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Bipolar neutrosophic soft generalized pre-closed sets and pre-open sets in topological space

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Abstract. Neutrosophy is one of the widely used tool to deal with uncertainty. In recent years, neutrosophic sets were applied in the field of topology. There are numerous types of neutrosophic topological spaces based on different kinds of neutrosophic sets are proposed by research community. Bipolar neutrosophic set and their topological spaces were proposed and analyzed by many researchers. In this paper, the generalization of bipolar neutrosophic soft set and their classifications are proposed. A bipolar neutrosophic soft generalized pre-closed sets and bipolar neutrosophic soft generalized pre-open sets are proposed along with their properties. Then, bipolar neutrosophic soft topology concepts are generalized to the proposed sets. Also, the relation between proposed sets and various conventional sets are discussed through theorems with examples.

Keywords: Bipolar neutrosophic soft set; BNGS-topology; BNGPCS; BNGPOS; Pre-closed set; Pre-open set.

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1. Introduction

Most of the real life problems has some uncertain information which makes difficult to retrieve the solution. In earlier days, researcher did not take the uncertainty into account while solving problems. But, those information makes significant difference in the final decision. Zadeh [1] were introduced fuzzy theory in 1968. Fuzzy theory were very useful to deal with uncertainty in real life problems. Since the introduction of fuzzy theory, many researchers were proposed different types of fuzzy concepts by extending and modifying the original fuzzy theory and applied to science and engineering problems. But the main drawback of fuzzy theory is, its uncertainty is dependent on the certainty of the problem. In many situations, uncertainty information may be independent. Many years later, Florentin Smarandache [2, 3] introduced the novel concept neutrosophy in 1998. Neutrosophy has three independent components namely, truth membership, indeterminacy and false membership each has the value in the interval $]^{-}0, 1^{+}[$. Neutrosophic sets were derived from neutrosophy and which is powerful than fuzzy sets. Molodtsov [5] introduced soft set theory in 1999 which is also deal with uncertainty in a parametric wise. In 2013, Pabitra Kumar Maji [14,15] proposed neutrosophic soft set which is the combination of both neutrosophic set and soft set. Neutrosophic soft sets were widely used in decision making problems by many researchers. Irfan Deli et al. [4] proposed bipolar neutrosophic sets and decision making technique in 2015. Mumtaz ali et al. [6] proposed bipolar neutrosophic soft sets and decision making method in 2017. After that, many different approaches on bipolar neutrosophic soft sets were proposed by several authors [7,8,16].

Neutrosophic sets were applied in almost all mathematics fields such as neutrosophic graph, neutrosophic statistics, neutrosophic algebra and so on. Neutrosophic sets were widely used many topology concepts; in particular, general topology. In 2012, A.A.Salama et al. [19] developed a new topological space namely, neutrosophic topology based on neutrosophic sets. Then, most of the general topology concepts were combined with neutrosophic sets and some new topologies were proposed [9–11, 16, 18]. In 1970's Norman Levine [12, 13] was defined generalized closed sets and many set theory concepts. In 1995, J.Dontchev [11] proposed generalized semi pre-open sets in topology. In 2018, Taha Yasin [20] have proposed some properties on bipolar soft topological space with appropriate examples; later, in 2020 [21], he introduced bipolar soft points which is very useful to investigate continuity, openness and closeness of topology mappings. In 2021, Simsekler Dizman and Taha Yasin [22] proposed a novel concept fuzzy bipolar soft topological spaces which is the extension of bipolar soft topology to fuzzy sets.

In this paper, the generalized set concept is applied to the bipolar neutrosophic soft set. As we discussed, the fusion of soft set and bipolar neutrosophic set gives bipolar neutrosophic soft set. In a similar manner, we take a fusion of generalized pre-sets (both open and closed) with bipolar neutrosophic soft set and defined new classes namely, bipolar neutrosophic soft generalized pre-cloesd sets and bipolar neutrosophic soft generalized pre-open set. Further, we investigate the reltion between former sets with the proposed sets.

This paper is organized as follows: section 1, gives introduction and previous works on the related topics. Section 2 consists required preliminary definitions. Section 3 deals with the notions on bipolar neutrosophic soft set topological space and their important results and properties. Section 4 and 5 deals with the proposed set, bipolar neutrosophic soft generalized pre-closed sets and related theorems and the following section consists, bipolar neutrosophic soft generalized pre-open sets and related theorems.

2. Preliminaries

Definition 2.1. [2,3] For a universal set X and every $x \in X$, the components $\mathcal{T}(x),\ \mathcal{I}(x)$ and $\mathcal{F}(x)$ represents truth, indeterminate and false degrees of x. Then the Neutrosophic set (NS) over X be defined as follows.
$N = \{\mathcal{T}(x),\ \mathcal{I}(x),\ \mathcal{F}(x)\ :\ x \in X\}$
Here, $\mathcal{T}(x),\ \mathcal{I}(x),\ \mathcal{F}(x)$ ranges in the non-standard interval $]^{-}0, 1^{+}[$ and their sum $^{-}0 \leq T + I + F \leq 3^{+}$. Further, single valued neutrosophic set is defined by replacing the interval $]^{-}0, 1^{+}[$ with $[0, 1]$ in the definition of neutrosophic set.

