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Generalized Open Sets in Neutrosophic Soft Bitopological Spaces

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Abstract. In this study, some generalized neutrosophic soft open sets are defined in neutrosophic soft bitopological spaces. Also, some theorems related to the subject have been given with their proofs and supported with examples for a better understanding of the subject.

Keywords: Neutrosophic set; Neutrosophic soft set; Neutrosophic soft bitopological space; Generalized neutrosophic soft closed (open) set.

1. Introduction

Neutrosophy was defined by Smarandache in 2013 for the first time [15]. After that, this topic became very popular in the scientific world, and many studies have been done in this area to date. Salama and Alblowi developed topological structure on neutrosophic sets in 2012 [14]. The concept of neutrosophic bitopological space was defined in 2019 by Ozturk and Alkan [13]. Then in 2020, neutrosophic interior, closure and boundary were defined in neutrosophic bitopological spaces by Mwchahary and Bhimraj [12]. Some generalized open sets were defined in neutrosophic bitopological spaces [5,6]. In 2013, neutrosophic soft set was defined by Maji [10]. The concept of neutrosophic soft topological space was defined in 2017 by Bera and Nirmal [2]. Neutrosophic soft bitopological space was defined in [4]. In this study some generalized open sets are defined in neutrosophic soft bitopological spaces.

2. Preliminaries

Let X be a space of points. A neutrosophic set (NS) A in X is characterized by a falsity-membership function F, a indeterminacy-membership function I and a truth-membership
function \(T\) where \(F, I, T : X \rightarrow [0, 1], 0 \leq T(x) + I(x) + F(x) \leq 3\). The set of all neutrosophic set in \(X\) is denoted by \(N^X\).

**Definition 2.1.** [14] Let \(B, D \in N^X\). Then

1. Subset: \(D \subseteq B\) if \(T_D(z) \leq T_B(z), I_D(z) \leq I_B(z), F_D(z) \geq F_B(z)\) for all \(z \in X\).
2. Equality: \(D = B\) if \(D \subseteq B\) and \(B \subseteq D\).
3. Intersection:
   \[
   D \cap B = \{< z, \min\{T_D(z), T_B(z)\}, \max\{I_D(z), I_B(z)\}, \max\{F_D(z), F_B(z)\}> : z \in X\}
   \]
4. Union:
   \[
   D \cup B = \{< z, \max\{T_D(z), T_B(z)\}, \min\{I_D(x), I_B(x)\}, \min\{F_D(z), F_B(z)\}> : z \in X\}
   \]
   The intersection and the union of a collection of NSs \(\{D_i\} \in I\) are defined by:
   \[
   \bigcap_{i \in I} D_i = \{< z, \inf\{T_{D_i}(z)\}, \sup\{I_{D_i}(z)\}, \sup\{F_{D_i}(z)\}> : z \in X\}
   \]
   \[
   \bigcup_{i \in I} D_i = \{< z, \sup\{T_{D_i}(z)\}, \inf\{I_{D_i}(z)\}, \inf\{F_{D_i}(z)\}> : z \in X\}
   \]
5. The neutrosophic set defined as \(T_D(z) = 1, I_D(z) = 1\) and \(F_D(z) = 0\) for all \(z \in X\) is called the universal NS denoted by \(1_X\). Also the neutrosophic set defined as \(T_D(z) = 0, I_D(z) = 0\) and \(F_D(z) = 1\) for all \(z \in X\) is called the empty NS denoted by \(0_X\).
6. Difference: \(D/B = \{< z, T_D(z) - T_B(z), I_D(z) - I_B(z), F_D(z) - F_B(z)> : z \in X\}\)
7. Complement: \(D^c = 1_X/D\)

Clearly, the complements of \(1_X\) and \(0_X\) are defined:

\[
(1_X)^c = 1_X/1_X = \{< z, 0, 1, 1> : z \in X\} = 0_X
\]

\[
(0_X)^c = 1_X/0_X = \{< z, 1, 0, 0> : z \in X\} = 1_X
\]

**Proposition 2.2.** Let \(D_1, D_2, D_3, D_4 \in N(X)\). Then the followings hold:

1. \(D_1 \cap D_3 \subseteq D_2 \cap D_4\) and \(D_1 \cup D_3 \subseteq D_2 \cup D_4\) if \(D_1 \subseteq D_2\) and \(D_3 \subseteq D_4\)
2. \((D_1^c)^c = D_1\) and \(D_1 \subseteq D_2\) if \(D_2^c \subseteq D_1^c\)
3. \((D_1 \cap D_2)^c = D_1^c \cup D_2^c\) and \((D_1 \cup D_2)^c = D_1^c \cap D_2^c\)

**Definition 2.3.** Let \(\Gamma^n \subset N(Y)\). Then \(\Gamma^n\) is named a neutrosophic topology (NT) on \(Y\) if the following conditions hold:

1. \(0_X\) and \(1_X\) are belong to \(\Gamma^n\).
2. Union of any number of NSs in \(\Gamma^n\) is again belong to \(\Gamma^n\).
3. Intersection of any two NSs in \(\Gamma^n\) is belong to \(\Gamma^n\).

Then the pair \((Y, \Gamma^n)\) is named neutrosophic topology on \(Y\).
2.1. Neutrosophic Soft Sets

Definition 2.4. Let $U$ be an initial universe set and $E$ be a set of parameters. Then the pair $(H, E)$ is called a neutrosophic soft set (NSS) over $U$, where $H$ is a mapping from $E$ to $N(U)$.

The set of all NSS over $U$ is denoted by $NSS(U, E)$. A neutrosophic set $(H, E)$ can be written as: $(H, E) = \{(e, \{<x, T_H(x), I_H(x), F_H(x) > : x \in X\}) : e \in E\}$.

Definition 2.5. Let $X$ be an initial universe set and $E$ be a set of parameters. Then the neutrosophic soft set $x^{e}_{(\alpha, \beta, \gamma)}$ defined as

$$x^{e}_{(\alpha, \beta, \gamma)}(e') = \begin{cases} 
(\alpha, \beta, \gamma) & \text{if } e = e' \text{ and } y = y' \\
(0, 0, 1) & \text{if } e \neq e' \text{ and } y \neq y'
\end{cases}$$

for all $x \in X, 0 < \alpha, \beta, \gamma \leq 1, e \in E$, is called a neutrosophic soft point.

