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Sibel Demiralp

Hasan Dadas

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Generalized Open Sets in Neutrosophic Soft Bitopological Spaces

Sibel Demiralp[1,*], Hasan Dadas[2]
[1]Department of Mathematics, University of Kastamonu,Turkey; sdemiralp@kastamonu.edu.tr
[2]Department of Mathematics, University of Kastamonu,Turkey; hsn.dad@gmail.com
*Correspondence: sdemiralp@kastamonu.edu.tr

Abstract. In this study, some generalized neutrosophic soft open sets are defined in neutrosophic soft bitopological spaces. Also, some theorems related to the subject have been given with their proofs and supported with examples for a better understanding of the subject.

Keywords: Neutrosophic set; Neutrosophic soft set; Neutrosophic soft bitopological space; Generalized neutrosophic soft closed (open) set.

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1. Introduction

Neutrosophy was defined by Smarandache in 2013 for the first time [15]. After that, this topic became very popular in the scientific world, and many studies have been done in this area to date. Salama and Alblowi developed topological structure on neutrosophic sets in 2012 [14]. The concept of neutrosophic bitopological space was defined in 2019 by Ozturk and Alkan [13]. Then in 2020, neutrosophic interior, closure and boundary were defined in neutrosophic bitopological spaces by Mwchahary and Bhimraj [12]. Some generalized open sets were defined in neutrosophic bitopological spaces [5,6]. In 2013, neutrosophic soft set was defined by Maji [10]. The concept of neutrosophic soft topological space was defined in 2017 by Bera and Nirmal [2]. Neutrosophic soft bitopological space was defined in [4].
In this study some generalized open sets are defined in neutrosophic soft bitopological spaces.

2. Preliminiaries

[1] Let X be a space of points. A neutrosophic set (NS) A in X is characterized by a falsity-membership function F, a indeterminacy-membership function I and a truth-membership

function T where $F, I, T : X \to [0,1]$, $0 \leq T(x) + I(x) + F(x) \leq 3$. The set of all neutrosophic set in X is denoted by N^X.

Definition 2.1. [14] Let $B, D \in N^X$. Then

(1) Subset: $D \subset B$ if $T_D(z) \leq T_B(z), I_D(z) \leq I_B(z), F_D(z) \geq F_B(z)$ for all $z \in X$.

(2) Equality: $D = B$ if $D \subset B$ and $B \subset D$.

(3) Intersection:

$$D \cap B = \{< z, min\{T_D(z), T_B(z)\}, max\{I_D(z), I_B(z)\}, max\{F_D(z), F_B(z)\} >: z \in X\}$$

(4) Union:

$$D \cup B = \{< z, max\{T_D(z), T_B(z)\}, min\{I_D(x), I_B(x)\}, min\{F_D(z), F_B(z)\} >: z \in X\}$$

The intersection and the union of a collection of NSs $\{D_i\} \in I$ are defined by:

$$\bigcap_{i \in I} D_i = \{< z, \inf\{T_i(z)\}, \sup\{I_{D_i}(z)\}, \sup\{F_{D_i}(z)\} >: z \in X\}$$

$$\bigcup_{i \in I} D_i = \{< z, \sup\{T_{D_i}(z)\}, \inf\{I_{D_i}(z)\}, \inf\{F_{D_i}(z)\} >: z \in X\}$$

(5) The neutrosophic set defined as $T_D(z) = 1, I_D(z) = 1$ and $F_D(z) = 0$ for all $z \in X$ is called the universal NS denoted by 1_X . Also the neutrosophic set defined as $T_D(z) = 0, I_D(z) = 0$ and $F_D(z) = 1$ for all $z \in X$ is called the empty NS denoted by 0_X .

(6) Difference: $D/B = \{< z, T_D(z) - T_B(z), I_D(z) - I_B(z), F_D(z) - F_B(z) >: z \in X\}$

(7) Complement: $D^c = 1_X/D$

Clearly, the complements of 1_X and 0_X are defined:

$$(1_X)^c = 1_X/1_X = \{< z, 0, 1, 1 >: z \in X\} = 0_X$$

$$(0_X)^c = 1_X/0_X = \{< z, 1, 0, 0 >: z \in X\} = 1_X$$

Proposition 2.2. *Let $D_1, D_2, D_3, D_4 \in N(X)$. Then the followings hold:*

(1) $D_1 \cap D_3 \subset D_2 \cap D_4$ *and* $D_1 \cup D_3 \subset D_2 \cup D_4$ *if* $D_1 \subset D_2$ *and* $D_3 \subset D_4$

(2) $(D_1^c)^c = D_1$ *and* $D_1 \subset D_2$ *if* $D_2^c \subset D_1^c$

(3) $(D_1 \cap D_2)^c = D_1^c \cup D_2^c$ *and* $(D_1 \cup D_2)^c = D_1^c \cap D_2^c$

Definition 2.3. Let $\Gamma^n \subset N(Y)$. Then Γ^n is named a neutrosophic topology (NT) on Y if the following conditions hold;

(1) 0_X and 1_X are belong to Γ^n.

(2) Union of any number of NSs in Γ^n is again belong to Γ^n.

(3) Intersection of any two NSs in Γ^n is belong to Γ^n.

Then the pair (Y, Γ^n) is named neutrosophic topology on Y.

2.1. *Neutrosophic Soft Sets*

Definition 2.4. Let U be an initial universe set and E be a set of parameters. Then the pair (H, E) is called as neutrosophic soft set (NSS) over U, where H is a mapping from E to $N(U)$.

The set of all NSS over U is denoted by $NSS(U, E)$. A neutrosophic set (H, E) can be written as: $(H, E) = \{(e, \{< x, T_H(x), I_H(x), F_H(x) >: x \in X\}) : e \in E\}$.

Definition 2.5. Let X be an initial universe set and E be a set of parameters. Then the neutrosophic soft set $x^e_{(\alpha,\beta,\gamma)}$ defined as

$$x^e_{(\alpha,\beta,\gamma)}(e')(y) = \begin{cases} (\alpha, \beta, \gamma) & \text{if } e = e' \text{ and } y = y' \\ (0, 0, 1) & \text{if } e \neq e' \text{ and } y \neq y' \end{cases}$$

for all $x \in X, 0 < \alpha, \beta, \gamma \leq 1, e \in E$, is called a neutrosophic soft point.

Definition 2.6. [2] Let $(H, E), (G, E) \in NSS(U, E)$. Then for all $x \in U$

(1) Subset: $(H, E) \subset (G, E)$ if $T_{H(e)}(x) \leq T_{G(e)}(x), I_{H(e)}(x) \leq I_{G(e)}(x)$ and $F_{H(e)}(x) \geq F_{G(e)}(x)$ for all $e \in E$.

