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Neutrosophic Entropy Based Heavy Metal Contamination Indices for Impact Assessment of Sarsa River Water Quality Within County of District Baddi, India

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Abstract: This study contributes a novel fuzzy and neutrosophic entropy-based procedure to identify the most contaminated sampling spot and to assess the impact of heavy metals concentration, before and after the amalgamation of pharmaceutical effluents treated in common effluent treatment plant. in river water samples. It is observed that the concentration of heavy metals, which were within permissible limits before amalgamation, dwindled gradually after amalgamation, owing to the decrease in fuzzy and neutrosophic entropy values. To identify the most contaminated sampling spot, responsible for heavy metal contamination, the proposed trigonometric fuzzy and single valued neutrosophic entropy measures are fascinated for assigning weights to each monitored heavy metal concentration reading with respect to four sampling spots and thereafter coupled with the relative sub-indices of each heavy metal to construct fuzzy and neutrosophic entropy weighted heavy metal contamination indices (FHCI and NHCI). The maximum (or minimum) FHCI and NHCI score among each sampling spot is designated to the "most contaminated" or "least contaminated" sampling spot accordingly. The proposed entropy-based contamination indices are superior in providing a better insight in classifying the desired contaminated sampling spot in comparison with the existing Deluca-Termini fuzzy entropy-based contamination index which may indicate uncertainty in the quality analysis of heavy metal contamination in river water samples.

Keywords: Neutrosophic Entropy, Deluca-Termini Entropy, Pharmaceutical Effluents, Heavy Metals Contamination.

1. Introduction

Heavy metals contamination in river water is a serious problem, not only in India, but also all over the world. The possible reasons behind this could be the increasing human population and excessive use of fertilizers in agriculture that causes pollution of fresh water resources with diverse and detrimental contaminants leading to the spread of water borne diseases. Contamination of water resources, available for domestic and drinking purposes with heavy metals and harmful bacteria, leads to health problems which, of course, may be life threatening. Waste water could be full of significant amounts of toxic heavy metals, that might not only pollute the soil, but also carries

deleterious effects on food quality after entering in to the food chain. All of the above issues reinforce the necessity of establishing an efficient methodology which can assess the impact of heavy metal concentration and enhance the quality analysis of heavy metal contamination in river water. Our main endeavor is to establish a novel entropy based heavy metal contamination evaluation methodology, the aftermaths of which can be utilized to control the spread of water borne deceases, reduce the risk of water and soil pollution, increase the ecological and aesthetical qualities of lakes and rivers, etc. The Baddi-Barotiwala region, better known as the industrial and commercial district of Himachal Pradesh, India, forms a part of the Shivalik's and lies in the lap of outer Himalayas. It has become an industrial hub due to huge investments in Baddi, Barotiwala, Nalagarh and Parwanoo districts. The Sarsa River-a tributary of the Sutlej River of district Baddi (Himachal Pradesh, India)- has turned out to be a grave for aquatic life due to uncontrolled industrial growth of the area and alleged discharge of toxic effluents from pharmaceutical industries. Likewise, flora, fauna and the surrounding environment are adversely affected by the harmful chemicals released from pharmaceutical industries and from the improperly treated effluents from waste water treatment plants (WWTPs). Besides industrial pollution, drug pollution is a major contributor in killing of fish, amphibians and amphibian path morphology. The ongoing research has shown that continuing exposure of compound pharmaceutical effluents to rivulet biota may result in severe and persistent problems, behavioral issues, buildup in tissues, effect on reproductive system, impact on cell propagation & multiplication. Severely elevated chloride concentrations in the water bodies are unfavorable for aquatic life and can amplify metal toxicity and another bioactive composites. High levels of nitrogen and phosphorus appearing from sewage, animal wastes, fertilizer and agriculture may affect rivulet biota. Excess of phosphates cause stepped up eutrophication as they overfertilize the aquatic plants; choke the waterways due to excessive algal growth and other wild plants. In warm weather, fast growth of algae and floating aquatic weeds is stimulated by the nutrients resulting into deterioration of water quality. All these suffocated activities decline the ecological and aesthetic qualities of lakes, rivers etc. A systematic analysis of the river Nile in Egypt was computed to assess the water suitability for aquatic life and domestic purposes. Studies have shown that applications of electrochemical technologies like Electro-coagulation & electro-floatation, in wastewater treatment are very effective in recuperating toxic heavy metals from wastewater. These techniques perform better than the conventional techniques being used in removing colloidal particles as well as other organic pollutants. Electro-oxidation is also being used in treatment of wastewater by combining it with other contamination techniques.

Recently, Alam et al. **[1]** deployed pollution indices and geographical accumulation index approach for evaluating heavy metal contamination in water resources of an open landfill area. The heavy metal accumulation in the samples was observed to be slightly higher than the standard one, indicating a danger to the humanity and environment. Vardhan et al. **[2]** discussed an environment friendly adsorption analysis approach for removing the toxicity among heavy metals available in the

aquatic system for the purpose of avoiding illness of human beings. Khangembam and Kshetrimayum **[3]** utilized some statistical measures including scatter diagrams, trilinear and Gibbs plots for evaluating the water quality index of ground water samples and indicated the unsuitability of ground water for drinking purposes because of the excessive availability of highly toxic heavy metals. Yang et al. **[4]** employed Information entropy and cloud model theory for assessing the complexity of heavy metal pollution in agriculture soils of mining zones. Huu et al. **[5]** investigated geo chemical and distribution factors to assess the contamination status of heavy metals in estuarine system, which indicated an increase of arsenic metal in the samples. Hussain et al. **[6]** experimented the heavy metal contamination through the analytical hierarchy process and validated that, among all heavy metals available in the river water samples of Godavari; the concentration of Zn was highest and of Cd was lowest. Sabbir et al. **[7]** calculated the concentration of arsenic, chromium, lead, mercury and cadmium available in freshwater fish muscles and sediments of the Rupsha river and evaluated the suitability of River water. Alidadi et al. **[8]** calculated the carcinogenic and non-carcinogenic risks of arsenic for adults and kids based on the LCR (life time cancer risk) factor, hazard index and hazard quotient by chemically analyzing and testing of toxicity of arsenic in water resources of north-east regions of Iran.

Recently, Singh et al. **[9]** determined some entropy weighted heavy metal contamination indices (EHCI) of various sampling spots of the Brahmaputra River by quantifying Shannon's probabilistic entropy and evaluated the impact of heavy metal contamination. Basset et al. **[10]** developed an aggregation operator based on neutrosophic numbers of type 2 and selected the best banking facilities. The authors also modified the existing TOPSIS method under neutrosophic environment and selected the best corporation importing supplier. In another work, Basset et al. **[11]** integrated the enduring ANP and VIKOR method by taking into consideration the triangular neutrosophic numbers and demonstrated a case study of selecting the best supplier for importing. Furthermore, Basset et al. **[12]** suggested a novel robust ranking procedure by integrating trapezoidal neutrosophic numbers with GSCM approach, intended to predict the environmental and economical practices to be implemented in industry. Also, Basset et al. **[13]** presented a hybrid model that could combine EDAS, DEMATEC and neutrosophic numbers for the purpose of classifying the most sustainable bioenergy technique under ambiguous and inconsistent situations. In addition, Basset et al. **[14]** utilized a hybrid approach by combining TOPSIS and VIKOR methods for classifying the most sustainable RESs under neutrosophic treatments.

1.1 Motivation

To represent the macroscopic state of heavy metal concentration in river water and to construct entropy weighted heavy metal contamination indices (EHCI) by deploying Shannon's entropy, there may occur a problematic situation because this fascinating entropy is facing a major drawback because of its assumption 0log0=0. Due to this fancy assumption, Shannon's entropy is facing

intrinsic conflicts and hence indicating lack of macroscopic view in the quality analysis of concentration and contamination of heavy metals in river water samples. Zadeh's **[15]** fuzzy set theory has become an indispensable tool for reflecting the complexity of heavy metal contamination. A fuzzy entropy measure can represent the macroscopic state of heavy metal concentration in a broader way. Dubois and Prade **[16]** established many variants of fuzzy sets, one of which is related to the grade or membership degree of the underlying fuzzy set. Thereafter, many equivalents of fuzzy sets have been developed and utilized for dealing with assessment problems of heavy metal contamination in river water samples. A neutrosophic set (NS), which is hinged on three variants-truth, indeterminacy and falsity membership degrees of a fuzzy set, can represent the macroscopic state of heavy metal contamination in an efficient way. Smarandache's neutrosophic set theory **[17]** can play a vital role in classifying the most contaminated sampling spots with respect to each heavy metal concentration in river water samples. Motivated by Zadeh's fuzzy set theory and Smarandache's neutrosophic set theory, an effort has been accomplished in this path way by construct trigonometric fuzzy and single valued neutrosophic entropy weighted heavy metal contamination indices. The desired goal is achieved by establishing a superior contamination evaluation methodology and its applicability which can provide a better insight in classifying the most contaminated sampling spot with respect to each heavy metal concentration in river water samples.

1.2 Novelties

The identification of the most contaminated sampling spot through the proposed methodology can help in reducing the risk of water and soil pollution. The following points have been addresses by the proposed research work.

- To construct a novel trigonometric fuzzy entropy measure.
- To construct a novel symmetric trigonometric fuzzy cross entropy (TFE) measure.
- To establish a novel trigonometric single valued neutrosophic entropy (TNE) measure.
- To assess the concentration of heavy metals in river water samples though the proposed TFE and TNE measures.
- To construct fuzzy and single valued neutrosophic entropy weighted heavy metal contamination indices (FHCI and NHCI).
- To identify the most contaminated sampling spot through the proposed FHCI and NHCI.

The major contributions delivered in this study can be summarized as follows.

- Because of the fancy assumption 0log0=0 deliberated to Shannon's probabilistic entropy as it may represent macroscopic view of contamination in a narrow way, our TFE measure has been found efficient in assessing the accurate impact of heavy metal concentration and thus representing the macroscopic view of heavy metal concentration in a broader way.

- Because of the limitation deliberated to Deluca and Termini's entropy measure as it may return meaningless results in certain mathematical treatments, especially, when the grade or degree of membership conceived by its membership function is zero or unity, our TFE measure can perform well under fuzzy environment and provide consistent and specified results in certain mathematical treatments.
- To expand the applicability of Smarandache's neutrosophic set theory **[17]** and to enhance the quality analysis of heavy metal contamination under neutrosophic environment, our TNE measure has been found capable in reckoning the most contaminated sampling spot with respect to each heavy metal concentration in river water samples.
- The findings of the proposed study can be utilized for controlling the spread of water borne deceases, reducing the risk of water and soil pollution, increasing the ecological and aesthetical qualities of lakes and rivers, etc. The rest of the proposed research work is organized as follows:

Section 2 introduces in brief the basic concepts of Information theory required for understanding the proposed heavy metal contamination evaluation methodology. **Sections 3-4** are dedicated for the establishment of novel trigonometric fuzzy entropy and single valued neutrosophic entropy measures consecutively. **Section 5** is devoted to assess the impact of heavy metal concentration through the experimental investigations and proposed trigonometric fuzzy entropy measure (TFE) and single valued neutrosophic entropy (TNE) measure consecutively**. Section 6** introduces a novel entropy-based heavy metal contamination evaluation methodology by means of fuzzy and single valued neutrosophic entropy weighted heavy metal contamination indices (FHCI and NHCI). **Section 7** validates the effectiveness of the proposed methodology by identifying the most contaminated sampling spot responsible for heavy metal contamination with respect to each heavy metal concentration in river water samples. **Section 8** finally summarizes the concrete conclusions of this study.

2. Preliminaries:

This section deals with the introduction of basic prerequisites required for understanding the propounded study.

Def. 2.1Fuzzy Set (FS) [18] A fuzzy set $A_{FS}^{w} \subseteq U$ in a finite discourse of universe $U = \left(x_1, x_2, ..., x_n\right)$ is an object of the form: $A_{FS}^{w} = \left(< x_i, \tilde{\mu}_{A^w}(x_i) > | \; x_i \in U\right)$, where $\tilde{\mu}_{A^w}(x_i) : U \to [0,1]$ represents true membership function and satisfy $0 \le \tilde{\mu}_{A^w}(x_i) \le 1$ Further, the complement $C\left(A_{FS}^{w}\right)$ of $A_{FS}^{w} \subseteq U$ is an object of the form defined by $C\left(A_{FS}^{w}\right) = \left(< x_i, 1 - \tilde{\mu}_{A^w}(x_i) > | \; x_i \in U\right)$.