Definition 2.6. Let $(H, E), (G, E) \in NSS(U, E)$. Then for all $x \in U$

1. Subset: $(H, E) \subset (G, E)$ if $T_{H(e)}(x) \leq T_{G(e)}(x), I_{H(e)}(x) \leq I_{G(e)}(x)$ and $F_{H(e)}(x) \geq F_{G(e)}(x)$ for all $e \in E$.
2. Equality: $(H, E) = (G, E)$ if $(H, E) \subset (G, E)$ and $(G, E) \subset (H, E)$.
3. Intersection:

$$(H, E) \cap (G, E) = \{(e, \{<x, \min \{T_{H(e)}(x), T_{G(e)}(x)\}, \max \{I_{H(e)}(x), I_{G(e)}(x)\}, \max \{F_{H(e)}(x), F_{G(e)}(x)\} > : e \in E\}.)$$

4. Union:

$$(H, E) \cup (G, E) = \{(e, \{<x, \max \{T_{H(e)}(x), T_{G(e)}(x)\}, \min \{I_{H(e)}(x), I_{G(e)}(x)\}, \min \{F_{H(e)}(x), F_{G(e)}(x)\}> : e \in E\})$$

The intersection and the union of a collection of $\{(H_i, E)\} \subset NSS(U, E)$ are defined by:

$$\bigcap_{i \in I}(H_i, E) = \left\{\left(e, \{<x, \inf \{T_{H_i(e)}(x)\}, \sup \{I_{H_i(e)}(x)\}, \sup \{F_{H_i(e)}(x)\} > \right) : e \in E\right\}$$

$$\bigcup_{i \in I}(H_i, E) = \left\{\left(e, \{<x, \sup \{T_{H_i(e)}(x)\}, \inf \{I_{H_i(e)}(x)\}, \inf \{F_{H_i(e)}(x)\} > \right) : e \in E\right\}$$

5. The NSS defined as $T_{H(e)}(x) = 1, I_{H(e)}(x) = 0$ and $F_{H(e)}(x) = 0$, for all $e \in E$ and $x \in U$ is called the universal NSS denoted by $1_{(U, E)}$. Also the neutrosophic set defined as $T_{H(e)}(x) = 0 I_{H(e)}(x) = 1$ and $F_{H(e)}(x) = 1$ for all $e \in E$ and $x \in U$ is called the empty NSS denoted by $0_{(U, E)}$.

6. Complement:

$$(H, E)^c = 1_{(X, E)}/(H, E) = \{(e, \{<x, F_{H(e)}(x), 1 - I_{H(e)}(x), T_{H(e)}(x) > : e \in E\})$$

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Clearly, the complements of $1_{(X, E)}$ and $0_{(X, E)}$ are defined:

\[
(1_{(X, E)})^c = 1_{(X, E)}/1_{(X, E)} = \{ (e, \{ x, 0, 1 \}) : e \in E \} = 0_{(X, E)} \\
(0_{(X, E)})^c = 1_{(X, E)}/0_{(X, E)} = \{ (e, \{ x, 1, 0 \}) : e \in E \} = 1_{(X, E)}
\]

**Definition 2.7.** Let $\tau \subset NSS(Y, E)$. Then $\tau$ is called as a neutrosophic soft topology on $Y$ if the following conditions hold:

NST1) $0_{(Y, E)}, 1_{(Y, E)} \in \tau$

NST2) Union of any number of NSSs in $\tau$ is belong to $\tau$.

NST3) Intersection of finite number of NSSs in $\tau$ is belong to $\tau$.

Then $(Y, E, \tau)$ is called as neutrosophic soft topological space. Any element of $\tau$ is called as $\tau$-neutrosophic soft open ($\tau$-NSO) set. A NSS is called as $\tau$-neutrosophic soft closed ($\tau$-NSC) if the complement of the set is $\tau$-NSO. The set of all neutrosophic soft closed sets is denoted by $(\tau)^c$.

**Definition 2.8.** Let $(Y, E, \tau)$ be a neutrosophic soft topological space and $(M, E) \in NSS(Y, E)$. Then the intersection of all $\tau$-NSC sets containing $(M, E)$ is called as closure of $(M, E)$ and denoted by $cl_\tau(M, E)$, i.e. $cl_\tau(M, E) = \bigcap\{(N, E) \in (\tau)^c : (M, E) \subset (N, E)\}$

**Theorem 2.9.** Let $(Y, E, \tau)$ be a neutrosophic soft topological space and $(M, E), (N, E) \in NSS(Y, E)$. Then

\[
\begin{align*}
cl_1(M, E) & \subset cl_\tau(M, E) \\
cl_2(M, E) & \subset (N, E) \text{ then } cl_\tau(M, E) \subset cl_\tau(N, E) \\
cl_3(cl_\tau((M, E) \cap (N, E)) & \subset cl_\tau(M, E) \cap cl_\tau(N, E) \\
cl_4(cl_\tau((M, E) \cup (N, E)) & = cl_\tau(M, E) \cup cl_\tau(N, E)
\end{align*}
\]

**Definition 2.10.** Let $\tau \subset NSS(Y, E)$. Then $\tau$ is called as a neutrosophic soft supra topology on $Y$ if it satisfies just NST1) and NST2).

**Definition 2.11.** Let $(Y, E, \tau)$ be a neutrosophic soft topological space and $(M, E) \in NSS(Y, E)$. Then the union of all $\tau$-NSO sets subset of $(M, E)$ is called as interior of $(M, E)$ and denoted by $int_\tau(M, E)$, i.e. $int_\tau(M, E) = \bigcup\{(N, E) \in \tau : (N, E) \subset (M, E)\}$

**Theorem 2.12.** Let $(Y, E, \tau)$ be a neutrosophic soft topological space and $(M, E), (N, E) \in NSS(Y, E)$. Then

\[
\begin{align*}
int_1(int_\tau(M, E) & \subset (M, E) \\
int_2(M, E) & \subset (N, E) \text{ then } int_\tau(M, E) \subset int_\tau(N, E) \\
int_3(int_\tau((M, E) \cap (N, E)) & = int_\tau(M, E) \cap int_\tau(N, E) \\
int_4(int_\tau(M, E) \cup int_\tau(N, E) & \subset int_\tau((M, E) \cup (N, E))
\end{align*}
\]
3. Neutrosophic Soft Bitopological Space

Definition 3.1. If \((Y, \tau_1, E)\) and \((Y, \tau_2, E)\) are two neutrosophic soft topological space, then \((Y, E, \tau_1, \tau_2)\) is named as neutrosophic soft bitopological space. The sets belong to \(\tau_i\) are called as neutrosophic soft \(\tau_i\)-open set for \(i = 1, 2\).

Definition 3.2. An operator \(C : NSS(X, E) \to NSS(X, E)\) is called a neutrosophic soft supra closure operator if it satisfies the following conditions for all \((N, E), (M, E) \in NSS(X, E),\)

\[
\begin{align*}
C_1) \ C(0_{(X,E)}) & = 0_{(X,E)} \\
C_2) \ (N, E) & \subset C(N, E) \\
C_3) \ C(N, E) & \cup C(M, E) \subset C(N \cup M) \\
C_4) \ C(C(N, E)) & = C(N, E).
\end{align*}
\]

Theorem 3.3. Let \((X, E, \tau_1, \tau_2)\) be a neutrosophic soft bitopological space. Then, the operator \(cl_{12} : NSS(X, E) \to NSS(X, E)\) defined as \(cl_{12}(N, E) = cl_{\tau_1}(N, E) \cap cl_{\tau_2}(N, E)\) is a neutrosophic soft supra closure operator on \((X, E)\) and induces the supra neutrosophic soft topology \(\tau_{12} = \{(M, E) \in NSS(X, E) : cl_{12}((M, E)^c) = (M, E)^c\}\).