(2) Equality: $(H, E) = (G, E)$ if $(H, E) \subset (G, E)$ and $(G, E) \subset (H, E)$.

(3) Intersection:

$$(H, E)\cap(G, E) = \{(e, \{< x, min\{T_{H(e)}(x), T_{G(e)}(x)\}, max\{I_{H(e)}(x), I_{G(e)}(x)\}, max\{F_{H(e)}(x), F_{G(e)}(x)\} >\} : e \in E\}.$$

(4) Union:

$$(H, E)\cup(G, E) = \{(e, \{< x, max\{T_{H(e)}(x),\ T_{G(e)}(x)\}, min\{I_{H(e)}(x), I_{G(e)}(x)\}, min\{F_{H(e)}(x), F_{G(e)}(x)\} >\} : e \in E\}.$$

The intersection and the union of a collection of $\{(H_i, E)\} \subset NSS(U, E)$ are defined by:

$$\bigcap_{i\in I}(H_i, E) = \left\{\left(e, \{< x, \inf\{T_{H_{i(e)}}(x)\}, \sup\{I_{H_i(e)}(x)\}, \sup\{F_{H_i(e)}(x)\} >\}\right) : e \in E\right\}$$

$$\bigcup_{i\in I}(H_i, E) = \left\{\left(e, \{< x, \sup\{T_{H_{i(e)}}(x)\}, \inf\{I_{H_{i(e)}}(x)\}, \inf\{F_{H_{i(e)}}(x)\} >\}\right) : e \in E\right\}$$

(5) The NSS defined as $T_{H(e)}(x) = 1, I_{H(e)}(x) = 0$ and $F_{H(e)}(x) = 0$, for all $e \in E$ and $x \in U$ is called the universal NSS denoted by $1_{(U,E)}$. Also the neutrosophic set defined as $T_{H(e)}(x) = 0$ $I_{H(e)}(x) = 1$ and $F_{H(e)}(x) = 1$ for all $e \in E$ and $x \in U$ is called the empty NSS denoted by $0_{(U;E)}$.

(6) Complement:

$$(H, E)^c = 1_{(X,E)}/(H, E) = \{(e, \{< x, F_{H(e)}(x), 1 - I_{H(e)}(x), T_{H(e)}(x) >\} : e \in E\}$$

Clearly, the complements of $1_{(X,E)}$ and $0_{(X,E)}$ are defined:

$$(1_{(X,E)})^c = 1_{(X,E)}/1_{(X,E)} = \{(e, \{< x, 0, 1, 1 >\} : e \in E\} = 0_{(X,E)}$$

$$(0_{(X,E)})^c = 1_{(X,E)}/0_{(X,E)} = \{(e, \{< x, 1, 0, 0 >\} : e \in E\} = 1_{(X,E)}$$

Definition 2.7. [1] Let $\tau \subset NSS(Y, E)$. Then τ is called as a neutrosophic soft topology on Y if the following conditions hold:

NST_1) $0_{(Y,E)}, 1_{(Y,E)} \in \tau$

NST_2) Union of any number of NSSs in τ is belong to τ.

NST_3) Intersection of finite number of NSSs in τ is belong to τ.

Then (Y, E, τ) is called as neutrosophic soft topological space. Any element of τ is called as τ-neutrosophic soft open (τ-NSO) set. A NSS is called as τ-neutrosophic soft closed (τ-NSC) if the complement of the set is τ-NSO. The set of all neutrosophic soft closed sets is denoted by $(\tau)^c$.

Definition 2.8. [1] Let (Y, E, τ) be a neutrosophic soft topological space and $(M, E) \in NSS(Y, E)$. Then the intersection of all τ-NSC sets containing (M, E) is called as closure of (M, E) and denoted by $cl_\tau(M, E)$, i.e. $cl_\tau(M, E) = \bigcap\{(N, E) \in (\tau)^c : (M, E) \subset (N, E)\}$

Theorem 2.9. *[1] Let (Y, E, τ) be a neutrosophic soft topological space and $(M, E), (N, E) \in NSS(Y, E)$. Then*

cl_1) $(M, E) \subset cl_\tau(M, E)$

cl_2) $(M, E) \subset (N, E)$ *then* $cl_\tau(M, E) \subset cl_\tau(N, E)$

cl_3) $cl_\tau((M, E) \cap (N, E)) \subset cl_\tau(M, E) \cap cl_\tau(N, E)$

cl_4) $cl_\tau((M, E) \cup (N, E)) = cl_\tau(M, E) \cup cl_\tau(N, E)$

Definition 2.10. Let $\tau \subset NSS(Y, E)$. Then τ is called as a neutrosophic soft supra topology on Y if it satisfies just NST_1) and NST_2).

Definition 2.11. Let (Y, E, τ) be a neutrosophic soft topological space and $(M, E) \in NSS(Y, E)$. Then the union of all τ-NSO sets subset of (M, E) is called as interior of (M, E) and denoted by $int_\tau(M, E)$, i.e. $int_\tau(M, E) = \bigcup\{(N, E) \in \tau : (N, E) \subset (M, E)\}$

Theorem 2.12. *[1] Let (Y, E, τ) be a neutrosophic soft topological space and $(M, E), (N, E) \in NSS(Y, E)$. Then*

int_1) $int_\tau(M, E) \subset (M, E)$

int_2) $(M, E) \subset (N, E)$ *then* $int_\tau(M, E) \subset int_\tau(N, E)$

int_3) $int_\tau((M, E) \cap (N, E)) = int_\tau(M, E) \cap int_\tau(N, E)$

int_4) $int_\tau(M, E) \cup int_\tau(N, E) \subset int_\tau((M, E) \cup (N, E))$

3. Neutrosophic Soft Bitopological Space

Definition 3.1. If (Y, τ_1, E) and (Y, τ_2, E) are two neutrosophic soft topological space, then (Y, E, τ_1, τ_2) is named as neutrosophic soft bitopological space. The sets belong to τ_i are called as neutrosophic soft τ_i-open set for $i = 1, 2$.

Definition 3.2. An operator $C : NSS(X, E) \to NSS(X, E)$ is called a neutrosophic soft supra closure operator if it satisfies the following conditions for all $(N, E), (M, E) \in NSS(X, E)$,

C_1) $C(0_{(X,E)}) = 0_{(X,E)}$
C_2) $(N, E) \subset C(N, E)$
C_3) $C(N, E) \cup C(M, E) \subset C(N \cup M)$
C_4) $C(C(N, E)) = C(N, E)$.

