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Abstract: The notion of Neutrosophic sets naturally plays a significant role in the study of Neutrosophic topology which was introduced by A.A. Salama. Chang also studied fuzzy continuity which was proved to be of fundamental importance in the realm of Neutrosophic topology. Since then various notions in classical topology have been extended to Neutrosophic topological spaces. Aim of this paper is to initiate and examine about new type of Neutrosophic closed set called Neutrosophic α-GS closed sets and Neutrosophic α-GS open sets. Further some of their properties are discussed.

Keywords: Neutrosophic, Topological, Closed Sets, $\boldsymbol{\alpha}$GS

1. Introduction

In 1980s the international movement called paradoxism based on contradictions in science and literature, was founded by Smarandache[18,19], who then extended it to neutrosophy, based on contradictions and their neutrals.Smarandache's[18,19] Neutrosophic sets have the components T, I, F which symbolize the membership, indeterminacy and non-membership values in that order. A.A. Salama [32] introduced Neutrosophic topological spaces by using Smarandache's Neutrosophic sets. Each year different kinds of Neutrosophic closed sets have been introduced by researchers.The concept of Neutrosophic semiopen sets and Neutrosophic semiclosed sets were first introduced in Neutrosophic topological space by P. Ishwarya [20] in 2016 and also studied the concept of Neutrosophic semi interior and closure properties in Neutrosophic topological spaces. V.VenkateswaraRao & Y.SrinivasaRao [35] extended the concepts of Neutrosophic preopen sets and Neutrosophic pre closed sets in Neutrosophic setting in 2017. I. Arokiarani [7] et al., introduced Neutrosophic α closed sets in Neutrosophic topological spaces. R. Dhavaseelan, and S.Jafari (2018)[17] introduced and studied the concept of Generalized Neutrosophic closed sets. V.K.Shanthi and S.Chandrasekar[34] et al (2018) introduced and established Neutrosophic Generalized semi closed sets. Another important Neutrosophic closed sets Neutrosophic $\boldsymbol{\alpha}$-generalized closed sets initiated by R. Dhavaseelan[17] et al., Aim of this paper is , We introduce the concepts of Neutrosophic $\boldsymbol{\alpha}$-generalized semi-closed sets and Neutrosophic α -generalized semi-open sets. we get results Every Neutrosophic closed set, Neutrosophic α-closed sets, Neutrosophic regular closed sets are Neutrosophic $\boldsymbol{\alpha}$-generalized semi-closed sets. Also, every Neutrosophic α-generalized semi-closed sets is Neutrosophic α-generalized closed sets, Neutrosophic generalized α- closed sets, and Neutrosophic generalized semi-closed sets. Neutrosophic α-generalized semi-closed sets independent with Neutrosophic pre-closed sets, Neutrosophic b closed sets, Neutrosophic semi pre-closed sets and Neutrosophic generalized closed sets. We obtain their properties and relationship between other Neutrosophic closed sets.. Also, we discussed their properties and relationships.

2. Preliminaries

Definition 1.1 [18,19] Let N^X be a non-empty fixed set. A Neutrosophic set V_1^* in N^X is a object having the form$V_1^* = \{\langle x, \mu_{V_1^*}(x), \sigma_{V_1^*}(x), \nu_{V_1^*}(x)\rangle | x \in N^x\}$ where the function $\mu_{V_1^*}(x): N^X \to [0,1]$ degree of membership (namely $\mu_{V_1^*}(x)$), $\sigma_{V_1^*}(x)$ denotes the indeterminancy and the function $\nu_{V_1^*}(x): N^X \to [0,1]$ denotes the degree of non-membership (namely $\nu_{V_1^*}(x)$) of each element $x \in N^X$ to the set V_1^* respectively.

Definition 1.2 [18,19]. Let V_1^* and V_2^* be NSs of the form $V_1^* = \{\langle x, \mu_{V_1^*}(x), \sigma_{V_1^*}(x), \nu_{V_1^*}(x)\rangle | x \in N^x\}$ and $V_2^* = \{\langle x, \mu_{V_2^*}(x), \sigma_{V_2^*}(x), \nu_{V_2^*}(x)\rangle | x \in N^x\}$. Then

1. $V_1^* \subseteq V_2^*$ iff $\mu_{V_1^*}(x) \le \mu_{V_2^*}(x)$, $\sigma_{V_1^*}(x) \le \sigma_{V_2^*}(x)$ and $\nu_{V_1^*}(x) \ge \nu_{V_2^*}(x)$ for all $x \in N^x$
2. $V_1^* = V_2^*$ iff $V_1^* \subseteq V_2^*$ and $V_2^* \subseteq V_1^*$
3. $V_1^{*c} = \{\langle x, \nu_{V_1^*}(x), 1 - \sigma_{V_1^*}(x), \mu_{V_1^*}(x)\rangle | x \in N^x\}$
4. $V_1^* \cap V_2^*$=$\{\langle x, \mu_{V_1^*}(x) \wedge \mu_{V_2^*}(x), \sigma_{V_1^*}(x) \wedge \sigma_{V_2^*}(x), \nu_{V_1^*}(x) \vee \nu_{V_2^*}(x)\rangle | x \in N^x\}$
5. $V_1^* \cup V_2^*$=$\{\langle x, \mu_{V_1^*}(x) \vee \mu_{V_2^*}(x), \sigma_{V_1^*}(x) \vee \sigma_{V_2^*}(x), \nu_{V_1^*}(x) \wedge \nu_{V_2^*}(x)\rangle | x \in N^x\}$

Definition 1.3 [32]. A Neutrosophic topology (NT in short) on N^x is a family $N^\tau$of NS in N^x satisfying the following axioms.

1. $0_N, 1_N \in N^\tau$
2. $J_1 \cap J_2 \in N^\tau$ for any $J_1, J_2 \in N^\tau$
3. $\cup J_i \in N^\tau$for any family $\{J_i | i \in j\} \subseteq N^\tau$

In this case, the pair (N^X, N^τ) is called a Neutrosophic topological space (NTS in short) and any NS in N^τ is known as an Neutrosophic open set (NOS) in N^X. The complement V_1^{*c} of a NOS V_1^* in a NTS (N^X, N^τ) is called a Neutrosphic closed set (NCS) in N^X.

