[Neutrosophic Sets and Systems](https://digitalrepository.unm.edu/nss_journal)

Volume 48 Article 4

2-15-2022

Neutrosophic Nano RW-Closed Sets in Neutrosophic Nano Topological Spaces

S. Jafari

C. Janaki

D. Savithiri

Neutrosophic Sets and Systems, Vol. 48, 2022
University of New Mexico

Neutrosophic Nano RW-Closed Sets in Neutrosophic Nano Topological Spaces

S.Jafari[1], C.Janaki[2], D. Savithiri[3]*

1. College of Vestsjaelland South, Herrestraede 11, 4200 Slagelse, Denmark. Email: jafaripersia@gmail.com.

2. Department of Mathematics, L.R.G. Govt. Arts College for women, Tirupur, TN, INDIA. Email: janakicsekar@gmail.com

*3. Assistant Professor, Sree Narayana Guru College, Coimbatore, TN, INDIA. Email: savithirisngcmat@gmail.com

Corresponding author email: savithirisngcmat@gmail.com

Abstract: The main objective of this study is to introduce a new class of closed sets namely Neutrosophic Nano RW-closed sets and Neutrosophic Nano RW-continuous functions in Neutrosophic Nano topological spaces. Some of its properties and interrelationship with some existing Neutrosophic nano closed sets have been discussed.

Keywords: N_NRW-closed set, N_NRW-open set, N_NRW$T_{1/2}$ space, N_NRW-connected space, N_NRW-continuous, N_NRW-irresolute, N_NRW-open and N_{N-} closed maps.

1. Introduction

The theory of neutrosophic sets with three components namely, membership T (Truth), Indeterminacy I, and non-membership F (Falsehood), one of the interesting generalizations of theory of fuzzy sets and Intuitionistic fuzzy sets introduced by F.Smarandache [8]. In 2012, A.A. Salama and S.A. Alblowi [13] introduced and studied the theory of neutrosophic topological spaces. Since then several mathematicians contributed many papers to this area. Various results in ordinary topological spaces have been put in the neutrosophic setting, and also various departures have been observed. Neutrosophic set is a powerful tool to deal with indeterminate and inconsistent data. The concept of nano topology explored by M. Lellis Thivagar et. al[11] can be described as a collection of nano approximations for which equivalence classes are building blocks. In 2018, M. Lellis Thivagar et. al. [12] introduced a new concept called as Neutrosophic Nano topology and discussed neutrosophic nano interior and neutrosophic nano closure.

In 2007, S.S. Benchalli and R.S. Wali [4] introduced RW-closed sets in topological spaces. The authors D. Savithiri and C. Janaki [15] introduced the concept of Neutrosophic RW-closed sets in Neutrosophic topological spaces. In this article we introduce Neutrosophic Nano RW-closed sets and discuss some of its properties.

2 PRELIMINARIES

The following recalls requisite ideas and preliminaries necessary in the sequel of our work.

Definition 2.1:[9] Let X be a non-empty fixed set a Neutrosophic set **(NS for short)** A is an object having the form A = < x, $\mu_A(x)$,$\sigma_A(x)$,$\gamma_A(x)$> , x∈X where $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$ which represents the degree of membership function, the degree of indeterminacy and the degree of non-membership function respectively of each element x∈X to the set A.

Definition 2.2:[11] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let X ⊆ U.

(i) The lower approximation of X with respect to R is the set of all objects, which can be classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = \bigcup_{x\in U}\{(R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$.

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) – L_R(X)$.

Remark 2.3:[11]

(i) $L_R(X) \subseteq X \subseteq U_R(X)$.

(ii) $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$.

(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$.

(iv) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$.

(v) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$.

(vi) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$.

(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$, whenever $X \subseteq Y$.

(viii) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$.

(ix) $U_RU_R(X) = L_RU_R(X) = U_R(X)$.

(x) $L_RL_R(X) = L_RU_R(X) = L_R(X)$.

Definition 2.4:[11] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. $\tau_R(X)$ satisfies the following axioms:

(i) U and $\phi \in \tau_R(X)$.

(ii) The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on U called the nano topology on U with respect to X. We call (U, $\tau_R(X)$) as the nano topological space. The elements of $\tau_R(X)$ are called nano-open sets.

Definition 2.5:[12] Let U be a non-empty set and R be an equivalence relation on U. Let S be a neutrosophic set in U with the membership function μ_S, the indeterminacy function σ_S, and the non-membership function γ_S. The neutrosophic nano lower, neutrosophic nano upper approximation and neutrosophic nano boundary of S in the approximation (U,R) denoted by $\underline{N}(S), \overline{N}(S) and\ B(S)$ are respectively defined as follows:

(i) $\underline{N}(S) = \{\langle x, \mu_{\underline{R}(A)}(x),\ \sigma_{\underline{R}(A)}(x), \gamma_{\underline{R}(A)}(x)\rangle / y \in [x]_R, x \in U\}$.

(ii) $\overline{N}(S) = \{\langle x, \mu_{\overline{R}(A)}(x), \sigma_{\overline{R}(A)}(x), \gamma_{\overline{R}(A)}(x)\rangle /\ y\ \in [x]_R, x \in U\}$.

(iii) B(S) = $\overline{N}(S) - \underline{N}(S)$.

where $\mu_{\underline{R}(A)}(x) = \ \wedge_{y\in[x]_R} \mu_A(y),\ \sigma_{\overline{R}(A)}(x) = \ \wedge_{y\in[x]_R} \sigma_A(y), \gamma_{\underline{R}(A)}(x) = \ \vee_{y\in[x]_R} \gamma_A(y)$,

$\mu_{\bar{R}(A)}(x) = \vee_{y\in[x]_R} \mu_A(y), \sigma_{\bar{R}(A)}(x) = \vee_{y\in[x]_R} \sigma_A(y), \gamma_{\bar{R}(A)}(x) = \wedge_{y\in[x]_R} \gamma_A(y)$.

Definition 2.6:[12] Let U be an universe, R be an equivalence relation on U and S be a neutrosophic set in U and if the collection $\tau_N(S)$ = {0_N, 1_N, $\underline{N}(S), \overline{N}(S), B(S)$} forms a topology then it is said to be a neutrosophic nano topology. We call (U, $\tau_N(S)$) as the neutrosophic nano topological space **(Briefly NNTS)**. The elements of $\tau_N(S)$ are called as neutrosophic nano open **(In Short N_NO)** sets.

Remark 2.7:[12]$[\tau_N(S)]^C$ is called as dual neutrosophic nano topology of $\tau_N(S)$. The elements of $[\tau_N(S)]^C$ are called neutrosophic nano closed **(In Short N_NC)** sets.

Remark 2.8:[12] In neutrosophic nano topological space, the neutrosophic nano boundary cannot be empty. Since the difference between neutrosophic nano upper and neutrosophic nano lower approximations is defined as the maximum and minimum of the values in the neutrosophic sets.

