-membership value of a predessor and T_δ is the truth-membership value of the transition. Also, $E_1^I : [0,1] \times [0,1] \to [0,1]$ is a t-norm and it is called the indeterminacy-membership function. $E_1^I(I, I_\delta)$ is motivated by two parameters I and I_δ, where I is the indeterminacy-membership value of a predessor and I_δ is the indeterminacy-membership value of the transition. Moreover, $E_1^F : [0,1] \times$

$[0,1] \to [0,1]$ is a t-conorm and it is called the falsity-membership function. $E_1^F(F, F_\delta)$ is motivated by two parameters F and F_δ, where F is the falsity-membership value of a predessor and F_δ is the falsity-membership value of the transition.

In this definition, the process that takes place upon the transition from the state q_i to q_j on an input a_k is represented by:

$$T^{t+1}(q_j) = \tilde{\delta}_1((q_i, T^t(q_i), I^t(q_i), F^t(q_i)), a_k, q_j) = E_1^T(T^t(q_i), \delta_1(q_i, a_k, q_j)),$$
$$I^{t+1}(q_j) = \tilde{\delta}_2((q_i, T^t(q_i), I^t(q_i), F^t(q_i)), a_k, q_j) = E_1^I(I^t(q_i), \delta_2(q_i, a_k, q_j)),$$
$$F^{t+1}(q_j) = \tilde{\delta}_3((q_i, T^t(q_i), I^t(q_i), F^t(q_i)), a_k, q_j) = E_1^F(T^t(q_i), \delta_3(q_i, a_k, q_j)),$$

where

$$\tilde{\delta}((q_i, T^t(q_i), I^t(q_i), F^t(q_i)), a_k, q_j) = (\tilde{\delta}_1((q_i, T^t(q_i), I^t(q_i), F^t(q_i)), a_k, q_j), \tilde{\delta}_2((q_i, T^t(q_i), I^t(q_i), F^t(q_i)), a_k, q_j), \tilde{\delta}_3((q_i, T^t(q_i), I^t(q_i), F^t(q_i)), a_k, q_j)),$$

and

$$\delta(q_i, a_k, q_j) = (\delta_1(q_i, a_k, q_j), \delta_2(q_i, a_k, q_j), \delta_3(q_i, a_k, q_j)).$$

6. $E_2 = (E_2^T, E_2^I, E_2^F)$, where $E_2^T : [0,1] \times [0,1] \to [0,1]$ is a T-conorm and it is called multi-truth-membership function, $E_2^I : [0,1] \times [0,1] \to [0,1]$ is a T-conorm and it is called multi-indetermincy-membership function, $E_2^F : [0,1] \times [0,1] \to [0,1]$ is a T-norm and it is called multi-falsity-membership function.

Example 3.2. Let the SVNGM $\mathcal{M} = (Q, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ such that $Q = \{q_0, q_1, q_2\}, \Sigma = \{a\}$, $\tilde{R} = \{(q_0, 0.4, 0.7, 0.3)\}$ and δ is defined as follows:

$$\delta(q_0, a, q_0) = (0.6, 0.7, 1), \qquad \delta(q_0, a, q_1) = (0.7, 0.5, 0.5),$$
$$\delta(q_0, a, q_2) = (0.9, 0.7, 0.4), \qquad \delta(q_1, a, q_1) = (0.4, 0.5, 0.2),$$
$$\delta(q_1, a, q_2) = (0.3, 0.7, 0.6), \qquad \delta(q_2, a, q_0) = (0.7, 0.9, 0.6),$$
$$\delta(q_2, a, q_1) = (0.7, 1, 1), \qquad \delta(q_2, a, q_2) = (0.6, 0.9, 0.5).$$

Now, we can consider E_1 as follows:

1. $E_1^T = T \wedge T_\delta,\ E_1^I = I \wedge I_\delta,\ E_1^F = F \vee F_\delta$,

$$T^{t+1}(q_m) = \bigvee_{i=1}^{n} E_1^T(T^t(q_i), \delta_1(q_i, a_k, q_m)),$$
$$I^{t+1}(q_m) = \bigvee_{i=1}^{n} E_1^I(I^t(q_i), \delta_2(q_i, a_k, q_m)),$$
$$F^{t+1}(q_m) = \bigwedge_{i=1}^{n} E_3^F(F^t(q_i), \delta_3(q_i, a_k, q_m)),$$

2. $E_1^T = T.T_\delta,\ E_1^I = I.I_\delta,\ E_1^F = F + F_\delta - F.F_\delta,$

$$T^{t+1}(q_m) = \bigvee_{i=1}^{n} E_1^T(T^t(q_i), \delta_1(q_i, a_k, q_m)),$$
$$I^{t+1}(q_m) = \bigvee_{i=1}^{n} E_1^I(I^t(q_i), \delta_2(q_i, a_k, q_m)),$$
$$F^{t+1}(q_m) = \bigwedge_{i=1}^{n} E_3^F(F^t(q_i), \delta_3(q_i, a_k, q_m)),$$

3. $E_1^T = T \wedge T_\delta,\ E_1^I = I \wedge I_\delta,\ E_1^F = F \vee F_\delta,$

$$T^{t+1}(q_m) = T_p(T_p(T^t(q_i), \delta_1(q_i, a_k, q_m))),$$
$$I^{t+1}(q_m) = T_p(T_p(I^t(q_i), \delta_2(q_i, a_k, q_m)),$$
$$F^{t+1}(q_m)) = S_p(S_p E_3^F(F^t(q_i), \delta_3(q_i, a_k, q_m))),$$

where T_p is the product t-norm and S_p is the product t-conorm.
If we choose the case 1, then we have

$$T^{t_1}(q_0) = E_1^T(T^{t_0}(q_0), \delta_1(q_0, a, q_0)) = 0.4 \wedge 0.6 = 0.4,$$
$$I^{t_1}(q_0) = E_1^I(I^{t_0}(q_0), \delta_2(q_0, a, q_0)) = 0.7 \wedge 0.7 = 0.7,$$
$$F^{t_1}(q_0) = E_1^F(F^{t_0}(q_0), \delta_3(q_0, a, q_0)) = 0.3 \vee 1 = 1,$$
$$T^{t_1}(q_1) = E_1^T(T^{t_0}(q_0), \delta_1(q_0, a, q_1)) = 0.4 \wedge 0.7 = 0.4,$$
$$I^{t_1}(q_1) = E_1^I(I^{t_0}(q_0), \delta_2(q_0, a, q_1)) = 0.7 \wedge 0.5 = 0.5,$$
$$F^{t_1}(q_1) = E_1^F(F^{t_0}(q_0), \delta_3(q_0, a, q_1)) = 0.3 \vee 0.5 = 0.5,$$
$$T^{t_1}(q_2) = E_1^T(T^{t_0}(q_0), \delta_1(q_0, a, q_2)) = 0.4 \wedge 0.9 = 0.4,$$
$$I^{t_1}(q_2) = E_1^I(I^{t_0}(q_0), \delta_2(q_0, a, q_2)) = 0.7 \wedge 0.7 = 0.7,$$
$$F^{t_1}(q_2) = E_1^F(F^{t_0}(q_0), \delta_3(q_0, a, q_2)) = 0.3 \vee 0.4 = 0.4,$$

$$T^{t_2}(q_0) = E_1^T(T^{t_1}(q_0), \delta_1(q_0, a, q_0)) \vee E_1^T(T^{t_1}(q_2), \delta_1(q_2, a, q_0)) = (0.4 \wedge 0.6) \vee (0.4 \wedge 0.7) = 0.4,$$

$$I^{t_2}(q_0) = E_1^I(I^{t_1}(q_0), \delta_2(q_0, a, q_0)) \vee E_1^I(I^{t_1}(q_2), \delta_2(q_2, a, q_0)) = (0.7 \wedge 0.7) \vee (0.7 \wedge 0.9) = 0.7,$$

$$F^{t_2}(q_0) = E_1^F(F^{t_1}(q_0), \delta_3(q_0, a, q_0)) \wedge E_1^F(F^{t_1}(q_2), \delta_3(q_2, a, q_0)) = (1 \vee 1) \wedge (0.4 \vee 0.6) = 0.6,$$

$$\begin{aligned} T^{t_2}(q_1) &= E_1^T(T^{t_1}(q_0), \delta_1(q_0, a, q_1)) \vee E_1^T(T^{t_1}(q_1), \delta_1(q_1, a, q_1)) \vee E_1^T(T^{t_1}(q_2), \delta_1(q_2, a, q_1)) \\ &= (0.4 \wedge 0.7) \vee (0.4 \wedge 0.4) \vee (0.4 \wedge 0.7) = 0.4, \end{aligned}$$

