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Chander Parkash

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Symmetric Neutrosophic Cross Entropy Based Fault Recognition of Turbine

Chander Parkash*[1]

1Department of Mathematics, Rayat Bahra University, Mohali, Punjab, India. Email: cpgandhi@rayatbahrauniversity.edu.in

***Correspondence: cchanderr@gmail.com

Abstract: This study introduces a novel fault recognition methodology for turbine faults through symmetric trigonometric fuzzy and neutrosophic cross entropy measures (FCEM and NCEM) consequently. After knowing the nethermost (lowest) and uppermost (highest) energy bounds of each real fault conditions, the energy interval ranges are constructed and then transformed into the form of single valued neutrosophic (SN) sets. Thereafter, the proposed symmetric trigonometric cross -entropy measures are deployed to recognize faults of turbine. The nethermost FCEM and NCEM values between familiar and unfamiliar fault conditions indicates that the unfamiliar fault condition is closer to the familiar one. The applicability of the proposed methodology is validated by taking into consideration the example of fault diagnosis of turbine. The repercussions of this study yield that the proposed symmetric trigonometric FCEM and NCEM cannot only recognize optimal fault, they can also provide meaningful and remarkable fault information. A comparison of the underlying FCEM and NCEM (based on SN sets) with the enduring cosine measures (based on vague sets) conclude that the latter sets may hide some fruitful fault information., when experimented under sensitive and intuitive criteria and thus resulting an incomplete fault evaluation criterion.

Keywords: Fuzzy Sets, Neutrosophic Sets, Cross Entropy, Fault Diagnosis, Turbine**.**

1. Introduction

A Turbine generator is an important mechanical device and is being widely used for converting heat energy of steam into electrical energy in thermal plants. It is a natural process for a huge steam turbine generator to have vibrations produced by many factors such as misalignment, heating of rotor element and lubricant oil etc. When a fault occurs, it not only damages the generator set but also disturbs the continuous and safe operation of internal machinery. Careful analysis of vibration signals of the generator set can reveal some useful evaluation information, which in turn, can avoid catastrophic mechanical disorder as well as huge economic losses. It is, therefore, necessary to reckon and fix the actual cause of the fault as early as possible. Over the past few years, researchers have developed some fault recognition methods, that works on cross entropy measures, for quantifying the non-linear relationship between unfamiliar and familiar turbine fault conditions. Recently, Ren et al. **[1]** extracted the fuzzy entropy of a series of mode components for observing the complexity of working condition and thereafter improved the fault diagnosis accuracy of wind

turbine. Lilian and Ye **[2,3]** modified the vague sets of enduring similarity measure and observed the non-linear and complex relationship between vibration signals and various fault conditions. Recently, Lilian Shi **[4]** constructed simplified neutrosophic sets by exercising the enduring Karl Pearson's coefficient of correlation and combined it with wavelet packet transforms for reckoning faults of rolling bearing. Tian et al. **[5]** established a systematic and comprehensive approach based on permutation entropy for automatic testimony of bearing defects under and time varying conditions. Recently, Martinez et al. **[6]** utilized Shannon's Information entropy for quantifying and extracting the fault information available in the vibration signals of broken bars in induction motors. Under multi fault severities and time-varied complexities, Fu et al. **[7]** combined approximate entropy and wavelet packet transforms for decomposing deterministic and stochastic power signals. Zhao et al. **[8]** extended the existing wavelet entropy to instantaneous wavelet singular entropy for extracting the sensor fault characteristics of a gas turbine. Zhao et al. **[8]** deployed multiscale fuzzy distribution entropy for understanding the nonlinear and non-consistent fault characteristics signals. Zhang et al. **[9]** understood the irregularity and complexity of vibration signals by extending Shannon's entropy to wavelet entropy and concluded that whenever wavelet entropy increases, the tightness conditions of bolted joints diverge to looseness. Leite et al. **[10]** deployed Shannon's entropy and Jenson-Renyi's directed divergence (JRDD) for constructing discrete probability mass function of a known time waveform and utilized it for identifying faults of rolling bearing elements. Many times, the approaches based on variants of Shannon's probabilistic entropy and JRDD have been found inefficacious in providing semantic output due to the difficulty in transforming fault characteristics of cumbersome signals. Hence, the above-mentioned fault diagnosis techniques may not be capable for extracting remarkable and accurate fault information from faults conditions of turbine. This reinforces the exigency for an effective fault diagnosis procedure which can make precise and fruitful analysis for a fault that occurs in turbine generator set because the same symptom of a fault may have variety of fault causations. A single valued neutrosophic (SN) set **[11]** is mainly portrayed by truth, indeterminacy and falsity membership functions and inherits its indeterminacy into the form of truth and falsity values. Kumar et al. **[12,13]** effectively identified bearing faults by decomposing vibrational signals

into eight different frequency modes under neutrosophic environment. However, the enduring research on neutrosophic sets and systems have mainly dealt with its theoretical or asymmetrical aspects and ignores those engineering problems which may exhibit symmetrical phenomenon or return inconsequential results under neutrosophic treatments. Neutrosophic cross entropy approach has been found significantly efficacious in tackling complex engineering problems under multi-faults severities. Till so far, no symmetric neutrosophic cross entropy measure has been developed and utilized for improving fault identification accuracy of turbine. Subsequently, an effort is accomplished in this direction which can overcome the above-mentioned shortcomings and effectively diagnose the faults of a huge steam turbine generator set. Moreover, the underlying symmetric trigonometric cross entropy measure of neutrosophic sets provides meaningful fault information whereas the enduring similarity measure may hide some fruitful fault information and thus resulting an ambiguous phenomenon. In addition,

Section 2 deals with pre-requisites of neutrosophic entropy measure, needed for the successive growth of the proposed research. **Section 3** is devoted to establish a novel symmetric trigonometric FCEM whereas **Section 4** expands the outcomes of **Section 3** to another novel symmetric cross entropy measure, hinged on two single valued neutrosophic sets. **Section 5** inaugurates the proposed neutrosophic cross entropy-based fault recognition methodology, the applicability and remarkability of which are exemplified in **Section 6.** Finally, **Section 6** contributes the concrete conclusions extracted from this study.

2. **Preliminaries: -**

This section deals with the introduction of some familiar apprehensions as follows:

Def. 2.1 SN Entropy Measure [11-13] A SN set, in any universal set X with its generic elements $x_1, x_2, ..., x_n$, is an entity of the form: $A=\left(< x_i, \mu_A(x_i), i_A(x_i), f_A(x_i) >| x_i \in X\right)$ where each $\mu_A(x_i): X \to [0,1], i_A(x_i): X \to [0,1], f_A(x_i): X \to [0,1]$ satisfy $0^- \le \mu_A(x_i)+i_A(x_i)+f_A(x_i) \le 3^+$. Suppose $T(X)$ represents the collection of all SN sets in X , Then $T_N(A): T(X) \to R$ is called as SN entropy measure if

$(i) T_N(A) \geq 0 \forall 0 \leq \mu_A(x_i), i_A(x_i), f_A(x_i) \in [0,1]$ with equality if either $\mu_A(x_i) = 1, i_A(x_i) = 0,$ $f_A(x_i) = 0$ or $\mu_A(x_i) = 0, i_A(x_i) = 0, f_A(x_i) = 1.$ $(ii) T_N(A^c) = T_N(A).$ If A^c denotes the complement of A, then $A^c = \left(< x_i, f_A(x_i), 1 - i_A(x_i), \mu_A(x_i) > | \, x_i \in X \right).$

(iii) $T_N(A)$ possesses concavity property for each $\mu_A(x_i), i_A(x_i), f_A(x_i).$

(iv) $T_N(A)$ admits its maximum value which arises when $\mu_A(x_i) = i_A(x_i) = f_A(x_i) = \frac{1}{2}.$

3. A Novel Symmetric Trigonometric FCEM (Fuzzy Cross Entropy Measure)

We first establish the following **Theorem 3.1,** the out coming of which will be a backbone for the proposed symmetric trigonometric fuzzy cross entropy measure, hinged on two fuzzy sets (**Theorem 3.2).**

Theorem.3.1 Set $T_0 = \sqrt{\mu_A(x_i)}, T_1 = \sqrt{1 - \mu_A(x_i)}, T_2 = \sqrt{\mu_A(x_i)(1 - \mu_A(x_i))}.$ Let $A \subseteq X$ be any fuzzy set **[14]**.Then

$$T_{FS}(A) = \sum_{i=1}^{n} \left[\tan\left(\frac{3\sqrt{2}}{3\sqrt{2} + 2} \right) - \tan\left(\frac{3\sqrt{2}}{3\sqrt{2} + 2(T_0 + T_1) - \sqrt{2}T_2} \right) \right] \quad \text{... (1)}$$

represents a valid measure of fuzzy entropy with $Max.T_{FS}(A) = \left(\tan \frac{3\sqrt{2}}{3\sqrt{2} + 2} - \tan \frac{2}{3} \right) n$ and minimum value as zero.