Definition 1.4 [32]. For any NSs V_1^* and V_2^* in (N^X, N^τ), we have

1. $N^{int}(V_1^*) \subseteq V_1^*$
2. $V_1^* \subseteq N^{cl}(V_1^*)$
3. $V_1^* \subseteq V_2^* \Longrightarrow N^{int}(V_1^*) \subseteq N^{int}(V_2^*)$ and $N^{cl}(V_1^*) \subseteq N^{cl}(V_2^*)$
4. $N^{int}(N^{int}(V_1^*)) = N^{int}(V_1^*)$
5. $N^{cl}(N^{cl}(V_1^*)) = N^{cl}(V_1^*)$
6. $N^{cl}(V_1^* \cup V_2^*) = N^{cl}(V_1^*) \cup N^{cl}(V_2^*)$
7. $N^{int}(V_1^* \cap V_2^*) = N^{int}(V_1^*) \cap N^{int}(V_2^*)$

Proposition 1.5 [32]. For any NS V_1^* in (N^X, N^τ), we have

1. $N^{int}(0_N) = 0_N$ and $N^{cl}(0_N) = 0_N$

2. $N^{int}(1_N) = 1_N$ and $N^{cl}(1_N) = 1_N$
3. $(N^{int}(V_1^*))^c = N^{cl}(V_1^{*c})$
4. $(N^{cl}(V_1^*))^c = N^{int}(V_1^{*c})$

Definition 1.6. A NS $V_1^* = \langle x, \mu_{V_1^*}, \sigma_{V_1^*}, \nu_{V_1^*} \rangle$ in a NTS (N^X, N^τ) is called as

1. Neutrosophic regular closed set [7] (N(R)CS in short) if $V_1^* = N^{cl}(N^{int}(V_1^*))$
2. Neutrosophic α-closed set [7] (N(α)CS in short) if $N^{cl}\big(N^{int}(N^{cl}(V_1^*))\big) \subseteq V_1^*$
3. Neutrosophic semi closed set [20] (N(S)CS in short) if $N^{int}(N^{cl}(V_1^*)) \subseteq V_1^*$
4. Neutrosophic pre-closed set [35] (N(P)CS in short) if $N^{cl}(N^{int}(V_1^*)) \subseteq V_1^*$
5. Neutrosophic b-closed set [23] (N(b)CS in short) if $N^{cl}(N^{int}(V_1^*)) \cap N^{int}(N^{cl}(V_1^*)) \subseteq V_1^*$

Definition 1.7. A NS V_1^* of a NTS (N^X, N^τ) is a

1. Neutrosophic semi preopen set [17] (N(SP)OS) if there exists a N(P)OS V_2^* such that $V_1^* \subseteq (V_1^*) \subseteq N^{cl}(V_2^*)V_1^*$
2. Neutrosophic semi pre closed set (N(SP)CS) if there exists a N(P)CS V_2^* such that $N^{int}(V_2^*) \subseteq V_1^* \subseteq V_2^*$

Definition 1.8. Let V_1^* be a NS in (N^X, N^τ), then Neutrosophic semi interior of V_1^* ($N^{Sint}(V_1^*)$ in short) and Neutrosophic semi closure of V_1^* ($N^{Scl}(V_1^*)$ in short) are defined as

1. $N^{Sint}(V_1^*) = \cup \{H | H \text{ is a } N(S)OS \text{ in } N^X \text{ and } H \subseteq V_1^*\}$
2. $N^{Scl}(V_1^*) = \cap \{G | G \text{ is a } N(S)CS \text{ in } N^X \text{ and } V_1^* \subseteq G\}$

Definition 1.9. Let V_1^* be a NS in (N^X, N^τ), then Neutrosophic semi pre interior of V_1^* ($N^{SPint}(V_1^*)$ in short) and Neutrosophic semi preclosure of V_1^* ($N^{SPcl}(V_1^*)$ in short) are defined as

1. $N^{SPint}(V_1^*) = \cup \{E | E \text{ is a } N(S)POS \text{ in } N^X \text{ and } E \subseteq V_1^*\}$
2. $N^{SPcl}(V_1^*) = \cap \{K | K \text{ is a } N(S)PCS \text{ in } N^X \text{ and } V_1^* \subseteq K\}$

Definition 1.10. Let V_1^* be an NS of a NTS (N^X, N^τ). Then

1. $N^{\alpha cl}(V_1^*) = \cap \{I | I \text{ is a } N(\alpha)CS \text{ in } N^X \text{ and } V_1^* \subseteq I\}$
2. $N^{\alpha int}(V_1^*) = \cup \{I | I \text{ is a } N(\alpha)OS \text{ in } N^X \text{ and } I \subseteq V_1^*\}$

Definition 1.11. A NS V_1^* of a NTS (N^X, N^τ) is a

1. Neutrosophic generalized closed set [15] (N(G)CS in short) if $N^{cl}(V_1^*) \subseteq \Psi$ whenever $V_1^* \subseteq \Psi$ and Ψ is a NOS in N^X.
2. Neutrosophic generalized semi closed set [34](N(GS)CS in short) if $N^{Scl}(V_1^*) \subseteq \Psi$ whenever $V_1^* \subseteq \Psi$ and Ψ is a NOS in N^X.
3. Neutrosophic alpha generalized closed set [21](N(α)GCS in short) if $N^{\alpha cl}(V_1^*) \subseteq \Psi$ whenever $V_1^* \subseteq \Psi$ and Ψ is a NOS in N^X.
4. Neutrosophic generalized alpha closed set [16](NGαCS in short) if $N^{\alpha cl}(V_1^*) \subseteq \Psi$ whenever $V_1^* \subseteq \Psi$ and Ψ is a NαOS in N^X.

The complement of the above mentioned Neutrosophic closed sets are called their relevant open sets.

Remark 1.12. Let V_1^* be a NS in (N^X, N^τ). Then

1. $N^{S-cl}(V_1^*) = V_1^* \cap N^{int}(N^{cl}(V_1^*))$
2. $N^{S-int}(V_1^*) = V_1^* \cup N^{cl}(N^{int}(V_1^*))$

If V_1^* is a NS of N^X then $N^{Scl}(V_1^{*c}) = (N^{Scl}(V_1^*))^c$

Definition 1.13. Let V_1^* be a NS in (N^X, N^τ). Then

1. $N^{\alpha cl}(V_1^*) = V_1^* \cup N^{cl}\left(N^{int}(N^{cl}(V_1^*))\right)$

2. $N^{\alpha int}(V_1^*) = V_1^* \cap N^{int}\left(N^{cl}(N^{int}(V_1^*))\right)$

2. Neutrosophic α Generalized Semi-Closed Sets

Definition 2.1. A NS V_1^* in (N^X, N^τ) is said to be a Neutrosophic α generalized semi-closed set (N(αGS)CS in short) if $N^{\alpha cl}(V_1^*) \subseteq \Psi$ whenever $V_1^* \subseteq \Psi$ and Ψ is a N(S)OS in (N^X, N^τ).