$$\begin{aligned} I^{t_2}(q_1) &= E_1^I(I^{t_1}(q_0), \delta_2(q_0, a, q_1)) \vee E_1^I(I^{t_1}(q_1), \delta_2(q_1, a, q_1)) \vee E_1^I(I^{t_1}(q_2), \delta_2(q_2, a, q_1)) \\ &= (0.7 \wedge 0.5) \vee (0.5 \wedge 0.5) \vee (0.7 \wedge 1) = 0.7, \end{aligned}$$

$$\begin{aligned} F^{t_2}(q_1) &= E_1^F(F^{t_1}(q_0), \delta_3(q_0, a, q_1)) \wedge E_1^F(F^{t_1}(q_1), \delta_3(q_1, a, q_1)) \wedge E_1^F(F^{t_1}(q_2), \delta_3(q_2, a, q_1)) \\ &= (1 \vee 0.5) \wedge (0.5 \vee 0.2) \wedge (0.4 \vee 1) = 0.5, \end{aligned}$$

$$\begin{aligned} T^{t_2}(q_2) &= E_1^T(T^{t_1}(q_0), \delta_1(q_0, a, q_2)) \vee E_1^T(T^{t_1}(q_1), \delta_1(q_1, a, q_2)) \vee E_1^T(T^{t_1}(q_2), \delta_1(q_2, a, q_2)) \\ &= (0.4 \wedge 0.9) \vee (0.4 \wedge 0.3) \vee (0.4 \wedge 0.6) = 0.4, \end{aligned}$$

$$\begin{aligned} I^{t_2}(q_2) &= E_1^I(I^{t_1}(q_0), \delta_2(q_0, a, q_2)) \vee E_1^I(I^{t_1}(q_1), \delta_2(q_1, a, q_2)) \vee E_1^I(I^{t_1}(q_2), \delta_2(q_2, a, q_2)) \\ &= (0.7 \wedge 0.7) \vee (0.5 \wedge 0.7) \vee (0.7 \wedge 0.9) = 0.7, \end{aligned}$$

$$\begin{aligned} F^{t_2}(q_2) &= E_1^F(F^{t_1}(q_0), \delta_3(q_0, a, q_2)) \wedge E_1^F(F^{t_1}(q_1), \delta_3(q_1, a, q_2)) \wedge E_1^F(F^{t_1}(q_2), \delta_3(q_2, a, q_2)) \\ &= (1 \vee 0.4) \wedge (0.5 \vee 0.6) \wedge (0.4 \vee 0.5) = 0.5. \end{aligned}$$

Clearly, we can see that there are three simultaneous transition to the action states q_0, q_1 and q_2 at time t_2.

Definition 3.3. Let $\mathcal{M} = (Q, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ be a SVNGM. We define max-min SVNGM $\mathcal{M} = (Q, \Sigma, \tilde{\delta}^*, \tilde{R}, E_1, E_2)$ such that $\tilde{\delta}^* : Q_{act} \times \Sigma^* \times Q \to [0,1] \times [0,1] \times [0,1]$, where $Q_{act} = \{Q_{act}(t_0), Q_{act}(t_1), ...\}$ and for every $i \geq 0$,

$$\tilde{\delta}_1^*((q, T^t(q), I^t(q), F^t(q)), \Lambda, p) = \begin{cases} 1 & \text{if p=q} \\ 0 & \text{otherwise} \end{cases}, \tag{1}$$

$$\tilde{\delta}_2^*((q, T^t(q), I^t(q), F^t(q)), \Lambda, p) = \begin{cases} 1 & \text{if p=q} \\ 0 & \text{otherwise} \end{cases}, \tag{2}$$

$$\tilde{\delta}_3^*((q, T^t(q), I^t(q), F^t(q)), \Lambda, p) = \begin{cases} 0 & \text{if p=q} \\ 1 & \text{otherwise} \end{cases}, \tag{3}$$

and for every $i \geq 1, \tilde{\delta}^*((q, T^t(q), I^t(q), F^t(q)), a, p) = \tilde{\delta}((q, T^t(q), I^t(q), F^t(q)), a, p)$ and recursively,

$$\begin{aligned} &\tilde{\delta}_1^*((q, T^t(q), I^t(q), F^t(q)), a_1a_2...a_n, p) = \vee\{\tilde{\delta}_1((q, T^t(q), I^t(q), F^t(q)), a_1, p_1) \wedge ... \\ &\quad \wedge \tilde{\delta}_1((p_{n-1}, T^t(p_{n-1}), I^t(p_{n-1}), F^t(p_{n-1})), a_n, p) \big| p_1 \in Q_{act}(t_1), p_2 \in Q_{act}(t_2), ..., p_{n-1} \in Q_{act}(t_{n-1})\}, \end{aligned}$$

$$\tilde{\delta}_2^*((q, T^t(q), I^t(q), F^t(q)), a_1a_2...a_n, p) = \vee\{\delta_2((q, T^t(q), I^t(q), F^t(q)), a_1, p_1) \wedge ...$$
$$\wedge\, \tilde{\delta}_2((p_{n-1}, T^t(p_{n-1}), I^t(p_{n-1}), F^t(p_{n-1})), a_n, p)\big|p_1 \in Q_{act}(t_1), p_2 \in Q_{act}(t_2), ..., p_{n-1} \in Q_{act}(t_{n-1})\},$$

$$\tilde{\delta}_3^*((q, T^t(q), I^t(q), F^t(q)), a_1a_2...a_n, p) = \wedge\{\tilde{\delta}_3((q, T^t(q), I^t(q), F^t(q)), a_1, p_1) \vee ...$$
$$\vee\, \tilde{\delta}_3((p_{n-1}, T^t(p_{n-1}), I^t(p_{n-1}), F^t(p_{n-1})), a_n, p)\big|p_1 \in Q_{act}(t_1), p_2 \in Q_{act}(t_2), ..., p_{n-1} \in Q_{act}(t_{n-1})\},$$

in which $a_i \in \Sigma$, for all $1 \leq i \leq n$ and assuming that the entered input at time t_i is a_i, for $1 \leq i \leq n-1$.

In the rest of paper, instead of max-min SVNGM we say that SVNGM.

Definition 3.4. Let $\mathcal{M} = (Q, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ be a SVNGM. Let N be a single-valued neutrosophic subset of Q. Then $\mathcal{M}' = (Q, N, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ is called a single-valued neutrosophic subgeneral machine (SVNSGM) of $\mathcal{M}$ if

$$T_N(q) \geq T_N(p) \wedge \tilde{\delta}_1((p, T^t(p), I^t(p), F^t(p)), a, q),$$
$$I_N(q) \geq I_N(p) \wedge \tilde{\delta}_2((p, T^t(p), I^t(p), F^t(p)), a, q),$$
$$F_N(q) \leq F_N(p) \vee \tilde{\delta}_3((p, T^t(p), I^t(p), F^t(p)), a, q),$$

for every $p, q \in Q$ and $a \in \Sigma$.

Theorem 3.5. *Let $\mathcal{M} = (Q, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ be a SVNGM and N be a SVNS of Q. Then $\mathcal{M}' = (Q, N, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ is a SVNSGM of $\mathcal{M}$ if and only if*

$$T_N(q) \geq T_N(p) \wedge \tilde{\delta}_1^*((p, T^t(p), I^t(p), F^t(p)), x, q),$$
$$I_N(q) \geq I_N(p) \wedge \tilde{\delta}_2^*((p, T^t(p), I^t(p), F^t(p)), x, q),$$
$$F_N(q) \leq F_N(p) \vee \tilde{\delta}_3^*((p, T^t(p), I^t(p), F^t(p)), x, q),$$

for every $p, q \in Q$ and $x \in \Sigma^*$.

Proof. Let $\mathcal{M}'$ be a SVNSGM of $\mathcal{M}$. We prove the claim by induction on $|x| = n$. If $n = 0$, then $x = \Lambda$. Let $p = q$. Then

$$T_N(p) \wedge \tilde{\delta}_1^*((p, T^t(p), I^t(p), F^t(p)), \Lambda, q) \leq T_N(p) = T_N(q),$$
$$I_N(p) \wedge \tilde{\delta}_2^*((p, T^t(p), I^t(p), F^t(p)), \Lambda, q) \leq I_N(p) = I_N(q),$$
$$F_N(p) \vee \tilde{\delta}_3^*((p, T^t(p), I^t(p), F^t(p)), \Lambda, q) \geq F_N(p) = F_N(q),$$

for every $p, q \in Q$ and $x \in \Sigma^*$. Now, let $p \neq q$. Then

$$T_N(p) \wedge \tilde{\delta}_1^*((p, T^t(p), I^t(p), F^t(p)), \Lambda, q) = 0 \leq T_N(q),$$
$$I_N(p) \wedge \tilde{\delta}_2^*((p, T^t(p), I^t(p), F^t(p)), \Lambda, q) = 0 \leq I_N(q),$$
$$F_N(p) \vee \tilde{\delta}_3^*((p, T^t(p), I^t(p), F^t(p)), \Lambda, q) = 1 \geq F_N(q).$$