Definition 2.2. [5] A soft set is a function which maps a parameter set to the power set of X. It is denoted by (f, E) and is defined by
$f : E \to P(x)$
Each member of X is parameterized with the parameter set E by the function f.

Definition 2.3. [4] For the universe set X and positive member values $T^{+},\ I^{+},\ F^{+}\ :\ E \to [0, 1]$, negative member values $T^{-},\ I^{-},\ F^{-}\ :\ E \to\ [-1, 0]$, A bipolar neutrosophic set (BNS) is defined by

$$B = \left\{ \langle x,\ \mathcal{T}^{+}(x),\ \mathcal{I}^{+}(x),\ \mathcal{F}^{+}(x),\ \mathcal{T}^{-}(x),\ \mathcal{I}^{-}(x),\ \mathcal{F}^{-}(x)\ \rangle :\ x \in\ X \right\}$$

Definition 2.4. [6,7] A bipolar neutrosophic soft set (BNSS) is the fusion of soft set and bipolar neutrosophic set and is defined as follows.

$$BNS = (f_A, E) = \{\langle e, f_A(x)\rangle : e \in A \subset E, f_A(x) \in BNS(X)\}$$

Here $f_A(x) = \left\{ \left\langle x, \mathcal{T}^{+}_{f_{A(e)}}(x), \mathcal{I}^{+}_{f_{A(e)}}(x), \mathcal{F}^{+}_{f_{A(e)}}(x), \mathcal{T}^{-}_{f_{A(e)}}(x), \mathcal{I}^{-}_{f_{A(e)}}(x), \mathcal{F}^{-}_{f_{A(e)}}(x) \right\rangle : x \in X \right\}$.

Definition 2.5. [6,7,16] Let B be a $BNSS$. Then the complement of B is defined as

$$B^{c} = \left\{ \left\langle e, \mathcal{F}^{+}_{f}(e), 1 - \mathcal{I}^{+}_{f}(e), \mathcal{T}^{+}_{f}(e), \mathcal{F}^{-}_{f}(e), -1 - \mathcal{I}^{-}_{f}(e), \mathcal{T}^{-}_{f}(e) \right\rangle \right\}.$$

Definition 2.6. [6,7,16] Let $\phi_{\mathbb{B}}$ be a null $BNSS$ and is defined as

$$\phi_{\mathbb{B}} = \left\{ \langle e_i, \{x_i, 0, 1, 1, 0, -1, -1\}\rangle : x \in X, e \in E \right\}$$

Definition 2.7. [6,7,16] Let $1_{\mathbb{B}}$ be a complete $BNSS$ and is defined as

$$1_{\mathbb{B}} = \left\{ \langle e_i, \{x_i, 1, 0, 0, -1, 0, 0\}\rangle : x \in X, e \in E \right\}$$

Definition 2.8. [6,7,16] Let B_1 and B_2 be two $BNSS$s. Then their union $B_1 \cup B_2$ is defined as

$$B_1 \cup B_2 = \left\{ \left\langle e, \cup_i f^{(i)}(e) \right\rangle \right\}.$$

Here,

$$\bigcup_i f^{(i)}(e) = \Big\{ \langle x, max\left[\mathcal{T}^+_{f^i}(e)(x)\right], min\left[\mathcal{I}^+_{f^i}(e)(x)\right], min\left[\mathcal{F}^+_{f^i}(e)(x)\right], min\left[\mathcal{T}^-_{f^i}(e)(x)\right], max\left[\mathcal{I}^-_{f^i}(e)(x)\right], max\left[\mathcal{F}^-_{f^i}(e)(x)\right] \rangle \Big\}$$

Definition 2.9. [6,7,16] Let B_1 and B_2 be two $BNSS$s. Then their intersection $B_1 \cap B_2$ is defined as

$$B_1 \cap B_2 = \left\{ \left\langle e, \cap_i f^{(i)}(e) \right\rangle \right\}.$$

Here,

$$\bigcap_i f^{(i)}(e) = \Big\{ \langle x, min\left[\mathcal{T}^+_{f^i}(e)(x)\right], max\left[\mathcal{I}^+_{f^i}(e)(x)\right], max\left[\mathcal{F}^+_{f^i}(e)(x)\right], max\left[\mathcal{T}^-_{f^i}(e)(x)\right], min\left[\mathcal{I}^-_{f^i}(e)(x)\right], min\left[\mathcal{F}^-_{f^i}(e)(x)\right] \rangle \Big\}$$

