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Vulnerability Parameters in Neutrosophic Graphs

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Abstract: Let $G = (U, V)$ be a Single valued Neutrosophic graph. A subset $S \in U(G)$ is a said to be score equitable set if the score value of any two nodes in $S$ differ by at most one. That is, $|s(u) - s(v)| \leq 1, u, v \in S$. If $e$ is an edge with end vertices $u$ and $v$ and score of $u$ is greater than or equal to score of $v$ then we say $u$ strongly dominates $v$. If every vertex of $V - S$ is strongly influenced by some vertex of $S$ then $S$ is called strong score set of $G$. The minimum cardinality of a strong dominating set is called the strong score number of $G$. The equitable integrity of Single valued Neutrosophic graph $G$ which is defined as $EI(G) = \min\{|S| + m(G - S)\}$, where $m(G - S)$ denotes the order of the largest component in $G - S$. The strong integrity of Single valued Neutrosophic graph $G$ which is defined as $SI(G) = \min\{|S| + m(G - S)\}$: $S$ is a strong score set in $G$. In this paper, we study the concepts of equitable integrity and strong equitable integrity in different classes of regular Neutrosophic graphs and discussed the upper and lower bounds.

Keywords: Score equitable sets, Strong Score Equitable Sets, Equitable integrity, Strong Equitable integrity

1. Introduction

Real-life problems in any communication network, social network, supply chain network and brain network analysis can be modelled as a graph. The objects and the relations between objects are represented by the vertices and edges of the graph. In many real life problems, loss of information, a lack of evidence, imperfect statistical data and insufficient information can be converted by using classical set theory, which was presented by Cantor. Any vertex or edge in the classical graphs is having two possibilities, is either in the graph or it is not in the graph. Therefore, uncertain optimization problems cannot be modelled as a classical graph. An extended version of the classical sets is the fuzzy sets, where the objects have varying membership degrees. It gives different membership degrees between zero and one to its objects. The membership describes membership in vaguely-defined sets but not the same as probability. Zadeh [1] introduced the degree of membership/truth (T) in 1965 and defined the fuzzy set. The concept of fuzziness in graph theory was described by Kaufmann [2] using the fuzzy relation. Rosenfeld [3] introduced some
concepts such as bridges, cycles, paths, trees, and the connectedness of the fuzzy graph and described some of the properties of the fuzzy graph. Samanta and Pal [4] and Rashmanlou and Pal [5] presented the concept of the irregular and regular fuzzy graph. They also described some applications of those graphs.

Intuitionistic fuzzy sets (IFS) considers not only the membership grade (degree), but also independent membership grade and non-membership grade for any entity, and the only requirement is that the sum of non-membership and membership degree values be no greater than one. The idea of the intuitionistic fuzzy set (IFS) as a modified version of the classical fuzzy set was introduced by Atanassov [6–8]. The idea of the IFS relation and the intuitionistic fuzzy graphs (IFG) and discussed many theorems, proofs, and properties were presented by Shannon and Atanassov [9]. Parvathi et al. [10–12] presented many different operations such as the join, union, and product of two IFGs. Some products such as strong, direct, and lexicographic products for two IFGs were presented by Rashmanlou et. al. [13]. In real-world problems, uncertainties due to inconsistent and indeterminate information about a problem cannot be represented properly by the fuzzy graph or IFG. To overcome this situation, a new concept introduced which is called the neutrosophic sets.

Smarandache [15] introduced the degree of indeterminacy/neutrality (I) as independent component in 1995 and defined the neutrosophic set on three components \((T, I, F)\) = (Truth, Indeterminacy, Falsity). Neutrosophic sets are identified by three functions called truth-membership (T), indeterminacy-membership (I) and falsity-membership (F) whose values are real standard or non-standard subset of unit interval \([-0, 1+\] such that \(-0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+ \) for all \(x \in X\).