Theorem 3.3. *Let (X, E, τ_1, τ_2) be a neutrosophic soft bitopological space. Then, the operator $cl_{12} : NSS(X, E) \to NSS(X, E)$ defined as $cl_{12}(N, E) = cl_{\tau_1}(N, E) \cap cl_{\tau_1}(N, E)$ is a neutrosophic soft supra closure operator on (X, E) and induces the supra neutrosophic soft topology $\tau_{12} = \{(M, E) \in NSS(X, E) : cl_{12}((M, E)^c) = (M, E)^c\}$.*

Proof. First let prove that cl_{12} is a neutrosophic soft supra closure operator.

C_1) $cl_{12}(0_{(X,E)}) = cl_{\tau_1}(0_{(X,E)}) \cap cl_{\tau_2}(0_{(X,E)}) = 0_{(X,E)} \cap 0_{(X,E)} = 0_{(X,E)}$
C_2) $(N, E) \subset cl_{\tau_1}(N, E)$ and $(N, E) \subset cl_{\tau_2}(N, E)$. Then $(N, E) \subset cl_{\tau_1}(N, E) \cap cl_{\tau_2}(N, E) = cl_{12}(N, E)$.
C_3)

$$\begin{aligned} cl_{12}(N, E) \cup cl_{12}(M, E) &= [cl_{\tau_1}(N, E) \cap cl_{\tau_2}(N, E)] \cup [cl_{\tau_1}(M, E) \cap cl_{\tau_2}(M, E)] \\ &= cl_{\tau_1}[(N, E) \cup (M, E)] \cap [cl_{\tau_2}(N, E) \cup cl_{\tau_1}(M, E)] \\ &\quad \cap [cl_{\tau_1}(N, E) \cup cl_{\tau_2}(M, E)] \cap cl_{\tau_2}[(N, E) \cup (M, E)] \\ &\subset cl_{\tau_1}[(N, E) \cup (M, E)] \cap cl_{\tau_2}[(N, E) \cup (M, E)] \\ &= cl_{12}[(N, E) \cup (M, E)]. \end{aligned}$$

C_4) From C_3, $cl_{12}(N, E) \subset cl_{12}(cl_{12}(N, E))$. Also

$$\begin{aligned} cl_{12}(cl_{12}(N, E)) &= cl_{12}(cl_{\tau_1}(N, E) \cap cl_{\tau_2}(N, E)) \\ &= cl_{\tau_1}((cl_{\tau_1}(N, E) \cap cl_{\tau_2}(N, E)) \cap cl_{\tau_2}((cl_{\tau_1}(N, E) \cap cl_{\tau_2}(N, E)) \\ &\subset cl_{\tau_1}((cl_{\tau_1}(N, E)) \cap cl_{\tau_1}(cl_{\tau_2}(N, E)) \cap cl_{\tau_2}(cl_{\tau_1}(N, E)) \cap cl_{\tau_2}(cl_{\tau_2}(N, E)) \\ &\subset cl_{\tau_1}(N, E) \cap cl_{\tau_2}(N, E) = cl_{12}(N, E). \end{aligned}$$

Therefore $cl_{12}(N, E) = cl_{12}(cl_{12}(N, E))$.

Now let prove that τ_{12} is a neutrosophic soft supra topology.

NST_1) Since $cl_{12}\left((1_{(X,E)})^c\right) = cl_{12}(0_{(X,E)}) = 0_{(X,E)}$, then $0_{(X,E)} \in \tau_{12}$. Also $cl_{12}\left((0_{(X,E)})^c\right) = cl_{12}(1_{(X,E)}) \subset 1_{(X,E)}$ and from (C_2), $1_{(X,E)} \subset cl_{12}(1_{(X,E)})$. Therefore $0_{(X,E)} \in \tau_{12}$.

NST_2) Let $(N_i, E) \in \tau_{12}$. Then $cl_{12}\left((N_i, E)^c\right) = (N_i, E)^c$.

$$
\begin{aligned}
cl_{12}\left(\bigcup_{i\in I}(N_i, E)\right)^c &= cl_{\tau_1}\left(\bigcup_{i\in I}(N_i, E)\right)^c \cap cl_{\tau_2}\left(\bigcup_{i\in I}(N_i, E)\right)^c \\
&= cl_{\tau_1}\left(\bigcap_{i\in I}(N_i, E)^c\right) \cap cl_{\tau_2}\left(\bigcap_{i\in I}(N_i, E)^c\right) \\
&\subset \bigcap_{i\in I}(cl_{\tau_1}(N_i, E)^c) \cap \bigcap_{i\in I}(cl_{\tau_2}(N_i, E)^c) \\
&= \bigcap_{i\in I}(cl_{\tau_1}(N_i, E)^c \cap cl_{\tau_2}(N_i, E)^c) \\
&= \bigcap_{i\in I}(cl_{12}(N_i, E)^c) = \bigcap_{i\in I}(N_i, E)^c = \left(\bigcup_{i\in I}(N_i, E)\right)^c.
\end{aligned}
$$

Also from (C_2), $\left(\bigcup_{i\in I}(N_i, E)\right)^c \subset cl_{12}\left(\bigcup_{i\in I}(N_i, E)\right)^c$. Therefore $cl_{12}\left(\bigcup_{i\in I}(N_i, E)\right)^c = \left(\bigcup_{i\in I}(N_i, E)\right)^c$, then $\bigcup_{i\in I}(N_i, E) \in \tau_{12}$.

Consequently τ_{12} is a neutrosophic soft supra topology on (X, E). □

Theorem 3.4. *Let (X, E, τ_1, τ_2) be a neutrosophic soft bitopological space and $(N, E) \in NSS(X, E)$. Then $(N, E) \in \tau_{12}$ if and only if there exists a τ_1-NSC set (N_1, E) and τ_2-NSC set (N_2, E) such that $(N, E) = (N_1, E) \cap (N_2, E)$.*

Proof. If we take $(N_1, E) = cl_{\tau_1}(N, E)$ and $(N_2, E) = cl_{\tau_2}(N, E)$, then proof is clear. □

Theorem 3.5. *Let (X, E, τ_1, τ_2) be a neutrosophic soft bitopological space and $(M, E), (N, E) \in NSS(X, E)$. Then*

1) *if $(M, E) \subset (N, E)$ then $cl_{12}(M, E) \subset cl_{12}(N, E)$.*
2) *$cl_{12}\left((M, E) \cap (N, E)\right) \subset cl_{12}(M, E) \cap cl_{12}(N, E)$.*

Proof. For any $(M, E), (N, E) \in NSS(X, E)$,

1) Let $(M, E) \subset (N, E)$. Then $cl_{\tau_1}(M, E) \subset cl_{\tau_1}(N, E)$ and $cl_{\tau_2}(M, E) \subset cl_{\tau_2}(N, E)$. Therefore $cl_{\tau_1}(M, E) \cap cl_{\tau_2}(M, E) \subset cl_{\tau_1}(N, E) \cap cl_{\tau_2}(N, E)$. So $cl_{12}(M, E) \subset cl_{12}(N, E)$.
2) $(M, E) \cap (N, E) \subset (M, E)$ and $(M, E) \cap (N, E) \subset (N, E)$. Then from (1), $cl_{12}((M, E) \cap (N, E)) \subset cl_{12}(M, E)$ and $cl_{12}((M, E) \cap (N, E)) \subset cl_{12}(N, E)$. Therefore $cl_{12}\left((M, E) \cap (N, E)\right) \subset cl_{12}(M, E) \cap cl_{12}(N, E)$.