Proof. First let prove that \(cl_{12}\) is a neutrosophic soft supra closure operator.

\[
\begin{align*}
C_1) \ cl_{12}(0_{(X,E)}) & = cl_{\tau_1}(0_{(X,E)}) \cap cl_{\tau_2}(0_{(X,E)}) = 0_{(X,E)} \cap 0_{(X,E)} = 0_{(X,E)} \\
C_2) \ (N, E) & \subset cl_{\tau_1}(N, E) \text{ and } (N, E) \subset cl_{\tau_2}(N, E). \text{ Then } (N, E) \subset cl_{\tau_1}(N, E) \cap cl_{\tau_2}(N, E) = cl_{12}(N, E). \\
C_3) \ cl_{12}(N, E) \cup cl_{12}(M, E) & = [cl_{\tau_1}(N, E) \cap cl_{\tau_2}(N, E)] \cup [cl_{\tau_1}(M, E) \cap cl_{\tau_2}(M, E)] \\
& = cl_{\tau_1}[(N, E) \cup (M, E)] \cap cl_{\tau_2}(N, E) \cup cl_{\tau_2}(M, E)] \\
& \quad \cap [cl_{\tau_1}(N, E) \cup cl_{\tau_2}(M, E)] \cap cl_{\tau_2}[(N, E) \cup (M, E)] \\
& \quad \subset cl_{\tau_1}[(N, E) \cup (M, E)] \cap cl_{\tau_2}[(N, E) \cup (M, E)] \\
& \quad = cl_{12}[(N, E) \cup (M, E)]. \\
C_4) \ From \ C_3, \ cl_{12}(N, E) & \subset cl_{12}(cl_{12}(N, E)). \ Also \\
cl_{12}(cl_{12}(N, E)) & = cl_{12}(cl_{\tau_1}(N, E) \cap cl_{\tau_2}(N, E)) \\
& = cl_{\tau_1}[(cl_{\tau_1}(N, E) \cap cl_{\tau_2}(N, E))] \cap cl_{\tau_2}[(cl_{\tau_1}(N, E) \cap cl_{\tau_2}(N, E))] \\
& \quad \subset cl_{\tau_1}[(cl_{\tau_1}(N, E)) \cap cl_{\tau_1}(cl_{\tau_2}(N, E))] \cap cl_{\tau_2}(cl_{\tau_1}(N, E)) \\
& \quad \subset cl_{\tau_1}(N, E) \cap cl_{\tau_2}(N, E) = cl_{12}(N, E).
\end{align*}
\]

Therefore \(cl_{12}(N, E) = cl_{12}(cl_{12}(N, E))\).
Now let prove that $\tau_{12}$ is a neutrosophic soft supra topology.

\textit{NST1)} Since $cl_{12}((1_{(X,E)})^c) = cl_{12}(0_{(X,E)}) = 0_{(X,E)}$, then $0_{(X,E)} \in \tau_{12}$. Also $cl_{12}((0_{(X,E)})^c) = cl_{12}(1_{(X,E)}) \subset 1_{(X,E)}$ and from $(C_2)$, $1_{(X,E)} \subset cl_{12}(1_{(X,E)})$. Therefore $0_{(X,E)} \in \tau_{12}$.

\textit{NST2)} Let $(N_i, E) \in \tau_{12}$. Then $cl_{12}((N_i, E)^c) = (N_i, E)^c$.

$$cl_{12}\left(\bigcup_{i \in I}(N_i, E)^c\right) = cl_{\tau_1}\left(\bigcup_{i \in I}(N_i, E)^c\right) \cap cl_{\tau_2}\left(\bigcup_{i \in I}(N_i, E)^c\right)$$

$$\subset \bigcap_{i \in I}(cl_{\tau_1}(N_i, E)^c) \cap \bigcap_{i \in I}(cl_{\tau_2}(N_i, E)^c)$$

$$= \bigcap_{i \in I}(cl_{\tau_1}(N_i, E)^c \cap cl_{\tau_2}(N_i, E)^c)$$

$$= \bigcap_{i \in I}(cl_{12}(N_i, E)^c) = \bigcap_{i \in I}(N_i, E)^c = \left(\bigcup_{i \in I}(N_i, E)^c\right)^c.$$ 

Also from $(C_2)$, $(\bigcup_{i \in I}(N_i, E))^c \subset cl_{12}\left(\bigcup_{i \in I}(N_i, E)^c\right)$. Therefore $cl_{12}\left(\bigcup_{i \in I}(N_i, E)^c\right) = \left(\bigcup_{i \in I}(N_i, E)^c\right)^c$, then $\bigcup_{i \in I}(N_i, E) \in \tau_{12}$.

Consequently $\tau_{12}$ is a neutrosophic soft supra topology on $(X, E).$ \hfill \Box

\textbf{Theorem 3.4.} Let $(X, E, \tau_1, \tau_2)$ be a neutrosophic soft bitopological space and $(N, E) \in NSS(X, E)$. Then $(N, E) \in \tau_{12}$ if and only if there exists a $\tau_1$-NSC set $(N_1, E)$ and $\tau_2$-NSC set $(N_2, E)$ such that $(N, E) = (N_1, E) \cap (N_2, E)$.

\textit{Proof.} If we take $(N_1, E) = cl_{\tau_1}(N, E)$ and $(N_2, E) = cl_{\tau_2}(N, E)$, then proof is clear. \hfill \Box

\textbf{Theorem 3.5.} Let $(X, E, \tau_1, \tau_2)$ be a neutrosophic soft bitopological space and $$(M, E), (N, E) \in NSS(X, E).$$ Then

1) if $(M, E) \subset (N, E)$ then $cl_{12}(M, E) \subset cl_{12}(N, E)$. 

2) $cl_{12}((M, E) \cap (N, E)) \subset cl_{12}(M, E) \cap cl_{12}(N, E)$. 

\textit{Proof.} For any $(M, E), (N, E) \in NSS(X, E)$,

1) Let $(M, E) \subset (N, E)$. Then $cl_{\tau_1}(M, E) \subset cl_{\tau_1}(N, E)$ and $cl_{\tau_2}(M, E) \subset cl_{\tau_2}(N, E)$. Therefore $cl_{\tau_1}(M, E) \cap cl_{\tau_2}(M, E) \subset cl_{\tau_1}(N, E) \cap cl_{\tau_2}(N, E)$. So $cl_{12}(M, E) \subset cl_{12}(N, E)$.