Section 2 briefly introduces the concepts and operations of NSs, SVNSs, and INSs. In Section 3, define a new set of vulnerability parameters based score functions and discussed some basic bounds. Then in Section 4, two examples are presented to illustrate the proposed parameters and its applications. Finally, Section 6 concludes the paper.

2. Preliminaries

In this section, we provide the basic concepts and definitions in neutrosophic sets and graphs and different types of neutrosophic sets and graphs. In 1999, Smarandache, F. introduced the following definition for Neutrosophic sets (NS)

2.1. Definition [14]

A Neutrosophic set \(A\) in \(X\) is defined by its “truth membership function” \((T_A)\), an “indeterminacy-membership function” \((I_A)\), and a “falsity membership function” \((F_A)\) where all are the subset of \([-0, 1]\) such that \(0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+ \) for all \(x \in X\).
2.2. Definition [40]

A NS $A$ in $X$ is defined as $A = \{ < x, T_A(x), l_A(x), F_A(x) > | x \in X \}$, and is called as SNS where $T_A(x), l_A(x), F_A(x) \in [0, 1]$. SNS is also denoted by $A = \{ < T_A(x), l_A(x), F_A(x) > | x \in X \}$ or $A = < a, b, c >$.

2.3. Definition [41]

An INS $A$ in $X$ is defined as

$A = \{ < [\inf T_A(x), \sup T_A(x)], [\inf l_A(x), \sup l_A(x)], [\inf F_A(x), \sup F_A(x)] > | x \in X \}$,

Where $T_A(x), l_A(x), F_A(x) \in [0, 1]$ and $0 \leq \inf T_A(x) + \sup l_A(x) + \sup F_A(x) \leq 3, x \in X$. An INS is also denoted by $A = < [a^L, a^U], [b^L, b^U], [c^L, c^U] >$.

3. Rank & Score Functions

Ranking of uncertainty numbers is an important issue in fuzzy set theory. Then numerical values are represented in uncertain nature termed as fuzzy numbers, a comparison of these numerical values is not easy. There are various methods have been introduced in literature to rank fuzzy numbers. An intuitionistic fuzzy number (IFN) is a generalization of fuzzy numbers. Many ranking methods for ordering of IFNs have been introduced in the literature. IFNs are treated as two families of metrics and developed a ranking method for IFNs by Grzegorzewski [42,43]. A ranking method to order triangular intuitionistic fuzzy numbers (TIFNs) proposed Mitchell [46] by accepting a statistical viewpoint and interpreting each IFN as ensemble of ordinary fuzzy numbers. Ranking of TIFN on the basis of value index to ambiguity index is proposed by Li [45] and solved a multi attribute decision-making problem.

A ranking function based on score function was proposed and the same used to solve intuitionistic fuzzy linear programming (IFLP), in which the data parameters are TIFNs. In the past, Nayagam et al. [47] introduced TIFNs and proposed a method to rank them. He has also [48] defined new intuitionistic fuzzy scoring method for the intuitionistic fuzzy number. Wang et al. in [49] proposed Intuitionistic trapezoidal fuzzy weighted arithmetic averaging operator and weighted geometric averaging operator.

The expected values, score function, and the accuracy function of intuitionistic trapezoidal fuzzy numbers are also defined. By comparing the score function and the accuracy function values of integrated fuzzy numbers, a ranking of the whole alternative set was attained. A ranking technique for TIFN using $a,b$-cut, score function and accuracy function was introduced by Nagoorgani et al. [51], is validated by applying the concept to solve the intuitionistic fuzzy variable linear programming problem. K. Arun Prakash et al. [52] introduced the method of ranking trapezoidal intuitionistic fuzzy numbers with centroid index uses the geometric center of a trapezoidal intuitionistic fuzzy number.

Decision making problems are one of the most widely used tools in any real time problems. In this process, several steps involve reaching the final destination and some of them may be vague in nature. The decision makers are facing several difficulties to make a decision within a reasonable time by using uncertain, imprecise, and vague information.