So, the claim is true for $n = 0$. Now, let the claim holds for every $y \in \Sigma^*$ such that $|y| = n-1, n \geq 1$. Let $x = ya$, $|y| = n-1, y \in \Sigma^*$ and $a \in \Sigma$. Then we have

$$\begin{aligned}
T_N(p) \wedge \tilde{\delta}_1^*((p, T^t(p), I^t(p), F^t(p)), x, q) &= T_N(p) \wedge \tilde{\delta}_1^*((p, T^t(p), I^t(p), F^t(p)), ya, q) \\
&= T_N(p) \wedge \big(\vee \{\tilde{\delta}_1^*((p, T^t(p), I^t(p), F^t(p)), y, r) \\
&\wedge \tilde{\delta}_1^*((r, T^{t+n-1}(r), I^{t+n-1}(r), F^{t+n-1}(r)), a, q) \big| r \in Q\}\big) \\
&= \vee\{T_N(p) \wedge \tilde{\delta}_1^*((p, T^t(p), I^t(p), F^t(p)), y, r) \wedge \\
&\tilde{\delta}_1^*((r, T^{t+n-1}(r), I^{t+n-1}(r), F^{t+n-1}(r)), a, q) \big| r \in Q\} \\
&\leq \vee\{T_N(r) \wedge \tilde{\delta}_1^*((r, T^{t+n-1}(r), I^{t+n-1}(r), F^{t+n-1}(r)), a, q) \big| r \in Q\} \\
&\leq T_N(q),
\end{aligned}$$

$$\begin{aligned}
I_N(p) \wedge \tilde{\delta}_2^*((p, T^t(p), I^t(p), F^t(p)), x, q) &= I_N(p) \wedge \tilde{\delta}_2^*((p, T^t(p), I^t(p), F^t(p)), ya, q) \\
&= I_N(p) \wedge \big(\vee \{\tilde{\delta}_2^*((p, T^t(p), I^t(p), F^t(p)), y, r) \\
&\wedge \tilde{\delta}_2^*((r, T^{t+n-1}(r), I^{t+n-1}(r), F^{t+n-1}(r)), a, q) \big| r \in Q\}\big) \\
&= \vee\{I_N(p) \wedge \tilde{\delta}_2^*((p, T^t(p), I^t(p), F^t(p)), y, r) \\
&\wedge \tilde{\delta}_2^*((r, T^{t+n-1}(r), I^{t+n-1}(r), F^{t+n-1}(r)), a, q) \big| r \in Q\} \\
&\leq \vee\{I_N(r) \wedge \tilde{\delta}_2^*((r, T^{t+n-1}(r), I^{t+n-1}(r), F^{t+n-1}(r)), a, q) \big| r \in Q\} \\
&\leq I_N(q),
\end{aligned}$$

$$\begin{aligned}
F_N(p) \vee \tilde{\delta}_3^*((p, T^t(p), I^t(p), F^t(p)), x, q) &= F_N(p) \vee \tilde{\delta}_3^*((p, T^t(p), I^t(p), F^t(p)), ya, q) \\
&= F_N(p) \vee \big(\wedge \{\tilde{\delta}_3^*((p, T^t(p), I^t(p), F^t(p)), y, r) \\
&\vee \tilde{\delta}_3^*((r, T^{t+n-1}(r), I^{t+n-1}(r), F^{t+n-1}(r)), a, q) \big| r \in Q\}\big) \\
&= \wedge\{F_N(p) \vee \tilde{\delta}_3^*((p, T^t(p), I^t(p), F^t(p)), y, r) \\
&\vee \tilde{\delta}_3^*((r, T^{t+n-1}(r), I^{t+n-1}(r), F^{t+n-1}(r)), a, q) \big| r \in Q\} \\
&\geq \wedge\{F_N(r) \vee \tilde{\delta}_1^*((r, T^{t+n-1}(r), I^{t+n-1}(r), F^{t+n-1}(r)), a, q) \big| r \in Q\} \\
&\geq F_N(q).
\end{aligned}$$

Hence, the claim holds. □

Example 3.6. Let $Q = \{p_1, p_2\}, \Sigma = \{a, b\}$, $\tilde{R} = \{(p_1, 0.7, 0.8, 0.4), (p_2, 0.7, 0.8, 0.4)\}$, $\delta(q, a, q') = (0.5, 0.6, 0.7)$, for every $q, q' \in Q$. Let $N = \{(p_1, 0.6, 0.8, 0.6), (p_2, 0.5, 0.6, 0.7)\}$. Then

$$\tilde{\delta}_1^*((q_1, T^{t_0}(q_1), I^{t_0}(q_1), F^{t_0}(q_1)), a, q_2) = 0.7 \wedge 0.5 = 0.5$$
$$\tilde{\delta}_2^*((q_1, T^{t_0}(q_1), I^{t_0}(q_1), F^{t_0}(q_1)), a, q_2) = 0.8 \wedge 0.6 = 0.6$$
$$\tilde{\delta}_3^*((q_1, T^{t_0}(q_1), I^{t_0}(q_1), F^{t_0}(q_1)), a, q_2) = 0.4 \vee 0.7 = 0.4,$$

so $\tilde{\delta}((q_1, T^{t_0}(q_1), I^{t_0}(q_1), F^{t_0}(q_1)), a, q_2) = (0.5, 0.6, 0.7)$, for every $q_1, q_2 \in Q$. Then

$$T_N(p_1) \wedge \tilde{\delta}_1^*((p_1, T^{t_0}(p_1), I^{t_0}(p_1), F^{t_0}(p_1)), a, p_2) = 0.5 = T_N(p_2)$$
$$I_N(p_1) \wedge \tilde{\delta}_2^*((p_1, T^{t_0}(p_1), I^{t_0}(p_1), F^{t_0}(p_1)), a, p_2) = 0.6 = I_N(p_2)$$
$$F_N(p_1) \vee \tilde{\delta}_3^*((p_1, T^{t_0}(p_1), I^{t_0}(p_1), F^{t_0}(p_1)), a, p_2) = 0.7 = F_N(p_2),$$

and

$$T_N(p_2) \wedge \tilde{\delta}_1^*((p_2, T^{t_0}(p_2), I^{t_0}(p_2), F^{t_0}(p_2)), a, p_1) = 0.5 \leq T_N(p_1)$$
$$I_N(p_2) \wedge \tilde{\delta}_2^*((p_2, T^{t_0}(p_2), I^{t_0}(p_2), F^{t_0}(p_2)), a, p_1) = 0.6 \leq I_N(p_1)$$
$$F_N(p_2) \vee \tilde{\delta}_3^*((p_2, T^{t_0}(p_2), I^{t_0}(p_2), F^{t_0}(p_2)), a, p_1) = 0.7 \geq F_N(p_1).$$

Therefore, $\mathcal{M}' = (Q, N, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ is a SVNSGM of $\mathcal{M} = (Q, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$.

Theorem 3.7. *Let $\mathcal{M} = (Q, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ be a SVNGM and $\mathcal{M}_1 = (Q, N_1, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ and $\mathcal{M} = (Q, N_2, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ be two SVNSGM of $\mathcal{M}$. Then the following hold:*

(1) *$\mathcal{M}_1 \cup \mathcal{M}_2 = (Q, N_1 \cup N_2, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ is a SVNSGM of $\mathcal{M}$, where*

$$N_1 \cup N_2 = (x, T_{N_1}(x) \vee T_{N_2}(x), I_{N_1}(x) \vee I_{N_2}(x), F_{N_1}(x) \wedge F_{N_2}(x)),$$

(2) *$\mathcal{M}_1 \cap \mathcal{M}_2 = (Q, N_1 \cap N_2, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ is a SVNSGM of $\mathcal{M}$, where*

$$N_1 \cap N_2 = (x, T_{N_1}(x) \wedge T_{N_2}(x), I_{N_1}(x) \wedge I_{N_2}(x), F_{N_1}(x) \vee F_{N_2}(x)).$$

Proof. The proofs 1 and 2 are clear. □

Definition 3.8. Let $\mathcal{M} = (Q, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ be a SVNGM and $\mathcal{M}' = (Q, N, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ be a SVNSGM of $\mathcal{M}$. Define the SVNS N_x of Q, for every $x \in \Sigma^*$, as follows: $N_x(q) =$

$(q, T_{N_x}(q), I_{N_x}(q), F_{N_x}(q))$, where

$$\begin{aligned}
T_{N_x}(q) &= \bigvee_{p\in Q} T_N(p) \wedge \tilde{\delta}_1^*((p, T^t(p), I^t(p), F^t(p)), a, q),\\
I_{N_x}(q) &= \bigvee_{p\in Q} I_N(p) \wedge \tilde{\delta}_2^*((p, T^t(p), I^t(p), F^t(p)), a, q)\\
F_{N_x}(q) &= \bigwedge_{p\in Q} F_N(p) \vee \tilde{\delta}_3^*((p, T^t(p), I^t(p), F^t(p)), a, q),
\end{aligned}$$

for every $q \in Q$.

