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On Neurosophic Delta Generated Per-Continuous Functions in Neutrosophic Topological Spaces

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Abstract: In this work, we investigate new type of neutrosophic continuity, it is called neutrosophic almost δgp −continuity functions, which is stronger than the conception of neutrosophic almost gpr-continuous functions. Also, new notions like neutrosophic δgp-compact, neutrosophic δgp-compact relative to neutrosophic space and neutrosophic strongly δgp −closed for graph of neutrosophic functions are shown. Furthermore, some of its interest properties are shown and studied.

Keywords: neutrosophic sets, neutrosophic topological space, neutrosophic δgp −continuity functions, neutrosophic almost gpr-continuous functions.

1. Introduction

As an expansion of Fuzzy sets given in 1965 by Zadeh [1] and Intuitionistic Fuzzy sets given in 1983 by Atanassav [2], the Neutrosophic sets (NSs) have been shown and explained by Smarandache. A (NS) is depicted by a truth value (membershis), an indeterminacy value and a falsity value (non-membershis). Salama and Alblowi [3] introduced the new concept of neutrosophic topological space (NTS) in 2012, which had been investigated recently. In 2018, Parimala M et al. explain the concept of Neutrosophic homeomorphism and Neutrosophic αψ homeomorphism in (NTS) [4]. In 2020, the notions of Ngpr homeomorphism and Nigpr homeomorphism in (NTS) are introduced and studied [5]. There are some sets in topological spaces their expansion in non-classical are studied, like soft sets [6-13], fuzzy sets [14-19], permutation sets [20-26], neutrosophic sets [27-30] nano sets [31,32] and others [33,34]. Here, we will use the conception of neutrosophic to study our expansion in non-classical. The neutrosophic closure and neutrosophic interior of any (NS) A in (NTS) (Ψ,τ) are defined as $Ncl(A) = \cap\{A \subseteq B;\ B^c \in \tau\}$ and $Nint(A) = \cup\{B \subseteq A;\quad B \in \tau\}$, respectively. The neutrosophic class of neutrosophic δgp-open (resp. neutrosophic δgp-closed, neutrosophic open, closed, neutrosophic regular closed, neutrosophic regular open, neutrosophic δ-preopen, neutrosophic δ-semiopen, neutrosophic preopen, neutrosophic semiopen, neutrosophic e^*-open and neutrosophic β-open) sets of (Ψ,τ) containing a point s∈ Ψ is denoted by NδGPO(Ψ,s) (resp. NδGPC(Ψ,s), NO(Ψ,s), NC(Ψ,s), NRC(Ψ,s), NRO(Ψ,s), NδPO(Ψ,s), NδSO(Ψ,s), NPO(Ψ,s), NSO(Ψ,s), Ne^*O(Ψ,s) and Nβ O(Ψ,s)). That means if A is neutrosophic q-open (q-closed) set in neutrosophic topological space (Ψ,τ), where q is any property for the neutrosophic set A and s∈A for some s∈ Ψ, then it is denoted by NqO(Ψ,s) (NqC(Ψ,s)). In this paper, We're looking into a new kind of neutrosophic continuity, it is known as neutrosophic almost $\delta gp-$continuity functions, which is stronger than the conception of neutrosophic almost gpr-continuous functions. Also, some characteristics of neutrosophic almost δgp $-$continuity functions are explained and discussed.

2. Preliminaries

Basic definitions and notations can be found here, which are used in this section are referred from the references [3,35-37].

Definition 2.1:

Assume $\Psi \neq \varphi$. A neutrosophic set (NS) θ is defined as $\theta = \{\langle\alpha, \partial_\theta(\alpha),\ \omega_\theta(\alpha),\ \ell_\theta(\alpha)\rangle : \alpha \in \Psi\}$ where $\partial_\theta(\alpha)$ is the degree of membership, $\omega_\theta(\alpha)$ is the degree of indeterminacy and $\ell_\theta(\alpha)$ is the degree of non-membership, $\forall\alpha \in \Psi$ to θ. Let $D = \{\langle\alpha, \partial_D(\alpha), \omega_D(\alpha), \ell_D(\alpha)\rangle : \alpha \in \Psi\}$ be the second (NS), then $\theta \cap D = \{\langle\alpha,\ \min\{\partial_\theta(\alpha), \partial_D(\alpha)\},\ \max\{\omega_\theta(\alpha), \omega_D(\alpha)\}, \max\{\ell_\theta(\alpha), \ell_D(\alpha)\}\rangle : \alpha \in \Psi\}$ and $\theta \cup D = \{\langle\alpha, \max\{\partial_\theta(\alpha), \partial_D(\alpha)\}, \min\{\omega_\theta(\alpha), \omega_D(\alpha)\}, \min\{\ell_\theta(\alpha), \ell_D(\alpha)$

$\}\rangle : \alpha \in \Psi\}$. Also, $\theta \subseteq D$ if and only if $\partial_\theta(\alpha) \leq \partial_D(\alpha)$, $\omega_\theta(\alpha) \geq \omega_D(\alpha)$ and $\ell_\theta(\alpha) \geq \ell_D(\alpha)$. The complement of θ is $\theta^c = \{\langle \alpha, \ell_\theta(\alpha), 1-\omega_\theta(\alpha), \partial_\theta(\alpha)\rangle : \alpha \in \Psi\}$

Definition 2.2: We say (Ψ, τ) is a neutrosophic topological space (NTS) if and only if τ is a collection of (NSs) in Ψ and it such that:

(1) $1_N, 0_N \in \tau$, where $0_N = \{\langle \alpha, (0,1,1)\rangle : \alpha \in \Psi\}$ and $1_N = \{\langle \alpha, (1,0,0)\rangle : \alpha \in \Psi\}$.

(2) $\theta \cap \beta \in \tau$ for any $\theta, \beta \in \tau$,

(3) $\bigcup_{i \in I} \theta_i \in \tau$ for any arbitrary family $\{\theta_i | i \in I\} \subseteq \tau$. Also, any $\theta \in \tau$ is called neutrosophic open set (NOS) and we say neutrosophic closed set (NCS) for its complement.

Definition 2.3. Let $\Gamma \subseteq X$be (NS) in (NTS) X. We say Γ is neutrosophic pre-closed (NP-C) (resp. neutrosophic regular-closed (NR-C), neutrosophic semi-closed (NS-C), neutrosophic β -closed (Nβ -C)) if $Ncl(int(\Gamma)) \subseteq \Gamma$ (resp. $\Gamma = Ncl(N\,int(\Gamma))$, $Ncl(Nint(\Gamma)) \subseteq \Gamma$ and $Nint(Ncl(N\,int(\Gamma)) \subseteq \Gamma$).

Definition 2.4. Let $\Gamma \subseteq X$be (NS) in (NTS) X. We say Γ is neutrosophic δ-closed ($N\delta$-C), if $\Gamma = Ncl_\delta(\Gamma)$ where $Ncl_\delta(\Gamma) = \{p \in X : Nint(Ncl(D) \cap \Gamma \neq \varphi,\ D \in \tau$ and $p \in D\}$.

Definition 2.5. Let $\Gamma \subseteq X$be (NS) in (NTS) X. We say Γ is neutrosophic δ-preclosed (NδP-C) (resp. neutrosophic e^*-closed (N$e^* - C$), neutrosophic δ -semiclosed (NδS-C) and neutrosophic α -closed (N α -C)) if $Ncl(Nint_\delta(\Gamma)) \subseteq \Gamma$ (resp. $Nint(Ncl(Nint_\delta(\Gamma)) \subseteq \Gamma$, $Nint(cl_\delta(\Gamma)) \subseteq \Gamma$ and $Ncl(Nint(Ncl_\delta(\Gamma))) \subseteq \Gamma$).