2) $(M, E) \cap (N, E) \subset (M, E)$ and $(M, E) \cap (N, E) \subset (N, E)$. Then from (1), $cl_{12}(M, E) \cap (N, E)) \subset cl_{12}(M, E)$ and $cl_{12}((M, E) \cap (N, E)) \subset cl_{12}(N, E)$. Therefore $cl_{12}((M, E) \cap (N, E)) \subset cl_{12}(M, E) \cap cl_{12}(N, E)$. 

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Remark 3.6. Let \((X, E, \tau_1, \tau_2)\) be a neutrosophic soft bitopological space. Then \(cl_{12}(M, E) \cap cl_{12}(N, E) \neq cl_{12}((M, E) \cap (N, E))\), in general.

Example 3.7. Let the neutrosophic soft bitopological space \((X, U, \tau)\) be defined as \(X = \{x_1, x_2, x_3\}\), \(U = \{e_1, e_2\}\), \(\tau_1 = \{0_{(X, U)}, 1_{(X, U)}, (A, U), (B, U), (C, U), (D, U)\}\), \(\tau_2 = \{0_{(X, U)}, 1_{(X, U)}, (D, U), (F, U), (G, U), (H, U)\}\) where the tabular representations of NSSs are as follows:

\[
\begin{array}{|c|c|c|}
\hline
(A, U) & X & \epsilon_1 & \epsilon_2 \\
\hline
 & x_1 & <0.2,0.3,0.8> & <0.9,0.1,0.3> \\
 & x_2 & <0.1,0.5,0.4> & <0.4,0.4,0.4> \\
 & x_3 & <0.8,0.1,0.5> & <0.2,0.8,0.1> \\
\hline
(B, U) & X & \epsilon_1 & \epsilon_2 \\
\hline
 & x_1 & <0.1,0.3,0.8> & <0.3,0.1,0.7> \\
 & x_2 & <0.1,0.1,0.4> & <0.1,0.2,0.5> \\
 & x_3 & <0.3,0.1,0.5> & <0.2,0.1,0.3> \\
\hline
(C, U) & X & \epsilon_1 & \epsilon_2 \\
\hline
 & x_1 & <0.2,0.3,0.4> & <0.9,0.1,0.3> \\
 & x_2 & <0.2,0.1,0.3> & <0.4,0.2,0.4> \\
 & x_3 & <0.8,0.1,0.5> & <0.6,0.1,0.1> \\
\hline
(D, U) & X & \epsilon_1 & \epsilon_2 \\
\hline
 & x_1 & <0.1,0.3,0.4> & <0.3,0.2,0.7> \\
 & x_2 & <0.2,0.1,0.3> & <0.1,0.2,0.5> \\
 & x_3 & <0.3,0.7,0.8> & <0.6,0.1,0.3> \\
\hline
(F, U) & X & \epsilon_1 & \epsilon_2 \\
\hline
 & x_1 & <0.7,0.1,0.1> & <0.2,0.5,0.5> \\
 & x_2 & <0.9,0.5,0.3> & <0.3,0.8,0.1> \\
 & x_3 & <0.1,0.8,0.1> & <0.8,0.2,0.7> \\
\hline
\end{array}
\]
Definition 3.8. An operator \( N, E \) interior operator if it satisfies the following conditions for all \((X, E, \tau)\):

\[
\begin{array}{c|c|c}
X & e_1 & e_2 \\
\hline
x_1 & <0.1,0.1,0.4> & <0.2,0.2,0.7> \\
x_2 & <0.2,0.1,0.3> & <0.1,0.2,0.5> \\
x_3 & <0.1,0.7,0.8> & <0.6,0.1,0.7> \\
\end{array}
\]

\[
\begin{array}{c|c|c}
X & e_1 & e_2 \\
\hline
x_1 & <0.7,0.1,0.1> & <0.3,0.2,0.5> \\
x_2 & <0.9,0.1,0.3> & <0.3,0.2,0.1> \\
x_3 & <0.3,0.7,0.1> & <0.8,0.1,0.3> \\
\end{array}
\]

Let two NSSs \((X_1, U)\) and \((X_2, U)\) are defined as

\[
\begin{array}{c|c|c}
X & e_1 & e_2 \\
\hline
x_1 & <0.8,0.5,0.1> & <0.7,0.1,0.3> \\
x_2 & <0.5,0.9,0.1> & <0.8,0.1,0.1> \\
x_3 & <0.5,0.8,0.2> & <0.5,0.9,0.2> \\
\end{array}
\]

\[
\begin{array}{c|c|c}
X & e_1 & e_2 \\
\hline
x_1 & <0.9,0.7,0.1> & <0.9,0.9,0.1> \\
x_2 & <0.4,0.5,0.1> & <0.5,0.8,0.1> \\
x_3 & <0.7,0.9,0.3> & <0.3,0.9,0.1> \\
\end{array}
\]

Then \( cl_{12}((X_1, U) \cap (X_2, U)) = (B, U)^c \) and \( cl_{12}(X_1, U) = cl_{12}(X_2, U) = 1_{(X,U)} \). So \( cl_{12}(X_1, U) \cap cl_{12}(X_2, U) \not\subset cl_{12}((X_1, U) \cap (X_2, U)) \).

**Definition 3.8.** An operator \( I : NSS(X, E) \rightarrow NSS(X, E) \) is called a neutrosophic soft supra interior operator if it satisfies the following conditions for all \((N, E), (M, E) \in NSS(X, E)\),

\[
I_1) \ I(0_{(X,E)}) = 0_{(X,E)} \\
I_2) \ I(N, E) \subset (N, E) \\
I_3) \ I(N, E) \cap I(M, E) \subset I(N \cap M) \\
I_4) \ I(I(N, E)) = I(N, E). \\
\]

**Theorem 3.9.** Let \((X, E, \tau_1, \tau_2)\) be a neutrosophic soft bitopological space. Then, the operator \( \text{int}_{12} : NSS(X, E) \rightarrow NSS(X, E) \) defined as \( \text{int}_{12}(N, E) = \text{int}_{\tau_1}(N, E) \cup \text{int}_{\tau_2}(N, E) \) is a neutrosophic soft supra interior operator on \((X, E)\) and induces the supra neutrosophic soft topology \( \tau_{12} = \{(M, E) \in NSS(X, E) : \text{int}_{12}(M, E) = (M, E)\}\).
Proof. First let prove that $int_{12}$ is a neutrosophic soft supra interior operator.

