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Neutrosophic Nano RW-Closed Sets in Neutrosophic Nano Topological Spaces

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Abstract: The main objective of this study is to introduce a new class of closed sets namely Neutrosophic Nano RW-closed sets and Neutrosophic Nano RW-continuous functions in Neutrosophic Nano topological spaces. Some of its properties and interrelationship with some existing Neutrosophic nano closed sets have been discussed.

Keywords: NeuRW-closed set, NeuRW-open set, NeuRW-T1/2 space, NeuRW-connected space, NeuRW-continuous, NeuRW-irresolute, NeuRW-open and Neu-closed maps.

1. Introduction

The theory of neutrosophic sets with three components namely, membership T (Truth), Indeterminacy I, and non-membership F (Falsehood), one of the interesting generalizations of theory of fuzzy sets and Intuitionistic fuzzy sets introduced by F.Smarandache [8]. In 2012, A.A. Salama and S.A. Alblowi [13] introduced and studied the theory of neutrosophic topological spaces. Since then several mathematicians contributed many papers to this area. Various results in ordinary topological spaces have been put in the neutrosophic setting, and also various departures have been observed. Neutrosophic set is a powerful tool to deal with indeterminate and inconsistent data. The concept of nano topology explored by M. Lellis Thivagar et. al[11] can be described as a collection of nano approximations for which equivalence classes are building blocks. In 2018, M. Lellis Thivagar et. al. [12] introduced a new concept called as Neutrosophic Nano topology and discussed neutrosophic nano interior and neutrosophic nano closure.


2 PRELIMINARIES

The following recalls requisite ideas and preliminaries necessary in the sequel of our work.

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Definition 2.1:[9] Let X be a non-empty fixed set a Neutrosophic set (NS for short) A is an object having the form $A = < x, \mu_A(x), \sigma_A(x), \gamma_A(x)>$, $x \in X$ where $\mu_A(x)$, $\sigma_A(x)$, $\gamma_A(x)$ which represents the degree of membership function, the degree of indeterminacy and the degree of non-membership function respectively of each element $x \in X$ to the set $A$.

Definition 2.2:[11] Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair $(U, R)$ is said to be the approximation space. Let $X \subseteq U$.

(i) The lower approximation of X with respect to R is the set of all objects, which can be classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = \bigcup_{x \in U} \{ R(x) : R(x) \subseteq X \}$, where R(x) denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = \bigcup_{x \in U} \{ R(x) : R(x) \cap X \neq \emptyset \}$.

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) \setminus L_R(X)$.

Remark 2.3:[11]

(i) $L_R(X) \subseteq X \subseteq U_R(X)$.

(ii) $L_R(\emptyset) = U_R(\emptyset) = \emptyset$ and $L_R(U) = U_R(U) = U$.

(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$.

(iv) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$.

(v) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$.

(vi) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$.

(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$, whenever $X \subseteq Y$.

(viii) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$.

(ix) $U_R L_R(X) = L_R U_R(X) = U_R(X)$.

(x) $L_R L_R(X) = L_R U_R(X) = L_R(X)$.

Definition 2.4:[11] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = [U, \emptyset, L_R(X), U_R(X), B_R(X)]$ where $X \subseteq U$. $\tau_R(X)$ satisfies the following axioms:

(i) $U$ and $\emptyset \in \tau_R(X)$.
(ii) The union of the elements of any sub-collection of $\tau(X)$ is in $\tau(X)$.

(iii) The intersection of the elements of any finite sub collection $\tau(X)$ is in $\tau(X)$.

That is, $\tau(X)$ forms a topology on $U$ called the nano topology on $U$ with respect to $X$. We call $(U, \tau(X))$ as the nano topological space. The elements of $\tau(X)$ are called nano-open sets.