Definition 2.6. Let $\Gamma \subseteq X$be (NS) in (NTS) X. We say Γ is;

(i) neutrosophic δgp -closed ($N\delta gp$-C) (resp. neutrosophic gpr -closed ($Ngpr$-C) and neutrosophic gp -closed (Ngp -C)) if N$pcl(\Gamma) \subseteq L$ whenever $\Gamma \subseteq L$ and L is neutrosophic δ

-open (Nδ -O) (resp. neutrosophic regular open (NR-O) and neutrosophic open (NO)) in X, where $\mathrm{N}pcl(\Gamma) = \cap \{\Gamma \subseteq B;\ B\ \ is\ (\mathrm{NP}-\mathrm{C})\}$

(ii) neutrosophic $g\delta s$ -closed (N $g\delta s$ -C) if $Nscl(\Gamma) \subseteq L$ whenever $\Gamma \subseteq L$ and L is (Nδ -O) in X, where $\mathrm{N}scl(\Gamma) = \cap \{\Gamma \subseteq B;\ B\ \ is\ (\mathrm{NS}-\mathrm{C})\}$.

The neutrosophic open sets are the complements of the previously described neutrosophic closed sets.

Definition 2.7. Assume $W$and V are (NTSs) and $h: W \to V$ is a neutrosophic map (NM). We say h is;

(i) Neutrosophic R -map (NR-M) (resp. neutrosophic δ -continuous (Nδ -CO), neutrosophic almost continuous (NA-CO), neutrosophic almost pre -continuous (NAP-CO), neutrosophic almost gp -continuous (NAgp -CO), neutrosophic almost G -continuous (NAG -CO) and neutrosophic almost $g\delta s$ -continuous (NA$g\delta s$ -CO) if $h^{-1}(L)$ of any (NR-O) set L of V is (NR-O) set (resp. (Nδ -O), (NO), (NP-O), (Ngp -O), (NG -O) and (N $g\delta s$ -O)) set in W,

(ii) Neutrosophic δgp -continuous (N δgp -CO) if $h^{-1}(L)$ of any (NO) set L of V is neutrosophic δgp -open (Nδgp -O) in W,

(iii) Neutrosophic almost contra continuous (NAC-CO) (resp. neutrosophic almost contra $sup\,e\,r$ -continuous (NACsup-CO) and neutrosophic contr R -map (NCR-M)) if $h^{-1}(L)$ of any (NR-C) set L of V is (NO) (resp. $(N\delta$ -O) and (NR-O)) in W,

(iv) Neutrosophic almost perfectly-continuous (NAperf-CO) if the inverse image of any (NR-C) set L of V is neutrosophic clopen in W,

(v) Neutrosophic almost contra δgp -continuous (NACδgp -CO) (resp. neutrosophic contra δgp -continuous (NCδGP-CO) and neutrosophic δgp -irresolute (Nδgp -IR), if $h^{-1}(L)$ of any (NR-O) (resp. (NO) and (Nδgp -C)) set L of V is (Nδgp -C) in W.

Definition 2.8. Let Ω be a (NTS), $NGPRO(\Omega) = \{A \subseteq \Omega \mid A\ is\ (NGPR - O) in\ \Omega\}$,

$N\delta GPO(\Omega) = \{A \subseteq \Omega \mid A\ is\ (N\,\delta gp - O) in\ \Omega\}$ and $NPO(\Omega) = \{A \subseteq \Omega \mid A\ is\ (NP - O)$ $in\ \Omega\}$. We say Ω is;

(i) Neutrosophic preregular $T_{\frac{1}{2}}$-space (Npr-reg-$T_{\frac{1}{2}}$-S) if $NGPRO(\Omega) = NPO(\Omega)$,

(ii) Neutrosophic $T_{\delta gp}$ -space (N$T_{\delta gp}$-S) if $N\delta GPO(\Omega) = NPO(\Omega)$

(iii) Neutrosophic $\delta gpT_{\frac{1}{2}}$-space (N$\delta gpT_{\frac{1}{2}}$-S) if N$\delta GPO(\Omega) = NPO(\Omega)$, ,

(iv) Neutrosophic extremely disconnected (NED) if the closure of any (NO) subset of Ω is (NO),

(v) Neutrosophic submaximal space (N-submax-S) if any (NP-O) set is (NO),

(vi) Neutrosophic strongly irresolvable (N-si) if any neutrosophic open subspace of Ω is irresolvable,

(vii) Neutrosophic nearly compact space (N-NCom-S) if any (NR-O) cover of Ω has a finite subcover,

(viii) Neutrosophic $r - T_1$-space (N-$r - T_1$-S) if for each $\sigma_1 \neq \sigma_2$ two points in Ω, there exist (NR-O) sets λ_1 and λ_2 such that $\sigma_1 \in \lambda_1$, $\sigma_2 \notin \lambda_1$ and $\sigma_1 \notin \lambda_2$, $\sigma_2 \in \lambda_2$,

(ix) Neutrosophic $r - T_2$-space (N-$r - T_2$-S) if for each $\sigma_1 \neq \sigma_2$ in Ω, there exist (NR-O) sets λ_1 and λ_2 such that $x \in U$, $y \in V$and $U \cap V = \varphi$,

(x) Neutrosophic $\delta gp - T_1$ -space (N $\delta gp - T_1$ -S) if for each $p \neq q$ in Ω, there exist $\Psi_1, \Psi_2 \in N\delta GPO(\Omega)$ such that $P \in \Psi_1$, $q \notin \Psi_1$and $q \in \Psi_2$, $p \notin \Psi_2$,

(xi) Neutrosophic Hausdorff space (NH-S) (resp., Neutrosophic δgp-Hausdorff, space (Nδgp-H-S)) if for each $\sigma_1 \neq \sigma_2$ in Ω, there exist $\Psi_1,\Psi_2 \in NO(\Omega)$ (resp., $\Psi_1,\Psi_2 \in \delta GPO(\Omega)$) such that $x \in G$, $y \in H$and $G \cap H = \varphi$

(xii) Neutrosophic δgp -additive space (N-δgp -add-S) if $\delta GPC(\Omega)$ is closed under arbitrary intersections.

Definition 2.9. Let Ω be a (NTS) and $\lambda \subseteq \Omega$. We say Ω is Neutrosophic N -closed relative (NN-Cl-R) to λ if any cover of λ by (NR-O) sets of Ω has a finite subcover.

Theorem 2.10. (i)If λ_1 and λ_2 are (Nδgp-O) subsets of a (N-submax-S)λ, then $\lambda_1 \cap \lambda_2$ is (Nδgp-O) in Ω.

(ii) Let Ω be a (N-δgp -add-S). Then $\lambda_1 \subseteq \Omega$is (N$\delta gp$-C) if and only if N$\delta gp - cl(\lambda_1) = \lambda_1$, where N$\delta gp - cl(\lambda_1)$ =$\cap \{\lambda_1 \subseteq B;\ B \ \ is\ (\mathrm{N}\delta gp - \mathrm{C})\}$.

Definition 2.11. Assume Ω is a (NTS). We say Ω is a neutrosophic locally indiscrete space (N-li-S) if $NO(\Omega) = NRO(\Omega)$, where $NO(\Omega) = \{A \subseteq \Omega \mid A\ is\ (NO)in\ \Omega\}$ and $NRO(\Omega) = \{A \subseteq \Omega \mid A\ is\ (NR - O)in\ \Omega\}$.