Def. 2.2 Fuzzy Entropy Measure [18] Suppose $S(U)$ represents the collection of all fuzzy sets in $U=(x_1,x_2,...,x_n)$ and $A_{FS}^{w}=(<x_i,\tilde{\mu}_{A^w}(x_i)>|\, x_i\in U)$ be any fuzzy set quantified by its truth membership functions $\tilde{\mu}_{A^w}(x_i):U\to[0,1]$ satisfying $0\le\tilde{\mu}_{A^w}(x_i)\le 1.$ Then a function $T_E:S(U)\to R^{+}(\text{set of positive reals})$ is called as fuzzy entropy measure if $(i)T_E(A_{FS}^{w})\ge 0\forall A_{FS}^{w}\subseteq U$ with equality if $\tilde{\mu}_{A^w}(x_i)=0$ or 1 *(ii)* $T_E(A_{FS}^{w})$ does not change whenever $\tilde{\mu}_{A^w}(x_i)$ is replaced by $1-\tilde{\mu}_{A^w}(x_i)$ *(iii)* $T_E(A_{FS}^{w})$ is a concave function of $\tilde{\mu}_{A^w}(x_i)$ *(iv)* $T_E(A_{FS}^{w})$ possesses its maximum value which arises when $\tilde{\mu}_{A^w}(x_i)=\frac{1}{2}.$

Def.2.3Symmetric Fuzzy Cross Entropy Measure [19] Let $A_{FS}^{w}=(<x_i,\tilde{\mu}_{A^w}(x_i)>|\, x_i\in U)$ and $B_{FS}^{w}=(<x_i,\tilde{\mu}_{B^w}(x_i)>|\, x_i\in U)$ are any two fuzzy sets in $U=(x_1,x_2,...,x_n)$ quantified by their truth membership functions $\tilde{\mu}_{A^w}(x_i),\tilde{\mu}_{A^w}(x_i):U\to[0,1]$ satisfying $0\le\tilde{\mu}_{A^w}(x_i),\tilde{\mu}_{B^w}(x_i)\le 1.$ Then a function $T_{CE}:S(U)\times S(U)\to R^{+}(\text{set of positive reals})$ is called as symmetric fuzzy cross entropy or discrimination information measure between two fuzzy sets A_{FS}^{w} and B_{FS}^{w} if $(i)\,T_{CE}(A_{FS}^{w},B_{FS}^{w})\ge 0\forall A_{FS}^{w},B_{FS}^{w}\in S(U)$ with equality if $A_{FS}^{w}=B_{FS}^{w}.$ $(ii)\,T_{CE}(A_{FS}^{w},B_{FS}^{w})=T_{CE}(B_{FS}^{w},A_{FS}^{w})$ and $(iii)T_{CE}(A_{FS}^{w},B_{FS}^{w})$ does not change whenever $\tilde{\mu}_{A^w}(x_i),\tilde{\mu}_{B^w}(x_i)$ are replaced by their counterparts $1-\tilde{\mu}_{A^w}(x_i),1-\tilde{\mu}_{B^w}(x_i).$

Def. 2.4 Single Valued Neutrosophic Set (SVNS) [17]. A SVNS $A_{FS}^{w}\subseteq U$ is defined as $A_{SV}^{w}=(<x_i,\tilde{\mu}_{A^w}(x_i),\tilde{i}_{A^w}(x_i),\tilde{f}_{A^w}(x_i)>|\, x_i\in U)$ where $\tilde{\mu}_{A^w}(x_i),\tilde{i}_{A^w}(x_i),\tilde{f}_{A^w}(x_i):U\to[0,1]$ satisfy $0\le\tilde{\mu}_{A^w}(x_i),\tilde{i}_{A^w}(x_i),\tilde{f}_{A^w}(x_i)\le 3$ and respectively called as truth, indeterminacy and falsity membership functions. Further, the complement $C(A_{SV}^{w})$ of $A_{SV}^{w}\subseteq U$ is defined as $C(A_{SV}^{w})=(<x_i,\tilde{f}_{A^w}(x_i),1-\tilde{i}_{A^w}(x_i),\tilde{\mu}_{A^w}(x_i)>|\, x_i\in U).$

Def. 2.5 Single Valued Neutrosophic Entropy Measure [17] Let $R(U)$ be a well-defined collection of all single valued neutrosophic sets $A_{FS}^{w} \subseteq U$,then a function $R_N : R(U) \to R^{+}$ is called as single valued neutrosophic entropy measure if

$(i)\, R_N\left(A_{SV}^{w}\right) \geq 0 \forall A_{SV}^{w} \in R(U)$ with equality if either $\tilde{\mu}_{A^w}(x_i) = 1, \tilde{i}_{A^w}(x_i) = 0, \tilde{f}_{A^w}(x_i) = 0$ or $\tilde{\mu}_{A^w}(x_i) = 0, \tilde{i}_{A^w}(x_i) = 0, \tilde{f}_{A^w}(x_i) = 1$ $(ii)\, R_N\left(C\left(A_{SV}^{w}\right)\right) = R_N\left(A_{SV}^{w}\right)$ $(iii) R_N\left(A_{SV}^{w}\right)$ exhibits its concavity property for each $\tilde{\mu}_{A^w}(x_i), \tilde{i}_{A^w}(x_i), \tilde{f}_{A^w}(x_i)$ $(iv) R_N\left(A_{SV}^{w}\right)$ possesses its maximum value which arises when each $\tilde{\mu}_{A^w}(x_i) = \tilde{i}_{A^w}(x_i) = \tilde{f}_{A^w}(x_i) = \frac{1}{2}$.

Def. 2.6 Symmetric Single Valued Neutrosophic Cross Entropy Measure [17]

A function $R_{CE} : R(U) \times R(U) \to R$ is called as symmetric single valued neutrosophic cross entropy measure between two SVNSs A_{SV}^{w} and B_{SV}^{w} if

$(i) R_{CE}\left(A_{SV}^{w}, B_{SV}^{w}\right) \geq 0 \forall A_{SV}^{w}, B_{SV}^{w} \in R(U)$ with equality if and only if $A_{SV}^{w} = B_{SV}^{w}$.

$(ii)\, R_{CE}\left(A_{SV}^{w}, B_{SV}^{w}\right) = R_{CE}\left(B_{SV}^{w}, A_{SV}^{w}\right)$ $(iii)\, R_{CE}\left(C\left(A_{SV}^{w}\right), C\left(B_{SV}^{w}\right)\right) = R_{CE}\left(A_{SV}^{w}, B_{SV}^{w}\right) \forall A_{SV}^{w}, B_{SV}^{w} \in R(U)$.

3. A Novel Trigonometric Fuzzy Entropy Measure

We shall, here, develop a novel trigonometric fuzzy entropy (TFE) measure (**Theorems. 3.1**) followed by trigonometric symmetric fuzzy cross entropy (FCE) measure hinged on two fuzzy sets (**Theorems. 3.2),** the outcomes of which will be utilized to establish the proposed single valued neutrosophic entropy (TNE) measure.

Theorem.3.1 Let $A_{FS}^{w} = \left(< x_i, \tilde{\mu}_{A^w}(x_i) > | \, x_i \in U\right)$ be any fuzzy set in U with cardinality n.

Then, $H_F(A_{FS}^{w})$ is an authentic trigonometric fuzzy entropy measure **[Def. 2.2]** defined as

$$H_F(A_{FS}^{w}) = -\sum_{i=1}^{n}\left[\tan\left(\frac{2\sqrt{2} + 2\sqrt{\tilde{\mu}_{A^w}^{2}(x_i) + \left(1 - \tilde{\mu}_{A^w}(x_i)\right)^{2}} - \sqrt{2\tilde{\mu}_{A^w}(x_i)\left(1 - \tilde{\mu}_{A^w}(x_i)\right)}}{5}\right) - \tan\left(\frac{2\sqrt{2}+2}{5}\right)\right] \quad \ldots (1)$$

with minimum zero and maximum as $\left(\tan\left(\frac{2\sqrt{2}+2}{5}\right) - \tan\left(\frac{1}{\sqrt{2}}\right)\right) n$.

Here, the generic entity 'x_i' represents the i^{th} macroscopic level of heavy metal contamination and $H_F(A_{FS}^{w})$ indicates the fuzzy entropy of heavy metal contamination indicated by the fuzzy set A_{FS}^{w}.

Proof In view of **[Def. 2.2],**

(i) $H_F^w\left(A_{FS}^w\right) \geq 0 \forall \tilde{\mu}_{A^w}(x_i) \in [0,1]$ with equality if $\tilde{\mu}_{A^w}(x_i) = 0$ or 1 for each $i = 1, 2, ..., n$.

(ii) $H_F(A_{FS}^w)$ remains unchanged whenever $\tilde{\mu}_{A^w}(x_i)$ is replaced by $1 - \tilde{\mu}_{A^w}(x_i)$.

(iii) **Concavity:** The fact that $H_F\left(A_{FS}^w\right)$ is concave in nature can be seen from its three-dimensional rotational plot (**Fig. 1).** Also, the finite series of positive terms in **(1)** can be partially differentiated with respect to each $\tilde{\mu}_{A^w}(x_i)$ because of its uniform and absolute convergence. Mathematica (software from Wolfram) yields $\dfrac{\partial^2 H_F\left(A_{FS}^w\right)}{\partial \tilde{\mu}_{A^w}^2(x_i)} < 0 \forall \tilde{\mu}_{A^w}(x_i) \in [0,1]$, which also justifies the concavity of $H_F\left(A_{FS}^w\right)$ with respect to each $\tilde{\mu}_{A^w}(x_i)$.

(iv)With the aid of concavity property of $H_F\left(A_{FS}^w\right)$ with respect to $\tilde{\mu}_{A^w}(x_i)$., there exists its maximum value which arises when

$$\frac{\partial H_F\left(A_{FS}^w\right)}{\partial \tilde{\mu}_{A^w}(x_i)} = \frac{1}{5}\left(\frac{1-2\tilde{\mu}_{A^w}(x_i)}{\sqrt{2\tilde{\mu}_{A^w}(x_i)\left(1-\tilde{\mu}_{A^w}(x_i)\right)}} + \frac{2-4\tilde{\mu}_{A^w}(x_i)}{\sqrt{\tilde{\mu}_{A^w}^2(x_i)+\left(1-\tilde{\mu}_{A^w}(x_i)\right)^2}}\right) \sec^2\left(\frac{2\sqrt{2}+2\sqrt{\tilde{\mu}_{A^w}^2(x_i)+\left(1-\tilde{\mu}_{A^w}(x_i)\right)^2}-\sqrt{2\tilde{\mu}_{A^w}(x_i)\left(1-\tilde{\mu}_{A^w}(x_i)\right)}}{5}\right) = 0$$

which yields $\tilde{\mu}_{A^w}(x_i) = \dfrac{1}{2}$. In view of **(1),**

$$\text{Max.} H_F\left(A_{FS}^w\right) = H_F\left(A_{FS}^w\right)\Big|_{\tilde{\mu}_{A^w}(x_i)=\frac{1}{2}} = \left(\tan\left(\frac{2\sqrt{2}+2}{5}\right) - \tan\left(\frac{1}{\sqrt{2}}\right)\right) n \qquad \ldots (2)$$

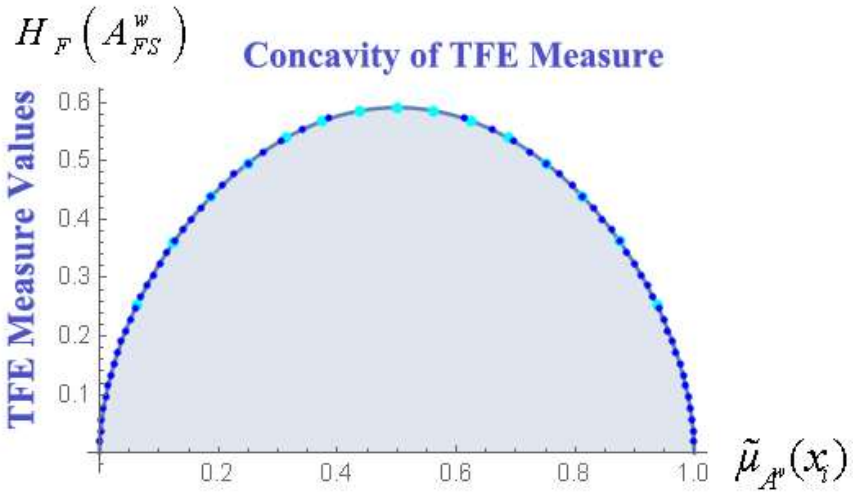


Fig.1 Concavity property exhibited by TFE measure $H_F\left(A_{FS}^w\right)$ with respect to $\tilde{\mu}_{A^w}(x_i)$

Theorem.3.2 Let $A_{FS}^w = \left(< x_i, \tilde{\mu}_{A^w}(x_i) > | \, x_i \in U\right)$ and $B_{FS}^w = \left(< x_i, \tilde{\mu}_{B^w}(x_i) > | \, x_i \in U\right)$ be any two fuzzy sets with same cardinality n. Show that $H_{CE}^{\mu}\left(A_{FS}^w, B_{FS}^w\right)$ is a valid trigonometric symmetric fuzzy cross entropy measure (**Def. 2.3)** between two fuzzy sets A_{FS}^w and B_{FS}^w defined by

$$H_{CE}^{\mu}\left(A_{FS}^w, B_{FS}^w\right)$$

$$= \sum_{i=1}^{n} \left[\begin{array}{l} -10\tan\frac{1}{\sqrt{2}} + \left(4 + \tilde{\mu}_{A^w}(x_i) + \tilde{\mu}_{B^w}(x_i)\right)\tan\left(\frac{2\sqrt{2} + 2\sqrt{\tilde{\mu}_{A^w}^2(x_i) + \tilde{\mu}_{B^w}^2(x_i)} - \sqrt{2\tilde{\mu}_{A^w}(x_i)\tilde{\mu}_{B^w}(x_i)}}{4 + \tilde{\mu}_{A^w}(x_i) + \tilde{\mu}_{B^w}(x_i)}\right) \\ + \left(6 - \tilde{\mu}_{A^w}(x_i) - \tilde{\mu}_{B^w}(x_i)\right)\tan\left(\frac{2\sqrt{2} + 2\sqrt{+\left(1-\tilde{\mu}_{A^w}(x_i)\right)^2 + \left(1-\tilde{\mu}_{B^w}(x_i)\right)^2} - \sqrt{\left(1-\tilde{\mu}_{A^w}(x_i)\right)\left(1-\tilde{\mu}_{B^w}(x_i)\right)}}{6 - \tilde{\mu}_{A^w}(x_i) - \tilde{\mu}_{B^w}(x_i)}\right) \end{array} \right] \quad \ldots (3)$$

Here, $H_{CE}^{\mu}\left(A_{FS}^w, B_{FS}^w\right)$ indicates the amount of true membership degree of symmetric discrimination of the fuzzy set A_{FS}^w against B_{FS}^w.