Definition 2.10. [6,7,16] Let B_1 and B_2 be two $BNSS$s. Then B_1 is called subset of B_2 (i.e. $B_1 \subseteq B_2$) only if the following condition hold.
For every $x \in X$ and $e \in E$,

$$\left[\mathcal{T}^+_{B_1}(x) \leq \mathcal{T}^+_{B_2}(x)\right], \left[\mathcal{I}^+_{B_1}(x) \geq \mathcal{I}^+_{B_2}(x)\right], \left[\mathcal{F}^+_{B_1}(x) \geq \mathcal{F}^+_{B_2}(x)\right]$$
$$\left[\mathcal{T}^-_{B_1}(x) \geq \mathcal{T}^-_{B_2}(x)\right], \left[\mathcal{I}^-_{B_1}(x) \leq \mathcal{I}^-_{B_2}(x)\right], \left[\mathcal{F}^-_{B_1}(x) \leq \mathcal{F}^-_{B_2}(x)\right]$$

Definition 2.11. [13] Let (X, τ) be a topological space. For any subset $Y \in X$,

i). $cl(Y) = Y$, then Y is closed set
ii). $int(cl(Y)) \subseteq Y$, then Y is semi closed set (SCS)
iii). $cl(int(Y)) \subseteq Y$, then Y is pre-closed set (PCS)
iv). $int(cl(int(Y))) \subseteq Y$, then Y is semi pre-closed set (SPCS)
v). $cl(int(cl(Y))) \subseteq Y$, then Y is α-closed set (α-CS)
vi). $Y = cl(int(Y))$, then Y is regular closed set (RCS)

Definition 2.12. [12] Let (X, τ) be a topological space. For any subset $Y \in X$ and $Y \subseteq U$ and U is open in X,

i). $cl(Y) \subseteq U$, then Y is generalized closed set (g-closed).
ii). $scl(Y) \subseteq U$, then Y is generalized semi closed set (gs-closed).

iii). $pcl(Y) \subseteq U$, then Y is generalized pre-closed set (gp-closed).
iv). $spcl(Y) \subseteq U$, then Y is generalized semi pre-closed set (gsp-closed).
v). $\alpha cl(Y) \subseteq U$, then Y is α- generalized closed set (αg-closed).

Definition 2.13. [16] A bipolar neutrosophic soft topology (BNST) on X is a collection τ of bipolar neutrosophic soft sets (BNSS) in X satisfying the following conditions:

1). $\phi_{\mathbb{B}}, 1_{\mathbb{B}} \in \tau_{\mathbb{B}}$
2). $\bigcup_{i\in n} \mathbb{B}_i \in \tau_{\mathbb{B}}$ for each $\mathbb{B}_i \in \tau_{\mathbb{B}}$
3). $\mathbb{B}_i \cap \mathbb{B}_j \in \tau_{\mathbb{B}}$ for any $\mathbb{B}_i, \mathbb{B}_j \in \tau_{\mathbb{B}}$

The pair $(X, \tau_{\mathbb{B}})$ is called $\mathbb{BNSS}$-topological space. The members of $\tau_{\mathbb{B}}$ are called bipolar neutrosophic soft open sets (BNOS) and their complements are called bipolar neutrosophic soft closed sets (BNCS).
The collection of all subsets of X [P(x)] along with null set and complete set, i.e. $\tau = \{\phi_{\mathbb{B}}, 1_{\mathbb{B}}, P(X)\}$ is called discrete topology on X. The collection of X and null set, i.e. $\tau = \{\phi_{\mathbb{B}}, X\}$ is called indiscrete topology.

Example 2.14. Let $X = x_1, x_2$ be set of alternatives and $E = e_1, e_2, e_3$ be a parameter set. Now let us define a topology on (X, E) as follows.

$$\tau_{\mathcal{B}} = \left\{\phi_{\mathbb{B}}, 1_{\mathbb{B}}, \mathbb{B}_1, \mathbb{B}_2, \mathbb{B}_3, \mathbb{B}_4\right\}$$

Here $\phi_{\mathbb{B}}, 1_{\mathbb{B}}$ are null and complete $\mathbb{BNSS}$ respectively. Also,

$$\mathbb{B}_1 = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 1, 0, 1, -1, 0, 0\rangle, \langle x_2,\ 0.5, 0.2, 0.4, -0.5, -0.4, -0.3\rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.4, 0.6, 0.3, -0.4, -0.7, -0.2\rangle, \langle x_2,\ 0.7, 0.2, 0.1, -0.3, -0.5, -0.7\rangle\} \right\rangle, \\ \left\langle e_3, \{\langle x_1,\ 0.5, 0.3, 0.7, -0.2, -0.4, -0.8\rangle, \langle x_2,\ 0.4, 0.3, 0.5, -0.1, -0.4, -0.6\rangle\} \right\rangle \end{array} \right\}$$

$$\mathbb{B}_2 = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 0.3, 0.1, 0.7, -0.5, -0.6, -0.3\rangle, \langle x_2,\ 0, 1, 1, -0.7, 0, -1\rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.2, 0.5, 0.7, -1, 0, -0.2\rangle, \langle x_2,\ 0.9, 0.1, 0.3, -0.1, -0.6, -0.3\rangle\} \right\rangle, \\ \left\langle e_3, \{\langle x_1,\ 0.3, 0.5, 0.3, -0.2, 0, -0.4\rangle, \langle x_2,\ 0.7, 0.4, 0.1, -0.3, -0.5, -0.1\rangle\} \right\rangle \end{array} \right\}$$