Lemma 2.12. Let Ω be a (NTS) and $\lambda \subseteq \Omega$. Then these terms are true:

(i) $\lambda \ \ \in NPO(\Omega)$ if and only if $Nscl(\lambda \ \ ) = Nint(Ncl(\lambda \ \ ))$.

(ii) $p \in N\delta gpcl(\lambda)$ if and only if $B \cap \lambda \neq \varphi$ for any (Nδgp -O) set B containing r.

Remark: 2.13: For any (NS) $\lambda \subseteq \Omega$ in (NTS) Ω, we consider that:

(1) $Ncl(N\,int_\delta(\lambda)) = N\,cl_\delta(\,(N\,int_\delta(\,\lambda)),$

(2) $Nint(N\,cl_\delta(\,\lambda)) = N\,int_\delta(\,(N\,cl_\delta(\,\lambda)).$

(3) $N\,int_\delta(\,\Omega \backslash \lambda) = \Omega(N\,cl_\delta(\,\lambda)) \in NRO(\Omega), if\ \lambda\ is\ (\mathrm{Ne}^* - O).$

3. Neutrosophic Almost δgp -Continuous Functions.

Definition 3.1. Let $h: \Omega \to \mu$ be a (NM). We say h is neutrosophic almost δgp -continuous (NAδgp -CO) if $h^{-1}(\lambda) \in N\delta GPC(\Omega)$ for each (NR-C) set $\boldsymbol{\lambda}$of $\boldsymbol{\mu}$.

Example 3.2. Define the neutrosophic sets D_1, D_2, D_3, D_4 and H_1, H_2, H_3, H_4, H_5 as follows:

D_1={$\langle a, (0,1,0.3)\rangle, \langle b, (0.3,0.5,1)\rangle, \langle c, (0,0.6,1)\rangle, \langle d, (0.5,1,0.8)\rangle$}

D_2={⟨a, (0.2,0.4,1)⟩, ⟨b, (0,1,0.3)⟩, ⟨c, (0.7,0.1,0.6)⟩, ⟨d, (0,0.5,1)⟩}

D_3={⟨a, (0.2,0.4,0.3)⟩, ⟨b, (0.3,0.5,0.3)⟩, ⟨c, (0.7,0.6,0.6)⟩, ⟨d, (0.5,0.5,0.8)⟩}

D_4={⟨a, (0.3,0.3,0.2)⟩, ⟨b, (0.4,0.4,0.3)⟩, ⟨c, (0.8,0.5,0.5)⟩, ⟨d, (0.6,0.4,0.7)⟩}

And

H_1={⟨a, (0.2,0.4,1)⟩, ⟨b, (0,1,0.3)⟩, ⟨c, (0.7,0.1,0.6)⟩, ⟨d, (0,0.5,1)⟩}

H_2={⟨a, (0,1,0.3)⟩, ⟨b, (0.3,0.5,1)⟩, ⟨c, (0,0.6,1)⟩, ⟨d, (0.5,1,0.8)⟩}

H_3={⟨a, (0.3,0.3,0.2)⟩, ⟨b, (0.4,0.4,0.3)⟩, ⟨c, (0.8,0.5,0.5)⟩, ⟨d, (0.6,0.4,0.7)⟩}

H_4={⟨a, (0.2,0.4,0.3)⟩, ⟨b, (0.3,0.5,0.3)⟩, ⟨c, (0.7,0.6,0.6)⟩, ⟨d, (0.5,0.5,0.8)⟩}

Now, let t=$\{1_N, 0_N, D_1, D_2, D_3, D_4\}$ and h=$\{1_N, 0_N, H_1, H_2, H_3, H_4\}$ then (X,t) and (Y,h) are (NTSs), where X = {a,b,c,d} = Y . Define $f: X \to Y$ by $f(a) = f(c) = b, f(b) = a, f(d) = c$. We consider that f is neutrosophic almost δgp -continuous.

Theorem 3.3. Let $h: X \to Y$ be (NM). Then h is (NAδgp -CO) if and only if $h^{-1}(\mu)$ of any (NR-O) set μ of *Y* is (Nδgp -O) in *X*.

Proof: since the complement for any (NO) is (NC) and by Definition (3.1). Then the theorem is held.

Example 3.4. Define the neutrosophic sets D_1, D_2, D_3, D_4 and H_1, H_2, H_3, H_4, H_5 as follows:

D_1={⟨a, (0.1,1,0.4)⟩, ⟨b, (0.4,0.6,1)⟩, ⟨c, (0.1,0.7,1)⟩, ⟨d, (0.6,1,0.9)⟩}

D_2={⟨a, (0.3,0.5,1)⟩, ⟨b, (0.1,1,0.4)⟩, ⟨c, (0.8,0.2,0.7)⟩, ⟨d, (0.1,0.6,1)⟩}

D_3={⟨a, (0.3,0.5,0.4)⟩, ⟨b, (0.4,0.6,0.4)⟩, ⟨c, (0.8,0.7,0.7)⟩, ⟨d, (0.6,0.6,0.9)⟩}

D_4={⟨a, (0.4,0.4,0.3)⟩, ⟨b, (0.5,0.5,0.4)⟩, ⟨c, (0.9,0.6,0.6)⟩, ⟨d, (0.7,0.5,0.8)⟩}

And

H_1={⟨a, (0.3,0.5,1)⟩, ⟨b, (0.1,1,0.4)⟩, ⟨c, (0.8,0.2,0.7)⟩, ⟨d, (0.1,0.6,1)⟩}

H_2={⟨a, (0.1,1,0.4)⟩, ⟨b, (0.4,0.6,1)⟩, ⟨c, (0.1,0.7,1)⟩, ⟨d, (0.6,1,0.9)⟩}

H_3={⟨a, (0.4,0.4,0.3)⟩, ⟨b, (0.5,0.5,0.4)⟩, ⟨c, (0.9,0.6,0.6)⟩, ⟨d, (0.7,0.5,0.8)⟩}

H_4={⟨a, (0.3,0.5,0.4)⟩, ⟨b, (0.4,0.6,0.4)⟩, ⟨c, (0.8,0.7,0.7)⟩, ⟨d, (0.6,0.6,0.9)⟩}

Now, let t={$1_N, 0_N, D_1, D_2, D_3, D_4$} and h={$1_N, 0_N, H_1, H_2, H_3, H_4$} then (X,t) and (Y,h) are (NTSs), where X = {a,b,c,d} = Y. Define $h: X \to Y$ by $h(a) = h(c) = b, h(b) = a, h(d) = c$. Then we consider that h is (NAδgp -CO). Also, $h^{-1}(\mu)$ is (Nδgp -O) in X for any (NR-O) set μ of Y .

Remark 3.4. Let $h: \Omega \to \mu$ be a (NM). Then by Definitions (2.7) and (3.1), we consider diagram (1) as follows:

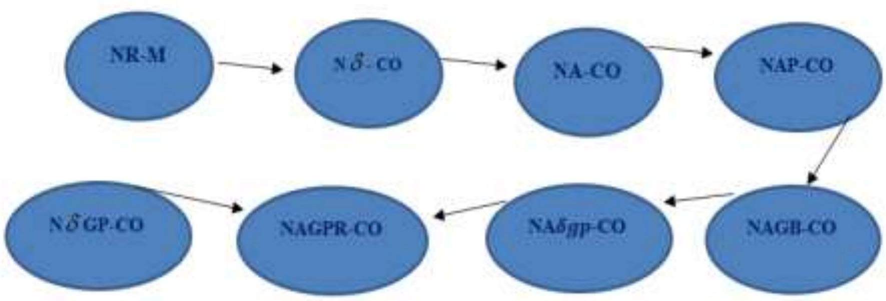

Diagram (1): The relationships among some classes of neutrosophic continuous

Theorem 3.5. If $\boldsymbol{f}: \boldsymbol{\mu} \to \boldsymbol{\eta}$ is (NA$\boldsymbol{\delta gp}$ -CO) and $\boldsymbol{\eta}$ is (N-li-S), then $\boldsymbol{f}$ is (N$\boldsymbol{\delta gp}$ -CO).