Proof. It is easy to verify that $(i)\, H_{CE}^{\mu}\left(C\left(A_{FS}^w\right), C\left(B_{FS}^w\right)\right) = H_{CE}^{\mu}\left(A_{FS}^w, B_{FS}^w\right)$ and $H_{CE}^{\mu}\left(A_{FS}^w, B_{FS}^w\right) = H_{CE}^{\mu}\left(B_{FS}^w, A_{FS}^w\right) \; \forall A_{FS}^w, B_{FS}^w \in S(U)$. To establish the non-negativity of $H_{CE}^{\mu}\left(A_{FS}^w, B_{FS}^w\right)$, we first divert to develop the following **Lemma 3.1.**

Lemma 3.1 Define

$A_1\left(\tilde{\mu}_{A^w}(x_i), \tilde{\mu}_{B^w}(x_i)\right) = \frac{\tilde{\mu}_{A^w}(x_i) + \tilde{\mu}_{B^w}(x_i)}{2}$, $N_1\left(\tilde{\mu}_{A^w}(x_i), \tilde{\mu}_{B^w}(x_i)\right) = \left(\frac{\sqrt{\tilde{\mu}_{A^w}(x_i)} + \sqrt{\tilde{\mu}_{B^w}(x_i)}}{2}\right)^2$,

$S_1\left(\tilde{\mu}_{A^w}(x_i), \tilde{\mu}_{B^w}(x_i)\right) = \sqrt{\frac{\tilde{\mu}_{A^w}^2(x_i) + \tilde{\mu}_{B^w}^2(x_i)}{2}}$. Then there exist the inequalities: $S_1 \geq A_1 \geq N_1$ with equality if and only if $\tilde{\mu}_{A^w}(x_i) = \tilde{\mu}_{B^w}(x_i) \forall \tilde{\mu}_{A^w}(x_i), \tilde{\mu}_{B^w}(x_i) \in [0,1] (i = 1, 2, ..., n)$

Proof. In view of our notations,

$$(i)\; S_1^2 - A_1^2 = \frac{\tilde{\mu}_{A^w}^2(x_i) + \tilde{\mu}_{B^w}^2(x_i)}{2} - \left(\frac{\tilde{\mu}_{A^w}(x_i) + \tilde{\mu}_{B^w}(x_i)}{2}\right)^2 = \left(\frac{\tilde{\mu}_{A^w}(x_i) - \tilde{\mu}_{B^w}(x_i)}{2}\right)^2 \geq 0 \Rightarrow S_1^2 - A_1^2 \Rightarrow S_1 \geq A_1 \quad \ldots (4)$$

$$(ii)\, A_1^2 - N_1^2 = \left(\frac{\tilde{\mu}_{A^w}(x_i) + \tilde{\mu}_{B^w}(x_i)}{2}\right)^2 - \left(\frac{\sqrt{\tilde{\mu}_{A^w}(x_i)} + \sqrt{\tilde{\mu}_{B^w}(x_i)}}{2}\right)^2 = \left(\frac{\sqrt{\tilde{\mu}_{A^w}(x_i)} - \sqrt{\tilde{\mu}_{B^w}(x_i)}}{2}\right)^2 \geq 0 \Rightarrow A_1^2 - N_1^2 \Rightarrow A_1 \geq N_1$$

… (5)

Combining the resulting inequalities **(4)** and **(5)** to obtain $S_1 \geq A_1 \geq N_1$ with equality if and only if $\tilde{\mu}_{A^w}(x_i) = \tilde{\mu}_{B^w}(x_i) \forall \tilde{\mu}_{A^w}(x_i), \tilde{\mu}_{B^w}(x_i) \in [0,1]$.

Thus, in view of Lemma **3.1,** the resulting inequality $S_1 \geq A_1 \geq N_1$ can be re-scheduled to give

$$S_1\left(\tilde{\mu}_{A^w}(x_i), \tilde{\mu}_{B^w}(x_i)\right) \geq N_1\left(\tilde{\mu}_{A^w}(x_i), \tilde{\mu}_{B^w}(x_i)\right)$$

$$\Rightarrow \sqrt{\frac{\tilde{\mu}_{A^w}^2(x_i) + \tilde{\mu}_{B^w}^2(x_i)}{2}} \geq \left(\frac{\sqrt{\tilde{\mu}_{A^w}(x_i)} + \sqrt{\tilde{\mu}_{B^w}(x_i)}}{2}\right)^2 = \frac{\tilde{\mu}_{A^w}(x_i) + \tilde{\mu}_{B^w}(x_i)}{4} + \frac{\sqrt{\tilde{\mu}_{A^w}(x_i)\tilde{\mu}_{B^w}(x_i)}}{2}$$

$$\Rightarrow \sqrt{\frac{\tilde{\mu}_{A^w}^2(x_i) + \tilde{\mu}_{B^w}^2(x_i)}{2}} - \frac{\sqrt{\tilde{\mu}_{A^w}(x_i)\tilde{\mu}_{B^w}(x_i)}}{2} + 1 \geq \frac{\tilde{\mu}_{A^w}(x_i) + \tilde{\mu}_{B^w}(x_i)}{4} + 1$$

$$\Rightarrow \frac{2\sqrt{2} + 2\sqrt{\tilde{\mu}_{A^w}^2(x_i) + \tilde{\mu}_{B^w}^2(x_i)} - \sqrt{2\tilde{\mu}_{A^w}(x_i)\tilde{\mu}_{B^w}(x_i)}}{4 + \tilde{\mu}_{A^w}(x_i) + \tilde{\mu}_{B^w}(x_i)} \geq \frac{1}{\sqrt{2}} \quad \text{… (6)}$$

Employing the monotonicity property of tangent function over $[0,1]$, the inequality **(6)** yields

$$\left(\left(4 + \tilde{\mu}_{A^w}(x_i) + \tilde{\mu}_{B^w}(x_i)\right)\tan\left(\frac{2\sqrt{2} + 2\sqrt{\tilde{\mu}_{A^w}^2(x_i) + \tilde{\mu}_{B^w}^2(x_i)} - \sqrt{2\tilde{\mu}_{A^w}(x_i)\tilde{\mu}_{B^w}(x_i)}}{4 + \tilde{\mu}_{A^w}(x_i) + \tilde{\mu}_{B^w}(x_i)}\right)\right) \geq \left(4 + \tilde{\mu}_{A^w}(x_i) + \tilde{\mu}_{B^w}(x_i)\right)\tan\left(\frac{1}{\sqrt{2}}\right)$$

… (7)

Replacement of $\tilde{\mu}_{A^w}(x_i), \tilde{\mu}_{B^w}(x_i)$ with $\left(1 - \tilde{\mu}_{A^w}(x_i)\right), \left(1 - \tilde{\mu}_{B^w}(x_i)\right)$ into **(7)** yields

$$\left(\begin{array}{l}\left(6 - \tilde{\mu}_{A^w}(x_i) - \tilde{\mu}_{B^w}(x_i)\right)\times \\ \tan\left(\dfrac{2\sqrt{2} + 2\sqrt{\left(1 - \tilde{\mu}_{A^w}(x_i)\right)^2 + \left(1 - \tilde{\mu}_{B^w}(x_i)\right)^2} - \sqrt{2\left(1 - \tilde{\mu}_{A^w}(x_i)\right)\left(1 - \tilde{\mu}_{B^w}(x_i)\right)}}{6 - \tilde{\mu}_{A^w}(x_i) - \tilde{\mu}_{B^w}(x_i)}\right)\end{array}\right) \geq \left(6 - \tilde{\mu}_{A^w}(x_i) - \tilde{\mu}_{B^w}(x_i)\right)\tan\left(\frac{1}{\sqrt{2}}\right)$$

… (8)

Simply adding the inequalities **(7) & (8)** and taking the sum over $i = 1$ to $i = n$ to obtain

$H_{CE}^{\mu}\left(A_{FS}^w, B_{FS}^w\right) \geq 0 \forall \tilde{\mu}_{A^w}(x_i), \tilde{\mu}_{B^w}(x_i) \in [0,1]$ with equality if $\tilde{\mu}_{A^w}(x_i) = \tilde{\mu}_{B^w}(x_i) \forall \tilde{\mu}_{A^w}(x_i), \tilde{\mu}_{B^w}(x_i) \in [0,1]$.

We next divert to discuss the situation under which out TFE measure $H_{CE}^{\mu}\left(A_{FS}^w, B_{FS}^w\right)$ admits its extreme values as shown in the following **Theorem. 3.3.**

Theorem3.3 Let A_{FS}^w and B_{FS}^w be any two fuzzy sets with same cardinality *n.* then there exists the inequality: $0 \leq H_{CE}^{\mu}\left(A_{FS}^w, B_{FS}^w\right) \leq 10\left(\tan\left(\frac{2\sqrt{2}+2}{5}\right) - \tan\left(\frac{1}{\sqrt{2}}\right)\right)n.$

Proof. In view of Def**. 2.1,** the resulting **Theorem 3.2** yields

$$H_{CE}^{\mu}\left(A_{FS}^{w},C\left(A_{FS}^{w}\right)\right)=\sum_{i=1}^{n}\left[-10\tan\left(\frac{1}{\sqrt{2}}\right)+10\tan\left(\frac{2\sqrt{2}+2\sqrt{\tilde{\mu}_{A^w}^{2}(x_i)+\left(1-\tilde{\mu}_{A^w}(x_i)\right)^{2}}-\sqrt{2\tilde{\mu}_{A^w}(x_i)\left(1-\tilde{\mu}_{A^w}(x_i)\right)}}{5}\right)\right]$$

$$=\sum_{i=1}^{n}\left[10\tan\left(\frac{2\sqrt{2}+2}{5}\right)-10Tan\left(\frac{1}{\sqrt{2}}\right)-10\left\{\begin{matrix}\tan\left(\frac{2\sqrt{2}+2}{5}\right)\\-\tan\left(\frac{2\sqrt{2}+2\sqrt{\tilde{\mu}_{A^w}^{2}(x_i)+\left(1-\tilde{\mu}_{A^w}(x_i)\right)^{2}}-\sqrt{2\tilde{\mu}_{A^w}(x_i)\left(1-\tilde{\mu}_{A^w}(x_i)\right)}}{5}\right)\end{matrix}\right\}\right]$$

$$=10\,\text{Max.}H_F(A_{FS}^{w})-10\,H_F(A_{FS}^{w}) \quad \ldots (9)$$

Since $H_F\left(A_{FS}^{w}\right)\geq 0$ (**Theorem. 3.1)**, therefore, the resulting expression **(9)** yields

$$H_F\left(A_{FS}^{w}\right)=\text{Max.}H_F\left(A_{FS}^{w}\right)-\frac{1}{10}H_{CE}^{\mu}\left(A_{FS}^{w},C\left(A_{FS}^{w}\right)\right)\geq 0 \quad \ldots. (10)$$

$$\Rightarrow 0\leq H_{CE}^{\mu}\left(A_{FS}^{w},C\left(A_{FS}^{w}\right)\right)\leq 10\left(\tan\left(\frac{2\sqrt{2}+2}{5}\right)-\tan\left(\frac{1}{\sqrt{2}}\right)\right)n \quad \ldots (11)$$

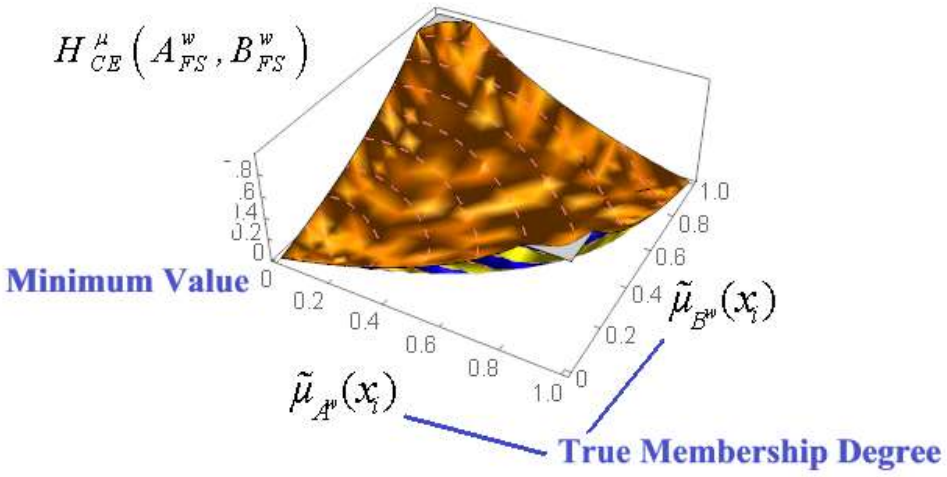
Inequality **(11)** suggests that $H_{CE}^{\mu}\left(A_{FS}^{w},C\left(A_{FS}^{w}\right)\right)$ is finite. Hence, it is easy to establish that $H_{CE}^{\mu}\left(A_{FS}^{w},B_{FS}^{w}\right)$ is also finite and satisfy $0\leq H_{CE}^{\mu}\left(A_{FS}^{w},B_{FS}^{w}\right)\leq 10\left(\tan\left(\frac{2\sqrt{2}+2}{5}\right)-\tan\left(\frac{1}{\sqrt{2}}\right)\right)n$ for a fixed n. This implies that $\text{Max.}H_{CE}^{\mu}\left(A_{FS}^{w},B_{FS}^{w}\right)=10\left(\tan\left(\frac{2\sqrt{2}+2}{5}\right)-\tan\left(\frac{1}{\sqrt{2}}\right)\right)n$ and this value completely depends only on the cardinality of U. The fact that, $H_{CE}^{\mu}\left(A_{FS}^{w},B_{FS}^{w}\right)$ affirms its minimum value zero can be seen from its three-dimensional plot as shown in **Fig.2 (a).** Furthermore, the three-dimensional plots represented in **Fig. 2(b)** depicts that $H_{CE}^{\mu}\left(A_{FS}^{w},B_{FS}^{w}\right)$ increases whenever $\left|A_{FS}^{w}-B_{FS}^{w}\right|$ increases, attains its maximum value as $10\left(\tan\left(\frac{2\sqrt{2}+2}{5}\right)-\tan\left(\frac{1}{\sqrt{2}}\right)\right)n$ at the points (1, 0) and (0, 1) and minimum value zero whenever $A_{FS}^{w}=B_{FS}^{w}$.