· □

Remark 3.6. Let (X, E, τ_1, τ_2) be a neutrosophic soft bitopological space. Then $cl_{12}(M, E) \cap cl_{12}(N, E) \neq cl_{12}((M, E) \cap (N, E))$, in general.

Example 3.7. Let the neutrosophic soft bitopological space (X, U, τ_1, τ_2) be defined as $X = \{x_1, x_2, x_3\}$, $U = \{e_1, e_2\}$, $\tau_1 = \{0_{(X,U)}, 1_{(X,U)}, (A, U), (B, U), (C, U), (D, U)\}$, $\tau_2 = \{0_{(X,U)}, 1_{(X,U)}, (D, U), (F, U), (G, U), (H, U)\}$ where the tabular representations of NSSs are as follows:

$(A, U) =$

X	e_1	e_2
x_1	$< 0.2, 0.3, 0.8 >$	$< 0.9, 0.1, 0.3 >$
x_2	$< 0.1, 0.5, 0.4 >$	$< 0.4, 0.4, 0.4 >$
x_3	$< 0.8, 0.1, 0.5 >$	$< 0.2, 0.8, 0.1 >$

$(B, U) =$

X	e_1	e_2
x_1	$< 0.1, 0.3, 0.8 >$	$< 0.3, 0.1, 0.7 >$
x_2	$< 0.1, 0.1, 0.4 >$	$< 0.1, 0.2, 0.5 >$
x_3	$< 0.3, 0.1, 0.5 >$	$< 0.2, 0.1, 0.3 >$

$(C, U) =$

X	e_1	e_2
x_1	$< 0.2, 0.3, 0.4 >$	$< 0.9, 0.1, 0.3 >$
x_2	$< 0.2, 0.1, 0.3 >$	$< 0.4, 0.2, 0.4 >$
x_3	$< 0.8, 0.1, 0.5 >$	$< 0.6, 0.1, 0.1 >$

$(D, U) =$

X	e_1	e_2
x_1	$< 0.1, 0.3, 0.4 >$	$< 0.3, 0.2, 0.7 >$
x_2	$< 0.2, 0.1, 0.3 >$	$< 0.1, 0.2, 0.5 >$
x_3	$< 0.3, 0.7, 0.8 >$	$< 0.6, 0.1, 0.3 >$

$(F, U) =$

X	e_1	e_2
x_1	$< 0.7, 0.1, 0.1 >$	$< 0.2, 0.5, 0.5 >$
x_2	$< 0.9, 0.5, 0.3 >$	$< 0.3, 0.8, 0.1 >$
x_3	$< 0.1, 0.8, 0.1 >$	$< 0.8, 0.2, 0.7 >$

$(G, U) =$

X	e_1	e_2
x_1	$< 0.1, 0.1, 0.4 >$	$< 0.2, 0.2, 0.7 >$
x_2	$< 0.2, 0.1, 0.3 >$	$< 0.1, 0.2, 0.5 >$
x_3	$< 0.1, 0.7, 0.8 >$	$< 0.6, 0.1, 0.7 >$

$(H, U) =$

X	e_1	e_2
x_1	$< 0.7, 0.1, 0.1 >$	$< 0.3, 0.2, 0.5 >$
x_2	$< 0.9, 0.1, 0.3 >$	$< 0.3, 0.2, 0.1 >$
x_3	$< 0.3, 0.7, 0.1 >$	$< 0.8, 0.1, 0.3 >$

Let two NSSs (X_1, U) and (X_2, U) are defined as

$(X_1, U) =$

X	e_1	e_2
x_1	$< 0.8, 0.5, 0.1 >$	$< 0.7, 0.1, 0.3 >$
x_2	$< 0.5, 0.9, 0.1 >$	$< 0.8, 0.1, 0.1 >$
x_3	$< 0.5, 0.8, 0.2 >$	$< 0.5, 0.9, 0.2 >$

$(X_2, U) =$

X	e_1	e_2
x_1	$< 0.9, 0.7, 0.1 >$	$< 0.9, 0.9, 0.1 >$
x_2	$< 0.4, 0.5, 0.1 >$	$< 0.5, 0.8, 0.1 >$
x_3	$< 0.7, 0.9, 0.3 >$	$< 0.3, 0.9, 0.1 >$

Then $cl_{12}((X_1, U) \cap (X_2, U)) = (B, U)^c$ and $cl_{12}(X_1, U) = cl_{12}(X_2, U) = 1_{(X,U)}$. So $cl_{12}(X_1, U) \cap cl_{12}(X_2, U) \not\subset cl_{12}((X_1, U) \cap (X_2, U))$

Definition 3.8. An operator $I : NSS(X, E) \to NSS(X, E)$ is called a neutrosophic soft supra interior operator if it satisfies the following conditions for all $(N, E), (M, E) \in NSS(X, E)$,

I_1) $I(0_{(X,E)}) = 0_{(X,E)}$

I_2) $I(N, E) \subset (N, E)$

I_3) $I(N, E) \cap I(M, E) \subset I(N \cap M)$

I_4) $I(I(N, E)) = I(N, E)$.

Theorem 3.9. *Let (X, E, τ_1, τ_2) be a neutrosophic soft bitopological space. Then, the operator $int_{12} : NSS(X, E) \to NSS(X, E)$ defined as $int_{12}(N, E) = int_{\tau_1}(N, E) \cup int_{\tau_1}(N, E)$ is a neutrosophic soft supra interior operator on (X, E) and induces the supra neutrosophic soft topology $\tau_{12} = \{(M, E) \in NSS(X, E) : int_{12}(M, E) = (M, E)\}$.*

Proof. First let prove that int_{12} is a neutrosophic soft supra interior operator.