Proof. Let $\boldsymbol{\lambda}$ be (NO) set in $\boldsymbol{\eta}$, then $\boldsymbol{\lambda}$ is (NR-O) in $\boldsymbol{\eta}$. Since $\boldsymbol{f}$ is (NA$\boldsymbol{\delta gp}$ -CO), then $\boldsymbol{f}^{-1}(\boldsymbol{\lambda})$ is (N$\boldsymbol{\delta gp}$ -O) in $\boldsymbol{\mu}$. Hence $\boldsymbol{f}$ is (N$\boldsymbol{\delta gp}$ -CO)

Theorem 3.6. Let $\boldsymbol{\Omega}$ be a (N-li-S), then these terms are

equivalent:

(i) $f: \Omega \to \mu$ is (Ngpr -CO),

(ii) $f: \Omega \to \mu$ is (NAδgp -CO),

(iii) $f: \Omega \to \mu$ is (NAgp -CO).

Proof. Follows from the Definitions (2.11), (2.7) and (3.1).

Remark 3.7. It is clear from the definitions in section 2, we consider that all of the theorems [(3.8)-(3.13)] are held.

Theorem 3.8. (i) If $f: \Omega \to \mu$ is (NA$g\delta s$ -CO) with Ω as (NED), then it is (NAδgP -CO).

(ii) If $f: \Omega \to \mu$ is (NAδgP -CO) with Ω as (N-si). Then it is (NA$g\delta s$ -CO).

Theorem 3.9. All of these terms are equivalent:

(i) $f: \Omega \to \eta$ is (NAperf-CO),

(ii) $f: \Omega \to \eta$ is (NAC-CO) and (NAP-CO),

(iii) $f: \Omega \to \eta$ is (NAC-CO) and (NAgp -CO),

(iv) $f: \Omega \to \eta$ is (NACsup-CO) and (NAδgp -CO),

(v) $f: \Omega \to \eta$ is (NCR-M) and (NAgpr -CO),

(vi) $f: \Omega \to \eta$ is (NCR-M) and (NAP-CO),

(vii) $f: \Omega \to \eta$ is (NACsup-CO) and (NAP-CO).

Theorem 3.10. Let Ω be a(N$\delta gpT_{\frac{1}{2}}$-S). Then all of these terms are equivalent:

(i) $f: \Omega \to \eta$ is (NAP-CO),

(ii) $f: \Omega \to \eta$ is (NAgp -CO),

(iii) $f: \Omega \to \eta$ is (NAδgp -CO).

Theorem 3.11. Let Ω be a (Npr-reg-$T_{\frac{1}{2}}$-S). Then All of these terms are equivalent:

(i) $f: \Omega \to \eta$ is (NAP-CO),

(ii) $f: \Omega \to \eta$ is (NAgp -CO),

(iii) $\boldsymbol{f}: \boldsymbol{\Omega} \rightarrow \boldsymbol{\eta}$ is (NA$\boldsymbol{\delta gp}$ -CO),

(iv) $\boldsymbol{f}: \boldsymbol{\Omega} \rightarrow \boldsymbol{\eta}$ is (NA$\boldsymbol{gpr}$ -CO).

Theorem 3.12. Let $\boldsymbol{\Omega}$ be a $\boldsymbol{T}_{\delta gp}$-space. Then these terms are equivalent:

(i) $\boldsymbol{f}: \boldsymbol{\Omega} \rightarrow \boldsymbol{\mu}$ is (NA-CO);

(ii) $\boldsymbol{f}: \boldsymbol{\Omega} \rightarrow \boldsymbol{\mu}$ is (NA$\boldsymbol{pre}$ -CO),

(iii) $\boldsymbol{f}: \boldsymbol{\Omega} \rightarrow \boldsymbol{\mu}$ is (NA$\boldsymbol{gp}$ -CO),

(iv) $\boldsymbol{f}: \boldsymbol{\Omega} \rightarrow \boldsymbol{\mu}$ is (NA$\boldsymbol{\delta gp}$ - CO),

(v) $\boldsymbol{f}: \boldsymbol{\Omega} \rightarrow \boldsymbol{\mu}$ is (NA$\boldsymbol{gpr}$ - CO).

Theorem 3.13. The following are equivalent:

(i) $\boldsymbol{f}: \boldsymbol{\Omega} \rightarrow \boldsymbol{\mu}$ is (NA$\boldsymbol{\delta gp}$ -CO) and $\boldsymbol{\Omega}$ is (N$\boldsymbol{\delta gp}$ -add-S),

(ii) for each $\boldsymbol{\sigma} \in \boldsymbol{\Omega}$ and each open set λ_1 containing $f(p)$, there exists (Nδgp -O) set λ_2 containing σ such that $f(\lambda_2) \subset$ N$int(\,Ncl(\lambda_1))$.

Theorem 3.14. All of these terms are equivalent:

(i) $\boldsymbol{f}: \boldsymbol{\Omega} \rightarrow \boldsymbol{\mu}$ is (NA$\boldsymbol{\delta gp}$ -CO) and $\boldsymbol{\Omega}$ is (N$\boldsymbol{\delta gp}$ -add-S),

(ii) For each $\sigma \in \Omega$ and each $\lambda_1 \in NO\big(\mu, f(\sigma)\big)$, there exists $\lambda_2 \in$ N$\delta GPO(\Omega, \sigma)$ such that $f(\lambda_2) \subset Nscl(\lambda_1)$;

(iii) For each $\sigma \in \Omega$ and each $\lambda_3 \in NO\big(\mu, f(\sigma)\big)$,there exists $\gamma_1 \in$N$\delta GPO(\Omega, \sigma)$ such that $f(\gamma_1) \subset \lambda_3$;

(iv) For each $\sigma \in \Omega$ and each $\gamma_2 \in N\delta O\big(\mu, f(\sigma)\big)$, there exists $\Sigma \in$N$\delta GPO(\Omega, \sigma)$ such that $f(\Sigma) \subset \gamma_2$;

(v) For each $\sigma \in \Omega$ and each $\gamma_2 \in N\delta C\big(\mu, f(\sigma)\big)$, there exists $\Sigma \in N\delta GPC(\Omega, \sigma)$ such that $f(\Sigma) \subset \gamma_2$;

Proof. (i) $\rightarrow$ (ii): Let $\sigma \in \Omega$and N be (NO) set of μ containing $f(\sigma)$. By (i) and Theorem 3.13, there exists $\lambda_2 \in$ N$\delta GPO(\Omega,\sigma)$ such that $f(\lambda_2) \subset Nint(Ncl(\lambda_1))$. Since λ_2 is preopen, then by Lemma 2.12(i), $f(\lambda_2) \subset Nscl(\lambda_1)$.