The findings of resulting **Theorems 3.1 & 3.2** will be utilized to establish one more important **Theorem 4.1,** the outcomes of which will play an eminent role in understanding the macroscopic state of heavy metal pollution as follows.

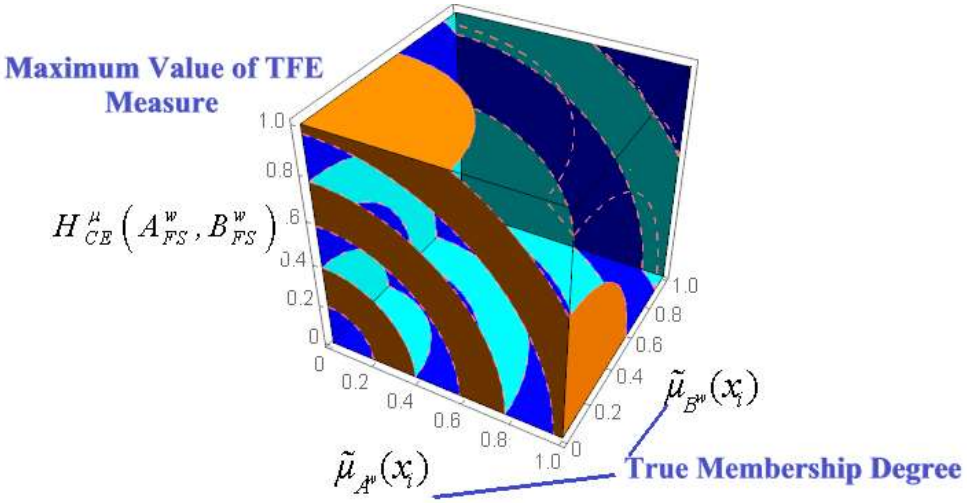
4. A Trigonometric Single Valued Neutrosophic Cross Entropy Measure

We shall now, equally will, extend the newly discovered trigonometric symmetric fuzzy cross entropy measure **(Theorem 3.2**) hinged on two fuzzy sets to this measure hinged on two single-valued neutrosophic sets.

Def.4.1 Let $A_{SV}^{w} \subseteq U$ and $B_{SV}^{w} \subseteq U$ be any two single valued neutrosophic sets given by $A_{SV}^{w} = \left(< x_i, \tilde{\mu}_{A^w}(x_i), \tilde{i}_{A^w}(x_i), \tilde{f}_{A^w}(x_i) > | \, x_i \in U \right); B_{SV}^{w} = \left(< x_i, \tilde{\mu}_{B^w}(x_i), \tilde{i}_{B^w}(x_i), \tilde{f}_{B^w}(x_i) > | \, x_i \in U \right)$. The amount of true membership degree between two fuzzy sets A_{FS}^{w} and B_{FS}^{w}, represented by $H_{CE}^{\mu}\left(A_{FS}^{w}, B_{FS}^{w}\right)$, is established in **Theorem 3.2.** Similarly, the amount of indeterminacy degree between two fuzzy sets A_{FS}^{w} and B_{FS}^{w} can be represented by $H_{CE}^{i}\left(A_{FS}^{w}, B_{FS}^{w}\right)$ and is defined as



(a)



(b)

Fig.2 Minimum value of symmetric fuzzy cross entropy measure $H^{\mu}_{CE}\left(A^{w}_{FS}, B^{w}_{FS}\right)$

$$H^{i}_{CE}\left(A^{w}_{FS}, B^{w}_{FS}\right)$$
$$= \sum_{i=1}^{n} \left[\begin{array}{l} -10\tan\frac{1}{\sqrt{2}} + \left(4 + \tilde{i}_{A^w}(x_i) + \tilde{i}_{B^w}(x_i)\right)\tan\left(\frac{2\sqrt{2} + 2\sqrt{\tilde{i}^2_{A^w}(x_i) + \tilde{i}^2_{B^w}(x_i)} - \sqrt{2\tilde{i}_{A^w}(x_i)\tilde{i}_{B^w}(x_i)}}{4 + \tilde{i}_{A^w}(x_i) + \tilde{i}_{B^w}(x_i)}\right) \\ + \left(6 - \tilde{i}_{A^w}(x_i) - \tilde{i}_{B^w}(x_i)\right)\tan\left(\frac{2\sqrt{2} + 2\sqrt{+\left(1-\tilde{i}_{A^w}(x_i)\right)^2 + \left(1-\tilde{i}_{B^w}(x_i)\right)^2} - \sqrt{\left(1-\tilde{i}_{A^w}(x_i)\right)\left(1-\tilde{i}_{B^w}(x_i)\right)}}{6 - \tilde{i}_{A^w}(x_i) - \tilde{i}_{B^w}(x_i)}\right) \end{array} \right] \quad \ldots (12)$$

Furthermore, the amount of falsity membership degree between two fuzzy sets A^{w}_{FS} and B^{w}_{FS} can be represented by $H^{f}_{CE}\left(A^{w}_{FS}, B^{w}_{FS}\right)$ and is defined as

$$H^{f}_{CE}\left(A^{w}_{FS}, B^{w}_{FS}\right)$$
$$= \sum_{i=1}^{n} \left[\begin{array}{l} -10\tan\frac{1}{\sqrt{2}} + \left(4 + \tilde{f}_{A^w}(x_i) + \tilde{f}_{B^w}(x_i)\right)\tan\left(\frac{2\sqrt{2} + 2\sqrt{\tilde{f}^2_{A^w}(x_i) + \tilde{f}^2_{B^w}(x_i)} - \sqrt{2\tilde{f}_{A^w}(x_i)\tilde{f}_{B^w}(x_i)}}{4 + \tilde{f}_{A^w}(x_i) + \tilde{f}_{B^w}(x_i)}\right) \\ + \left(6 - \tilde{f}_{A^w}(x_i) - \tilde{f}_{B^w}(x_i)\right)\tan\left(\frac{2\sqrt{2} + 2\sqrt{+\left(1-\tilde{f}_{A^w}(x_i)\right)^2 + \left(1-\tilde{f}_{B^w}(x_i)\right)^2} - \sqrt{\left(1-\tilde{f}_{A^w}(x_i)\right)\left(1-\tilde{f}_{B^w}(x_i)\right)}}{6 - \tilde{f}_{A^w}(x_i) - \tilde{f}_{B^w}(x_i)}\right) \end{array} \right] \quad \ldots (13)$$

Hence, the proclaimed single valued neutrosophic cross entropy measure hinged on two single-valued neutrosophic sets (SVNSs) A^{w}_{SV} and B^{w}_{SV} can be obtained by simply adding the resulting equations **(3), (12)** and **(13).** Thus,

$$R_{SV}\left(A^{w}_{SV}, B^{w}_{SV}\right) = H^{\mu}_{CE}\left(A^{w}_{FS}, B^{w}_{FS}\right) \textbf{(Eq.3)} + H^{i}_{CE}\left(A^{w}_{FS}, B^{w}_{FS}\right) \textbf{(Eq.12)} + H^{i}_{CE}\left(A^{w}_{FS}, B^{w}_{FS}\right) \quad \textbf{(Eq.13)} \qquad \ldots (14)$$

Here, $R_{SV}\left(A_{SV}^{w}, B_{SV}^{w}\right)$ satisfies all the conditions (i), (ii) and (iii) of **Def. 2.6** and hence a valid single valued neutrosophic entropy measure hinged on two single-valued neutrosophic sets A_{SV}^{w} and B_{SV}^{w}.

Theorem 4.1 Let

$A_{SV}^{w} = \left(< x_i, \tilde{\mu}_{A^w}(x_i), \tilde{i}_{A^w}(x_i), \tilde{f}_{A^w}(x_i) >| x_i \in U\right); B_{SV}^{w} = \left(< x_i, \tilde{\mu}_{B^w}(x_i), \tilde{i}_{B^w}(x_i), \tilde{f}_{B^w}(x_i) >| x_i \in U\right)$ be any two single-valued neutrosophic sets with same cardinality n. There exist the inequality

$$0 \le R_{SV}\left(A_{SV}^{w}, B_{SV}^{w}\right) \le 30\left(\tan\left(\frac{2\sqrt{2}+2}{5}\right) - \tan\left(\frac{1}{\sqrt{2}}\right)\right)n.$$

Proof. In view of equations **(3), (12)** and **(13)** and replacement of B_{SV}^{w} with $C\left(A_{SV}^{w}\right)$ into **(14)** yields

$$R_{SV}\left(A_{SV}^{w}, C\left(A_{SV}^{w}\right)\right)$$

$$= \sum_{i=1}^{n}\left[\begin{array}{l} 30\tan\left(\frac{2\sqrt{2}+2}{5}\right) - 30\tan\left(\frac{1}{\sqrt{2}}\right) \\ -10\left(\begin{array}{l} 3\tan\left(\frac{2\sqrt{2}+2}{5}\right) - \left(\frac{4+\tilde{\mu}_{A^w}(x_i)+\tilde{f}_{A^w}(x_i)}{5}\right)\tan\left(\frac{2\sqrt{2}+2\sqrt{\tilde{\mu}_{A^w}^2(x_i)+\tilde{f}_{A^w}^2(x_i)}-\sqrt{2\tilde{\mu}_{A^w}(x_i)\tilde{f}_{A^w}(x_i)}}{4+\tilde{\mu}_{A^w}(x_i)+\tilde{f}_{A^w}(x_i)}\right) \\ -\left(\frac{6-\tilde{\mu}_{A^w}(x_i)-\tilde{f}_{A^w}(x_i)}{5}\right)\tan\left(\frac{2\sqrt{2}+2\sqrt{\left(1-\tilde{\mu}_{A^w}(x_i)\right)^2+\left(1-\tilde{f}_{A^w}(x_i)\right)^2}-\sqrt{2\left(1-\tilde{\mu}_{A^w}(x_i)\right)\left(1-\tilde{f}_{A^w}(x_i)\right)}}{6-\tilde{\mu}_{A^n}(x_i)-\tilde{f}_{A^n}(x_i)}\right) \\ -\tan\left(\frac{2\sqrt{2}+2\sqrt{\tilde{i}_{A^w}^2(x_i)+\left(1-\tilde{i}_{A^w}(x_i)\right)^2}-\sqrt{2\tilde{i}_{A^w}(x_i)\left(1-\tilde{i}_{A^w}(x_i)\right)}}{5}\right) \end{array}\right) \end{array}\right]$$

$$= 10\,\text{Max.}R_N\left(A_{SV}^{w}\right) - 10\,R_N\left(A_{SV}^{w}\right); \text{where} \qquad \ldots (15)$$

$$R_N\left(A_{SV}^{w}\right)$$

$$= \sum_{i=1}^{n}\left[\begin{array}{l} 3\tan\left(\frac{2\sqrt{2}+2}{5}\right) - \left(\frac{4+\tilde{\mu}_{A^w}(x_i)+\tilde{f}_{A^w}(x_i)}{5}\right)\tan\left(\frac{2\sqrt{2}+2\sqrt{\tilde{\mu}_{A^w}^2(x_i)+\tilde{f}_{A^w}^2(x_i)}-\sqrt{2\tilde{\mu}_{A^w}(x_i)\tilde{f}_{A^w}(x_i)}}{4+\tilde{\mu}_{A^w}(x_i)+\tilde{f}_{A^w}(x_i)}\right) \\ -\left(\frac{6-\tilde{\mu}_{A^w}(x_i)-\tilde{f}_{A^w}(x_i)}{5}\right)\tan\left(\frac{2\sqrt{2}+2\sqrt{\left(1-\tilde{\mu}_{A^w}(x_i)\right)^2+\left(1-\tilde{f}_{A^w}(x_i)\right)^2}-\sqrt{2\left(1-\tilde{\mu}_{A^w}(x_i)\right)\left(1-\tilde{f}_{A^w}(x_i)\right)}}{6-\tilde{\mu}_{A^n}(x_i)-\tilde{f}_{A^n}(x_i)}\right) \\ -\tan\left(\frac{2\sqrt{2}+2\sqrt{\tilde{i}_{A^w}^2(x_i)+\left(1-\tilde{i}_{A^w}(x_i)\right)^2}-\sqrt{2\tilde{i}_{A^w}(x_i)\left(1-\tilde{i}_{A^w}(x_i)\right)}}{5}\right) \end{array}\right]. \qquad \ldots (16)$$

