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On Neurosophic Delta Generated Per-Continuous Functions in Neutrosophic Topological Spaces

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Abstract: In this work, we investigate new type of neutrosophic continuity, it is called neutrosophic almost δgp –continuity functions, which is stronger than the conception of neutrosophic almost gpr-continuous functions. Also, new notions like neutrosophic δgp-compact, neutrosophic δgp-compact relative to neutrosophic space and neutrosophic strongly δgp –closed for graph of neutrosophic functions are shown. Furthermore, some of its interest properties are shown and studied.

Keywords: neutrosophic sets, neutrosophic topological space, neutrosophic δgp –continuity functions, neutrosophic almost gpr-continuous functions.

1. Introduction
As an expansion of Fuzzy sets given in 1965 by Zadeh [1] and Intuitionistic Fuzzy sets given in 1983 by Atanassav [2], the Neutrosophic sets (NSs) have been shown and explained by Smarandache. A (NS) is depicted by a truth value (memberships), an indeterminacy value and a falsity value (non-memberships). Salama and Alblowi [3] introduced the new concept of neutrosophic topological space (NTS) in 2012, which had been investigated recently. In 2018, Parimala M et al. explain the concept of Neutrosophic homeomorphism and Neutrosophic αψ homeomorphism in (NTS) [4]. In 2020, the notions of Ngpr homeomorphism and Nigpr homeomorphism in (NTS) are introduced and studied [5]. There are some sets in topological spaces their expansion in non-classical are studied, like soft sets [6-13], fuzzy sets [14-19], permutation sets [20-26], neutrosophic sets [27-30] nano sets [31,32] and others [33,34]. Here, we will use the conception of neutrosophic to study our
expansion in non-classical. The neutrosophic closure and neutrosophic interior of any (NS) $A$ in (NTS) $(\Psi, \tau)$ are defined as $\text{Ncl}(A) = \cap \{A \subseteq B; B^c \in \tau \}$ and $\text{Nint}(A) = \cup \{B \subseteq A; B \in \tau \}$, respectively. The neutrosophic class of neutrosophic $\delta\text{gp}$-open (resp. neutrosophic $\delta\text{gp}$-closed, neutrosophic open, closed, neutrosophic regular closed, neutrosophic regular open, neutrosophic $\delta$-preopen, neutrosophic $\delta$-semiopen, neutrosophic preopen, neutrosophic semiopen, neutrosophic $\epsilon^*$-open and neutrosophic $\beta$-open) sets of $(\Psi, \tau)$ containing a point $s \in \Psi$ is denoted by $\text{N}\delta\text{GPO}(\Psi, s)$ (resp. $\text{N}\delta\text{GPC}(\Psi, s)$, $\text{N}\delta\text{O}(\Psi, s)$, $\text{N}\delta\text{C}(\Psi, s)$, $\text{N}\delta\text{R}(\Psi, s)$, $\text{N}\delta\text{RO}(\Psi, s)$, $\text{N}\delta\text{PO}(\Psi, s)$, $\text{N}\delta\text{SO}(\Psi, s)$, $\text{N}\delta\text{PO}(\Psi, s)$, $\text{N}\delta\text{SO}(\Psi, s)$, $\text{N}\delta\text{PO}(\Psi, s)$) and $\text{N}\beta\text{O}(\Psi, s)$). That means if $A$ is neutrosophic $q$-open ($q$-closed) set in neutrosophic topological space $(\Psi, \tau)$, where $q$ is any property for the neutrosophic set $A$ and $s \in A$ for some $s \in \Psi$, then it is denoted by $\text{N}q\text{O}(\Psi, s)$ ($\text{N}q\text{C}(\Psi, s)$).

In this paper, we're looking into a new kind of neutrosophic continuity, it is known as neutrosophic almost $\delta\text{gp}$ — continuity functions, which is stronger than the conception of neutrosophic almost $g\text{pr}$-continuous functions. Also, some characteristics of neutrosophic almost $\delta\text{gp}$ — continuity functions are explained and discussed.

2. Preliminaries

Basic definitions and notations can be found here, which are used in this section are referred from the references [3,35-37].

**Definition 2.1:**

Assume $\Psi \neq \varnothing$. A neutrosophic set (NS) $\theta$ is defined as $\theta = \{(\alpha, \vartheta_\theta(\alpha), \omega_\theta(\alpha), \ell_\theta(\alpha)); \alpha \in \Psi \}$ where $\vartheta_\theta(\alpha)$ is the degree of membership, $\omega_\theta(\alpha)$ is the degree of indeterminacy and $\ell_\theta(\alpha)$ is the degree of non-membership, $\forall \alpha \in \Psi$ to $\theta$. Let $D = \{(\alpha, \vartheta_D(\alpha), \omega_D(\alpha), \ell_D(\alpha)); \alpha \in \Psi \}$ be the second (NS), then $\theta \cap D = \{(\alpha, \min \{\vartheta_\theta(\alpha), \vartheta_D(\alpha)\}, \max \{\omega_\theta(\alpha), \omega_D(\alpha)\}, \max \{\ell_\theta(\alpha), \ell_D(\alpha)\)); \alpha \in \Psi \}$ and $\theta \cup D = \{(\alpha, \max \{\vartheta_\theta(\alpha), \vartheta_D(\alpha)\}, \min \{\omega_\theta(\alpha), \omega_D(\alpha)\}, \min \{\ell_\theta(\alpha), \ell_D(\alpha)\)}; \alpha \in \Psi \}$.
\[ \{\alpha \in \Psi\} \]. Also, \( \theta \subseteq D \) if and only if \( \partial_\theta (\alpha) \leq \partial_D (\alpha) , \ \omega_\theta (\alpha) \geq \omega_D (\alpha) \) and \( \ell_\theta (\alpha) \geq \ell_D (\alpha) \). The complement of \( \theta \) is \( \theta^c = \{ (\alpha, \ell_\theta (\alpha), 1 - \omega_\theta (\alpha), \partial_\theta (\alpha)) ; \alpha \in \Psi \} \)

**Definition 2.2:** We say \((\Psi, \tau)\) is a neutrosophic topological space (NTS) if and only if \( \tau \) is a collection of (NSs) in \( \Psi \) and it such that:

1. \( 1_N, 0_N \in \tau \) where \( 0_N = \{ (\alpha, (0,0,1)) ; \alpha \in \Psi \} \) and \( 1_N = \{ (\alpha, (1,0,0)) ; \alpha \in \Psi \} \).
2. \( \theta \cap \beta \in \tau \) for any \( \theta, \beta \in \tau \).
3. \( \bigcup_{i \in I} \theta_i \in \tau \) for any arbitrary family \( \{ \theta_i | i \in I \} \subseteq \tau \). Also, any \( \theta \in \tau \) is called neutrosophic open set (NOS) and we say neutrosophic closed set (NCS) for its complement.

**Definition 2.3.** Let \( \Gamma \subseteq X \) be (NS) in (NTS) \( X \). We say \( \Gamma \) is neutrosophic pre-closed (NP-C) (resp. neutrosophic regular-closed (NR-C), neutrosophic semi-closed (NS-C), neutrosophic \( \beta \) -closed (N\( \beta \) -C)) if \( \text{Ncl}(\text{int}(\Gamma)) \subseteq \Gamma \) (resp. \( \Gamma = \text{Ncl}(\text{N int}(\Gamma)) \), \( \text{Ncl}(\text{N int}(\Gamma)) \subseteq \Gamma \) and \( \text{N int}(\text{Ncl}(\text{N int}(\Gamma))) \subseteq \Gamma \)).