$I_1)$ $int_{12}(0_{(X,E)}) = int_{\tau_1}(0_{(X,E)}) \cup int_{\tau_2}(0_{(X,E)}) = 0_{(X,E)} \cup 0_{(X,E)} = 0_{(X,E)}$

$I_2)$ $int_{\tau_1}(N,E) \subset (N,E)$ and $int_{\tau_2}(N,E) \subset (N,E)$. Then $int_{\tau_1}(N,E) \cup int_{\tau_2}(N,E) \subset (N,E)$. Therefore $int_{\tau_{12}}(N,E) \subset (N,E)$

$I_3)$

$$\begin{aligned} int_{12}(N,E) \cap int_{12}(M,E) &= [int_{\tau_1}(N,E) \cup int_{\tau_2}(N,E)] \cap [int_{\tau_1}(M,E) \cup int_{\tau_2}(M,E)] \\ &= int_{\tau_1}[(N,E) \cap (M,E)] \cup [int_{\tau_2}(N,E) \cap int_{\tau_1}(M,E)] \\ &\quad \cup [int_{\tau_1}(N,E) \cap int_{\tau_2}(M,E)] \cup int_{\tau_2}[(N,E) \cap (M,E)] \\ &= int_{\tau_{12}}[(N,E) \cap (M,E)] \cup [int_{\tau_2}(N,E) \cap int_{\tau_1}(M,E)] \\ &\quad \cup [int_{\tau_1}(N,E) \cap int_{\tau_2}(M,E)] \\ &\subset int_{12}[(N,E) \cap (M,E)]. \end{aligned}$$

$I_4)$ From (I_3), $int_{12}(int_{12}(N,E)) \subset int_{12}(N,E)$. Also

$$\begin{aligned} int_{12}(N,E) &= int_{\tau_1}(N,E) \cup int_{\tau_2}(N,E)) \\ &= int_{\tau_1}(int_{\tau_1}(N,E)) \cup int_{\tau_2}(int_{\tau_2}(N,E)) \\ &\subset int_{\tau_1}(int_{\tau_1}(N,E)) \cup int_{\tau_1}(int_{\tau_2}(N,E)) \cup int_{\tau_2}(int_{\tau_1}(N,E)) \cup int_{\tau_2}(int_{\tau_2}(N,E)) \\ &\subset int_{\tau_1}(int_{\tau_1}(N,E)) \cup int_{\tau_2}(N,E)) \cup int_{\tau_2}(int_{\tau_1}(N,E)) \cup int_{\tau_2}(N,E)) \\ &= int_{\tau_1}(int_{12}(N,E)) \cup int_{\tau_2}(int_{12}(N,E)) \\ &= int_{12}(int_{12}(N,E)). \end{aligned}$$

Therefore $int_{12}(N,E) = int_{12}(int_{12}(N,E))$.

Now let prove that τ_{12} is a neutrosophic soft supra topology.

$NST_1)$ From (I_1), $int_{12}(0_{(X,E)}) = 0_{(X,E)}$, then $0_{(X,E)} \in \tau_{12}$. Also $int_{12}((1_{(X,E)})) = int_{\tau_1}(1_{(X,E)}) \cup int_{\tau_2}(1_{(X,E)}) = 1_{(X,E)}$. Therefore $1_{(X,E)} \in \tau_{12}$.

$NST_2)$ Let $(N_i, E) \in \tau_{12}$. Then $int_{12}((N_i, E)) = (N_i, E)$.

$$\begin{aligned}
\bigcup_{i\in I}(N_i, E) &= \bigcup_{i\in I} int_{12}(N_i, E) \\
&= \bigcup_{i\in I} (int_1(N_i, E) \cup int_2(N_i, E)) \\
&= \left(\bigcup_{i\in I} int_1(N_i, E)\right) \cup \left(\bigcup_{i\in I} int_2(N_i, E)\right) \\
&\subset int_1\left(\bigcup_{i\in I}(N_i, E)\right) \cup int_2\left(\bigcup_{i\in I}(N_i, E)\right) \\
&= int_{12}\left(\bigcup_{i\in I}(N_i, E)\right).
\end{aligned}$$

Also from (I_2),

$$int_{12}\left(\bigcup_{i\in I}(N_i, E)\right) \subset \left(\bigcup_{i\in I}(N_i, E)\right).$$

Therefore $int_{12}\left(\bigcup_{i\in I}(N_i, E)\right) = \left(\bigcup_{i\in I}(N_i, E)\right)$, then $\bigcup_{i\in I}(N_i, E) \in \tau_{12}$.

Consequently τ_{12} is a neutrosophic soft supra topology on (X, E). □

Theorem 3.10. *Let (X, E, τ_1, τ_2) be a neutrosophic soft bitopological space and $(N, E) \in NSS(X, E)$. Then $(N, E) \in \tau_{12}$ if and only if there exists a τ_1-NSO set (N_1, E) and τ_2-NSO set (N_2, E) such that $(N, E) = (N_1, E) \cup (N_2, E)$.*

Proof. If we take $(N_1, E) = int_{\tau_1}(N, E)$ and $(N_2, E) = int_{\tau_2}(N, E)$, then proof is clear. □

Theorem 3.11. *Let (X, E, τ_1, τ_2) be a neutrosophic soft bitopological space and $(M, E), (N, E) \in NSS(X, E)$. Then*

1) *if $(M, E) \subset (N, E)$ then $int_{12}(M, E) \subset int_{12}(N, E)$.*
2) *$int_{12}(M, E) \cup int_{12}(N, E) \subset int_{12}((M, E) \cup (N, E))$.*

Proof. For any $(M, E), (N, E) \in NSS(X, E)$,

1) Let $(M, E) \subset (N, E)$. Then $int_{\tau_1}(M, E) \subset int_{\tau_1}(N, E)$ and $int_{\tau_2}(M, E) \subset int_{\tau_2}(N, E)$. Therefore $int_{\tau_1}(M, E) \cap int_{\tau_2}(M, E) \subset int_{\tau_1}(N, E) \cap int_{\tau_2}(N, E)$. So $int_{12}(M, E) \subset int_{12}(N, E)$.

2) $(M, E) \subset (M, E) \cup (N, E)$ and $(N, E) \subset (M, E) \cup (N, E)$. Then from (1), $int_{12}(M, E) \subset int_{12}((M, E) \cup (N, E))$ and $int_{12}(N, E) \subset int_{12}((M, E) \cup (N, E))$. Therefore $int_{12}(M, E) \cup int_{12}(N, E) \subset int_{12}((M, E) \cup (N, E))$. □

Remark 3.12. Let (X, E, τ_1, τ_2) be a neutrosophic soft bitopological space. Then $int_{12}(M, E) \cup int_{12}(N, E) \neq int_{12}((M, E) \cup (N, E))$, in general.

