Neutrosophic Sets and Systems

Volume 49 Article 3

4-12-2022

Pentapartitioned Neutrosophic Probability Distributions

Suman Das

Bimal Shil

Rakhal Das

Huda E. Khaled

A. A. Salama

Neutrosophic Sets and Systems, Vol. 49, 2022
University of New Mexico

Pentapartitioned Neutrosophic Probability Distributions

Suman Das[1], Bimal Shil[2], Rakhal Das[3], Huda E. Khalid[4], and A. A. Salama[5*]

[1]Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India.
Email: suman.mathematics@tripurauniv.in , sumandas18842@gmail.com
[2]Department of Statistics, Tripura University, Agartala, 799022, Tripura, India.
Email: bimalshil738@gmail.com, bimal.statistics@tripurauniv.in
[3]Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India.
Email: rakhal.mathematics@tripurauniv.in , rakhaldas95@gmail.com
[4] Telafer University, The Administration Assistant for the President of the Telafer University, Telafer, Iraq. ; https://orcid.org/0000-0002-0968-5611 , dr.huda-ismael@uotelafer.edu.iq
[5]Department of Mathematics and Computer Science, Port Said University, Egypt.
Email: drsalama44@gmail.com
***Correspondence:** drsalama44@gmail.com

Abstract: In this manuscript, we introduce and study some pentapartitioned neutrosophic probability distributions. The study is done through the generalization of some classical probability distributions as Poisson distribution, Exponential distribution, Uniform distribution etc. This study opens the way for dealing with issues that follow the classical distributions and at the same time contains data not specified accurately.

Keywords: Neutrosophic Set; Probability Distributions; Pentapartitioned Neutrosophic Probability.

---

1. Introduction: The term "Neutrosophy" was first proposed by Prof. Florentin Smarandache [5] in the year 1995. Neutrosophy is a new branch of philosophy, where one can study origin, nature and scope of neutralities. This theory considers every notion or idea <A> together with its opposite or negation <Anti-A>. The <neut-A> and <Anti-A> ideas together called as a <non-A>. Neutrosophic logic is a general framework for unification of many existing logics, fuzzy logic, intuitionistic logic, paraconsistent logic etc. The core objective of neutrosophic logic is to characterize each logical statement in a 3D-neutrosophic space, where each dimension of space represents respectively the truth(T), falsehood(F) and indeterminacies (I) of the statements under consideration, where T, I, F are standard or non-standard real subset of]-0,1+[without necessary connection between them. The classical distribution is extended neutrosophically. Which means that there is some indeterminacy related to the probabilistic experiment. Each experimental observation can result in an outcome of each trial labelled by failure (F) or some indeterminacies (I), in addition to some truthiness (T). Neutrosophic statistics is an extended form of classical statistics, dealing with values holding some vague, or indeterminacy, or incompleteness information. The fundamental concepts of neutrosophic set, introduced by Smarandache, et al [5-9] and Salama et al [10-14]. Recently, using neutrosophic theory, dozens of applications were re-analyzed and re-evaluated, including but not limited to the E-Learning that was raised due to quarantine situations of Coved-19 and its Omicron mutation, the integration system of renewable energy using various resources such as (Photovoltaic panels and Wind Turbines), and the neutrosophic treatment of the static model for inventory management with a safety reserve…etc. [15-27]. In this article, we will discuss a discrete random distribution such as Binomial distribution by approaching neutrosophically. Before shed the light on this context, we should familiar with the following notions: Neutrosophic statistical number 'N' has the form $N = a + I$; where the component a refers to the determinate part of N, while I refers to the indeterminate part of N. Recently, Mallick and Pramanik [2] introduced the concept of pentapartitioned neutrosophic set as an extension of neutrosophic set.

2. Some Relevant Definitions:

In this section, we recall some basic preliminaries and definitions which are relevant to the main results of this paper.

Definition 2.1. [1] Assume that 'w' be a continuous variable. A neutrosophic uniform distribution of w, is a classical uniform distribution, with imprecise distribution parameters c or d $(c < d)$.

Example 2.1. Assume that w be a variable represents a man waiting time lift (in minutes), lift arrival time is not specified, another man said:

1. the lift arrival time is either from now to 3 minutes [0,3] or will arrive after 13 to 17 minutes [13,17] , then $c = [0, 3]$, $d = [13, 17]$

Then, the probability density function:

$f_N(W) = \frac{1}{d-c} = \frac{1}{[13,17]-[0,3]} = \frac{1}{[10,17]}$ = [0.059, 0.1].