$I_1$ ) $int_{12}(0_{(X,E)}) = int_{\tau_1}(0_{(X,E)}) \cup int_{\tau_2}(0_{(X,E)}) = 0_{(X,E)} \cup 0_{(X,E)} = 0_{(X,E)}$

$I_2$ ) $int_{\tau_1}(N,E) \subset (N,E)$ and $int_{\tau_2}(N,E) \subset (N,E)$. Then $int_{\tau_1}(N,E) \cup int_{\tau_2}(N,E) \subset (N,E)$. Therefore $int_{\tau_{12}}(N,E) \subset (N,E)$

$I_3$

\[
int_{12}(N,E) \cap int_{12}(M,E) = [int_{\tau_1}(N,E) \cup int_{\tau_2}(N,E)] \cap [int_{\tau_1}(M,E) \cup int_{\tau_2}(M,E)]
\]
\[
= int_{\tau_1}[(N,E) \cap (M,E)] \cup [int_{\tau_2}(N,E) \cap int_{\tau_1}(M,E)]
\]
\[
\quad \cup [int_{\tau_1}(N,E) \cap int_{\tau_2}(M,E)] \cup int_{\tau_2}[(N,E) \cap (M,E)]
\]
\[
= int_{\tau_1_2}[(N,E) \cap (M,E)] \cup [int_{\tau_2}(N,E) \cap int_{\tau_1}(M,E)]
\]
\[
\quad \cup [int_{\tau_1}(N,E) \cap int_{\tau_2}(M,E)] 
\subset int_{12}[(N,E) \cap (M,E)] .
\]

$I_4$ ) From ($I_3$), $int_{12}(int_{12}(N,E)) \subset int_{12}(N,E)$. Also

\[
int_{12}(N,E) = int_{\tau_1}(N,E) \cup int_{\tau_2}(N,E)
\]
\[
= int_{\tau_1}(int_{\tau_1}(N,E)) \cup int_{\tau_2}(int_{\tau_2}(N,E))
\]
\[
\subset int_{\tau_1}(int_{\tau_1}(N,E)) \cup int_{\tau_2}(int_{\tau_2}(N,E)) \cup int_{\tau_2}(int_{\tau_1}(N,E)) \cup int_{\tau_2}(int_{\tau_2}(N,E))
\]
\[
\subset int_{\tau_1}(int_{\tau_1}(N,E)) \cup int_{\tau_2}(int_{\tau_2}(N,E)) \cup int_{\tau_2}(int_{\tau_1}(N,E)) \cup int_{\tau_2}(N,E))
\]
\[
= int_{\tau_1}(int_{12}(N,E)) \cup int_{\tau_2}(int_{12}(N,E))
\]
\[
= int_{12}(int_{12}(N,E)).
\]

Therefore $int_{12}(N,E) = int_{12}(int_{12}(N,E))$.

Now let prove that $\tau_{12}$ is a neutrosophic soft supra topology.

NST1) From ($I_1$), $int_{12}(0_{(X,E)}) = 0_{(X,E)}$, then $0_{(X,E)} \in \tau_{12}$. Also $int_{12}(1_{(X,E)}) = int_{\tau_1}(1_{(X,E)}) \cup int_{\tau_2}(1_{(X,E)}) = 1_{(X,E)}$. Therefore $1_{(X,E)} \in \tau_{12}$. 

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Let \((N, E) \in \tau_{12}\). Then \(\text{int}_{12}((N_i, E)) = (N_i, E)\).

\[
\bigcup_{i \in I} (N_i, E) = \bigcup_{i \in I} \text{int}_{12}(N_i, E)
\]

\[= \bigcup_{i \in I} (\text{int}_1(N_i, E) \cup \text{int}_2(N_i, E))
\]

\[= \left( \bigcup_{i \in I} \text{int}_1(N_i, E) \right) \cup \left( \bigcup_{i \in I} \text{int}_2(N_i, E) \right)
\]

\[\subseteq \text{int}_1\left( \bigcup_{i \in I} (N_i, E) \right) \cup \text{int}_2\left( \bigcup_{i \in I} (N_i, E) \right)
\]

\[= \text{int}_{12}\left( \bigcup_{i \in I} (N_i, E) \right).
\]

Also from \((J_2)\),

\[\text{int}_{12}\left( \bigcup_{i \in I} (N_i, E) \right) \subset \left( \bigcup_{i \in I} (N_i, E) \right).
\]

Therefore \(\text{int}_{12}\left( \bigcup_{i \in I} (N_i, E) \right) = \left( \bigcup_{i \in I} (N_i, E) \right)\), then \(\bigcup_{i \in I} (N_i, E) \in \tau_{12}\).

Consequently \(\tau_{12}\) is a neutrosophic soft supra topology on \((X, E)\).

**Theorem 3.10.** Let \((X, E, \tau_1, \tau_2)\) be a neutrosophic soft bitopological space and \((N, E) \in NSS(X, E)\). Then \((N, E) \in \tau_{12}\) if and only if there exists a \(\tau_1\text{-NSO set} (N_1, E)\) and \(\tau_2\text{-NSO set} (N_2, E)\) such that \((N, E) = (N_1, E) \cup (N_2, E)\).

**Proof.** If we take \((N_1, E) = \text{int}_{\tau_1}(N, E)\) and \((N_2, E) = \text{int}_{\tau_2}(N, E)\), then proof is clear.

**Theorem 3.11.** Let \((X, E, \tau_1, \tau_2)\) be a neutrosophic soft bitopological space and \((M, E), (N, E) \in NSS(X, E)\). Then

1) if \((M, E) \subset (N, E)\) then \(\text{int}_{12}(M, E) \subset \text{int}_{12}(N, E)\).

2) \(\text{int}_{12}(M, E) \cup \text{int}_{12}(N, E) \subset \text{int}_{12}(\{M, E\} \cup \{N, E\})\).

**Proof.** For any \((M, E), (N, E) \in NSS(X, E),\)

1) Let \((M, E) \subset (N, E)\). Then \(\text{int}_{\tau_1}(M, E) \subset \text{int}_{\tau_1}(N, E)\) and \(\text{int}_{\tau_2}(M, E) \subset \text{int}_{\tau_2}(N, E)\). Therefore \(\text{int}_{\tau_1}(M, E) \cap \text{int}_{\tau_2}(M, E) \subset \text{int}_{\tau_1}(N, E) \cap \text{int}_{\tau_2}(N, E)\). So \(\text{int}_{12}(M, E) \subset \text{int}_{12}(N, E)\).

2) \((M, E) \subset (M, E) \cup (N, E)\) and \((N, E) \subset (M, E) \cup (N, E)\). Then from \((1)\), \(\text{int}_{12}(M, E) \subset \text{int}_{12}((M, E) \cup (N, E))\) and \(\text{int}_{12}(N, E) \subset \text{int}_{12}((M, E) \cup (N, E))\). Therefore \(\text{int}_{12}(M, E) \cup \text{int}_{12}(N, E) \subset \text{int}_{12}((M, E) \cup (N, E))\).
Remark 3.12. Let \((X, \tau_1, \tau_2)\) be a neutrosophic soft bitopological space. Then 
\[
\text{int}_{12}(M, E) \cup \text{int}_{12}(N, E) \neq \text{int}_{12}((M, E) \cup (N, E)),
\]
in general.