Proposition 2.9:[12] Let U be a non-empty finite universe and S be a neutrosophic set on U. Then the following statements hold:

(i) The collection $\tau_N(S)$ = {0_N, 1_N}, is the indiscrete neutrosophic nano topology on U.

(ii) If $\underline{N}(S) = \overline{N}(S) = B(S)$, then the neutrosophic nano topology, $\tau_N(S)$ = {0_N, 1_N, $\underline{N}(S), B(S)$}.

(iii) If $\underline{N}(S) = B(S)$, then $\tau_N(S)$ = {0_N, 1_N, $\underline{N}(S), \overline{N}(S)$} is a neutrosophic nano topology.

(iv) If $\overline{N}(S) = B(S)$, then $\tau_N(S)$ = {0_N, 1_N, $\underline{N}(S), B(S)$}.

(v) The collection $\tau_N(S) = \{0_N, 1_N, \underline{N}(S), \overline{N}(S), B(S)\}$ is the discrete neutrosophic nano topology on U

Definition 2.10:[12] Let (U, $\tau_N(S)$) be NNTS and A = < x, $\mu_A(x)$,$\sigma_A(x)$,$\gamma_A(x)$, $x \in$ U> be a NNS in X. Then the neutrosophic nano closure and neutrosophic nano interior of A are defined by

N_NCl (A) = $\cap$ { K : K is a N_NCS in X and A $\subseteq$ K }

N_NInt (A) = $\cup$ { G : G is a N_NOS in X and G $\subseteq$ A }.

Definition 2.11:[12] A subset A of a neutrosophic nano topological space Let (U, $\tau_N(S)$) is said to be

(i) a neutrosophic nano pre closed **($N_N$pre-closed)** set if N_NCl(N_NInt(A)) $\subseteq$ A.

(ii) a neutrosophic nano semi-closed **($N_N$semi-closed)** set if N_NInt(N_NCl(A)) $\subseteq$ A.

(iii) a neutrosophic nano regular open **(In short N_NRO)** set if A =N_NInt(N_NCl(A)) and regular closed **(In short N_NRC)** set if A = N_NCl(N_NInt(A)) .

(iv) a neutrosophic regular semi open **(In short NRSO)** if there exists a NRO set U such that U $\subseteq$ A $\subseteq$ NCl(A)

(v) a neutrosophic nano α-closed **($N_N\alpha$-closed)** set if N_NCl(N_NInt(N_NCl(A))) $\subseteq$ A.

(vi) a neutrosophic nano g-closed **(N_N g-closed)** set if N_NCl(A) $\subseteq$ F whenever A $\subseteq$ F and F is NNO in U.

Definition 2.11:[6] The difference between two neutrosophic nano sets A and B is defined as

A$\setminus$B (S) = {x, min [($\mu_A(x)$, $\gamma_B(x)$)], min [($\sigma_A(x)$,1- $\sigma_B(x)$)], max [$\gamma_A(x)$, $\mu_B(x)$)].

3. NEUTROSOPHIC NANO RW-CLOSED SETS

Definition 3.1: A subset A of a neutrosophic nano topological space (U, $\tau_N(S)$) is called as neutrosophic nano regular weakly closed **(In short N_NRW-closed)** set, if N_NCl(A) $\subseteq$ V whenever A $\subseteq$ V and V is a neutrosophic nano regular open in U.

Definition 3.2: The neutrosophic nano RW-closure and neutrosophic nano RW-interior of A are defined by

N_NRWCl (A) = $\cap$ { K : K is a N_NRWCS in X and A $\subseteq$ K }

N_NRWInt (A) = $\cup$ { G : G is a N_NRWOS in X and G $\subseteq$ A }.

Definition 3.3: (i) neutrosophic nano RG- Closed set **(shortly N_NRG – closed set)** of X if there exists a neutrosophic nano regular open set U such that N_NCl(A) $\subseteq$ U whenever A $\subseteq$ U.

(ii) neutrosophic nano RWG- closed set **(shortly N_NRWG – closed set)** of X if there exists a neutrosophic nano regular open set U such that N_NCl(N_NInt(A)) $\subseteq$ U whenever A $\subseteq$ U.

(iii) neutrosophic nano W-closed set **(shortly NNW – closed set)** of X if there exists a neutrosophic nano semi-open set U such that NNCl(A) ⊆ U whenever A ⊆ U.

(iv) neutrosophic nano g-closed set **(shortly NNG – closed set)** of X if there exists a neutrosophic open set U such that NNCl(A) ⊆ U whenever A ⊆ U.

Proposition 3.3: (i) Every NN-closed set is NNRW-closed.

(ii) Every NN- regular closed set is NNRW-closed.

(iii) Every NN- πclosed set is NNRW-closed.

(iv) Every NNW-closed set is NNRW-closed.

Proof: Follows from [4].

The following example makes clear that the converse of the Proposition 3.3 need not be true.

Example 3.4: Let U = $\{p_1, p_2, p_3\}$ be the universe set and the equivalence relation U\R = $\{\{p_1, p\}, \{p_2\}\}$. Let

$S = \left\{\langle\frac{p_1}{(0.1,0.2,0.3)}\rangle, \langle\frac{p_2}{(0.2,0.3,0.4)}\rangle, \langle\frac{p_3}{(0.1,0.6,0.4)}\rangle\right\}$ be a neutrosophic nano subset of U. Then $\overline{N}(S) = \left\{\langle\frac{p_1,p_3}{(0.1,0.6,0.3)}\rangle, \langle\frac{p_2}{(0.2,0.3,0.4)}\rangle\right\}$, $\underline{N}(S) = \left\{\langle\frac{p_1,p_3}{(0.1,0.2,0.4)}\rangle, \langle\frac{p_2}{(0.2,0.3,0.4)}\rangle\right\}$ and B $(S) = \left\{\langle\frac{p_1,p_3}{(0.1,0.6,0.3)}\rangle, \langle\frac{p_2}{(0.2,0.3,0.4)}\rangle\right\}$. So the neutrosophic nano topology τN = $\{0_N, 1_N, \underline{N}, B\}$ where the neutrosophic closed sets are τN^C = $\{0_N, 1_N, \underline{N}^C, B^C\}$. Let $Q_1 = \left\{\langle\frac{p_1}{(0.2,0.1,0.3)}\rangle, \langle\frac{p_2}{(0.3,0.1,0.2)}\rangle, \langle\frac{p_3}{(0.1,0.2,0.3)}\rangle\right\}$, then Q_1 is NNRW-closed but it is not an NN-closed set in U. $Q_2 = \left\{\langle\frac{p_1}{(0.2,0.3,0.5)}\rangle, \langle\frac{p_2}{(0.3,0.6,0.5)}\rangle, \langle\frac{p_3}{(0.2,0.3,0.3)}\rangle\right\}$, Q_2 is NNRW-closed but it is neither NN Regular-closed nor NNπ-closed set and $Q_3 = \left\{\langle\frac{p_1}{(0.1,0.3,0.6)}\rangle, \langle\frac{p_2}{(0.2,0.6,0.6)}\rangle, \langle\frac{p_3}{(0.1,0.2,0.6)}\rangle\right\}$, then Q_3 is NNRW-closed but not NNW-closed set.