Proof (i) The expressions denoted by T_0, T_1, T_2 are non0negative because $\mu_A(x_i) \in [0,1].$ This justifies that $T_F(A) \geq 0 \ \forall \mu_A(x_i) \in [0,1]$ with equality $T_0 = 0, T_1 = 1, T_2 = 1$ or $T_0 = 1, T_1 = 0, T_2 = 0.$ In other words, $T_F(A)$ vanishes whenever $\mu_A(x_i) = 0 \text{or} 1.$

(ii) If we replace $\mu_A(x_i)$ with its counterpart $1 - \mu_A(x_i),$ then T_0 changes to T_1, $T_1 \to T_0, T_2 \to T_2,$ which means $T_F(A^c) = T_F(A).$

(iii) **Concavity:** To establish the concavity of $T_{FS}(A),$ differentiating **(1)** partially with respect to $\mu_A(x_i)$ to get

$$\frac{\partial T_{FS}(A)}{\partial \mu_A(x_i)} = \frac{3\sqrt{2}\left(\frac{T_1^2 - T_0^2}{T_0 T_1 (T_0 + T_1)} - \frac{1-2T_0^2}{\sqrt{2}T_2}\right)\sec^2 \frac{3\sqrt{2}}{3\sqrt{2}+2(T_0+T_1)-\sqrt{2}T_2}}{\left(3\sqrt{2}+2(T_0+T_1)-\sqrt{2}T_2\right)^2} \quad \ldots (2)$$

It is informative to point out that $T_1^2 - T_0^2 = 1 - 2\mu_A(x_i) = 1 - 2T_0^2$. With this information in hand, the above equality simplifies to

$$\frac{\partial T_{FS}(A)}{\partial \mu_A(x_i)} = \frac{3\sqrt{2}(1-2\mu_A(x_i))\left(\frac{1}{T_0 T_1 (T_0 + T_1)} - \frac{2}{\sqrt{2}T_2}\right)\sec^2 \frac{3\sqrt{2}}{3\sqrt{2}+2(T_0+T_1)-\sqrt{2}T_2}}{\left(3\sqrt{2}+2(T_0+T_1)-\sqrt{2}T_2\right)^2} \quad \ldots (3)$$

Again, partial differentiation of **(2)** with respect to $\mu_A(x_i)$ yields

$$\frac{\partial^2 T_{FS}(A)}{\partial \mu_A^2(x_i)} = -\frac{9\left(\begin{array}{l}\frac{\left(\sqrt{2}-2T_1+2T_1T_0^2-2T_0^3\right)\left(3\sqrt{2}+2T_1+2T_0-\sqrt{2}T_2\right)^2}{2\sqrt{2}T_1\left(T_0^2-1\right)T_0^3} \\ +\left(\frac{1}{T_0}-\frac{1}{T_1}-\frac{1-2T_0^2}{\sqrt{2}T_2}\right)^2\left(\begin{array}{l}2\sqrt{2}\left(3\sqrt{2}+2(T_0+T_1)-\sqrt{2}T_2\right) \\ +12\tan\frac{3\sqrt{2}}{3\sqrt{2}+2(T_0+T_1)-\sqrt{2}T_2}\end{array}\right)\end{array}\right)\sec^2 \frac{3\sqrt{2}}{3\sqrt{2}+2(T_0+T_1)-\sqrt{2}T_2}}{\left(3\sqrt{2}+2(T_0+T_1)-\sqrt{2}T_2\right)^2} \leq 0$$

for each $\mu_A(x_i) \in [0,1]$, This establishes that $T_{FS}(A)$ exhibits the concavity property with respect to $\mu_A(x_i)$. This motivates $T_F(A)$ to admit its maximum value which can occur if $\frac{\partial T_{FS}(A)}{\partial \mu_A(x_i)} = 0$ and hence **(1)** yields $\mu_A(x_i) = \frac{1}{2}$. Thus,

$$\text{Max.} T_{FS}(A) = T_{FS}(A)\big|_{\mu_A(x_i)=\frac{1}{2}.} = \left(\tan\frac{3\sqrt{2}}{3\sqrt{2}+2} - \tan\frac{2}{3}\right)n. \quad \ldots (4)$$

Also, the graphical representation of $T_{FS}(A)$ as shown in **Fig 2** justifies that it admits its minimum value as zero.

Theorem.3.2 Set $E_0 = \sqrt{\mu_B(x_i)}, E_1 = \sqrt{1-\mu_B(x_i)}$. Let A and B belongs to $T(X)$ X, then $T_{FS}^{\mu}(A,B)$ is a correct symmetric trigonometric FCEM (fuzzy cross entropy measure **[15-16]**) given as

$$T_{FS}^{\mu}(A,B) = \sum_{i=1}^{n}\left[\begin{array}{l} -6\tan\left(\frac{2}{3}\right) \\ +\left(2+T_0^2+E_0^2\right)\tan\left(\frac{2+T_0^2+E_0^2}{3+2\left(T_0+E_0\right)\sqrt{\frac{T_0^2+E_0^2}{2}}-T_0E_0}\right)+\left(4-T_0^2-E_0^2\right)\tan\left(\frac{4-T_0^2-E_0^2}{3+2\left(T_1+E_1\right)\sqrt{\frac{2-T_0^2-E_0^2}{2}}-T_1E_1}\right) \end{array}\right] \quad \ldots (5)$$

Here, $T_{FS}^{\mu}(A,B)$ represents the subjective value of symmetric discrimination of A against B.

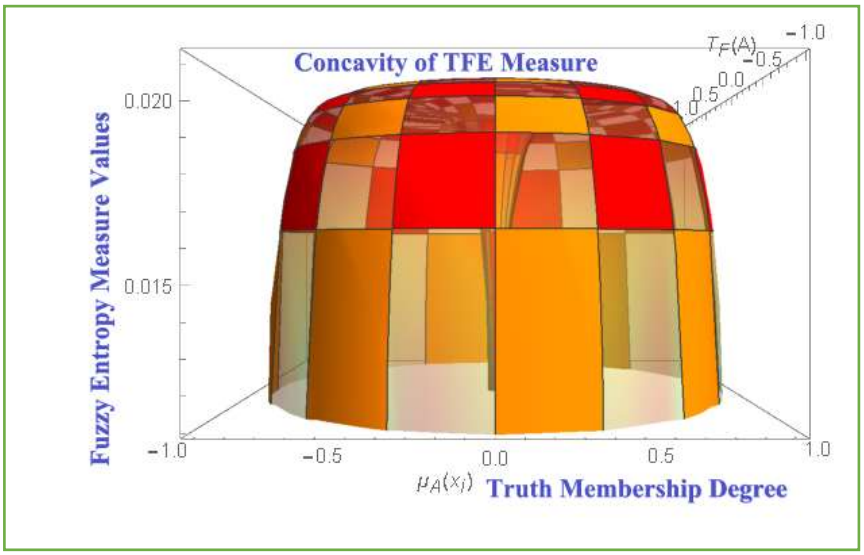


Fig. 2 A Revolution Plot for the Concavity Property and Minimum value of $T_{FS}(A)$

Proof. (i) Since $T_{FS}^{\mu}(A,B)$ does not change after the replacement of $\mu_A(x_i)$ with $\mu_B(x_i)$, this validates the symmetric nature of $T_{FS}^{\mu}(A,B)$.

(ii) Since $T_{FS}^{\mu}(A,B)$ remains unchanged after the replacement of A,B with A^c,B^c, this suggests that $T_{FS}^{\mu}(A^c,B^c)=T_{FS}^{\mu}(A,B)$. The fact, that $T_{FS}^{\mu}(A,B)$ is non-negative, can be established if we first inculcate the following **Lemma 3.1.**

Lemma 3.1 Define $N=\left(\frac{T_0+E_0}{2}\right)\left(\sqrt{\frac{T_0^2+E_0^2}{2}}\right), A=\frac{T_0^2+E_0^2}{2}, G=T_0E_0$. There exists the inequality: $4N\le 3A+G$ with equality if $T_0^2=\mu_A(x_i)=E_0^2=\mu_B(x_i)$. … (6)