Proposition 3.13. *Let (X, E, τ_1, τ_2) be a neutrosophic soft bitopological space and $(N, E) \in NSS(X, E)$. Then*

1) $\tau_1, \tau_2 \subset \tau_{12}$
2) $cl_{12}(N, E) = (int_{12}(N, E)^c)^c$
3) $int_{12}(N, E) = (cl_{12}(N, E)^c)^c$

Proof. 1) Let $(N, E) \in \tau_1$. Then $(N, E) = int_{\tau_1}(N, E)$. Therefore

$$int_{12}(N, E) = int_{\tau_1}(N, E) \cup int_{\tau_2}(N, E) = (N, E)$$

So $\tau_1 \subset \tau_{12}$. Similar for $\tau_2 \subset \tau_{12}$.

2)

$$\begin{aligned} cl_{12}(N, E) = cl_{\tau_1}(N, E) \cap cl_{\tau_2}(N, E) &= \left(\bigcap_{j\in I}(F_j^1, E)\right) \cap \left(\bigcap_{j\in J}(F_j^2, E)\right) \\ &= \left[\bigcup_{j\in I}(F_j^1, E)^c\right]^c \cap \left[\bigcup_{j\in J}(F_j^2, E)^c\right]^c \\ &= \left[\left(\bigcup_{j\in I}(F_j^1, E)^c\right) \cup \left(\bigcup_{j\in J}(F_j^2, E)^c\right)\right]^c \\ &= [int_{\tau_1}(N, E)^c \cup int_{\tau_2}(N, E)^c]^c = (int_{12}(N, E)^c)^c \end{aligned}$$

where $(N, E) \subset (F_j^i, E)$, $(F_j^i, E)^c \in \tau_i$ for all $j \in I, J$ and $i = 1, 2$.

3)

$$\begin{aligned} int_{12}(N, E) = int_{\tau_1}(N, E) \cup int_{\tau_2}(N, E) &= \left(\bigcup_{j\in I}(U_j^1, E)\right) \cup \left(\bigcup_{j\in J}(U_j^2, E)\right) \\ &= \left[\bigcap_{j\in I}(U_j^1, E)^c\right]^c \cup \left[\bigcap_{j\in J}(U_j^2, E)^c\right]^c \\ &= \left[\left(\bigcap_{j\in I}(U_j^1, E)^c\right) \cap \left(\bigcap_{j\in J}(U_j^2, E)^c\right)\right]^c \\ &= [cl_{\tau_1}(N, E)^c \cap cl_{\tau_2}(N, E)^c]^c = (cl_{12}(N, E)^c)^c \end{aligned}$$

where $(U_j^i, E) \subset (N, E)$, $(U_j^i, E) \in \tau_i$ for all $j \in I, J$ and $i = 1, 2$.

. □

4. Some Generalized Open Sets in Neutrosophic Soft Bitopological Spaces

Throughout this section, $i, j = 1, 2$ and $i \neq j$.

Definition 4.1. Let (X, E, τ_1, τ_2) be a neutrosophic soft bitopological space and $(N, E) \in NSS(X, E)$. Then (N, E) is called as

1) $ij-$ neutrosophic soft preopen $(ij - NSPO)$ if $(N, E) \subset int_{\tau_i}\left(cl_{\tau_j}(N, E)\right)$
2) $ij-$ neutrosophic soft semi-open $(ij - NSSO)$ if $(N, E) \subset cl_{\tau_j}\left(int_{\tau_i}(N, E)\right)$
3) $ij-$ neutrosophic soft b-open $(ij - NSbO)$ if $(N, E) \subset cl_{\tau_i}\left(int_{\tau_j}(N, E)\right) \cup int_{\tau_j}\left(cl_{\tau_i}(N, E)\right)$.
4) $ij-$ neutrosophic soft β-open $(ij - NS\beta O)$ if $(N, E) \subset cl_{\tau_j}\left(int_{\tau_i}(cl_{\tau_j}(N, E))\right)$.

Example 4.2. Let the neutrosophic soft bitopological space (X, U, τ_1, τ_2) be defined as $X = \{x_1, x_2, x_3\}$, $U = \{e_1, e_2\}$, $\tau_1 = \{0_{(X,U)}, 1_{(X,U)}, (A, U), (B, U), (C, U), (D, U)\}$, $\tau_2 = \{0_{(X,U)}, 1_{(X,U)}, (E, U), (F, U), (G, U), (H, U)\}$ where the tabular representations of NSSs are as follows:

$(A, U) =$

X	e_1	e_2
x_1	$< 0.2, 0.1, 0.9 >$	$< 0.6, 0.1, 0.7 >$
x_2	$< 0.1, 0.8, 0.4 >$	$< 0.1, 0.1, 0.8 >$
x_3	$< 0.3, 0.4, 0.8 >$	$< 0.5, 0.1, 0.4 >$

$(B, U) =$

X	e_1	e_2
x_1	$< 0.1, 0.3, 0.4 >$	$< 0.2, 0.7, 0.8 >$
x_2	$< 0.2, 0.1, 0.5 >$	$< 0.5, 0.6, 0.7 >$
x_3	$< 0.3, 0.3, 0.7 >$	$< 0.1, 0.8, 0.8 >$

$(C, U) =$

X	e_1	e_2
x_1	$< 0.2, 0.1, 0.4 >$	$< 0.6, 0.1, 0.7 >$
x_2	$< 0.2, 0.1, 0.4 >$	$< 0.5, 0.1, 0.7 >$
x_3	$< 0.3, 0.3, 0.7 >$	$< 0.5, 0.1, 0.4 >$

$(D, U) =$

X	e_1	e_2
x_1	$< 0.1, 0.3, 0.9 >$	$< 0.2, 0.7, 0.8 >$
x_2	$< 0.1, 0.8, 0.5 >$	$< 0.1, 0.6, 0.8 >$
x_3	$< 0.3, 0.4, 0.8 >$	$< 0.1, 0.8, 0.8 >$

$(E,U) =$

X	e_1	e_2
x_1	$< 0.2, 0.6, 0.3 >$	$< 0.1, 0.1, 0.9 >$
x_2	$< 0.3, 0.7, 0.4 >$	$< 0.3, 0.4, 0.6 >$
x_3	$< 0.3, 0.5, 0.8 >$	$< 0.6, 0.1, 0.8 >$

