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Abstract. In this article the notion of two quadripartitioned single valued weighted dombi prioritized operators, namely, quadripartitioned single valued weighted dombi prioritized average (QSVNWDPA) operator and quadripartitioned single valued weighted dombi prioritized geometric (QSVNWDPG) operator have been developed which are based on quadripartitioned single valued neutrosophic (QSVN) sets. Further some important properties of these two operators are studied. Finally a multi-attribute decision making (MADM) problem has been solved using QSVNWDPA operator and QSVNWDPG operator.

Keywords: Quadripartitioned single valued neutrosophic set; Aggregation operator; Dombi operator; Prioritized operator; QSVN weighted Dombi prioritized average operator; QSVN weighted Dombi prioritized geometric operator; Multi-attributive decision making.

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1. **Introduction**

Smarandache [3, 13, 20] introduced Neutrosophic set (NS) theory in which each element of this set is assigned with a truth value (T), a indeterminacy value (I) and a falsity value (F) which are independent of each other. Later many authors have introduced several types of generalizations of NS along with their various types of applications [4–12, 14, 15]. An extension of Neutrosophic set, called QSVN set, was further developed in [16] which was motivated by Belnap's four valued logic [1]. Here every element in a set have four values associated with it namely truth value T, a contradiction value C, an ignorance value U and a falsity value F. Thus QSVN sets are equipped with better tool for solving various types of decision making problems in comparison with other types of neutrosophic sets. The idea of quadripartitioned neutrosophic numbers ($\mathcal{QNN}$ number) which are based on QSVN sets are introduced and studied along with some well known properties of $\mathcal{QNN}$-numbers in [19]. Currently the application of information aggregation operators in the area of multi-attribute decision making process has become a popular topic of research. Aggression operator based decision making methods are preferred than ordinary decision making methods because these operators readily combines data into one single entity from which one could easily make decisions. Several researchers have proposed new aggregation operators or have extended known operators to new settings. On contrary Dombi [2] presented the operations of Dombi T -norms (D_T) and T-conorms ($\widehat{D_T}$) for fuzzy sets way back in 1982. Both the norms have wide applications as an operator as they have good advantage of flexibility to tackle the operational parameters. Also Dombi aggregation operators make the optimal outcomes more accurate and definite when used properly in any MADM problem. Many researchers extended the idea of Dombi norms together with Prioritized operator to IFS [18], NS [17, 21] theories and applied to different MADM problems. In this paper we have applied weighted Dombi Prioritized norms on $\mathcal{QNN}$ and applied them to solve a very relevant MADM problem. The rest of this paper is constructed as follows: In Section 2 we have discussed some basic theories which will be used throughout the rest of the article. We have defined some order relations on $\mathcal{QNN}$ in Section 3. In the next section some Dombi operations on $\mathcal{QNN}$ are defined. Section 5 introduces the QSVNWDPA and QSVNWDPG operators and studied their properties. Next a MADM problem is solved using QSVNWDA and QSVNWDG operators in section 6 along with sensitivity analysis of these two methods. Then Section 7 concludes the article.

2. **Some Basics**

For better understanding of this article we need some terminologies from literature of NS sets.

Definition 2.1. [3] A neutrosophic set (NS) A in $Y \neq \phi$ is characterized by a truth-membership function A_t, an indeterminacy membership function A_i and a falsity-membership function A_f. Here for each $y \in Y$, $A_t(y), A_i(y)$ and $A_f(y)$ are real non-standard elements of $]0^-, 1^+[$. A can be written as:

$$A = \{(y, A_t(y), A_i(y), A_f(y)) : y \in Y, A_t(y), A_i(y), A_f(y) \in]0^-, 1^+[\}.$$

Definition 2.2. [16] A QSVN set M over a set $Y \neq \phi$ distinguishes each element y in Y by a truth-value M_t, a contradiction value M_c, an ignorance-value M_u and a falsity value M_f s.t. for each $y \in Y$, $M_t(y), M_c(y), M_u(y), M_f(y) \in [0, 1]$, $0 \leq M_t(y) + M_c(y) + M_u(y) + M_f(y) \leq 4$.

Based on QSVN set Prof. R. Chatterjee et. al. introduced the QSVN numbers together with some operations in their paper [19] in 2019.

Definition 2.3. [19] An QSVN element $\omega = \langle \omega_t, \omega_c, \omega_u, \omega_f \rangle \in [0,1]^4$ is said to be a QSVN number. We represent the set of QSVN numbers as $\mathcal{QNN}$.

Definition 2.4. [19] Consider $\varsigma, \tau, \upsilon \in \mathcal{QNN}$ and $k \in \mathbb{N}$. Then the following basic operations hold on $\mathcal{QNN}$:

(i) $\varsigma \bigoplus \tau = \langle \varsigma_t + \tau_t - \varsigma_t\tau_t, \varsigma_c + \tau_c - \varsigma_c\tau_c, \varsigma_u\tau_u, \varsigma_f\tau_f \rangle$,
(ii) $\varsigma \bigodot \tau = \langle \varsigma_t\tau_t, \varsigma_c\tau_c, \varsigma_u + \tau_u - \varsigma_u\tau_u, \varsigma_f + \tau_f - \varsigma_f\tau_f \rangle$,
(iii) $(\varsigma)^k = \langle (\varsigma_t)^k, (\varsigma_c)^k, 1-(1-\varsigma_u)^k, 1-(1-\varsigma_f)^k \rangle$,
(iv) $k\varsigma = \langle 1-(1-\varsigma_t)^k, 1-(1-\varsigma_c)^k, (\varsigma_u)^k, (\varsigma_f)^k \rangle$,
(v) $\varsigma \bigoplus \tau = \tau \bigoplus \varsigma$,
(vi) $(\varsigma \bigoplus \tau) \bigoplus \upsilon = \tau \bigoplus (\varsigma \bigoplus \upsilon)$,
(vii) $\varsigma \bigodot \tau = \tau \bigodot \varsigma$,
(viii) $(\varsigma \bigodot \tau) \bigodot \upsilon = \tau \bigodot (\varsigma \bigodot \upsilon)$,

2.1. *Dombi T-norm and T-conorm*

Dombi Operator was introduced by J. Dombi in 1982 in [2]. In 2008 Prof Yager firstly introduced the Prioritized aggregation operators in [4]. For convenience of the readers of this article we request you to follow the articles [2] and [4] respectively.