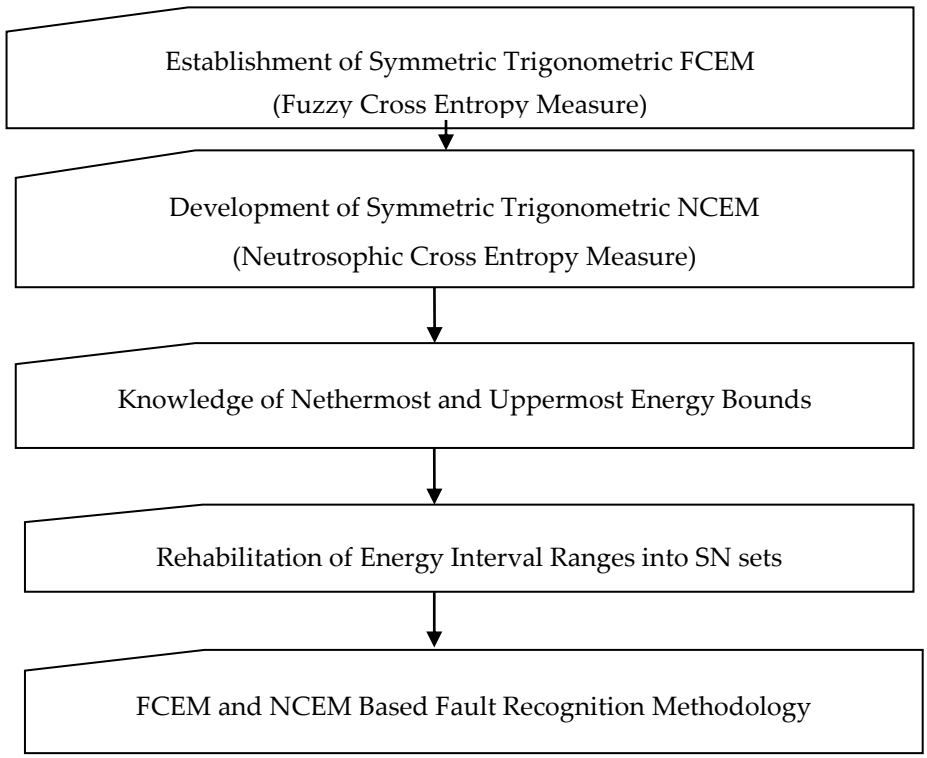


Fig. 1 A Step-wise Flow Chart of FCEM and NCEM Based urbine Fault Recognition Methodology

Proof. In our notations, we have

$$N=\left(\frac{T_0+E_0}{2}\right)\left(\sqrt{\frac{T_0^2+E_0^2}{2}}\right), A=\frac{T_0^2+E_0^2}{2}, G=T_0E_0.$$

The undergoing inequality **(6)** could be true if $2\left(T_0+E_0\right)\left(\sqrt{\frac{T_0^2+E_0^2}{2}}\right)\le\frac{3}{2}\left(T_0^2+E_0^2\right)+T_0E_0$

$$\Leftrightarrow 8\left(T_0^2+E_0^2+2T_0E_0\right)\left(T_0^2+E_0^2\right)\le 9\left(T_0^2+E_0^2\right)^2+4T_0^2E_0^2+12\left(T_0^2+E_0^2\right)T_0E_0$$

$$\Leftrightarrow 8\left(T_0^2+E_0^2\right)^2+16T_0E_0\left(T_0^2+E_0^2\right)\le 9\left(T_0^2+E_0^2\right)^2+4T_0^2E_0^2+12\left(T_0^2+E_0^2\right)T_0E_0$$

$$\Leftrightarrow \left(T_0^2+E_0^2\right)^2-4\left(T_0^2+E_0^2\right)T_0E_0+4T_0^2E_0^2\ge 0$$

$\Leftrightarrow \left(T_0^2+E_0^2-2T_0E_0\right)^2\ge 0 \Leftrightarrow \left(T_0-E_0\right)^4\ge 0$ which is obviously true.

Thus, in view of the resulting **Lemma 3.1,** the inequality **(6)** can be rescheduled as

$$\frac{4N-G}{3}\le A\Rightarrow\frac{4N-G}{3}+1\le A+1 \quad\Rightarrow\frac{2\left(T_0+E_0\right)\left(\sqrt{\frac{T_0^2+E_0^2}{2}}\right)-T_0E_0}{3}+1\le\frac{T_0^2+E_0^2}{2}+1$$

$$\Rightarrow \frac{2+T_0^2+E_0^2}{3+2\left(T_0+E_0\right)\sqrt{\frac{T_0^2+E_0^2}{2}}-T_0E_0} \geq \frac{2}{3} \qquad \text{.. (7)}$$

Since tangent function exhibits the monotonicity property over $\left[0,1\right]$, the resulting inequality **(7)** can be rescheduled as

$$\left(2+T_0^2+E_0^2\right)\tan\frac{2+T_0^2+E_0^2}{3+2\left(T_0+E_0\right)\sqrt{\frac{T_0^2+E_0^2}{2}}-T_0E_0} \geq \left(2+T_0^2+E_0^2\right)\tan\frac{2}{3} \qquad \text{… (8)}$$

With the replacement of $\mu_A(x_i)$ with $1-\mu_A(x_i)$ and of $\mu_B(x_i)$ with $1-\mu_B(x_i)$ into **(8),** we observe that

T_0^2 changes to $T_1^2=1-T_0^2;\ E_0^2 \to E_1^2=1-E_0^2;\ T_0+E_0 \to T_1+E_1;\ \ T_0E_0 \to T_1E_1.$ Thus, **(8)** yields

$$\left(4-T_0^2-E_0^2\right)\tan\frac{4-T_0^2-E_0^2}{3+2\left(T_1+E_1\right)\sqrt{\frac{2-T_0^2-E_0^2}{2}}-T_1E_1} \geq \left(4-T_0^2-E_0^2\right)\tan\frac{2}{3} \qquad \text{... (9)}$$

We can simply add the resulting inequalities **(8, 9)** and then take the summation over $i=1$ to n to yield $T_{FS}^{\mu}(A,B)\geq 0$ as desired. Moreover, when $\mu_A(x_i)=\mu_B(x_i)$, then $T_0=E_0, T_1=E_1, 1-T_0^2=T_1^2, \sqrt{1-T_0^2}=T_1.$ Also,

$$T_{FS}^{\mu}(A,A)=\sum_{i=1}^{n}\left[-6\tan\left(\frac{2}{3}\right)+\left(2+2T_0^2\right)\tan\left(\frac{2+2T_0^2}{3+3T_0^2}\right)+\left(4-2T_0^2\right)\tan\left(\frac{4-2T_0^2}{6-3T_0^2}\right)\right]=0 \qquad \text{... (10)}$$

The equality (10) justifies that $T_{FS}^{\mu}(A,B)=0$ whenever $\mu_A(x_i)=\mu_B(x_i)$ as desired.

After the establishment of proposed fuzzy cross entropy measure $T_{FS}^{\mu}(A,B)$, the next Theorem 3.3 argues the urgent situation under which it will admits its extreme values.

Theorem 3.3 If $n\in N$ is the cardinality of X, then

$$0\leq T_{FS}^{\mu}(\mathrm{A},\mathrm{B})\leq 6\left(\tan\frac{3\sqrt{2}}{3\sqrt{2}+2}-\tan\frac{2}{3}\right)n. \qquad \text{… (11)}$$

Proof. If we replace the fuzzy set B with A^c, we observe that E_0^2 changes to $1-T_0^2;\ E_1\to T_0, E_0\to T_1, T_0T_1\to T_2.$

Thus, after the replacement of $\mu_B(x_i)$ with $1-\mu_A(x_i)$, the undergoing equality measure **(5)** yields

$$T_{FS}^{\mu}(A,A^c)=\sum_{i=1}^{n}\left[6\tan\frac{3\sqrt{2}}{3\sqrt{2}+2}-6\tan\left(\frac{2}{3}\right)-6\left(\tan\left(\frac{3\sqrt{2}}{3\sqrt{2}+2}\right)-\tan\left(\frac{3\sqrt{2}}{3\sqrt{2}+2\left(T_0+T_1\right)-\sqrt{2}T_2}\right)\right)\right]$$

$$= 6\text{Max.T}_F(A) - 6\text{T}_F(A) \quad \ldots (12)$$

Because $T_F(\text{A})$ is non-negative (T**heorem 3.1),** this motivates **(12)** to yield

$$T_F(\text{A}) = \text{Max.T}_F(A) - \frac{1}{6}T_{FS}^{\mu}(\text{A},\text{A}^c) \geq 0 \Rightarrow 0 \leq T_{FS}^{\mu}(\text{A},\text{A}^c) \leq 6\left(\tan\frac{3\sqrt{2}}{3\sqrt{2}+2} - \tan\frac{2}{3}\right)n \quad \ldots (13)$$

With the establishment of resulting inequality **(13),** it is informative to know that $T_{FS}\left(A, A^c\right)$ is finite for a fixed n. This justifies the finiteness of our proposed symmetric trigonometric FCEM (fuzzy cross entropy measure) which ranges as $0 \leq T_{FS}^{\mu}(\text{A},\text{B}) \leq 6\left(\tan\frac{3\sqrt{2}}{3\sqrt{2}+2} - \tan\frac{2}{3}\right)n$. Thus, $\text{Max}.T_{FS}^{\mu}(A,B) = \left(6\tan\frac{3\sqrt{2}}{3\sqrt{2}+2} - \tan\frac{2}{3}\right)n$ which clarifies that this maximum value does not depend upon its truth membership degree, but completely depends upon the cardinality of X. Also, the surface plot of $T_{FS}^{\mu}(A,B)$, represented by **Fig 3(a, b),** justifies the fact that this measure, because of its convexity, admits its $\text{Min}.T_{FS}^{\mu}(A,B) = 0$ Also, it is evident that $T_{FS}^{\mu}(A,B)$ gets increased as soon as $|A-B|$ increases, attains $\text{Max}.T_{FS}^{\mu}(A,B) = 6\left(\tan\frac{3\sqrt{2}}{3\sqrt{2}+2} - \tan\frac{2}{3}\right)n.$

The establishment of FCEM $T_{FS}^{\mu}(A,B)$, resulted from **Theorem 3.2** in the overhead discussion, will lead to develop the proposed NCEM (represented by $T_{\text{NC}}(\text{A},\text{B})$), the repercussions of which will be utilized to meet our goal of recognizing fault conditions of turbine.