2. The lift arrives after seven minutes or will arrive after 13 to 17 minutes [13, 17]

Then, $c = 7, d = [13,17]$

Hence, the probability density function:

$f_N(w) = \frac{1}{d-c} = \frac{1}{[13,17]-7} = \frac{1}{[6,10]} = [0.1,0.167]$

Definition 2.2.[4] Assume that w be a random variable, which represents the number of success when events performs more than or equal to one times. Then, the corresponding probability distribution of w, is called a neutrosophic binomial distribution.

i. Neutrosophic Binomial Random Variable: The random variable 'w' represents the number of success more than or equal to one times.

ii. Neutrosophic Binomial Probability Distribution: The neutrosophic binomial probability distribution of 'w' is represented by n.b.p.d.

iii. Indeterminacy: It is not sure about the success or failure of an experiment output.

iv. Indeterminacy Threshold: Outcome of an event are indeterminate form**.** Where $th \in \{0,1,2 \dots m\}$, m is the sample size. Consider, $P(S)$ = The chance of a particular event outcome in the case of success. $P(F)$ = The chance of a particular event outcome in the case of failure, for both S and F different from indeterminacy. $P(I)$ = The chance of a particular event outcome in the case of an indeterminacy.

Let, $w \in \{0,1,2,\dots,m\}, NP = (T_w, I_w, F_w)$ with

T_w: Chances of 'w' success and the value of $(n-w)$ represents the number of failures and indeterminacy, such that the number of indeterminacy is less than or equal to the indeterminacy threshold. Where, n represents the population size.

F_w: Chances of 'v' success, with $v \neq w$ and the value of $(n-v)$ represents the number of failures and indeterminacy, and it is less than the indeterminacy threshold.

I_w: Chances of 'u' indeterminacy, where 'u' is strictly greater than the indeterminacy threshold.

$T_w + I_w + F_w = (P(S) + P(I) + P(F))^m$

For complete probability we have $P(S) + P(I) + P(F) = 1$; while if the probability was incomplete then, $0 \leq P(S) + P(I) + P(F) < 1$; however, for the paraconsistent probability we have $1 < P(S) + P(I) + P(F) \leq 3$;

$$T_w = \frac{m!}{w!\,(m-w)!} P(S)^w \sum_{r=0}^{th} \frac{r!}{(m-w)!\ (r-m+w)!} P(I)^r P(F)^{m-w-r}$$

$$= \frac{m!}{w!\,(m-x)!} P(S)^w \sum_{r=0}^{th} \frac{(m-w)!}{(m-w+r)!} P(I)^r P(F)^{m-w-r}$$

$$= \frac{m!}{w!} P(S)^w \sum_{r=0}^{th} \frac{\mathrm{P(I)}^r\ \mathrm{P(F)}^{\mathrm{m-w-r}}}{r!\ (m-w+r)!}$$

$$F_w = \sum_{v=0}^{m} T_v = \sum_{v=0, v\neq w}^{th} \frac{m!}{v!} \mathrm{P(S)}^v \sum_{r=0}^{th} \frac{\mathrm{P(S)}^r\ \mathrm{P(F)}^{\mathrm{m-v-r}}}{r!\ (m-v+r)!}$$

$$I_w = \sum_{u=th+1}^{m} \frac{m!}{u!\,(m-u)!} \mathrm{P(I)}^u \sum_{r=0}^{m-u} \frac{(m-u)!}{(m-u)!\ (m-u+r)!} \mathrm{P(S)}^r\ \mathrm{P(F)}^{\mathrm{m-u-r}}$$

$$= \sum_{u=th+1}^{m} \frac{m!}{u!} P(I)^u \sum_{r=0}^{m-u} \frac{\mathrm{P(S)}^r\ \mathrm{P(F)}^{\mathrm{m-u-r}}}{r!\ (m-u+r)!}$$

It is worthy to mention that $T_w, I_w, F_w, P(S), P(I), P(F)$ have their usual meaning.

Definition 2.3. [3] Neutrosophic Poisson distribution of a discrete variable 'w' is a classical Poisson distribution of w, but its parameter is imprecise. For example, λ can be a set contains two or more elements. The most common such distribution can be defined as follow:

$NP(w) = e^{-\lambda_N} \frac{(\lambda_N)^w}{w!}$; where λ_N is a set, and $w = 0,1, \ldots$

λ_N : Is a parameter of the distribution, also, λ_N represents the mean (the expectation) of the distribution , and at the same time it represents the variance value of the distribution. In symbols we can write, $NE(w) = NV(w) = \lambda_N$; where $N = d + I$; is a neutrosophic statistical number.

Definition 2.4. [1] Let 'w' be a continuous random variable is said to be neutrosophic exponential distribution, with parameter λ_N having some imprecise events which represent intervals, then the neutrosophic probability distribution function is given by

$W_N \sim \exp(\lambda_N) = \mathrm{f_N}(w) = \lambda_N . e^{-w.\lambda_N}$; $0 < w < \infty$,

$\exp(\lambda_N)$: Neutrosophic Exponential Distribution.