Proposition 3.5: (i) Every NNRW-closed set is NNRG-closed.

(ii) Every NNRW-closed set is NNGPR-closed.

(iii) Every NNRW-closed set is NNRWG-closed.

Proof: Follows from [4].

The converse of the Proposition 3.4 need not be true.

Example 3.6: * Let U = $\{p_1, p_2, p_3, p_4, p_5\}$ be the universe set and the equivalence relation U\R = $\{\{p_1, p_3\}, \{p_2\}, \{p_4, p_5\}\}$. Let S $= \left\{\langle\frac{p_1}{(0.4,0.3,0.4)}\rangle, \langle\frac{p_2}{(0.5,0.3,0.5)}\rangle, \langle\frac{p_3}{(0.5,0.3,0.5)}\rangle, \langle\frac{p_4}{(0.6,0.3,0.1)}\rangle, \langle\frac{p_5}{(0.5,0.3,0.1)}\rangle\right\}$ be a neutrosophic nano subset of U $\overline{N}(S) = \left\{\langle\frac{p_1,p_3}{(0.5,0.3,0.2)}\rangle, \langle\frac{p_2}{(0.5,0.3,0.5)}\rangle, \langle\frac{p_4,p_5}{(0.6,0.3,0.1)}\rangle\right\}$, $\underline{N}(S) =$

$\left\{\langle\frac{p_1,p_3}{(0.4,0.3,0.4)}\rangle,\langle\frac{p_2}{(0.5,0.3,0.5)}\rangle,\langle\frac{p_4,p_5}{(0.5,0.3,0.1)}\rangle\right\}$ and B (S) = $\left\{\langle\frac{p_1,p_3}{(0.4,0.3,0.4)}\rangle,\langle\frac{p_2}{(0.5,0.3,0.5)}\rangle,\langle\frac{p_4,p_5}{(0.1,0.3,0.6)}\rangle\right\}$. The neutrosophic nano topology τN = $\{0_N, 1_N, \underline{N}, \overline{N}, B\}$. Let R_1 = $\left\{\langle\frac{p_1}{(0.3,0.3,0.7)}\rangle,\langle\frac{p_2}{(0.2,0.3,0.6)}\rangle,\langle\frac{p_3}{(0.2,0.3,0.5)}\rangle,\langle\frac{p_4}{(0.1,0.2,0.7)}\rangle,\langle\frac{p_5}{(0.1,0.3,0.8)}\rangle\right\}$. Then R_1 is both NNGPR-closed and NNRWG – closed but it is not an NNRW-closed.

* In example 3.4, let R_2 = $\left\{\langle\frac{p_1}{(0.3,0.7,0.5)}\rangle,\langle\frac{p_2}{(0.3,0.4,0.6)}\rangle,\langle\frac{p_3}{(0.2,0.5,0.5)}\rangle,\langle\frac{p_4}{(0.1,0.5,0.6)}\rangle,\langle\frac{p_5}{(0.1,0.6,0.7)}\rangle\right\}$, then R_2 is NNRG-closed but not an NNRW-closed.

Proposition 3.7: The finite union of NNRW – closed subsets of U is also an NNRW – closed subset of U.

Proof: Assume that P and Q are NNRW –closed sets in U. Let R be an NNRSO set in X such that P ∪ Q ⊆ R. Then P ⊆ R and Q ⊆ R. Since P and Q are NNRW – closed sets, NNCl(P) ⊆ R and NNCl (Q) ⊆ R. Then NNCl(P∪Q) = NNCl(P) ∪ NNCl(Q) ⊆ R. Hence P ∪ Q is an NNRW – closed set in U.

Remark 3.8: The intersection of two NNRW-closed sets in (U, τN(S)) need not be an NNRW-closed set in U.

Example 3.9: Let U $=\{p_1, p_2, p_3, p_4, p_5\}$ be the universe set and the equivalence relation U\R = $\{\{p_1, p_3\}, \{p_2\}, \{p_4, p_5\}\}$. Let S= $\left\{\langle\frac{p_1}{(0.4,0.3,0.4)}\rangle,\langle\frac{p_2}{(0.5,0.3,0.5)}\rangle,\langle\frac{p_3}{(0.5,0.3,0.5)}\rangle,\langle\frac{p_4}{(0.6,0.3,0.1)}\rangle,\langle\frac{p_5}{(0.5,0.3,0.1)}\rangle\right\}$ be a neutrosophic nano subset of U$\overline{N}(S)$ = $\left\{\langle\frac{p_1,p_3}{(0.5,0.3,0.2)}\rangle,\langle\frac{p_2}{(0.5,0.3,0.5)}\rangle,\langle\frac{p_4,p_5}{(0.6,0.3,0.1)}\rangle\right\}$, $\underline{N}(S)$ = $\left\{\langle\frac{p_1,p_3}{(0.4,0.3,0.4)}\rangle,\langle\frac{p_2}{(0.5,0.3,0.5)}\rangle,\langle\frac{p_4,p_5}{(0.5,0.3,0.1)}\rangle\right\}$ and B(S) = $\left\{\langle\frac{p_1,p_3}{(0.4,0.3,0.4)}\rangle,\langle\frac{p_2}{(0.5,0.3,0.5)}\rangle,\langle\frac{p_4,p_5}{(0.1,0.3,0.6)}\rangle\right\}$. The neutrosophic nano topology τN = $\{0_N, 1_N, \underline{N}, \overline{N}, B\}$. R_1 = $\left\{\langle\frac{p_1}{(0.6,0.3,0.3)}\rangle,\langle\frac{p_2}{(0.5,0.3,0.3)}\rangle,\langle\frac{p_3}{(0.5,0.2,0.3)}\rangle,\langle\frac{p_4}{(0.3,0.3,0.1)}\rangle,\langle\frac{p_5}{(0.4,0.4,0.1)}\rangle\right\}$, R_2 = $\left\{\langle\frac{p_1}{(0.2,0.3,0.5)}\rangle,\langle\frac{p_2}{(0.3,0.5,0.7)}\rangle,\langle\frac{p_3}{(0.2,0.3,0.5)}\rangle,\langle\frac{p_4}{(0.1,0.5,0.6)}\rangle,\langle\frac{p_5}{(0.1,0.7,0.6)}\rangle\right\}$. Then R_1 and R_2 are NNRW-closed sets but $R_1 \cap R_2$ is not an NNRW-closed set.

Proposition 3.10: If a subset A of U is NNRW – closed set in U, then NNCl(A)\A does not contain any non-empty neutrosophic nano regular semi-open set in U.