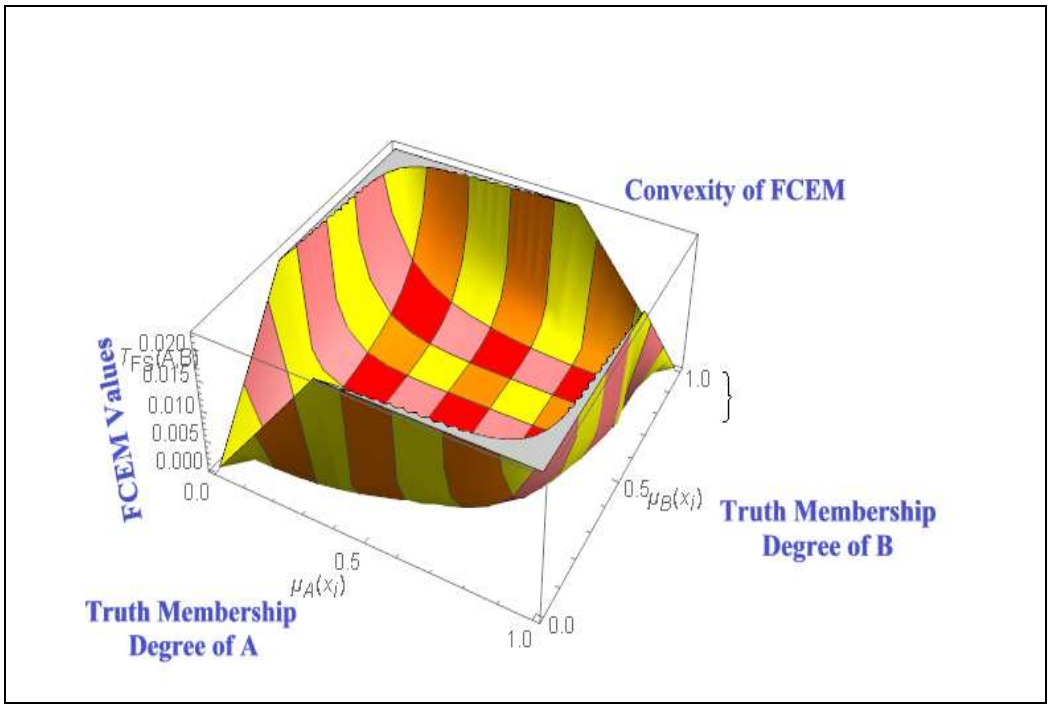
4. A Novel Symmetric Trigonometric NCEM (Neutrosophic Cross Entropy Measure)

In the resulting **Theorem 3.2,** the entity $T_{FS}^{\mu}(A,B)$ represents the amount of fuzziness which arises due to true membership degree for symmetric discrimination of fuzzy set A against B. Similarly, if we set $I_0 = \sqrt{i_A(x_i)}, I_1 = \sqrt{1-i_A(x_i)}, J_0 = \sqrt{i_B(x_i)}, J_1 = \sqrt{1-i_B(x_i)},$ then the amount of fuzziness which arises due to indeterminancy membership degree of A against B is established as

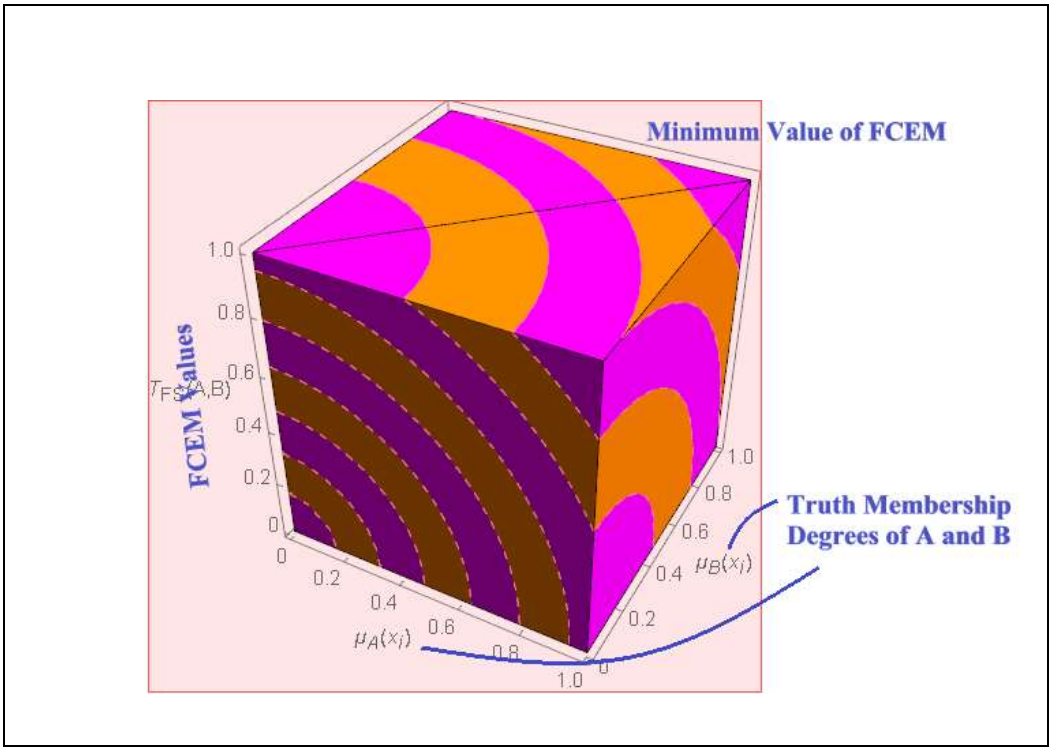
$$T_{FS}^{i}(A,B)$$

$$= \sum_{i=1}^{n} \left[\begin{array}{l} -6\tan\left(\frac{2}{3}\right) \\ +\left(2+I_0^2+J_0^2\right)\tan\left(\frac{2+I_0^2+J_0^2}{3+2\left(I_0+J_0\right)\sqrt{\frac{I_0^2+J_0^2}{2}}-I_0J_0}\right)+\left(4-I_0^2-J_0^2\right)\tan\left(\frac{4-I_0^2-J_0^2}{3+2\left(I_1+J_1\right)\sqrt{\frac{2-I_0^2-J_0^2}{2}}-I_1J_1}\right) \end{array} \right]$$

… (14)



(a)



(b)

Fig 3 (a) Convexity and (b) Minimum Value of the proposed of FCEM $T_{FS}^{\mu}(A,B)$

Again, we set $F_0=\sqrt{f_A(x_i)}, F_1=\sqrt{1-f_A(x_i)}, H_0=\sqrt{f_B(x_i)}, H_1=\sqrt{1-f_B(x_i)}$, then the amount of fuzziness which arises due to falsity membership degree for symmetric discrimination of A against B can also be established as

$$T_{FS}^{f}(A,B)$$
$$=\sum_{i=1}^{n}\left[\begin{array}{l}-6\tan\left(\frac{2}{3}\right)\\ +\left(2+F_0^2+H_0^2\right)\tan\left(\frac{2+F_0^2+H_0^2}{3+2\left(F_0+H_0\right)\sqrt{\frac{F_0^2+H_0^2}{2}}-F_0H_0}\right)+\left(4-F_0^2-H_0^2\right)\tan\left(\frac{4-F_0^2-H_0^2}{3+2\left(F_1+H_1\right)\sqrt{\frac{2-F_0^2-H_0^2}{2}}-F_1H_1}\right)\end{array}\right] \quad \ldots (15)$$

Def.4.1 Let A and B are any two SN sets in $X=\left(x_1,x_2,...,x_n\right)$. The desired NCEM (symmetric trigonometric neutrosophic cross entropy measure of SN sets can be constructed by simply adding **(12), (14)** and **(15)** as below.

$$T_{\mathrm{NC}}(\mathrm{A,B})=T_{FS}^{\mu}(\mathrm{A,B})+T_{FS}^{i}(\mathrm{A,B})+T_{FS}^{f}(\mathrm{A,B}) \quad \ldots (16)$$

Following same procedure as deployed in **Theorem 3.3,** readers can easily establish that if A and B are any SN sets with same cardinality n, then there exists the inequality:

$$0 \le T_{\mathrm{NC}}(\mathrm{A,B}) \le 18\left(\tan\frac{3\sqrt{2}}{3\sqrt{2}+2} - \tan\frac{2}{3}\right)n.$$

We shall now authenticate the applicability of our newly discovered NCEM $T_{\mathrm{NC}}(\mathrm{A,B})$ by recognizing the optimal fault condition of some huge steam turbine generator as follows.