$(F,U) =$

X	e_1	e_2
x_1	$< 0.1, 0.1, 0.8 >$	$< 0.1, 0.8, 0.9 >$
x_2	$< 0.3, 0.2, 0.5 >$	$< 0.3, 0.5, 0.7 >$
x_3	$< 0.7, 0.5, 0.6 >$	$< 0.1, 0.7, 0.7 >$

$(G,U) =$

X	e_1	e_2
x_1	$< 0.2, 0.1, 0.3 >$	$< 0.1, 0.1, 0.9 >$
x_2	$< 0.3, 0.2, 0.4 >$	$< 0.3, 0.4, 0.6 >$
x_3	$< 0.7, 0.5, 0.6 >$	$< 0.6, 0.1, 0.7 >$

$(H,U) =$

X	e_1	e_2
x_1	$< 0.1, 0.6, 0.8 >$	$< 0.1, 0.8, 0.9 >$
x_2	$< 0.3, 0.7, 0.5 >$	$< 0.3, 0.5, 0.7 >$
x_3	$< 0.3, 0.5, 0.8 >$	$< 0.1, 0.7, 0.8 >$

Let an NSSs (W,U) is defined as

$(W,U) =$

X	e_1	e_2
x_1	$< 0.3, 0.7, 0.2 >$	$< 0.8, 0.2, 0.3 >$
x_2	$< 0.4, 0.8, 0.3 >$	$< 0.6, 0.4, 0.6 >$
x_3	$< 0.5, 0.5, 0.7 >$	$< 0.7, 0.2, 0.3 >$

Then $int_{\tau_1}(W,U) = (A,U)$, $int_{\tau_2}(W,U) = (E,U)$.

$(W,U) \subset cl_{\tau_2}(A,U) =$

X	e_1	e_2
x_1	$< 0.8, 0.9, 0.1 >$	$< 0.9, 0.2, 0.1 >$
x_2	$< 0.5, 0.8, 0.3 >$	$< 0.7, 0.5, 0.3 >$
x_3	$< 0.6, 0.5, 0.7 >$	$< 0.7, 0.3, 0.1 >$

Then (W,U) is a $12-NSSO$ set.

$(W,U) \subset cl_{\tau_1}(E,U) =$

X	e_1	e_2
x_1	$< 0.4, 0.7, 0.1 >$	$< 0.8, 0.3, 0.2 >$
x_2	$< 0.5, 0.9, 0.2 >$	$< 0.7, 0.4, 0.5 >$
x_3	$< 0.7, 0.7, 0.3 >$	$< 0.8, 0.2, 0.1 >$

Then (W,U) is also a $21-NSSO$ set.

$cl_{\tau_1}(W,U) =$

X	e_1	e_2
x_1	$< 0.4, 0.9, 0.2 >$	$< 0.7, 0.9, 0.6 >$
x_2	$< 0.4, 0.9, 0.2 >$	$< 0.7, 0.9, 0.5 >$
x_3	$< 0.7, 0.7, 0.3 >$	$< 0.4, 0.9, 0.5 >$

$cl_{\tau_1}(W,U) =$

X	e_1	e_2
x_1	$< 0.4, 0.9, 0.2 >$	$< 0.7, 0.9, 0.6 >$
x_2	$< 0.4, 0.9, 0.2 >$	$< 0.7, 0.9, 0.5 >$
x_3	$< 0.7, 0.7, 0.3 >$	$< 0.4, 0.9, 0.5 >$

Definition 4.3. Let (X,E,τ_1,τ_2) be a neutrosophic soft bitopological space and $(N,E) \in NSS(X,E)$. Then (N,E) is called as

1) $ij-$ neutrosophic soft preclosed $(ij-NSPC)$ if $(N,E)^c$ is a $ij-NSPO$ set. Equivalently (N,E) is called as $ij-NSPC$ if $(N,E) \supset cl_{\tau_i}\left(int_{\tau_j}(N,E)\right)$
2) $ij-$ neutrosophic soft semi-closed $(ij-NSSC)$ if $(N,E)^c$ is a $ij-NSSO$ set. Equivalently (N,E) is called as $ij-NSSC$ if $(N,E) \supset int_{\tau_j}\left(cl_{\tau_i}(N,E)\right)$
3) $ij-$ neutrosophic soft b-closed $(ij-NSbC)$ $(N,E)^c$ is a $ij-NSbC$ set. Equivalently (N,E) is called as $ij-NSbC$ if $(N,E) \supset int_{\tau_i}\left(cl_{\tau_j}(N,E)\right) \cap cl_{\tau_j}\left(int_{\tau_i}(N,E)\right)$.
4) $ij-$ neutrosophic soft β-closed $(ij-NS\beta C)$ $(N,E)^c$ is a $ij-NS\beta O$ set. Equivalently (N,E) is called as $ij-NS\beta C$ if $(N,E) \supset int_{\tau_j}\left(cl_{\tau_i}(int_{\tau_j}(N,E))\right)$.

Theorem 4.4. *Let (X,E,τ_1,τ_2) be a neutrosophic soft bitopological space and $(N,E) \in NSS(X,E)$. If $(N,E) \in \tau_j^c$ and $ij-NSPO$ then (N,E) is a $ij-NSSO$ set.*

Proof. Let $(N,E) \in \tau_j^c$ and $ij-NSPO$. Then $(N,E) = cl_{\tau_j}(N,E)$ and $(N,E) \subset int_{\tau_i}\left(cl_{\tau_j}(N,E)\right)$. Therefore $(N,E) \subset int_{\tau_i}(N,E) \subset cl_{\tau_j}\left(int_{\tau_i}(N,E)\right)$. □

Theorem 4.5. *Let (X,E,τ_1,τ_2) be a neutrosophic soft bitopological space and $(N,E) \in NSS(X,E)$. If $(N,E) \in \tau_j^c$ and $ij-NSPO$ then (N,E) is a $ij-NSSO$ set.*

Proof. Let (N,E) be $ij-NSPO$. Then $(N,E)\subset int_{\tau_i}\left(cl_{\tau_j}(N,E)\right)$. Since $(N,E)\in\tau_j^c$, then $(N,E)=cl_{\tau_j}(N,E)$. Therefore $(N,E)\subset int_{\tau_i}(N,E)\subset cl_{\tau_j}\left(int_{\tau_i}(N,E)\right)$. □

Theorem 4.6. *Let (X,E,τ_1,τ_2) be a neutrosophic soft bitopological space and $(N,E),(M,E)\in NSS(X,E)$. If (N,E) is $ij-NSPO$ and $(M,E)\in\tau_1\cap\tau_2$ then $(N,E)\cup(M,E)$ is $ij-NSPO$.*