Proposition 3.13. Let \((X, E, \tau_1, \tau_2)\) be a neutrosophic soft bitopological space and \((N, E) \in NSS(X, E)\). Then

1) \(\tau_1, \tau_2 \subset \tau_{12}\)
2) \(cl_{12}(N, E) = (\text{int}_{12}(N, E)^c)^c\)
3) \(\text{int}_{12}(N, E) = (cl_{12}(N, E)^c)^c\)

Proof. 1) Let \((N, E) \in \tau_1\). Then \((N, E) = \text{int}_{\tau_1}(N, E)\). Therefore
\[
\text{int}_{12}(N, E) = \text{int}_{\tau_1}(N, E) \cup \text{int}_{\tau_2}(N, E) = (N, E)
\]
So \(\tau_1 \subset \tau_{12}\). Similar for \(\tau_2 \subset \tau_{12}\).

2)
\[
cl_{12}(N, E) = cl_{\tau_1}(N, E) \cap cl_{\tau_2}(N, E) = \left(\bigcap_{j \in I} (F_{j_1}^1, E)\right) \cap \left(\bigcap_{j \in J} (F_{j_2}^2, E)\right)
\]
\[
= \left[\bigcup_{j \in I} (F_{j_1}^1, E)^c\right] \cap \left[\bigcup_{j \in J} (F_{j_2}^2, E)^c\right]
\]
\[
= \left[\left(\bigcup_{j \in I} (F_{j_1}^1, E)^c\right) \cup \left(\bigcup_{j \in J} (F_{j_2}^2, E)^c\right)\right]^c
\]
where \((N, E) \subset (F_{j_1}^i, E), (F_{j_2}^i, E)^c \in \tau_i\) for all \(j \in I, J\) and \(i = 1, 2\).

3)
\[
\text{int}_{12}(N, E) = \text{int}_{\tau_1}(N, E) \cup \text{int}_{\tau_2}(N, E) = \left(\bigcup_{j \in I} (U_{j_1}^1, E)\right) \cup \left(\bigcup_{j \in J} (U_{j_2}^2, E)\right)
\]
\[
= \left[\bigcap_{j \in I} (U_{j_1}^1, E)^c\right] \cap \left[\bigcap_{j \in J} (U_{j_2}^2, E)^c\right]
\]
\[
= \left[\left(\bigcap_{j \in I} (U_{j_1}^1, E)^c\right) \cap \left(\bigcap_{j \in J} (U_{j_2}^2, E)^c\right)\right]^c
\]
where \((U_{j_1}^i, E) \subset (N, E), (U_{j_2}^i, E) \in \tau_i\) for all \(j \in I, J\) and \(i = 1, 2\).

\[\Box\]
4. Some Generalized Open Sets in Neutrosophic Soft Bitopological Spaces

Throughout this section, \(i, j = 1, 2\) and \(i \neq j\).

**Definition 4.1.** Let \((X, E, \tau_1, \tau_2)\) be a neutrosophic soft bitopological space and \((N, E) \in NSS(X, E)\). Then \((N, E)\) is called as

1) \(ij\)- neutrosophic soft preopen \((ij - NSPO)\) if \((N, E) \subset int_{\tau_i} (cl_{\tau_j}(N, E))\)
2) \(ij\)- neutrosophic soft semi-open \((ij - NSSO)\) if \((N, E) \subset cl_{\tau_j}(int_{\tau_i}(N, E))\)
3) \(ij\)- neutrosophic soft b-open \((ij - NSbO)\) if \((N, E) \subset cl_{\tau_i}(int_{\tau_j}(N, E)) \cup int_{\tau_j}(cl_{\tau_i}(N, E))\).
4) \(ij\)- neutrosophic soft \(\beta\)-open \((ij - NSF\beta O)\) if \((N, E) \subset cl_{\tau_j}(int_{\tau_i}(cl_{\tau_{i}}(N, E)))\).

**Example 4.2.** Let the neutrosophic soft bitopological space \((X, U, \tau_{1}, \tau_{2})\) be defined as \(X = \{x_1, x_2, x_3\}\), \(U = \{e_1, e_2\}\), \(\tau_{1} = \{0_{(X,U)}, 1_{(X,U)}\}, (A, U), (B, U), (C, U), (D, U)\}\), \(\tau_{2} = \{0_{(X,U)}, 1_{(X,U)}\}, (E, U), (F, U), (G, U), (H, U)\}\) where the tabular representations of NSSs are as follows:

\[
\begin{array}{|c|c|c|}
\hline
(A, U) & X & e_1 & e_2 \\
\hline
x_1 & < 0.2, 0.1, 0.9 > & < 0.6, 0.1, 0.7 > \\
\hline
x_2 & < 0.1, 0.8, 0.4 > & < 0.1, 0.1, 0.8 > \\
\hline
x_3 & < 0.3, 0.4, 0.8 > & < 0.5, 0.1, 0.4 > \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
(B, U) & X & e_1 & e_2 \\
\hline
x_1 & < 0.1, 0.3, 0.4 > & < 0.2, 0.7, 0.8 > \\
\hline
x_2 & < 0.2, 0.1, 0.5 > & < 0.5, 0.6, 0.7 > \\
\hline
x_3 & < 0.3, 0.3, 0.7 > & < 0.1, 0.8, 0.8 > \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
(C, U) & X & e_1 & e_2 \\
\hline
x_1 & < 0.2, 0.1, 0.4 > & < 0.6, 0.1, 0.7 > \\
\hline
x_2 & < 0.2, 0.1, 0.4 > & < 0.5, 0.1, 0.7 > \\
\hline
x_3 & < 0.3, 0.3, 0.7 > & < 0.5, 0.1, 0.4 > \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
(D, U) & X & e_1 & e_2 \\
\hline
x_1 & < 0.1, 0.3, 0.9 > & < 0.2, 0.7, 0.8 > \\
\hline
x_2 & < 0.1, 0.8, 0.5 > & < 0.1, 0.6, 0.8 > \\
\hline
x_3 & < 0.3, 0.4, 0.8 > & < 0.1, 0.8, 0.8 > \\
\hline
\end{array}
\]
\[ (E, U) = \begin{array}{|c|c|c|} \hline X & e_1 & e_2 \\ \hline x_1 & <0.2,0.6,0.3> & <0.1,0.1,0.9> \\ x_2 & <0.3,0.7,0.4> & <0.3,0.4,0.6> \\ x_3 & <0.3,0.5,0.8> & <0.6,0.1,0.8> \\ \hline \end{array} \]