**Definition 2.4.** Let \( \Gamma \subseteq X \) be (NS) in (NTS) \( X \). We say \( \Gamma \) is neutrosophic \( \delta \) -closed (N\( \delta \) -C), if \( \Gamma = \text{Ncl}_\delta (\Gamma) \) where \( \text{Ncl}_\delta (\Gamma) = \{ p \in X ; \text{N int}(\text{Ncl}(D)) \cap \Gamma \neq \emptyset , \ D \in \tau \ \text{and} \ p \in D \} \).

**Definition 2.5.** Let \( \Gamma \subseteq X \) be (NS) in (NTS) \( X \). We say \( \Gamma \) is neutrosophic \( \delta \) -preclosed (N\( \delta \)P-C) (resp. neutrosophic \( \varepsilon '' \) -closed (N\( \varepsilon '' \) - C), neutrosophic \( \delta \) -semiclosed (N\( \delta \)S-C) and neutrosophic \( \alpha \) -closed (N\( \alpha \) - C)) if \( \text{Ncl}(\text{N int}_\delta (\Gamma)) \subseteq \Gamma \) (resp. \( \text{N int}(\text{Ncl}(\text{N int}_\delta (\Gamma))) \subseteq \Gamma \), \( \text{N int}(\text{cl}_\delta (\Gamma))) \subseteq \Gamma \) and \( \text{Ncl}(\text{N int}(\text{Ncl}_\delta (\Gamma))) \subseteq \Gamma \)).

**Definition 2.6.** Let \( \Gamma \subseteq X \) be (NS) in (NTS) \( X \). We say \( \Gamma \) is;

(i) neutrosophic \( \delta gp \) -closed (N\( \delta gp \) -C) (resp. neutrosophic \( gpr \) -closed (N\( gpr \) -C) and neutrosophic \( gp \) -closed (N\( gp \) -C)) if \( \text{Npcl}(\Gamma) \subseteq L \) whenever \( \Gamma \subseteq L \) and \( L \) is neutrosophic \( \delta \).
-open (N\(\delta\) -O) (resp. neutrosophic regular open (NR-O) and neutrosophic open (NO)) in \(X\),

where \(Npcl(\Gamma) = \bigcap \{\Gamma \subseteq B; \ B \text{ is } (NP - C)\}\)

(ii) neutrosophic \(g\delta s\) -closed (N \(g\delta s\) -C) if \(Nsccl(\Gamma) \subseteq L\) whenever \(\Gamma \subseteq L\) and \(L\) is (N\(\delta\) -O) in \(X\), where \(Nsccl(\Gamma) = \bigcap \{\Gamma \subseteq B; \ B \text{ is } (NS - C)\}\).

The neutrosophic open sets are the complements of the previously described neutrosophic closed sets.

**Definition 2.7.** Assume \(W\) and \(V\) are (NTSs) and \(h: W \rightarrow V\) is a neutrosophic map (NM). We say \(h\) is:

(i) Neutrosophic \(R\) -map (NR-M) (resp. neutrosophic \(\delta\) -continuous (N\(\delta\) -CO), neutrosophic almost continuous (NA-CO), neutrosophic almost \(pre\) -continuous (NAP-CO), neutrosophic almost \(gp\) -continuous (NAGp -CO), neutrosophic almost \(G\) -continuous (NAG -CO) and neutrosophic almost \(g\delta s\) -continuous (NAG\(g\delta s\) -CO) if \(h^{-1}(L)\) of any (NR-O) set \(L\) of \(V\) is (NR-O) set (resp. (N\(\delta\) -O), (NO), (NP-O), (N\(g\)p -O), (NG -O) and (N \(g\delta s\) -O)) set in \(W\).

(ii) Neutrosophic \(\delta gp\) -continuous (N \(\delta gp\) -CO) if \(h^{-1}(L)\) of any (NO) set \(L\) of \(V\) is neutrosophic \(\delta gp\) -open (N\(\delta gp\) -O) in \(W\).

(iii) Neutrosophic almost contra continuous (NAC-CO) (resp. neutrosophic almost contra \(sup\) -continuous (NACsup-CO) and neutrosophic contr \(R\) -map (NCR-M)) if \(h^{-1}(L)\) of any (NR-C) set \(L\) of \(V\) is (NO) (resp. (N\(\delta\) -O) and (NR-O)) in \(W\).

(iv) Neutrosophic almost perfectly-continuous (NAperf-CO) if the inverse image of any (NR-C) set \(L\) of \(V\) is neutrosophic clopen in \(W\),

(v) Neutrosophic almost contra \(\delta gp\) -continuous (NAC\(\delta gp\) -CO) (resp. neutrosophic contra \(\delta gp\) -continuous (NC\(\delta GP\) -CO) and neutrosophic \(\delta gp\) -resolute (N\(\delta gp\) -IR), if \(h^{-1}(L)\) of any (NR-O) (resp. (NO) and (N\(\delta gp\) -C)) set \(L\) of \(V\) is (N\(\delta gp\) -C) in \(W\).
Definition 2.8. Let $\Omega$ be a (NTS). $\text{NGPR}(\Omega) = \{ A \subseteq \Omega \mid A \text{ is (NGPR - O) in } \Omega \}$, $\text{NGPGPO}(\Omega) = \{ A \subseteq \Omega \mid A \text{ is (NGPGPO - O) in } \Omega \}$ and $\text{NPO}(\Omega) = \{ A \subseteq \Omega \mid A \text{ is (NPO - O) in } \Omega \}$.

(i) Neutrosophic preregular $\mathcal{T}_{\frac{1}{2}}$-space (Npr-reg-$\mathcal{T}_{\frac{1}{2}}$-S) if $\text{NGPR}(\Omega) = \text{NPO}(\Omega)$,

(ii) Neutrosophic $\mathcal{T}_{\frac{1}{2}}$-space (NT$\mathcal{T}_{\frac{1}{2}}$-S) if $\text{NGPGPO}(\Omega) = \text{NPO}(\Omega)$,

(iii) Neutrosophic $\mathcal{T}_{\frac{1}{2}}$-space (N$\mathcal{T}_{\frac{1}{2}}$-S) if $\text{NGPGPO}(\Omega) = \text{NPO}(\Omega)$,

(iv) Neutrosophic extremely disconnected (NED) if the closure of any (NO) subset of $\Omega$ is (NO),

(v) Neutrosophic submaximal space (N-submax-S) if any (NP-O) set is (NO),

(vi) Neutrosophic strongly irresolvable (N-si) if any neutrosophic open subspace of $\Omega$ is irresolvable,

(vii) Neutrosophic nearly compact space (N-NCom-S) if any (NR-O) cover of $\Omega$ has a finite subcover,

(viii) Neutrosophic $r - \mathcal{T}_{1}$-space (N$r - \mathcal{T}_{1}$-S) if for each $\sigma_1 \neq \sigma_2$ two points in $\Omega$, there exist (NR-O) sets $\lambda_1$ and $\lambda_2$ such that $\sigma_1 \in \lambda_1, \sigma_2 \notin \lambda_1$ and $\sigma_1 \notin \lambda_2, \sigma_2 \in \lambda_2$.

(ix) Neutrosophic $r - \mathcal{T}_{2}$-space (N$r - \mathcal{T}_{2}$-S) if for each $\sigma_1 \neq \sigma_2$ in $\Omega$, there exist (NR-O) sets $\lambda_1$ and $\lambda_2$ such that $x \in U, y \in V \text{ and } U \cap V = \varnothing$.