6. FCEM and NCEM Based Fault Recognition Methodology

To achieve the desired goal, we shall, equally well, establish a neutrosophic cross entropy-based methodology which has the necessary capability of identifying various fault conditions of some turbine. A schematic flow chart explaining our fault recognition methodology has been provided in **Fig. 1** and discussed as below.

Step:-1 Construction of Energy Interval Ranges

The applicability of the underlying methodology is exemplified by taking into consideration the illustration **[12].** Suppose the ten familiar fault conditions experienced by some huge steam turbine generator set is represented by $B_K = (B_1, B_2, B_3, ...B_{10})$ where the fault condition "Unbalance" is abbreviated as B_1. Similarly, the conditions $B_2, B_3, ..., B_{10}$ have been provided in **[2,3].** Also, the nine frequency intervals $C_1 = [0.01, 0.3\,f], C_2 = [0.4, 0.49\,f], ..., C_9 = \text{higher frequency} > 5f$, of frequency spectrum , resulted from vibration signals of turbine, are available in **[2,3].**

Let $\mu_{B_K}(x_i)$ (lower bound) and $U_{B_K}(x_i)$ (upper bound) represent the amount of fuzziness resulted from the truth membership degree of K^{th} fault condition at i^{th} range of frequency spectrum. Then

$$B_K = \left(< x_1, \left[\mu_{B_K}(x_1), U_{B_K}(x_1)\right] >, < x_2, \left[\mu_{B_K}(x_2), U_{B_K}(x_2)\right] >, ... < x_9, \left[\mu_{B_K}(x_9), U_{B_K}(x_9)\right] >\right); K = 1, 2, ..., 10. \quad \ldots (17)$$

Generally, the acquitted vibration data may be non-commensurate and conflicting, it becomes essential for us to transform the energy interval ranges **(17)** into the form of SN sets. This conversion, however may be problematic, but can be done as follows.

Step:-2 Transformation of Interval Ranges (energy) by the Form of Neutrosophic Sets

The amount of fuzziness based on falsity membership degree of K^{th} familiar fault condition at i^{th} range of frequency spectrum is denoted by $f_{B_K}(x_i)$ where $f_{B_K}(x_i) = 1 - U_{B_K}(x_i)$. Similarly, the

amount of fuzziness based on indeterminacy membership degree of K^{th} familiar fault condition at i^{th} range of frequency spectrum is denoted by $i_{B_K}(x_i)$ where $i_{B_K}(x_i)=1-f_{B_K}(x_i)-U_{B_K}(x_i)$. We have restricted the value of $i_{B_K}(x_i)$ to $0{\cdot}001$ in case it if returns any other value less than or equal to $0{\cdot}001$.Then, the interval ranges (energy), represented by **(17),** for each B_K can be transformed into the forms of SN sets is described below.

$$B_K=\begin{pmatrix}\left(<x_1,\left[\mu_{B_K}(x_1),i_{B_K}(x_1),f_{B_K}(x_1)\right]>\right),\left(<x_2,\left[\mu_{B_K}(x_2),i_{B_K}(x_2),f_{B_K}(x_2)\right]>\right),\\ \left(<x_3,\left[\mu_{B_K}(x_3),i_{B_K}(x_3),f_{B_K}(x_3)\right]>\right),\ldots\left(<x_9,\left[\mu_{B_K}(x_9),i_{B_K}(x_9),f_{B_K}(x_9)\right]>\right)\end{pmatrix};K=1,2,\ldots,10 \quad \ldots(18)$$

Also, the unfamiliar fault conditions, represented by F_{T_j}, can also be transformed into the forms of SN sets as below:

$$F_{T_J}=\begin{pmatrix}\left(<x_1,\left[\mu_{F_{T_J}}(x_1),i_{F_{T_J}}(x_1),f_{F_{T_J}}(x_1)\right]>\right),\left(<x_2,\left[\mu_{F_{T_J}}(x_2),i_{F_{T_J}}(x_2),f_{F_{T_J}}(x_2)\right]>\right),\\ \left(<x_3,\left[\mu_{F_{T_J}}(x_3),i_{F_{T_J}}(x_3),f_{F_{T_J}}(x_3)\right]>\right),\ldots,\left(<x_9,\left[\mu_{F_{T_J}}(x_9),i_{F_{T_J}}(x_9),f_{F_{T_J}}(x_9)\right]>\right)\end{pmatrix} \quad \ldots(19)$$

Step: -3 Computation of FCEM and NCEM Values between familiar and unfamiliar fault conditions

The cross-entropy values $T_{\text{NC}}(B_K,F_{T_J}),T_{FS}^{\mu}(B_K,F_{T_J})$ between each B_K and F_{T_J} can be evaluated as follows. Replacement of introduced notations $T_0,F_0,H_0\ldots,$ etc., with their original values and then taking $i=1,2,\ldots,9$ into **(5,16)** yields

$$T_{FS}^{\mu}(B_K,F_{T_J})$$

$$=\sum_{i=1}^{9}\left[\begin{array}{l}-6\tan\frac{2}{3}+\left(2+\mu_{B_K}(x_i)+\mu_{F_{T_J}}(x_i)\right)\tan\left(\frac{2+\mu_{B_K}(x_i)+\mu_{F_{T_J}}(x_i)}{3+2\left(\sqrt{\mu_{B_K}(x_i)}+\sqrt{\mu_{F_{T_J}}(x_i)}\right)\left(\frac{\sqrt{\mu_{B_K}(x_i)+\mu_{F_{T_J}}(x_i)}}{2}\right)-\sqrt{\mu_{B_K}(x_i)\mu_{F_{T_J}}(x_i)}}\right)\\ -\left(4-\mu_{B_K}(x_i)-\mu_{F_{T_J}}(x_i)\right)\tan\left(\frac{4-\mu_{B_K}(x_i)-\mu_{F_{T_J}}(x_i)}{3+2\left(\sqrt{1-\mu_{B_K}(x_i)}+\sqrt{1-\mu_{F_{T_J}}(x_i)}\right)\left(\frac{\sqrt{2-\mu_{B_K}(x_i)-\mu_{F_{T_J}}(x_i)}}{2}\right)-\sqrt{\left(1-\mu_{B_K}(x_i)\right)\left(1-\mu_{F_{T_J}}(x_i)\right)}}\right)\end{array}\right]$$

... (20)

$$T_{NC}\left(B_K,F_{T_J}\right)$$

$$=\sum_{i=1}^{9}\left[\begin{array}{l}-6\tan\frac{2}{3}+\left(2+\mu_{B_K}(x_i)+\mu_{F_{T_J}}(x_i)\right)\tan\left(\frac{2+\mu_{B_K}(x_i)+\mu_{F_{T_J}}(x_i)}{3+2\left(\sqrt{\mu_{B_K}(x_i)}+\sqrt{\mu_{F_{T_J}}(x_i)}\right)\left(\frac{\sqrt{\mu_{B_K}(x_i)+\mu_{F_{T_J}}(x_i)}}{2}\right)-\sqrt{\mu_{B_K}(x_i)\mu_{F_{T_J}}(x_i)}}\right)\\-\left(4-\mu_{B_K}(x_i)-\mu_{F_{T_J}}(x_i)\right)\tan\left(\frac{4-\mu_{B_K}(x_i)-\mu_{F_{T_J}}(x_i)}{3+2\left(\sqrt{1-\mu_{B_K}(x_i)}+\sqrt{1-\mu_{F_{T_J}}(x_i)}\right)\left(\frac{\sqrt{2-\mu_{B_K}(x_i)-\mu_{F_{T_J}}(x_i)}}{2}\right)-\sqrt{\left(1-\mu_{B_K}(x_i)\right)\left(1-\mu_{F_{T_J}}(x_i)\right)}}\right)\end{array}\right]$$

$$+\sum_{i=1}^{9}\left[\begin{array}{l}-6\tan\frac{2}{3}+\left(2+i_{B_K}(x_i)+i_{F_{T_J}}(x_i)\right)\tan\left(\frac{2+i_{B_K}(x_i)+i_{F_{T_J}}(x_i)}{3+2\left(\sqrt{i_{B_K}(x_i)}+\sqrt{i_{F_{T_J}}(x_i)}\right)\left(\frac{\sqrt{i_{B_K}(x_i)+i_{F_{T_J}}(x_i)}}{2}\right)-\sqrt{i_{B_K}(x_i)i_{F_{T_J}}(x_i)}}\right)\\-\left(4-i_{B_K}(x_i)-i_{F_{T_J}}(x_i)\right)\tan\left(\frac{4-i_{B_K}(x_i)-i_{F_{T_J}}(x_i)}{3+2\left(\sqrt{1-i_{B_K}(x_i)}+\sqrt{1-i_{F_{T_J}}(x_i)}\right)\left(\frac{\sqrt{2-i_{B_K}(x_i)-i_{F_{T_J}}(x_i)}}{2}\right)-\sqrt{\left(1-i_{B_K}(x_i)\right)\left(1-i_{F_{T_J}}(x_i)\right)}}\right)\end{array}\right]$$

$$+\sum_{i=1}^{9}\left[\begin{array}{l}-6\tan\frac{2}{3}+\left(2+f_{B_K}(x_i)+f_{F_{T_J}}(x_i)\right)\tan\left(\frac{2+f_{B_K}(x_i)+f_{F_{T_J}}(x_i)}{3+2\left(\sqrt{f_{B_K}(x_i)}+\sqrt{f_{F_{T_J}}(x_i)}\right)\left(\frac{\sqrt{f_{B_K}(x_i)+f_{F_{T_J}}(x_i)}}{2}\right)-\sqrt{f_{B_K}(x_i)f_{F_{T_J}}(x_i)}}\right)\\-\left(4-f_{B_K}(x_i)-f_{F_{T_J}}(x_i)\right)\tan\left(\frac{4-f_{B_K}(x_i)-f_{F_{T_J}}(x_i)}{3+2\left(\sqrt{1-f_{B_K}(x_i)}+\sqrt{1-f_{F_{T_J}}(x_i)}\right)\left(\frac{\sqrt{2-f_{B_K}(x_i)-f_{F_{T_J}}(x_i)}}{2}\right)-\sqrt{\left(1-f_{B_K}(x_i)\right)\left(1-f_{F_{T_J}}(x_i)\right)}}\right)\end{array}\right]$$