λ_N: Neutrosophic distribution parameter.

Definition 2.5. [7,8] Let the continuous random variable 'w' be a classical normal distribution, is said to be neutrosophic normal distribution with mean μ_N and variance σ_N, both contain intervals. Which probability distribution function is given by

$$W_N \sim N_N(\mu_N, \sigma_N) = \frac{1}{\sigma_N \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{w-\mu_N}{\sigma_N}\right)^2}$$

N_N: Neutrosophic Normal Distribution.

W_N: w Neutrosophic Random Variable.

3. A New Concepts:

In this section, we introduce new pentapartitioned neutrosophic probability distributions, which are first introduced and well defined supported by concrete examples.

Definition 3.1. Let w be a continuous random variable, and w is followed classical uniform distribution, with imprecise distribution parameters $r \, and \, s$ $(r < s)$, this kind of distribution is said to be a pentapartitioned neutrosophic uniform distribution, where pentapartitioned neutrosophic probability distribution function is given by

$f_{PN}(w) = \frac{1}{s-r}$, where r ≤ w ≤ s

Example 3.2.

Assume that w be a variable represents a person waiting time lift (in minutes/ seconds), where the lift arrival time is not specified, another person said:

1. The lift arrival time is either from now to 3 minutes $[0, 3]$, or will arrive after 13 to 17 minutes [13, 17], then: r= [0, 3], s = [13, 17]; Then, the probability density function:

   $$f_{PN}(w) = \frac{1}{s-r} = \frac{1}{[13,17]-[0,3]} = \frac{1}{[10,17]} = [0.059, 0.1000]$$

2. The lift arrives after seven minutes or will arrive after 13 to 17 minutes $[13, 17]$,

   then: $r = 7,\ s = [13,17]$

Then, the probability density function:

$f_{PN}(w) = \frac{1}{s-r} = \frac{1}{[13,17]-7} = \frac{1}{[6,10]} = [0.1, 0.167]$.

Now, solving this problem by using pentapartitioned neutrosophic uniform distribution,

1. The lift arrival time is either from now to 3 minutes $[0, 3]$ with contradiction $C \in [0, 0.2]$, ignorance $G \in [0, 0.04]$, unknown $U \in [0, 0.03]$, or will arrive after 13 to 17 minutes (i.e. [13,17]), then, $r = [0, 3] + \frac{[0,0.2]+[0,0.04]+[0,0.03]}{3} = [0, 3] + [0, 0.09] = [0, 3.09]$, $s = [13, 17]$ Then, the probability density function:

$f_{PN}(w) = \frac{1}{s-r} = \frac{1}{[13,17]-[0,3.09]} = \frac{1}{[9.91,17]} = [0.059, 0.1009]$

2. The lift arrives after seven minutes along with contradiction $C \in [0, 0.2]$, ignorance $G \in [0, 0.04]$, unknown $U \in [0, 0.03]$ or will arrive after 13 to 17 minutes [13, 17],

Then, $r = (7 + 0.09) = 7.09$, $s = [13, 17]$, and the probability density function:

$f_{PN}(w) = \frac{1}{s-r} = \frac{1}{[13,17]-7.09} = \frac{1}{[5.91,9.91]} = [0.1009, 0.1692]$.

Definition 3.3. A pentapartitioned neutrosophic random variable w is said to follow the pentapartition neutrosophic binomial distribution, if it is assuming non-negative variable and the number of success of an experiment is more than or equal to one time.

i. **Pentapartitioned Neutrosophic Binomial Random Variable:** is the random variable 'w' represents the number of success is more than or equal to one time.

ii. **Pentapartitioned Neutrosophic Binomial Probability Distribution:** The pentapartitioned neutrosophic probability distribution of w with pentapartitioned neutrosophic probability density function.

iii. **Contradiction:** it is a contradiction part of success and failure in which the event results cannot be confined.

iv. **Ignorance:** it is an ignorance part of success and failure in which the event results cannot be confined.

v. **Unknown:** it is an unknown part of success and failure in which the event results cannot be confined.

vi. **C.G.U. Threshold:** represents the number of events whose outcome is imprecise. In this study C. G. U. is $th \in \{0,1,2 \dots m\}$. In other words, C. G. U. is the number of events whose outcomes belong to contradiction, ignorance and unknown events.

Let P(S) = The scope of a particular event in which the output will be fully successful.

P(C) = The scope of a particular event in which the output will be a contradiction.

P(G) = The scope of a particular event in which the output will be ignored.

P(U) = The scope of a particular event in which the output will be unknown.

P(F) = The scope of a particular event in which the output will be failure, for both S and F, except the indeterminacy (I).