(x) Neutrosophic $\mathcal{D}_{1} - \mathcal{T}_{1}$-space (N$\mathcal{D}_{1} - \mathcal{T}_{1}$-S) if for each $p \neq q$ in $\Omega$, there exist $\Psi_1, \Psi_2 \in \text{NGPGPO}(\Omega)$ such that $P \in \Psi_1, q \notin \Psi_1$ and $q \in \Psi_2, p \notin \Psi_2$.

(xi) Neutrosophic Hausdorff space (NH-S) (resp., Neutrosophic $\mathcal{D}_{1}$-Hausdorff, space (N$\mathcal{D}_{1}$-H-S)) if for each $\sigma_1 \neq \sigma_2$ in $\Omega$, there exist $\Psi_1, \Psi_2 \in \text{NPO}(\Omega)$ (resp., $\Psi_1, \Psi_2 \in \text{NGPO}(\Omega)$) such that $x \in G, y \in H \text{ and } G \cap H = \varnothing$.

(xii) Neutrosophic $\mathcal{D}_{1}$-additive space (N$\mathcal{D}_{1}$-add-S) if $\mathcal{D}_{1}\text{GPC}(\Omega)$ is closed under arbitrary intersections.
Definition 2.9. Let $\Omega$ be a (NTS) and $\lambda \subseteq \Omega$. We say $\Omega$ is Neutrosophic $N$-closed relative (NN-CI-R) to $\lambda$ if any cover of $\lambda$ by (NR-O) sets of $\Omega$ has a finite subcover.

Theorem 2.10. (i) If $\lambda_1$ and $\lambda_2$ are (N$\delta$gp-O) subsets of a (N-submax-S)$\lambda$, then $\lambda_1 \cap \lambda_2$ is (N$\delta$gp-O) in $\Omega$.

(ii) Let $\Omega$ be a (N-$\delta$gp-add-S). Then $\lambda_2 \subseteq \Omega$ is (N$\delta$gp-C) if and only if $N\delta gp - cl(\lambda_2) = \lambda_2$, where $N\delta gp - cl(\lambda_1) = \cap \{\lambda_1 \subseteq B; B$ is (N$\delta$gp - C)$\}$.

Definition 2.11. Assume $\Omega$ is a (NTS). We say $\Omega$ is a neutrosophic locally indiscrete space (N-li-S) if $N O(\Omega) = N R O(\Omega)$, where $N O(\Omega) = \{A \subseteq \Omega | A$ is (NO)in $\Omega\}$ and $N R O(\Omega) = \{A \subseteq \Omega | A$ is (NR - O)in $\Omega\}$.

Lemma 2.12. Let $\Omega$ be a (NTS) and $\lambda \subseteq \Omega$. Then these terms are true:

(i) $\lambda \in N P O(\Omega)$ if and only if $N s c l(\lambda) = N i n t(\{N c l(\lambda)\})$.

(ii) $p \in N \delta gp c l(\lambda)$ if and only if $B \cap \lambda \neq \phi$ for any (N$\delta$gp - O) set $B$ containing $r$.

Remark: 2.13: For any (NS) $\lambda \subseteq \Omega$ in (NTS) $\Omega$, we consider that:

1. $N c l(N i n t_{\delta}(\lambda)) = N c l_{\delta}(N i n t_{\delta}(\lambda))$.

2. $N i n t(N c l_{\delta}(\lambda)) = N i n t_{\delta}(N c l_{\delta}(\lambda))$.

3. $N i n t_{\delta}(\Omega \lambda) = \Omega(N c l_{\delta}(\lambda)) \in N R O(\Omega)$, if $\lambda$ is (Ne$^*$ - O).

3. Neutrosophic Almost $\delta$gp-Continuous Functions.

Definition 3.1. Let $h: \Omega \rightarrow \mu$ be a (NM). We say $h$ is neutrosophic almost $\delta$gp-continuous (NA$\delta$gp-CO) if $h^{-1}(\lambda) \in N \delta G P C(\Omega)$ for each (NR-C) set $\lambda$ of $\mu$.

Example 3.2. Define the neutrosophic sets $D_1, D_2, D_3, D_4$ and $H_1, H_2, H_3, H_4, H_5$ as follows:

$D_1 = \{(a, (0,1,0.3)), (b, (0.3,0.5,1)), (c, (0,0.6,1)), (d, (0.5,1,0.8))\}$
\[ D_2 = \{(a, (0.2, 0.4, 0.1)), (b, (0.1, 0.3)), (c, (0.7, 0.1, 0.6)), (d, (0.0, 0.5, 1))\} \]

\[ D_3 = \{(a, (0.2, 0.4, 0.3)), (b, (0.3, 0.5, 0.3)), (c, (0.7, 0.6, 0.6)), (d, (0.5, 0.5, 0.8))\} \]

\[ D_4 = \{(a, (0.3, 0.3, 0.2)), (b, (0.4, 0.4, 0.3)), (c, (0.8, 0.5, 0.5)), (d, (0.6, 0.4, 0.7))\} \]

And

\[ H_1 = \{(a, (0.2, 0.4, 0.4)), (b, (0.1, 0.3)), (c, (0.7, 0.1, 0.6)), (d, (0.0, 0.5, 1))\} \]

\[ H_2 = \{(a, (0.1, 0.3)), (b, (0.3, 0.5, 1)), (c, (0.0, 0.6, 1)), (d, (0.5, 0.1, 0.8))\} \]

\[ H_3 = \{(a, (0.3, 0.3, 0.2)), (b, (0.4, 0.4, 0.3)), (c, (0.8, 0.5, 0.5)), (d, (0.6, 0.4, 0.7))\} \]

\[ H_4 = \{(a, (0.2, 0.4, 0.3)), (b, (0.3, 0.5, 0.3)), (c, (0.7, 0.6, 0.6)), (d, (0.5, 0.5, 0.8))\} \]

Now, let \( t = \{1, 0, D_1, D_2, D_3, D_4\} \) and \( h = \{1, 0, H_1, H_2, H_3, H_4\} \) then \((X, t)\) and \((Y, h)\) are (NTSs), where \( X = \{a, b, c, d\} = Y \). Define \( f: X \to Y \) by \( f(a) = f(c) = b, f(b) = a, f(d) = c \). We consider that \( f \) is neutrosophic almost \( \delta_{gp} \)-continuous.

**Theorem 3.3.** Let \( h: X \to Y \) be (NM). Then \( h \) is (NA \( \delta_{gp} \) -CO) if and only if \( h^{-1}(\mu) \) of any (NR-O) set \( \mu \) of \( Y \) is (N\( \delta_{gp} \) -O) in \( X \).