Definition 2.5. [2] Suppose $r, s \in \mathbb{R}$. The D_T and $\widehat{D_T}$ between r and s are defined respectively as below:

$$D_T(r,s) = \frac{1}{1 + \{(\frac{1-r}{r})^\lambda + (\frac{1-s}{s})^\lambda\}^{\frac{1}{\lambda}}}$$
$$\widehat{D_T}(r,s) = \frac{1}{1 + \{(\frac{r}{1-r})^\lambda + (\frac{s}{1-s})^\lambda\}^{\frac{1}{\lambda}}},$$

$\lambda \geq 1$ and $(r,s) \in [0,1] \times [0,1]$.

3. **Order properties in $\mathcal{QNN}$**

Now we will discuss some order relations of $\mathcal{QNN}$.

Definition 3.1. The score function of $\omega = \langle \omega_t, \omega_c, \omega_u, \omega_f \rangle : \mathcal{QNN} \to [0,1]$ is defined as

$$S(\omega) = \frac{3 + \omega_t + \omega_c - \omega_u - \omega_f}{4}$$

We now define a few accuracy functions $A_i : \mathcal{QNN} \to [0,1], i = \infty, \in, \ni$ of $\omega = \langle \omega_t, \omega_c, \omega_u, \omega_f \rangle \in \mathcal{QNN}$ as follows:

$$\begin{aligned}\mathcal{A}_{\infty}(\omega) &= \frac{(\omega_t+\omega_c)-(\omega_u+\omega_f)}{2}\\ \mathcal{A}_{\in}(\omega) &= \frac{\omega_t-\omega_c}{4}\\ \mathcal{A}_{\ni}(\omega) &= \frac{\omega_u-\omega_f}{4}.\end{aligned}$$

Remark 3.2. From Definition 3.1, the following properties of score function and accuracy functions of a QSVN number $\omega \in \mathcal{QNN}$ can be obtained:

(i) $0 \le S(\omega) \le 1.25$.

(ii) $-1 \le \mathcal{A}_{\infty}(\omega) \le 1$.

(iii) $-0.25 \le \mathcal{A}_{\in}(\omega) \le 0.25$.

(iv) $-0.25 \le \mathcal{A}_{\ni}(\omega) \le 0.25$.

Definition 3.3. Suppose $\mu, \nu \in \mathcal{QNN}$. We define the order relation between any two $\mu, \nu \in \mathcal{QNN}$ as following:

(i) If $S(\mu) < S(\nu)$, then $\mu \le \nu$.

(ii) If $S(\mu) = S(\nu)$, then

(a) $\mathcal{A}_{\infty}(\mu) < \mathcal{A}_{\infty}(\nu) \Rightarrow \mu \le \nu$ else if

(b) $\mathcal{A}_{\infty}(\mu) = \mathcal{A}_{\infty}(\nu)$ with $\mathcal{A}_{\in}(\mu) < \mathcal{A}_{\in}(\nu) \Rightarrow \mu \le \nu$ else if

(c) $\mathcal{A}_{\infty}(\mu) = \mathcal{A}_{\infty}(\nu), \mathcal{A}_{\in}(\mu) = \mathcal{A}_{\in}(\nu)$ with $\mathcal{A}_{\ni}(\mu) < \mathcal{A}_{\ni}(\nu) \Rightarrow \mu \le \nu$ else if

(d) $\mathcal{A}_{\infty}(\mu) = \mathcal{A}_{\infty}(\nu), \mathcal{A}_{\in}(\mu) = \mathcal{A}_{\in}(\nu)$ and $\mathcal{A}_{\ni}(\mu) = \mathcal{A}_{\ni}(\nu) \Rightarrow \mu = \nu$.

Here $\mu \le \nu$ denotes μ proceeds ν.

4. Some QSVN Dombi operations

Definition 4.1. Let $\mu = \langle m_1, n_1, p_1, q_1 \rangle \in \mathcal{QNN}$ and $\nu = \langle m_2, n_2, p_2, q_1 \rangle \in \mathcal{QNN}$, $\lambda \ge 1$ and $k > 0$. Then the D_T and $\widehat{D_T}$ operations on $\mathcal{QNN}$ are defined as below:

(i)

$$\mu \oplus \nu = \left\langle 1-\frac{1}{1+\left((\frac{m_1}{1-m_1})^{\lambda}+(\frac{m_2}{1-m_2})^{\lambda}\right)^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left((\frac{n_1}{1-n_1})^{\lambda}+(\frac{n_2}{1-n_2})^{\lambda}\right)^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left((\frac{1-p_1}{p_1})^{\lambda}+(\frac{1-p_2}{p_2})^{\lambda}\right)^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left((\frac{1-q_1}{q_1})^{\lambda}+(\frac{1-q_2}{q_2})^{\lambda}\right)^{\frac{1}{\lambda}}} \right\rangle$$

(ii)

$$\mu \odot \nu = \left\langle 1-\frac{1}{1+\left((\frac{1-m_1}{m_1})^{\lambda}+(\frac{1-m_2}{m_2})^{\lambda}\right)^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left((\frac{1-n_1}{n_1})^{\lambda}+(\frac{1-n_2}{n_2})^{\lambda}\right)^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left((\frac{p_1}{1-p_1})^{\lambda}+(\frac{p_2}{1-p_2})^{\lambda}\right)^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left((\frac{q_1}{1-q_1})^{\lambda}+(\frac{q_2}{1-q_2})^{\lambda}\right)^{\frac{1}{\lambda}}} \right\rangle$$