Researchers give more attention to the fuzzy set (FS) theory and corresponding extensions such as intuitionistic fuzzy set (IFS) theory, interval-valued IFS (IVIFS), Neutrosophic set (NS), etc.
for handling these situations. IFSs and IVIFSs have been widely applied by the various researchers in different decision-making problems. An aggregation operator for handling the different preferences of the decision makers towards the alternatives under IFS environment proposed by some of the authors proposed. Garg [26-30] presented a generalized score function for ranking the IVIFSs. Garg presented some series of geometric aggregation operator under an intuitionistic multiplicative set environment. He also presented [33] an accuracy function for interval-valued Pythagorean fuzzy sets. Garg studied a novel correlation coefficient between the Pythagorean fuzzy sets.

3.1. Definition [36]
Consider $SNS_A = \langle a, b, c \rangle$ then in order to rank the NS, score functions [35] have been defined as

$$K(A) = \frac{1 + a - 2b - c}{2}; K(A) \in [0,1]$$

$$I(A) = a - 2b - c; I(A) \in [-3,1].$$

These score functions $I(A)$ and $K(A)$ are unable to give the best alternative under some special cases. So, a new score function for ranking NS and INS by overcoming the shortcoming of the above functions has been proposed by Nancy & Harish Garg [36].

3.2. Definition [36]
Let $A = \langle a, b, c \rangle$ be a SNS, a score function $N(\cdot)$, based on the “truth-membership degree” $(a)$, “indeterminacy-membership degree” $(b)$, and “falsity-membership degree” $(c)$ which is defined as

$$N(A) = \frac{1 + (a - 2b - c)(2 - a - c)}{2}$$

Clearly, if in some cases SNS has $a + c = 1$ then $N(A)$ reduces to $K(A)$. Based on it, a prioritized comparison method for any two SNSs $A_1$ and $A_2$ is defined as

(i) if $K(A_1) < K(A_2)$ then $A_1 \prec A_2$,

(ii) if $K(A_1) = K(A_2)$ then

- if $N(A_1) < N(A_2)$ then $A_1 \prec A_2$
- if $N(A_1) > N(A_2)$ then $A_1 \succ A_2$
- if $N(A_1) = N(A_2)$ then $A_1 \sim A_2$

3.3. Definition [36]
Let $G = (U, V)$ be a SVNG, where $U$ is a single-valued neutrosophic vertex set of $G$ and $V$ is called single-valued neutrosophic edge set of $G$, such that $U = \{T_U(x), I_U(x), F_U(x) : x \in X\}$ is a SVN. The score function of SVNG is computed using the value of truth membership $T_U(x)$, indeterminacy membership $I_U(x)$ and falsity membership $F_U(x)$ and is defined by

$$S(u) = \frac{1 + pq}{2} \quad \ldots \ldots \ (1)$$

Where $p = T_U(x) - 2I_U(x) - F_U(x)$ and $q = 2 - T_U(x) - F_U(x)$

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3.4. Observations
Case 1: if \( B = (1,0,0) \) then \( S(B) = 1 \)
Case 2: if \( B = (0,0,1) \) then \( S(B) = 0 \)
Case 3: if \( B = (0,1,0) \) then \( S(B) = -1.5 \)
Case 4: if \( B = (1,1,0) \) then \( S(B) = 0 \)
Case 5: if \( B = (0,1,1) \) then \( S(B) = -1 \)
Case 6: if \( B = (1,0,1) \) then \( S(B) = 0.5 \)
Case 7: if \( B = (0,0,0) \) then \( S(B) = 0.5 \)
Case 8: if \( B = (1,1,1) \) then \( S(B) = 0.5 \)
Therefore the bounds are sharp \( -1.5 \leq S(B) \leq 1 \)