(ii) $\rightarrow$ (iii): Let $\sigma \in \Omega$ and $\lambda_1 \in NRO\big(\mu, f(\sigma)\big)$. Then $\lambda_1 \in NO\big(\mu, f(\sigma)\big)$. By (ii), there exists $\lambda_2 \in N\delta GPO(\Omega,\sigma)$ such that $f(\lambda_2) \subset Nscl(\lambda_1)$. Since λ_3 is (NP-O), then by Lemma 2.12 (i), $f(\lambda_2) \subset N\,int(Ncl(\lambda_1)) = \lambda_1$.

(iii) $\rightarrow$ (iv): Let $\sigma \in \Omega$ and $\lambda_1 \in N\delta O\big(\mu, f(\sigma)\big)$, then there exists $\lambda_2 \in NO\big(\Omega, f(\sigma)\big)$ such that $M \subset N\,int(Ncl(M)) \subset N$. Since $N\,int(Ncl(M)) \in NRO\big(Y, f(p)\big)$, by (iii), there exists $\Sigma \in N\delta GPO(\Omega,\sigma)$ such that $f(\Sigma) \subset N\,int(N\,cl(\lambda_2)) \subset \lambda_1$.

(iv)$\rightarrow$ (i): Let $\sigma \in \Omega$ and $\lambda_1 \in NO\big(\mu, f(\sigma)\big)$. Then $N\,int(Ncl(\lambda_1)) \in N\delta O\big(\mu, f(\sigma)\big)$. By (iv), there exists $\lambda_2 \in N\delta GPO(\Omega,\sigma)$ such that $f(\lambda_2) \subset N\,int(Ncl(\lambda_1))$. Hence f is (NAδgp -CO).

(iv) $\leftrightarrow$ (v): Obvious.

Remark 3.15. If Ω is a (Nδgp -add-S), then $\lambda \subseteq \Omega$ is (Nδgp -C) (resp. (Nδgp -O)) if and only if N $\delta gp - cl(\lambda) = \lambda$ (resp. N $\delta gp - int(\lambda) = \lambda$)., where N$\delta gp - cl(\lambda) = \cap\,\{\lambda \subseteq B;\ B\ \ is\ (\text{N}\delta gp - \text{C})\}$ and N$\delta gp - int(\lambda) = \cap\,\{B \subseteq \lambda;\ B\ \ is\ (\text{N}\delta gp - \text{O})\}$

Theorem 3.16. All of these terms are equivalent:

(i) $f{:}\,\Omega \rightarrow \mu$ is (NAδgp -CO) and $\Omega$is (Nδgp -add-S),

(ii) $f\big(N\delta gp - cl(\lambda_2)\big) \subset Ncl_\delta\big(f(\lambda_1)\big)$ for each $\lambda_1 \subseteq \Omega$;

(iii) N$\delta gp - cl\big(f^{-1}(\lambda_2)\big) \subset f^{-1}\big(Ncl_\delta(\lambda_2)\big)$ for each $\lambda_2 \subseteq \mu$;

(iv) $f^{-1}(\beta_1) \in N\delta GPC(\Omega)$ for each $\beta_2 \in N\delta C(\mu)$;

(v) $f^{-1}(\gamma_1) \in N\delta GPO(\Omega)$ for each $\gamma_1 \in N\delta O(\mu)$;

Proof. (i) $\rightarrow$ (ii) Suppose that $\lambda_2 \in N\delta C(\mu)$; such that $f(\lambda_1) \subset \lambda_2$. Observe that $\lambda_1 = Ncl_\delta(\lambda_1) = \cap\{\gamma_2 : \lambda_2 \subset \gamma_2$ and $\gamma_2 \in NRC(\mu)\}$ and so $f^{-1}(\lambda_2) = \cap\{f^{-1}(\gamma_2) : \lambda_2 \subset \gamma_2\}$. By (i) and Definition 2.8 (xii), we have $f^{-1}(\lambda_2) \in N\delta GPC(\Omega)$ and $\lambda_1 \subset f^{-1}(\lambda\ \ )$. Hence N $\delta gp - cl(\lambda_1)$ $\subset f^{-1}(\lambda_2)$, and it follows that $f(N\delta gp - cl(\lambda_1)) \subset \lambda_2$. Since this is true for any (Nδ -C) set λ_2 containing $f(\lambda_1)$, we have $f(N\delta gp - cl(\lambda_1)) \subset Ncl_\delta(f(\lambda_1))$..

(ii) $\rightarrow$ (iii) Let $\beta_1 \subset \mu$, then $f^{-1}(\beta_1) \subset \Omega$. By (ii),

$f\left(N\delta gp - cl(f^{-1}(\beta_1))\right) \subset$ N $cl_\delta\left(f(f^{-1}(\beta_1))\right) \subset$ N $\delta gp - cl(\beta_1)$. So that N$\delta gp - cl(f^{-1}(\beta_1)) \subset f^{-1}(Ncl_\delta(\beta_1))$

(iii) $\rightarrow$ (iv) Let $\beta_2 \in N\delta C(\mu)$. Then by (iii), N $\delta gp - cl(f^{-1}(\beta_2))$ $\subset f^{-1}(Ncl_\delta(\beta_2))$ $= f^{-1}(\beta_2)$.In consequence, N$\delta gp - cl(f^{-1}(\beta_2))$ $= f^{-1}(\beta_2)$ and hence by remark (3.15), $f^{-1}(\beta_2) \in$ N$\delta GPC(\Omega)$.

(iv) $\rightarrow$ (v): Clear.

(v) $\rightarrow$ (i): Let $\lambda_2 \in NRO(\mu)$ Then $\lambda_2 \in N\delta O(\mu)$. By (v), $f^{-1}(\lambda_2) \in$ N$\delta GPO(\Omega)$. Hence by Theorem 3.3, f is (NAδgp -CO).

Theorem 3.17. All of these terms are equivalent:

(i) $f : \Omega \rightarrow \eta$ is almost δgp -continuous and $\Omega$is (Nδgp -add-S),

(ii) For any $\lambda \in NO(\eta)$,$f^{-1}(N\,int\,(Ncl(\lambda) \in N\delta GPO(\Omega)$;

(iii) For any $\gamma \in NC(\eta)$,$f^{-1}(Ncl(N\,int\,(\gamma) \in N\delta GPC(\Omega)$;

(iv) For any $\lambda \in N\beta O(\eta)$, N$\delta gpcl(f^{-1}(\lambda)) \subset f^{-1}(Ncl(\lambda))$;

(v) For any $\gamma \in N\beta C(\eta)$,$f^{-1}(N\,int\,(\gamma) \subset N\delta gp\,int(^{-1}(\gamma))$;

(vi) For any $\gamma \in NSC(\eta)$,$f^{-1}(N\,int(\gamma)) \subset N\delta gp\,int(f^{-1}(\gamma))$;

(vii) For any $\lambda \in NSO(\eta)$, N$\delta gpcl(f^{-1}(\lambda)) \subset f^{-1}(Ncl(\lambda))$;

(viii) For any $\gamma \in NPO(\eta))$, $f^{-1}(\gamma) \subset N\delta gp\,int(f^{-1}(N\,int(Ncl(\gamma))$

Proof. (i) $\leftrightarrow$ (ii): Let $\lambda \in NO(\eta)$ .Since N $int(N\,cl(\lambda)) \in NRO(\eta)$ Then by (i), $f^{-1}(N\,int(Ncl(\lambda)) \in N\delta GPO(\Omega)$. The converse is similar.

(i) $\leftrightarrow$ (iii) It is similar to (i) $\leftrightarrow$ (ii).