**Definition 2.5** Let $U$ be a non-empty set and $R$ be an equivalence relation on $U$. Let $S$ be a neutrosophic set in $U$ with the membership function $\mu_S$, the indeterminacy function $\sigma_S$, and the non-membership function $\gamma_S$. The neutrosophic nano lower, neutrosophic nano upper approximation and neutrosophic nano boundary of $S$ in the approximation $(U, R)$ denoted by $N(S), N(S)$ and $B(S)$ are respectively defined as follows:

(i) $N(S) = \{(x, \mu_R(x), y_R(x), y_R(\gamma(x))) / x \in [x], x \in U\}.$

(ii) $N(S) = \{(x, \mu_R(x), y_R(x), y_R(\gamma(x))) / x \in [x], x \in U\}.$

(iii) $B(S) = N(S) - N(S).$

where $\mu_R(x) = \bigwedge_{y \in [x]} \mu_A(y), \sigma_R(x) = \bigwedge_{y \in [x]} \sigma_A(y), y_R(x) = \bigvee_{y \in [x]} y_A(y),$

$\mu_R(x) = \bigvee_{y \in [x]} \mu_A(y), \sigma_R(x) = \bigwedge_{y \in [x]} \sigma_A(y), y_R(x) = \bigwedge_{y \in [x]} y_A(y).$

**Definition 2.6** Let $U$ be an universe, $R$ be an equivalence relation on $U$ and $S$ be a neutrosophic set on $U$. Then the collection $\tau(S) = \{0_n, 1_n, N(S), N(S), B(S)\}$ forms a topology then it is said to be a neutrosophic nano topology. We call $(U, \tau(S))$ as the neutrosophic nano topological space (Briefly NNTS). The elements of $\tau(S)$ are called as neutrosophic nano open (In Short NnO) sets.

**Remark 2.7** $[\tau(S)]^C$ is called as dual neutrosophic nano topology of $\tau(S)$. The elements of $[\tau(S)]^C$ are called neutrosophic nano closed (In Short NnC) sets.

**Remark 2.8** In neutrosophic nano topological space, the neutrosophic nano boundary cannot be empty. Since the difference between neutrosophic nano upper and neutrosophic nano lower approximations is defined as the maximum and minimum of the values in the neutrosophic sets.

**Proposition 2.9** Let $U$ be a non-empty finite universe and $S$ be a neutrosophic set on $U$. Then the following statements hold:

(i) The collection $\tau(S) = [0_n, 1_n]$, is the indiscrete neutrosophic nano topology on $U$.

(ii) If $N(S) = N(S) = B(S)$, then the neutrosophic nano topology, $\tau(S) = [0_n, 1_n, N(S), N(S), B(S)].$

(iii) If $N(S) = B(S)$, then $\tau(S) = [0_n, 1_n, N(S), N(S)]$ is a neutrosophic nano topology.

(iv) If $N(S) = B(S)$, then $\tau(S) = [0_n, 1_n, N(S), B(S)].$
(v) The collection \( \tau_N(S) = \{0_N, 1_N, \overline{N}(S), B(S)\} \) is the discrete neutrosophic nano topology on \( U \).

**Definition 2.10:** Let \((U, \tau_N(S))\) be NNTS and \( A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x) \rangle, x \in U \rangle \) be a NNS in \( X \). Then the neutrosophic nano closure and neutrosophic nano interior of \( A \) are defined by

\[
N_N\text{Cl}(A) = \bigcap \{ K : K \text{ is a } N_N\text{CS in } X \text{ and } A \subseteq K \} \\
N_N\text{Int}(A) = \bigcup \{ G : G \text{ is a } N_N\text{OS in } X \text{ and } G \subseteq A \}.
\]

**Definition 2.11:** A subset \( A \) of a neutrosophic nano topological space \((U, \tau_N(S))\) is said to be

(i) a neutrosophic nano pre closed (\( N_N\text{pre-closed} \)) set if \( N_N\text{Cl}(N_N\text{Int}(A)) \subseteq A \).

(ii) a neutrosophic nano semi-closed (\( N_N\text{semi-closed} \)) set if \( N_N\text{Int}(N_N\text{Cl}(A)) \subseteq A \).

(iii) a neutrosophic nano regular open (\( \text{In short } N_N\text{RO} \)) set if \( A = N_N\text{Int}(N_N\text{Cl}(A)) \).

(iv) a neutrosophic regular semi open (\( \text{In short } N_N\text{RSO} \)) if there exists a NRO set \( U \) such that \( U \subseteq A \subseteq N_N\text{Cl}(A) \).