$$\mathbb{B}_3 = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 1, 0, 0.7, -1, 0, 0\rangle, \langle x_2,\ 0.5, 0.2, 0.4, -0.7, 0, -0.3\rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.4, 0.5, 0.3, -1, 0, -0.2\rangle, \langle x_2,\ 0.9, 0.1, 0.1, -0.3, -0.5, -0.3\rangle\} \right\rangle, \\ \left\langle e_3, \{\langle x_1,\ 0.5, 0.3, 0.3, -0.2, 0, -0.4\rangle, \langle x_2,\ 0.7, 0.3, 0.1, -0.3, -0.4, -0.1\rangle\} \right\rangle \end{array} \right\}$$

$$\mathbb{B}_4 = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 0.3, 0.1, 1, -0.5, -0.6, -0.3\rangle, \langle x_2,\ 0, 1, 1, -0.5, -0.4, -1\rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.2, 0.6, 0.7, -0.4, -0.7, -0.2\rangle, \langle x_2,\ 0.7, 0.2, 0.3, -0.1, -0.6, -0.7\rangle\} \right\rangle, \\ \left\langle e_3, \{\langle x_1,\ 0.3, 0.5, 0.7, -0.2, -0.4, -0.8\rangle, \langle x_2,\ 0.4, 0.4, 0.5, -0.1, -0.5, -0.6\rangle\} \right\rangle \end{array} \right\}$$

The $\tau_{\mathbb{B}}$ satisfies all three conditions of topology. So $\tau_{\mathbb{B}}$ is a $\mathbb{BNSS}$-topology.

3. Notions of Bipolar neutrosophic soft topological spaces

Taha Yasin et al. [16] proposed bipolar neutrosophic soft topological space in 2019. Here, we defined some notions and properties of the bipolar neutrosophic soft topological spaces. However, we redefined some of the existing results in order to make suitable for the bipolar neutrosophic soft set which was defined by Arulpandy et al. [7] in 2019. Since the proposed bipolar neutrosophic soft set by Arulpandy et al. [7] is modified version of Mumtaz Ali's [6], there should be some changes in the corresponding topological spaces are also needed.

Definition 3.1. Let $(X, \tau_{\mathbb{B}})$ be a $BNST$ and $B = \left\{\langle e, f(x)\rangle : e \in E, f(x) \in BNS(X)\right\}$ be $BNSS$ in X. Then the bipolar neutrosophic soft interior and bipolar neutrosophic soft closure are defined by

$BNint(B) = \bigcup\left\{U\ :\ U\ is\ a\ BNOS\ in\ U \subseteq B\right\}$

$BNcl(B) = \bigcap\left\{V\ :\ V\ is\ a\ BNCS\ in\ V \subseteq B\right\}$

Note 3.2. Let B be BNS of a $BNTS(X, \tau)$. Then

1. $BN\alpha cl(B) = B \cup BNcl(BNint(BNcl(B)))$
2. $BN\alpha int(B) = B \cap BNint(BNcl(BNint(B)))$

Remark 3.3. Following relations hold for any BNS set $B \in (X, \tau)$.

1. $BNcl(B^c) = (BNint(B))^c$ and $BNint(B^c) = (BNcl(B))^c$.
2. $BNcl(B)$ is a $BNCS$ and $BNint(B)$ is a $BNOS$ in X.
3. B is $BNCS$ in X if and only if $BNcl(B) = B$.
4. B is $BNOS$ in X if and only if $BNint(B) = B$.

Proposition 3.4. *Let (X, τ) be a $BNSTS$ and A, B be $BNSSs$ in X. Then the following relations hold.*

i). $BNint(A) \subseteq A$; $A \subseteq BNcl(A)$

ii). $A \subseteq B \Rightarrow BNint(A) \subseteq BNint(B)$; $A \subseteq B \Rightarrow BNcl(A) \subseteq BNcl(B)$

iii). $BNint(BNint(A)) = BNint(A)$; $BNcl(BNcl(A)) = BNcl(A)$

iv). $BNint(A \cap B) = BNint(A) \cap BNint(B)$; $BNcl(A \cup B) = BNcl(A) \cup BNcl(B)$

v). $BNint(1_{BN}) = 1_{BN}$; $BNcl(0_{BN}) = 0_{BN}$

Definition 3.5. A $BNSS$set B in $BNSTS(X, \tau)$ is said to be

1). Bipolar neutrosophic soft semi closed set (BNSCS) if $BNint(BNcl(B)) \subseteq B$,

2). Bipolar neutrosophic soft semi open set (BNSOS) if $B \subseteq BNcl(BNint(B))$,

3). Bipolar neutrosophic soft pre-closed set (BNPCS) if $BNcl(BNint(B)) \subseteq B$,

4). Bipolar neutrosophic soft pre-open set (BNPOS) if $B \subseteq BNint(BNcl(B))$,

5). Bipolar neutrosophic soft α-closed set (BNαCS) if $BNcl(BNint(BNcl(B))) \subseteq B$,

6). Bipolar neutrosophic soft α-open set (BNαOS) if $B \subseteq BNint(BNcl(BNint(B)))$,

7). Bipolar neutrosophic soft semi pre-closed set (BNSPCS) if $BNint(BNcl(BNint(B))) \subseteq B$,

8). Bipolar neutrosophic soft semi pre-open set (BNSPOS) if $B \subseteq BNcl(BNint(BNcl(B)))$,

9). Bipolar neutrosophic soft regular open set (BNROS) if $B = BNint(BNcl(B))$,

10). Bipolar neutrosophic soft regular closed set (BNRCS) if $B = BNcl(BNint(B))$.