(iii) $$k\mu = \left\langle 1-\frac{1}{1+\left(k(\frac{m_1}{1-m_1})^{\lambda}\right)^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left(k(\frac{n_1}{1-n_1})^{\lambda}\right)^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left(k(\frac{1-p_1}{p_1})^{\lambda}\right)^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left(k(\frac{1-q_1}{q_1})^{\lambda}\right)^{\frac{1}{\lambda}}} \right\rangle,$$

(iv) $$\mu^k = \left\langle 1-\frac{1}{1+\left(k(\frac{1-m_1}{m_1})^{\lambda}\right)^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left(k(\frac{1-n_1}{n_1})^{\lambda}\right)^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left(k(\frac{p_1}{1-p_1})^{\lambda}\right)^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left(k(\frac{q_1}{1-q_1})^{\lambda}\right)^{\frac{1}{\lambda}}} \right\rangle.$$

5. Dombi prioritized average operators on $\mathcal{QNN}$

Definition 5.1. Let $\gamma_j = \langle m_j, n_j, p_j, q_j\rangle$ $(j = 1, 2, \ldots, l)$ be a collection on $\mathcal{QNN}$. A QSVN Dombi prioritized average (QSVNDPA) operator of dimension l is a function $s_1 : \mathcal{QNN}^l \to \mathcal{QNN}$ defined by:

$$s_1(\gamma_1, \gamma_2, \ldots, \gamma_l) = \bigoplus_{j=1}^{l}\left(\frac{T_j\gamma_j}{\sum\limits_{j=1}^{l} T_j}\right)$$

where $T_j = \prod\limits_{k=1}^{j-1} S(\gamma_k)\ \forall\, k,\ T_1 = 1$ and $S(\gamma_j) = \frac{3+m_j+n_j-p_j-q_j}{4}$

Theorem 5.2. *Suppose $\gamma_j = \langle m_j, n_j, p_j, q_j\rangle$ $\forall j \in \mathbb{N}$ be a collection on $\mathcal{QNN}$. Then*

$$s_1(\gamma_1, \gamma_2, \ldots, \gamma_l) = \bigoplus_{j=1}^{l}\left(\frac{T_j\gamma_j}{\sum\limits_{j=1}^{l} T_j}\right)$$

$$=\left\langle 1-\frac{1}{1+\left\{\sum\limits_{j=1}^{l}\frac{T_j\left(\frac{m_j}{1-m_j}\right)^{\lambda}}{\sum\limits_{j=1}^{l}T_j}\right\}^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left\{\sum\limits_{j=1}^{l}\frac{T_j\left(\frac{n_j}{1-n_j}\right)^{\lambda}}{\sum\limits_{j=1}^{l}T_j}\right\}^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left\{\sum\limits_{j=1}^{l}\frac{T_j\left(\frac{1-p_j}{p_j}\right)^{\lambda}}{\sum\limits_{j=1}^{l}T_j}\right\}^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left\{\sum\limits_{j=1}^{l}\frac{T_j\left(\frac{1-q_j}{q_j}\right)^{\lambda}}{\sum\limits_{j=1}^{l}T_j}\right\}^{\frac{1}{\lambda}}}\right\rangle.$$

Proof. Here $\gamma_1 \in \mathcal{QNN}$. Now we have $\frac{T_1\gamma_1}{T_1} = \gamma_1$ $\left\langle 1-\frac{1}{1+\left\{\left(\frac{m_1}{1-m_1}\right)^{\lambda}\right\}^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left\{\left(\frac{n_1}{1-n_1}\right)^{\lambda}\right\}^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left\{\left(\frac{1-p_1}{p_1}\right)^{\lambda}\right\}^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left\{\left(\frac{1-q_1}{q_1}\right)^{\lambda}\right\}^{\frac{1}{\lambda}}}\right\rangle$. Hence the above equation trivially holds for $l = 1$. In a parallel way for $\gamma_2 \in \mathcal{QNN}$, we have $\frac{T_2\gamma_2}{T_1+T_2} =$
$$\left\langle 1-\frac{1}{1+\left\{\frac{T_2\left(\frac{m_2}{1-m_2}\right)^{\lambda}}{T_1+T_2}\right\}^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left\{\frac{T_2\left(\frac{n_2}{1-n_2}\right)^{\lambda}}{T_1+T_2}\right\}^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left\{\frac{T_2\left(\frac{1-p_2}{p_2}\right)^{\lambda}}{T_1+T_2}\right\}^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left\{\frac{T_2\left(\frac{1-q_2}{q_2}\right)^{\lambda}}{T_1+T_2}\right\}^{\frac{1}{\lambda}}}\right\rangle.$$
Therefore

$$s_1(\gamma_1, \gamma_2) = \bigoplus_{j=1}^{2}\left(\frac{T_j\gamma_j}{\sum\limits_{j=1}^{2} T_j}\right)$$

$$=\left\langle 1-\frac{1}{1+\left\{\sum\limits_{j=1}^{2}\frac{T_j\left(\frac{m_j}{1-m_j}\right)^{\lambda}}{\sum\limits_{j=1}^{2}T_j}\right\}^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left\{\sum\limits_{j=1}^{2}\frac{T_j\left(\frac{n_j}{1-n_j}\right)^{\lambda}}{\sum\limits_{j=1}^{2}T_j}\right\}^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left\{\sum\limits_{j=1}^{2}\frac{T_j\left(\frac{1-p_j}{p_j}\right)^{\lambda}}{\sum\limits_{j=1}^{2}T_j}\right\}^{\frac{1}{\lambda}}}, 1-\frac{1}{1+\left\{\sum\limits_{j=1}^{2}\frac{T_j\left(\frac{1-q_j}{q_j}\right)^{\lambda}}{\sum\limits_{j=1}^{2}T_j}\right\}^{\frac{1}{\lambda}}}\right\rangle.$$

Hence the equation is valid for $l = 1, 2$. We assume that the equation is valid for $l = s$ i.e.