**Proof:** since the complement for any (NO) is (NC) and by Definition (3.1). Then the theorem is held.

**Example 3.4.** Define the neutrosophic sets \( D_1, D_2, D_3, D_4 \) and \( H_1, H_2, H_3, H_4, H_5 \) as follows:

\[ D_1 = \{(a, (0.1, 0.4)), (b, (0.4, 0.6, 1)), (c, (0.1, 0.7, 1)), (d, (0.6, 1, 0.9))\} \]

\[ D_2 = \{(a, (0.3, 0.5, 1)), (b, (0.1, 0.4)), (c, (0.8, 0.2, 0.7)), (d, (0.1, 0.6, 1))\} \]

\[ D_3 = \{(a, (0.3, 0.5, 0.4)), (b, (0.4, 0.6, 0.4)), (c, (0.8, 0.7, 0.7)), (d, (0.6, 0.6, 0.9))\} \]

\[ D_4 = \{(a, (0.4, 0.4, 0.3)), (b, (0.5, 0.5, 0.4)), (c, (0.9, 0.6, 0.6)), (d, (0.7, 0.5, 0.8))\} \]

And
\[H_1=\{(a, (0.3,0.5,1)), \langle b, (0.1,1,0.4)\rangle, \langle c, (0.8,0.2,0.7)\rangle, \langle d, (0.1,0.6,1)\rangle\}\]

\[H_2=\{(a, (0.1,1,0.4)), \langle b, (0.4,0.6,1)\rangle, \langle c, (0.1,0.7,1)\rangle, \langle d, (0.6,1,0.9)\rangle\}\]

\[H_3=\{(a, (0.4,0.4,0.3)), \langle b, (0.5,0.5,0.4)\rangle, \langle c, (0.9,0.6,0.6)\rangle, \langle d, (0.7,0.5,0.8)\rangle\}\]

\[H_4=\{(a, (0.3,0.5,0.4)), \langle b, (0.4,0.6,0.4)\rangle, \langle c, (0.8,0.7,0.7)\rangle, \langle d, (0.6,0.6,0.9)\rangle\}\]

Now, let \(t=\{1_N, 0_N, D_2, D_2, D_2, D_4\}\) and \(h=\{1_N, 0_N, H_1, H_2, H_2, H_4\}\) then \((X,t)\) and \((Y,h)\) are (NTSs), where \(X = \{a,b,c,d\} = Y\). Define \(h: X \rightarrow Y\) by \(h(a) = h(c) = b, h(b) = a, h(d) = c\).

Then we consider that \(h\) is (NA\(\delta\)gp -CO). Also, \(h^{-1}(\mu)\) is (N\(\delta\)gp -O) in \(X\) for any (NR-O) set \(\mu\) of \(Y\).

**Remark 3.4.** Let \(h: \Omega \rightarrow \mu\) be a (NM). Then by Definitions (2.7) and (3.1), we consider diagram (1) as follows:

**Theorem 3.5.** If \(f: \mu \rightarrow \eta\) is (NA\(\delta\)gp -CO) and \(\eta\) is (N-li-S), then \(f\) is (N\(\delta\)gp -CO).

**Proof.** Let \(\lambda\) be (NO) set in \(\eta\), then \(\lambda\) is (NR-O) in \(\eta\). Since \(f\) is (NA\(\delta\)gp -CO), then \(f^{-1}(\lambda)\) is (N\(\delta\)gp -O) in \(\mu\). Hence \(f\) is (N\(\delta\)gp -CO)

**Theorem 3.6.** Let \(\Omega\) be a (N-li-S), then these terms are equivalent:
(i) \( f: \Omega \to \mu \) is \((N_{gpr} \text{-CO})\),

(ii) \( f: \Omega \to \mu \) is \((NA_{dp} \text{-CO})\),

(iii) \( f: \Omega \to \mu \) is \((NA_{gp} \text{-CO})\).

**Proof.** Follows from the Definitions (2.11), (2.7) and (3.1).

**Remark 3.7.** It is clear from the definitions in section 2, we consider that all of the theorems [(3.8)–(3.13)] are held.

**Theorem 3.8.** (i) If \( f: \Omega \to \mu \) is \((NA_{dp} \text{-CO})\) with \( \Omega \) as \((N_{E}\)D), then it is \((NA_{gp} \text{-CO})\).

(ii) If \( f: \Omega \to \mu \) is \((NA_{dp} \text{-CO})\) with \( \Omega \) as \((N-si)\). Then it is \((NA_{dp} \text{-CO})\).

**Theorem 3.9.** All of these terms are equivalent:

(i) \( f: \Omega \to \eta \) is \((NA_{perf}-CO)\),

(ii) \( f: \Omega \to \eta \) is \((NA_{perf}-CO)\) and \((NA_{gp} \text{-CO})\),

(iii) \( f: \Omega \to \eta \) is \((NA_{perf}-CO)\) and \((NA_{gp} \text{-CO})\),

(iv) \( f: \Omega \to \eta \) is \((NA_{perf}-CO)\) and \((NA_{gp} \text{-CO})\),

(v) \( f: \Omega \to \eta \) is \((NCR-M)\) and \((NA_{gpr} \text{-CO})\),

(vi) \( f: \Omega \to \eta \) is \((NCR-M)\) and \((NA_{gp} \text{-CO})\),

(vii) \( f: \Omega \to \eta \) is \((NCR-M)\) and \((NA_{gp} \text{-CO})\).

**Theorem 3.10.** Let \( \Omega \) be \((N_{gpr} \text{T}\_2\text{-S})\). Then all of these terms are equivalent:

(i) \( f: \Omega \to \eta \) is \((NA_{gp} \text{-CO})\),

(ii) \( f: \Omega \to \eta \) is \((NA_{gp} \text{-CO})\),

(iii) \( f: \Omega \to \eta \) is \((NA_{gp} \text{-CO})\).

**Theorem 3.11.** Let \( \Omega \) be \((N_{reg} \text{T}\_2\text{-S})\). Then All of these terms are equivalent:

(i) \( f: \Omega \to \eta \) is \((NA_{gp} \text{-CO})\),

(ii) \( f: \Omega \to \eta \) is \((NA_{gp} \text{-CO})\),
(iii) \( f: \Omega \to \eta \) is (NA\(\delta\)gp -CO),

(iv) \( f: \Omega \to \eta \) is (NA\(g\)pr -CO).

**Theorem 3.12.** Let \( \Omega \) be a \( T_{\delta gp} \)-space. Then these terms are equivalent:

(i) \( f: \Omega \to \mu \) is (NA-CO);

(ii) \( f: \Omega \to \mu \) is (NA\(pr\)v -CO),

(iii) \( f: \Omega \to \mu \) is (NA\(gp\) -CO),

(iv) \( f: \Omega \to \mu \) is (NA\(\delta\)gp -CO),

(v) \( f: \Omega \to \mu \) is (NA\(g\)pr -CO).

**Theorem 3.13.** The following are equivalent:

(i) \( f: \Omega \to \mu \) is (NA\(\delta\)gp -CO) and \( \Omega \) is (N\(\delta\)gp -add-S),

(ii) for each \( \sigma \in \Omega \) and each open set \( \lambda_1 \) containing \( f(p) \), there exists (N\(\delta\)gp -O) set \( \lambda_2 \) containing \( \sigma \) such that \( f(\lambda_2) \subset \text{Nint}(\text{Ncl}(\lambda_1)) \).

**Theorem 3.14.** All of these terms are equivalent:

(i) \( f: \Omega \to \mu \) is (NA\(\delta\)gp -CO) and \( \Omega \) is (N\(\delta\)gp -add-S),

(ii) For each \( \sigma \in \Omega \) and each \( \lambda_1 \in NO(\mu, f(\sigma)) \), there exists \( \lambda_2 \in N\delta GPO(\Omega, \sigma) \) such that \( f(\lambda_2) \subset \text{Nscl}(\lambda_1) \);

(iii) For each \( \sigma \in \Omega \) and each \( \lambda_3 \in NO(\mu, f(\sigma)) \), there exists \( \gamma_1 \in N\delta GPO(\Omega, \sigma) \) such that \( f(\gamma_1) \subset \lambda_3 \);

(iv) For each \( \sigma \in \Omega \) and each \( \gamma_2 \in N\delta O(\mu, f(\sigma)) \), there exists \( \Sigma \in N\delta GPO(\Omega, \sigma) \) such that \( f(\Sigma) \subset \gamma_2 \);

(v) For each \( \sigma \in \Omega \) and each \( \gamma_2 \in N\delta C(\mu, f(\sigma)) \), there exists \( \Sigma \in N\delta GPC(\Omega, \sigma) \) such that \( f(\Sigma) \subset \gamma_2 \);
**Proof.** (i) $\Rightarrow$ (ii): Let $\sigma \in \Omega$ and $N$ be (NO) set of $\mu$ containing $f(\sigma)$. By (i) and Theorem 3.13, there exists $\lambda_2 \in N\delta GPO(\Omega, \sigma)$ such that $f(\lambda_2) \subseteq N\text{int}(N\text{cl}(\lambda_1))$. Since $\lambda_2$ is preopen, then by Lemma 2.12(i), $f(\lambda_2) \subseteq N\text{sc}(\lambda_2)$.