Assume that $w \in \{0,1,2,\dots,m\}$, where m represents sample size, $NP = (T_w, C_w, G_w, U_w, F_w)$ with

T_w: Chances of 'w' success, and $(n-w)$ is the number of failures, contradiction, ignorance, and unknown such that the events summation of contradiction, ignorance and unknown is less than or equal to C.G.U. Threshold. It is well known that n represents population size.

F_w: Chances of 'z' success, with $z \neq w$, and $(m-z)$ is the number of failures and contradiction, while the summation of ignorance and unknown events is less than the C.G.U. threshold.

C_w: Chances of 'u' contradiction, where 'u' is strictly greater than C.G.U. threshold.

G_w: Chances of 'v' ignorance, where 'v' is strictly greater than C.G.U. threshold.

U_w: Chances of 't' unknown, where 't' is strictly greater than C.G.U. threshold.

$T_w + F_w + C_w + G_w + U_w = (P(S) + P(C) + P(G) + P(U) + P(F))^m$.

For the complete probability, we have $P(S) + P(C) + P(G) + P(U) + P(F) = 1$;

for incomplete probability, $0 \leq P(S) + P(C) + P(G) + P(U) + P(F) < 1$;

for paraconsistent probability, $1 < P(S) + P(C) + P(G) + P(U) + P(F) \leq 5$;

$$T_w = \frac{m!}{w!\,(m-w)!}\,P(S)^w \sum_{r=0}^{th} \frac{r!}{(m-w)!\,(r-m+w)!}\,(P(C)+P(G)+P(U))^r\ \ \mathrm{P(F)}^{m-w-r}$$

$$= \frac{m!}{w!\,(m-w)!}\,P(S)^w \sum_{r=0}^{th} \frac{(m-w)!}{(m-w+r)!}\,(P(C)+P(G)+P(U))^r\ \ \mathrm{P(F)}^{m-w-r}$$

$$= \frac{m!}{w!}\,\mathrm{P(S)}^w \sum_{r=0}^{th} \frac{(P(C)+P(G)+P(U))^r\ \ \mathrm{P(F)}^{m-w-r}}{r!\ (m-w+r)!}$$

$$F_w = \sum_{z=0}^{m} \mathrm{T}_z = \sum_{z=0,z\neq w}^{th} \frac{m!}{z!}\mathrm{P(S)}^z \sum_{r=0}^{th} \frac{\mathrm{P(S)}^r\ \ \mathrm{P(F)}^{\mathrm{m-z-r}}}{r!\ (m-z+r)!}$$

$$C_w = \sum_{u=th+1}^{m} \frac{m!}{m!\,(m-u)!}\mathrm{P(C)}^u \sum_{r=0}^{n-u} \frac{(m-u)!}{(n-u)!\ (n-u+r)!}\mathrm{P(S)}^r\ \ \mathrm{P(F)}^{\mathrm{m-u-r}}$$

$$= \sum_{u=th+1}^{m} \frac{m!}{u!}\ P(C)^u \sum_{r=0}^{m-u} \frac{\mathrm{P(S)}^r\ \ \mathrm{P(F)}^{\mathrm{m-u-r}}}{r!\ (m-u+r)!}$$

$$G_w = \sum_{v=th+1}^{m} \frac{m!}{v!\,(m-v)!}\mathrm{P(G)}^v \sum_{r=0}^{m-v} \frac{(m-v)!}{(m-v)!\ (m-v+r)!}\mathrm{P(S)}^r\ \ \mathrm{P(F)}^{\mathrm{m-v-r}}$$

$$= \sum_{v=th+1}^{m} \frac{m!}{v!}\ P(G)^v \sum_{r=0}^{m-v} \frac{\mathrm{P(S)}^r\ \ \mathrm{P(F)}^{\mathrm{m-v-r}}}{r!\ (m-v+r)!}$$

$$U_w = \sum_{t=th+1}^{m} \frac{m!}{t!\,(m-t)!}\mathrm{P(U)}^t \sum_{r=0}^{m-t} \frac{(m-t)!}{(m-t)!\ (m-t+r)!}\mathrm{P(S)}^r\ \ \mathrm{P(F)}^{\mathrm{m-t-r}}$$

$$= \sum_{t=th+1}^{m} \frac{m!}{t!}\ P(U)^t \sum_{r=0}^{m-t} \frac{\mathrm{P(S)}^k\ \ \mathrm{P(F)}^{\mathrm{m-t-r}}}{r!\ (m-t+r)!}$$

Where, T_w, $I_w, F_w, P(S), P(C), P(G), P(U), P(F)$ have their usual meaning.

Example 3.4.

In a certain hospital, there are (6) patients suffering a particular disease, monitoring cases showed that 70% of patients are die, and 20% of patients recover, due to medicine, inexperienced doctors, the contradiction of availability of oxygen occurs 8% percentage, ignorance occurs 5% percentage, and unknown reasons occurs 2%. What is the probability that from three random selection patients, two will recover, with C.G.U. Threshold 3.