$$s_1(\gamma_1, \gamma_2, \ldots, \gamma_s) = \bigoplus_{j=1}^{s} \left(\frac{T_j \gamma_j}{\sum_{j=1}^{s} T_j} \right)$$

$$= \left\langle 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{s} \frac{T_j \left(\frac{m_j}{1-m_j} \right)^\lambda}{\sum_{j=1}^{s} T_j} \right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{s} \frac{T_j \left(\frac{n_j}{1-n_j} \right)^\lambda}{\sum_{j=1}^{s} T_j} \right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{s} \frac{T_j \left(\frac{1-p_j}{p_j} \right)^\lambda}{\sum_{j=1}^{s} T_j} \right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{s} \frac{T_j \left(\frac{1-q_j}{q_j} \right)^\lambda}{\sum_{j=1}^{s} T_j} \right\}^{\frac{1}{\lambda}}} \right\rangle.$$

Finally for $l = s + 1$, one can easily see that

$$s_1(\gamma_1, \gamma_2, \ldots, \gamma_s) = \bigoplus_{j=1}^{s} \left(\frac{T_j \gamma_j}{\sum_{j=1}^{s} T_j} \right) \bigoplus \left(\frac{T_{s+1} \gamma_{s+1}}{\sum_{j=1}^{s+1} T_j} \right)$$

$$= \left\langle 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{s} \frac{T_j \left(\frac{m_j}{1-m_j} \right)^\lambda}{\sum_{j=1}^{s} T_j} \right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{s} \frac{T_j \left(\frac{n_j}{1-n_j} \right)^\lambda}{\sum_{j=1}^{s} T_j} \right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{s} \frac{T_j \left(\frac{1-p_j}{p_j} \right)^\lambda}{\sum_{j=1}^{s} T_j} \right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{s} \frac{T_j \left(\frac{1-q_j}{q_j} \right)^\lambda}{\sum_{j=1}^{s} T_j} \right\}^{\frac{1}{\lambda}}} \right\rangle \oplus \left(\frac{T_{s+1} \gamma_{s+1}}{\sum_{j=1}^{s+1} T_j} \right)$$

$$= \left\langle 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{s+1} \frac{T_j \left(\frac{m_j}{1-m_j} \right)^\lambda}{\sum_{j=1}^{s+1} T_j} \right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{s+1} \frac{T_j \left(\frac{n_j}{1-n_j} \right)^\lambda}{\sum_{j=1}^{s+1} T_j} \right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{s+1} \frac{T_j \left(\frac{1-p_j}{p_j} \right)^\lambda}{\sum_{j=1}^{s+1} T_j} \right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{s+1} \frac{T_j \left(\frac{1-q_j}{q_j} \right)^\lambda}{\sum_{j=1}^{s+1} T_j} \right\}^{\frac{1}{\lambda}}} \right\rangle.$$

Finally the equation

$$s_1(\gamma_1, \gamma_2, \ldots, \gamma_s) = \bigoplus_{j=1}^{s} \left(\frac{T_j \gamma_j}{\sum_{j=1}^{s} T_j} \right)$$

$$= \left\langle 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{s} \frac{T_j \left(\frac{m_j}{1-m_j} \right)^\lambda}{\sum_{j=1}^{s} T_j} \right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{s} \frac{T_j \left(\frac{n_j}{1-n_j} \right)^\lambda}{\sum_{j=1}^{s} T_j} \right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{s} \frac{T_j \left(\frac{1-p_j}{p_j} \right)^\lambda}{\sum_{j=1}^{s} T_j} \right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{s} \frac{T_j \left(\frac{1-q_j}{q_j} \right)^\lambda}{\sum_{j=1}^{s} T_j} \right\}^{\frac{1}{\lambda}}} \right\rangle.$$

holds for all $s \in \mathbb{N}$. □

Theorem 5.3. *The QSVNDPA operator s_1 satisfies the following properties:*

(i) *Consistency:* $s_1(\gamma_1, \gamma_2, \ldots, \gamma_l) \in \mathcal{QNN}$.

(ii) *Idempotency:* $s_1(\gamma, l \text{ times } \ldots, \gamma) = \gamma$.

(iii) *Commutativity:* $s_1(\gamma_1, \gamma_2, \ldots, \gamma_l) = s_1(\gamma_l, \gamma_{l-1}, \ldots, \gamma_1)$.

(iv) $s_1(\gamma_{\pi(1)}, \gamma_{\pi(2)}, \ldots, \gamma_{\pi(l)}) = s_1(\gamma_1, \gamma_2, \ldots, \gamma_l)$ *where π is a permutation on* $\{1, 2, \ldots, l\}$.

Proof. The basic two properties of QSVNDPA operator i.e. consistency and commutativity properties are quite easy. We will prove the property (ii) and (iv) respectively. If $\gamma_j = \gamma \; \forall \; j$

then

$$s_1(\gamma, l \text{ times} \ldots, \gamma) = \bigoplus_{j=1}^{s}\left(\frac{T_j\gamma_j}{\sum\limits_{j=1}^{s} T_j}\right) = \bigoplus_{j=1}^{s}\left(\frac{T_j}{\sum\limits_{j=1}^{s} T_j}\right)\gamma \quad = \gamma.$$

Finally consider π as a permutation on $\{1, 2, \ldots, l\}$. Now due to additive commutativity in $\mathcal{QNN}$

$$s_1(\gamma_{\pi(1)}, \gamma_{\pi(2)}, \ldots, \gamma_{\pi(s)}) = \bigoplus_{j=1}^{s}\left(\frac{T_{\pi(j)}\gamma_{\pi(j)}}{\sum\limits_{j=1}^{s} T_{\pi(j)}}\right) = \bigoplus_{j=1}^{s}\left(\frac{T_j\gamma_j}{\sum\limits_{j=1}^{s} T_j}\right).$$

Hence we are done. □

Theorem 5.4. *Consider* $\gamma_j = \langle m_j, n_j, p_j, q_j\rangle$ $(j = 1, 2, \ldots, l)$ *and* $\delta_j = \langle \widetilde{m_j}, \widetilde{n_j}, \widetilde{p_j}, \widetilde{q_j}\rangle$ $(j = 1, 2, \ldots, l)$ *are two collections on* $\mathcal{QNN}$ *such that* $m_j \leq \widetilde{m_j}, n_j \leq \widetilde{n_j}, p_j \geq \widetilde{p_j}, q_j \geq \widetilde{q_j} \forall j$. *Then* $s_1(\gamma_1, \gamma_2, \ldots, \gamma_l) \leq s_1(\delta_1, \delta_2, \ldots, \delta_l)$.