(ii) $\Rightarrow$ (iii): Let $\sigma \in \Omega$ and $\lambda_1 \in N\text{NO}(\mu, f(\sigma))$. Then $\lambda_1 \in N\text{NO}(\mu, f(\sigma))$. By (ii), there exists $\lambda_2 \in N\delta GPO(\Omega, \sigma)$ such that $f(\lambda_2) \subseteq N\text{sc}(\lambda_2)$. Since $\lambda_3$ is (NP-O), then by Lemma 2.12 (i), $f(\lambda_2) \subseteq N\text{int}(N\text{cl}(\lambda_1)) = \lambda_1$.

(iii) $\Rightarrow$ (iv): Let $\sigma \in \Omega$ and $\lambda_1 \in N\delta O(\mu, f(\sigma))$, then there exists $\lambda_2 \in N\text{NO}(\Omega, f(\sigma))$ such that $M \subseteq N\text{int}(N\text{cl}(M)) \subseteq N$. Since $N\text{int}(N\text{cl}(M)) \subseteq N\text{RO}(Y, f(p))$, by (iii), there exists $\Sigma \in N\delta GPO(\Omega, \sigma)$ such that $f(\Sigma) \subseteq N\text{int}(N\text{cl}(\lambda_2)) \subseteq \lambda_1$.

(iv) $\Rightarrow$ (i): Let $\sigma \in \Omega$ and $\lambda_1 \in N\text{NO}(\mu, f(\sigma))$. Then $N\text{int}(N\text{cl}(\lambda_1)) \in N\delta O(\mu, f(\sigma))$. By (iv), there exists $\lambda_2 \in N\delta GPO(\Omega, \sigma)$ such that $f(\lambda_2) \subseteq N\text{int}(N\text{cl}(\lambda_1))$. Hence $f$ is (NA$\delta gp$ -CO).

(iv) $\Leftrightarrow$ (v): Obvious.

**Remark 3.15.** If $\Omega$ is a (N$\delta gp$ -add-S), then $\lambda \subseteq \Omega$ is (N$\delta gp$ -C) (resp. (N$\delta gp$ -O)) if and only if $\lambda$ is (N$\delta gp$ -CO) and $\lambda$ is (N$\delta gp$ -add-S).

where $N\delta gp - cl(\lambda) = \bigcap \{\lambda \subseteq B; B \ in (N\delta gp - C)\}$ and $N\delta gp - int(\lambda) = \bigcap \{B \subseteq \lambda; B \ in (N\delta gp - O)\}$

**Theorem 3.16.** All of these terms are equivalent:

(i) $f: \Omega \rightarrow \mu$ is (NA$\delta gp$ -CO) and $\Omega$ is (N$\delta gp$ -add-S),

(ii) $f(N\delta gp - cl(\lambda_2)) \subseteq N\text{cl}_g(f(\lambda_1))$ for each $\lambda_1 \subseteq \Omega$;

(iii) $N\delta gp - cl(f^{-1}(\lambda_2)) \subseteq f^{-1}(N\text{cl}_g(\lambda_2))$ for each $\lambda_2 \subseteq \mu$;

(iv) $f^{-1}(\beta_2) \in N\delta GPC(\Omega)$ for each $\beta_2 \in N\delta C(\mu)$;
(v) \( f^{-1}(\gamma_1) \in N\delta GPO(\Omega) \) for each \( \gamma_1 \in N\delta O(\mu) \);

**Proof.** (i) \( \rightarrow \) (ii) Suppose that \( \lambda_2 \in N\delta C(\mu) \); such that \( f(\lambda_1) \subseteq \lambda_2 \). Observe that 
\[
\lambda_1 = Ncl_\delta(\lambda_1) = \cap \{ \gamma_2 : \lambda_2 \subseteq \gamma_2 \quad \text{and} \quad \gamma_2 \in NRC(\mu) \}
\]
and so 
\[
f^{-1}(\lambda_2) = \cap \{ f^{-1}(\gamma_2) : \lambda_2 \subseteq \gamma_2 \} \quad \text{by (i) and Definition 2.8 (xii), we have} \quad f^{-1}(\lambda_2) \subseteq N\delta GPC(\Omega) \quad \text{and} \quad \lambda_2 \subseteq f^{-1}(\lambda_1) \quad \text{Hence} \quad N \quad \delta gp - cl(\lambda_2) \subseteq f^{-1}(\lambda_2) \quad \text{and it follows that} \quad f(N\delta gp - cl(\lambda_1)) \subseteq \lambda_2 \quad \text{Since this is true for any (N\delta-C) set} \quad \lambda_2 \quad \text{containing} \quad f(\lambda_1) \quad \text{we have} \quad f(N\delta gp - cl(\lambda_1)) \subseteq Ncl_\delta(f(\lambda_1)).
\]

(ii) \( \rightarrow \) (iii) Let \( \beta_1 \subseteq \mu \), then \( f^{-1}(\beta_1) \subseteq \Omega \). By (ii),
\[
f(N\delta gp - cl(f^{-1}(\beta_1))) \subseteq N \quad cl_\delta(f(f^{-1}(\beta_1))) \quad \subseteq N \delta gp - cl(\beta_1) \quad \text{So that} \quad N\delta gp - cl(f^{-1}(\beta_1)) \subseteq f^{-1}(Ncl_\delta(\beta_1))
\]

(iii) \( \rightarrow \) (iv) Let \( \beta_2 \in N\delta C(\mu) \). Then by (iii), \( N \delta gp - cl(f^{-1}(\beta_2)) \subseteq f^{-1}(Ncl_\delta(\beta_2)) \)
\[
= f^{-1}(\beta_2) \quad \text{In consequence,} \quad N\delta gp - cl(f^{-1}(\beta_2)) = f^{-1}(\beta_2) \quad \text{and hence by remark (3.15),} \quad f^{-1}(\beta_2) \in N\delta GPC(\Omega).
\]

(iv) \( \rightarrow \) (v): Clear.

(v) \( \rightarrow \) (i): Let \( \lambda_2 \in NRO(\mu) \) Then \( \lambda_2 \in N\delta O(\mu) \). By (v), \( f^{-1}(\lambda_2) \in N\delta GPO(\Omega) \). Hence by Theorem 3.3, \( f \) is (N\(\delta\)gp - CO).