3.5. Definition [19,40]
A single-valued neutrosophic (SVNG) graph on a nonempty set \( X \) is a pair \( G = (U,V) \), where \( U \) is single-valued neutrosophic set in \( X \) and \( V \) is single-valued Neutrosophic relation on \( X \) such that
\[
T_U(x,y) \leq \min\{T_U(x),T_U(y)\},
\]
\[
I_U(x,y) \leq \min\{I_U(x),I_U(y)\},
\]
\[
F_U(x,y) \leq \max\{F_U(x),F_U(y)\},
\]
For all \( x, y \in X \). \( U \) is said to be single-valued neutrosophic vertex set of \( G \) and \( V \) is called single-valued neutrosophic edge set of \( G \), respectively.

3.6. Definition
The order and the size of a SVNG \( G \) are denoted by \( O(G) \) and \( S(G) \), respectively and are defined by
\[
O(G) = \left( \sum_{x \in X} T_U(x), \sum_{x \in X} I_U(x), \sum_{x \in X} F_U(x) \right),
\]
\[
S(G) = \left( \sum_{x \in X} T_U(x,y), \sum_{x \in X} I_U(x,y), \sum_{x \in X} F_U(x,y) \right).
\]

3.7. Definition
The degree and the total degree of a vertex \( x \) of a SVNG \( G \) are defined by
\[
d_G(x) = \left( \sum_{x \neq y} T_U(x,y), \sum_{x \neq y} I_U(x,y), \sum_{x \neq y} F_U(x,y) \right),
\]
and
\[
Td_G(x) = \left( \sum_{x \neq y} T_U(x,y) + T_U(x), \sum_{x \neq y} I_U(x,y) + I_U(x), \sum_{x \neq y} F_U(x,y) + F_U(x) \right).
\]
For \( xy \in V \) and \( x \in X \), is denoted by \( d_G(x) = (d_T(x),d_I(x),d_F(x)) \) and \( Td_G(x) = (Td_T(x),Td_I(x),Td_F(x)) \), respectively.

3.8. Definition
The maximum degree of a SVNG \( G \) is defined as \( \Delta(G) = (\Delta_T(G), \Delta_I(G), \Delta_F(G)) \), where
\[
\Delta_T(G) = \max\{d_T(x) : x \in X\}
\]
\[
\Delta_I(G) = \max\{d_I(x) : x \in X\}
\]
\[
\Delta_F(G) = \max\{d_F(x) : x \in X\}
\]

3.9. Definition

The minimum degree of a SVNG \( G \) is defined as \( \delta(G) = (\delta_T(G), \delta_I(G), \delta_F(G)) \), where
\[
\delta_T(G) = \min\{d_T(x) : x \in X\}
\]
\[
\delta_I(G) = \min\{d_I(x) : x \in X\}
\]
\[
\delta_F(G) = \min\{d_F(x) : x \in X\}
\]

3.10. Definition

A SVNG \( G \) is called a regular if each vertex has same degree, (i.e.)
\[
d_G(x) = (m_1, m_2, m_3), \quad \text{for all} \ x \in X
\]

Example:

![Fig. 1. Regular SVNG](image)

<table>
<thead>
<tr>
<th>Vertices</th>
<th>T</th>
<th>I</th>
<th>F</th>
<th>p</th>
<th>q</th>
<th>S(u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0.3</td>
<td>0.8</td>
<td>0.4</td>
<td>-1.7</td>
<td>1.3</td>
<td>-0.605</td>
</tr>
<tr>
<td>S2</td>
<td>0.1</td>
<td>0.8</td>
<td>0.9</td>
<td>-2.4</td>
<td>1</td>
<td>-0.7</td>
</tr>
<tr>
<td>S3</td>
<td>0.3</td>
<td>0.6</td>
<td>0.4</td>
<td>-1.3</td>
<td>1.3</td>
<td>-0.345</td>
</tr>
<tr>
<td>S4</td>
<td>0.1</td>
<td>0.9</td>
<td>0.5</td>
<td>-2.2</td>
<td>1.4</td>
<td>-1.04</td>
</tr>
</tbody>
</table>