Proof. Here,

$$s_1(\gamma_1, \gamma_2, \ldots, \gamma_l) = \bigoplus_{j=1}^{l}\left(\frac{T_j\gamma_j}{\sum\limits_{j=1}^{l} T_j}\right)$$

$$= \left\langle 1 - \frac{1}{1 + \left\{\sum\limits_{j=1}^{l}\frac{T_j\left(\frac{m_j}{1-m_j}\right)^{\lambda}}{\sum\limits_{j=1}^{l} T_j}\right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{\sum\limits_{j=1}^{l}\frac{T_j\left(\frac{n_j}{1-n_j}\right)^{\lambda}}{\sum\limits_{j=1}^{l} T_j}\right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{\sum\limits_{j=1}^{l}\frac{T_j\left(\frac{1-p_j}{p_j}\right)^{\lambda}}{\sum\limits_{j=1}^{l} T_j}\right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{\sum\limits_{j=1}^{l}\frac{T_j\left(\frac{1-q_j}{q_j}\right)^{\lambda}}{\sum\limits_{j=1}^{l} T_j}\right\}^{\frac{1}{\lambda}}}\right\rangle.$$

$$s_1(\delta_1, \delta_2, \ldots, \delta_l) = \bigoplus_{j=1}^{l}\left(\frac{T_j\delta_j}{\sum\limits_{j=1}^{l} T_j}\right)$$

$$= \left\langle 1 - \frac{1}{1 + \left\{\sum\limits_{j=1}^{l}\frac{T_j\left(\frac{\widetilde{m_j}}{1-\widetilde{m_j}}\right)^{\lambda}}{\sum\limits_{j=1}^{l} T_j}\right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{\sum\limits_{j=1}^{l}\frac{T_j\left(\frac{\widetilde{n_j}}{1-\widetilde{n_j}}\right)^{\lambda}}{\sum\limits_{j=1}^{l} T_j}\right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{\sum\limits_{j=1}^{l}\frac{T_j\left(\frac{1-\widetilde{p_j}}{\widetilde{p_j}}\right)^{\lambda}}{\sum\limits_{j=1}^{l} T_j}\right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{\sum\limits_{j=1}^{l}\frac{T_j\left(\frac{1-\widetilde{q_j}}{\widetilde{q_j}}\right)^{\lambda}}{\sum\limits_{j=1}^{l} T_j}\right\}^{\frac{1}{\lambda}}}\right\rangle.$$

Firstly we consider that $m_j < \widetilde{m_j}, n_j < \widetilde{n_j}, p_j > \widetilde{p_j}, q_j > \widetilde{q_j} \forall j \in \{1, \ldots, l\}$. Then

$$1 - m_j > 1 - \widetilde{m_j} \quad \forall j \in \{1, \ldots, l\}$$
$$\Rightarrow \left(\frac{1-m_j}{m_j}\right) > \left(\frac{1-\widetilde{m_j}}{\widetilde{m_j}}\right)$$
$$\Rightarrow T_j\left(\frac{m_j}{1-m_j}\right)^{\lambda} < T_j\left(\frac{\widetilde{m_j}}{1-\widetilde{m_j}}\right)^{\lambda}$$

$$\Rightarrow \frac{1}{1+\left\{\sum_{j=1}^{l}\frac{T_j\left(\frac{m_j}{1-m_j}\right)^{\lambda}}{\sum_{j=1}^{l}T_j}\right\}^{\frac{1}{\lambda}}} > \frac{1}{1+\left\{\sum_{j=1}^{l}\frac{T_j\left(\frac{\widetilde{m_j}}{1-\widetilde{m_j}}\right)^{\lambda}}{\sum_{j=1}^{l}T_j}\right\}^{\frac{1}{\lambda}}}$$

$$\Rightarrow 1-\frac{1}{1+\left\{\sum_{j=1}^{l}\frac{T_j\left(\frac{m_j}{1-m_j}\right)^{\lambda}}{\sum_{j=1}^{l}T_j}\right\}^{\frac{1}{\lambda}}} < 1-\frac{1}{1+\left\{\sum_{j=1}^{l}\frac{T_j\left(\frac{\widetilde{m_j}}{1-\widetilde{m_j}}\right)^{\lambda}}{\sum_{j=1}^{l}T_j}\right\}^{\frac{1}{\lambda}}}$$

In a same way we can observe that

$$\Rightarrow \frac{1}{1+\left\{\sum_{j=1}^{l}\frac{T_j\left(\frac{n_j}{1-n_j}\right)^{\lambda}}{\sum_{j=1}^{l}T_j}\right\}^{\frac{1}{\lambda}}} > \frac{1}{1+\left\{\sum_{j=1}^{l}\frac{T_j\left(\frac{\widetilde{n_j}}{1-\widetilde{n_j}}\right)^{\lambda}}{\sum_{j=1}^{l}T_j}\right\}^{\frac{1}{\lambda}}}$$

$$\Rightarrow \frac{1}{1+\left\{\sum_{j=1}^{l}\frac{T_j\left(\frac{1-p_j}{p_j}\right)^{\lambda}}{\sum_{j=1}^{l}T_j}\right\}^{\frac{1}{\lambda}}} > \frac{1}{1+\left\{\sum_{j=1}^{l}\frac{T_j\left(\frac{1-\widetilde{p_j}}{\widetilde{p_j}}\right)^{\lambda}}{\sum_{j=1}^{l}T_j}\right\}^{\frac{1}{\lambda}}}$$

$$\Rightarrow \frac{1}{1+\left\{\sum_{j=1}^{l}\frac{T_j\left(\frac{1-q_j}{q_j}\right)^{\lambda}}{\sum_{j=1}^{l}T_j}\right\}^{\frac{1}{\lambda}}} > \frac{1}{1+\left\{\sum_{j=1}^{l}\frac{T_j\left(\frac{1-\widetilde{q_j}}{\widetilde{q_j}}\right)^{\lambda}}{\sum_{j=1}^{l}T_j}\right\}^{\frac{1}{\lambda}}}$$