**Theorem 3.17.** All of these terms are equivalent:

(i) \( f : \Omega \rightarrow \eta \) is almost \( \delta gp \) -continuous and \( \Omega \) is (N\(\delta\)gp - add-S),

(ii) For any \( \lambda \in N\delta O(\eta) \), \( f^{-1}(N \int (Ncl(\lambda)) \in N\delta GPO(\Omega) \);

(iii) For any \( \gamma \in N\delta C(\eta) \), \( f^{-1}(Ncl(N \int (\gamma)) \in N\delta GPC(\Omega) \);

(iv) For any \( \lambda \in N\beta O(\eta) \), \( N\delta gp cl(f^{-1}(\lambda)) \subseteq f^{-1}(Ncl(\lambda)) \);

(v) For any \( \gamma \in N\beta C(\eta) \), \( f^{-1}(N \int (\gamma)) \subseteq N\delta gp int^{-1}(\gamma) \);
(vi) For any \( \gamma \in \text{NSC}(\eta), f^{-1}(N \text{int}(\gamma)) \subseteq N \delta g p \text{int}(f^{-1}(\gamma)) \);

(vii) For any \( \lambda \in \text{NSO}(\eta), N \delta \text{gpc}(f^{-1}(\lambda)) \subseteq f^{-1}(N \text{cl}(\lambda)) \);

(viii) For any \( \gamma \in \text{NPO}(\eta), f^{-1}(\gamma) \subseteq N \delta g p \text{int}(f^{-1}(N \text{int}(N \text{cl}(\gamma))) \)

Proof. (i) \( \Leftrightarrow \) (ii): Let \( \lambda \in \text{NO}(\eta) \). Since \( N \text{int}(N \text{cl}(\lambda)) \in N \text{RO}(\eta) \), then by (i),

\[
N \delta g p \text{int}(f^{-1}(N \text{int}(N \text{cl}(\lambda)))) \in N \delta g p \text{int}(f^{-1}(N \text{int}(N \text{cl}(\lambda))) \) \( \subseteq N \delta g p \text{int}(f^{-1}(N \text{int}(N \text{cl}(\lambda)))) \).

The converse is similar.

(i) \( \Leftrightarrow \) (iii) It is similar to (i) \( \Leftrightarrow \) (ii).

(i) \( \rightarrow \) (iv): Let \( \lambda \in \text{NPO}(\eta) \), then \( N \text{cl}(\lambda) \in N \text{RC}(\eta) \) so by (i),

\[
N \delta g p \text{int}(f^{-1}(N \text{cl}(\lambda)) \subseteq f^{-1}(N \text{cl}(\lambda)) \) \( \subseteq N \delta g p \text{int}(f^{-1}(N \text{cl}(\lambda)))) \) \( \in N \delta g p \text{int}(f^{-1}(N \text{cl}(\lambda)))) \).

Since \( f^{-1}(\lambda) \subseteq f^{-1}(N \text{cl}(\lambda)) \) which implies \( N \delta g p \text{cl}(f^{-1}(\lambda)) \subseteq f^{-1}(N \text{cl}(\lambda)) \).

(iv) \( \rightarrow \) (v) and (vi) \( \rightarrow \) (vii): Obvious.

(v) \( \rightarrow \) (vi): It follows from the fact that \( \text{NSC}(\eta) \subseteq \text{NBC}(\eta) \).

(vii) \( \rightarrow \) (i): It follows from the fact that \( \text{NRC}(\eta) \subseteq \text{NSO}(\eta) \).

(i) \( \Leftrightarrow \) (viii): Let \( \lambda \in \text{NPO}(\eta) \). Since \( N \text{int}(N \text{cl}(\lambda)) \in N \text{RO}(\eta) \), then by (i),

\[
f^{-1}(N \text{int}(N \text{cl}(\lambda))) \subseteq N \delta g p \text{int}(f^{-1}(N \text{int}(N \text{cl}(\lambda)))) \text{ and hence } f^{-1}(\lambda) \subseteq f^{-1}(N \text{int}(N \text{cl}(\lambda))) \)

\[
\subseteq N \delta g p \text{int}(f^{-1}(N \text{int}(N \text{cl}(\lambda)))) \) \( \subseteq N \delta g p \text{int}(f^{-1}(\lambda)) \) \( \subseteq N \delta g p \text{int}(f^{-1}(\lambda)) \), in consequence,

\[
N \delta g p \text{int}(f^{-1}(\lambda)) = f^{-1}(\lambda) \) and by remark (3.15), \( f^{-1}(\lambda) \in N \delta g p \text{int}(f^{-1}(\lambda)) \).

Theorem 3.18. The following are equivalent:

(i) \( f: \mu \rightarrow \eta \) is \( (\text{NA} \delta g p -\text{CO}) \) and \( \mu \) is \( (\text{N} \delta g p -\text{S}) \),

(ii) For any \( (\text{Na} \delta g p -\text{O}) \) set \( \alpha \) of \( \eta, f^{-1}(N \text{cl}_{\delta}(\alpha)) \) is \( (\text{N} \delta g p -\text{C}) \) in \( \mu \).

(iii) For any \( (\text{N} \delta g p -\text{O}) \) subset \( \alpha \) of \( \eta, f^{-1}(N \text{cl}_{\delta}(\alpha)) \) is \( (\text{N} \delta g p -\text{C}) \) in \( \mu \).

(iv) For any \( (\text{N} \delta p -\text{O}) \) subset \( \alpha \) of \( \eta, f^{-1}(N \text{int}(N \text{cl}_{\delta}(\alpha))) \) is \( (\text{N} \delta g p -\text{O}) \) in \( \mu \).

(v) For any \( (\text{NO}) \) subset \( \alpha \) of \( \eta, f^{-1}(N \text{int}(N \text{cl}_{\delta}(\alpha))) \) is \( (\text{N} \delta g p -\text{O}) \) in \( \mu \).
(vi) For any (NC) subset \( \alpha \) of \( Y \), \( f^{-1}(\text{Ncl}(N \text{int}_{\delta}(\alpha))) \) is (N\( \delta \)gp -C) in \( \mu \).

**Proof.** (i) \( \rightarrow \) (ii): Let \( \alpha \in N_{\alpha^*O}(\eta) \). Then by remark (2.13), \( \text{Ncl}_{\delta}(\alpha) \in N_{\text{RC}(\eta)} \). By (i), 
\[
 f^{-1}(\text{Ncl}_{\delta}(\eta)) \in N_{\delta \text{GPC}}(\mu).
\]

(ii) \( \rightarrow \) (iii): Obvious since \( N_{\delta \text{SO}}(\eta) \subset N_{\alpha^*O}(\eta) \).

(iii) \( \rightarrow \) (iv): Let \( \alpha \in N_{\delta \text{PO}}(\eta) \), then \( \text{Nint}_{\delta}(\eta \setminus \alpha) \in N_{\delta \text{SO}}(\eta) \). By (iii),
\[
 f^{-1}(\text{Ncl}_{\delta}(\text{Nint}_{\delta}(\eta \setminus \alpha))) \in N_{\delta \text{GPC}}(\mu) \text{ which implies } f^{-1}(\text{Nint}(\text{Ncl}_{\delta}(\alpha))) \in N_{\delta \text{PO}}(\mu).
\]

(iv) \( \rightarrow \) (v): Obvious since \( N_{\text{O}}(\eta) \subset N_{\delta \text{PO}}(\eta) \).