Example 2.2. Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ be a NT on N^X, where $J_1^* = \langle x, \left(\frac{6}{10}, \frac{1}{2}, \frac{4}{10}\right), \left(\frac{7}{10}, \frac{1}{2}, \frac{3}{10}\right) \rangle$. Let us consider the NS $V_1^* = \langle x, \left(\frac{1}{10}, \frac{1}{2}, \frac{9}{10}\right), \left(\frac{1}{5}, \frac{1}{2}, \frac{4}{5}\right) \rangle$.

Since $N^{\alpha cl}(V_1^*) = V_1^*$, V_1^* is N(αGS)CS in (N^X, N^τ).

Theorem 2.3 Every NCS in (N^X, N^τ) is a N(αGS)CS.

Proof: Assume that V_1^* is a NCS in (N^X, N^τ). Let us consider a NS $V_1^* \subseteq \Psi$ and Ψ is a N(S)OS in N^X. Since $N^{\alpha cl}(V_1^*) \subseteq N^{cl}(V_1^*)$ and V_1^* is a NCS in N^X, $N^{\alpha cl}(V_1^*) \subseteq N^{cl}(V_1^*) = V_1^* \subseteq \Psi$ and Ψ is N(S)OS. That is $N^{\alpha cl}(V_1^*) \subseteq \Psi$. Therefore V_1^* is N(αGS)CS in N^X.

Example 2.4. Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ be a NT on N^X. Here $J_1^* = \langle x, \left(\frac{2}{5}, \frac{1}{2}, \frac{3}{5}\right), \left(\frac{1}{5}, \frac{1}{2}, \frac{7}{10}\right) \rangle$.

Then the NS $V_1^* = \langle x, \left(\frac{7}{10}, \frac{1}{2}, \frac{3}{10}\right), \left(\frac{4}{5}, \frac{1}{2}, \frac{1}{5}\right) \rangle$ is N(αGS)CS but not NCS. Since $N^{\alpha cl}(V_1^*) = 1_N$ and possible $\Psi = 1_N$.

Theorem 2.5 Every NαCS in (N^X, N^τ) is a N(αGS)CS in (N^X, N^τ).

Proof: Let V_1^* be a NαCS in N^X. Let us consider a NS $V_1^* \subseteq \Psi$ and Ψ be a N(S)OS in (N^X, N^τ). Since V_1^* is a NαCS, $N^{\alpha cl}(V_1^*) = V_1^*$. Hence $N^{\alpha cl}(V_1^*) \subseteq \Psi$ whenever $V_1^* \subseteq \Psi$ and Ψ is N(S)OS. Therefore V_1^* is a N(αGS)CS in N^X.

Example 2.6. Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ be a NT on N^X. Here $J_1^* = \langle x, \left(\frac{2}{5}, \frac{1}{2}, \frac{3}{5}\right), \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right) \rangle$.

Consider NS $V_1^* = \langle x, \left(\frac{4}{5}, \frac{1}{2}, \frac{1}{5}\right), \left(\frac{3}{5}, \frac{1}{2}, \frac{2}{5}\right) \rangle$ is N(αGS)CS but not NαCS since $N^{cl}\left(N^{int}(N^{cl}(V_1^*))\right) = 1_N \nsubseteq V_1^*$.

Theorem 2.7 Every N(R)CS in (N^X, N^τ) is a N(αGS)CS in (N^X, N^τ).

Proof: Let V_1^* be a N(R)CS in (N^X, N^τ). Since every N(R)CS is a NCS ,V_1^* is a NCS in N^X. By Theorem 2.3, V_1^* is a N(αGS)CS in N^X.

Example 2.8. Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ be a NT on N^X.

Here $J_1^* = \langle x, \left(\frac{4}{5}, \frac{1}{2}, \frac{1}{5}\right), \left(\frac{7}{10}, \frac{1}{2}, \frac{3}{10}\right)\rangle$. Consider a NS $V_1^* = \langle x, \left(0, \frac{1}{2}, \frac{9}{10}\right), \left(\frac{1}{5}, \frac{1}{2}, \frac{4}{5}\right)\rangle$ which is a N(αGS)CS but not N(R)CS in N^X as $N^{cl}(N^{int}(V_1^*)) = 0_N \neq V_1^*$.

Remark 2.9. A N(G) closedness is independent of a N(αGS) closedness.

Example 2.10. Let $N^X = \{v_1, v_2\}$.Let $N^\tau = \{0_N, J_1^*, 1_N\}$ be a NT on N^X.

Here $J_1^* = \langle x, \left(\frac{2}{5}, \frac{1}{2}, \frac{1}{2}\right), \left(\frac{7}{10}, \frac{1}{2}, \frac{3}{10}\right)\rangle$. Then the NS $V_1^* = \langle x, \left(\frac{3}{10}, \frac{1}{2}, \frac{3}{5}\right), \left(\frac{1}{5}, \frac{1}{2}, \frac{4}{5}\right)\rangle$ is a N(αGS)CS but not NGCS in N^X as $N^{cl}(V_1^*) \nsubseteq G$ eventhough $V_1^* \subseteq G$ and G is a GSOS in N^X.

Example 2.11. Let $N^X = \{v_1, v_2\}$.Let $N^\tau = \{0_N, J_1^*, 1_N\}$ be a NT on N^X.

Here $J_1^* = \langle x, \left(\frac{3}{5}, \frac{1}{2}, \frac{2}{5}\right), \left(\frac{4}{5}, \frac{1}{2}, \frac{1}{5}\right)\rangle$. Then the NS $V_1^* = \langle x, \left(\frac{7}{10}, \frac{1}{2}, \frac{3}{10}\right), \left(\frac{9}{10}, \frac{1}{2}, \frac{1}{10}\right)\rangle$ is a NGCS but not N(αGS)CS since $N^{cl}(V_1^*) = 1_N \nsubseteq V_2^* = \langle x, \left(\frac{4}{5}, \frac{1}{2}, \frac{1}{5}\right), \left(\frac{9}{10}, \frac{1}{2}, \frac{1}{10}\right)\rangle$ whenever $V_1^* \subseteq V_2^*$ and V_2^* is a N(S)OS in N^X.

Theorem 2.12. Every N(αGS)CS in (N^X, N^τ) is a NGSCS in (N^X, N^τ).