Combining all the above we get $s_1(\gamma_1, \gamma_2, \ldots, \gamma_l) < s_1(\delta_1, \delta_2, \ldots, \delta_l)$. Now if $m_j = \widetilde{m_j}, n_j = \widetilde{n_j}, p_j = \widetilde{p_j}, q_j = \widetilde{q_j}\ \forall j \in \{1, \ldots, l\}$, then all the equalities as well as the score functions become equal. Finally $s_1(\gamma_1, \gamma_2, \ldots, \gamma_l) \le s_1(\delta_1, \delta_2, \ldots, \delta_l)$. □

Theorem 5.5. *Consider a collection of* $\gamma_j = \langle m_j, n_j, p_j, q_j\rangle, j \in \mathbb{N}$ *in* $\mathcal{QNN}$. *Then*

$$\underline{\gamma} \le s_1(\gamma_1, \gamma_2, \ldots, \gamma_l) \le \overline{\gamma}, \textit{where}$$

$$\underline{\gamma} = \langle \min_j(m_j), \min_j(n_j), \min_j(p_j), \min_j(q_j)\rangle = \langle \underline{m_j}, \underline{n_j}, \underline{p_j}, \underline{q_j}\rangle \textit{ and}$$

$$\overline{\gamma} = \langle \max_j(m_j), \max_j(n_j), \max_j(p_j), \max_j(q_j)\rangle = \langle \overline{m_j}, \overline{n_j}, \overline{p_j}, \overline{q_j}\rangle.$$

Proof. From Definition of $\mathcal{QNN}$ we have $\forall j = \{1, 2, \ldots, l\}$,

$$\underline{m_j} \le m_j \le \overline{m_j}, \underline{n_j} \le n_j \le \overline{n_j} \text{ and}$$

$$\underline{p_j} \ge p_j \ge \overline{p_j}, \underline{q_j} \ge q_j \ge \overline{q_j}.$$

Then

$$s_(\underline{\gamma}, l\ times, \underline{\gamma}) \le s_(\gamma_1, \gamma_2, \ldots, \gamma_l) \le s_(\overline{\gamma}, l\ times, \overline{\gamma}), i.e$$

$$\underline{\gamma} \le s_(\gamma_1, \gamma_2, \ldots, \gamma_l) \le \overline{\gamma}.$$

□

Definition 5.6. Consider the mass associated with γ_j as $M_j > 0\ \forall\, j = 1, \ldots, l$, where $M = (M_1, M_2, \ldots, M_l)^T$ is the mass vector such that $\sum_{j=1}^{l} M_j = 1$. Then the QSVNWDPA (QSVN weighted DPA) can be defined as follows:

$$s_M(\gamma_1, \gamma_2, \ldots, \gamma_l) = \bigoplus_{j=1}^{l} \left(\frac{M_j T_j}{\sum_{j=1}^{l} M_j T_j} \gamma_j \right)$$

where $T_j = \prod_{k=1}^{j-1} S(\gamma_k)$ $(k = 1, 2, \ldots, l), T_1 = 1$ and $S(\gamma_j) = \frac{3+m_j+n_j-p_j-q_j}{4}$

Definition 5.7. Suppose $\gamma_j = \langle m_j, n_j, p_j, q_j \rangle,\ (j = 1, 2, \ldots, l)$ be a collection on $\mathcal{QNN}$. Then a QSVNDPA operator s_1 of dimension l can be written as follows

$$s_1(\gamma_1, \gamma_2, \ldots, \gamma_l) = \bigoplus_{j=1}^{l} \left(\frac{T_j \gamma_j}{\sum_{j=1}^{l} T_j} \right)$$

Now if $T_j = \frac{1}{l}\ \forall\ j$ then

$$s_1(\gamma_1, \gamma_2, \ldots, \gamma_l) = \frac{1}{l} \bigoplus_{j=1}^{l} \gamma_j.$$

is called average QSVNDPA operator of $\gamma_j = \langle m_j, n_j, p_j, q_j \rangle$ $(j = 1, 2, \ldots, l)$.

Definition 5.8. Let $\gamma_j = \langle m_j, n_j, p_j, q_j \rangle$ $(j = 1, 2, \ldots, l)$ be a collection on $\mathcal{QNN}$. A QSVN Dombi prioritized geometric (QSVNDPG) operator of dimension l is a function $s_2 : \mathcal{QNN}^l \to \mathcal{QNN}$ defined by:

$$s_2(\gamma_1, \gamma_2, \ldots, \gamma_l) = \bigodot_{j=1}^{l} \gamma_j^{\frac{T_j}{\sum_{j=1}^{l} T_j}}$$

where $T_j = \prod_{k=1}^{j-1} S(\gamma_k)$ $(k = 1, 2, \ldots, l)$, $T_1 = 1$ and $S(\gamma_j) = \frac{3+m_j+n_j-p_j-q_j}{4}$.