Definition 3.6. Let B be a $BNSS$ in $BNSTS(X, \tau)$. Then

1). Bipolar neutrosophic soft semi interior of B (BNsint(B)) is
$BNsint(B) = \cup\{U \mid U\ is\ a\ BNSOS\ in\ X\ and\ U \subseteq B\}$

2). Bipolar neutrosophic soft semi closure of B (BNscl(B)) is
$BNscl(B) = \cap\{V \mid V\ is\ a\ BNSCS\ in\ X\ and\ B \subseteq V\}$

3). Bipolar neutrosophic soft alpha interior of B (BNαint(B)) is
$BN\alpha int(B) = \cup\{U \mid U\ is\ a\ BN\alpha OS\ in\ X\ and\ U \subseteq B\}$

4). Bipolar neutrosophic soft alpha closure of B (BNαcl(B)) is
$BN\alpha cl(B) = \cap\{V \mid V\ is\ a\ BN\alpha CS\ in\ X\ and\ B \subseteq V\}$

5). Bipolar neutrosophic soft semi pre-interior of B (BNspint(B)) is
$BNspint(B) = \cup\{U \mid U\ is\ a\ BNSPOS\ in\ X\ and\ U \subseteq B\}$

6). Bipolar neutrosophic soft semi pre-closure of B (BNspcl(B)) is
$BNspcl(B) = \cap\{V \mid V\ is\ a\ BNSPCS\ in\ X\ and\ B \subseteq V\}$

7). Bipolar neutrosophic soft pre-interior of$B$ (BNpint(B)) is
$BNpint(B) = \cup\{U \mid U\ is\ a\ BNPOS\ in\ X\ and\ U \subseteq B\}$

8). Bipolar neutrosophic soft pre-closure of B (BNpcl(B)) is
$BNspcl(B) = \cap\{V \mid V\ is\ a\ BNPCS\ in\ X\ and\ B \subseteq V\}$

Remark 3.7. For a $BNSS$ B in (X, τ),

1. $BNscl(B) = B \cup BNint(BNcl(B))$
2. $BNsint(B) = B \cap BNcl(BNint(B))$
3. $BN\alpha cl(B) = B \cup BNcl(BNint(BNcl(B)))$
4. $BN\alpha int(B) = B \cap BNint(BNcl(BNint(B)))$
5. $BNpcl(B) = B \cup BNcl(BNint(B))$
6. $BNpint(B) = B \cap BNint(BNcl(B))$

Definition 3.8. A $BNSS$set B in $BNSTS(X, \tau)$ is said to be

1). Bipolar neutrosophic soft generalized closed set (BNGCS) if $BNcl(B) \subseteq U$ whenever $B \subseteq U$ and U is $BNOS$ in X.

2). Bipolar neutrosophic soft generalized semi closed set (BNGSCS) if $BNscl(B) \subseteq U$ whenever $B \subseteq U$ and U is $BNOS$ in X.

3). Bipolar neutrosophic soft α generalized closed set (BNαGCS) if $BN\alpha cl(B) \subseteq U$ whenever $B \subseteq U$ and U is $BNOS$ in X.

4. Bipolar neutrosophic soft generalized pre-closed sets

In this section, a new class of sets namely, bipolar neutrosophic soft generalized pre-closed sets are proposed. Also, we have investigated some properties of the proposed set with appropriate examples.

Definition 4.1. A $BNSS$set B is said to be bipolar neutrosophic soft generalized pre-closed set (BNGPCS) in (X, τ) if $BNpcl(B) \subseteq U$ whenever $B \subseteq U$ and U is $BNOS$ in X. The collection of all $BNGPCS$s of a $BNSTS$ (X, τ) is denoted by $BNGPC(X)$.

Example 4.2. Consider the BNS-topology (X, τ) in Example 2.14.
Let

$$B = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 0.3, 0.1, 1, -0.5, -0.6, -0.3\rangle, \langle x_2,\ 0, 1, 1, -0.5, -0.4, -1\rangle\}\right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.2, 0.6, 0.7, -0.4, -0.7, -0.2\rangle, \langle x_2,\ 0.7, 0.2, 0.3, -0.1, -0.6, -0.7\rangle\}\right\rangle, \\ \left\langle e_3, \{\langle x_1,\ 0.3, 0.5, 0.7, -0.2, -0.4, -0.8\rangle, \langle x_2,\ 0.4, 0.4, 0.5, -0.1, -0.5, -0.6\rangle\}\right\rangle \end{array} \right\}$$

Here, $BNint(B) = \phi_{\mathbb{B}}$ and $BNcl(BNint(B)) = B \subseteq \mathbb{B}_2$ whereas $\mathbb{B}_2$ is a $BNOS$ in (X, τ). Hence B is a $BNGPCS$ in X.

