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Abstract: The simulation process depends on generating a series of random numbers subject to the uniform probability distribution in the interval [0, 1]. The generation of these numbers is starting from the cumulative distribution function of the uniform distribution. Through previous studies in classical logic, we found any random number R_0, met with a cumulative distribution function value equal to R_0, but these specific numbers may not have sufficient accuracy, which leads to obtaining results that are not sufficiently accurate when doing the simulation. To bypass this case, in this paper, we present a study that enables us to generate as accurate as possible random numbers, using neutrosophic logic ' this Logic given by American mathematician Florentin Smarandache in 1995'. The first step in the study is, define the cumulative distribution function of the neutrosophic uniform distribution, depending on definition of the neutrosophic integral and definition of the neutrosophic uniform distribution. We used the new definition to generate random numbers subject to a neutrosophic uniform distribution on the interval [0, 1]. The result was that each random number R_0 corresponds to a interval of the distribution function related to R_0, So that it preserves enough precision for the random numbers, and thus we get a more accurate simulation of any system we want to simulate.

Keywords: Neutrosophic uniform distribution; Simulations; Cumulative distribution function of neutrosophic uniform distribution; neutrosophic random numbers.

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1. Introduction

Because of the great difficulty that can face us when studying the work of any real system. As well as the high cost of studying. In addition, some systems we cannot be directly studied. Here comes the importance of the simulation process in all branches of science. The simulation depends on the application of the study on systems similar to the real systems, and then projecting these results if they are appropriate on the real system.

The simulation based on generating a series of random numbers that are subject to a uniform probability distribution in the interval [0, 1]. Then converting these numbers into random variables subject to the probability distribution in which the system to be simulated operates, based on the cumulative distribution function of the probability distribution [9].

In front of the great revolution brought about by neutrosophic logic in all fields of science, after the American philosopher and mathematician Florentin Smarandache laid its foundations in 1995 [2,4,5,6,8], who put it forward as a generalization of fuzzy logic, especially Intuitionistic fuzzy logic [3], and an extension of the theory of fuzzy sets presented by Lotfi Zadeh 1965 [1] . By extension to that, A.A. Salama presented the neutrosophic classical set theory as a generalization of the classical set theory and developed, introduced and formulated new concepts in the fields of mathematics, statistics, computer science and classical information systems using the neutrosophic [7,15,16]. The neutrosophic is the logic that studies the origin, nature and field of indeterminacy, taking into account every idea with its opposite and with the spectrum of indeterminacy.

This logic has developed in recent years, and most of the known concepts in classical logic have been reformulated according to neutrosophic logic [10,12,13,17,19,20,21,22,23,24]. Among these concepts is the study and formulation of most of the known probability distributions in classical logic [18]. In this paper, we present a study of the cumulative distribution function of the neutrosophic uniform distribution on the interval [a, b]. Depending on what researchers have found in the field of neutrosophic [11,14], such as the definition of the neutrosophic uniform distribution on the interval [a, b] and the definition of the neutrosophic integration [25]. Especially in the case where the indeterminacy is related to the upper and lower bounds of the interval [a, b]. We used the results as a basis for generating random numbers subject to a neutrosophic uniform distribution on the interval [0, 1].

The importance of this research stems from the importance of simulation in all fields of science, especially when we need accuracy in the results during the simulation process for any system. This accuracy not provided by classical logic. The limits of this study include all scientific fields that may contain indefinite cases and need simulation to represent them, and aim to reach the most accurate possible results

2. Experimental and Theoretical Part:

Based on the importance of simulation in all fields of science. Since the simulation depends primarily on generating a series of random numbers that are subject to a uniform distribution on the interval [0, 1]. It was necessary to keep pace with the neutrosophic revolution by presenting a study that generates neutrosophic random numbers that are subject to a neutrosophic uniform distribution on the interval [0, 1]. That is by finding mathematical relationships that describe the cumulative distribution function of the neutrosophic uniform distribution on the interval [a, b], especially when the indeterminacy relates to both ends of the interval. Where we studied the following three cases:

The first case: if the indeterminacy related to the lower limit of interval.

The second case: if the indeterminacy related to the upper limit of interval.

The third case: if the indeterminacy related to both the lower and upper limits of interval.

In addition, we arrived at mathematical formulas for the cumulative distribution function for each of the previous cases. We then used these formulas to generate random numbers that are subject to a neutrosophic uniform distribution. Which provides us with the most accurate results possible while performing the simulation process for any system.