(i) $\rightarrow$ (iv): Let $\lambda \in N\beta O(\eta)$, then N$cl(\lambda) \in NRC(\eta)$ so by (i), $f^{-1}(Ncl(\lambda)) \in N\delta GPC(\Omega)$. Since $f^{-1}(\lambda) \subset f^{-1}(Ncl(\lambda))$ which implies $N\delta gpcl(f^{-1}(\lambda)) \subset f^{-1}(Ncl(\lambda))$.

(iv) $\rightarrow$ (v) and (vi) $\rightarrow$ (vii):Obvious

(v) $\rightarrow$ (vi): It follows from the fact that N$SC(\eta) \subset N\beta C(\eta)$.

(vii) $\rightarrow$ (i): It follows from the fact that N$RC(\eta) \subset NSO(\eta)$.

(i) $\leftrightarrow$ (viii): Let $\lambda \in NPO(\eta)$. Since N$int(Ncl(\lambda)) \in NRO(\eta)$, then by (i), $f^{-1}(Nint(Ncl(N)) \in N\delta GPO(X)$ and hence $f^{-1}(\lambda) \subset f^{-1}(N\,int(Ncl(\lambda)))$ $= N\delta gp\,int(f^{-1}(N\,int(Ncl(\lambda))))$. Conversely, let $\lambda \in NRO(\eta)$. Since $\lambda \in NPO(\eta)$, $f^{-1}(\eta) \subset N\delta gp\,int(f^{-1}(N\,int(cl(\eta)))) = N\delta gp\,int(f^{-1}(\lambda))$, in consequence, N$\delta gp\,int(f^{-1}(\lambda)) = f^{-1}(\lambda)$ and by remark (3.15), $f^{-1}(\lambda) \in N\delta GPO(\Omega)$.

Theorem 3.18. The following are equivalent:

(i) $f: \mu \rightarrow \eta$ is (NAδgp -CO) and μ is (Nδgp -add-S),

(ii) For any (Ne^*-O) set α of η, $f^{-1}(Ncl_\delta(\alpha))$ is (Nδgp -C) in μ,

(iii) For any (NδS-O) subset α of η, $f^{-1}(Ncl_\delta(\alpha))$ is (Nδgp -C) in μ;

(iv) For any (NδP-O) subset α of η, $f^{-1}(N\,int(N\,cl_\delta(\alpha)))$ is (Nδgp -O)in μ;

(v) For any (NO) subset α of α, $f^{-1}(N\,int(N\,cl_\delta(\alpha)))$ is (Nδgp -O) in μ;

(vi) For any (NC) subset α of Y, $f^{-1}(Ncl(N\,int_{\delta}(\alpha)))$ is (Nδgp -C) in μ.

Proof. (i) $\to$ (ii): Let $\alpha \in Ne^{*}O(\eta)$. Then by remark (2.13), N$cl_{\delta}(\alpha) \in NRC(\eta)$. By (i), $f^{-1}\big(Ncl_{\delta}(\eta)\big) \in N\delta GPC(\mu)$.

(ii) $\to$ (iii): Obvious since N$\delta SO(\eta) \subset Ne^{*}O(\eta)$.

(iii) $\to$ (iv): Let $\alpha \in N\delta PO(\eta)$,then N$int_{\delta}(\eta\backslash\alpha) \in N\delta SO(\eta)$). By (iii), $f^{-1}\big(Ncl_{\delta}(N\,int_{\delta}(\eta\backslash\alpha)\big) \in N\delta GPC(\mu)$ which implies $f^{-1}\big(N\,int(Ncl_{\delta}(\alpha)\big) \in N\delta GPO(\mu)$.

(iv) $\to$ (v): Obvious since N$O(\eta) \subset N\delta PO(\eta)$.

(v) $\to$ (vi):Clear

(vi) $\to$ (i): Let N$\alpha \in NRO(\eta)$. Then $\alpha = Nint(Ncl_{\delta}(\alpha))$ and hence $(\eta\backslash\alpha) \in NC(\eta)$. By (vi), $f^{-1}(\eta\backslash\alpha) = \mu\backslash f^{-1}(N\,int(N\,cl_{\delta}(\alpha)) = f^{-1}(Ncl(N\,int_{\delta}(\eta\backslash\alpha))) \in N\delta GPC(\mu)$. Thus $f^{-1}(\alpha) \in N\delta GPO(\mu)$.

Theorem 3.19. If $\boldsymbol{f}:\boldsymbol{\Omega} \to \boldsymbol{\mu}$ is (NA$\boldsymbol{\delta gp}$ -CO) injective function and $\boldsymbol{\mu}$ is (N-$r - T_1$-S), then $\boldsymbol{X}$ is (N$\delta gp - T_1$-S).

Proof. Let (μ,σ) be (N-$r - T_1$-S) and $p, q \in \Omega$, with $p \neq q$. Then there exist (NR-O) subsets λ, γ in Y such that $f(p) \in \lambda, f(q) \notin \lambda$, $f(p) \notin \gamma$ and $f(q) \in \gamma$. Since f is (NAδgp -CO), $f^{-1}(\lambda)$ and $f^{-1}(\gamma) \in$ N$\delta GPO(\Omega)$ satisfy $p \in f^{-1}(\lambda)$, $q \notin f^{-1}(\lambda)$, $p \notin f^{-1}(\gamma)$ and $q \in f^{-1}(\gamma)$. Hence Ω is (N$\delta gp - T_1$-S).

Theorem 3.20. If $\boldsymbol{f}:\boldsymbol{\Omega} \to \boldsymbol{\eta}$ is (NA$\boldsymbol{\delta gp}$ -CO) injective function and $\boldsymbol{\eta}$ is(N-$\boldsymbol{r} - \boldsymbol{T_2} - S$), then Ω is $(N\delta gp - T_2 - S)$.

Proof. The proof is the same way of Theorem (3.20).

Theorem 3.21. If $\boldsymbol{f}, \boldsymbol{g}:\boldsymbol{\Omega} \to \boldsymbol{\eta}$ are (NA$\boldsymbol{\delta gp}$ -CO) with $\boldsymbol{\Omega}$ as (N-submax-S) and (N$\boldsymbol{\delta gp}$ -add-S) and $\boldsymbol{\eta}$ is (NH-S), then the set $\{\boldsymbol{x} \in \Omega : \boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{g}(\boldsymbol{x})\}$ is $\boldsymbol{\delta gp}$ -closed in $\boldsymbol{\Omega}$.

Proof. Let $E = \{x \in \Omega: f(x) = g(x)\}$ and $x \notin (\Omega \backslash \lambda)$. Then $f(x) \neq g(x)$. Since η (NH-S), there exist (NO) sets λ_1 and λ_2 of η satisfy $f(x) \in \lambda_1$, $g(x) \in \lambda_2$ and $\lambda_1 \cap \lambda_2 = \varphi$, hence N $int(N\,cl(\lambda_1)) \cap Nint(\,Ncl(\lambda_2)) = \varphi$. Since f and g are (NA δgp -CO), there exist $\gamma_1, \gamma_2 \in N\delta GPO(\Omega, x))$ satisfy $f(\gamma_1) \subseteq N\,int(N\,cl(\lambda_1))$ and $g(\gamma_2) \subseteq Nint(\,Ncl(\lambda_2))$. Now, put $\Sigma = \gamma_1 \cap \gamma_2$, then $\Sigma \in N\delta GPO(\Omega, x)$ and $f(\Sigma) \cap g(\Sigma) \subseteq N\,int(\,Ncl(\lambda_1)) \cap Nint(N\,cl(\lambda_2)) = \varphi$. Thus, we get $\Sigma \cap \lambda = \varphi$ and hence $x \notin N\delta gp - cl(E)$ then $\lambda = N\delta gp - cl(\lambda)$. Since Ω is (Nδgp -add-S), $\lambda$is (Nδgp -C) in Ω.