Proof. Let (N,E) is $ij-NSPO$ and $(M,E)\in\tau_1\cap\tau_2$. Then $(N,E)\subset int_{\tau_i}\left(cl_{\tau_j}(N,E)\right)$ and $int_{\tau_i}(M,E)=(M,E)$. So

$$\begin{aligned}(N,E)\cup(M,E) &\subset int_{\tau_i}\left(cl_{\tau_j}(N,E)\right)\cup int_{\tau_i}(M,E)\\ &\subset int_{\tau_i}\left(cl_{\tau_j}(N,E)\cup(M,E)\right)\subset int_{\tau_i}\left(cl_{\tau_j}(N,E)\cup cl_{\tau_j}(M,E)\right)\\ &= int_{\tau_i}\left(cl_{\tau_j}\left((N,E)\cup(M,E)\right)\right).\end{aligned}$$

Therefore $(N,E)\cup(M,E)$ is $ij-NSPO$. □

Theorem 4.7. *Let (X,E,τ_1,τ_2) be a neutrosophic soft bitopological space. Then*

1) *Every $ij-NSPO$ set is $ji-NSbO$.*
2) *Every $ij-NSSO$ set is $ji-NSbO$.*
3) *Every $ij-NSSO$ set is $ij-NS\beta O$.*

Proof.

1) Let $(N,E)\in NSS(X,E)$ be $ij-NSPO$ set. Then $(N,E)\subset int_{\tau_i}\left(cl_{\tau_j}(N,E)\right)\subset cl_{\tau_j}\left(int_{\tau_i}(N,E)\right)\cup int_{\tau_i}\left(cl_{\tau_j}(N,E)\right)$.
2) Let (N,E) be a $ij-NSSO$ set. Then $(N,E)\subset cl_{\tau_j}\left(int_{\tau_i}(N,E)\right)\subset cl_{\tau_j}\left(int_{\tau_i}(N,E)\right)\cup int_{\tau_i}\left(cl_{\tau_j}(N,E)\right)$.
3) Let (N,E) be a $ij-NSSO$ set. Then since $(N,E)\subset cl_{\tau_j}(N,E)$,

$$(N,E)\subset cl_{\tau_j}\left(int_{\tau_i}(N,E)\right)\subset cl_{\tau_j}\left(int_{\tau_i}(cl_{\tau_j}(N,E))\right).$$

□

Theorem 4.8. *Let (X,E,τ_1,τ_2) be a neutrosophic soft bitopological space. Then*

1) *Union of any $ij-NSPO$ set is $ij-NSPO$.*
2) *Union of any $ij-NSSO$ set is $ij-NSSO$.*
3) *Union of any $ij-NSbO$ set is $ij-NSbO$.*
4) *Union of any $ij-NS\beta O$ set is $ij-NS\beta O$.*
5) *Intersection of any $ij-NSPC$ set is $ij-NSPC$.*
6) *Intersection of any $ij-NSSO$ set is $ij-NSSO$.*

7) *Intersection of any* $ij - NSbO$ *set is* $ij - NSbC$.

8) *Intersection of any* $ij - NS\beta O$ *set is* $ij - NS\beta C$.

Proof.

1) Let (N_k, E) be $ij - NSPO$ set in (X, E, τ_1, τ_2) for all $k \in I$. Then

$$\bigcup_{k\in I}(N_k,E) \subset \bigcup_{k\in I} int_{\tau_i}\left(cl_{\tau_j}(N_k,E)\right) \subset int_{\tau_i}\left(\bigcup_{k\in I}\left(cl_{\tau_j}(N_k,E)\right)\right) = int_{\tau_i}\left(cl_{\tau_j}\left(\bigcup_{k\in I}(N_k,E)\right)\right)$$

2) Let (N_k, E) be $ij - NSSO$ set in (X, E, τ_1, τ_2) for all $k \in I$. Then

$$\bigcup_{k\in I}(N_k,E) \subset \bigcup_{k\in I} cl_{\tau_j}\left(int_{\tau_i}(N_k,E)\right) = cl_{\tau_j}\left(\bigcup_{k\in I}\left(int_{\tau_i}(N_k,E)\right)\right) \subset cl_{\tau_j}\left(int_{\tau_i}\left(\bigcup_{k\in I}(N_k,E)\right)\right)$$

3) Let (N_k, E) be $ij - NSbO$ set in (X, E, τ_1, τ_2) for all $k \in I$. Then

$$\begin{aligned}\bigcup_{k\in I}(N_k,E) &\subset \bigcup_{k\in I}\left(cl_{\tau_i}\left(int_{\tau_j}(N_k,E)\right) \cup int_{\tau_j}\left(cl_{\tau_i}(N_k,E)\right)\right)\\ &= \left(\bigcup_{k\in I} cl_{\tau_i}\left(int_{\tau_j}(N_k,E)\right)\right) \cup \left(\bigcup_{k\in I} int_{\tau_j}\left(cl_{\tau_i}(N_k,E)\right)\right)\\ &\subset cl_{\tau_i}\left(int_{\tau_j}\left(\bigcup_{k\in I}(N_k,E)\right)\right) \cup int_{\tau_j}\left(cl_{\tau_i}\left(\bigcup_{k\in I}(N_k,E)\right)\right)\end{aligned}$$

4) Let (N_k, E) be $ij - NS\beta O$ set in (X, E, τ_1, τ_2) for all $k \in I$. Then

$$\bigcup_{k\in I}(N_k,E) \subset \bigcup_{k\in I} cl_{\tau_j}\left(int_{\tau_i}(cl_{\tau_j}(N_k,E))\right) = cl_{\tau_j}\left(\bigcup_{k\in I} int_{\tau_i}(cl_{\tau_j}(N_k,E))\right) \subset cl_{\tau_j}\left(int_{\tau_i}(cl_{\tau_j}\left(\bigcup_{k\in I}(N_k,E)\right))\right)$$

The rest of the theorem can be proved easily by taking the complement of 1-4. □

5. Conclusion

In this paper, we defined neutrosophic soft supra closure operator in a neutrosphic soft bitopological space and investigated some properties of it. Then we obtained a neutrosophic soft topology with this closure operator. Also we defined neutrosophic soft supra interior operator and obtained a neutrosophic soft topology with this interior operator. In the section 4, we defined some new generalized open sets in neutrosophic soft bitopological spaces such as $ij-$ neutrosophic soft preopen, $ij-$ neutrosophic soft semi-open, $ij-$ neutrosophic soft b-open, $ij-$ neutrosophic soft β-open set. We examined the relationships between these newly defined open sets.

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