Solution:

$$T_w = \frac{6!}{3!(6-3)!}(0.7)^3 \sum_{r=0}^{3} \frac{r!}{(6-3)!(r-3)!} (0.08+0.05+0.02)^r (0.2)^{6-3-r} =$$

$$\frac{6!}{3!\,3!} (0.7)^3 \sum_{r=0}^{3} \frac{r!}{(6-3)!(r-3)!} (0.15)^r (0.2)^{3-r} = 0.023153$$

$$C_w = \sum_{u=4}^{6} \frac{6!}{4!\,2!}(0.08)^u \sum_{r=0}^{2} \frac{2!}{2!\ (2-r)!}(0.7)^r (0.2)^{2-\mathrm{r}} = 0.000327$$

$$G_w = \sum_{v=4}^{6} \frac{6!}{4!\,2!}(0.05)^v \sum_{r=0}^{2} \frac{2!}{2!\ (2-r)!}(0.7)^r (0.2)^{2-\mathrm{r}} = 0.0000000657$$

$$U_w = \sum_{t=4}^{6} \frac{6!}{4!\,2!}(0.02)^t \sum_{r=0}^{2} \frac{2!}{2!\ (2-r)!}(0.7)^r \ (0.2)^{2-\mathrm{r}} = 0.0000000199$$

$$F_w = \sum_{z=0}^{m} \mathrm{T}_z = \sum_{z=0,z\neq w}^{th} \frac{n!}{z!} \mathrm{P(S)}^z \sum_{r=0}^{th} \frac{\mathrm{P(S)}^r\ \mathrm{P(F)}^{\mathrm{m-z-r}}}{r!\,(m-z+r)!} = \big(P(S)+P(C)+P(G)+P(U)+P(F)\big)^n - T_w -$$

$$C_w - G_w - U_w = (0.7+0.08+0.05+0.02+0.2)^6 - 0.023153 - 0.000327 - 0.0000000657 -$$

$$0.0000000199 = 1.317269726$$

Definition 3.5. Consider a random variable 'w' follows Poisson distribution with imprecise parameter λ_{PN} represented by an interval is said to be pentapartitioned neutrosophic Poisson distribution , if the probability mass function is given by:

$$\mathrm{NP(w)} = e^{-\lambda_{PN}}.\frac{(\lambda_{PN})^w}{w!} \ ;\ \mathrm{w} = 0,1,\ldots$$

The mean and the variance of this distribution are:

$NE(w) = NV(w) = \lambda_{PN}$.

Example 3.6.

The rate numbers of cars crossing over the bridge are $\lambda_{PN} = [4, 6]$ cars per minute. We want to calculate the probability that only one car crosses through a particular minute.

Solution: Assume z be the number of cars passing within minutes.

$$\mathrm{NP(w=1)} = e^{-\lambda_{PN}} \frac{(\lambda_{PN})^1}{1!} = e^{-\lambda_{PN}} . \lambda_{PN} = \lambda_{PN} . e^{-[4,6]}$$

When $\lambda_{PN} = 4$; NP(w = 1) $= e^{-\lambda_{PN}} \frac{(\lambda_{PN})^1}{1!} = \lambda_{PN}. e^{-4} = 0.0182\,.(4) = 0.0733$

When $\lambda_{PN} = 6$; NP(w=1) = $e^{-\lambda_{PN}} \frac{(\lambda_{PN})^1}{1!} = \lambda_{PN}. e^{-6} = 0.0025.(6) = 0.0148$

Therefore, the probability that only one car crossed in a minute be within ranges between $[0.0148, 0.0733]$.

Definition 3.7. Assume that 'w' be a continuous random variable , and it follows exponential distribution with imprecise distribution parameter θ_{PN} represented by an interval is said to be pentapartitioned neutrosophic exponential distribution, if the probability density function is given by:

$W_N \sim \exp(\theta_{PN}) = f_N(w) = \theta_{PN}. e^{-w.\theta_{PN}}; 0 < w < \infty$,

$exp(\theta_{PN})$: pentapartitioned neutrosophic exponential distribution, θ_{PN} : pentapartitioned neutrosophic distribution parameter.

3.8 Properties of Pentapartitioned Neutrosophic Exponential Distribution:

1. The values of the expectation and variance are: $E(w) = \frac{1}{\theta_{PN}}$, $Var(w) = \frac{1}{(\theta_{PN})^2}$;
2. The distribution function: $NF(w) = NP(W \le w) = (1 - e^{-w.\theta_{PN}})$

Example 3.9.

The time required to terminate a taxi service in a particular taxi online taxi booking app follows an exponential distribution, with an average of one minute, let us write a density function that represents the time required for terminating taxi service, and then calculate the probability of terminating taxi service in less than one minute.