Definition 3.22. A space $\boldsymbol{\mu}$ is called neutrosophic $\boldsymbol{\delta gp}$ -compact (N$\boldsymbol{\delta gp}$ -Com) if any cover of $\boldsymbol{\mu}$ by $\boldsymbol{\delta gp}$ -open sets has a finite subcover.

Definition 3.23. Let $\boldsymbol{\lambda}$ be (NS) in (NTS) $\boldsymbol{\Omega}$**.** We say $\boldsymbol{\lambda}$ is neutrosophic $\boldsymbol{\delta gp}$ -compact relative (N$\boldsymbol{\delta gp}$ -Com-R) to $\boldsymbol{\Omega}$ if any cover of $\boldsymbol{\lambda}$ by (N$\boldsymbol{\delta gp}$ -O) sets of $\boldsymbol{\Omega}$ has a finite subcover.

Theorem 3.24. If $\boldsymbol{f}: \boldsymbol{\mu} \to \boldsymbol{\eta}$ is (NA$\boldsymbol{\delta gp}$ -CO) and $\boldsymbol{\lambda}$ is (N$\boldsymbol{\delta gp}$ -Com-R) to $\boldsymbol{\mu}$, then $\boldsymbol{f(\lambda)}$ is (NN-Cl-R) to $\boldsymbol{\eta}$.

Proof. Let $\{\boldsymbol{A_\alpha}: \boldsymbol{\alpha} \in \boldsymbol{\Omega}\}$ be any cover of $\boldsymbol{f(\lambda)}$ by (NR-O) sets of $\boldsymbol{\eta}$. Then $\{\boldsymbol{f^{-1}(A_\alpha)}: \boldsymbol{\alpha} \in \boldsymbol{\Omega}\}$ is a cover of $\boldsymbol{\lambda}$ by (N$\boldsymbol{\delta gp}$ -O) sets of $\boldsymbol{\mu}$. Hence there exists a finite subset $\boldsymbol{\Omega_0}$ of $\boldsymbol{\Omega}$ such that $\boldsymbol{\lambda} \subset \cup \{\boldsymbol{f^{-1}(A\alpha)}: \boldsymbol{\alpha} \in \boldsymbol{\Omega_0}\}$. Therefore, we obtain $\boldsymbol{f(\lambda)} \subset \{\boldsymbol{A_\alpha}: \boldsymbol{\alpha} \in \boldsymbol{\Omega_0}\}$. This shows that $\boldsymbol{f(\lambda)}$ is (NN-Cl-R) to $\boldsymbol{\eta}$.

Corollary 3.25. If $\boldsymbol{f}: \boldsymbol{\Omega} \to \boldsymbol{\mu}$ is (NA$\boldsymbol{\delta gp}$ -CO) surjection and $\boldsymbol{\Omega}$ is (N$\boldsymbol{\delta gp}$ -Com) and (N$\boldsymbol{\delta gp}$ -add-S), then $\boldsymbol{\mu}$ is (N-NCom-S).

Lemma 3.26. Let $\boldsymbol{\mu}$ be (N$\boldsymbol{\delta gp}$ -Com). If $\boldsymbol{\lambda} \subset \boldsymbol{\mu}$ is (N$\boldsymbol{\delta gp}$ -C), then $\boldsymbol{\lambda}$ is (N$\boldsymbol{\delta gp}$ -Com-R) to $\boldsymbol{\mu}$.

Proof. Let $\{\boldsymbol{\beta_\alpha}: \boldsymbol{\alpha} \in \boldsymbol{\Omega}\}$ be a cover of $\boldsymbol{N}$ by (N$\boldsymbol{\delta gp}$ -O) sets of $\boldsymbol{\mu}$. Note that $(\boldsymbol{\mu} - \boldsymbol{N})$ is (N$\boldsymbol{\delta gp}$ -O) and that the (NS) $(\boldsymbol{\mu} - \boldsymbol{N}) \cup \{\boldsymbol{\beta_\alpha}: \boldsymbol{\alpha} \in \boldsymbol{\Omega}\}$ is a cover of $\boldsymbol{\mu}$ by (N$\boldsymbol{\delta gp}$ -O) sets. Since $\boldsymbol{\mu}$ is (N$\boldsymbol{\delta gp}$ -Com), there exists a finite $\boldsymbol{\Omega_0}$ subset of $\boldsymbol{\Omega}$ such that the (NS) $(\boldsymbol{\mu} - \boldsymbol{N}) \cup \{\boldsymbol{\beta_\alpha}: \boldsymbol{\alpha} \in \boldsymbol{\Omega_0}\}$ is a cover of $\boldsymbol{\mu}$ by (N$\boldsymbol{\delta gp}$ -O) sets in $\boldsymbol{\mu}$. Hence $\{\boldsymbol{\beta_\alpha}: \boldsymbol{\alpha} \in \boldsymbol{\Omega_0}\}$ is a finite cover of $\boldsymbol{N}$ by (N$\boldsymbol{\delta gp}$ -O) sets in $\boldsymbol{\mu}$.

Theorem 3.27 If the graph function $\boldsymbol{g}: \boldsymbol{\Omega} \to \boldsymbol{\Omega} \times \boldsymbol{\mu}$ of $\boldsymbol{f}: \boldsymbol{\Omega} \to \boldsymbol{\mu}$, defined by $\boldsymbol{g}(\boldsymbol{\sigma}) = (\boldsymbol{\sigma}, \boldsymbol{f}(\boldsymbol{\sigma}))$ for each $\boldsymbol{\sigma} \in \boldsymbol{\Omega}$ is (NA$\boldsymbol{\delta gp}$ -CO) Then $\boldsymbol{f}$ is (NA$\boldsymbol{\delta gp}$ -CO)

Proof. Let $\boldsymbol{\lambda} \in NRO(\mu)$, then $\Omega \times \mu \in NRO(\Omega \times \mu)$. As g is (NA δgp -CO), $f^{-1}(\lambda) = g^{-1}(\Omega \times \lambda) \in N\delta GPO(\Omega)$.

Theorem 3.28. Let Ω, η be (NTSs) and $g: \Omega \to \Omega \times \eta$ be graph neutrosophic function of $f: \Omega \to \eta$, defined by $g(\sigma) = (\sigma, f(\sigma))$ for each $\sigma \in \Omega$. If Ω is a (N-submax-S) and (Nδgp -add-S), then g is (NAδgp -CO) if and only if f is (NAδgp -CO).

Proof. We only prove the sufficiency. Let $\boldsymbol{\sigma} \in \boldsymbol{\Omega}$ and $\boldsymbol{W} \in \boldsymbol{RO}(\boldsymbol{\Omega} \times \boldsymbol{\eta})$. Then there exist (NR-O) sets $\boldsymbol{\lambda_1}$ and $\boldsymbol{V}$ in $\boldsymbol{\Omega}$ and $\boldsymbol{\eta}$, respectively such that $\boldsymbol{\lambda_1} \times \boldsymbol{V} \subset \boldsymbol{W}$. If $\boldsymbol{f}$ is (NA$\boldsymbol{\delta gp}$ -CO), so there exists a (N$\boldsymbol{\delta gp}$ -O) set $\boldsymbol{\lambda_2}$in $\boldsymbol{\Omega}$ satisfies $\boldsymbol{\sigma} \in \boldsymbol{\lambda_2}$ and $\boldsymbol{f}(\boldsymbol{\lambda_2}) \subset \boldsymbol{V}$. Put $\boldsymbol{\lambda} = (\boldsymbol{\lambda_1} \cap \boldsymbol{\lambda_2})$.Then $\boldsymbol{\lambda}$ is (N$\boldsymbol{\delta gp}$ -O) and $\boldsymbol{g}(\boldsymbol{\lambda}) \subset \boldsymbol{\lambda_1} \times \boldsymbol{V} \subset \boldsymbol{W}$. Thus $\boldsymbol{g}$ is (NA$\boldsymbol{\delta gp}$ -CO).