(v) \( \rightarrow \) (vi): Clear

(vi) \( \rightarrow \) (i): Let \( N_{\alpha} \in N_{\text{RO}}(\eta) \). Then \( \alpha = N\text{int}(N_{\text{cl}_{\delta}}(\alpha)) \) and hence \( (\eta \setminus \alpha) \in N_{\text{C}}(\eta) \). By (vi),
\[
 f^{-1}(\eta \setminus \alpha) = \mu \setminus f^{-1}(N\text{int}(N_{\text{cl}_{\delta}}(\alpha))) = f^{-1}(N_{\text{cl}}(N\text{int}_{\delta}(\eta \setminus \alpha))) \in N_{\delta \text{GPC}}(\mu) \text{. Thus } f^{-1}(\alpha) \in N_{\delta \text{PO}}(\mu).
\]

**Theorem 3.19.** If \( f: \Omega \rightarrow \mu \) is (NA\( \delta \)gp -CO) injective function and \( \mu \) is \( (N\gamma - T_{1}\text{-}S) \), then \( X \) is \( (N\delta \text{gp} - T_{1}\text{-}S) \).

**Proof.** Let \( (\mu, \sigma) \) be \( (N\gamma - T_{1}\text{-}S) \) and \( p, q \in \Omega \) with \( p \neq q \). Then there exist (NR-O) subsets \( \lambda, \gamma \) in \( Y \) such that \( f(p) \in \lambda, f(q) \notin \lambda, f(p) \notin \gamma \) and \( f(q) \in \gamma \). Since \( f \) is (NA\( \delta \)gp -CO), \( f^{-1}(\lambda) \) and \( f^{-1}(\gamma) \in N_{\delta \text{GPC}}(\Omega) \) satisfy \( p \notin f^{-1}(\lambda), q \notin f^{-1}(\gamma) \) and \( q \in f^{-1}(\gamma) \). Hence \( \Omega \) is \( (N\delta \text{gp} - T_{1}\text{-}S) \).

**Theorem 3.20.** If \( f: \Omega \rightarrow \eta \) is (NA\( \delta \)gp -CO) injective function and \( \eta \) is \( (N\gamma - T_{2} - S) \), then \( \Omega \) is \( (N\delta \text{gp} - T_{2} - S) \).

**Proof.** The proof is the same way of Theorem (3.20).

**Theorem 3.21.** If \( f, g: \Omega \rightarrow \eta \) are (NA\( \delta \)gp -CO) with \( \Omega \) as \( (N\text{-submax}-S) \) and \( (N\delta \text{gp} - \text{add}-S) \) and \( \eta \) is \( (N\gamma \text{-}S) \), then the set \( \{ x \in \Omega : f(x) = g(x) \} \) is \( \delta \text{gp} \) -closed in \( \Omega \).
Proof. Let $E = \{x \in \Omega: f(x) = g(x)\}$ and $x \in (\Omega \setminus \lambda)$. Then $f(x) \neq g(x)$. Since $\eta$ (NH-S), there exist (NO) sets $\lambda_1$ and $\lambda_2$ of $\eta$ satisfy $f(x) \in \lambda_1$, $g(x) \in \lambda_2$ and $\lambda_1 \cap \lambda_2 = \varnothing$, hence $N \text{ int}(N \text{ cl}(\lambda_1)) \cap N \text{ int}(N \text{ cl}(\lambda_2)) = \varnothing$. Since $f$ and $g$ are (NA $\delta gp$ -CO), there exist $\gamma_1, \gamma_2 \in N \delta GPO(\Omega, x)$ satisfy $f(\gamma_1) \subseteq N \text{ int}(N \text{ cl}(\lambda_1))$ and $g(\gamma_2) \subseteq N \text{ int}(N \text{ cl}(\lambda_2))$. Now, put $\Sigma = \gamma_1 \cap \gamma_2$, then $\Sigma \in N \delta GPO(\Omega, x)$ and $f(\Sigma) \cap g(\Sigma) \subseteq N \text{ int}(N \text{ cl}(\lambda_1)) \cap N \text{ int}(N \text{ cl}(\lambda_2)) = \varnothing$. Thus, we get $\Sigma \cup \lambda = \varnothing$ and hence $x \in N \delta gp - \text{cl}(E)$ then $\lambda = N \delta gp - \text{cl}(\lambda)$. Since $\Omega$ is (N$\delta gp$ -add-S), $\lambda$ is (N$\delta gp$ -C) in $\Omega$.

Definition 3.22. A space $\mu$ is called neutrosophic $\delta gp$ -compact (N$\delta gp$ -Com) if any cover of $\mu$ by $\delta gp$ -open sets has a finite subcover.

Definition 3.23. Let $\lambda$ be (NS) in (NTS) $\Omega$. We say $\lambda$ is neutrosophic $\delta gp$ -compact relative (N$\delta gp$ -Com-R) to $\Omega$ if any cover of $\lambda$ by (N$\delta gp$ -O) sets of $\Omega$ has a finite subcover.

Theorem 3.24. If $f: \mu \rightarrow \eta$ is (NA $\delta gp$ -CO) and $\lambda$ is (N$\delta gp$ -Com-R) to $\mu$, then $f(\lambda)$ is (NN-CI-R) to $\eta$.

Proof. Let $\{A_\alpha: \alpha \in \Omega\}$ be any cover of $f(\lambda)$ by (NR-O) sets of $\eta$. Then $\{f^{-1}(A_\alpha): \alpha \in \Omega\}$ is a cover of $\lambda$ by (N$\delta gp$ -O) sets of $\mu$. Hence there exists a finite subset $\Omega_0$ of $\Omega$ such that $\lambda \subseteq \bigcup \{f^{-1}(A_\alpha): \alpha \in \Omega_0\}$. Therefore, we obtain $f(\lambda) \subseteq \{A_\alpha: \alpha \in \Omega_0\}$. This shows that $f(\lambda)$ is (NN-CI-R) to $\eta$.

Corollary 3.25. If $f: \Omega \rightarrow \mu$ is (NA $\delta gp$ -CO) surjection and $\Omega$ is (N$\delta gp$ -Com) and (N$\delta gp$ -add-S), then $\mu$ is (N-NCom-S).

Lemma 3.26. Let $\mu$ be (N$\delta gp$ -Com). If $\lambda \subseteq \mu$ is (N$\delta gp$ -C), then $\lambda$ is (N$\delta gp$ -Com-R) to $\mu$. 
Proof. Let \( \{ \beta_a: \alpha \in \Omega \} \) be a cover of \( \mathcal{N} \) by (N\( \delta \)gp -O) sets of \( \mu \). Note that (\( \mu - \mathcal{N} \)) is (N\( \delta \)gp -O) and that the (NS) (\( \mu - \mathcal{N} \)) \( \cup \) \( \{ \beta_a: \alpha \in \Omega \} \) is a cover of \( \mu \) by (N\( \delta \)gp -O) sets. Since \( \mu \) is (N\( \delta \)gp -Com), there exists a finite \( \Omega_0 \) subset of \( \Omega \) such that the (NS) (\( \mu - \mathcal{N} \)) \( \cup \) \( \{ \beta_a: \alpha \in \Omega_0 \} \) is a cover of \( \mu \) by (N\( \delta \)gp -O) sets in \( \mu \). Hence \( \{ \beta_a: \alpha \in \Omega_0 \} \) is a finite cover of \( \mathcal{N} \) by (N\( \delta \)gp -O) sets in \( \mu \).

Theorem 3.27 If the graph function \( g: \Omega \rightarrow \Omega \times \mu \) of \( f: \Omega \rightarrow \mu \), defined by \( g(\sigma) = (\sigma, f(\sigma)) \) for each \( \sigma \in \Omega \) is (NA\( \delta \)gp -CO) Then \( f \) is (NA\( \delta \)gp -CO).