2.1. In classical logic:

To generate the random numbers subject to a uniform distribution in the interval [0, 1] (according to the Monte–Carlo method), we assume R is the continuous random variable that is subject to the continuous uniform distribution defined on the interval [a, b], which is given by:

$$f(x) = \frac{1}{b-a} \quad ; \quad a \le x \le b$$

Then it would be:

$$f(R) = \frac{1}{1-0} = 1 \qquad 0 \le R \le 1$$

The distribution function is of the form:

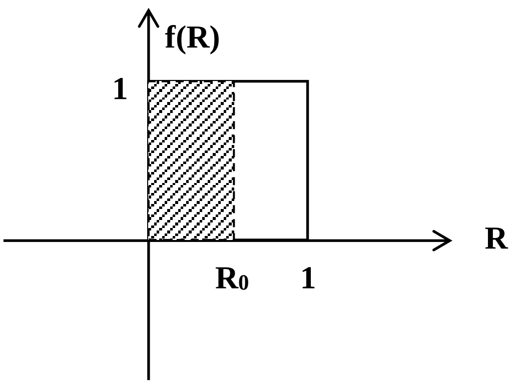

Figure (1)

We take the cumulative distribution function for this distribution. We will call it F(R), then:

$$P(R < R_0) = F(R_0) = \int_0^{R} f(R)dR = \int_0^{R_0} 1.dR = R_0$$

That is, every random number R_0 corresponds to a value of the distribution function equal to R_0.

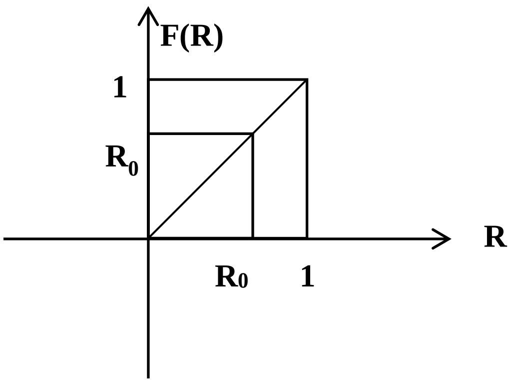

Figure (2)

2.2. In Neutrosophic logic:

The uniform distribution in neutrosophic logic takes the following form:

$$f_N(x) = \frac{1}{b-a}$$

Where b, a: one or both of them is not precisely defined, we find it in the form of set or interval, etc., we can consider all possible cases for b, a. While keeping the condition a < b.

Accordingly, we can write the interval [a, b] in one of the following forms **(*)**:

1. $[a_1 + \varepsilon, b]$; $a = a_N = a_1 + \varepsilon$

2. $[a, b_1 + \varepsilon]$; $b = b_N = b_1 + \varepsilon$

3. $[a_1 + \varepsilon, b_1 + \varepsilon]$; $a = a_N = a_1 + \varepsilon$ & $b = b_N = b_1 + \varepsilon$

Where $\varepsilon \in [0, n]$, with $a < n < b$.

In order to define the cumulative distribution function of the neutrosophic uniform distribution on the interval [a, b], it is necessary to define the neutrosophic integral.

We define the neutrosophic integral in the case of indeterminacy related to the lower limit of interval as follows:

Suppose we want to integrate the function **f: X → R** on the interval [a, b] of X , but we are not sure of the lower limit **a** , since **a** has a finite part (a_1) and an indefinite part (ε), i.e.:

$a = a_N = a_1 + \varepsilon$; $\varepsilon \in [0, n]$; $a < n < b$

Then we write the integral of f(x) on the interval [a, b], as follows:

$$\int_a^b f(x)dv = \int_a^b f(x)dx - i_1$$

Where: $i_1 \in \left[0, \int_{a_1}^{a_1+n} f(x)dx\right]$

In another way: $\int_a^b f(x)dv = \int_{a_1+n}^b f(x)dx + i_2$

Where: $i_2 \in \left[0, \int_{a_1}^{a_1+n} f(x)dx\right]$

2.3. Neutrosophical cumulative distribution function with states (*):

First case:

The indeterminate is in the lower limit of the interval $a = a_1 + \varepsilon$; $\varepsilon \in [0, n]$; $a < n < b$