Theorem 3.9. *Let $\mathcal{M} = (Q, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ be a SVNGM. Then for every SVNS N of Q and for every $x, y \in \Sigma^*$, we have $(N_x)_y = N_{xy}$.*

Proof. Let N be a SVNS of Q and $x, y \in \Sigma^*$. We prove the claim by induction on $|y| = n$. Let $n = 0$. Then $y = \Lambda$. Let $q \in Q$. Then

$$\begin{aligned}
T_{(N_x)_\Lambda}(q) &= \bigvee_{p\in Q} T_{N_x}(p) \wedge \tilde{\delta}_1^*((p, T^t(p), I^t(p), F^t(p)), \Lambda, q)\\
&= T_{N_x}(q) \wedge \tilde{\delta}_1^*((q, T^t(q), I^t(q), F^t(q)), \Lambda, q) = T_{N_x}(q),\\
I_{(N_x)_\Lambda}(q) &= \bigvee_{p\in Q} I_{N_x}(p) \wedge \tilde{\delta}_2^*((p, T^t(p), I^t(p), F^t(p)), \Lambda, q)\\
&= I_{N_x}(q) \wedge \tilde{\delta}_2^*((q, T^t(q), I^t(q), F^t(q)), \Lambda, q) = I_{N_x}(q),\\
F_{(N_x)_\Lambda}(q) &= \bigwedge_{p\in Q} F_{N_x}(p) \vee \tilde{\delta}_3^*((p, T^t(p), I^t(p), F^t(p)), \Lambda, q)\\
&= F_{N_x}(q) \vee \tilde{\delta}_3^*((q, T^t(q), I^t(q), F^t(q)), \Lambda, q) = F_{N_x}(q).
\end{aligned}$$

Therefore, $(N_x)_\Lambda = N_{x\Lambda}$. Now, let the claim holds for every SVNS N and for every $y \in \Sigma^*$ such that $|y| = n-1, n \geq 1$. Let $y = wa$, where $w \in \Sigma^*, |w| = n-1$ and $a \in \Sigma$. Then

$$\begin{aligned}
T_{N_{xy}}(q) &= T_{N_{x(wa)}}(q) \\
&= T_{N_{(xw)a}}(q) \\
&= \bigvee_{p\in Q} T_{N_{xw}}(p) \wedge \tilde{\delta}_1^*((p, T^t(p), I^t(p), F^t(p)), a, q) \\
&= \bigvee_{p\in Q} \bigvee_{r\in Q} T_{N_x}(r) \wedge \tilde{\delta}_1^*((r, T^{t-n+1}(r), I^{t-n+1}(r), F^{t-n+1}(r)), w, p) \\
&\wedge \tilde{\delta}_1^*((p, T^t(p), I^t(p), F^t(p)), a, q) \\
&= \bigvee_{r\in Q} T_{N_x}(r) \wedge \bigvee_{p\in Q} \big(\tilde{\delta}_1^*((r, T^{t-n+1}(r), I^{t-n+1}(r), F^{t-n+1}(r)), w, p) \\
&\wedge \tilde{\delta}_1^*((p, T^t(p), I^t(p), F^t(p)), a, q)\big) \\
&= \bigvee_{r\in Q} T_{N_x}(r) \wedge \tilde{\delta}_1^*((r, T^{t-n+1}(r), I^{t-n+1}(r), F^{t-n+1}(r)), wa, r) \\
&= T_{N_{x(wa)}}(q),
\end{aligned}$$

and

$$\begin{aligned}
I_{N_{xy}}(q) &= I_{N_{x(wa)}}(q) \\
&= I_{N_{(xw)a}}(q) \\
&= \bigvee_{p\in Q} I_{N_{xw}}(p) \wedge \tilde{\delta}_2^*((p, T^t(p), I^t(p), F^t(p)), a, q) \\
&= \bigvee_{p\in Q} \bigvee_{r\in Q} I_{N_x}(r) \wedge \tilde{\delta}_2^*((r, T^{t-n+1}(r), I^{t-n+1}(r), F^{t-n+1}(r)), w, p) \\
&\wedge \tilde{\delta}_2^*((p, T^t(p), I^t(p), F^t(p)), a, q) \\
&= \bigvee_{r\in Q} I_{N_x}(r) \wedge \bigvee_{p\in Q} \big(\tilde{\delta}_2^*((r, T^{t-n+1}(r), I^{t-n+1}(r), F^{t-n+1}(r)), w, p) \\
&\wedge \tilde{\delta}_2^*((p, T^t(p), I^t(p), F^t(p)), a, q)\big) \\
&= \bigvee_{r\in Q} I_{N_x}(r) \wedge \tilde{\delta}_2^*((r, T^{t-n+1}(r), I^{t-n+1}(r), F^{t-n+1}(r)), wa, r) \\
&= I_{N_{x(wa)}}(q),
\end{aligned}$$

similarly, $F_{N_{xy}}(q) = I_{(N_x)_y}(q)$, for every $q \in Q$. Hence, the claim holds. □

Definition 3.10. Let A and B be two SVNS of Q. Then $A \subseteq B$ if and only if for every $q \in Q$, $T_A(q) \leq T_B(q), I_A(q) \leq I_B(q)$ and $F_A(q) \geq F_B(q)$.

Theorem 3.11. *Let $\mathcal{M} = (Q, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ be a SVNGM and $\mathcal{M}' = (Q, N, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ be a SVNSGM of $\mathcal{M}$. Then $\mathcal{M}'$ is a SVNSGM of $\mathcal{M}$ if and only if $N_x \subseteq N$, for every $x \in \Sigma^*$.*

Proof. Let $\mathcal{M}'$ be a SVNSGM of $\mathcal{M}$. For every $x \in \Sigma^*$ and $q \in Q$, we have

$$T_{N_x}(q) = \bigvee_{p\in Q} T_N(p) \wedge \tilde{\delta}_1^*((p, T^t(p), I^t(p), F^t(p)), x, q) \leq T_N(q),$$
$$I_{N_x}(q) = \bigvee_{p\in Q} I_N(p) \wedge \tilde{\delta}_2^*((p, T^t(p), I^t(p), F^t(p)), x, q) \leq I_N(q),$$

and

$$F_{N_x}(q) = \bigwedge_{p\in Q} F_N(p) \vee \tilde{\delta}_3^*((p, T^t(p), I^t(p), F^t(p)), x, q) \geq F_N(q).$$

So, $N_x \subseteq N$. Now, let $N_x \subseteq N$, for every $x \in \Sigma^*$. Then for every $x \in \Sigma^*$ and $q \in Q$, we have

$$\begin{aligned} T_N(q) \geq T_{N_x}(q) &= \bigvee_{p\in Q} T_N(p) \wedge \tilde{\delta}_1^*((p, T^t(p), I^t(p), F^t(p)), x, q) \\ &\geq T_N(p) \wedge \tilde{\delta}_1^*((p, T^t(p), I^t(p), F^t(p)), x, q), \\ I_N(q) \geq I_{N_x}(q) &= \bigvee_{p\in Q} I_N(p) \wedge \tilde{\delta}_2^*((p, T^t(p), I^t(p), F^t(p)), x, q) \\ &\geq I_N(p) \wedge \tilde{\delta}_2^*((p, T^t(p), I^t(p), F^t(p)), x, q), \end{aligned}$$

and

$$\begin{aligned} F_N(q) \leq F_{N_x}(q) &= \bigwedge_{p\in Q} F_N(p) \vee \tilde{\delta}_3^*((p, T^t(p), I^t(p), F^t(p)), x, q) \\ &\leq F_N(p) \vee \tilde{\delta}_3^*((p, T^t(p), I^t(p), F^t(p)), x, q), \end{aligned}$$

for every $p \in Q$. Hence, $\mathcal{M}' = (Q, N, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ is a SVNSGM of $\mathcal{M}$. □

Definition 3.12. Let $\mathcal{M} = (Q, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ be a SVNGM, $t \in (0, 1]$ and $q \in Q$. Define the SVNS $q_t\Sigma$ by $q_t\Sigma = (p, T_{q_t\Sigma}(p), I_{q_t\Sigma}(p), F_{q_t\Sigma}(p))$, where

$$T_{q_t\Sigma}(p) = \bigvee_{a\in\Sigma} t \wedge \tilde{\delta}_1((q, T^t(q), I^t(q), F^t(q)), a, p),$$
$$I_{q_t\Sigma}(p) = \bigvee_{a\in\Sigma} t \wedge \tilde{\delta}_2((q, T^t(q), I^t(q), F^t(q)), a, p),$$
$$F_{q_t\Sigma}(p) = \bigwedge_{a\in\Sigma} t \vee \tilde{\delta}_3((q, T^t(q), I^t(q), F^t(q)), a, p),$$

for every $p \in Q$.