Proof: Assume that V_1^* is a N(αGS)CS in (N^X, N^τ). Let a NS $V_1^* \subseteq \Psi$ and Ψ be a NOS in N^X. By hypothesis $N^{cl}(V_1^*) \subseteq \Psi$, that is $V_1^* \cup N^{cl}\left(N^{int}(N^{cl}(V_1^*))\right) \subseteq \Psi$. This implies $V_1^* \cup N^{int}(N^{cl}(V_1^*)) \subseteq \Psi$. But $N^{scl}(V_1^*) = V_1^* \cup N^{int}(N^{cl}(V_1^*))$. Therefore $N^{scl}(V_1^*) = V_1^* \cup N^{int}(N^{cl}(V_1^*)) \subseteq \Psi$ whenever $V_1^* \subseteq \Psi$ and Ψ is NOS. Hence V_1^* is NGSCS.

Example 2.13. Let $N^X = \{v_1, v_2\}$.Let $N^\tau = \{0_N, J_1^*, 1_N\}$ be a NT on N^X.Here $J_1^* = \langle x, \left(\frac{7}{10}, \frac{1}{2}, \frac{3}{10}\right), \left(\frac{4}{5}, \frac{1}{2}, \frac{1}{10}\right)\rangle$. Then the NS $V_1^* = \langle x, \left(\frac{4}{5}, \frac{1}{2}, \frac{1}{5}\right), \left(\frac{4}{5}, \frac{1}{2}, 0\right)\rangle$ is a NGCS but not N(αGS)CS as $N^{\alpha cl}(V_1^*) = 1_N \nsubseteq V_2^* = \langle x, \left(\frac{9}{10}, \frac{1}{2}, \frac{1}{10}\right), \left(\frac{9}{10}, \frac{1}{2}, 0\right)\rangle$ whenever $V_1^* \subseteq V_2^*$ and V_2^* is a N(S)OS in N^X.

Remark 2.14. A NP closedness is independent of NαGS closedness.

Example 2.15. Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ be a NT on N^X. Here $J_1^* = \langle x, \left(\frac{2}{5}, \frac{1}{2}, \frac{7}{10}\right), \left(\frac{1}{10}, \frac{1}{2}, \frac{4}{5}\right)\rangle$. Then the NS $V_1^* = \langle x, \left(\frac{1}{5}, \frac{1}{2}, \frac{7}{10}\right), \left(\frac{1}{10}, \frac{1}{2}, \frac{4}{5}\right)\rangle$ is a N(P)CS but not N(αGS)CS. Since $N^{\alpha cl}(V_1^*) \nsubseteq G$ even though $V_1^* \subseteq G$ and G is N(S)OS.

Example 2.16. Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, J_2^*, 1_N\}$ is NT on N^X . Here $J_1^* = \langle x, \left(\frac{3}{10}, \frac{1}{2}, \frac{7}{10}\right), \left(\frac{2}{5}, \frac{1}{2}, \frac{3}{5}\right)\rangle$ and $J_2^* = \langle x, \left(\frac{2}{5}, \frac{1}{2}, \frac{3}{5}\right), \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)\rangle$. Then the NS $V_1^* = \langle x, \left(\frac{3}{10}, \frac{1}{2}, \frac{7}{10}\right), \left(\frac{3}{5}, \frac{1}{2}, \frac{2}{5}\right)\rangle$ is a N(αGS)CS. Since $N^{cl}(N^{int}(V_1^*)) \subseteq V_1^*, V_1^*$ is not a N(P)CS.

Remark 2.17. N(SP) closedness is independent of a NαGS closedness.

Example 2.18. Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ is NT on N^X. Here $J_1^* = \langle x, \left(\frac{3}{10}, \frac{1}{2}, \frac{3}{5}\right), \left(\frac{2}{5}, \frac{1}{2}, \frac{1}{2}\right)\rangle$.

Then the NS $V_1^* = \langle x, \left(\frac{2}{5}, \frac{1}{2}, \frac{1}{2}\right), \left(\frac{3}{10}, \frac{1}{2}, \frac{3}{5}\right)\rangle$ is a NSPCS but not N(αGS)CS. Since $N^{\alpha cl}(V_1^*) \nsubseteq V_2^*$, $V_2^* = \langle x, \left(\frac{1}{2}, \frac{1}{2}, \frac{2}{5}\right), \left(\frac{2}{5}, \frac{1}{2}, \frac{1}{2}\right)\rangle$ where $V_1^* \nsubseteq V_2^*$ and V_2^* is N(S)OS.

Example 2.19. Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, J_2^*, 1_N\}$ is NT on N^X . Here $J_1^* = \langle x, \left(\frac{3}{10}, \frac{1}{2}, \frac{7}{10}\right), \left(\frac{2}{5}, \frac{1}{2}, \frac{3}{5}\right)\rangle$ and $J_2^* = \langle x, \left(\frac{2}{5}, \frac{1}{2}, \frac{3}{5}\right), \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)\rangle$. Then the NS $V_1^* = \langle x, \left(\frac{3}{10}, \frac{1}{2}, \frac{7}{10}\right), \left(\frac{3}{5}, \frac{1}{2}, \frac{2}{5}\right)\rangle$ is a N(αGS)CS but not NSPCS as $N^{int}\left(N^{cl}(N^{int}(V_1^*))\right) \nsubseteq V_1^*$.

Diagram- I

$$\begin{tikzcd}
 & & & NGSCS & & & \\
N(R)CS \arrow[r] & NCS \arrow[r] & N\alpha CS \arrow[ur] \arrow[rr] \arrow[dr] & & N(\alpha GS)CS \arrow[ul] \arrow[dl] \arrow[r] & NG\alpha CS \arrow[r] & N\alpha GCS \\
 & & & NGSPCS & & &
\end{tikzcd}$$

Diagram-II

$$\begin{tikzcd}
 & N(b)CS \arrow[d, leftrightarrow, "/" marking] & \\
NPCS \arrow[r, leftrightarrow, "/" marking] & N(\alpha GS)CS \arrow[r, leftrightarrow, "/" marking] \arrow[d, leftrightarrow, "/" marking] & N(SP)CS \\
 & N(G)CS &
\end{tikzcd}$$

Remark 2.20. *Nb* closedness is independent of NαGS closedness.

Example 2.21. Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ is NT on N^X,Here $J_1^* = \langle x, \left(\frac{2}{5}, \frac{1}{2}, \frac{4}{5}\right), \left(\frac{2}{5}, \frac{1}{2}, \frac{3}{5}\right)\rangle$.

Then the NS $V_1^* = \langle x, \left(\frac{3}{5}, \frac{1}{2}, \frac{2}{5}\right), \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)\rangle$ is NbCS but not N(α GS)CS. Since $N^{\alpha cl}(V_1^*) \nsubseteq V_2^* =$

$\langle x, \left(\frac{7}{10}, \frac{1}{2}, \frac{3}{10}\right), \left(\frac{3}{5}, \frac{1}{2}, \frac{2}{5}\right)\rangle$, where $V_1^* \nsubseteq V_2^*$and V_2^* is N(S)OS in N^X.