Solution: Assume w the time required to terminating taxi service per minute:

The average $\frac{1}{\theta} = 1 \Rightarrow \theta = \mathbf{1}$

The probability density function: $f(w) = e^{-w}; 0 < w < \infty$

The possibility of taxi terminated in less than a minute:

P(W$\le$ 1) = (1- e^{-w}) = (1- $e^{-(1)}$) = 0.63

Practically, the above one is a simple example, if we change it in the neutrosophic form then we get the following context:

The time required to terminate to taxi service follows an exponential distribution, with an average of $[0.69, 3]$ minutes. We know that when data are accurate then classical exponential distribution is performed. Remember the average here is an interval value, now, try to solve this problem by a neutrosophic exponential distribution with an average $[0.69,3]$ minutes, we get:

$\frac{1}{\theta_N} = [0.69,3] \Rightarrow \theta_N = \frac{1}{[0.69,\ 3]} = [0.33, 1.45]$

The probability density function become:

$f_N(\mathrm{w}) = \theta_N .\ e^{-w.\theta_N}\ ; 0 < \mathrm{w} < \infty,$

$f_N(\mathrm{w}) = [0.33, 1.45].e^{-[0.33,\ 1.45]w};\ 0 < \mathrm{w} < \infty,$

Now, the probability to terminate the taxi service in less than one minute:

$NF(w) = NP(W \le w) = (1 - e^{-w.\theta_N})$

$NP(W \le 1) = \left(1 - e^{-[0.33,\ 1.45](1)}\right) = 1 - e^{-[0.33,\ 1.45]}$

Noticed that, for $\theta = 0.33$,

$NP(W \le 1) = 1 - e^{-0.33} = 0.281,$

For $\theta = 1.45$,

$NP(W \le 1) = 1 - e^{-1.45} = 0.765$

Therefore, the probability of terminating the taxi service in less than one minute is within the range $[0.281, 0.765]$.

Hence, the value of classical probability to terminate the taxi service in less than one minute is one of the domain values of neutrosophic probability.

$P(W \le 1) = 0.63 \in [\,0.281, 0.765] = NP(\mathrm{W} \le 1).$

Now, applying the pentapartitioned neutrosophic exponential distribution with contradiction $C \in [0, 0.1]$, ignorance $G \in [0, 0.06]$, unknown $U \in [0, 0.05]$, and $\frac{[0,0.1]+[0,0.06]+[0,0.05]}{3} = [0, 0.07]$; average become $[0.7, 3]$, so we get:

$\frac{1}{\theta_{PN}} = [0.7, 3] \Rightarrow \theta_{PN} = \frac{1}{[0.7,\ 3]} = [0.33, 1.43]$

The probability density function become:

$f_N(w) = \theta_N.e^{-w.\theta_N}; 0 < w < \infty,$

$f_N(\mathrm{w}) = [0.33, 1.43].\, e^{-[0.33,\ 1.43]\ w}\ ;\ 0 < \mathrm{w} < \infty,$

Now, the probability to terminate the taxi service in less than one minute is:

$NF(\mathrm{w}) = NP(\mathrm{W} \leq w) = (1 - e^{-w.\theta_N})$

$NP(\mathrm{W} \leq 1) = (1 - e^{-[0.33,\ 1.43](1)}) = 1 - e^{-[0.33,\ 1.43]}$

Noticed that, for $\theta = 0.33$,

$NP(\mathrm{W} \leq 1) = 1 - e^{-0.33} \cong 0.281$

For $\theta = 1.43$,

$NP(\mathrm{W} \leq 1) = 1 - e^{-1.43} \cong 0.761$

Therefore, the probability of terminating the taxi service in less than one minute is within the range $[\,0.281, 0.761\,]$.

The value of the classical probability to terminate the taxi service in less than one minute is one of the domain values of pentapartitioned neutrosophic probability, and it is quite closer to classical probability than neutrosophic probability.

Definition 3.10. Assume that w be a continuous random variable is follows classical normal distribution, with imprecise distribution parameters μ and σ , where they may contain some particular events such as contradiction, or ignorance, or unknown , all of these parameters represent intervals, this kind of distributions is said to be pentapartitioned neutrosophic normal distribution if the probability density function is given by:

$$\mathrm{W}_{\mathrm{PN}} \sim \mathrm{N}_{\mathrm{PN}}(\mu_{PN}, \sigma_{PN}) = \frac{1}{\sigma_{PN}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{w-\mu_{PN}}{\sigma_{PN}}\right)^2}$$

Where μ_{PN}, σ_{PN} both are set contain two or more elements.

N_{PN}: Pentapartitioned Neutrosophic Normal Distribution.

W_{PN}: Pentapartitioned Neutrosophic Continuous Random Variable.