Table 1: Score value of regular SVNG

3.11. Definition

Let \( G = (U, V) \) be an SVNG. \( G \) is said to be a strong SVNG if:
\[
T_V(x, y) = \min (T_U(x), T_U(y))
\]
\[
I_V(x, y) = \min (I_U(x), I_U(y))
\]
\[
F_V(x, y) = \max (F_U(x), F_U(y)), \forall (x, y) \in E
\]

Example:
3.12. Definition

A SVNG $G = (U, V)$ is called complete if the following conditions are satisfied:

$$
T_V(x, y) = \min\{T_U(x), T_U(y)\} \\
I_V(x, y) = \min\{I_U(x), I_U(y)\} \\
F_V(x, y) = \max\{F_U(x), F_U(y)\}, \forall (x, y) \in E
$$

Example:
A SVNG \( G = (U, V) \) is called complete bipartite neutrosophic graph if the vertex set \( V \) can be divided into two nonempty sets, such that for every \( v_1, v_2 \in V_1 \) or \( V_2 \) and for every \( u \in V_1 \) and \( v \in V_2 \).

\[ \text{Fig. 4. Complete bipartite SVNG} \]

<table>
<thead>
<tr>
<th>Vertices</th>
<th>T</th>
<th>I</th>
<th>F</th>
<th>p</th>
<th>q</th>
<th>S(u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>0.2</td>
<td>0.3</td>
<td>0.6</td>
<td>-1</td>
<td>1.2</td>
<td>-0.1</td>
</tr>
<tr>
<td>b</td>
<td>0.2</td>
<td>0.3</td>
<td>0.6</td>
<td>-1</td>
<td>1.2</td>
<td>-0.1</td>
</tr>
<tr>
<td>c</td>
<td>0.2</td>
<td>0.3</td>
<td>0.6</td>
<td>-1</td>
<td>1.2</td>
<td>-0.1</td>
</tr>
<tr>
<td>d</td>
<td>0.2</td>
<td>0.3</td>
<td>0.6</td>
<td>-1</td>
<td>1.2</td>
<td>-0.1</td>
</tr>
<tr>
<td>e</td>
<td>0.2</td>
<td>0.3</td>
<td>0.6</td>
<td>-1</td>
<td>1.2</td>
<td>-0.1</td>
</tr>
</tbody>
</table>

Table: 4 Score value of complete bipartite SVNG

4. **Score Equitable Integrity and Strong Score Equitable Integrity of SVNG**

4.1 **Definition**

Let \( G = (U, V) \) be a Single valued Neutrosophic graph. A subset \( S \in U(G) \) is said to be score equitable set if the score value of any two nodes in \( S \) differ by at most one. (i.e.) \(|s(u)-s(v)| \leq 1, u, v \in S\). If \( e \) is an edge with end vertices \( u \) and \( v \) and score of \( u \) is greater than or equal to score of \( v \) then we say \( u \) strongly dominates \( v \). If every vertex of \( V - S \) is strongly influenced by some vertex of \( S \) then \( S \) is called strong score set of \( G \). The minimum cardinality of a strong dominating set is called the strong score number of \( G \).

4.2 **Definition**

The equitable integrity of Single valued Neutrosophic graph \( G \) which is defined as \( EI(G) = \min(|S| + m(G - S)) : S \) is a score equitable set in \( G \), where \( m(G - S) \) denotes the order of the largest component in \( G - S \).

4.3 **Definition**

The strong integrity of Single valued Neutrosophic graph \( G \) which is defined as \( SI(G) = \min(|S| + m(G - S)) : S \) is a strong score set in \( G \), where \( m(G - S) \) denotes the order of the largest component in \( G - S \).