The interval becomes $[a_1 + \varepsilon , b]$, and we have:

$$a = a_N = a_1 + \varepsilon = a_1 + [0, n] = [a_1, \quad a_1 + n]$$

The cumulative distribution function for a classical uniform distribution given as:

$$P(X < x_s) = F(x_s) = \int_a^{x_s} f(x)dx$$

The Cumulative distribution function for a neutrosophic uniform distribution:

$$NP(X < x_s) = NF(x_s) = \int_a^{x_s} f(x)dv = \int_{a_1}^{x_s} f(x)dx - i$$

Where: $i \in \left[0, \int_{a_1}^{a_1+n} f(x)dx\right]$

In the case of a neutrosophic uniform distribution, we get:

$$NF(x_s) = \int_{a_1}^{x_s} \frac{1}{b-a} dx - i = \frac{x_s - a_1}{b-a} - i$$

We define the interval for **i** , let's calculate the integral:

$$\int_{a_1}^{a_1+n} \frac{1}{b-a}dx = \frac{1}{b-a}[x]_{a_1}^{a_1+n} = \frac{a_1+n-a_1}{b-a} = \frac{n}{b-a}$$

This means that the indeterminate **i** belongs to the interval:

$$\mathrm{i} \in \left[0, \frac{n}{b-a_N}\right]$$

<u>Second case:</u>

The indeterminate is in the upper limit of the interval $b = b_1 + \varepsilon$; $\varepsilon \in [0, n]$; $a < n < b$

The interval becomes **$[a , b_1 + \varepsilon]$**, and we have:

$$b = b_N = b_1 + \varepsilon = b_1 + [0 , n] = [b_1 , \quad b_1 + n]$$

The cumulative distribution function for a classical uniform distribution given as:

$$P(X < x_s) = F(x_s) = \int_a^{x_s} f(x)dx$$

The Cumulative distribution function for a neutrosophic uniform distribution:

$$NP(X < x_s) = NF(x_s) = \int_a^{x_s} f(x)dv$$

$$NF(x_s) = \int_a^{x_s} \frac{1}{b-a}dx = \frac{x_s - a}{b_N - a}$$

<u>Third case:</u>

The Indeterminacy exists in the lower and upper limit of the interval

$=[a_N, b_N]$ $[a_1 + \varepsilon, \quad b_1 + \varepsilon]$; $\varepsilon \in [0, n]$; $a < n < b$. i.e.:

$$a_N = a_1 + \varepsilon = a_1 + [0 , n] = [a_1 , a_1 + n]$$

$$b_N = b_1 + \varepsilon = b_1 + [0 , n] = [b_1 , \ b_1 + n]$$

So we write:

$$NP(X < x_s) = NF(x_s) = \int_{a}^{x_s} f(x)dv = \int_{a_1}^{x_s} f(x)dx - i$$

Where: $i \in \left[0, \int_{a_1}^{a_1+n} f(x)dx\right]$

In addition, it will be:

$$NF(x_s) = \int_{a_1}^{x_s} \frac{1}{b-a}dx - i = \frac{x_s - a_1}{b-a} - i$$

Where: $i \in \left[0, \frac{n}{b_N - a_N}\right]$

3. Results and Discussion

Generation of neutrosophic random numbers that are subject to a uniform distribution in the interval [0, 1]:

Based on what we have put forward, we get the following forms of the interval [0,1] with cases of the Indeterminacy:

1. $[0 + \varepsilon \ , \ 1]$
2. $[0 \ , \ 1 + \varepsilon]$
3. $[0 + \varepsilon \ , \ 1 + \varepsilon]$

Where: $\varepsilon \in [0, n]$; $0 < n < 1$.

First case:

$$NP(R < R_0) = NF(R_0) = \frac{R_0 - \varepsilon}{1 - \varepsilon} \quad ; \varepsilon \in [0, n]$$

$$\Rightarrow NP(R < R_0) = NF(R_0) = \frac{R_0 - \varepsilon}{1 - \varepsilon} = \frac{[R_0 , R_0 - n]}{[1 , 1 - n]} = \left[R_0, \frac{R_0 - n}{1 - n}\right]$$

Second case:

$$NP(R < R_0) = NF(R_0) = \frac{R_0}{1 + \varepsilon} \quad ; \varepsilon \in [0, n]$$

$$\Rightarrow NP(R < R_0) = NF(R_0) = \frac{R_0}{1 + \varepsilon} = \frac{R_0}{[1 , \ n+1]} = \left[R_0 , \frac{R_0}{n+1}\right]$$

Third case:

$NP(R < R_0) = NF(R_0) = R_0 - \varepsilon \quad ; \ \varepsilon \in [0, n]$

$\Rightarrow NP(R < R_0) = NF(R_0) = R_0 - \varepsilon \ = [\, R_0 \,, R_0 - n]$

From the above we conclude that each random number R_0 corresponds to an interval of the cumulative distribution function related to R_0.

4. Application example:

Using "mean of the square" method (for von Neumann), we generate the random numbers R_0, R_1, R_2, R_3, R_4.