Example 3.11.

1- In a shopping mall 55% shirt was not sell in Christmas, the average price of shirt is 55, and the standard deviation is 7 with contradiction $C \in [0,0.02]$, ignorance $G \in [0,0.03]$, unknown $U \in [0,0.05]$, the manager decide to give discount to show the owner that the percentage of sells will raise to 70% . Find the lowest discount amounts to show the percentage of sell is 70%. (Discount are normally distributed).

Solution: Given $\mu = 55, \sigma = 7$ with contradiction $C \in [0, 0.03]$, ignorance $G \in [0, 0.02]$, unknown $U \in [0, 0.04]$, so $\sigma = 7 + [[0, 0.02] + [0, 0.03] + [0, 0.05] = [7, 7.1]$. therefore, $\mu \pm \sigma = 55 \pm [7,7.1] = [55 - 7.1, 55 + 7] = [47.9, 62]$.

Now, $0.7 = P(W_{PN} \geq \alpha_{NP})$

$= 1 - \mathrm{P}(\mathrm{W_{PN}} \leq \alpha_{NP})$

$= 1\text{- P}\left(\frac{\mathrm{W_{NP}} - \mu_{\mathrm{NP}}}{\sigma_{NP}} \leq \frac{\alpha_{\mathrm{NP}} - \mu_{\mathrm{NP}}}{\sigma_{NP}} \right)$

$=1\text{- P}\left(\mathrm{Z_{NP}} \leq \frac{\alpha_{\mathrm{NP}} - 55}{[7,\ 7.1]} \right)$

Therefore, $\mathrm{P}\left(\mathrm{Z_{NP}} \leq \frac{\alpha_{\mathrm{NP}} - 55}{[7,\ 7.1]} \right) = 0.3$ clearly $\frac{\alpha_{\mathrm{NP}} - 55}{[7,\ 7.1]} < 0$; so, $\mathrm{P}(\mathrm{Z_{NP}} \leq \mathrm{Z_{0.3}}) = 0.7$

$\mathrm{Z_{0.3}} = \left(\frac{\alpha_{\mathrm{NP}} - 55}{[7,\ 7.1]} \right)$

2-The monthly electricity bill of a certain university is follows pentapartitioned neutrosophic normal distribution with mean 30,000 and standard deviation 5,000. find the following: $\mu \pm \sigma$, $\mu \pm 2\sigma$, where, $C \in [0, 0.08], G \in [0, 0.07], U \in [0, 0.05]$,

Solution: Given $\mu = 30{,}000$, $\sigma = 5{,}000 + [0, 0.08] + [0, 0.07] + [0, 0.05] = [5000, 5000.2]$

So, $\mu \pm \sigma = 55 \pm [7, 7.1] = [55-7.1, 55+7] = [47.9, 62]$.

$\mu \pm \sigma = 30000 \pm [5000, 5000.2] = [30000-5000.2, 30000+5000] = [24999.8, 35000]$.

$\mu \pm 2\sigma = 30000 \pm 2[5000, 5000.2] = [30000-10000.4, 30000+10000] = [19999.6, 40000]$.

4. Conclusion:

In this article, we introduce different types of pentapartitioned neutrosophic probability distributions as an extension to the neutrosophic probability distributions. Three original events parameters are ignorance, unknown and contradiction, have been presented in this article, in addition to the fully successes and fully failures. Depending upon this new vision, we got new five probability density/ mass functions are T_w, F_w, C_w, G_w, U_w, which led to more accurate in analyzing practical problems that have been explained by well explained examples. We hope that, based on the notion of pentapartitioned neutrosophic probability distributions so many new investigations can be carried out in future.

References:

1. Alhabib, R., Ranna, M.M., Farah, H., & Salama A.A. (2018). Some Neutrosophic Probability Distributions. *Neutrosophic Sets and Systems*, 22, 30-38.
2. Mallick, R., & Pramanik, S. (2020). Pentapartitioned neutrosophic set and its properties. *Neutrosophic Sets and Systems*, 36, 184-192.
3. Mukherjee, A., & Das, R. (2020). Neutrosophic Bipolar Vague Soft Set and Its Application to Decision Making Problems. *Neutrosophic Sets and Systems*, 32, 410-424.
4. Patro, S.K., & Smarandache, F. (2016). The neutrosophic statistical distribution, more problems, more solutions. *Neutrosophic Sets and Systems*, 12, 73-79.
5. Smarandache, F. (1998). A unifying field in logics, neutrosophy: neutrosophic probability, set and logic. Rehoboth, *American Research Press*.
6. Smarandache F. (2014) ''Introduction to Neutrosophic Statistics'' Sitech & Education Publishing.
7. Smarandache F. (2002). Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA.
8. Khalid H. E., Essa A. K. (2020) ''Introduction to Neutrosophic Statistics '' Translated Arabic Version, Pons Publishing House, Brussels, ISBN: 978-1-59973-906-9.
9. Wang, H., Smarandache, F., Zhang, Y. Q., & Sunderraman, R. (2010). Single valued neutrosophic sets. *Multispace and Multistructure*, 4, 410–413.
10. Alhabib, Rafif; Salama A. A.(2020), "The Neutrosophic Time Series-Study Its Models (Linear Logarithmic) and test the Coefficients Significance of Its linear model." Neutrosophic Sets and Systems 33. Pp 105-115.
11. Salama, A. A., Smarandache F (2015), Neutrosophic Crisp Set Theory, Educational. Education Publishing 1313 Chesapeake, Avenue, Columbus, Ohio 43212..