Definition 3.13. Let $\mathcal{M} = (Q, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ be a SVNGM and $t \in (0, 1]$. For every $q \in Q$ define the SVNS $q_t\Sigma^*$ by $q_t\Sigma^* = (p, T_{q_t\Sigma^*}(p), I_{q_t\Sigma^*}(p), F_{q_t\Sigma^*}(p))$, where

$$T_{q_t\Sigma^*}(p) = \bigvee_{y\in\Sigma^*} t \wedge \tilde{\delta}_1^*((q, T^t(q), I^t(q), F^t(q)), y, p),$$
$$I_{q_t\Sigma^*}(p) = \bigvee_{y\in\Sigma^*} t \wedge \tilde{\delta}_2^*((q, T^t(q), I^t(q), F^t(q)), y, p),$$
$$F_{q_t\Sigma^*}(p) = \bigwedge_{y\in\Sigma^*} t \vee \tilde{\delta}_3^*((q, T^t(q), I^t(q), F^t(q)), y, p),$$

for every $p \in Q$.

Theorem 3.14. *Let $\mathcal{M} = (Q, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ be a SVNGM and $t \in (0, 1]$. Then $\mathcal{M} = (Q, q_t\Sigma^*, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ is a SVNSGM of $\mathcal{M}$.*

Proof. Let $p' \in Q$ and $x \in \Sigma^*$. First we show that $(q_t\Sigma^*)_x \subseteq q_t\Sigma^*$.

$$\begin{aligned}
T_{(q_t\Sigma^*)_x}(p') &= \bigvee_{p\in Q} T_{q_t\Sigma^*}(p) \wedge \tilde{\delta}_1^*((p, T^t(p), I^t(p), F^t(p)), x, p') \\
&= \bigvee_{p\in Q} \Big(\bigvee_{y\in\Sigma^*} t \wedge \tilde{\delta}_1^*((q, T^{t-|y|}(q), I^{t-|y|}(q), F^{t-|y|}(q)), y, p) \\
&\wedge \tilde{\delta}_1^*((p, T^t(p), I^t(p), F^t(p)), x, p')\Big) \\
&= \bigvee_{y\in\Sigma^*} t \wedge \tilde{\delta}_1^*((q, T^{t-|y|}(q), I^{t-|y|}(q), F^{t-|y|}(q)), yx, p') \\
&\leq \bigvee_{w\in\Sigma^*} t \wedge \tilde{\delta}_1^*((q, T^{t-|y|}(q), I^{t-|y|}(q), F^{t-|y|}(q)), w, p') \\
&= T_{q_t\Sigma^*}(p'),
\end{aligned}$$

$$\begin{aligned}
I_{(q_t\Sigma^*)_x}(p') &= \bigvee_{p\in Q} I_{q_t\Sigma^*}(p) \wedge \tilde{\delta}_2^*((p, T^t(p), I^t(p), F^t(p)), x, p') \\
&= \bigvee_{p\in Q} \Big(\bigvee_{y\in\Sigma^*} t \wedge \tilde{\delta}_2^*((q, T^{t-|y|}(q), I^{t-|y|}(q), F^{t-|y|}(q)), y, p) \\
&\wedge \tilde{\delta}_2^*((p, T^t(p), I^t(p), F^t(p)), x, p')\Big) \\
&= \bigvee_{y\in\Sigma^*} t \wedge \tilde{\delta}_2^*((q, T^{t-|y|}(q), I^{t-|y|}(q), F^{t-|y|}(q)), yx, p') \\
&\leq \bigvee_{w\in\Sigma^*} t \wedge \tilde{\delta}_2^*((q, T^{t-|y|}(q), I^{t-|y|}(q), F^{t-|y|}(q)), w, p') \\
&= I_{q_t\Sigma^*}(p'),
\end{aligned}$$

$$
\begin{aligned}
F_{(q_t\Sigma^*)_x}(p') &= \bigwedge_{p\in Q} F_{q_t\Sigma^*}(p) \vee \tilde{\delta}_3^*((p, T^t(p), I^t(p), F^t(p)), x, p') \\
&= \bigwedge_{p\in Q} \Big(\bigwedge_{y\in\Sigma^*} t \vee \tilde{\delta}_3^*((q, T^{t-|y|}(q), I^{t-|y|}(q), F^{t-|y|}(q)), y, p) \\
&\vee \tilde{\delta}_3^*((p, T^t(p), I^t(p), F^t(p)), x, p')\Big) \\
&= \bigwedge_{y\in\Sigma^*} t \vee \tilde{\delta}_3^*((q, T^{t-|y|}(q), I^{t-|y|}(q), F^{t-|y|}(q)), yx, p') \\
&\geq \bigwedge_{w\in\Sigma^*} t \vee \tilde{\delta}_3^*((q, T^{t-|y|}(q), I^{t-|y|}(q), F^{t-|y|}(q)), w, p') \\
&= F_{q_t\Sigma^*}(p'),
\end{aligned}
$$

Therefore, $(q_t\Sigma^*)_x \subseteq q_t\Sigma^*$. Then by Theorem 3.11, $\mathcal{M}'$ is a SVNSGM of $\mathcal{M}$. □

Theorem 3.15. *Let $\mathcal{M} = (Q, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ be a SVNGM and N ba a SVNS of Q. Then the following assertions are equivalent:*

(1) $\mathcal{M}' = (Q, N, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ *is a SVNSGM of* $\mathcal{M}$,

(2) $q_t\Sigma^* \subseteq N$, *for every* $q_t \subseteq N$, *where* $q \in Q$ *and* $t \in (0, 1]$,

(3) $q_t\Sigma \subseteq N$, *for every* $q_t \subseteq N$, *where* $q \in Q$ *and* $t \in (0, 1]$,

Proof. $1 \to 2$. Let $q_t \subseteq N$, where $q \in Q$ and $t \in (0, 1]$. Let $p \in Q$ and $y \in \Sigma^*$. Then

$$
\begin{aligned}
\tilde{\delta}_1^*((q, T^t(q), I^t(q), F^t(q)), y, p) \wedge t &= \tilde{\delta}_1^*((q, T^t(q), I^t(q), F^t(q)), y, p) \wedge T_{(q_t)\Lambda}(q) \\
&\leq \tilde{\delta}_1^*((q, T^t(q), I^t(q), F^t(q)), y, p) \wedge T_N(q) \\
&\leq T_N(p),
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\delta}_2^*((q, T^t(q), I^t(q), F^t(q)), y, p) \wedge t &= \tilde{\delta}_2^*((q, T^t(q), I^t(q), F^t(q)), y, p) \wedge I_{(q_t)\Lambda}(q) \\
&\leq \tilde{\delta}_2^*((q, T^t(q), I^t(q), F^t(q)), y, p) \wedge I_N(q) \\
&\leq I_N(p),
\end{aligned}
$$

$$
\begin{aligned}
\tilde{\delta}_3^*((q, T^t(q), I^t(q), F^t(q)), y, p) \vee t &= \tilde{\delta}_3^*((q, T^t(q), I^t(q), F^t(q)), y, p) \vee F_{(q_t)\Lambda}(q) \\
&\geq \tilde{\delta}_3^*((q, T^t(q), I^t(q), F^t(q)), y, p) \vee F_N(q) \\
&\geq F_N(p).
\end{aligned}
$$

Therefore, $q_t\Sigma^* \subseteq N$.

$2 \to 3$. It is clear.

$3 \to 1$. Let $p, q \in Q$ and $a \in \Sigma$. If $T_N(q) = 0$ or $\tilde{\delta}_1^*((q, T^t(q), I^t(q), F^t(q)), a, p) = 0$, then $T_N(p) \geq T_N(q) \wedge \tilde{\delta}_1^*((q, T^t(q), I^t(q), F^t(q)), a, p)$. Also, if $I_N(q) = 0$ or $\tilde{\delta}_2^*((q, T^t(q), I^t(q), F^t(q)), a, p) = 0$, then $I_N(p) \geq I_N(q) \wedge \tilde{\delta}_2^*((q, T^t(q), I^t(q), F^t(q)), a, p)$ and if $F_N(q) = 1$ or $\tilde{\delta}_3^*((q, T^t(q), I^t(q), F^t(q)), a, p) = 1$, then $F_N(p) \leq F_N(q) \vee$