Example 2.22. Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, J_2^*, 1_N\}$ is NT on N^X ,Here $J_1^* = \langle x, \left(\frac{3}{10}, \frac{1}{2}, \frac{7}{10}\right), \left(\frac{2}{5}, \frac{1}{2}, \frac{3}{5}\right)\rangle$ and $J_2^* = \langle x, \left(\frac{2}{5}, \frac{1}{2}, \frac{3}{5}\right), \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)\rangle$. Then the NS $V_1^* = \langle x, \left(\frac{3}{10}, \frac{1}{2}, \frac{7}{10}\right), \left(\frac{3}{5}, \frac{1}{2}, \frac{2}{5}\right)\rangle$ is a N(αGS)CS but not NbCS as $N^{int}\left(N^{cl}(V_1^*)\right) \cap N^{cl}(N^{int}(V_1^*)) \nsubseteq V_1^*$.

Theorem 2.23. Every N(αGS)CS in (N^X, N^τ) is a NαGCS in (N^X, N^τ).
Proof: Assume that V_1^* is a N(αGS)CS in (N^X, N^τ). Let us consider a NS $V_1^* \subseteq \Psi$ and Ψ is NOS in (N^X, N^τ) . By hypothesis, $N^{\alpha cl}(V_1^*) \nsubseteq \Psi$ whenever, $V_1^* \subseteq \Psi$ and Ψ is N(S)OS. This implies $N^{\alpha cl}(V_1^*) \nsubseteq \Psi$ whenever $V_1^* \subseteq \Psi$ and Ψ is NOS. Therefore V_1^* is a N(αG)CS in (N^X, N^τ).

Example 2.24. Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ is NT on N^X ,Here $J_1^* = \langle x, \left(\frac{1}{10}, \frac{1}{2}, \frac{7}{10}\right), \left(\frac{3}{10}, \frac{1}{2}, \frac{3}{5}\right)\rangle$. Then the NS $V_1^* = \langle x, \left(\frac{3}{10}, \frac{1}{2}, \frac{3}{5}\right), \left(\frac{2}{5}, \frac{1}{2}, \frac{1}{2}\right)\rangle$ is NαGCS but not N(αGS)CS. Since

$N^{\alpha cl}(V_1^*) \nsubseteq V_2^* = \langle x, \left(\frac{2}{5}, \frac{1}{2}, \frac{1}{2}\right), \left(\frac{2}{5}, \frac{1}{2}, \frac{2}{5}\right)\rangle$, eventhough $V_1^* \nsubseteq V_2^*$and V_2^* is N(S)OS .

Theorem 2.25. Every N(αGS)CS in (N^X, N^τ) is a NGαCS in (N^X, N^τ).
Proof: Assume that V_1^* is a N(αGS)CS in (N^X, N^τ). Let $V_1^* \subseteq \Psi$ and Ψ is NαOS in N^X. By hypothesis, $N^{\alpha cl}(V_1^*) \subseteq \Psi$ whenever, $V_1^* \subseteq \Psi$ and Ψ is N(S)OS. This implies $N^{\alpha cl}(V_1^*) \subseteq \Psi$ whenever $V_1^* \subseteq \Psi$ and Ψ is NαOS. Therefore V_1^* is a NGαCS in (N^X, N^τ).

Example 2.26. Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ is NT on N^X.

Here $J_1^* = \langle x, \left(\frac{2}{5}, \frac{1}{2}, \frac{3}{5}\right), \left(\frac{1}{5}, \frac{1}{2}, \frac{4}{5}\right)\rangle$. Then the NS $V_1^* = \langle x, \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right), \left(\frac{3}{5}, \frac{1}{2}, \frac{2}{5}\right)\rangle$ is NGαCS but not

N(αGS)CS. Since $N^{\alpha cl}(V_1^*) \nsubseteq V_2^* = \langle x, \left(\frac{11}{20}, \frac{1}{2}, \frac{9}{20}\right), \left(\frac{7}{10}, \frac{1}{2}, \frac{3}{10}\right)\rangle$, eventhough $V_1^* \nsubseteq V_2^*$and V_2^* is N(S)OS .

Remark 2.27. The intersection of any two N(αGS)CS is not a N(αGS)CS in general as seen from the following example.

Example 2.28. Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ is NT on N^X.

Here $J_1^* = \langle x, \left(\frac{1}{5}, \frac{1}{2}, \frac{7}{10}\right), \left(\frac{3}{10}, \frac{1}{2}, \frac{3}{5}\right)\rangle$. Then the NS $V_1^* = \langle x, \left(\frac{1}{10}, \frac{1}{2}, \frac{7}{10}\right), \left(\frac{4}{5}, \frac{1}{2}, \frac{1}{5}\right)\rangle$, $V_2^* =$

$\langle x, \left(\frac{4}{5}, \frac{1}{2}, \frac{1}{10}\right), \left(\frac{1}{5}, \frac{1}{2}, \frac{7}{10}\right)\rangle$ are N(αGS)CS . Now $V_1^* \cap V_2^* = \langle x, \left(\frac{1}{10}, \frac{1}{2}, \frac{7}{10}\right), \left(\frac{1}{5}, \frac{1}{2}, \frac{7}{10}\right)\rangle$. Since $N^{\alpha cl}(V_1^* \cap V_2^*) \nsubseteq G$, eventhough $V_1^* \subseteq G$ and G is N(S)OS in N^X, $V_1^* \cap V_2^*$ is not a N(αGS)CS in N^X.

Theorem 2.29. Every (N^X, N^τ) is a NTS. Then for every $V_1^* \in$ N(αGS)C(N^X) and for every $V_2^* \in$ NS(N^X),$V_1^* \subseteq V_2^* \subseteq N^{\alpha cl}(V_1^*)$ implies $V_2^* \in$ N(αGS)C(N^X).

Proof: Let $V_2^* \subseteq \Psi$ and Ψ is N(S)OS in N^X . Since $V_1^* \subseteq V_2^*$, $V_1^* \subseteq \Psi$ and V_1^* is a N(α GS)CS, $N^{\alpha cl}(V_1^*) \subseteq \Psi$. By hypothesis, $V_2^* \subseteq N^{\alpha cl}(V_1^*)$, $N^{\alpha cl}(V_2^*) \subseteq N^{\alpha cl}(V_1^*) \subseteq \Psi$. Therefore $N^{\alpha cl}(V_2^*) \subseteq \Psi$. Hence V_2^* is N(αGS)CS of N^X.