Proof: Suppose that A is an NNRW –closed set in U. We shall prove by contradiction. Let R be an NNRSO set such that NNCl(A)\A ⊃ R which implies R ⊆ U\A i.e., A ⊆ U\R. Since R is NNRSO, U\R is also NNRSO set in U. Since A is an NNRW – closed set, NNCl(A) ⊆ U\R. So R ⊆ U\NNCl(A) also R ⊆ NNCl(A) implies R = ϕ. Hence NNCl (A)\A does not contain any non-empty NNRSO set in U.

The converse of the Proposition 3.10 need not be true as shown in the following example.

Example 3.11: In example 3.9, in the neutrosophic nano topological space (U, τN(S)), let A = $\left\{\langle\frac{p_1}{(0.3,0.2,0.5)}\rangle,\langle\frac{p_2}{(0.3,0.2,0.6)}\rangle,\langle\frac{p_3}{(0.2,0.3,0.5)}\rangle,\langle\frac{p_4}{(0.1,0.2,0.7)}\rangle,\langle\frac{p_5}{(0.1,0.3,0.8)}\rangle\right\}$, then NNCl(A)\A does not contain any non-empty NNRSO set, but A is not an NNRW-closed set in U.

Corollary 3.12: If a subset A of U is N_NRW – closed set in U, then N_NCl(A)\A does not contain any non-empty neutrosophic nano regular- open set in U.

Proof: Follows from the Proposition 3.10 and the fact that every N_NRO set is N_NRSO in U.

Proposition 3.13: If A is N_NRO and N_NRW-closed, then A is N_NRC set and hence N_N-clopen.

Proof: Suppose A is N_NRO and N_NRW – closed. As every N_NRO set is N_NRSO and A ⊂ A, we have N_NCl(A) ⊂ A. Also A ⊂ N_NCl(A), thus N_NCl(A) = A. Hence A is a N_NC set. Since A is N_NRO it is N_NO set. Now N_NCl(N_NInt(A) = N_NCl(A) = A. Therefore A is N_NRC and Neutrosophic nano clopen.

Proposition 3.14: If A is an N_NRW – closed subset of U such that A ⊆ B ⊆ NCl(A), then B is an N_NRW – closed set in U.

Proof: Let A be an N_NRW – closed set of U such that A ⊆ B ⊆ N_NCl(A). Let R be N_NRSO set of U such that B ⊆ R. Then A ⊆ R. Since A is N_NRW –closed set, we have N_NCl(A) ⊆ R and N_NCl(B) ⊆ N_NCl(N_NCl(A)) ⊆ R. Therefore B is also an N_NRW – closed set in U.

The following example shows that the converse of the Proposition 3.13 need not be true.

Example 3.15: Let U = $\{n_1, n_2, n_3\}$ be the universe set and the equivalence relation U\R = $\{\{n_1, n_3\}, \{n_2\}\}$. Let $S = \left\{\langle\frac{x_1}{(0.1,0.2,0.3)}\rangle, \langle\frac{x_2}{(0.2,0.3,0.4)}\rangle, \langle\frac{x_3}{(0.1,0.6,0.4)}\rangle\right\}$ be a neutrosophic nano subset of U. Then $\overline{N}(S) = \left\{\langle\frac{x_1,x_3}{(0.1,0.6,0.3)}\rangle, \langle\frac{x_2}{(0.2,0.3,0.4)}\rangle\right\}$, $\underline{N}(S) = \left\{\langle\frac{x_1,x_3}{(0.1,0.2,0.4)}\rangle, \langle\frac{x_2}{(0.2,0.3,0.4)}\rangle\right\}$ and $B(S) = \left\{\langle\frac{x_1,x_3}{(0.1,0.6,0.3)}\rangle, \langle\frac{x_2}{(0.2,0.3,0.4)}\rangle\right\}$. So the neutrosophic nano topology $\tau_N = \{0_N, 1_N, \underline{N}, B\}$ and the neutrosophic closed sets are $\tau_N{}^C = \{0_N, 1_N, \underline{N}^C, B^C\}$. Let A = $\left\{\langle\frac{x_1}{(0.1,0.3,0.6)}\rangle, \langle\frac{x_2}{(0.2,0.6,0.6)}\rangle, \langle\frac{x_3}{(0.1,0.2,0.6)}\rangle\right\}$ and B = $\left\{\langle\frac{x_1}{(0.2,0.3,0.5)}\rangle, \langle\frac{x_2}{(0.3,0.6,0.5)}\rangle, \langle\frac{x_3}{(0.2,0.3,0.3)}\rangle\right\}$. Then A and B are N_NRW-closed sets in (U, τ_N(S)), but A ⊂ B is not a subset of N_NCl(A).

Proposition 3.16: Let A be an N_NRW-closed in (U, τ_N(S)). Then A is N_N-closed if and only if N_NCl(A)\A is N_NRSO.

Proof: Let A be an N_N-closed in (U, τ_N(S)). Then N_NCl(A)\A = ϕ which is N_NRSO.

Conversely, suppose N_NCl(A)\A is N_NRSO in U. By hypothesis, A is N_NRW-closed implies N_NCl(A)\A does not contain any non-empty N_NRSO in U. Then N_NCl(A)\A = ϕ which implies that A is N_N-closed in U.

Proposition 3.17: If A is N_NRO and N_NRG closed, then A is N_NRW-closed in U.

Proof: Let A be an N_NRO and N_NRG-closed. Let Q be any N_NRSO set in U such that A ⊆ R. since A is N_NRO and N_NRG we have N_NCl(A) ⊆ A ⊆ R. Therefore A is N_NRW-closed.

Proposition 3.18: If a subset A of a neutrosophic nano topological space U is both NNRSO and NNRW-closed, then it is NN-closed.

Proof: Suppose A be a subset of a neutrosophic nano topological space U is both NNRSO and NNRW-closed. Then A ⊂ A and NNCl(A) ⊆ A which implies A is NN-closed.

Remark 3.19: The concept of NNRW-closed set is independent with the concepts of (i) NNsemi –closed (ii) NNRW-preclosed (iii) NNα-closed (iv) NNWG - closed sets which is shown by the following example.