… (21)

Step: -4 Identification of Turbine Faults

The Smallest value of $T_{NC}(B_K,F_{T_J});T_{FS}^{\mu}(B_K,F_{T_J})$ indicate that the familiar fault condition B_K is closer to the unfamiliar fault condition F_{T_J}. In other words, a typical selection of turbine fault will be designated as optimal fault type selection owing to the smallest NCEM $T_{NC}(B_K,F_{T_J})$ or FCEM $T_{FS}^{\mu}(B_K,F_{T_J})$ value.

7. APPLICATION TO FAULT DIAGNOSIS OF TURBINE

In order to validate the applicability of FCEM and NCEM based fault recognition methodology, the energy interval ranges for each familiar fault condition at various ranges of frequency spectrum is provided in **Table 1.**

Table 1. The Nethermost and Uppermost Energy Bounds of Each B_K at Nine Ranges of frequency Spectrum

B_K	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
B_1	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.85,1.00]	[0.04,0.06]	[0.04,0.07]	[0.00,0.00]	[0.00,0.00]
B_2	[0.00,0.00]	[0.03,0.31]	[0.90,0.12]	[0.55,0.70]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.08,0.13]
B_3	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.30,0.58]	[0.40,0.62]	[0.08,0.13]	[0.00,0.00]	[0.00,0.00]
B_4	[0.09,0.11]	[0.78,0.82]	[0.00,0.00]	[0.08,0.11]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,1.00]
B_5	[0.09,0.12]	[0.09,0.11]	[0.08,0.12]	[0.09,0.12]	[0.18,0.21]	[0.08,0.13]	[0.08,0.13]	[0.08,0.22]	[0.08,0.12]
B_6	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.18,0.22]	[0.12,0.17]	[0.37,0.45]	[0.00,0.00]	[0.22,0.28]
B_7	[0.00,0.00]	[0.00,0.00]	[0.08,0.12]	[0.86,0.93]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
B_8	[0.00,0.00]	[0.27,0.32	[0.08,0.12]	[0.54,0.62]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]
B_9	[0.85,0.93]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.08,0.12]	[0.00,0.00]
B_{10}	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.00,0.00]	[0.77,0.83]	[0.19,0.30]	[0.00,0.00]	[0.00,0.00]

Table 2. Transforming B_K into the forms of Single valued neutrosophic (SN) Sets

C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.85,0.15,0.00]	[0.04,0.02,0.94]	[0.04,0.03,0.93]	[0.00,0.01,1.00]	[0.00,0.01,1.00]
[0.00,0.01,1.00]	[0.03,0.01,0.69]	[0.90,0.03,0.88]	[0.55,0.15,0.30]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.08,0.05,0.87]
[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.30,0.28,0.42]	[0.40,0.22,0.38]	[0.08,0.05,0.87]	[0.00,0.01,1.00]	[0.00,0.01,1.00]
[0.09,0.02,0.89]	[0.78,0.04,0.18]	[0.00,0.01,1.00]	[0.08,0.03,0.89]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]
[0.09,0.03,0.88]	[0.09,0.02,0.89]	[0.08,0.04,0.88]	[0.09,0.03,0.88]	[0.18,0.03,0.79]	[0.08,0.05,0.87]	[0.08,0.05,0.87]	[0.08,0.04,0.88]	[0.08,0.04,0.88]
[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.18,0.04,0.78]	[0.12,0.05,0.83]	[0.37,0.08,0.55]	[0.00,0.01,1.00]	[0.22,0.06,0.72]
[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.08,0.04,0.88]	[0.86,0.07,0.07]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]
[0.00,0.01,1.00]	[0.27,0.05,0.68]	[0.08,0.04,0.88]	[0.54,0.08,0.38]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]
[0.85,0.08,0.07]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.08,0.04,0.88]	[0.00,0.01,1.00]
[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.00,0.01,1.00]	[0.77,0.06,0.17]	[0.19,0.04,0.77]	[0.00,0.01,1.00]	[0.00,0.01,1.00]

Step:-2 The nethermost (lowest) and uppermost (highest) energy bounds of each real fault conditions (B_K) have been extracted and thereafter rehabilitated into the forms of SN sets as

shown in **Table 2.** The fault testing samples $F_{T_J}\ (J=1,2)$ in this study can also be transformed into the forms of SN sets as follows.

$$F_{T_1}=\left\langle\begin{matrix}[0.000,0.010,1.000],[0.000,0.010,1.000],[0.100,0.010,0.900],[0.000,0.010,1.000],\\ [0.000,0.010,1.000],[0.000,0.010,1.000],[0.000,0.010,1.000],[0.000,0.010,1.000],\\ [0.000,0.010,1.000]\end{matrix}\right\rangle \quad \ldots \ (22)$$

$$F_{T_2}=\left\langle\begin{matrix}[0.390,0.010,0.610],[0.070,0.010,0.930],[0.000,0.010,1.000],[0.060,0.010,0.940],\\ [0.000,0.010,1.000],[0.130,0.010,0.870],[0.000,0.010,1.000],[0.000,0.010,1.000],\\ [0.350,0.010,0.650]\end{matrix}\right\rangle \quad \ldots (23)$$

Step:- 3 The FCEM $T_{FS}^{\mu}(B_K,F_{T_J})$ and NCEM $T_{NC}(B_K,F_{T_J})$ values between each B_K (provided in **Table 2**) and F_{T_J} (represented by **(22,23)**) can be computed employing the resulting equations **(20,21)**. The fault diagnosis order obtained through the proposed FCEM and NCEM as well as by the existing cosine similarity measure **[3]** is represented in **Table 3.**

Diagnosis Result 1. The fuzzy as well as neutrosophic cross entropy values between each familiar fault condition B_K and the first testing sample F_{T_1}, as can be seen from **Table 3,** are

$$T_{FS}^{\mu}\left(B_1,F_{T_1}\right)=0.1382,T_{FS}^{\mu}\left(B_2,F_{T_1}\right)=0.0082,T_{FS}^{\mu}\left(B_3,F_{T_1}\right)=0.1222;T_{FS}^{\mu}\left(B_4,F_{T_1}\right)=0.0830,T_{FS}^{\mu}\left(B_5,F_{T_1}\right)=0.0595,$$

$$T_{FS}^{\mu}\left(B_6,F_{T_1}\right)=0.1289,T_{FS}^{\mu}\left(B_7,F_{T_1}\right)=0.0000,T_{FS}^{\mu}\left(B_8,F_{T_1}\right)=0.0184,T_{FS}^{\mu}\left(B_9,F_{T_1}\right)=0.1382,T_{FS}^{\mu}\left(B_{10},F_{T_1}\right)=0.1368.$$

$$T_{NC}\left(B_1,F_{T_1}\right)=0.3491,T_{NC}\left(B_2,F_{T_1}\right)=0.0385,T_{NC}\left(B_3,F_{T_1}\right)=0.2913;T_{NC}\left(B_4,F_{T_1}\right)=0.1676,T_{NC}\left(B_5,F_{T_1}\right)=0.1327,$$

$$T_{NC}\left(B_6,F_{T_1}\right)=0.2737,T_{NC}\left(B_7,F_{T_1}\right)=0.0006,T_{NC}\left(B_8,F_{T_1}\right)=0.0401,T_{NC}\left(B_9,F_{T_1}\right)=0.2931,T_{NC}\left(B_{10},F_{T_1}\right)=0.2824.$$

In view of Minimum Argument Principle, the minimum symmetric trigonometric FCEM and NCEM values are 0.0000 and 0.0006 respectively. Clearly, these values confirm that vibration fault in turbine occurs due to the defect in anti-thrust bearing $\left(B_7\right)$, which is an optimal turbine fault selection, as it can also be experienced from **Fig. 4(a).** The next smallest FCEM and NCEM values are 0.0082,0.0184 and 0.0385,0.0401 respectively which correspond to the fault types B_2 and B_8. This indicates that there is a high possibility of pneumatic force couple and surge faults in the generator. The fault type B_5 (radial impact friction of rotor) has low possibility owing to the next smaller

FCEM and NCEM values (0.0595,0.1327). Similarly, the fault types B_4 , B_6 , B_{10} , B_3 , B_9 and B_1 have very low possibility owing to their smaller FCEM and NCEM entropy values.