Theorem 4.3. *Every $BNCS$ is $BNGCS$ but converse not true.*

Proof. Let B be $BNCS$ in X. Suppose U in $BNOS$ in X, such that $B \subseteq U$. Then $BNcl(B) = B \subseteq U$. Hence B is $BNGCS$. Conversely, let B be a $BNGCS$; so $B \in U$ and U is some open set such that $cl(B) \subseteq U$. From this, $cl(B)$ only closed and B is not necessarily closed. Hence, B may or may not be $BNCS$. □

Example 4.4. Consider the BNS-topology in Example 2.14. Let

$$B = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 0.2, 0.3, 0.8, 0.2, 0.7, 0.8\rangle, \langle x_2,\ 0, 1, 1, 0.5, 0.4, 1\rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.1, 0.6, 0.8, 0.3, 0.5, 0.6\rangle, \langle x_2,\ 0.3, 0.4, 0.6, 0, 0.7, 0.5\rangle\} \right\rangle, \\ \left\langle e_3, \{\langle x_1,\ 0.1, 0.6, 0.5, 0.1, 0.4, 0.5\rangle, \langle x_2,\ 0.5, 0.5, 0.4, 0.2, 0.7, 0.4\rangle\} \right\rangle \end{array} \right\}$$

Then $BNcl(B) \neq B$. So B is not a $BNCS$.

Theorem 4.5. *Every $BNCS$ is $BNGPCS$ but converse not true.*

Proof. Let B be $BNCS$ in X and let $B \subseteq U$ and U be $BNOS$ in X. Since $BNpcl(B) \subseteq BNcl(B)$ and A is $BNCS$ in X, $BNpcl(B) \subseteq BNcl(B) = B \subseteq U$. So B is $BNGPCS$ in X. Conversely, if B is a $BNGPCS$, then $BNpcl(B) \subseteq U$. This means, only $BNpcl(B)$ is $BNCS$ and not necessarily B. Hence proved. □

Example 4.6. We proved earlier that every $BNCS$ is not necessarily be a $BNGCS$. By definition, every $BNGPCS$ must be a $BNGCS$ first. This implies that, every $BNCS$ not necessarily be a $BNGPCS$.

Theorem 4.7. *Every $BNGCS$ is $BNGPCS$ but converse not true.*

Proof. By definition of $BNGCS$, for some $BNOS$ U, $cl(B) \subseteq U$. Since B is closed by default, $cl(int(B)) = cl(B)$. So $cl(in(B)) \subseteq U$. Hence B is $BNGPCS$. Conversely, let B be $BNGPCS$ in X. Then, B is not necessarily closed. So B may or may not be $BNGCS$. □

Example 4.8. Consider the topology in Example 2.14 and $BNGCS$ in Example 4.4. Since B is closed set by default, $BNint(B) \neq B$ in most of the cases (equal in some cases). So, B is not $BNGCS$.

Theorem 4.9. *Every $BN\alpha CS$ is $BNGPCS$ but converse not true.*

Proof. Let B be a $BN\alpha CS$ in X and let $B \subseteq U$ and U be $BNOS$ in X. Since $B \subseteq BNcl(B)$, $BNcl(BNint(B)) \subseteq BNcl(BNint(BNcl(B))) \subseteq B$. Hence $BNpcl(B) \subseteq B \subseteq U$. So B is $BNGPCS$ in X. By converse, let B be $BNGPCS$ in X. By default, $BN\alpha CS$ is a subset of $BNpcs$. So it is obvious that every $BNGPCS$ is not necessarily be a $BN\alpha CS$. □

Example 4.10. Since every PCS is not necessarily be a $\alpha - CS$. By definition, every PCS must be a $GPCS$ first. From this, every, $GPCS$ not necessarily be a $\alpha - CS$. So that every $BNGPCS$ not necessarily a $BN\alpha CS$.

Theorem 4.11. *Every $BNPCS$ is $BNGPCS$ but converse not true.*

Proof. Let B be $BNPCS$ in X and let $B \subseteq U$ for some $BNOS$ U in X. By definition of $BNPCS$, $BNcl(BNint(B)) \subseteq B$. This gives, $BNpcl(B) = B \cup BNcl(BNint(B)) \subseteq B$. Hence $BNpcl(B) \subseteq U$. So B is $BNGPCS$ in X. □

Example 4.12. Since every $BNGPCS$ not necessarily be closed. But every $BNPCS$ is closed. So that, every $BNGPCS$ not necessarily be a $BNPCS$.