Method explanation: We choose a fractional random number R_0. Consisting of four places (called the seed) and does not contain zero. Then we square that number (R0). Choose the middle four digits of the fractional part then put a new fraction and consider it the random number R_1. And so on... until we get the required random numbers. i.e. we apply the rule:

$$R_{i+1} = \text{Mid} \left\lfloor R_i^2 \right\rfloor \quad i = 0, 1, 2, \ldots$$

We denote by "Mid" the middle four ranks of R_i^2.

For example, we choose, $R_0 = 0.1276$ then:

$R_1 = \text{Mid} \left\lfloor R_0^2 \right\rfloor = \text{Mid}\,[0.01628176] = 0.6281$

$R_2 = 0.4509$, $R_3 = 0.3310$, $R_4 = 0.0951$

We use these numbers to generate neutrosophic random numbers that follow a uniform distribution in the interval [0, 1], according to the three cases.

First case: In this case, each random number R_0 corresponds to $\dfrac{R_0 - \varepsilon}{1 - \varepsilon}$

We found that:

$$R_0 \to \frac{R_0 - \varepsilon}{1 - \varepsilon} \qquad ; \ \varepsilon \in [0, n]$$

We take, for example: $[0, n] = [0, 0.03]$

$$R_0 \to \frac{0.1276 - [0, 0.03]}{1 - [0, 0.03]} = \frac{[0.1276, 0.1276 - 0.03]}{[0.97, 1]} = \frac{[0.0976, 0.1276]}{[0.971]} = [0.1006, 0.1276]$$

$$R_1 \to \frac{R_1-\varepsilon}{1-\varepsilon} = \frac{0.6281-[0, 0.03]}{[1, 1\text{-}0.03]} = \frac{[0.5981, 0.6281]}{[0.97, 1]} = [0.6281, 0.6169]$$

$$R_2 \to \frac{R_2-\varepsilon}{1-\varepsilon} = \frac{0.4509-[0, 0.03]}{[1, 1\text{-}0.03]} = \frac{[0.4209, 0.4509]}{[0.97, 1]} = [0.4339, 0.4509]$$

$$R_3 \to \frac{R_3-\varepsilon}{1-\varepsilon} = \frac{0.3310-[0, 0.03]}{[0.97, 1]} = \frac{[0.301, 0.3310]}{[0.97, 1]} = [0.3103, 0.3310]$$

$$R_4 \to \frac{R_4-\varepsilon}{1-\varepsilon} = \frac{0.0956-[0, 0.03]}{[0.97, 1]} = \frac{[0.0656, 0.956]}{[0.97, 1]} = [0.0676, 0.956]$$

Second case: In this case, each random number R_0 corresponds to $\frac{R_0}{1+\varepsilon}$

Where $\varepsilon = [0, 0.03]$, $R_0 = 0.1276$

$$R_0 \to \frac{0.1276}{1+[0,0.03]} = \frac{0.1276}{[1, 1.03]} = [0.1238, 0.1276]$$

$$R_1 \to \frac{0.6281}{[1, 1.03]} = [0.6098, 0.6281]$$

$$R_2 \to \frac{0.4509}{[1, 1.03]} = [0.4377, 0.4509]$$

$$R_3 \to \frac{0.0956}{[1, 1.03]} = [0.0928, 0.0956]$$

$$R_4 \to \frac{0.0956}{[1, 1.03]} = [0.0928, 0.0956]$$

Third case: In this case, each random number R_0 corresponds to $R_0 - \varepsilon$, where:

$\varepsilon \in [0, n]$; $0 < n < 1$

For $\varepsilon = [0, 0.03]$ and $R_0 = 0.1276$, we have:

$R_0 \rightarrow 0.1276 - [0, 0.03] = [0.0976, 0.1276]$

$R_1 \rightarrow 0.6281 - [0, 0.03] = [0.5981, 0.6281]$

$R_2 \rightarrow 0.4509 - [0, 0.03] = [0.4209, 0.4509]$

$R_3 \rightarrow 0.3310 - [0, 0.03] = [0.301, 0.3310]$

$R_4 \rightarrow 0.0956 - [0, 0.03] = [0.0656, 0.0956]$

5. **Conclusions**:

Through this paper, we found that when we use neutrosophic logic to generate random numbers, we get a series of numbers that are more accurate than the numbers we get when using classical logic. This is due to the margin of freedom offered by neutrosophic logic through the indeterminacy spectrum.

We are looking forward in the near future , to preparing a study in which we can generate the random numbers that are subject to non-uniform distributions, by converting the regular random numbers in the interval [0, 1] into random numbers that are subject to the appropriate probability distribution for the case under study.

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