\[ (F, U) = \begin{array}{|c|c|c|} \hline X & e_1 & e_2 \\ \hline x_1 & <0.1,0.1,0.8> & <0.1,0.8,0.9> \\ x_2 & <0.3,0.2,0.5> & <0.3,0.5,0.7> \\ x_3 & <0.7,0.5,0.6> & <0.1,0.7,0.7> \\ \hline \end{array} \]

\[ (G, U) = \begin{array}{|c|c|c|} \hline X & e_1 & e_2 \\ \hline x_1 & <0.2,0.1,0.3> & <0.1,0.1,0.9> \\ x_2 & <0.3,0.2,0.4> & <0.3,0.4,0.6> \\ x_3 & <0.7,0.5,0.6> & <0.6,0.1,0.7> \\ \hline \end{array} \]

\[ (H, U) = \begin{array}{|c|c|c|} \hline X & e_1 & e_2 \\ \hline x_1 & <0.1,0.6,0.8> & <0.1,0.8,0.9> \\ x_2 & <0.3,0.7,0.5> & <0.3,0.5,0.7> \\ x_3 & <0.3,0.5,0.8> & <0.1,0.7,0.8> \\ \hline \end{array} \]

Let an NSSs \((W, U)\) is defined as

\[ (W, U) = \begin{array}{|c|c|c|} \hline X & e_1 & e_2 \\ \hline x_1 & <0.3,0.7,0.2> & <0.8,0.2,0.3> \\ x_2 & <0.4,0.8,0.3> & <0.6,0.4,0.6> \\ x_3 & <0.5,0.5,0.7> & <0.7,0.2,0.3> \\ \hline \end{array} \]

Then \( \text{int}_{\tau_1}(W, U) = (A, U), \text{int}_{\tau_2}(W, U) = (E, U) \).

\[ (W, U) \subset \text{cl}_{\tau_2}(A, U) = \begin{array}{|c|c|c|} \hline X & e_1 & e_2 \\ \hline x_1 & <0.8,0.9,0.1> & <0.9,0.2,0.1> \\ x_2 & <0.5,0.8,0.3> & <0.7,0.5,0.3> \\ x_3 & <0.6,0.5,0.7> & <0.7,0.3,0.1> \\ \hline \end{array} \]

Then \((W, U)\) is a 12-NSSO set.
$$\begin{array}{|c|c|c|} 
\hline
X & e_1 & e_2 \\
\hline
x_1 & <0.4,0.7,0.1> & <0.8,0.3,0.2> \\
\hline
x_2 & <0.5,0.9,0.2> & <0.7,0.4,0.5> \\
\hline
x_3 & <0.7,0.7,0.3> & <0.8,0.2,0.1> \\
\hline
\end{array}$$

Then \((W,U)\) is also a \(21-NSSO\) set.

$$\begin{array}{|c|c|c|} 
\hline
X & e_1 & e_2 \\
\hline
x_1 & <0.4,0.9,0.2> & <0.7,0.9,0.6> \\
\hline
x_2 & <0.4,0.9,0.2> & <0.7,0.9,0.5> \\
\hline
x_3 & <0.7,0.7,0.3> & <0.4,0.9,0.5> \\
\hline
\end{array}$$

**Definition 4.3.** Let \((X,E,\tau_1,\tau_2)\) be a neutrosophic soft bitopological space and \((N,E) \in NSS(X,E)\). Then \((N,E)\) is called as

1) \(ij\) – neutrosophic soft preclosed \((ij-NSPC)\) if \((N,E)^c\) is a \(ij-NSPO\) set. Equivalently \((N,E)\) is called as \(ij-NSPC\) if \((N,E) \supset cl_{\tau_1} (int_{\tau_1} (N,E))\)

2) \(ij\) – neutrosophic soft semi-closed \((ij-NSSC)\) if \((N,E)^c\) is a \(ij-NSSO\) set. Equivalently \((N,E)\) is called as \(ij-NSSC\) if \((N,E) \supset int_{\tau_1} (cl_{\tau_1} (N,E))\)

3) \(ij\) – neutrosophic soft b-closed \((ij-NSbC)\) \((N,E)^c\) is a \(ij-NSbC\) set. Equivalently \((N,E)\) is called as \(ij-NSbC\) if \((N,E) \supset int_{\tau_1} (cl_{\tau_1} (N,E)) \cap cl_{\tau_1} (int_{\tau_1} (N,E))\).

4) \(ij\) – neutrosophic soft \(\beta\)-closed \((ij-NS\beta C)\) \((N,E)^c\) is a \(ij-NS\beta O\) set. Equivalently \((N,E)\) is called as \(ij-NS\beta C\) if \((N,E) \supset int_{\tau_1} (cl_{\tau_1} (int_{\tau_1} (N,E)))\).

**Theorem 4.4.** Let \((X,E,\tau_1,\tau_2)\) be a neutrosophic soft bitopological space and \((N,E) \in NSS(X,E)\). If \((N,E) \in \tau_j^c\) and \(ij-NSPO\) then \((N,E)\) is a \(ij-NSSO\) set.

**Proof.** Let \((N,E) \in \tau_j^c\) and \(ij-NSPO\). Then \((N,E) = cl_{\tau_j} (N,E)\) and \((N,E) \subset int_{\tau_1} (cl_{\tau_j} (N,E))\). Therefore \((N,E) \subset int_{\tau_1} (N,E) \subset cl_{\tau_j} (int_{\tau_1} (N,E))\). \(\square\)

**Theorem 4.5.** Let \((X,E,\tau_1,\tau_2)\) be a neutrosophic soft bitopological space and \((N,E) \in NSS(X,E)\). If \((N,E) \in \tau_j^c\) and \(ij-NSPO\) then \((N,E)\) is a \(ij-NSSO\) set.
Proof. Let \((N, E)\) be \(ij - NSPO\). Then \((N, E) \subseteq int_{\tau_1}(cl_{\tau_j}(N, E))\). Since \((N, E) \in \tau_j^c\), then \((N, E) = cl_{\tau_j}(N, E)\). Therefore \((N, E) \subseteq int_{\tau_1}(N, E) \subseteq cl_{\tau_1}(int_{\tau_1}(N, E))\). \(\square\)

**Theorem 4.6.** Let \((X, E, \tau_1, \tau_2)\) be a neutrosophic soft bitopological space and \((N, E), (M, E) \in NSS(X, E)\). If \((N, E)\) is \(ij - NSPO\) and \((M, E) \in \tau_1 \cap \tau_2\) then \((N, E) \cup (M, E)\) is \(ij - NSPO\).