Proof. Let \( \lambda \in NRO(\mu) \), then \( \Omega \times \mu \in NRO(\Omega \times \mu) \). As \( g \) is (NA \( \delta \)gp -CO), \( f^{-1}(\lambda) = g^{-1}(\Omega \times \lambda) \in N\delta GPO(\Omega) \).

Theorem 3.28. Let \( \Omega, \eta \) be (NTSs) and \( g: \Omega \rightarrow \Omega \times \eta \) be graph neutrosophic function of \( f: \Omega \rightarrow \eta \), defined by \( g(\sigma) = (\sigma, f(\sigma)) \) for each \( \sigma \in \Omega \). If \( \Omega \) is a (N-submax-S) and (N\( \delta \)gp -add-S), then \( g \) is (NA\( \delta \)gp -CO) if and only if \( f \) is (NA\( \delta \)gp -CO).

Proof. We only prove the sufficiency. Let \( \sigma \in \Omega \) and \( W \in RO(\Omega \times \eta) \). Then there exist (NR-O) sets \( \lambda_1 \) and \( V \) in \( \Omega \) and \( \eta \), respectively such that \( \lambda_1 \times V \subseteq W \). If \( f \) is (NA\( \delta \)gp -CO), so there exists a (N\( \delta \)gp -O) set \( \lambda_2 \) in \( \Omega \) satisfies \( \sigma \in \lambda_2 \) and \( f(\lambda_2) \subseteq V \). Put \( \lambda = (\lambda_1 \cap \lambda_2) \). Then \( \lambda \) is (N\( \delta \)gp -O) and \( g(\lambda) \subseteq \lambda_1 \times V \subseteq W \). Thus \( g \) is (NA\( \delta \)gp -CO).

Definition 3.29. A graph \( G_f = \{ (\Omega, F(\sigma)): \sigma \in \Omega \} \subseteq \Omega \times \eta \) of a neutrosophic function \( f: \Omega \rightarrow \mu \) is said to be neutrosophic strongly \( \delta \)gp -closed (N-Str-\( \delta \)gp -C) if for each \( (\rho, \theta) \notin G_f \), there exist \( \lambda \in N\delta GPO(\Omega, \rho) \) and \( V \in NRO(\mu, \theta) \) satisfy \( (\lambda \times V) \cap G_f = \emptyset \).

Lemma 3.30. For a graph \( G_f \) of a neutrosophic function \( f: \Omega \rightarrow \mu \) the following properties are equivalent:

(i) \( G_f \) is (N-Str-\( \delta \)gp -C) in \( \Omega \times \mu \);
(ii) For each \((\rho, \theta) \in C_f\), there exist \(\lambda \in \mathbb{N}\delta\mathbb{GPO}(\Omega, \rho)\) and \(V \in \mathbb{N}\mathbb{R}\mathbb{O}(\mu, \theta)\) such that \(f(\lambda) \cap V = \emptyset\).

**Theorem 3.31.** Let \(f: \Omega \rightarrow \mu\) have a \((\text{N-Str}, \delta \mathbb{g} \mathbb{p}, \text{C})\) graph \(G_f\). If \(f\) is injective, then \(\Omega\) is \((\mathbb{N}\delta \mathbb{g} \mathbb{p} - T_1)\).

**Proof.** Let \(\sigma_1, \sigma_2 \in \Omega\) with \(\sigma_1 \neq \sigma_2\). Then \(f(\sigma_1) \neq f(\sigma_2)\) as \(f\) is injective so that \((\sigma_1, f(\sigma_2)) \notin G_f\). Thus there exist \(\lambda_1 \in \mathbb{N}\delta\mathbb{GPO}(\Omega, \sigma_1)\) and \(\lambda_2 \in \mathbb{N}\mathbb{R}\mathbb{O}(\mu, f(\sigma_2))\) such that \(f(\lambda_1) \cap \lambda_2 = \emptyset\). Then \(f(\sigma_2) \notin f(\lambda_1)\) implies \(\sigma_2 \notin \lambda_1\) and it follows that \(\Omega\) is \((\mathbb{N}\delta \mathbb{g} \mathbb{p} - T_1)\).

**Theorem 3.32.**

(i) If \(f: \Omega \rightarrow \mu\) is \((\mathbb{N}\delta \mathbb{g} \mathbb{p}, \text{CO})\) and \(g: \mu \rightarrow \eta\) is \((\mathbb{N}\text{R}, \text{M})\), then \(g \circ f: \Omega \rightarrow \eta\) is \((\mathbb{N}\delta \mathbb{g} \mathbb{p}, \text{CO})\).

(ii) If \(f: \Omega \rightarrow \mu\) is \((\mathbb{N}\delta \mathbb{g} \mathbb{p}, \text{CO})\) and \(g: \mu \rightarrow \eta\) is \((\mathbb{N}\text{CO})\), then \(g \circ f: \Omega \rightarrow \eta\) is \((\mathbb{N}\delta \mathbb{g} \mathbb{p}, \text{CO})\).

(iii) If \(f: \Omega \rightarrow \mu\) is \((\mathbb{N}\delta \mathbb{g} \mathbb{p}, \text{IR})\) and \(g: \mu \rightarrow \eta\) is \((\mathbb{N}\delta \mathbb{g} \mathbb{p}, \text{CO})\), then \(g \circ f: \Omega \rightarrow \eta\) is \((\mathbb{N}\delta \mathbb{g} \mathbb{p}, \text{CO})\).

**Proof.** (i) Let \(\lambda \in \mathbb{N}\mathbb{R}\mathbb{O}(\eta)\). Then \(g^{-1}(\lambda) \in \mathbb{N}\mathbb{R}\mathbb{O}(\mu)\) since \(g\) is \((\mathbb{N}\text{R}, \text{M})\). The \((\mathbb{N}\delta \mathbb{g} \mathbb{p}, \text{CO})\) of \(f\) implies \(f^{-1}[g^{-1}(\lambda)] = (g \circ f)^{-1}(\lambda)\) \(\in \mathbb{N}\delta\mathbb{GPO}(\Omega)\). Hence \(g \circ f\) is \((\mathbb{N}\delta \mathbb{g} \mathbb{p}, \text{CO})\).

The proofs of (ii) and (iii) are similar to (i).

**Theorem 3.33.** If \(f: \Omega \rightarrow \mu\) is a pre \(\delta \mathbb{g} \mathbb{p}\) -open surjection and \(g: \mu \rightarrow \eta\) is a function such that \(g \circ f: \mu \rightarrow \eta\) is \((\mathbb{N}\delta \mathbb{g} \mathbb{p}, \text{CO})\), then \(g\) is \((\mathbb{N}\delta \mathbb{g} \mathbb{p}, \text{CO})\).

**Proof.** Let \(\theta \in \eta\) and \(\sigma \in \Omega\) such that \(f(\sigma) = \theta\). Let \(G \in \mathbb{R}\mathbb{O}(\eta, (g \circ f)(\sigma))\). Then there exists \(U \in \delta\mathbb{GPO}(\Omega, \sigma)\) such that such that \(g(f(U)) \subseteq G\). Since \(f\) is pre \(\delta \mathbb{g} \mathbb{p}\) -open in \(\mu\), we have that \(g\) is \((\mathbb{N}\delta \mathbb{g} \mathbb{p}, \text{CO})\) at \(\mu\).
Conclusion

In this paper, some new notions of neurosophic delta generated pre-continuous functions in neutrosophic topological spaces are given and discussed, which is a very interesting topic in nature. It will open up many avenues for the researchers work neutrosophic topological spaces, we can in future work extend and study these our notions for this paper in soft setting form.

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