Theorem 2.30. If V_1^* **is** both N(S)OS and N(αGS)CS in (N^X, N^τ), then V_1^* is a N(α)CS in N^X.

Proof: Let V_1^* is N(S)OS in N^X. Since $V_1^* \subseteq V_1^*$, by hypothesis $N^{\alpha cl}(V_1^*) \subseteq V_1^*$. But $V_1^* \subseteq N^{\alpha cl}(V_1^*)$. Therefore $N^{\alpha cl}(V_1^*) = V_1^*$. Hence V_1^* is a NαCS in N^X.

Theorem 2.31. The union of two N(αGS)CS is a N(αGS)CS in (N^X, N^τ), if they are NCS in (N^X, N^τ).

Proof: Assume that V_1^* and V_2^* are N(αGS)CS in (N^X, N^τ). Since V_1^* and V_2^* are NCS in N^X, $N^{cl}(V_1^*) = V_1^*$ and $N^{cl}(V_2^*) = V_2^*$. Let $V_1^* \cup V_2^* \subseteq \Psi$ and Ψ is N(S)OS in N^X. Then $N^{cl}\left(N^{int}(N^{cl}(V_1^* \cup V_2^*))\right) = N^{cl}\left(N^{int}(V_1^* \cup V_2^*)\right) \subseteq N^{cl}(V_1^* \cup V_2^*) = V_1^* \cup V_2^* \subseteq \Psi$, i.e., $N^{\alpha cl}(V_1^* \cup V_2^*) \subseteq \Psi$. Therefore $V_1^* \cup V_2^*$ is N(αGS)CS.

Theorem 2.32. Let (N^X, N^τ) is NTS and V_1^* is NS in N^X. Then V_1^* is a N(αGS)CS if and only if $V_1^* \bar{q}$F implies $N^{\alpha cl}(V_1^*)\overline{q}$F for every N(S)CS of N^X.

Proof: Necessary Part: Let F_1^* is N(S)CS in N^X and let $V_1^* \bar{q} F_1^*$. Then $V_1^* \subseteq F_1^{*c}$, Here F_1^{*c} is a N(S)OS in N^X. Therefore by hypothesis, $N^{\alpha cl}(V_1^*) \subseteq F_1^{*c}$. Hence $N^{\alpha cl}(V_1^*)\overline{q} F_1^*$.

Sufficient Part: Let F_1^* is N(S)CS in N^X and let V_1^* is NS in N^X. By hypothesis, $V_1^* \bar{q}$F implies $N^{\alpha cl}(V_1^*)\overline{q} F_1^*$. Then $N^{\alpha cl}(V_1^*) \subseteq F_1^{*c}$ whenever $V_1^* \subseteq F_1^{*c}$ and F_1^{*c} is a N(S)OS in N^X. Hence V_1^* is a N(αGS)CS in N^X.

3.Neutrosophic α Generalized Semi-Open Sets

In this section we introduce Neutrosophic α Generalized Semi-Open Sets and study some of its properties.

Definition 3.1. A NS V_1^* is said to be Neutrosophic α generalized semi-open set (NαGSOS in short) in (N^X, N^τ), if the complement V_1^{*c} is a N(αGS)CS in N^X. The family of all N(αGS)OS of a NTS (N^X, N^τ) is denoted by NαGSO(N^X).

Theorem 3.2 For any NTS (N^X, N^τ), every NOS is a N(αGS)OS.

Proof: Let V_1^* is NOS in N^X. Then V_1^{*c} is a NCS in N^X, By Theorem 2.3, V_1^{*c} is a N(αGS)CS in N^X. Hence V_1^* is a N(αGS)OS in N^X.

Example 3.3. Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ is NT on N^X. Here $J_1^* = \langle x, \left(\frac{1}{5}, \frac{1}{2}, \frac{7}{10}\right), \left(\frac{1}{10}, \frac{1}{2}, \frac{4}{5}\right)\rangle$.

Then the NS $V_1^* = \langle x, \left(\frac{1}{10}, \frac{1}{2}, \frac{4}{5}\right), \left(0, \frac{1}{2}, \frac{9}{10}\right)\rangle$. Sinc e V_1^{*c} is a N(αGS)CS, V_1^* is a N(αGS)OS, but not NOS.

Theorem 3.4 For any NTS (N^X, N^τ),every N(α)OS is a N(αGS)OS.
Proof: Let V_1^* is N(α)OS in N^X. Then V_1^{*c} is a N(α)CS in N^X, By Theorem 2.5, V_1^{*c} is a N(αGS)CS in N^X.

Example 3.5 Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ is NT on N^X,Here $J_1^* = \langle x, \left(\frac{2}{5}, \frac{1}{2}, \frac{3}{5}\right), \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)\rangle$.

Then the NS $V_1^* = \langle x, \left(\frac{1}{5}, \frac{1}{2}, \frac{4}{5}\right), \left(\frac{2}{5}, \frac{1}{2}, \frac{3}{5}\right)\rangle$ is a N(αGS)OS in N^X , V_1^* is not a NαOS in N^X.

Theorem 3.6 For any NTS (N^X, N^τ),every N(R)OS is a N(αGS)OS.
Proof: Let V_1^* is N(R)OS in N^X. Then V_1^{*c} is a N(R)CS in N^X, By Theorem 2.7, V_1^{*c} is a N(αGS)CS in N^X.

Example 3.7 Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ is NT on N^X.

Here $J_1^* = \langle x, \left(\frac{3}{5}, \frac{1}{2}, \frac{3}{10}\right), \left(\frac{7}{10}, \frac{1}{2}, \frac{1}{10}\right)\rangle$. Then the NS $V_1^* = \langle x, \left(\frac{7}{10}, \frac{1}{2}, \frac{1}{5}\right), \left(\frac{4}{5}, \frac{1}{2}, 0\right)\rangle$ is a N(αGS)OS in N^X , V_1^* is not a N(R)OS in N^X.

Remark 3.8. N(αGS)OS and N(G)OS are independent in general.

Example 3.9 Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ is NT on N^X.

Here $J_1^* = \langle x, \left(\frac{2}{5}, \frac{1}{2}, \frac{1}{2}\right), \left(\frac{7}{10}, \frac{1}{2}, \frac{3}{10}\right)\rangle$. Then the NS $V_1^* = \langle x, \left(\frac{3}{5}, \frac{1}{2}, \frac{3}{10}\right), \left(\frac{4}{5}, \frac{1}{2}, \frac{1}{5}\right)\rangle$ is a N(αGS)OS in N^X , V_1^* is not a NGOS in N^X.