The resulting mathematical expression **(16)** is the desired single valued neutrosophic

entropy measure since it satisfies all the necessary conditions (i), (ii) and (iii) as laid down in **Def. 2.6.**

Since $\mathrm{R}_{\mathrm{N}}\left(A_{\mathrm{SV}}^{w}\right)\geq 0$ (in view of **Fig.3)** for each $A_{\mathrm{SV}}^{w}\subseteq R(U)$, the resulting inequality **(15)** yields

$$\mathrm{R}_{\mathrm{N}}\left(A_{\mathrm{SV}}^{w}\right)=\mathrm{Max}.\mathrm{R}_{\mathrm{N}}\left(A_{\mathrm{SV}}^{w}\right)-\frac{1}{10}\mathrm{R}_{\mathrm{SV}}\left(A_{\mathrm{SV}}^{w},C\left(A_{\mathrm{SV}}^{w}\right)\right)\geq 0 \qquad \ldots (17)$$

$$\Rightarrow 0\leq R_{\mathrm{SV}}\left(A_{\mathrm{SV}}^{w},C\left(A_{\mathrm{SV}}^{w}\right)\right)\leq 30\left(\tan\left(\frac{2\sqrt{2}+2}{5}\right)-\tan\left(\frac{1}{\sqrt{2}}\right)\right)n \qquad \ldots (18)$$

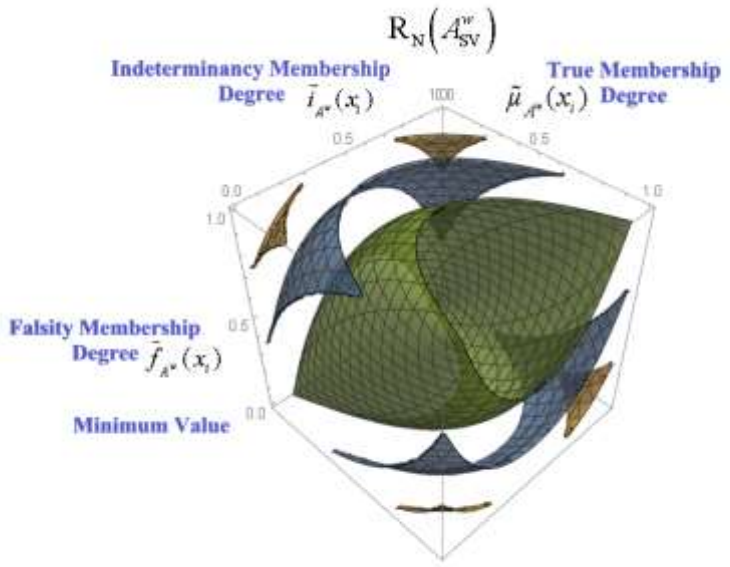


Fig.3 Three -dimensional contour plot for non-negativity of $\mathrm{R}_{\mathrm{N}}\left(A_{\mathrm{SV}}^{w}\right)$

Discussion The resulting inequality **(18)** justifies that $R_{\mathrm{SV}}\left(A_{\mathrm{SV}}^{w},C\left(A_{\mathrm{SV}}^{w}\right)\right)$ is a finite quantity. Following the similar pattern as deploying to obtain **(18),** it is reasonable to establish that $0\leq R_{\mathrm{SV}}\left(A_{\mathrm{SV}}^{w},B_{\mathrm{SV}}^{w}\right)\leq 30\left(\tan\left(\frac{2\sqrt{2}+2}{5}\right)-\tan\left(\frac{1}{\sqrt{2}}\right)\right)n$, where n is a fixed natural number. Thus, $\mathrm{Max}.R_{\mathrm{SV}}\left(A_{\mathrm{SV}}^{w},B_{\mathrm{SV}}^{w}\right)=30\left(\tan\left(\frac{2\sqrt{2}+2}{5}\right)-\tan\left(\frac{1}{\sqrt{2}}\right)\right)n$ and this value completely depends on the cardinality of U.

The overhead discussion has put us in a conclusive position to deploy the newly discovered trigonometric fuzzy entropy (TFE) and single valued neutrosophic entropy (TNE) measures,

represented by **(1)** and **(14),** to assess the impact of heavy metal concentration in river water as follows.

5. Impact Assessment of Heavy Metal

The underlying research work is initiated by collecting Sarsa River water samples, before and after the amalgamation of pharmaceutical effluents (PE) treated in common effluent treatment plant (CETP). To reckon the quality of river water for drinking purposes, we have done a lot of experimenatation investigations, data comparsion and expressed the concentration of each heavy metal in terms of $\mu g / mL$ (**Fig. 4(a-b))**. In this study, the impact of concentration of heavy metals like cadmium (Cd), manganese (Mn), cobalt (Co), lead (Pb), copper (Cu), zinc (Zn) and iron (Fe), have been done through experimental investigations as well as the proposed TFE and TNE measures.

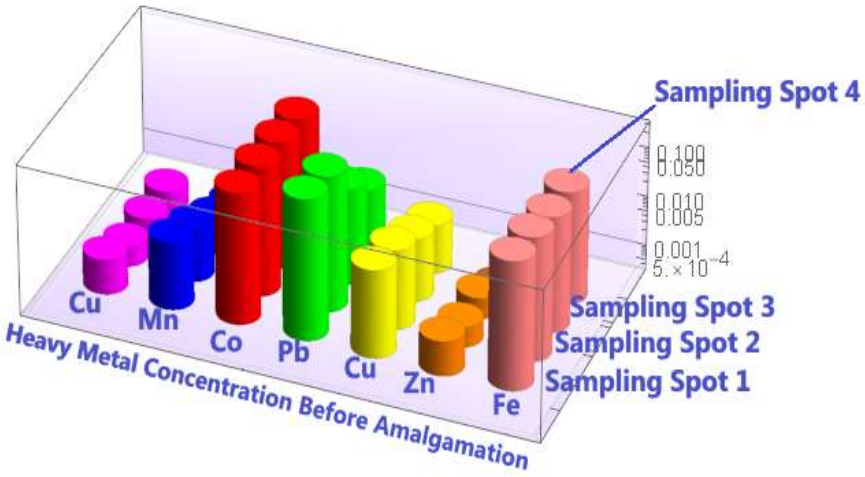
5.1 Assessment of Heavy Metal Concentration Based on Experimental Observations: The following observations were made before and after amalgamating pharmaceutical effluents (PE) treated in common effluent treatment plant (CETP) into river water samples.

(a) Cadmium. Before amalgamation, the concentration of cadmium, depcited in **Fig. 4(a),** was $0.002\ (S_1)$, followed by $0.001\ (S_2, S_3, S_4)$ with an average concentration of $0.00125\ \mu g / mL.$. After amalgamation, no cadmium was detected in the recorded observations as can be seen in **Fig. 4 (b).**

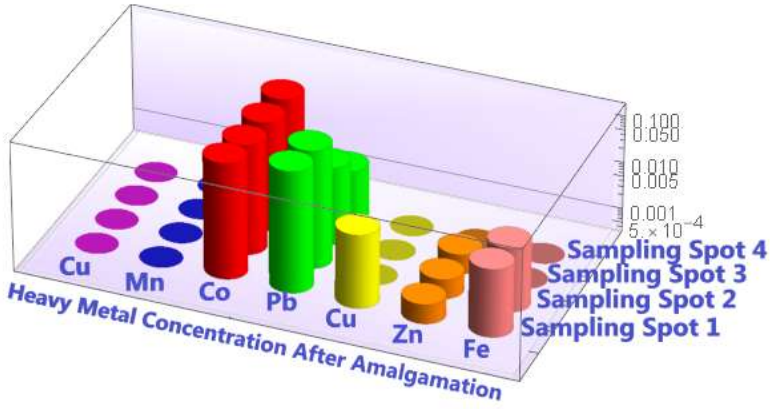
(b) Manganese. Spatial variations were observed in the concentration of manganese, before and after amalgamation, as depicited in **Fig. 4(a, b).** Before amalgamation, the concentration of Mn was $0.01\ (S_1), 0.005\ (S_2), 0.002\ (S_3)$, and $0.001\ (S_4)$ with an average concentration of $0.0045\ \mu g / mL.$ After introducing pharmaceutical effluents into river water samples, Mn was found to be absent.

(c) Cobalt. The cobalt's concentration varied spatially in Sarsa river water samples, taken before and after amalgamation, as shown in **Fig. 4(a, b)**. The concentration of cobalt decreased from 0.225 to $0.143\ (S_1), 0.214$ to $0.0107\ (S_2)$, 0.18 to $0.1\ (S_3)$ and 0.147 to $0.1\ (S_4)$ respectively and it was within permissible limits**.**

(d) Lead. The water of Sarsa river recorded lead concentration of 0.36 before amalgamation and it decreased to $0.19\ (S_1)$ after amalgamation, as indicated by **Fig.4(a, b**). Similarly, the treatment was effective in reducing lead concentration from 0.28 to $0.158\ (S_2)$, 0.04 to $0.02\ (S_3)$ and 0.01 to $0.005\ (S_4)$ respectively.



(a)



(b)

Fig 4. Heavy metal concentration of (a) Cd, (b) Mn, (c) Co, (d) Pb, (e) Cu, (f) Zn and (g) Fe in Sarsa river water samples

(e) Copper. The results depcited by **Fig. 4 (a, b**) indicate that before amalgamation of pharmaceutical effluents into river water samples, the concentration of Cu was $0.027\ (S_1)$ which reduced to $0.016\ (S_1)$ after amalgamation. The concentration further decreased from 0.018 to nil (S_2), 0.008 to nil $((S_3))$ and 0.004 to nil (S_4) respectively. Dilution of the effluents with river water could be the factor responsible for complete copper removal in the river water.

(f) Zinc. The results of **Fig. 4(a, b**) clearly indicate that before and after amalgamation, the concentration of zinc decreased from 0.002 to $0.001\ (S_1)$, from 0.001 to nil (S_2, S_3) and no zinc was detected in S_4 before and after amalgamation.

(g) Iron. While comparing the concentration of iron (Fe) represented by **Fig. 4(a, b**), it was found that, before and after amalgamation, the concentration of Fe decreased from 0.195 to 0.012 (S_1), 0.155 to 0.01 S_2, 0.104 to nil (S_3) and 0.09 to nil (S_4) consecutively.

The overhead discussion concludes that the concentration of heavy metals in Sarsa river water samples, which were within the permissible limits before amalgamation, dwindled gradually after the amalgamation of pharmaceutical effluents into river water samples **(Fig.4a, 4b**). The effectiveness of the proposed TFE and TNE measures can be confirmed only if these entropy measures can be proven capable in justifying the similar ressutls as obtained through experimental investigations.

5.2Assessment of Heavy Metal Concentration Based on TFE and TNE Measures

To evaluate the impact of heavy metals concentration in river water samples through our proposed trigonometric fuzzy and single valued neutrosophic entropy measures, we have represented each heavy metal by the set $B=(B_1,B_2,B_3,B_4,B_5,B_6,B_7)$ where B_1 = Cadmium (Cd), B_2 = Manganese (Mn), B_3 = Cobalt (Co), B_4 =Lead (Pb), B_5 = Copper (Cu), B_6 = Zinc (Zn) and B_7 = Iron (Fe) consecutively. After doing a lot of data comparison and experimental investigations, we have, equally well, extracted the lower (minimum) and upper (maximum) bounds for each monitored heavy metal concentration reading. Let $\tilde{\mu}_{B_K^w}(x_1)$ and $\tilde{U}_{B_K^w}(x_1)$ respectively be the lower and upper bounds extracted from K^{th} heavy metal concentration. In this study, we have constructed the concentration intervals $\left[\tilde{\mu}_{B_K^w}(x),\tilde{U}_{B_K^w}(x)\right]$, before and after amalgamation, for each heavy metal concentration represented by $B_K(K=1,2,3,4,5,6,7)$ and the results are displayed in **Table. 1(a).**

Let $\tilde{f}_{B_K^w}(x)=1-\tilde{U}_{B_K^w}(x)$, $\tilde{i}_{B_K^w}(x)=1-\tilde{f}_{B_K^w}(x)-\tilde{U}_{B_K^w}(x)$ denote the amount of fuzziness based on the falsity and indeterminacy membership degree of K^{th} heavy metal concentration. If we restrict the value(s) of $\tilde{i}_{B_K^w}(x)$ to $0\cdot0001$ if it is less than or equal to zero, then the set B_K can be extended into the forms of single valued neutrosophic set (SVNS) represented by $\left[\tilde{\mu}_{B_K^w}(x),\tilde{i}_{B_K^w}(x),\tilde{f}_{B_K^w}(x)\right]$ and the results are displayed in **Table. 1(b).** Let $H_F(B_K)$ and $R_N(B_K)$ denote the trigonometric fuzzy entropy and single valued neutrosophic entropy measures value of K^{th} heavy metal concentration.