$\tilde{\delta}_3^*((q, T^t(q), I^t(q), F^t(q)), a, p)$. Now, let $T_N(q) = t$ and $\tilde{\delta}_1^*((q, T^t(q), I^t(q), F^t(q)), a, p) \neq 0$. Then

$$\begin{aligned} T_N(p) \geq T_{q_t\Sigma}(p) &= \bigvee_{y\in\Sigma} t \wedge \tilde{\delta}_1^*((q, T^t(q), I^t(q), F^t(q)), y, p) \\ &\geq t \wedge \tilde{\delta}_1^*((q, T^t(q), I^t(q), F^t(q)), y, p) \\ &= T_N(q) \wedge \tilde{\delta}_1^*((q, T^t(q), I^t(q), F^t(q)), y, p), \end{aligned}$$

also, let $I_N(q) = t$ and $\tilde{\delta}_2^*((q, T^t(q), I^t(q), F^t(q)), a, p) \neq 0$. Then

$$\begin{aligned} I_N(p) \geq I_{q_t\Sigma}(p) &= \bigvee_{y\in\Sigma} t \wedge \tilde{\delta}_2^*((q, T^t(q), I^t(q), F^t(q)), y, p) \\ &\geq t \wedge \tilde{\delta}_2^*((q, T^t(q), I^t(q), F^t(q)), y, p) \\ &= I_N(q) \wedge \tilde{\delta}_2^*((q, T^t(q), I^t(q), F^t(q)), y, p), \end{aligned}$$

and if $F_N(q) = t \neq 1$ and $\tilde{\delta}_1^*((q, T^t(q), I^t(q), F^t(q)), a, p) \neq 1$, then

$$\begin{aligned} F_N(p) \leq F_{q_t\Sigma}(p) &= \bigwedge_{y\in\Sigma} t \vee \tilde{\delta}_3^*((q, T^t(q), I^t(q), F^t(q)), y, p) \\ &\leq t \vee \tilde{\delta}_3^*((q, T^t(q), I^t(q), F^t(q)), y, p) \\ &= F_N(q) \vee \tilde{\delta}_3^*((q, T^t(q), I^t(q), F^t(q)), y, p). \end{aligned}$$

Hence, $\mathcal{M}'$ is a SVNSGM of $\mathcal{M}$. □

Definition 3.16. Let $\mathcal{M}_1 = (Q_1, \Sigma_1, \tilde{\delta}, \tilde{R}, E_1, E_2)$ and $\mathcal{M}_2 = (Q_2, \Sigma_2, \tilde{\delta}', \tilde{R}', E_1, E_2)$ be two SVNGM. A pair (f, g) of mappings $f : Q_1 \to Q_2$ and $g : \Sigma_1 \to \Sigma_2$ is called a homomorphism, written $(f, g) : \mathcal{M}_1 \to \mathcal{M}_2$ if

(1) $\tilde{\delta}_1^*((q, T^t(q), I^t(q), F^t(q)), a, p) \leq \tilde{\delta}_1'^*((f(q), T^t(f(q)), I^t(f(q)), F^t(f(q))), g(a), f(p))$,
(2) $\tilde{\delta}_2^*((q, T^t(q), I^t(q), F^t(q)), a, p) \leq \tilde{\delta}_2'^*((f(q), T^t(f(q)), I^t(f(q)), F^t(f(q))), g(a), f(p))$,
(3) $\tilde{\delta}_3^*((q, T^t(q), I^t(q), F^t(q)), a, p) \geq \tilde{\delta}_3'^*((f(q), T^t(f(q)), I^t(f(q)), F^t(f(q))), g(a), f(p))$,
(4) for every $(q, T(q), I(q), F(q)) \in \tilde{R}$, we have $T(f(q)) \geq T(q), I(f(q)) \geq I(q)$ and $F(f(q)) \leq F(q)$,

for every $p, q \in Q_1$ and $x \in \Sigma_1$. The pair (f, g) is called strong homomorphism if

(1)

$$\begin{aligned} \tilde{\delta}_1'^*((f(q), &T^t(f(q)), I^t(f(q)), F^t(f(q))), g(a), f(p)) \\ &= \bigvee\{\tilde{\delta}_1^*((q, T^t(q), I^t(q), F^t(q)), a, r) \big| r \in Q_1, f(r) = f(p)\}, \end{aligned}$$

(2)

$$\tilde{\delta}_2'^*((f(q),T^t(f(q)),I^t(f(q)),F^t(f(q))),g(a),f(p)) \\ = \bigvee\{\tilde{\delta}_2^*((q,T^t(q),I^t(q),F^t(q)),a,r)\big|r\in Q_1, f(r)=f(p)\},$$

(3)

$$\tilde{\delta}_3'^*((f(q),T^t(f(q)),I^t(f(q)),F^t(f(q))),g(a),f(p)) \\ = \bigwedge\{\tilde{\delta}_3^*((q,T^t(q),I^t(q),F^t(q)),a,r)\big|r\in Q_1, f(r)=f(p)\},$$

(4) for every $(q,T(q),I(q),F(q))\in\tilde{R}$, we have $(f(q),T(f(q)),I(f(q)),F(f(q)))\in\tilde{R}'$,

where $p,q\in Q_1$ and $x\in\Sigma_1$.

Definition 3.17. Let $\mathcal{M}_1=(Q_1,\Sigma_1,\tilde{\delta},\tilde{R},E_1,E_2)$ and $\mathcal{M}_2=(Q_2,\Sigma_2,\tilde{\delta}',\tilde{R}',E_1,E_2)$ be two SVNGM. Let $(f,g):\mathcal{M}_1\to\mathcal{M}_2$ be a homomorphism and N be a SVNS of Q_1. Define the SVNS $f(N)$ of Q_2 as follows:

$$T_{f(N)}(q')=\begin{cases}\bigvee\{T_N(q)\big|q\in Q_1, f(q)=q'\} & if\ f^{-1}(q')\neq\emptyset\\ 0 & f^{-1}(q')=\emptyset\end{cases},$$

$$I_{f(N)}(q')=\begin{cases}\bigvee\{I_N(q)\big|q\in Q_1, f(q)=q'\} & if\ f^{-1}(q')\neq\emptyset\\ 0 & f^{-1}(q')=\emptyset\end{cases},$$

$$F_{f(N)}(q')=\begin{cases}\bigwedge\{F_N(q)\big|q\in Q_1, f(q)=q'\} & if\ f^{-1}(q')\neq\emptyset\\ 0 & f^{-1}(q')=\emptyset\end{cases},$$

for every $q'\in Q_2$.

Theorem 3.18. *Let $\mathcal{M}_1=(Q_1,\Sigma,\tilde{\delta},\tilde{R},E_1,E_2)$ and $\mathcal{M}_2=(Q_2,\Sigma,\tilde{\delta}',\tilde{R}',E_1,E_2)$ be two SVNGM and $f:\mathcal{M}_1\to\mathcal{M}_2$ be an onto strong homomorphism. If $\mathcal{M}_1'=(Q_1,N,\Sigma,\tilde{\delta},\tilde{R},E_1,E_2)$ is a SVNSGM of $\mathcal{M}_1$, then $\mathcal{M}_2'=(Q_2,f(N),\Sigma,\tilde{\delta}',\tilde{R}',E_1,E_2)$ is a SVNSGM of $\mathcal{M}_2$.*

Proof. Let $p, q \in Q_1, p', q' \in Q_2, a \in X$ and $f(p) = p'$ and $f(q) = q'$. Then

$$\begin{aligned}
&T_{f(N)}(p') \wedge \tilde{\delta}_1'^*((p', T^t(p'), I^t(p'), F^t(p')), a, q') \\
&\quad = (\bigvee\{T_N(p) \big| p \in Q_1, f(p) = p'\}) \wedge \tilde{\delta}_1'^*((p', T^t(p'), I^t(p'), F^t(p')), a, q') \\
&\quad = \bigvee\{T_N(p) \wedge \tilde{\delta}_1'^*((p', T^t(p'), I^t(p'), F^t(p')), a, q') \big| p \in Q_1, f(p) = p'\} \\
&\quad = \bigvee\{T_N(p) \wedge \tilde{\delta}_1'^*((f(p), T^t(f(p)), I^t(f(p)), F^t(f(p))), a, f(q)) \big| p \in Q_1, f(p) = p'\} \\
&\quad = \bigvee\{T_N(p) \wedge \bigvee\{\tilde{\delta}_1^*((p, T^t(p), I^t(p), F^t(p)), a, r) \big| p, r \in Q_1, f(p) = p', f(r) = f(q)\}\} \\
&\quad = \bigvee\{\bigvee\{T_N(p) \wedge \tilde{\delta}_1^*((p, T^t(p), I^t(p), F^t(p)), a, r) \big| r \in Q_1, f(r) = f(q)\} \big| p \in Q_1, f(p) = p'\} \\
&\quad \leq \bigvee\{\bigvee\{T_N(r) \big| r \in Q_1, f(r) = f(q)\} \big| p \in Q_1, f(p) = p'\} \\
&\quad = \bigvee\{T_{f(N)}(q') \big| p \in Q_1, f(p) = p'\} \\
&\quad = T_{f(N)}(q'),
\end{aligned}$$

also, similarly

$$I_{f(N)}(p') \wedge \tilde{\delta}_2'^*((p', T^t(p'), I^t(p'), F^t(p')), a, q') \leq I_{f(N)}(q'),$$