Example 3.20: Let U = $\{n_1, n_2, n_3\}$ be the universe set. U\R = $\{\{n_1\}, \{n_2, n_3\}\}$ be an equivalence relation. Let $S = \left\{\langle\frac{n_1}{(0.1,0.2,0.3)}\rangle, \langle\frac{n_2}{(0.3,0.4,0.5)}\rangle, \langle\frac{n_3}{(0.6,0.4,0.1)}\rangle\right\}$ be a neutrosophic nano subset of U. Then $\overline{N}(S) =$

$\left\{\langle\frac{n_1,n_3}{(0.1,0.2,0.3)}\rangle, \langle\frac{n_2}{(0.6,0.4,0.1)}\rangle\right\}$, $\underline{N}(S) = \left\{\langle\frac{n_1,n_3}{(0.1,0.2,0.3)}\rangle, \langle\frac{n_2}{(0.3,0.4,0.5)}\rangle\right\}$ and $B(S) = \left\{\langle\frac{n_1,n_3}{(0.1,0.2,0.3)}\rangle, \langle\frac{n_2}{(0.1,0.4,0.6)}\rangle\right\}$. So the neutrosophic nano topology τN = $\{0_N, 1_N, \underline{N}, \overline{N}, B\}$. In the neutrosophic nano topology (U, τN(S)),

- Let A = $\left\{\langle\frac{n_1}{(0.2,0.5,0.3)}\rangle, \langle\frac{n_2}{(0.1,0.5,0.6)}\rangle, \langle\frac{n_3}{(0.1,0.4,0.7)}\rangle\right\}$ and B = $\left\{\langle\frac{n_1}{(0.2,0.7,0.4)}\rangle, \langle\frac{n_2}{(0.5,0.6,0.4)}\rangle, \langle\frac{n_3}{(0.4,0.5,0.4)}\rangle\right\}$, then A is NNsemi-closed but not an NNRW-closed and B is NNRW-closed but it is not an NNsemi-closed.
- Let C = $\left\{\langle\frac{n_1}{(0.1,0.2,0.4)}\rangle, \langle\frac{n_2}{(0.3,0.3,0.6)}\rangle, \langle\frac{n_3}{(0.1,0.3,0.5)}\rangle\right\}$ and D = $\left\{\langle\frac{n_1}{(0.1,0.4,0.7)}\rangle, \langle\frac{n_2}{(0.1,0.6,0.7)}\rangle, \langle\frac{n_3}{(0.3,0.4,0.4)}\rangle\right\}$, then C is both NNpre-closed set and NNWG-closed but not an NNRW-closed and D is NNRW-closed but it is neither NNpre-closed nor an NNWG-closed sets.
- In example 3.8, in the topological space (U,τN(S)), E = $\left\{\langle\frac{n_1}{(0.6,0.3,0.3)}\rangle, \langle\frac{n_2}{(0.5,0.3,0.3)}\rangle, \langle\frac{n_3}{(0.5,0.2,0.3)}\rangle, \langle\frac{n_4}{(0.3,0.3,0.1)}\rangle, \langle\frac{n_5}{(0.4,0.4,0.1)}\rangle\right\}$ and F = $\left\{\langle\frac{n_1}{(0.3,0.3,0.7)}\rangle, \langle\frac{n_2}{(0.2,0.3,0.6)}\rangle, \langle\frac{n_3}{(0.2,0.3,0.5)}\rangle, \langle\frac{n_4}{(0.1,0.2,0.7)}\rangle, \langle\frac{n_5}{(0.1,0.3,0.8)}\rangle\right\}$, E is NNRW-closed set but not an NNα-closed set and F is NNα-closed but it is not an NNRW-closed set.

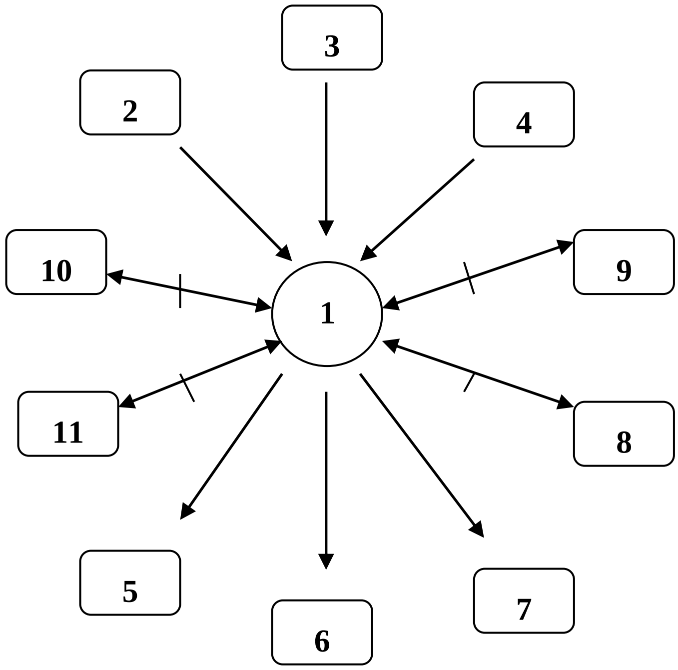
Proposition 3.21: If an NN subset A is both NN-open and NNG-closed in (U, τN(S)), then it is NNRW-closed in U.

Proof: Let A be NN-open and NNG-closed in U. Let A ⊂ U and U be an NNRSO in U. Now, A ⊂ A. By hypothesis, NNCl(A) ⊂ U. Thus A is NNRW-closed.

Remark 3.22: If A is both NN-open and NNRW-closed in U, then A need not be NNG-closed in general which is shown in the following example.

Example 3.23: In example 3.8, the NN-open set B is NNRW-closed but it is not an NNG-closed set.

The above discussions are implicated in the following diagram.



1. N_NRW-closed 2. N_N-closed 3. N_NR-closed 4. $N_N\pi$-closed 5. N_NRG-closed 6. N_NRWG-closed 7. N_NGPR-closed 8. $N_N$semi-closed 9. $N_N$pre-closed 10. $N_N\alpha$-closed 11.N_NWG-closed.

Proposition 3.24: If a subset A of a neutrosophic nano topological space U is both N_N-open and N_NWG-closed, then it is N_NRW-closed.

Proof: Suppose a subset A of U is both NN-open and N_NWG-closed. Let $A \subset U$ and U is N_NRSO. Then N_NCl (N_NInt(A)) = $A \subset A$, since A is NN-open. Hence N_NCl(A) $\subset U$ implies that A is an N_NRW-closed in U.

Definition 3.25: A neutrosophic nano subset A of a neutrosophic nano topological space (U,τ_N(S)) is called an N_NRW-open if and only if its complement A^C is N_NRW-closed.

Proposition 3.26: An N_N set A of a topological space (U,τ_N(S)) is N_NRW-open if F $\subseteq$ NNInt(A) whenever F is N_NRSO and $F \subset A$.

Proof: Follows from the definition 3.1.

Proposition 3.27: Let A be an N_NRW-open set of neutrosophic nano topological space (U,τ_N(S)) and N_NInt(A) $\subseteq B \subseteq A$. Then B is N_NRW-open.

Proof: Suppose that A is an N_NRW-open in U and N_NInt(A) $\subseteq B \subseteq A$ implies $A^c \subseteq B^c \subseteq N_N$Cl($A^c$). Since A^c is N_NRW-closed, by Proposition 3.14, B^c is N_NRW-closed. Hence B is N_NRW-open.