4.4 **Example**

Consider the SVNG in Figure 5.
Using Eq. 1 we can compute score value of all the nodes. Figure 1.(b) shows the score value of each node. The score equitable sets are $S_1 = \{u_1, u_3\}, S_2 = \{u_1, u_4\}, S_3 = \{u_2, u_5\}, S_4 = \{u_3, u_4\}, S_5 = \{u_1, u_3, u_4\}$ and score equitable integrity is calculated by $EI(G) = \min(\{[2 + 3 = 5], [2 + 3 = 5], [2 + 2 = 4], [2 + 3 = 5], [3 + 2 = 5]\}) = 4$. From this the score equitable integrity value is 4 and corresponding set is $S_3 = \{u_2, u_5\}$. The strong score equitable set is $S_3 = \{u_2, u_5\}$ and also strong equitable integrity is 4.

4.5 Theorem:
Let $G$ be SVNG then
(i) $EI(G) = n$ if and only if $G \cong K_n$
(ii) $SI(G) = n$ if and only if $G \cong K_n$

Proof: --------------
Proposition: Every score equitable integrity and strong score equitable integrity of complete SVNG is equal to score equitable integrity and strong score equitable integrity of regular SVNG.

5. Case Study
5.1 Detection of a Safe Root for an Airline Journey
We consider a neutrosophic set of five countries: Germany, China, USA, Brazil and Mexico. Suppose we want to travel between these countries through an airline journey. The airline companies aim to facilitate their passengers with high quality of services. Air traffic controllers have to make sure that company planes must arrive and depart at right time. This task is possible by planning efficient routes for the planes. A neutrosophic graph of airline network among these five countries is shown in Fig.6 in which vertices and edges represent the countries and flights, respectively.
Table: 5 Score value of Airline Network

The truth-membership degree of each vertex indicates the strength of that country’s airline system. The indeterminacy-membership degree of each vertex demonstrates how much the system is uncertain. The falsity-membership degree of each vertex tells the flaws of that system. The truth-membership degree of each edge interprets that how much the flight is safe. The indeterminacy-membership degree of each edge shows the uncertain situations during a flight such as weather conditions, mechanical error and sabotage. The falsity-membership degree of each edge indicates the flaws of that flight. For example, the edge between Germany and China indicates that the flight chosen for this travel is 80% safe, 10% depending on uncertain systems and 20% unsafe.

The truth-membership degree, the indeterminacy-membership degree and the falsity-membership degree of each edge are calculated by using the following relations.

\[
T_v(x,y) \leq \min(T_u(x), T_u(y)), \\
I_v(x,y) \leq \min(I_u(x), I_u(y)), \\
F_v(x,y) \leq \max(F_u(x), F_u(y)),
\]

Sometimes due to weather conditions, technical issues a passenger missed his direct flight between two particular countries. So, if he has to go somewhere urgently, then he has to choose indirect route as there are indirect routes between these countries.

Using Eq. 1 we can compute score value of all the nodes. Table 5. shows the score value of each node. We observe that all the sets are score and strong score equitable sets, and by computation the equitable integrity is, \( EI(G) = \min(|S| + m(G - S)) = 4 \), where \( S = \{\text{China, USA}\} \) The strong score equitable set is \( S = \{\text{China, USA}\} \) and strong score equitable integrity is \( SI(G) = \min(|S| + m(G - S)) = 4 \).

6. Conclusion

In this paper, Score Equitable Integrity and Strong Score Equitable Integrity of SVNG is introduced as a new vulnerability parameter in Neutrosophic graphs and some fundamental results in some standard graphs are established. Also the application on airline systems related to EI and SI parameters are dealt with real time scenario pertaining to the safety measures of flights connecting...
any two countries. We will focus on the study of EI and SI regular strong SVNG, $d_m$ regular SVNG, $td_m$ regular SVNG, soft graphs and so on.

7. References:
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