Example 3.10 Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ is NT on N^X. Here $J_1^* = \langle x, \left(\frac{3}{5}, \frac{1}{2}, \frac{2}{5}\right), \left(\frac{4}{5}, \frac{1}{2}, \frac{1}{5}\right)\rangle$.

Then the NS $V_1^* = \langle x, \left(\frac{3}{10}, \frac{1}{2}, \frac{7}{10}\right), \left(\frac{1}{10}, \frac{1}{2}, \frac{9}{10}\right)\rangle$ is a NGOS in N^X , but V_1^* is not a N(αGS)OS in N^X.

Theorem 3.11 Every N(αGS)OS in (N^X, N^τ) is a N(αGS)OS in (N^X, N^τ).
Proof: Let V_1^* is N(αGS)OS in N^X. Then V_1^{*c} is a N(αGS)CS in N^X, By Theorem 2.12, V_1^{*c} is a NGSCS in N^X.

Example 3.12 Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ is NT on N^X.

Here $J_1^* = \langle x, \left(\frac{7}{10}, \frac{1}{2}, \frac{3}{10}\right), \left(\frac{4}{5}, \frac{1}{2}, \frac{1}{10}\right)\rangle$. Then the NS $V_1^* = \langle x, \left(\frac{1}{5}, \frac{1}{2}, \frac{4}{5}\right), \left(0, \frac{1}{2}, \frac{4}{5}\right)\rangle$ is a NGSOS in N^X, but V_1^* is not a N(αGS)OS in N^X.

Remark 3.13. N(SP)OS is independent of N(αGS)OS.

Example 3.14 Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ is NT on N^X.

Here $J_1^* = \langle x, \left(\frac{3}{10}, \frac{1}{2}, \frac{3}{5}\right), \left(\frac{2}{5}, \frac{1}{2}, \frac{1}{2}\right)\rangle$. Then the NS $V_1^* = \langle x, \left(\frac{1}{2}, \frac{1}{2}, \frac{2}{5}\right), \left(\frac{3}{5}, \frac{1}{2}, \frac{3}{10}\right)\rangle$ is a N(SP)OS in N^X, but V_1^* is not a N(αGS)OS in N^X.

Example 3.15 Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ is NT on N^X.

Here $J_1^* = \langle x, \left(\frac{3}{10}, \frac{1}{2}, \frac{7}{10}\right), \left(\frac{2}{5}, \frac{1}{2}, \frac{3}{5}\right)\rangle$ and $J_2^* = \langle x, \left(\frac{2}{5}, \frac{1}{2}, \frac{3}{5}\right), \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)\rangle$. Then the NS $V_1^* =$

$\langle x, \left(\frac{7}{10}, \frac{1}{2}, \frac{3}{10}\right), \left(\frac{2}{5}, \frac{1}{2}, \frac{3}{5}\right)\rangle$ is a N(αGS)OS in N^X but V_1^* is not a N(SP)OS in N^X.

Theorem 3.16 Every N(αGS)OS in (N^X, N^τ) is a NαGOS in (N^X, N^τ).
Proof: Let V_1^* is N(αGS)OS in N^X. Then V_1^{*c} is a N(αGS)CS in N^X, By Theorem 2.23, V_1^{*c} is a N(αG)CS in N^X. Hence V_1^* is a NαGOS in N^X.

Example 3.17 Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ is NT on N^X.

Here $J_1^* = \langle x, \left(\frac{1}{10}, \frac{1}{2}, \frac{7}{10}\right), \left(\frac{3}{10}, \frac{1}{2}, \frac{3}{5}\right)\rangle$. Then the NS $V_1^* = \langle x, \left(\frac{3}{5}, \frac{1}{2}, \frac{3}{10}\right), \left(\frac{1}{2}, \frac{1}{2}, \frac{2}{5}\right)\rangle$ is NαGOS in N^X, but not N(αGS)OS in N^X.

Theorem 3.18 Every N(αGS)OS in (N^X, N^τ) is a NGαOS in (N^X, N^τ).
Proof: Let V_1^* is N(αGS)OS in N^X. Then V_1^{*c} is a N(αGS)CS in N^X, By Theorem 2.25, V_1^{*c} is a NGαCS in N^X. Hence V_1^* is a NGαOS in N^X.
Example 3.19 Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ is NT on N^X.

Here $J_1^* = \langle x, \left(\frac{2}{5}, \frac{1}{2}, \frac{3}{5}\right), \left(\frac{1}{5}, \frac{1}{2}, \frac{4}{5}\right)\rangle$. Then the NS $V_1^* = \langle x, \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right), \left(\frac{2}{5}, \frac{1}{2}, \frac{3}{5}\right)\rangle$ is NGαOS in N^X, but not N(αGS)OS in N^X.

Theorem 3.2 Let (N^X, N^τ) is NTS. If V_1^* is a NS of N^X followed by consequences are equal:
1. $V_1^* \in \mathrm{N\alpha GSO}(N^X)$

2. $V \subseteq N^{int}\left(N^{cl}(N^{int}(V_1^*))\right)$ whenever $V \subseteq V_1^*$ and V is a N(S)CS in N^X

3. There exists NOS $G_1 \subseteq V \subseteq N^{int}(N^{cl}(G))$ where $G = N^{int}(V_1^*); V \subseteq V_1^*$ and V is a N(S)CS in N^X
Proof: (1)$\Longrightarrow$ (2) Let $V_1^* \in \mathrm{N(\alpha GS)O}(N^X)$. Then V_1^{*c} is a N(αGS)CS in N^X, Therefore $N^{\alpha cl}(V_1^{*c}) \subseteq \Psi$, whenever $V_1^{*c} \subseteq \Psi$ and Ψ is a N(S)OS in N^X. i.e.,$N^{cl}\left(N^{int}(N^{cl}(V_1^{*c}))\right) \subseteq \Psi$. Taking complement on

both sides, we get $\left(N^{cl}\left(N^{int}(N^{cl}(V_1^{*c}))\right)\right)^c = \left(N^{int}\left(N^{int}(N^{cl}(V_1^{*c}))\right)\right)^c = N^{int}N^{int}(N^{cl}(N^{cl}(V_1^{*c}))^c =$ $N^{int}(N^{cl}(N^{int}(V_1^{*c})^c) = N^{int}\left(N^{cl}(N^{int}(V_1^{*}))\right) \supseteq \Psi^c$. This implies $\Psi^c \subseteq N^{int}\left(N^{cl}(N^{int}(V_1^{*}))\right)$ whenever $\Psi^c \subseteq V_1^*$ and Ψ^c is a N(S)CS in N^X. Replace Ψ^c by V, $V \subseteq N^{int}\left(N^{cl}(N^{int}(V_1^{*}))\right)$ whenever $V \subseteq V_1^*$ and V is a N(S)CS in N^X.