Proposition 3.28: Let (U,τ_N(S)) be a neutrosophic nano topological space and N_NRSO(X) and N_NC(X) be the family of all N_NRSO sets and N_NC sets respectively. Then N_NRSO(X) $\subseteq N_N$C(X) if and only if every neutrosophic nano set of U is N_NRW-closed.

Proof: Necessity: Suppose that N_NRSO(X) $\subseteq$ N_NC(X) and let A be an N_N – set of U such that A $\subseteq$ R $\in$ N_NRSO(X). Then N_NCl(A) $\subseteq$ N_NCl(R) = R, by hypothesis. Hence N_NCl(A) $\subseteq$ R when A $\subseteq$ R and R is N_NRSO which implies that A is N_NRW-closed.

Sufficiency: Assume that every neutrosophic nano set of U is N_NRW-closed. Let R $\in$ N_NRSO(X). Then since R $\subseteq$ R and R is N_NRW-closed, N_NCl(R) $\subseteq$ R then R $\in$ N_NCl(X). Therefore N_NRSO(X) $\subseteq$ N_NCl(X).

Definition 3.29: A neutrosophic nano topological space (U,τ_N(S)) is called as N_NRW-connected if there is no proper N_N-subset of U which is both N_NRW-open N_NRW-closed.

Proposition 3.30: Every N_NRW-connected space is N_N-connected.

Proof: Let (U,τ_N(S)) be an N_NRW-connected and suppose that (U,τ_N(S)) is not N_N-connected. Then there exists a proper N_N-set A (A $\neq$ 0_N, A $\neq$ 1_N) such that A is both N_N-open and N_N-closed set. Since every N_N-open and N_N-closed set is N_NRW-open and N_NRW-closed, (U,τ_N(S)) is not an N_NRW-connected which is a contradiction. This shows that U is N_N-connected.

Proposition 3.31: A N_NT space is N_NRW- connected if and only if there exists no non-zero N_NRW- open sets A and B in X such that A = B^C.

Proof: Necessity: Suppose that A and B are N_NRW-open sets such that A $\neq$ 0_N $\neq$ B. and A = B^C. Since B = A^C, A is N_NRW-closed set and B $\neq$ 0_N implies B^C $\neq$ 1_N, i.e., A $\neq$ 1_N. Hence there exists a proper N_N –set A which is both N_NRW-open and N_NRW-closed which is a contradiction to the fact that U is N_NRW-connected.

Sufficiency: Let (U,τ_N(S)) be an N_NTS and A is both N_NRW-open and N_NRW-closed set in U such that 0_N $\neq$ A $\neq$ 1_N. Take B = A^C implies that B is N_NRW-open and A $\neq$ 1_N $\Rightarrow$B = A^C $\neq$ 0_N which is a contradiction. Hence there is no proper N_N-subset of U which is both N_NRW-open and N_NRW-closed. Therefore N_NTS (U,τ_N(S)) is N_NRW-connected.

Definition 3.32: A neutrosophic nano topological space(U,τ_N(S)) is said to be an N_NRW$T_{1/2}$-space if every N_NRW-closed set in U is N_N-closed in U.

Proposition 3.33: A neutrosophic nano topological space (U,τ_N(S)) is N_NRW$T_{1/2}$ space, then the following statements are equivalent:

(i) U is N_NRW-connected (ii) U is N_N-connected.

Proof: (i) $\Rightarrow$ (ii): Follows from the Proposition 3.29.

(ii) $\Rightarrow$ (i): Assume that U is N_NRW$T_{1/2}$-space, and N_N-connected. Suppose that U is not an N_NRW-connected, then there exists a proper N_N-set A which is both N_NRW-open and N_NRW-closed. Since (U,τ_N(S)) is N_NRW$T_{1/2}$, A is both N_N-open and N_N-closed which is a contradiction to the fact that U is N_N-connected. This shows that U is N_NRW-connected.

4. N$_N$RW-CONTINUOUS FUNCTIONS

Definition 4.1: (i) A function f: (U,τ_N(S)) → (V,τ_N(T)) is said to be a neutrosophic nano RW-continuous **(In short N$_N$RW-continuous)** if the inverse image of N$_N$-closed set of V is N$_N$RW-closed in (U,τ_N(S)).

(ii) A function f: (U,τ_N(S)) → (V,τ_N(T)) is said to be a neutrosophic nano RW-irresolute **(In short N$_N$RW-irresolute)** if the inverse image of N$_N$RW-closed set of V is N$_N$RW-closed in (U,τ_N(S)).

Proposition 4.2: A mapping f: (U,τ_N(S)) → (V,τ_N(T)) is N$_N$RW-continuous if and only if the inverse image of every N$_N$-open set of V is N$_N$RW-open in U.

Proof: It is obvious because $f^{-1}(A^c) = [f^{-1}(A)]^c$ for every N$_N$-set A of V.

Proposition 4.3: If f: (U,τ_N(S)) → (V,τ_N(T)) is N$_N$RW-continuous, then f(N$_N$RWCl(A)) ⊆ N$_N$Cl(f(A)) for every N$_N$-set A of U.

Proof: Let A be an N$_N$-set of U. Then N$_N$Cl(f(A)) is an N$_N$-closed set of V. Since f is an N$_N$RW-continuous function, f^{-1}(N$_N$Cl(f(A)) is N$_N$RW-closed in U. Clearly A ⊆ f^{-1}(N$_N$Cl(f(A)). Therefore N$_N$RWCl(A) ⊆ N$_N$RWCl (f^{-1}(N$_N$Cl f(A))) = f^{-1}(N$_N$Cl(f(A)). Hence f(N$_N$RWCl(A)) ⊆ N$_N$Cl(f(A)) for every N$_N$-set A of U.

Proposition 4.4: (i) Every N$_N$-continuous map is N$_N$RW-continuous.

(ii) Every N$_N$- regular continuous map is N$_N$RW-continuous.

(iii) Every N$_N$- π-continuous set is N$_N$RW-continuous.

(iv) Every N$_N$W-continuous map is N$_N$RW-continuous.

(v) Every N$_N$RW-irresolute map is N$_N$RW-continuous.

Proof: Obvious.

Remark 4.4: The following example makes clear that the converse of the Proposition 4.4 may not be true.

Example 4.5: Let U = $\{n_1, n_2, n_3\}$ = V be the universe sets. U\R$_1$ = $\{\{n_1\},\{n_2,n_3\}\}$ and U\R$_2$ = $\{\{n_1,n_3\},\{n_2\}\}$ be equivalence relations. Let $S_1 = \left\{\langle\frac{n_1}{(0.3,0.4,0.3)}\rangle,\langle\frac{n_2}{(0.6,0.3,0.1)}\rangle,\langle\frac{n_3}{(0.2,0.6,0.2)}\rangle\right\}$, $S_2 = \left\{\langle\frac{n_1}{(0.1,0.2,0.3)}\rangle,\langle\frac{n_2}{(0.2,0.3,0.4)}\rangle,\langle\frac{n_3}{(0.1,0.6,0.4)}\rangle\right\}$ be a neutrosophic nano subsets of U. Then $\tau_N(S_1)$ = $\{0_N, \overline{N}(S_1), \underline{N}(S_1), B(S_1), 1_N\}$, $\tau_N(S_2)$ = $\{0_N, \overline{N}(S_2), B(S_2), 1_N\}$ be the neutrosophic nano topologies on U and V respectively. Define an identity map f: (U,τ_N(S$_1$)) → (V.τ_N(S$_2$)). Then f is N$_N$RW-continuous but is neither N$_N$-continuous nor N$_N$W-continuous. Similarly it's not an N$_N$R-continuous, N$_N$π -continuous and N$_N$RW-irresolute.