Taking $i=1$ and replacing $\tilde{\mu}_{A^w}(x_i)$ with $\tilde{\mu}_{B_K^w}(x)$ into the **(1),** Then, $H_F(B_K)(K=1,2,3,4,5,6,7)$, after modification, takes the form as shown in **(17).** The results are displayed in **Table 1(a).** Thus,

$$H_F(B_K)=\tan\left(\frac{1}{4}\right)\tan\left[\tan\left(\frac{2\sqrt{2}+2}{5}\right)-\tan\left(\frac{2\sqrt{2}+2\sqrt{\tilde{\mu}^2_{B_K^w}(x)+\left(1-\tilde{\mu}_{B_K^w}(x)\right)^2}-\sqrt{2\tilde{\mu}_{B_K^w}(x)\left(1-\tilde{\mu}_{B_K^w}(x)\right)}}{5}\right)\right] \quad \ldots(19)$$

Similarly, $R_N(B_K)$; $(K=1,2,...,7)$ can also be modified by taking $i=1$ and replacing $\tilde{\mu}_{A^w}(x_1),\tilde{i}_{A^w}(x_1),\tilde{f}_{A^w}(x_1)$ with $\tilde{\mu}_{B_K^w}(x),\tilde{\mu}_{B_K^w}(x),\tilde{f}_{B_K^w}(x)$ into the resulting equation **(16). T**he results are displayed in **Table 1(b).** Thus,

$$\begin{aligned}R_N(B_K)=3\tan\left(\frac{2\sqrt{2}+2}{5}\right)&-\left(\frac{4+\tilde{\mu}_{B_K^w}(x)+\tilde{f}_{B_K^w}(x)}{5}\right)\tan\left(\frac{2\sqrt{2}+2\sqrt{\tilde{\mu}^2_{B_K^w}(x)+\tilde{f}^2_{B_K^w}(x)}-\sqrt{2\tilde{\mu}_{B_K^w}(x)\,\tilde{f}_{B_K^w}(x)}}{4+\tilde{\mu}_{B_K^w}(x)+\tilde{f}_{B_K^w}(x)}\right)\\&-\left(\frac{6-\tilde{\mu}_{B_K^w}(x)-\tilde{f}_{B_K^w}(x)}{5}\right)\tan\left(\frac{2\sqrt{2}+2\sqrt{\left(1-\tilde{\mu}_{B_K^w}(x)\right)^2+\left(1-\tilde{f}_{B_K^w}(x)\right)^2}-\sqrt{2\left(1-\tilde{\mu}_{B_K^w}(x)\right)\left(1-\tilde{f}_{B_K^w}(x)\right)}}{6-\tilde{\mu}_{B_K^w}(x)-\tilde{f}_{B_K^w}(x)}\right)\\&-\tan\left(\frac{2\sqrt{2}+2\sqrt{\tilde{i}^2_{B_K^w}(x)+\left(1-\tilde{i}_{B_K^w}(x)\right)^2}-\sqrt{2\tilde{i}_{B_K^w}(x)\left(1-\tilde{i}_{B_K^w}(x)\right)}}{5}\right)\end{aligned} \quad (20)$$

The trigonometric fuzzy entropy (TFE) measure values $H_F(B_K)$ as well as single valued neutrosophic entropy (TNE) measure values $R_N(B_K)$ for each heavy metal B_K can be evaluated employing equations **(17)** and **(18)**. A comparative analysis of the results depicted in **Table. 1)a)** and **Table. 1(b)** reveal that, before amalgamation, the heavy metal concentration was found to be more macroscopic (owing to high TFE and TNE values as shown in **Fig. 5**) which became less macroscopic after amalgamation (owing to low TFE and TNE values as shown in **Fig. 5**). In other words, the concentration of each heavy metal, which was within the permissible limits before amalgamation, dwindled gradually (owing to negative change in TFE and TNE values) after amalgamating pharmaceutical effluents into River water samples.

Table 1(a). Concentration intervals and TFE values of each B_K before and after amalgamation of pharmaceutical effluents

Heavy Metal	Concentration Interval Before Amalgamation	Concentration Interval After Amalgamation	TFE Value Before Amalgamation	TFE Value After Amalgamation	Change in TFE Values
Cadmium	[0.001,0.002]	[0.000,0.000]	0.0073	0.0000	-0.0073
Manganese	[0.001,0.010]	[0.000,0.000]	0.0073	0.0000	-0.0073
Cobalt	[0.147,0.225]	[0.100,0.143]	0.1000	0.0825	-0.0174
Lead	[0.010,0.360]	[0.005,0.190]	0.0073	0.0000	-0.0073
Copper	[0.004,0.027]	[0.000,0.016]	0.0073	0.0168	0.0095
Zinc	[0.000,0.002]	[0.000,0.001]	0.0000	0.0000	0.0000
Iron	[0.090,0.195]	[0.000,0.012]	0.0073	0.0000	-0.0073

Table 1(b). Conversion of concentration intervals into the forms of SVNSs TNE values of each B_K before and after amalgamation of pharmaceutical effluents

Heavy Metal	SVNSs Before Amalgamation	SVNSs After Amalgamation	TNE Values Before Amalgamation	TNE Values After Amalgamation	Change in TNE Values
Cadmium	[0.0010,0.0010,0.9980]	[0.0010,0.0001,0.9999]	0.0249	0.0045	-0.0204
Manganese	[0.0010,0.0090,0.9900]	[0.0010,0.0001,0.9999]	0.0546	0.0045	-0.0500
Cobalt	[0.1470,0.0780,0.7750]	[0.1000,0.0430,0.8570]	0.2953	0.2347	-0.0606
Lead	[0.0010,0.3590,0.6400]	[0.0050,0.1850,0.8100]	0.3334	0.2541	-0.0793
Copper	[0.0010,0.0017,0.9973]	[0.0000,0.0160,0.9840]	0.0290	0.0628	0.0338
Zinc	[0.0000,0.0020,0.9980]	[0.0000,0.0010,0.9990]	0.0208	0.0146	0.0063
Iron	[0.0010,0.1940,0.8050]	[0.0000,0.0120,0.9880]	0.2508	0.0537	-0.1971

Discussion After amalgamation, the concentration of cadmium and manganese was found to be negligible **(Fig.4)**. The possible reasons for absence of these heavy metals in river water samples could be the use of physico-chemical processes-adsorption, membrane filtration, electro dialysis etc., which further diluted the river water after amalgamation and made heavy metal presence almost negligible in the river.

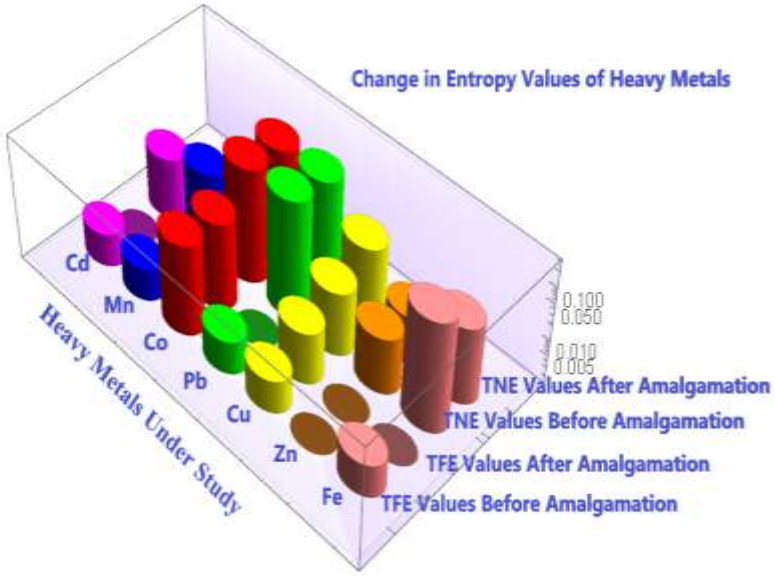


Fig.5 TFE and TNE values of each heavy metal concentration before and after amalgamating pharmaceutical effluents into river water samples

The similar observations were experienced from the experimental investigations. Hence, the effectiveness and validity of our proposed TFE and TNE measures have been justified.

We next switch to establish the proclaimed heavy metal contamination evaluation methodology, intended to identify the most contaminated sampling spot responsible for heavy metal contamination in Sarsa river water.

6. Heavy Metal Contamination Evaluation Methodology

To reckon the most contaminated sampling spot by the proposed methodology, we proceed as follows.

Step: -1 Collection of River Water Samples

The water samples were collected from Sarsa river by covering a stretch of 20 km from four sampling spots S_1, S_2, S_3 & S_4, before and after the amalgamation of CETP treated pharmaceutical effluents (PE) into river water samples. The samples were stored in high-grade polythene bottles of one-liter capacity. Representative water sample from selected sites was collected and transported to the laboratory for experimentation investigations by keeping in mind that the comparative concentrations of all related components were same in all the samples. All precautions were taken to avoid any significant alteration in sample composition before experiments were performed. Analytical studies were carried out by the methods of American Public Health Association **[20]**.

Step: -2 Normalization of Monitored Heavy Metal Concentration

Suppose the number of parameters (heavy metals) to be studied is denoted by "n". Let the number of sampling spots under study is denoted by "m". Let l_{ji} denotes the monitored concentration reading of j^{th} heavy metal at i^{th} sampling spot. To ensure the quality of various quantity grades,

it becomes essential, before fuzzification, to normalize each monitored heavy metal concentration reading. Let p_{ji} denotes the normalization concentration function for the concentration of j^{th} heavy metal at i^{th} sampling spot. Then

$$p_{ji} = \frac{l_{ji} - \text{Min}.l_{ji}}{\text{Max}.l_{ji} - \text{Min}.l_{ji}}; j = 1,2,3,4...,n; i = 1,2,3,4...,m. \quad \ldots (21)$$

Step: 3 Determination of Fuzzy and Neutrosophic Entropy Weights

After the introduction of fuzzy sets theory by Zadeh **[15]**, Information theory started receiving recognition from different quarters. In the existing literature, many fuzzy entropy measures have been investigated and characterized by researchers, but with some demerits and limitations. De Luca and Termini **[19]** suggested the first non-additive measure of fuzzy entropy:

$$H(A) = -\frac{1}{\log m}\sum_{j=1}^{n}\left[\mu_A(x_j)\log\mu_A(x_j) + \left(1-\mu_A(x_j)\right)\log\left(1-\mu_A(x_j)\right)\right] \quad \ldots (22)$$

where $A = \left(< x_i, \mu_A(x_i) > | \, x_i \in U\right)$ is the corresponding fuzzy set satisfying $\mu_A(x_j): X \to [0,1]$ and "m" is any fixed natural number. The fuzzy entropy measure **(22)** has been found capable for analyzing the macroscopic view of heavy metal pollution in river water samples**.** Unfortunately, the entropy measure **(22)** is facing a major drawback as it is unknowingly based on the fancy assumption $0 \times log0 = 0$ and hence indicates less macroscopic view of heavy metal contamination. To represent macroscopic view of heavy metal contamination in a broader way and to meet the exigency, we have successfully deployed our proposed TFE and TNE measures to construct fuzzy and single valued neutrosophic entropy weights for various sampling spot with respect to each heavy metal concentration as follows.

Let T_{ji} denotes the amount of fuzziness based on true membership degree of j^{th} heavy metal concentration at i^{th} sampling spot. Then,

$$T_{ji} = \frac{p_{ji}}{\sum_{j=1}^{n} p_{ji}}; j = 1,2,...,n, i = 1,2,..,m \quad \ldots (23)$$

(a) The weights $w_{ji}^{(0)}$ for j^{th} heavy metal concentration at i^{th} sampling spot employing Deluca and Termini **(22)** can be evaluated as follows. Let "m" be the number of sampling spots, then

$$w_{ji}^{(0)} = \frac{1 - E_{ji}^{(0)}}{\sum_{j=1}^{n} E_{ji}^{(0)}}, \text{ where} \quad \ldots (24)$$

$$E_{ji}^{(0)} = -\frac{1}{\log m}\sum_{j=1}^{n}\left[T_{ji}\log T_{ji} + \left(1-T_{ji}\right)\log\left(1-T_{ji}\right)\right] \quad \ldots (25)$$

(b) The weights $w_{ji}^{(1)}$ for j^{th} heavy metal concentration at i^{th} sampling spot employing the proposed trigonometric fuzzy entropy (TFE) measure (**1**) can be evaluated as follows: Let "m" be the number of sampling spots, then

$$w_{ji}^{(1)} = \frac{1-E_{ji}^{(1)}}{\sum_{j=1}^{n} E_{ji}^{(1)}}, \text{ where} \quad \ldots (26)$$

$$E_{ji}^{(1)} = -\tan\left(m^{-1}\right)\sum_{j=1}^{n}\left[\tan\left(\frac{2\sqrt{2}+2\sqrt{T_{ji}^{2}+\left(1-T_{ji}\right)^{2}}-\sqrt{2T_{ji}\left(1-T_{ji}\right)}}{5}\right)-\tan\left(\frac{2\sqrt{2}+2}{5}\right)\right] \quad \ldots (27)$$

(c) The weights $w_{ji}^{(2)}$ for j^{th} heavy metal concentration at i^{th} sampling spot employing the proposed single valued neutrosophic entropy (TNE) measure (**16)** can be evaluated as follows.