Theorem 5.9. *Suppose $\gamma_j = \langle m_j, n_j, p_j, q_j \rangle$ $(j = 1, 2, \ldots, l)$ be a collection on $\mathcal{QNN}$. Then*

$$s_2(\gamma_1, \gamma_2, \ldots, \gamma_l) = \bigodot_{j=1}^{l} \gamma_j^{\frac{T_j}{\sum_{j=1}^{l} T_j}}$$

$$= \left\langle 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{l} \frac{T_j \left(\frac{1-m_j}{m_j} \right)^{\lambda}}{\sum_{j=1}^{l} T_j} \right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{l} \frac{T_j \left(\frac{1-n_j}{n_j} \right)^{\lambda}}{\sum_{j=1}^{l} T_j} \right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{l} \frac{T_j \left(\frac{p_j}{1-p_j} \right)^{\lambda}}{\sum_{j=1}^{l} T_j} \right\}^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{l} \frac{T_j \left(\frac{q_j}{1-q_j} \right)^{\lambda}}{\sum_{j=1}^{l} T_j} \right\}^{\frac{1}{\lambda}}} \right\rangle.$$

Proof. The above theorem can be proved using the same proof procedure of Theorem 5.2. □

Theorem 5.10. *The QSVNDPG operator s_2 satisfies properties as defined below:*

(i) *Consistency:* $s_2(\gamma_1, \gamma_2, \ldots, \gamma_l) \in \mathcal{QNN}$.

(ii) *Idempotency:* $s_2(\gamma, l \text{ times } \ldots, \gamma) = \gamma$.

(iii) *Commutativity:* $s_2(\gamma_1, \gamma_2, \ldots, \gamma_l) = s_2(\gamma_l, \gamma_{l-1}, \ldots, \gamma_1)$.

(iv) $s_2(\gamma_{\pi(1)}, \gamma_{\pi(2)}, \ldots, \gamma_{\pi(l)}) = s_2(\gamma_1, \gamma_2, \ldots, \gamma_l)$ *where π is a permutation on* $\{1, 2, \ldots, l\}$.

Proof. We have omitted it due to similarity with Theorem 5.3. □

Theorem 5.11. *Consider $\gamma_j = \langle m_j, n_j, p_j, q_j\rangle$ $(j = 1, 2, \ldots, l)$ and $\widetilde{\gamma_j} = \langle \widetilde{m_j}, \widetilde{n_j}, \widetilde{p_j}, \widetilde{q_j}\rangle$ $(j = 1, 2, \ldots, l)$ are two collections on $\mathcal{QNN}$ such that $m_j \le \widetilde{m_j}, n_j \le \widetilde{n_j}, p_j \ge \widetilde{p_j}, q_j \ge \widetilde{q_j} \forall j$. Then $s_2(\gamma_1, \gamma_2, \ldots, \gamma_l) \le s_2(\widetilde{\gamma_1}, \widetilde{\gamma_2}, \ldots, \widetilde{\gamma_l})$.*

Proof. Here the proof is similar with Theorem 5.4, hence we have omitted it. □

Theorem 5.12. *Consider a collection of $\gamma_j = \langle m_j, n_j, p_j, q_j\rangle, j = 1, 2, \ldots, l$ in $\mathcal{QNN}$. Then*

$$\underline{\gamma} \le s_2(\gamma_1, \gamma_2, \ldots, \gamma_l) \le \overline{\gamma}, \text{ where}$$
$$\underline{\gamma} = \langle \min_j(m_j), \min_j(n_j), \min_j(p_j), \min_j(q_j)\rangle = \langle \underline{m_j}, \underline{n_j}, \underline{p_j}, \underline{q_j}\rangle \text{ and}$$
$$\overline{\gamma} = \langle \max_j(m_j), \max_j(n_j), \max_j(p_j), \max_j(q_j)\rangle = \langle \overline{m_j}, \overline{n_j}, \overline{p_j}, \overline{q_j}\rangle.$$

Proof. Again proof is not done due to its similarity with Theorem 5.5. □

Definition 5.13. Suppose $\gamma_j = \langle m_j, n_j, p_j, q_j\rangle$, $(j = 1, 2, \ldots, l)$ be a collection on $\mathcal{QNN}$. Then a QSVNDPG operator s_2 of dimension l can be written as follows:

$$s_2(\gamma_1, \gamma_2, \ldots, \gamma_l) = \bigodot_{j=1}^{l} \gamma_j^{\frac{T_j}{\sum_{j=1}^{l} T_j}}.$$

If $T_j = \frac{1}{l}\ \forall\ j \in \{1, 2, \ldots, l\}$ then

$$s_2(\gamma_1, \gamma_2, \ldots, \gamma_l) = (\bigodot_{j=1}^{l} \gamma_j)^{\frac{1}{l}}.$$

is called average QSVNDPG operator of $\gamma_j = \langle m_j, n_j, p_j, q_j\rangle$ $(j = 1, 2, \ldots, l)$.

Definition 5.14. Consider the mass associated with γ_j as $M_j > 0 \; \forall \, j = 1, \ldots, l$, where $M = (M_1, M_2, \ldots, M_l)^T$ is the mass vector such that $\sum_{j=1}^{l} M_j = 1$. Then the QSVNWDPG (SVN weighted DPG) can be defined as follows:

$$s_{\mathcal{M}}(\gamma_1, \gamma_2, \ldots, \gamma_l) = \bigodot_{j=1}^{l} \gamma_j^{\frac{M_j T_j}{\sum_{j=1}^{l} M_j T_j}},$$

where $T_j = \prod_{k=1}^{j-1} S(\gamma_k)(k = 1, 2, \ldots, l), T_1 = 1$ and $S(\gamma_j) = \frac{3+m_j+n_j-p_j-q_j}{4}$