Table 3. Fault Recognition of Turbine employing (a) FCEM (b) NCEM and (b) Existing Cosine Similarity Measure **[3]**

Description	**Measure Values**					**Recognized Fault Condition**	**Actual Fault Condition**
FCEM	**FCEM Values**						
$T_{FS}^{\mu}\left(B_K, F_{T_1}\right)$	0.1382	0.0082	0.1222	0.0830	0.0595	Antithrust Bearing	Antithrust Bearing
	0.1289	**0.0000**	0.0184	0.13821	0.1368		
$T_{FS}^{\mu}\left(B_K, F_{T_2}\right)$	0.1170	0.0445	0.0787	0.0424	**0.0282**	Radial Impact Friction	Radial Impact Friction
	0.0670	0.0818	0.0651	0.0448	0.0720		
NCEM	**NCEM Values**						
$T_{NC}\left(B_K, F_{T_1}\right)$	0.3491	0.0385	0.2913	0.1676	0.1327	Antithrust Bearing	Antithrust Bearing
	0.2737	**0.0006**	0.0401	0.2931	0.2824		
$T_{NC}\left(B_K, F_{T_2}\right)$	0.3053	0.0901	0.1916	0.0867	**0.0584**	Radial Impact Friction	Radial Impact Friction
	0.1428	0.1679	0.1284	0.0967	0.1493		
Cosine	**Cosine Similarity Measure Value []**						
$C_{VS}\left(B_K, F_{T_1}\right)$	0.7891	0.9799	0.8282	0.8236	0.9057	Antithrust Bearing	Antithrust Bearing
	0.8714	**0.9995**	0.9774	0.7974	0.8099		
$C_{VS}\left(B_K, F_{T_2}\right)$	0.8563	0.9128	0.9066	0.8953	**0.9738**	Radial Impact Friction	Radial Impact Friction
	0.9567	0.8720	0.9201	0.9403	0.8968		

Thus, the optimal fault recognition order is

$B_7 \succ B_2 \succ B_8 \succ B_5 \succ B_4 \succ B_3 \succ B_6 \succ B_{10} \succ B_1 \succ B_9$ (Obtained from FCEM)

$B_7 \succ B_2 \succ B_8 \succ B_5 \succ B_4 \succ B_6 \succ B_{10} \succ B_3 \succ B_9 \succ B_1$ (Obtained from NCEM)

Diagnosis Result 2. The FCEM and NCEM values between second real testing sample F_{T_2} and B_K are

$$T_{FS}^{\mu}\left(B_1, F_{T_2}\right)=0.1170, T_{FS}^{\mu}\left(B_2, F_{T_2}\right)=0.0445, T_{FS}^{\mu}\left(B_3, F_{T_2}\right)=0.0787, T_{FS}^{\mu}\left(B_4, F_{T_2}\right)=0.0424, T_{FS}^{\mu}\left(B_5, F_{T_2}\right)=0.0282,$$

$$T_{FS}^{\mu}\left(B_6, F_{T_2}\right)=0.0670, T_{FS}^{\mu}\left(B_7, F_{T_2}\right)=0.0818, T_{FS}^{\mu}\left(B_8, F_{T_2}\right)=0.0651, T_{FS}^{\mu}\left(B_9, F_{T_2}\right)=0.0448, T_{FS}^{\mu}\left(B_{10}, F_{T_2}\right)=0.0720.$$

$$T_{NC}\left(B_1, F_{T_2}\right)=0.3053, T_{NC}\left(B_2, F_{T_2}\right)=0.0901, T_{NC}\left(B_3, F_{T_2}\right)=0.1916, T_{NC}\left(B_4, F_{T_2}\right)=0.0867, T_{NC}\left(B_5, F_{T_2}\right)=0.0584,$$

$$T_{NC}\left(B_6, F_{T_2}\right)=0.1428, T_{NC}\left(B_7, F_{T_2}\right)=0.1679, T_{NC}\left(B_8, F_{T_2}\right)=0.1284, T_{NC}\left(B_9, F_{T_2}\right)=0.0967, T_{NC}\left(B_{10}, F_{T_2}\right)=0.1493.$$

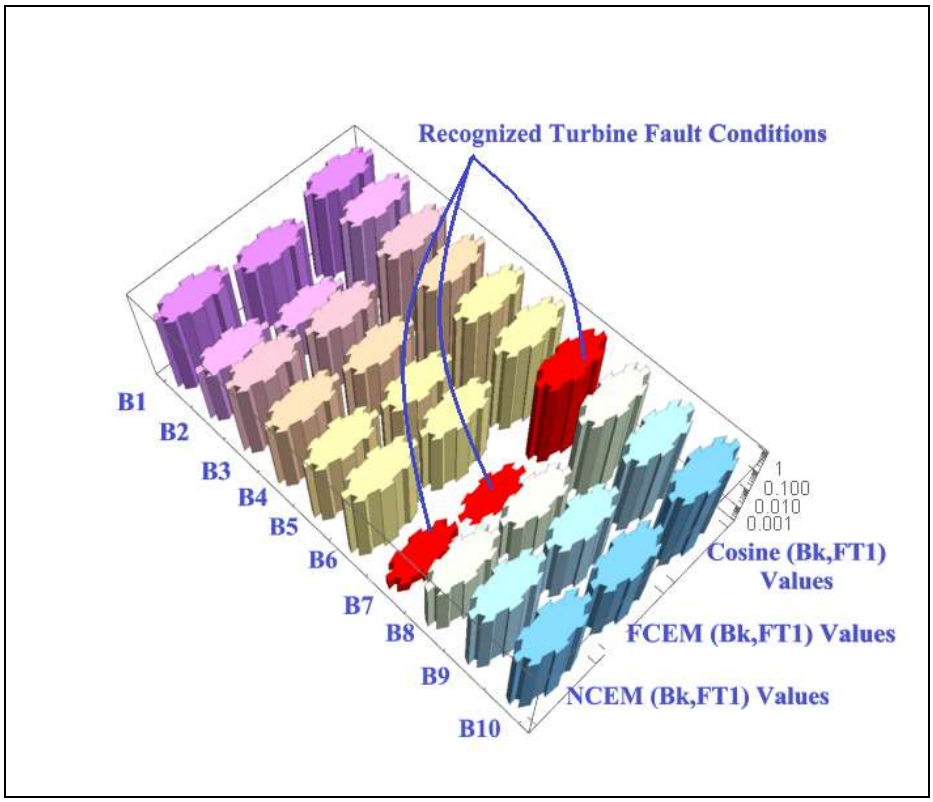


Fig.4(a) Recognized Optimal Fault Condition Employing Proposed Fuzzy, Neutrosophic Cross Entropy and Existing Cosine Similarity Measures **[3]**

In this case, the minimum symmetric trigonometric FCEM and NCEM values are 0.0282 and 0.0584 respectively. Clearly, these values confirm that vibration fault in turbine occurs due to the defect in radial impact friction of the rotor (B_5), which is an optimal turbine fault selection, as it can also be experienced from **Fig. 4(b).** The next smallest FCEM and NCEM values are 0.0424,0.0445 and 0.0584,0.0901 respectively which correspond to the fault types B_4 and B_2. This indicates that there is a high possibility of pneumatic force couple and oil membrane oscillation. The fault type B_9 (looseness of bearing block) has low possibility owing to its smaller FCEM and NCEM values (0.0448,0.0967). Similarly, the fault types B_8, B_6, B_{10}, B_7, B_3 and B_1 have very low possibility owing to their smaller cross entropy values. Thus, the optimal fault recognition order is $B_5 \succ B_4 \succ B_2 \succ B_9 \succ B_8 \succ B_6 \succ B_{10} \succ B_7 \succ B_3 \succ B_1$.