Proof. Let \((N, E)\) is \(ij - NSPO\) and \((M, E) \in \tau_1 \cap \tau_2\). Then \((N, E) \subseteq int_{\tau_1}(cl_{\tau_j}(N, E))\) and \(int_{\tau_1}(M, E) = (M, E)\).

\[
(N, E) \cup (M, E) \subseteq int_{\tau_1}(cl_{\tau_j}(N, E)) \cup int_{\tau_1}(M, E)
\]
\[
\subseteq int_{\tau_1}(cl_{\tau_j}(N, E) \cup (M, E)) \subseteq int_{\tau_1}(cl_{\tau_j}(N, E) \cup cl_{\tau_j}(M, E))
\]
\[
= int_{\tau_1}(cl_{\tau_j}((N, E) \cup (M, E)))
\]

Therefore \((N, E) \cup (M, E)\) is \(ij - NSPO\). \(\square\)

**Theorem 4.7.** Let \((X, E, \tau_1, \tau_2)\) be a neutrosophic soft bitopological space. Then

1) Every \(ij - NSPO\) set is \(ji - NSbO\).

2) Every \(ij - NSSO\) set is \(ji - NSbO\).

3) Every \(ij - NSSO\) set is \(ij - NS\beta O\).

Proof.

1) Let \((N, E) \in NSS(X, E)\) be \(ij - NSPO\) set. Then \((N, E) \subseteq int_{\tau_1}(cl_{\tau_j}(N, E)) \subseteq cl_{\tau_j}(int_{\tau_1}(N, E)) \cup int_{\tau_1}(cl_{\tau_j}(N, E))\).

2) Let \((N, E)\) be a \(ij - NSSO\) set. Then \((N, E) \subseteq cl_{\tau_j}(int_{\tau_1}(N, E)) \subseteq cl_{\tau_j}(int_{\tau_1}(N, E)) \cup int_{\tau_1}(cl_{\tau_j}(N, E))\).

3) Let \((N, E)\) be a \(ij - NSSO\) set. Then since \((N, E) \subseteq cl_{\tau_j}(N, E)\),

\[
(N, E) \subseteq cl_{\tau_j}(int_{\tau_1}(N, E)) \subseteq cl_{\tau_j}(int_{\tau_1}(cl_{\tau_j}(N, E)))
\]

\(\square\)

**Theorem 4.8.** Let \((X, E, \tau_1, \tau_2)\) be a neutrosophic soft bitopological space. Then

1) Union of any \(ij - NSPO\) set is \(ij - NSPO\).

2) Union of any \(ij - NSSO\) set is \(ij - NSSO\).

3) Union of any \(ij - NSbO\) set is \(ij - NSbO\).

4) Union of any \(ij - NS\beta O\) set is \(ij - NS\beta O\).

5) Intersection of any \(ij - NSPC\) set is \(ij - NSPC\).

6) Intersection of any \(ij - NSSO\) set is \(ij - NSSO\).
7) **Intersection of any** \( ij - NSbO \) **set is** \( ij - NSbC \).
8) **Intersection of any** \( ij - NSβO \) **set is** \( ij - NSβC \).

**Proof.**

1) Let \((N_k, E)\) be \( ij - NSPO \) set in \((X, E, \tau_1, \tau_2)\) for all \( k \in I \). Then

\[
\bigcup_{k \in I} (N_k, E) \subset \bigcup_{k \in I} \text{int}_{\tau_i} \left( \text{cl}_{\tau_j}(N_k, E) \right) \subset \text{int}_{\tau_i} \left( \bigcup_{k \in I} \left( \text{cl}_{\tau_j}(N_k, E) \right) \right) = \text{int}_{\tau_i} \left( \text{cl}_{\tau_j} \left( \bigcup_{k \in I} (N_k, E) \right) \right)
\]

2) Let \((N_k, E)\) be \( ij - NSSO \) set in \((X, E, \tau_1, \tau_2)\) for all \( k \in I \). Then

\[
\bigcup_{k \in I} (N_k, E) \subset \bigcup_{k \in I} \text{cl}_{\tau_j} \left( \text{int}_{\tau_i}(N_k, E) \right) = \text{cl}_{\tau_j} \left( \bigcup_{k \in I} \left( \text{int}_{\tau_i}(N_k, E) \right) \right) \subset \text{cl}_{\tau_j} \left( \text{int}_{\tau_i} \left( \bigcup_{k \in I} (N_k, E) \right) \right)
\]

3) Let \((N_k, E)\) be \( ij - NSbO \) set in \((X, E, \tau_1, \tau_2)\) for all \( k \in I \). Then

\[
\bigcup_{k \in I} (N_k, E) \subset \bigcup_{k \in I} \left( \text{cl}_{\tau_i} \left( \text{int}_{\tau_j}(N_k, E) \right) \cup \text{int}_{\tau_j} \left( \text{cl}_{\tau_i}(N_k, E) \right) \right)
\]

\[
= \left( \bigcup_{k \in I} \text{cl}_{\tau_i} \left( \text{int}_{\tau_j}(N_k, E) \right) \right) \cup \left( \bigcup_{k \in I} \text{int}_{\tau_j} \left( \text{cl}_{\tau_i}(N_k, E) \right) \right)
\]

\[
\subset \text{cl}_{\tau_i} \left( \text{int}_{\tau_j} \left( \bigcup_{k \in I} (N_k, E) \right) \right) \cup \text{int}_{\tau_j} \left( \text{cl}_{\tau_i} \left( \bigcup_{k \in I} (N_k, E) \right) \right)
\]

4) Let \((N_k, E)\) be \( ij - NSβO \) set in \((X, E, \tau_1, \tau_2)\) for all \( k \in I \). Then

\[
\bigcup_{k \in I} (N_k, E) \subset \bigcup_{k \in I} \text{cl}_{\tau_j} \left( \text{int}_{\tau_i}(\text{cl}_{\tau_j}(N_k, E)) \right) = \text{cl}_{\tau_j} \left( \bigcup_{k \in I} \text{int}_{\tau_i}(\text{cl}_{\tau_j}(N_k, E)) \right) \subset \text{cl}_{\tau_j} \left( \text{int}_{\tau_i} \left( \bigcup_{k \in I} (N_k, E) \right) \right)
\]

The rest of the theorem can be proved easily by taking the complement of 1-4. \( \square \)

5. **Conclusion**

In this paper, we defined neutrosophic soft supra closure operator in a neutrosophic soft bitopological space and investigated some properties of it. Then we obtained a neutrosophic soft topology with this closure operator. Also we defined neutrosophic soft supra interior operator and obtained a neutrosophic soft topology with this interior operator. In the section 4, we defined some new generalized open sets in neutrosophic soft bitopological spaces such as \( ij - \) neutrosophic soft preopen, \( ij - \) neutrosophic soft semi-open, \( ij - \) neutrosophic soft b-open, \( ij - \) neutrosophic soft \( β\)-open set. We examined the relationships between these newly defined open sets.

Received: date / Accepted: date
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Received: Nov 26, 2021. Accepted: Feb 3, 2022