Let $F_{ji}=1-T_{ji}$ and $I_{ji}=1-T_{ji}-F_{ji}$ denote the amount of fuzziness based on the indeterminacy and falsity membership degree of j^{th} heavy metal concentration at i^{th} sampling spot. Here, the values of I_{ji} are restricted to $0{\cdot}001$ if it is less than or equal to zero and "m" is the number of sampling spots. Then,

$$w_{ji}^{(2)} = \frac{1-E_{ji}^{(2)}}{\sum_{j=1}^{n} E_{ji}^{(2)}}, \text{where} \quad \ldots (28)$$

$$E_{ji}^{(2)} = \tan\left(m^{-1}\right)\sum_{j=1}^{n}\left[\begin{array}{l} 3\tan\left(\frac{2\sqrt{2}+2}{5}\right)-\left(\frac{4+T_{ji}+F_{ji}}{5}\right)\tan\left(\frac{2\sqrt{2}+2\sqrt{T_{ji}^{2}+F_{ji}^{2}}-\sqrt{2T_{ji}F_{ji}}}{4+T_{ij}+F_{ij}}\right) \\ -\left(\frac{6-T_{ji}-F_{ji}}{5}\right)\tan\left(\frac{2\sqrt{2}+2\sqrt{\left(1-T_{ji}\right)^{2}+\left(1-F_{ji}\right)^{2}}-\sqrt{2\left(1-T_{ji}\right)\left(1-F_{ji}\right)}}{6-T_{ij}-F_{ij}}\right) \\ -\tan\left(\frac{2\sqrt{2}+2\sqrt{I_{ji}^{2}+\left(1-I_{ji}\right)^{2}}-\sqrt{2I_{ji}\left(1-I_{ji}\right)}}{5}\right) \end{array}\right] \quad \ldots (29)$$

Step: -4 Calculations of Relative Sub-Indices

The quality of river water parameters (heavy metals) can be well described by means of two types of sub-indices-absolute and relative-which are being used by the eminent researchers. Since absolute (or relative) sub-indexing approaches are fully independent (or dependent) on water quality standards, the relative sub-indexing approach has been empowered in this study. Let $Q_{ji}=$ Relative

sub-index, S_{ji} = Maximum permissible concentration limit and I_{ji} = Monitored concentration reading of the j^{th} heavy metal at i^{th} sampling spot respectively. Then, the relative sub-indices of each heavy metal with respect to various sampling spots are assigned as

$$Q_{ji} = \frac{I_{ji}}{S_{ji}} \times 100;\ j = 1,2,3,4...n, i = 1,2,3,4..,m. \quad \ldots (30)$$

Step: -5 Constructions of FHCI and NHCI

The enduring Deluca and Termini entropy **([19]**, proposed trigonometric fuzzy entropy weighted and single valued neutrosophic entropy weighted heavy metal contamination indices (EHCI, FHCI and NHCI), before and after the amalgamation of pharmaceutical effluents into river water samples, can be constructed as follows:

$$\text{EHCI at } i^{th} \text{ Sampling Spot} = \sum_{j=1}^{n} w_{ji}^{(0)} Q_{ji} \quad \ldots (31)$$

$$\text{FHCI at } i^{th} \text{ Sampling Spot} = \sum_{j=1}^{n} w_{ji}^{(1)} Q_{ji} \quad \ldots (32)$$

$$\text{NHCI at } i^{th} \text{ Sampling Spot} = \sum_{j=1}^{n} w_{ji}^{(2)} Q_{ji} \quad \ldots (33)$$

Step: -6 Identifying the Most Contaminated Sampling Spot

The maximum(or minimum) EHCI, FHCI or NHCI score among each sampling spot is designated to the"most (or least) contaminated sampling spot" accordingly.

We finally deploy the proclaimed fuzzy entropy weighted heavy metal contamination index (EHCI) and single valued neutrosophic entropy weighted heavy metal contamination index (NHCI) to identify the most contaminated sampling spot responsible for heavy metal contamination in Sarsa river water.

7. Application of TFE and TNE Based Heavy Metal Contamination Evaluation Methodology

To reckon the most contaminated sampling spot, responsible for heavy metal contamination in Sarsa river water, we have computed the enduring Deluca and Termini entropy **[19]**, proposed trigonometric fuzzy entropy and single valued neutrosophic entropy weighted heavy metal contamination indices (FHCI, EHCI and NHCI) through the proposed methodology as explained in **Section. 6** and the results are displayed in **Tables.2-4(a, b).**

7.1 Identification of Most Contaminated Sampling Spot Through EHCI

The Deluca and Termini fuzzy entropy weighted heavy metal contamination index (EHCI) at each sampling spot S_1, S_2, S_3, S_4, before and after the amalgamation of pharmaceutical effluents into river water samples, is computed by deploying **(31)** and the results are depicted in **Table.2(a, b).** The monitored concentration reading of each heavy metal is expressed in terms of mg/L. In this study, the number of parameters (heavy metals) is seven $(n = 7)$ and the number of sampling spots is four $(m = 4)$. The normalization concentration function $p_{ji}\ (j = 1,2,...,7; i = 1,2,3,4)$ for each heavy

metal at various sampling spots and the results are depicted in **Table.2(a, b).** As per W.H.O. **[20]**, the maximum permissible limits $S_{ji}\ (j=1,2,..7, i=1,2,3,4)$ of each heavy metal at various sampling spots are considered as 0.005(Cd), 0.1(Mn), 2(Co), 0.05(Pb), 1(Cu), 5(Zn), 0.1(Fe)(mg/L).

Observations. The results depicted in **Table 2(a, b)** and **Fig. 5 i**ndicate that the trend of EHCI scores reduced gradually, from 0.3623 to0.1673 (S_1), from 0.2816 to0.1559 (S_2), from 0.1067 to 0.0440 (S_3) and from 0.0766 to 0.0468 (S_4) consecutively.

7.2 Identification of Most Contaminated Sampling Spot Through FHCI

The trigonometric fuzzy entropy weighted heavy metal contamination index (EHCI) at each sampling spot S_1, S_2, S_3, S_4, before and after the amalgamation of pharmaceutical effluents into river water samples, is computed by deploying **(32)** and the results are depicted in **Table.3(a, b).**

Observations. The results depicted in **Table 3(a, b)** and **Fig. 5** reveal that the proposed EHCI exhibited the similar trend as returned by Deluca and Termini entropy weighted heavy metal contamination index (FHCI). The FHCI scores at each sampling spot S_1, S_2, S_3, S_4 reduced gradually, from 1.6865 to0.8343 (S_1), from 1.3276 to0.8237(S_2), from 0.4186 to 0.1895 (S_3) and from 0.3064 to 0.1266 (S_4) consecutively.

7.3 Identification of Most Contaminated Sampling Spot Through NHCI

The single valued neutrosophic entropy weighted heavy metal contamination index (NHCI) at each sampling spot S_1, S_2, S_3, S_4, before and after the amalgamation of pharmaceutical effluents into river water samples, is computed by deploying **(33)** and the results are depicted in **Table.4(a, b).**

Observations The results depicted in **Table 4(a, b)** and **Fig. 5** reveal that NHCI scores at each sampling spot S_1, S_2, S_3, S_4 reduced gradually, from 0.7093 to 0.3216 (S_1), from 0.5575 to 0.3031(S_2), from 0.1841 to 0.0673 (S_3) and from 0.1342 to 0.0481(S_4) consecutively.

Results and Discussions The accumulated trend of EHCI, FHCI and NHCI scores depcited by **Table. 2-4(a, b)** and **Fig 6** leads to wind-up the conclusion that, before amalgamation, the EHCI, FHCI and NHCI scores, which were on higher side, dwindled gradually after amalgamation. In other words, before amalgamation, the quality of river water was imperfect" or "unfavourable"

but after amalgamation, resulted into " impeccable" and " favourable" (still not suitable for drinking purposes and could be owed to dilution of the river water). In view of and **Fig 6**, the sampling spot S_1 was found to be most contaminated owing to its maximum EHCI (before amalgamation: 0.3623, after amalgamation: 0.1673) scores, FHCI (before amalgamation: 1.6865, after amalgamation: 0.8343) scores and NHCI (before amalgamation: 0.7093, after amalgamation: 0.3216) scores.

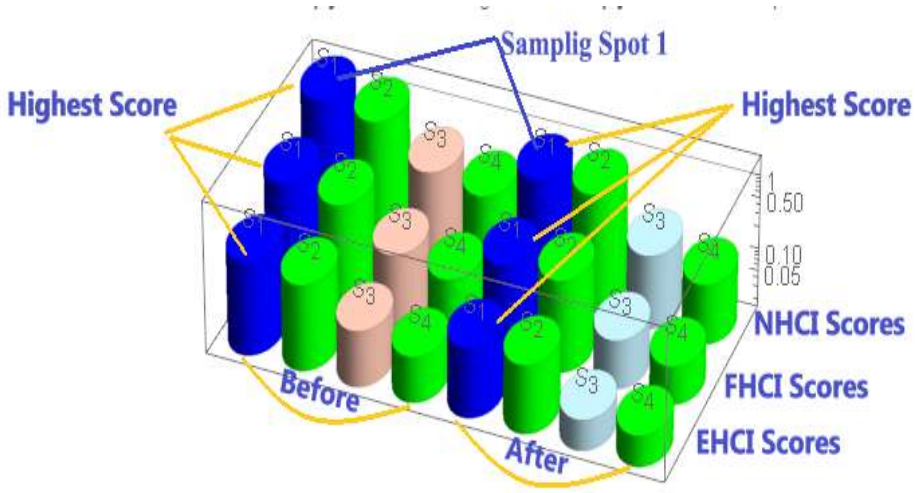


Fig.6 Trend of EHCI, FHCI and NHCI at four sampling spots before andafter amalgamating pharmaceutical effluents

Table 2(a): Calculation of EHCI scores employing Deluca and Termini entropy **[19]** before amalgamation

Heavy Metals	Construction Function	Fuzziness Values	Entropy Values	Assigned Weights	Relative Sub-Indices	EHCI Score
			Sampling Spot 1			
Cd	0.0050	0.0024	0.0123	0.6629	0.0400	
Mn	0.0250	0.0122	0.0475	0.6393	0.0100	
Co	0.5625	0.2741	0.4236	0.3868	0.0113	
Pb	0.9000	0.4385	0.4945	0.3392	0.7200	**0.3623**
Cu	0.0675	0.0329	0.1043	0.6011	0.0027	
Zn	0.0050	0.0024	0.0123	0.6629	0.0000	
Fe	0.4875	0.2375	0.3954	0.4057	0.1950	
			Sampling Spot 2			
Cd	0.0003	0.0015	0.0080	0.6767	0.0200	
Mn	0.0017	0.0074	0.0316	0.6606	0.0050	

Co	0.0713	0.3175	0.4508	0.3746	0.0107	
Pb	0.0933	0.4154	0.4896	0.3482	0.5600	0.2816
Cu	0.0060	0.0267	0.0888	0.6216	0.0018	
Zn	0.0003	0.0015	0.0080	0.6767	0.0000	
Fe	0.0517	0.2300	0.3890	0.4168	0.1550	
Sampling Spot 3						
Cd	0.0005	0.0030	0.0146	0.7329	0.0200	
Mn	0.0010	0.0060	0.0263	0.7242	0.0020	
Co	0.0900	0.5357	0.4982	0.3733	0.0090	
Pb	0.0200	0.1190	0.2633	0.5479	0.0800	0.1067
Cu	0.0040	0.0238	0.0812	0.6834	0.0008	
Zn	0.0005	0.0030	0.0146	0.7329	0.0000	
Fe	0.0520	0.3095	0.4463	0.4118	0.1040	
Sample Spot 4						
Cd	0.0100	0.0040	0.0186	0.8346	0.0200	
Mn	0.0100	0.0040	0.0186	0.8346	0.0010	
Co	1.4700	0.5810	0.4905	0.4333	0.0074	
Pb	0.1000	0.0395	0.1201	0.7483	0.0200	0.0766
Cu	0.0400	0.0158	0.0586	0.8006	0.0004	
Zn	0.0000	0.0000	0.0000*	0.8504*	0.0000	
Fe	0.9000	0.3557	0.4695	0.4511	0.0900	

*At S_4, the entropy value 0.8504 of Zinc is based on the assumption: $0 \times \log 0 = 0$.

Table 2(b): Calculation of EHCI scores employing Deluca and Termini **[19]** entropy after amalgamation

Heavy Metals	Monitored Values	Fuzziness Values	Entropy Values	Assigned Weights	Relative Sub-Indices	EHCI Score
Sampling Spot 1						
Cd	0.0000	0.0000	0.0000*	0.8114*	0.0000	
Mn	0.0000	0.0000	0.0000*	0.8114*	0.0000	
Co	0.7150	0.3950	0.4840	0.4187	0.0072	
Pb	0.9500	0.5249	0.4991	0.4064	0.3800	**0.1673**
Cu	0.0800	0.0442	0.1306	0.7054	0.0016	
Zn	0.0050	0.0028	0.0137	0.8003	0.0000	
Fe	0.0600	0.0331	0.1050	0.7262	0.0120	
Sampling Spot 2						
Cd	0.0000	0.0000	0.0000*	0.9060*	0.0000	
Mn	0.0000	0.0000	0.0000*	0.9060*	0.0000	
Co	0.5350	0.3877	0.4816	0.4697	0.0054	

Pb	0.7900	0.5725	0.4924	0.4599	0.3160	0.1559
Cu	0.0000	0.0000	0.0000	0.9060*	0.0000	
Zn	0.0000	0.0036	0.0173	0.8904	0.0000	
Fe	0.0500	0.0362	0.1124	0.8042	0.0100	
Sampling Spot 3						
Cd	0.0000	0.0000	0.0000*	1.4476*	0.0000	
Mn	0.0000	0.0000	0.0000*	1.4476*	0.0000	
Co	1.0000	0.8264	0.3329	0.9657	0.0050	
Pb	0.2000	0.1653	0.3234	0.9794	0.0400	0.0440
Cu	0.0000	0.0000	0.0000*	1.4476*	0.0000	
Zn	0.0000	0.0083	0.0345	1.3976	0.0000	
Fe	0.0000	0.0000	0.0000*	1.4476*	0.0000	
Sampling Spot 4						
Cd	0.0000	0.0000	0.0000*	3.6206*	0.0000	
Mn	0.0000	0.0000	0.0000*	3.6206*	0.0000	
Co	1.0000	0.9524	0.1381	3.1206	0.0050	
Pb	0.0500	0.0476	0.1381	3.1206	0.0100	0.0468
Cu	0.0000	0.0000	0.0000*	3.6206*	0.0000	
Zn	0.0000	0.0000	0.0000*	3.6206*	0.0000	
Fe	0.0000	0.0000	0.0000*	3.6206*	0.0000	

* Values are based on the assumption: $0 \times \log 0 = 0$ during calculation of $E_j^{(0)}$.