Definition 3.29. A graph $\boldsymbol{G_f} = \{(\boldsymbol{\Omega}, \boldsymbol{F}(\boldsymbol{\sigma}): \boldsymbol{\sigma} \in \boldsymbol{\Omega}\} \subset \boldsymbol{\Omega} \times \boldsymbol{\eta}$of a neutrosophic function $\boldsymbol{f}: \boldsymbol{\Omega} \to \boldsymbol{\mu}$ is said to be neutrosophic strongly $\boldsymbol{\delta gp}$ -closed (N-Str-$\boldsymbol{\delta gp}$ **-C)** if for each $(\boldsymbol{\rho}, \boldsymbol{\theta}) \notin \boldsymbol{G_f}$, there exist $\lambda \in N\delta GPO(\Omega, \rho)$ and $V \in NRO(\mu, \theta)$ satisfy $(\lambda \times V) \cap G_f = \varphi$.

Lemma 3.30. For a graph $\boldsymbol{G_f}$ of a neutrosophic function $\boldsymbol{f}: \boldsymbol{\Omega} \to \boldsymbol{\mu}$, the following properties are equivalent:

(i) $\boldsymbol{G_f}$ is (N-Str-δgp -C) in $\Omega \times \mu$;

(ii)For each $(\rho,\theta) \notin G_f$, there exist $\lambda \in N\delta GPO(\Omega,\rho)$ and $V \in$ N $RO(\mu,\theta)$ such that $f(\lambda) \cap V = \varphi$.

Theorem 3.31. Let $\boldsymbol{f}: \boldsymbol{\Omega} \rightarrow \boldsymbol{\mu}$ have a (N-Str-δgp -C) graph G_f . If f is injective, then Ω is (N$\delta gp - T_1$-S).

Proof. Let $\boldsymbol{\sigma_1}, \boldsymbol{\sigma_2} \in \boldsymbol{\Omega}$ with $\boldsymbol{\sigma_1} \neq \boldsymbol{\sigma_2}$ .Then $\boldsymbol{f}(\boldsymbol{\sigma_1}) \neq \boldsymbol{f}(\boldsymbol{\sigma_2})$ as $\boldsymbol{f}$ is injective so that $(\boldsymbol{\sigma_1}, \boldsymbol{f}(\boldsymbol{\sigma_2})) \notin \boldsymbol{G_f}$. Thus there exist $\lambda_1 \in N\delta GPO(\Omega,\sigma_1)$ and $\lambda_2 \in NRO\big(\mu, f(\sigma_2)\big)$ such that $f(\lambda_1) \cap \lambda_2 = \varphi$. Then $f(\sigma_2) \notin f(\lambda_1)$ implies $\sigma_2 \notin \lambda_1$ and it follows that Ω is (N$\delta gp - T_1$-S).

Theorem 3.32.

(i) If $\boldsymbol{f}: \boldsymbol{\Omega} \rightarrow \boldsymbol{\mu}$ is (NA$\boldsymbol{\delta gp}$ -CO) and $\boldsymbol{g}: \boldsymbol{\mu} \rightarrow \boldsymbol{\eta}$is (N$\boldsymbol{R}$ -M), then $\boldsymbol{g} \circ \boldsymbol{f}: \boldsymbol{\Omega} \rightarrow \boldsymbol{\eta}$is (NA$\boldsymbol{\delta gp}$ -CO).

(ii) If $\boldsymbol{f}: \boldsymbol{\Omega} \rightarrow \boldsymbol{\mu}$ is (N$\boldsymbol{\delta gp}$ -CO) and $\boldsymbol{g}: \boldsymbol{\mu} \rightarrow \boldsymbol{\eta}$ is (NA-CO), then $\boldsymbol{g} \circ \boldsymbol{f}: \boldsymbol{\Omega} \rightarrow \boldsymbol{\eta}$ is (NA$\boldsymbol{\delta gp}$ -CO).

(iii) If $\boldsymbol{f}: \boldsymbol{\Omega} \rightarrow \boldsymbol{\mu}$ is (Nδgp -IR) and $\boldsymbol{g}: \boldsymbol{\mu} \rightarrow \boldsymbol{\eta}$ is (NA$\boldsymbol{\delta gp}$ -CO), then $\boldsymbol{g} \circ \boldsymbol{f}: \boldsymbol{\Omega} \rightarrow \boldsymbol{\eta}$ is (NA$\boldsymbol{\delta gp}$ -CO).

Proof. (i) Let $\boldsymbol{\lambda} \in NRO(\eta)$. Then $g^{-1}(\lambda) \in NRO(\mu)$ since g is (NR -M). The (NAδgp -CO) of f implies $f^{-1}[g^{-1}(\lambda))] = (g \circ f)^{-1}((\lambda)) \in N\delta GPO(\Omega)$. Hence $g \circ f$ is (NAδgp -CO).

The proofs of (ii) and (iii) are similar to (i).

Theorem 3.33. If $\boldsymbol{f}: \boldsymbol{\Omega} \rightarrow \boldsymbol{\mu}$ is a pre $\boldsymbol{\delta gp}$ -open surjection and $\boldsymbol{g}: \boldsymbol{\mu} \rightarrow \boldsymbol{\eta}$ is a function such that $\boldsymbol{g} \circ \boldsymbol{f}: \boldsymbol{\mu} \rightarrow \boldsymbol{\eta}$ is (NA$\boldsymbol{\delta gp}$ -CO), then g is (NA$\boldsymbol{\delta gp}$ -CO).

Proof. Let $\boldsymbol{\theta} \in \boldsymbol{\eta}$ and $\boldsymbol{\sigma} \in \boldsymbol{\Omega}$ such that $\boldsymbol{f}(\boldsymbol{\sigma}) = \boldsymbol{\theta}$. Let $\boldsymbol{G} \in \boldsymbol{RO}(\boldsymbol{\eta}, (\boldsymbol{g} \circ \boldsymbol{f})(\boldsymbol{\sigma}))$. Then there exists $\boldsymbol{U} \in \boldsymbol{\delta GPO}(\boldsymbol{\Omega}, \boldsymbol{\sigma})$ such that such that $\boldsymbol{g}(\boldsymbol{f}(\boldsymbol{U})) \subset \boldsymbol{G}$. Since $\boldsymbol{f}$ is pre $\boldsymbol{\delta gp}$ -open in$\boldsymbol{\mu}$, we have that $\boldsymbol{g}$ is (NA$\boldsymbol{\delta gp}$ -CO) at $\boldsymbol{\mu}$.

Conclusion

In this paper, some new notions of neurosophic delta generated pre-continuous functions in neutrosophic topological spaces are given and discussed, which is a very interesting topic in nature. It will open up many avenues for the researchers work neutrosophic topological spaces, we can in future work extend and study these our notions for this paper in soft setting form.

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