(2) $\Rightarrow$ (3) Let $V \subseteq N^{int}\left(N^{cl}(N^{int}(V_1^{*}))\right)$ whenever $V \subseteq V_1^*$ and V is a N(S)CS in N^X. Hence $N^{int}(V) \subseteq V \subseteq N^{int}\left(N^{cl}(N^{int}(V_1^{*}))\right)$. Then there exists NOS J_1^* in N^X such that $G_1 \subseteq V \subseteq N^{int}(N^{cl}(G))$ where $G = N^{int}(V_1^*)$ and $J_1^* = N^{int}(V)$.

(3) $\Rightarrow$ (1) Suppose that there exists NOS J_1^* such that $J_1^* \subseteq V \subseteq N^{int}(N^{cl}(G))$ where $G = N^{int}(V_1^*)$; $V \subseteq V_1^*$ and V is a N(S)CS in N^X. It is clear that $(N^{int}(N^{cl}(G)))^c \subseteq V^c$. That is $\left(N^{int}\left(N^{cl}(N^{int}(V_1^{*}))\right)\right)^c \subseteq V^c$. This implies $N^{cl}\left(N^{cl}(N^{int}(V_1^{*}))\right)^c \subseteq V^c$. Therefore $N^{cl}\left(N^{int}(N^{int}(V_1^{*c}))\right) \subseteq V^c, V_1^{*c} \subseteq V^c$ and V^c is N(S)OS in N^X. Hence $\alpha N^{cl}(V_1^{*c}) \subseteq V^c$.i.e, V_1^{*c} is a N(αGS)CS in N^X. This implies $V_1^* \in$ NαGSO(N^X).

Theorem 3.21 Let (N^X, N^τ) is NTS. Then for every $V_1^* \in \mathrm{N\alpha GSO}(N^X)$ and for every $V_1^* \in NS(N^X), N^{\alpha int}(V_1^*) \subseteq V_2^* \subseteq V_1^*$ implies $V_2^* \in \mathrm{N\alpha GSO}(N^X)$.

Proof: By hypothesis, $N^{\alpha int}(V_1^*) \subseteq V_2^* \subseteq V_1^*$. Taking complement on both sides, we get $V_1^{*c} \subseteq V_2^{*c} \subseteq (N^{\alpha int}(V_1^*))^c$. Let $V_2^{*c} \subseteq \Psi$ and Ψ is N(S)OS in N^X. Since $V_1^{*c} \subseteq V_2^{*c}. V_1^{*c} \subseteq \Psi$. Since V_1^{*c} is a $\mathrm{N(\alpha GS)CS}, N^{\alpha int}(V_1^{*c}) \subseteq \Psi$. Also $V_2^{*c} \subseteq (N^{\alpha int}(V_1^*))^c = N^{\alpha \mathrm{cl}}(V_1^{*c})$. Therefore $N^{\alpha\mathrm{cl}}(V_2^{*c}) \subseteq N^{\alpha\mathrm{cl}}(V_1^{*c}) \subseteq \Psi$. Hence V_2^{*c} is a $\mathrm{N(\alpha GS)CS}$ in N^X. This implies V_2^* is a $\mathrm{N(\alpha GS)OS}$ in N^X.i.e.,$V_2^* \in \mathrm{N\alpha GSO}(N^X)$.

Remark 3.22. The union of any two N(αGS)OS in (N^X, N^τ) is not a N(αGS)OS in (N^X, N^τ).

Example 3.23 Let $N^X = \{v_1, v_2\}$. Let $N^\tau = \{0_N, J_1^*, 1_N\}$ is NT on N^X.

Here $J_1^* = \langle x, \left(\frac{3}{10}, \frac{1}{2}, \frac{3}{5}\right), \left(\frac{1}{5}, \frac{1}{2}, \frac{7}{10}\right)\rangle$. Then the NS $V_1^* = \langle x, \left(\frac{3}{5}, \frac{1}{2}, \frac{1}{5}\right), \left(\frac{1}{10}, \frac{1}{2}, \frac{4}{5}\right)\rangle$ and

$V_2^* = \langle x, \left(\frac{1}{5}, \frac{1}{2}, \frac{7}{10}\right), \left(\frac{4}{5}, \frac{1}{2}, \frac{1}{10}\right)\rangle$ are N(αGS)OS in N^X, but $V_1^* \cup V_2^* = \langle x, \left(\frac{3}{5}, \frac{1}{2}, \frac{1}{5}\right), \left(\frac{4}{5}, \frac{1}{2}, \frac{1}{10}\right)\rangle$ is not an N(αGS)OS in N^X.

Theorem 3.24 A NS V_1^* of a NTS (N^X, N^τ) is a N(αGS)OS if and only if $F \subseteq N^{\alpha int}(V_1^*)$ whenever $F \subseteq V_1^*$ and F is a N(S)CS in N^X.

Proof: Necessary Part: Suppose V_1^* is a N(αGS)OS in N^X. Let F is N(S)CS in N^X and $F \subseteq V_1^*$. Then F^c is a N(S)OS in N^X such that $V_1^{*c} \subseteq F^c$. Since V_1^{*c} is a N(αGS)CS, we have $N^{\alpha\text{cl}}(V_1^{*c}) \subseteq F^c$. Hence $\left(N^{\alpha\text{int}}(V_1^*)\right)^c \subseteq F^c$. Therefore $F \subseteq N^{\alpha int}(V_1^*)$.

Sufficient Part: Let V_1^* is NS in N^X and let $F \subseteq N^{\alpha\text{int}}(V_1^*)$ whenever F is a N(S)CS in N^X and $F \subseteq V_1^*$. Then $V_1^{*c} \subseteq F^c$ and F^c is a N(S)OS. By hypothesis , $\left(N^{\alpha\text{int}}(V_1^*)\right)^c \subseteq F^c$,which implies $N^{\alpha\text{cl}}(V_1^{*c}) \subseteq F^c$. Therefore V_1^{*c} is a N(αGS)CS in N^X. Hence V_1^* is a N(αGS)OS in N^X.

4. Conclusion

In this paper, Neutrosophic αGS closed sets and Neutrosophic αGS open sets are introduced and discussed some of its basic properties and their relationships with existing Neutrosophic closed and open sets. In future, this set can be extended with various results and their applications. this is a very initial work it can be applicable in Neutrosophic supra topological spaces, Neutrosophic crisp topological spaces and Neutrosophic n -topological spaces

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