Validity Test: In order to perform the validity of NCEM under validity criteria **[11]**, we inter-change the degree of true and falsity membership of non-optimal (B_9) alternative and worse (B_1) alternatives. The new symmetric trigonometric FCEM and NCEM values can be recalculated employing (21) and are given below.

$$T_{NC}\left(B_1,F_{T_1}\right)=1.4895, T_{NC}\left(B_2,F_{T_1}\right)=0.0385, T_{NC}\left(B_3,F_{T_1}\right)=0.2913; T_{NC}\left(B_4,F_{T_1}\right)=0.1676, T_{NC}\left(B_5,F_{T_1}\right)=0.1327,$$

$$T_{NC}\left(B_6,F_{T_1}\right)=0.2737, T_{NC}\left(B_7,F_{T_1}\right)=0.0006, T_{NC}\left(B_8,F_{T_1}\right)=0.0401, T_{NC}\left(B_9,F_{T_1}\right)=1.5678, T_{NC}\left(B_{10},F_{T_1}\right)=0.2824.$$

$$T_{NC}\left(B_1,F_{T_2}\right)=1.1789,T_{NC}\left(B_2,F_{T_2}\right)=0.0976,T_{NC}\left(B_3,F_{T_2}\right)=1.1882,T_{NC}\left(B_4,F_{T_2}\right)=0.0867,T_{NC}\left(B_5,F_{T_2}\right)=0.0660,$$

$$T_{NC}\left(B_6,F_{T_2}\right)=0.1428,T_{NC}\left(B_7,F_{T_2}\right)=0.1754,T_{NC}\left(B_8,F_{T_2}\right)=0.1360,T_{NC}\left(B_9,F_{T_2}\right)=0.0967,T_{NC}\left(B_{10},F_{T_2}\right)=0.1493.$$

The results clearly indicate that the optimal fault selection does not change whenever we interchange the non-optimal and worse alternatives. This justifies that our proposed NCEM is capable of holding the best fault selection whenever worse and non-optimal are interchanged. However, the existing measures **[2,3]** are insufficient for holding the best fault selection. This indicates some ambiguity in the enduring fault recognition methods

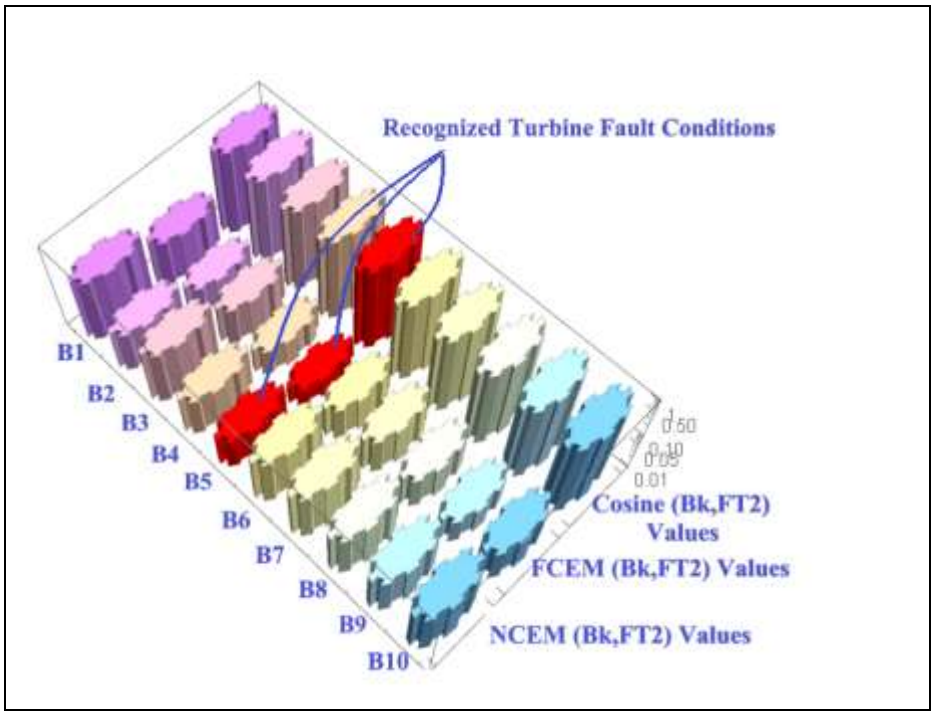


Fig.4(b) Recognized Optimal Fault Condition Employing Proposed Fuzzy, Neutrosophic Cross Entropy and Existing Cosine Similarity Measures

Table 4. Fault Recognition of Turbine employing (a) FCEM (b) NCEM and (b) Existing Cosine Similarity Measure **[3]** Under Sensitive Analysis

Description	**Measure Values**					**Optimal, Worse Alternatives Under Sensitive Analysis**	
NCEM	**NCEM Values**					**Before**	**After**
$T_{NC}\left(B_K,F_{T_1}\right)$	0.3497	0.0391	0.2920	0.1682	0.1284	B_9,B_1	B_3,B_1
	0.2743	**0.0012**	0.0407	0.2887	0.2831		
$T_{NC}\left(B_K,F_{T_2}\right)$	0.3059	0.0907	0.1922	0.0873	**0.0541**	B_3,B_1	B_3,B_1
	0.1434	0.1685	0.1290	0.0924	0.1499		

Sensitive Analysis In order to demonstrate the effectiveness of NCEM under sensitive analysis**[11]** ,we slightly change the value $\left(< x_8, [0.00, 0.01, 1.00] >\right)$ of F_{T_1} to $\left(< x_8, [0.010, 0.010, 1.000] >\right)$. Next, we again compute $T_{NC}\left(B_K, F_{T_J}\right)(J = 1, 2)$ employing **(21)** and represent the results in ranking order of all ten knowledge of system faults is provided in **Table 4**. A The comparison of the results depicted in **Table 3** and **Table 4** indicate that the optimal and worse alternatives remain unchanged whenever there is a small change in the SN set $\left(< x_8, [0.00, 0.01, 1.00] >\right)$. This clarifies that our symmetric trigonometric NCEM is an insensitive measure when subjected to a little change in the evaluation values. However, the enduring measures **[2,3]** have been found sensitive under this experiment.

Intuitive Analysis For the performance of FCEM, NCEM and existing measures **[12]** under intuitive analysis, we have assumed two fuzzy sets (F_1, F_2) and SN sets (T_1, T_2) as depicted in **Table**. In this experiment, we have fixed the value of F_2 as $[1.000]$, T_2 as $[1.000, 0.010, 0.000]$ meanwhile, the value of F_1, T_1 are increased gradually as presented in Table. The FCEM and NCEM values along with existing measure values **[]** are calculated using **(20,21)** and the results are presented **Table 5.** The tabulated results reveal that $T_{FS}^{\mu}(F_1, F_2); T_{NC}(T_1, T_2)$ values decrease whenever there is a slight increase in the values of F_1, T_1. However, a constant or undefined trend was experienced while repeating this phenomenon with the enduring measures **[2,3]**. This justifies that fault information conveyed by proposed cross entropy measures are feasible and meaningful. Moreover, this also justifies the superiority and remarkability of proposed methodology over the enduring methods **[2,3],** under intuitive analysis.

Table 5. Intuitive analysis of (a) FCEM (b) NCEM (c) Existing Measures **[2,3]**

Gp.	**Fuzzy Set**		**SN Set**		**FCEM**	**NCEM**	**Cosine[3]**	**Measure [3]**
No.	F_1	F_2	T_1	T_2	**Values**	**Values**	**Values**	**Values**
1	0.000	1.000	[0.000,0.010,0.000]	[1.000,0.010,0.000]	0.1272	0.1272	0.0009	#NUM!
2	0.100	1.000	[0.100,0.010,0.000]	[1.000,0.010,0.000]	0.0672	0.0672	0.0905	1.1371
3	0.200	1.000	[0.200,0.010,0.000]	[1.000,0.010,0.000]	0.0544	0.0544	0.0908	1.1262
4	0.300	1.000	[0.300,0.010,0.000]	[1.000,0.010,0.000]	0.0457	0.0457	0.0909	1.1163

5	0.400	1.000	[0.400,0.010,0.000]	[1.000,0.010,0.000]	0.0384	0.0384	0.0909	1.1071
6	0.500	1.000	[0.500,0.010,0.000]	[1.000,0.010,0.000]	0.0316	0.0316	0.0909	1.0984
7	0.600	1.000	[0.600,0.010,0.000]	[1.000,0.010,0.000]	0.0251	0.0251	0.0909	1.0901
8	0.700	1.000	[0.700,0.010,0.000]	[1.000,0.010,0.000]	0.0188	0.0188	0.0909	1.0822
9	0.800	1.000	[0.800,0.010,0.000]	[1.000,0.010,0.000]	0.0124	0.0124	0.0909	1.0745
10	0.900	1.000	[0.900,0.010,0.000]	[1.000,0.010,0.000]	0.0062	0.0062	0.0909	1.0671
11	1.000	1.000	[1.000,0.010,0.000]	[1.000,0.010,0.000]	0.0000	0.0000	0.0909	#NUM!

Conclusion

This study has propounded the establishment of novel symmetric trigonometric fuzzy as well as single valued neutrosophic cross entropy measures (FCEM and NCEM). To overcome the shortcomings faced by non-fuzzy and asymmetrical cross entropy measures and to obtain meaningful fault information, the proposed symmetric FCEM and NCEM has the necessary capability for recognizing the optimal fault conditions such as antithrust bearing and radial impact of friction of rotor, of a huge steam turbine generator. The proposed variants of neutrosophic cross entropy measures are compatible for further mathematical treatments under sensitive and intuitive analysis because of their symmetric quintessence whereas the enduring measures exhibit inconsequential results indicating ambiguity in the evaluation information of fault features

Credit Authorship Contribution Statement:

C.P. Gandhi: Writing Original Draft, Methodology.

Declaration of Competing Interest The authors declare no conflict of interest.

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