Theorem 4.13. *Every $BN\alpha GCS$ is $BNGPCS$ but converse not true.*

Proof. Let B be $BN\alpha GCS$ in X and let $B \subseteq U$ for some $BNOS$ U in (X, τ). From Note3.2, $B \cup BNcl(BNint(BNcl(A))) \subseteq U$. So $BNcl(BNint(BNcl(B))) \subseteq U$ and $BNcl(BNint(B)) \subseteq U$. Thus $BNpcl(B) = B \cup BNcl(BNint(A)) \subseteq U$. Hence B is $BNGPCS$ in X. □

Example 4.14. Since every $BNGPCS$ not necessarily a $BN\alpha CS$ and every $BN\alpha GCS$ not necessarily be a $BN\alpha CS$, so that every $BNGPCS$ not necessarily be $BN\alpha GCS$.

Theorem 4.15. *Every $BNGPCS$ is $BNSPCS$ but converse not true.*

Proof. Let B be $BNGPCS$ in X, then $BNpcl(B) \subseteq U$ when $B \subseteq U$ for some $BNOS$ U in X. By definition, $BNcl(BNint(B)) \subseteq B$. Therefore $BNint(BNcl(BNint(B))) \subseteq BNint(B) \subseteq B$. So $BNint(BNcl(BNint(B))) \subseteq B$. Hence B is $BNSPCS$ in X. □

Example 4.16. Since every $BNSPCS$ not necessarily a $BNPCS$ and every $BNPCS$ must be a $BNGPCS$, so that every $BNSPCS$ not necessarily be $BNGPCS$.

5. Bipolar neutrosophic soft generalized pre-open sets

In this section, bipolar neutrosophic soft generalized pre-open sets as the complement of bipolar neutrosophic soft generalized pre-closed sets are proposed. Also, we have investigated some properties of the proposed set with appropriate examples.

Definition 5.1. A $BNSS$ set B is said to bipolar neutrosophic soft generalized pre-open set (BNGPOS) in (X, τ) if the complement B^c is $BNGPCS$ in X. The collection of all $BNGPOS$s of $BNST(X, \tau)$ is denoted by $BNGPO(X)$.

Example 5.2. Consider the BNS-topology (X, τ) in Example 2.14. Let

$$B = \left\{ \begin{array}{c} \left\langle e_1, \{\langle x_1,\ 1, 0, 0.7, -1, 0, 0\rangle, \langle x_2,\ 0.5, 0.2, 0.4, -0.7, 0, -0.3\rangle\} \right\rangle, \\ \left\langle e_2, \{\langle x_1,\ 0.4, 0.5, 0.3, -1, 0, -0.2\rangle, \langle x_2,\ 0.9, 0.1, 0.1, -0.3, -0.5, -0.3\rangle\} \right\rangle, \\ \left\langle e_3, \{\langle x_1,\ 0.5, 0.3, 0.3, -0.2, 0, -0.4\rangle, \langle x_2,\ 0.7, 0.3, 0.1, -0.3, -0.4, -0.1\rangle\} \right\rangle \end{array} \right\}$$

Here, $BNcl(B) = 1_{\mathbb{B}}$ and $BNcl(BNint(B)) = 1_{\mathbb{B}} \supseteq B$ whereas $1_{\mathbb{B}}$ is a $BNOS$ in (X, τ). Hence, by definition, B is a $BNGPOS$ in X.

Theorem 5.3. *Let (X, τ) be a $BNSTS$. Then the following relations are hold.*

1). Every $BNOS$ is $BNGPOS$ but converse not true.

2). Every $BNROS$ is $BNGPOS$ but converse not true.

3). Every $BN\alpha OS$ is $BNGPOS$ but converse not true.

4). Every $BNPOS$ is $BNGPOS$ but not converse not true.

Theorem 5.4. *Let (X, τ) be a $BNSTS$. If $B \in BNGPO(X)$, then $V \subseteq BNint(BNcl(B))$ whenever $V \subseteq B$ and V is $BNCS$ in X.*

Proof. Let $B \in BNGPO(X)$. Then B^c be a $BNGPCS$ in X. So $BNpcl(B^c) \subseteq U$ whenever $B^c \subseteq U$ and U is $NVOS$ in X. Therefore $BNcl(BNint(B^c)) \subseteq U$. This implies that $U^c \subseteq BNint(BNcl(B))$ whenever $U^c \subseteq B$ and U^c is $BNCS$ in X. Substituting U^c by V, we get $V \subseteq BNint(BNcl(B))$ whenever $V \subseteq B$ and V is $BNCS$ in X. □

Theorem 5.5. *Let (X, τ) be $BNSTS$. Then for every $B \in BNGPO(X)$ and for every $N \in BNS(X)$, $BNpint(B) \subseteq N \subseteq B$ implies $N \in BNGPO(X)$.*

Proof. By hypothesis $B^c \subseteq N^c \subseteq (BNpint(B))^c$. Let $N^c \subseteq U$ and U be $BNOS$. Since $B^c \subseteq N^c \subseteq, B^c \subseteq U$. But B^c is $BNGPCS$, $BNpcl(B^c) \subseteq U$. Also $B^c \subseteq (BNpint(B))^c = BNpcl(B^c)$. Therefore $BNpcl(N^c) \subseteq BNpcl(B^c) \subseteq U$. Hence N^c is $BNGPCS$ which implies B is $BNGPOS$ in X. □