and

$$\begin{aligned}
&F_{f(N)}(p') \vee \tilde{\delta}_3'^*((p', T^t(p'), I^t(p'), F^t(p')), a, q') \\
&\quad = (\bigwedge\{F_N(p) \big| p \in Q_1, f(p) = p'\}) \vee \tilde{\delta}_3'^*((p', T^t(p'), I^t(p'), F^t(p')), a, q') \\
&\quad = \bigwedge\{F_N(p) \vee \tilde{\delta}_3'^*((p', T^t(p'), I^t(p'), F^t(p')), a, q') \big| p \in Q_1, f(p) = p'\} \\
&\quad = \bigwedge\{F_N(p) \vee \tilde{\delta}_3'^*((f(p), T^t(f(p)), I^t(f(p)), F^t(f(p))), a, f(q)) \big| p \in Q_1, f(p) = p'\} \\
&\quad = \bigwedge\{F_N(p) \vee \bigwedge\{\tilde{\delta}_3^*((p, T^t(p), I^t(p), F^t(p)), a, r) \big| p, r \in Q_1, f(p) = p', f(r) = f(q)\}\} \\
&\quad = \bigwedge\{\bigwedge\{F_N(p) \vee \tilde{\delta}_3^*((p, T^t(p), I^t(p), F^t(p)), a, r) \big| r \in Q_1, f(r) = f(q)\} \big| p \in Q_1, f(p) = p'\} \\
&\quad \geq \bigwedge\{\bigwedge\{F_N(r) \big| r \in Q_1, f(r) = f(q)\} \big| p \in Q_1, f(p) = p'\} \\
&\quad = \bigwedge\{F_{f(N)}(q') \big| p \in Q_1, f(p) = p'\} \\
&\quad = F_{f(N)}(q').
\end{aligned}$$

Hence, $\mathcal{M}_2'$ is a SVNSGM of $\mathcal{M}_2$. □

Now, in the following example we show that Theorem 3.18 is not true if f is not onto.

Example 3.19. Let $Q_1 = Q_2 = \{p_1, p_2\}, \Sigma = \{a\}$ and $\tilde{R} = \tilde{R}' = \{(p_1, 1, 1, 0), (p_2, 1, 1, 0)\}$ and $\delta(p, a, q) = (1, 1, 0) = \delta'(p, a, q)$, for every $p, q \in Q_1$. Then $\mathcal{M} = (Q_1, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ is a SVNGM. Let $f : Q_1 \to Q_2$ be a mapping such that $f(p_1) = f(p_2) = p_1$. Then f is not onto. It is clear that, f is a strong homomorphism. Let N be a SVNS of Q such that $N(p_1) = N(p_2) = (\frac{1}{2}, \frac{1}{2}, 0)$. Then

$$T_N(p) = \frac{1}{2} = T_N(q) \wedge \tilde{\delta}_1^*((q, T^t(q), I^t(q), F^t(q)), a, p),$$
$$I_N(p) = \frac{1}{2} = I_N(q) \wedge \tilde{\delta}_2^*((q, T^t(q), I^t(q), F^t(q)), a, p),$$
$$F_N(p) = 0 = F_N(q) \vee \tilde{\delta}_3^*((q, T^t(q), I^t(q), F^t(q)), a, p),$$

for every $p, q \in Q$. Then $\mathcal{M}' = (Q_1, N, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ is a SVNSGM of $\mathcal{M}_1$. Now, we have

$$T_{f(N)}(p_2) = 0 \leq \frac{1}{2} = T_{f(N)}(p_1) \wedge \tilde{\delta}_1^*((p_1, T^t(p_1), I^t(p_1), F^t(p_1)), a, p_2),$$
$$I_{f(N)}(p_2) = 0 \leq \frac{1}{2} = I_{f(N)}(p_1) \wedge \tilde{\delta}_2^*((p_1, T^t(p_1), I^t(p_1), F^t(p_1)), a, p_2),$$
$$F_{f(N)}(p_2) = 1 \geq \frac{1}{2} = F_N(p_1) \vee \tilde{\delta}_3^*((p_1, T^t(p_1), I^t(p_1), F^t(p_1)), a, p_2).$$

So, $\mathcal{M}_2' = (Q_2, f(N), \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ is not a SVNSGM of $\mathcal{M}_2 = (Q_2, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$.

4. Single-Valued Neutrosophic Strong Sub-General Machine

Definition 4.1. Let $\mathcal{M} = (Q, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ be a SVNGM and N be a SVNS of Q. Then we say that $\mathcal{M}' = (Q, N, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ is a single-valued neutrosophic strong sub-general machine (SVNSSGM) of $\mathcal{M}$ if and only if for every $p, q \in Q$ if there exists $a \in \Sigma$ such that

$$\tilde{\delta}_1((p, T^t(p), I^t(p), F^t(p)), a, q) > 0,$$
$$\tilde{\delta}_2((p, T^t(p), I^t(p), F^t(p)), a, q) > 0,$$
$$\tilde{\delta}_3((p, T^t(p), I^t(p), F^t(p)), a, q) < 1,$$

then $T_N(q) \geq T_N(p), I_N(q) \geq I_N(p), F_N(q) \leq F_N(p)$.

Theorem 4.2. *Let $\mathcal{M} = (Q, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ be a SVNGM and N be a SVNS of Q. Then $\mathcal{M}' = (Q, N, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ is a SVNSSGM of $\mathcal{M}$ if and only if for every $p, q \in Q$ if there exists $x \in \Sigma^*$ such that*

$$\tilde{\delta}_1^*((p, T^t(p), I^t(p), F^t(p)), a, q) > 0,$$
$$\tilde{\delta}_2^*((p, T^t(p), I^t(p), F^t(p)), a, q) > 0,$$
$$\tilde{\delta}_3^*((p, T^t(p), I^t(p), F^t(p)), a, q) < 1,$$

then $T_N(q) \geq T_N(p), I_N(q) \geq I_N(p), F_N(q) \leq F_N(p)$.

Proof. Let $\mathcal{M}' = (Q, N, \Sigma, \delta, R, E_1, E_2)$ be a SVNSSGM of $\mathcal{M}$. We prove the claim by induction on $|x| = n$. Let $n = 0$. Then $x = \Lambda$. If $p = q$, then

$$\tilde{\delta}_1((p, T^t(p), I^t(p), F^t(p)), \Lambda, q) = 1 > 0,$$
$$\tilde{\delta}_2((p, T^t(p), I^t(p), F^t(p)), \Lambda, q) = 1 > 0,$$
$$\tilde{\delta}_3((p, T^t(p), I^t(p), F^t(p)), \Lambda, q) = 0 < 1,$$

so, $T_N(q) = T_N(p), I_N(q) = I_N(p), F_N(q) = F_N(p)$. Now, let $p \neq q$. Then

$$\tilde{\delta}_1((p, T^t(p), I^t(p), F^t(p)), \Lambda, q) = 0,$$
$$\tilde{\delta}_2((p, T^t(p), I^t(p), F^t(p)), \Lambda, q) = 0,$$
$$\tilde{\delta}_3((p, T^t(p), I^t(p), F^t(p)), \Lambda, q) = 1,$$

so, the claim is true for $n = 0$. Now, let the result is true for every $y \in \Sigma^*$ such that $|y| = n - 1$, $n \geq 1$. Suppose that $x = ya, y \in \Sigma^*, a \in \Sigma$ and $|y| = n - 1$. Let $\tilde{\delta}_1((p, T^t(p), I^t(p), F^t(p)), x, q) = \tilde{\delta}_1((p, T^t(p), I^t(p), F^t(p)), ya, q)$. Then there exists $r \in Q$ such that

$$\tilde{\delta}_1((p, T^t(p), I^t(p), F^t(p)), y, r) \wedge \tilde{\delta}_1((r, T^{t+n-1}(r), I^{t+n-1}(r), F^{t+n-1}(r)), a, q) > 0.$$

So, $\tilde{\delta}_1((p, T^t(p), I^t(p), F^t(p)), y, r) > 0$ and $\tilde{\delta}_1((r, T^{t+n-1}(r), I^{t+n-1}(r), F^{t+n-1}(r)), a, q) > 0$. Therefore, $T_N(r) \geq T_N(p)$ and $T_N(q) \geq T_N(r)$. So, $T_N(q) \geq T_N(p)$. Also, let $\tilde{\delta}_2((p, T^t(p), I^t(p), F^t(p)), x, q) = \tilde{\delta}_2((p, T^t(p), I^t(p), F^t(p)), ya, q) > 0$. Then there exists $r \in Q$ such that $\tilde{\delta}_2((p, T^t(p), I^t(p), F^t(p)), y, r) > 0$ and $\tilde{\delta}_2((r, T^{t+n-1}(r), I^{t+n-1}(r), F^{t+n-1}(r)), a, q) > 0$. So, $I_N(r) \geq I_N(p)$ and $I_N(q) \geq I_N(r)$. So, $I_N(q) \geq I_N(p)$. Moreover, let $\tilde{\delta}_3((p, T^t(p), I^t(p), F^t(p)), x, q) < 1$. Then

$$\tilde{\delta}_3((p, T^t(p), I^t(p), F^t(p)), ya, q) = \bigwedge_{r \in Q} \tilde{\delta}_3((p, T^t(p), I^t(p), F^t(p)), y, r) \vee \tilde{\delta}_3((r, T^{t+n-1}(r), I^{t+n-1}(r), F^{t+n-1}(r)), a, q) < 1.$$