Heavy Metals	Construction Function	Fuzziness Values	Entropy Values	Assigned Weights	Relative Sub-Indices	FHCI Score
Sampling Spot 1						
Cd	0.0050	0.0024	0.0115	1.9734	0.0400	
Mn	0.0250	0.0122	0.0269	1.9427	0.0100	
Co	0.5625	0.2741	0.1313	1.7342	0.0113	
Pb	0.9000	0.4385	0.1496	1.6978	0.7200	**1.6865**
Cu	0.0675	0.0329	0.0459	1.9048	0.0027	
Zn	0.0050	0.0024	0.0115	1.9734	0.0000	
Fe	0.4875	0.2375	0.1242	1.7485	0.1950	
Sampling Spot 2						
Cd	0.0003	0.0015	0.0089	2.0284	0.0200	
Mn	0.0017	0.0074	0.0207	2.0043	0.0050	

Co	0.0713	0.3175	0.1383	1.7636	0.0107	
Pb	0.0933	0.4154	0.1483	1.7432	0.5600	1.3276
Cu	0.0060	0.0267	0.0410	1.9627	0.0018	
Zn	0.0003	0.0015	0.0089	2.0284	0.0000	
Fe	0.0517	0.2300	0.1225	1.7959	0.1550	
Sampling Spot 3						
Cd	0.0005	0.0030	0.0128	2.1447	0.0200	
Mn	0.0010	0.0060	0.0184	2.1325	0.0020	
Co	0.0900	0.5357	0.1505	1.8455	0.0090	
Pb	0.0200	0.1190	0.0902	1.9766	0.0800	0.4186
Cu	0.0040	0.0238	0.0386	2.0887	0.0008	
Zn	0.0005	0.0030	0.0128	2.1447	0.0000	
Fe	0.0520	0.3095	0.1371	1.8746	0.1040	
Sampling Spot 4						
Cd	0.0100	0.0040	0.0148	2.4452	0.0200	
Mn	0.0100	0.0040	0.0148	2.4452	0.0010	
Co	1.4700	0.5810	0.1485	2.1134	0.0074	
Pb	0.1000	0.0395	0.0506	2.3564	0.0200	0.3064
Cu	0.0400	0.0158	0.0309	2.4052	0.0004	
Zn	0.0000	0.0000	0.0000	2.4820	0.0000	
Fe	0.9000	0.3557	0.1431	2.1268	0.0900	

Table 3(a): Calculation of FHCI scores employing the proposed trigonometric fuzzy entropy (TFE) measure before amalgamation

Table 3(b): Calculation of FHCI score employing the proposed trigonometric fuzzy entropy (TFE) measure after amalgamation

Heavy Metals	Construction Function	Fuzziness Values	Entropy Values	Assigned Weights	Relative Sub-Indices	FHCI Score
Sampling Spot 1						
Cd	0.0000	0.0000	0.0000	2.4408	0.0400	
Mn	0.0000	0.0000	0.0000	2.4408	0.0100	
Co	0.7150	0.3950	0.1468	2.0824	0.0113	
Pb	0.9500	0.5249	0.1507	2.0729	0.7200	**0.8343**
Cu	0.0800	0.0442	0.0537	2.3096	0.0027	
Zn	0.0050	0.0028	0.0123	2.4108	0.0000	
Fe	0.0600	0.0331	0.0461	2.3284	0.1950	
Sampling Spot 2						
Cd	0.0000	0.0000	0.0000	2.7956	0.0200	

Mn	0.0000	0.0000	0.0000	2.7956	0.0050	
Co	0.5350	0.3877	0.1462	2.3869	0.0107	
Pb	0.7900	0.5725	0.1490	2.3791	0.5600	0.8237
Cu	0.0000	0.0000	0.0000	2.7956	0.0018	
Zn	0.0000	0.0036	0.0142	2.7560	0.0000	
Fe	0.0500	0.0362	0.0483	2.6606	0.1550	
Sampling Spot 3						
Cd	0.0000	0.0000	0.0000	4.2409	0.0200	
Mn	0.0000	0.0000	0.0000	4.2409	0.0020	
Co	1.0000	0.8264	0.1082	3.7821	0.0090	
Pb	0.2000	0.1653	0.1057	3.7925	0.0800	0.1895
Cu	0.0000	0.0000	0.0000	4.2409	0.0008	
Zn	0.0000	0.0083	0.0219	4.1480	0.0000	
Fe	0.0000	0.0000	0.0000	4.2409	0.1040	
Sampling Spot 4						
Cd	0.0000	0.0000	0.0000	8.9366	0.0200	
Mn	0.0000	0.0000	0.0000	8.9366	0.0010	
Co	1.0000	0.9524	0.0559	8.4368	0.0074	
Pb	0.0500	0.0476	0.0559	8.4368	0.0200	0.1266
Cu	0.0000	0.0000	0.0000	8.9366	0.0004	
Zn	0.0000	0.0000	0.0000	8.9366	0.0000	
Fe	0.0000	0.0000	0.0000	8.9366	0.0900	

Table 4(a) calculation of NHCI score employing the proposed single valued neutrosophic entropy (TNE) measure before amalgamation

Heavy Metals	Construction Function	Fuzziness Values	Entropy Values	Assigned Weights	Relative Sub-Indices	NHCI Score
Sampling Spot 1						
Cd	0.0050	0.0024	0.0230	0.9753	0.0400	
Mn	0.0250	0.0122	0.0538	0.9446	0.0100	
Co	0.5625	0.2741	0.2627	0.7361	0.0113	
Pb	0.9000	0.4385	0.2991	0.6997	0.7200	**0.7093**
Cu	0.0675	0.0329	0.0917	0.9067	0.0027	
Zn	0.0050	0.0024	0.0230	0.9753	0.0000	
Fe	0.4875	0.2375	0.2483	0.7504	0.1950	
Sampling Spot 2						
Cd	0.0003	0.0015	0.0178	1.0051	0.0200	

Mn	0.0017	0.0074	0.0414	0.9810	0.0050	
Co	0.0713	0.3175	0.2766	0.7403	0.0107	
Pb	0.0933	0.4154	0.2966	0.7198	0.5600	0.5575
Cu	0.0060	0.0267	0.0820	0.9394	0.0018	
Zn	0.0003	0.0015	0.0178	1.0051	0.0000	
Fe	0.0517	0.2300	0.2450	0.7726	0.1550	
Sampling Spot 3						
Cd	0.0005	0.0030	0.0256	1.0583	0.0200	
Mn	0.0010	0.0060	0.0368	1.0462	0.0020	
Co	0.0900	0.5357	**0.3010**	0.7592	0.0090	0.1841
Pb	0.0200	0.1190	0.1803	0.8903	0.0800	
Cu	0.0040	0.0238	0.0771	1.0024	0.0008	
Zn	0.0005	0.0030	0.0256	1.0583	0.0000	
Fe	0.0520	0.3095	0.2743	0.7882	0.1040	
Sampling Spot 4						
Cd	0.0100	0.0040	0.0297	1.2043	0.0200	
Mn	0.0100	0.0040	0.0297	1.2043	0.0010	
Co	1.4700	0.5810	0.2970	0.8725	0.0074	
Pb	0.1000	0.0395	0.1012	1.1155	0.0200	0.1342
Cu	0.0400	0.0158	0.0619	1.1643	0.0004	
Zn	0.0000	0.0000	0.0000	1.2412	0.0000	
Fe	0.9000	0.3557	0.2862	0.8859	0.0900	

Table4(b): Calculation of NHCI score employing the proposed single valued neutrosophic

Heavy Metals	Construction Function	Fuzziness Values	Entropy Values	Assigned Weights	Relative Sub-Indices	NHCI Score
Sampling Spot 1						
Cd	0.0000	0.0000	0.0073	1.1407	0.0400	
Mn	0.0000	0.0000	0.0073	1.1407	0.0100	
Co	0.7150	0.3950	0.3009	0.8033	0.0113	
Pb	0.9500	0.5249	0.3088	0.7943	0.7200	**0.3216**
Cu	0.0800	0.0442	0.1148	1.0172	0.0027	
Zn	0.0050	0.0028	0.0319	1.1124	0.0000	
Fe	0.0600	0.0331	0.0994	1.0348	0.1950	

Sampling Spot 2						
Cd	0.0000	0.0000	0.0073	1.2955	0.0200	
Mn	0.0000	0.0000	0.0073	1.2955	0.0050	
Co	0.5350	0.3877	0.2997	0.9138	0.0107	
Pb	0.7900	0.5725	0.3053	0.9066	0.5600	0.3031
Cu	0.0000	0.0000	0.0073	1.2955	0.0018	
Zn	0.0000	0.0036	0.0356	1.2585	0.0000	
Fe	0.0500	0.0362	0.1039	1.1694	0.1550	
Sampling Spot 3						
Cd	0.0000	0.0000	0.0073	1.9000	0.0200	
Mn	0.0000	0.0000	0.0073	1.9000	0.0020	
Co	1.0000	0.8264	0.2236	1.4859	0.0090	
Pb	0.2000	0.1653	0.2187	1.4952	0.0800	0.0673
Cu	0.0000	0.0000	0.0073	1.9000	0.0008	
Zn	0.0000	0.0083	0.0511	1.8162	0.0000	
Fe	0.0000	0.0000	0.0073	1.9000	0.1040	
Sampling Spot 4						
Cd	0.0000	0.0000	0.0073	3.6152	0.0200	
Mn	0.0000	0.0000	0.0073	3.6152	0.0010	
Co	1.0000	0.9524	0.1191	3.2078	0.0074	
Pb	0.0500	0.0476	0.1191	3.2078	0.0200	0.0481
Cu	0.0000	0.0000	0.0073	3.6152	0.0004	
Zn	0.0000	0.0000	0.0073	3.6152	0.0000	
Fe	0.0000	0.0000	0.0073	3.6152	0.0900	

entropy (TNE) measure after amalgamation

It is informative to note that while calculating EHCI score, as depcited in **Table.2 (a, b)**, the values $E_1^{(0)}, E_2^{(0)}$ at sampling spots S_1, S_2, S_3, S_4; $E_5^{(0)}$ at S_2, S_3, S_4; $E_5^{(0)}$ at S_3, S_4 and $E_7^{(0)}$ at S_4 are based on the fancy assumption $0 \times \log 0 = 0.$ This indicates major conflicts and lack of macroscopic view in the quality analysis of heavy metal contamination in river water samples. However, our trigonometric fuzzy entropy (TFE) and single valued neutrosophic entropy (TNE) measures have been proven capable for providing a superior contamination evaluation methodology.

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CONCLUSIONS

To assess the impact of concentration of heavy metals (cadmium, manganese, cobalt, lead, copper, zinc and iron) and to identify the most contaminated sampling spot responsible for heavy metal contamination in river water samples, a novel trigonometric fuzzy entropy as well as single valued neutrosophic entropy measures are established and thereafter deployed to construct fuzzy and neutrosophic entropy weighted heavy metal contamination indices (FHCI and NHCI). The novelty of our contaminated sampling spot identification methodology lies in the fact that our heavy metal contamination indices are superior and capable in classifying the most contaminated sampling spot, whereas the existing Deluca-Termini fuzzy entropy weighted heavy metal contamination index (EHCI) exhibits assumption-based results which can affect the identification accuracy of the selected contaminated sampling spot with respect to each heavy metal concentration in river water samples. It is concluded that

- The concentration of each heavy metal, which was within the permissible limits before amalgamation, dwindled gradually (owing to negative change in TFE and TNE values), after amalgamating pharmaceutical effluents into the river water samples.
- The concentration of cadmium and manganese is found to be negligible. The possible reasons could be the use of physico-chemical processes which further diluted the river water after amalgamation.
- The sampling spot S_1 was found to be the most contaminated, owing to its maximum EHCI (before amalgamation: 0.3623, after amalgamation: 0.1673) score, FHCI (before amalgamation: 1.6865, after amalgamation: 0.8343) score and NHCI (before amalgamation: 0.7093, after amalgamation: 0.3216) scores.

Moreover, the findings of the underlying study can be utilized for controlling the spread of water borne deceases, reducing the risk of water and soil pollution, increasing the ecological and aesthetical qualities of lakes and rivers, etc.

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