Proposition 4.6: (i) Every N$_N$RW-continuous map is N$_N$RG-continuous.

(ii) Every N$_N$RW- continuous map is N$_N$GPR- continuous.

(iii) Every N_NRW- continuous map is N_NRWG- continuous.

Proposition 4.7: If f: $(U,\tau_N(S)) \rightarrow (V,\tau_N(T))$ is N_NRW-continuous and g: $(V,\tau_N(T)) \rightarrow (W,\tau_N(R))$ is N_N-continuous. Then g°f: f: $(U,\tau_N(S)) \rightarrow (W,\tau_N(R))$ is N_NRW-continuous.

Proof: Let A be an N_N-closed in W. Then $g^{-1}(A)$ is N_N-closed in V, because g is N_N-continuous. Therefore $(g°f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is N_NRW-closed in U. Hence g°f is N_NRW-continuous.

Proposition 4.8: If f: $(U,\tau_N(S)) \rightarrow (V,\tau_N(T))$ is N_NRW-continuous and g: $(V,\tau_N(T)) \rightarrow (W,\tau_N(R))$ is N_NG-continuous and $(V,\tau_N(T))$ is $N_NT_{1/2}$ then g°f: $(U,\tau_N(S)) \rightarrow (W,\tau_N(R))$ is N_NRW-continuous.

Proof: Let A be an N_N-closed set in W, then $g^{-1}(A)$ is N_NG-closed in V. Since V is $N_NT_{1/2}$ then $g^{-1}(A)$ is N_N-closed in V. Hence, $(g°f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is N_NRW-closed in U. Hence g°f is N_NRW-continuous.

Proposition 4.9: If f: $(U,\tau_N(S)) \rightarrow (V,\tau_N(T))$ is N_NRG - irresolute and g: $(V,\tau_N(V)) \rightarrow (W,\tau_N(R))$ is N_NRW-continuous, then g°f: $(U,\tau_N(S)) \rightarrow (W,\tau_N(R))$ is N_NRG-continuous.

Proof: Let A be an N_N-closed set in W, then $g^{-1}(A)$ is N_NRW-closed in V, since g is N_NRW-continuous. Every N_NRW-closed set is N_NRG-closed, $g^{-1}(V)$ is N_NRG-closed set in V. Then $(g°f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is NNRG-closed in U, by hypothesis. Hence g°f: $(U,\tau_N(S)) \rightarrow (W,\tau_N(R))$ is N_NRG-continuous.

Proposition 4.10: If f: $(U,\tau_N(S)) \rightarrow (V,\tau_N(T))$ is N_NRW-continuous surjection and U is N_NRW-connected then V is N_N-connected.

Proof: Assume that V is not an N_N-connected space. Then there exists a proper N_N-subset F of V which is both N_N-open and N_N-closed. Therefore, by hypothesis, $f^{-1}(F)$ is a proper N_N-set of U which is both N_NRW-open and N_NRW-closed in U implies that U is not an N_NRW-connected which is a contradiction. This shows that V is N_N-connected.

Definition 4.11: (i) A mapping f: $(U,\tau_N(S)) \rightarrow (V,\tau_N(T))$ is said to be N_NRW-open map if the image of every N_N-open set of U is N_NRW-open set in V.

(ii) A mapping f: $(U,\tau_N(S)) \rightarrow (V,\tau_N(T))$ is said to be N_NRW-closed map if the image of every N_N-closed set of U is N_NRW-closed set in V.

Proposition 4.12: A mapping f: $(U,\tau_N(S)) \rightarrow (V,\tau_N(T))$ is N_NRW-open if and only if for every N_N-set A of U, $f(N_N\text{Int}(A)) \subseteq N_N\text{RW Int}(f(A))$.

Proof: Necessity: Let f be an N_NRW-open map and A is an N_N-open set in U, $N_N\text{Int}(A) \subseteq A$ which implies that $f(N_N\text{Int}(A)) \subseteq f(A)$. Since f is an N_NRW-open mapping, $f(N_N\text{Int}(A))$ is N_NRW-open set in V such that $f(N_N\text{Int}(A)) \subseteq f(A)$. Therefore $f(N_N\text{Int}(A)) \subseteq N_N\text{RWInt } f(A)$.

Sufficiency: Suppose that A is an N_N-open set of U. Then $f(A) = f(N_N\text{Int}(A) \subseteq N_N\text{RWInt } f(A)$. But $N_N\text{RWInt } (f(A)) \subseteq f(A)$. Consequently $f(A) = N_N\text{RWInt}(A)$ which implies that f(A) is an N_NRW-open set of V and hence f is an N_NRW-open map.

Proposition 4.13: A mapping f: $(U,\tau_N(S)) \rightarrow (V,\tau_N(T))$ is N_NRW-open if and only if for every neutrosophic nano set A of V and for each N_N-closed set B of U containing $f^{-1}(A)$ there is a N_NRW-closed set F of V such that $A \subseteq F$ and $f^{-1}(F) \subseteq B$.

Proof: Necessity: Suppose that f is N_NRW-open map. Let A be a N_N-closed set of V and B be a N_NC set of U such that $f^{-1}(A) \subseteq B$. Then $F = f^{-1}(B^c)^c$ is a N_NRW- closed set of V such that $f^{-1}(F) \subseteq B$.

Sufficiency: Let F be a N_NO set of U. Then $f^{-1}(f(F))^c \subseteq F^c$ and F^c is a N_NC set in X. By hypothesis there is an N_NRW- closed set G of V such that $(f(F))^c \subseteq G$ and $f^{-1}(G) \subseteq F^c$. Therefore $F \subseteq (f^{-1}(G))^c$. Hence $G^c \subseteq f(F) \subseteq f((f^{-1}(G))^c) \subseteq G^c$ i.e., $f(F) = G^c$ which is N_NRW-open in V and thus f is N_NRW-open map.

Proposition 4.14: If a mapping f: $(U,\tau_N(S)) \rightarrow (V,\tau_N(T))$ is N_NRW-open, then $N_N\text{Int}(f^{-1}(G)) \subseteq f^{-1}(N_N\text{RWInt}(G))$ for every neutrosophic nano set G of Y.