Theorem 5.6. *A BNS B of $BNSTS(X, \tau)$ is $BNGPOS$ if and only if $V \subseteq BNpint(B)$ whenever V is $BNCS$ and $V \subseteq B$.*

Proof. Suppose B is $BNGPOS$ in X. Let V be $BNCS$ and $V \subseteq B$. Then V^c is $BNOS$ in X such that $B^c \subseteq V^c$. Since B^c is $BNGPCS$, we have $BNpcl(B^c) \subseteq V^c$. Hence $(BNpint(B))^c \subseteq V^c$. Therefore $V \subseteq BNpint(B)$.

On the other hand, let B be BNS in X and let $V \subseteq BNpint(B)$ whenever V is $BNCS$ and

$V \subseteq B$. Then $B^c \subseteq N^c$ and N^c is $BNOS$. By hypothesis, $(BNpint(B))^c \subseteq N^c$ which implies $BNpcl(B^c) \subseteq N^c$. Therefore B^c is $BNGPCS$ of X. Hence B is $BNGPOS$ in X. □

Theorem 5.7. *A BNS B of a* $BNSTS(X,\tau)$ *is BNGPOS if and only if* $V \subseteq BNint(BNcl(B))$ *whenever V is BNCS and* $V \subseteq B$.

Proof. Suppose B is $BNGPOS$ in X. Let V be $NVCS$ and $V \subseteq B$. Then V^c is $BNOS$ in X such that $B^c \subseteq V^c$. Since B^c is $BNGPCS$, we have $BNpcl(B^c) \subseteq V^c$. Therefore $BNcl(BNint(B^c)) \subseteq V^c$. Hence $(BNint(BNcl(B)))^c \subseteq V^c$. This implies $V \subseteq BNint(BNcl(B))$.
On the other hand, let B be BNS of X and let $V \subseteq BNint(BNcl(B))$ whenever V is $BNCS$ and $V \subseteq B$. Then $B^c \subseteq V^c$ and V^c is $BNOS$. By hypothesis, $(BNint(BNcl(B)))^c \subseteq V^c$. Hence $BNcl(BNint(B^c)) \subseteq V^c$, which implies $BNpcl(B^c) \subseteq V^c$. Hence B is $BNGPOS$ of X. □

Theorem 5.8. *For any BNS B, B is BNOS and BNGPCS in X if and only if B is BNROS in X.*

Proof. Let B be $BNOS$ and $BNGPCS$ in X. Then $BNpcl(B) \subseteq B$. This implies $BNcl(BNint(B)) \subseteq B$. Since B is $BNOS$, it is $BNPOS$. Hence $B \subseteq BNint(BNcl(B))$. Therefore $B = BNint(BNcl(B))$. Hence B is $BNROS$ in X.
On the other hand, let B be $BNROS$ in X. So $B = BNint(BNcl(B))$. Let $B \subseteq U$ and U is $BNOS$ in X. This implies that $BNpcl(B) \subseteq B$. Hence B is $BNGPCS$ in X. □

Remark 5.9. There are few limitations of the proposed works. The proposed bipolar neutrosophic soft generalized pre-closed sets and pre-open sets are purely based on point set topology (i.e. general topology). So it is quite difficult to apply in real world problems unlike neutrosophic sets. On the other hand, along with neutrosophic topology, we can explore many applied mathematics problems such as decision making technique, image processing, data analytics and so on. Also, the soft sets are parametrized sets in nature. So obviously, the proposed topology and proposed sets are based on parameters. There are few drawbacks when applying soft sets in real world problems such as choosing correct number of parameters and choosing only the essential parameters. It will create an impact in final results. To overcome this, the user can decide the number of parameters and choice of parameters depends on the problem's nature.

6. Conclusion

Bipolar neutrosophic soft set is the base for many topological spaces. In topology, the topological structures such as closedness and openness are the important concepts. It helps to determine the continuity of a mapping between to topologies. Many researchers have proposed various types of closed and open sets for a specific topological space. In this paper, we introduced new family of sets namely, bipolar neutrosophic soft generalized pre-closed sets and bipolar neutrosophic soft generalized pre-open sets for the bipolar neutrosophic soft topological space. Further, some important relations between proposed sets and many other type of sets have been discussed through theorems. Development of bipolar neutrosophic soft generalized pre-sets is thought to contribute to the development of bipolar neutrosophic soft continuity in the topology as well as algebra, geometry and analysis of other sub-branches of mathematics. We expect that the proposed sets will serve contributions to some future works about bipolar neutrosophic soft topology. Our future work will consist applications of the proposed sets and topology in decision making problems. There are numerous neutrosophy based decision making algorithms available. In future, we will explore decision making scenarios and try to define novel algorithms by applying proposed concepts. Also, image processing is one of the field which uses neutrosophic logic. We will try to develop image processing algorithms based on proposed neutrosophic topology such as image denoising, segmentation, edge detection and so on.

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