Therefore, there exists $r \in Q$ such that $\tilde{\delta}_3((p, T^t(p), I^t(p), F^t(p)), y, r) < 1$ and

$$\tilde{\delta}_3((r, T^{t+n-1}(r), I^{t+n-1}(r), F^{t+n-1}(r)), a, q) < 1.$$

So, $F_N(r) \leq F_N(p)$ and $F_N(q) \leq F_N(r)$. Therefore, $F_N(q) \leq F_N(p)$. Then the claim holds. The converse is clear. □

Theorem 4.3. *Let $\mathcal{M} = (Q, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ be a SVNGM and N be a SVNS of Q. If $\mathcal{M}' = (Q, N, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ is a SVNSSGM of $\mathcal{M}$, then $\mathcal{M}'$ is a SVNSGM of $\mathcal{M}$.*

Proof. Let $\mathcal{M}'$ be a SVNSSGM of $\mathcal{M}$. Let

$$\tilde{\delta}_1((p, T^t(p), I^t(p), F^t(p)), a, q) > 0,$$
$$\tilde{\delta}_2((p, T^t(p), I^t(p), F^t(p)), a, q) > 0,$$
$$\tilde{\delta}_3((p, T^t(p), I^t(p), F^t(p)), a, q) < 1.$$

Then $T_N(q) \geq T_N(p), I_N(q) \geq I_N(p)$ and $F_N(q) \leq F_N(p)$. So,

$$T_N(q) \geq T_N(p) \wedge \tilde{\delta}_1((p, T^t(p), I^t(p), F^t(p)), a, q),$$
$$I_N(q) \geq I_N(p) \wedge \tilde{\delta}_2((p, T^t(p), I^t(p), F^t(p)), a, q) > 0,$$
$$F_N(q) \leq F_N(p) \vee \tilde{\delta}_3((p, T^t(p), I^t(p), F^t(p)), a, q) < 1.$$

Hence, $\mathcal{M}'$ is a SVNSGM of $\mathcal{M}$. □

In the next example, we show that the reverse of the Theorem 4.3, is incorrect.

Example 4.4. Let SVNGM $\mathcal{M}$ be as defined in Example 3.6. $\mathcal{M}'$ is a SVNGSM of $\mathcal{M}$.

$$\tilde{\delta}_1((p_1, T^t(p_1), I^t(p_1), F^t(p_1)), a, p_2) = 0.5 > 0,$$

but $T_N(p_2) = 0.5 < T_N(p_1) = 0.6$. So, $\mathcal{M}'$ is not a SVNSSGM of $\mathcal{M}$.

Theorem 4.5. *Let $\mathcal{M} = (Q, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ be a SVNGM. Let $\mathcal{M}_1 = (Q, N_1, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ and $\mathcal{M}_2 = (Q, N_2, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ be two SVNSSGM of $\mathcal{M}$. Then the following hold:*

1. *$\mathcal{M}_1 \cap \mathcal{M}_2$ is a SVNSSGM of $\mathcal{M}$.*
2. *$\mathcal{M}_1 \cup \mathcal{M}_2$ is a SVNSSGM of $\mathcal{M}$.*

Proof. The proofs 1 and 2 are clear. □

Theorem 4.6. *Let $\mathcal{M} = (Q, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ and $\mathcal{M}' = (Q', \Sigma, \tilde{\delta}', \tilde{R}', E_1, E_2)$ be two SVNGM. Let $f : \mathcal{M}_1 \to \mathcal{M}_2$ be an onto strong homomorphism. Let $\mathcal{M}_1 = (Q, N, \Sigma, \tilde{\delta}, \tilde{R}, E_1, E_2)$ be a SVNSSGM of $\mathcal{M}$. Then $\mathcal{M}'_1 = (Q', f(N), \Sigma, \tilde{\delta}', \tilde{R}', E_1, E_2)$ is a SVNSSGM of $\mathcal{M}'$.*

Proof. Let $p_1, p_2 \in Q_1$ and $a \in \Sigma$ be such that

$$\tilde{\delta}'_1((f(p_1), T^t(f(p_1)), I^t(f(p_1)), F^t(f(p_1))), a, f(p_2)) > 0,$$
$$\tilde{\delta}'_2((f(p_1), T^t(f(p_1)), I^t(f(p_1)), F^t(f(p_1))), a, f(p_2)) > 0,$$
$$\tilde{\delta}_3((f(p_1), T^t(f(p_1)), I^t(f(p_1)), F^t(f(p_1))), a, f(p_2)) < 1.$$

Also,

$$T_{f(N)}(f(p_1)) = \vee\{T_N(q_1)\big|f(p_1) = f(q_1), q_1 \in Q_1\},$$
$$I_{f(N)}(f(p_1)) = \vee\{I_N(q_1)\big|f(p_1) = f(q_1), q_1 \in Q_1\},$$
$$F_{f(N)}(f(p_1)) = \wedge\{F_N(q_1)\big|f(p_1) = f(q_1), q_1 \in Q_1\},$$

and

$$T_{f(N)}(f(p_2)) = \vee\{T_N(q_2)\big|f(p_2) = f(q_2), q_2 \in Q_1\},$$
$$I_{f(N)}(f(p_2)) = \vee\{I_N(q_2)\big|f(p_2) = f(q_2), q_2 \in Q_1\},$$
$$F_{f(N)}(f(p_2)) = \wedge\{F_N(q_2)\big|f(p_2) = f(q_2), q_2 \in Q_1\}.$$

Now, let $r \in Q_1$ and $T_N(r) > 0$ and $f(r) = f(p_1)$. Let

$$\tilde{\delta}'_1((f(r), T^t(f(r)), I^t(f(r)), F^t(f(r))), a, f(q)) = \tilde{\delta}'_1((f(p_1), T^t(f(p_1)), I^t(f(p_1)), F^t(f(p_1))), a, f(p_2)) > 0,$$
$$\tilde{\delta}'_2((f(r), T^t(f(r)), I^t(f(r)), F^t(f(r))), a, f(q)) = \tilde{\delta}'_2((f(p_1), T^t(f(p_1)), I^t(f(p_1)), F^t(f(p_1))), a, f(p_2)) > 0,$$
$$\tilde{\delta}'_3((f(r), T^t(f(r)), I^t(f(r)), F^t(f(r))), a, f(q)) = \tilde{\delta}'_3((f(p_1), T^t(f(p_1)), I^t(f(p_1)), F^t(f(p_1))), a, f(p_2)) < 1.$$

Then

$$\vee\{\tilde{\delta}_1((r, T^t(r), I^t(r), F^t(r)), a, s)\big|s \in Q_1, f(s) = f(q)\} > 0,$$
$$\vee\{\tilde{\delta}_2((r, T^t(r), I^t(r), F^t(r)), a, s)\big|s \in Q_1, f(s) = f(q)\} > 0,$$
$$\wedge\{\tilde{\delta}_3((r, T^t(r), I^t(r), F^t(r)), a, s)\big|s \in Q_1, f(s) = f(q)\} < 1.$$

Then there exists $q \in Q_1$ such that $f(q) = f(s)$ and

$$\tilde{\delta}_1((r, T^t(r), I^t(r), F^t(r)), a, q) > 0,$$
$$\tilde{\delta}_2((r, T^t(r), I^t(r), F^t(r)), a, q) > 0,$$
$$\tilde{\delta}_3((r, T^t(r), I^t(r), F^t(r)), a, q) < 1.$$

Then $T_N(q) \geq T_N(r), I_N(q) \geq I_N(r), F_N(q) \leq F_N(r)$. So, $T_{f(N)}(f(q)) \geq T_N(r), I_{f(N)}(f(q)) \geq I_N(r), F_{f(N)}(f(q)) \leq F_N(r)$. Therefore, $T_{f(N)}(f(q)) \geq T_{f(N)}(f(p_1)), I_{f(N)}(f(q)) \geq I_{f(N)}(f(p_1)), F_{f(N)}(f(q)) \leq F_{f(N)}(f(p_1))$. Hence, $\mathcal{M}'_1$ is a SVNSSGM of $\mathcal{M}_1$. □

5. **Conclusion**

In this study, for a given SVNGM $\mathcal{M}$ the notion of single-valued neutrosophic sub-general machine of $\mathcal{M}$ has been introduced and examined in details. Accordingly, the research has shown that the operators have some interesting properties under homomorphism. Moreover, the notion of single-valued neutrosophic strong sub-general machine has been presented. In addition, it has been shown that for a given SVNGM $\mathcal{M}$ if $\mathcal{M}'$ is a SVNSSGM of $\mathcal{M}$, then $\mathcal{M}'$ is a SVNSGM of $\mathcal{M}$, but the converse is not held.

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