Proof: Let G be neutrosophic nano set of V. Then $N_N\text{Int}f^{-1}(G)$ is a N_NO set in U. Since f is N_NRW – open $f(N_N\text{Int}f^{-1}(G)) \subseteq N_N\text{RWInt}(f(f^{-1}(G)) \subseteq N_N\text{RWInt}(G))$. Thus $N_N\text{Int}(f^{-1}(G)) \subseteq f^{-1}(N_N\text{RWInt}(G))$.

Proposition 4.15: A mapping f: $(U,\tau_N(S)) \rightarrow (V,\tau_N(T))$ is N_NRW-closed if and only if for every neutrosophic nano set A of V and for each N_NO set B of U containing $f^{-1}(A)$ there is a N_NRW -open set F of V such that $A \subseteq F$ and $f^{-1}(F) \subseteq B$.

Proof: Necessity: Suppose that f is N_NRW -closed map. Let A be a N_NC set of V and B be a N_NO set of U such that $f^{-1}(A) \subseteq B$. Then $F = V \setminus f^{-1}(B^c)$ is a N_NRW –open set of V such that $f^{-1}(F) \subseteq B$.

Sufficiency: Let F be a N_NC set of X. Then $f^{-1}(f(F))^c \subseteq F^c$ and F^c is a N_NO set in U. By hypothesis there is an N_NRW - open set R of V such that $(f(F))^c \subseteq R$ and $f^{-1}(R) \subseteq F^c$. Therefore $F \subseteq (f^{-1}(R))^c$. Hence $R^c \subseteq f(F) \subseteq f((f^{-1}(R))^c) \subseteq R^c$ i.e., $f(F) = R^c$ which is N_NRW -closed in V. Thus f is N_NRW -closed map.

Proposition 4.16: If f: $(U,\tau_N(S)) \rightarrow (V,\tau_N(T))$ is NN-almost irresolute and N_NRW-closed map. If A is N_NRW-closed set of U, then f(A) is N_NRW-closed in V.

Proof: Let $f(A) \subseteq R$ where R is an N_NRSO set of V. since f is an N_N-almost irresolute, $f^{-1}(R)$ is an N_NSO set of U such that $A \subseteq f^{-1}(R)$. Since A is N_NW-closed set of U which implies that $N_N\text{Cl}(A) \subseteq f^{-1}(R) \Rightarrow f(N_N\text{Cl}(A)) \subseteq R$, i.e., $N_N\text{Cl}(f(N_N\text{Cl}(A)) \subseteq R$. Therefore $N_N\text{Cl}(f(A)) \subseteq R$ whenever $f(A) \subseteq R$ where R is an N_NRSO set of V. Hence f(A) is an N_NRW-closed set of V.

Proposition 4.17: If f: $(U,\tau_N(S)) \rightarrow (V,\tau_N(T))$ is N_N-closed and g: $(V,\tau_N(T)) \rightarrow (W,\tau_N(R))$ is N_NRW-closed then $g°f: (U,\tau_N(S)) \rightarrow (W,\tau_N(R))$ is N_NRW-closed.

Proof: Let F be an N_N-closed set of neutrosophic nano topological space $(U, {}_N(S))$. Then f(F) is an N_N-closed set of $(V,\tau_N(T))$. By hypothesis, $g°f(F) = g(f(F))$ is an N_NRW-closed set in N_N-topological space W. Thus $g°f: (U,\tau_N(S)) \rightarrow (W,\tau_N(R))$ is N_NRW-closed.

Conclusions: In this article, the authors have introduced and studied the concepts such as, Neutrosophic nano RW- closed set, N_NRW-open set, $N_N\text{RWT}_{1/2}$ space, N_NRW-connected space, N_NRW-continuous, N_NRW-irresolute, N_NRW-open and N_N- closed maps. In future it can be extended to some new forms of continuous functions and homeomorphisms.

Funding: This research received no external funding.

Acknowledgements: The authors are highly grateful to the Referees for their valuable suggestions.

Conflicts of Interest: The authors declare no conflict of interest.

REFERENCES:

1. K. Atanassov, Intuitionistic fuzzy sets, in V.Sgurev,ed, Vii ITKRS Session, Sofia (June 1983 central Sci. and Techn. Library, Bulg Academy of Sciences (1984).

2. K. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and systems 20 (1986), 87-96.
3. K. Atanassov, Review and new result on intuitionistic fuzzy sets, preprint IM-MEAIS 1-88, Sofia, 1988.
4. S. S. Benchalli and R.S. Wali, On RW- closed sets in Topological Spaces, Bull. Malays. Math. Sci. Soc. (2) 30(2) (2007), 99-110.
5. Chang, C.L. Fuzzy topological spaces, *J Math. Anal.Appl. 1968;24, pp. 182-190*.
6. M. Dhanapackiam, M. Trinita Pricilla, A new class of Neutrosophic Nano gb-closed sets in Neutrosophic Nano topological spaces, Journal of Information and Computer Science, Vol 10, Issue 10(2020), 407-416.
7. R. Dhavaseelan, S. Jafari, C. Ozel and M. A.Al-Shumrani, Generalized Neutrosophic Contra-Continuity, New trends in Neutrosophic Theory and Applications, Volume II, 355-370.
8. F. Smarandache, A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. American Research Press, Rehoboth, NM, 1999.
9. F. Smarandache, Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy, Neutrosophic Logic, Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA (2002).
10. Florentin Smarandache, Neutrosophic Set - A Generalization of Intuitionistic Fuzzy sets, Journal of defense Sources Management 1 (2010), 107-116.
11. M. Lellis Thivagar, Carmel Richard, On nano forms of weakly open sets, International journal of mathematics and statistics invention, Volume 1, Issue 1, August 2013, 31-37.
12. M. Lellis Thivagar, Saied Jafari , V. Sudhadevi and V. Antonysamy, A novel approach to nano topology via neutrosophic sets, Neutrosophic sets and systems, Vol 20, 2018, 86-94.
13. A.A. Salama and S.A. Albowi, Generalized Neutrosophic Set and Generalized Neutrosophic topological spaces, Journal Computer Sci. Engineering, Vol (2) No. (7) (2012)
14. A.A Salama and S.A. Albowi, Neutrosophic set and Neutrosophic topological space, ISOR J. Mathematics, Vol (3), issue (4), 2012 pp- 31-35.
15. D.Savithiri, C.Janaki, Neutrosophic RW closed sets in neutrosophic topological spaces, *International Journal of Research and Analytical Reviews (IJRAR),* June 2019, Vol. 6, Issue2, 242-249.
16. D.Savithiri, C.Janaki, Neutrosophic RW-homeomorphism in Neutrosophic topological spaces, Aegaeum Journal, Vol9, Issue1,(2021), 410-418.
17. L. A. Zadeh, Fuzzy sets, Inform. and Control 8 (1965).

Received: Nov 1, 2021. Accepted: Feb 5, 2022