12. Alhabib R., Ranna M., Farah H., Salama A. A. (2017),"Foundation of Neutrosophic Crisp Probability Theory", Neutrosophic Operational Research, Volume III , Edited by Florentin Smarandache, Mohamed Abdel-Basset and Dr. Victor Chang (Editors), pp.49-60.
13. Salama, A. A., Al-Din, A. S., Alhabib, R., & Badran, M. (2020). Introduction to Decision Making for Neutrosophic Environment "Study on the Suez Canal Port, Egypt". Neutrosophic Sets and Systems, 35, 22-44.
14. Alhabib, R., & Salama, A. A. (2020). Using Moving Averages To Pave The Neutrosophic Time Series, International Journal of Neutrosophic Science (IJNS),3,1, pp14-20.
15. H. E. Khalid, F. Smarandache, A. K. Essa, (2018). The Basic Notions for (over, off, under) Neutrosophic Geometric Programming Problems. Neutrosophic Sets and Systems, 22, 50-62.
16. H. E. Khalid, (2020). Geometric Programming Dealt with a Neutrosophic Relational Equations Under the $(\boldsymbol{max-min})$ Operation. Neutrosophic Sets in Decision Analysis and Operations Research, chapter four. IGI Global Publishing House.
17. H. E. Khalid, "Neutrosophic Geometric Programming (NGP) with (max-product) Operator, An Innovative Model", Neutrosophic Sets and Systems, vol. 32, 2020.
18. Maissam Jdid, Rafif Alhabib, A. A. Salama, The static model of inventory management without a deficit with Neutrosophic logic, International Journal of Neutrosophic Science, Vol. 16 (1),2021, pp 42-48.
19. Maissam Jdid, A. A. Salama, Huda E. Khalid, Neutrosophic Handling of the Simplex Direct Algorithm to Define the Optimal Solution in Linear Programming, International Journal of Neutrosophic Science, Vol. 18 (1),2022, pp 30-41.
20. Maissam Jdid, A. A. Salama, Rafif Alhabib, Huda E. Khalid, Fatima Al Suleiman, Neutrosophic Treatment of the Static Model of Inventory Management with Deficit, International Journal of Neutrosophic Science, Vol. 18 (1),2022, pp 20-29.
21. Maissam Jdid , Rafif Alhabib, Ossama Bahbouh, A. A. Salama, Huda E. Khalid, The Neutrosophic Treatment for Multiple Storage Problem of Finite Materials and Volumes, International Journal of Neutrosophic Science, Vol. 18 (1),2022, pp 42-56.
22. F. Smarandache, H. E. Khalid, A. K. Essa, M. Ali, "The Concept of Neutrosophic Less Than or Equal To: A New Insight in Unconstrained Geometric Programming", Critical Review, Volume XII, 2016, pp. 72-80.
23. H. E. Khalid, "An Original Notion to Find Maximal Solution in the Fuzzy Neutrosophic Relation Equations (FNRE) with Geometric Programming (GP)", Neutrosophic Sets and Systems, vol. 7, 2015, pp. 3-7.

24. H. E. Khalid, "The Novel Attempt for Finding Minimum Solution in Fuzzy Neutrosophic Relational Geometric Programming (FNRGP) with (max, min) Composition", Neutrosophic Sets and Systems, vol. 11, 2016, pp. 107-111.
25. H. E. Khalid, F. Smarandache, A. K. Essa, (2016). A Neutrosophic Binomial Factorial Theorem with their Refrains. Neutrosophic Sets and Systems, 14, 50-62.
26. H. E. Khalid, A. K. Essa, (2021). The Duality Approach of the Neutrosophic Linear Programming. Neutrosophic Sets and Systems, 46, 9-23.
27. Maissam Jdid, Rafif Alhabib, A. A. Salama, Fundamentals of Neutrosophic Simulation for Generating Random Numbers Associated with Uniform Probability Distribution, Neutrosophic Sets and Systems, acceptable to publish in the upcoming Vol.49.

Received: Dec. 4, 2021. Accepted: April 2, 2022.