6. An application in MADM of QSVNWDPA and QSVNWDPG operator

For smooth understanding of QSVN operators it is better to apply our operators in MADM problems. Without a real life application any researcher cannot get any interest of studying this. In this regard we have tried to formulate a real life problem with the help of QSVNWDPA and QSVNWDPG operator. Suppose Govt of India wants to stop the spread of the second wave of Covid-19 virus. For this reason Govt of India has 4 ways of lockdown process in their policy i.e. L_1: Complete lockdown , L_2: Statewise lockdown, L_3: District wise lockdown , L_4: Specific area wise lockdown. However there are four attributes $A_j, j =, 2, 3, 4$ which are to be considered for choosing a particular process i.e. (A_1) : the economic growth of the country, (A_2) : the migrant workers, (A_3): The small industry (A_4): The poor people. In order to get a suitable choice L_i after consideration of all attributes A_j we have represented these MADM problems in the form of a decision making matrix $D(l_{ij})$ on $\mathcal{QNN}$ as following:

$$D(l_{ij}) = \begin{bmatrix} \langle 0.4, 0.6, 0.2, 0.3\rangle & \langle 0.4, 0.8, 0.7, 0.9\rangle & \langle 0.5, 0.6, 0.4, 0.2\rangle & \langle 0.1, 0.5, 0.2, 0.3\rangle \\ \langle 0.7, 0.5, 0.7, 0.6\rangle & \langle 0.2, 0.8, 0.3, 0.5\rangle & \langle 0.6, 0.6, 0.1, 0.4\rangle & \langle 0.3, 0.4, 0.5, 0.1\rangle \\ \langle 0.8, 0.5, 0.4, 0.6\rangle & \langle 0.3, 0.6, 0.1, 0.4\rangle & \langle 0.2, 0.5, 0.5, 0.3\rangle & \langle 0.6, 0.6, 0.2, 0.1\rangle \\ \langle 0, 7, 0.1, 0.6, 0.9\rangle & \langle 0.8, 0.3, 0.4, 0.6\rangle & \langle 0.5, 0.2, 0.8, 0.6\rangle & \langle 0.6, 0.4, 0.4, 0.9\rangle \end{bmatrix}.$$

Case-I: Firstly we take the help of QSVNWDPA operator to find out a possible way out of our MADM. Here we take $\lambda = 1, M = (0.4, 0.3, 0.2, 0.1)$ and derive the collection of QSVNs say L_i to find suitable way out among $L_i(i = 1, 2, 3, 4)$ by the help of Definition 5.1 as follows:

$$s_1(L_1) = \langle 0.473, 0.712, 0.731, 0.673\rangle$$

$$s_1(L_2) = \langle 0.639, 0.682, 0.716, 0.6324\rangle$$

$$s_1(L_3) = \langle 0.702, 0.578, 0.798, 0.654\rangle$$

$$s_1(L_4) = \langle 0.806, 0.406, 0.171, 0.615\rangle.$$

Based on the Definition 3.1 the scores are as follows:

$$S(L_1) = 0.69525, S(L_2) = 0.74335, S(L_3) = 0.70694, S(L_4) = 0.8567.$$

From above we have the priority order of lockdown process as $L_4 > L_2 > L_3 > L_1$. .

Case-II: Secondly we take the help of QSVNWDPG operator to find out a possible solution to our problem. Again we take $\lambda = 1, M = (0.4, 0.3, 0.2, 0.1)$ and derive the collective QSVNs L_i with the help of Definition 5.8 as follows:

$$s_2(L_1) = \langle 0.616, 0.434, 0.568, 0.769\rangle$$

$$s_2(L_2) = \langle 0.649, 0.518, 0.625, 0.586\rangle$$

$$s_2(L_3) = \langle 0.631, 0.518, 0.461, 0.542\rangle$$

$$s_2(L_4) = \langle 0.676, 0.873, 0.735, 0.919\rangle.$$

Based on the Definition 3.1 the scores are as follows:

$$S(L_1) = 0.6779, S(L_2) = 0.7378, S(L_3) = 0.786, S(L_4) = 0.724.$$

According to obtained scores, the priority order of lockdown process is $L_3 > L_2 > L_4 > L_1$.

6.1. *Sensitivity analysis*

In this section we have done a sensitivity analysis based on our method. For this purpose we have change the value of our parameter λ in an increasing manner starting from 0.2 to 1 with an increment 0.2. For both the operators i.e. QSVNWDPA and QSVNWDPG operator the following results are obtained. Tabular representation in case of QSVNWDPA operator

λ	$S(L_1), S(L_2), S(L_3), S(L_4)$
0.2	0.583, 0.643, 0.616, 0.677
0.4	0.549, 0.632, 0.607, 0.658
0.6	0.536, 0.617, 0.594, 0.634
0.8	0.493, 0.536, 0.511, 0.577
1.0	0.462, 0.514, 0.473, 0.543

Result: $L_4 > L_2 > L_3 > L_1$. Tabular representation in case of QSVNWDPG operator:

λ	$S(L_1), S(L_2), S(L_3), S(L_4)$
0.2	0.613, 0.649, 0.677, 0.627
0.4	0.553, 0.625, 0.652, 0.586
0.6	0.497, 0.531, 0.568, 0.519
0.8	0.468, 0.511, 0.547, 0.487
1.0	0.421, 0.473, 0.489, 0.445

Result: $L_3 > L_2 > L_4 > L_1$. Considering all the above cases we observed that the priority order of lockdown process remains unaltered irrespective of the values of λ. According to us that either specific area wise lockdown or district wise lock down will be the suitable process against the spread of corona virus second wave in India. But in all the above cases complete lockdown will not be proffered. The above procedure help our Government to choose a multi-solution based on the current situation at that time.

7. Conclusion

Benlap introduced the four valued logic in [1] and applied it in different areas. The QSVN sets are developed on Benlap's Model and they are very good in modeling uncertainty because they can single handedly tackle consistent, inconsistent, vague etc. information. Based on the QSVN set, $\mathcal{QNN}$ is introduced in 2019. In this article two prioritized aggregation operators i.e. QSVNWDPA and QSVNWDPG operator based on Dombi operations on $\mathcal{QNN}$ sets are studied. These aggregation operators are better than other available aggregation operators because they have combined effects of neutrosophy, four valued logic and the power of Dombi. We have also added weights in our operators to add flexibility in them. We have also shown the applicability of our operators by solving a MADM problem where we have utilized the score functions of $\mathcal{QNN}$ to finding the order of priority of different parameters. In future one can develop more advanced type of operators on $\mathcal{QNN}$ and apply them to solve real life MADM problems.

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