(v) a neutrosophic nano \( \alpha \)-closed (\( N_N\alpha\text{-closed} \)) set if \( N_N\text{Cl}(N_N\text{Int}(N_N\text{Cl}(A))) \subseteq A \).

(vi) a neutrosophic nano \( g \)-closed (\( N_N\text{g-closed} \)) set if \( N_N\text{Cl}(A) \subseteq F \) whenever \( A \subseteq F \) and \( F \) is NNO in \( U \).

**Definition 2.11:** The difference between two neutrosophic nano sets \( A \) and \( B \) is defined as

\[
A \setminus B = \{x, \min[\{\mu_A(x),\gamma_B(x)\}], \min[\{\sigma_A(x),1-\sigma_B(x)\}], \max[\{\gamma_A(x),\mu_B(x)\}]\}.
\]

### 3. NEUTROSOPHIC NANO RW-CLOSED SETS

**Definition 3.1:** A subset \( A \) of a neutrosophic nano topological space \((U, \tau_N(S))\) is called as neutrosophic nano regular weakly closed (\( \text{In short } N_N\text{RW-closed} \)) set, if \( N_N\text{Cl}(A) \subseteq V \) whenever \( A \subseteq V \) and \( V \) is a neutrosophic nano regular open in \( U \).

**Definition 3.2:** The neutrosophic nano RW-closure and neutrosophic nano RW-interior of \( A \) are defined by

\[
N_N\text{RWCl}(A) = \bigcap \{ K : K \text{ is a } N_N\text{RWCS in } X \text{ and } A \subseteq K \} \\
N_N\text{RWInt}(A) = \bigcup \{ G : G \text{ is a } N_N\text{RWOS in } X \text{ and } G \subseteq A \}.
\]

**Definition 3.3:** (i) neutrosophic nano RG- closed set (\( \text{shortly } N_N\text{RG – closed set} \)) of \( X \) if there exists a neutrosophic nano regular open set \( U \) such that \( N_N\text{Cl}(A) \subseteq U \) whenever \( A \subseteq U \).

(ii) neutrosophic nano RWG- closed set (\( \text{shortly } N_N\text{RWG – closed set} \)) of \( X \) if there exists a neutrosophic nano regular open set \( U \) such that \( N_N\text{Cl}(N_N\text{Int}(A)) \subseteq U \) whenever \( A \subseteq U \).
(iii) neutrosophic nano W-closed set (shortly N\textsubscript{\textbullet}W-closed set) of X if there exists a neutrosophic nano semi-open set U such that N\textsubscript{\textbullet}Cl(A) \subseteq U whenever A \subseteq U.

(iv) neutrosophic nano g-closed set (shortly N\textsubscript{\textbullet}G-closed set) of X if there exists a neutrosophic open set U such that N\textsubscript{\textbullet}Cl(A) \subseteq U whenever A \subseteq U.

**Proposition 3.3:** (i) Every N\textsubscript{\textbullet}N-closed set is N\textsubscript{\textbullet}RW-closed.

(ii) Every N\textsubscript{\textbullet}\pi regular closed set is N\textsubscript{\textbullet}RW-closed.

(iii) Every N\textsubscript{\textbullet}\pi closed set is N\textsubscript{\textbullet}RW-closed.

(iv) Every N\textsubscript{\textbullet}W-closed set is N\textsubscript{\textbullet}RW-closed.

**Proof:** Follows from [4].

The following example makes clear that the converse of the Proposition 3.3 need not be true.

**Example 3.4:** Let U = \{p\subscript{1}, p\subscript{2}, p\subscript{3}\} be the universe set and the equivalence relation U \setminus R = \{\{p\subscript{1}, p\}\}. Let 
\[ S = \left\{ \left(\frac{p\subscript{1}}{(0.1,0.2,0.3)}, \frac{p\subscript{2}}{(0.2,0.3,0.4)}, \frac{p\subscript{3}}{(0.1,0.2,0.3)} \right) \right\} \] be a neutrosophic nano subset of U. Then \[ N(S) = \left\{ \left(\frac{p\subscript{1}}{(0.1,0.2,0.3)}, \frac{p\subscript{2}}{(0.2,0.3,0.4)}, \frac{p\subscript{3}}{(0.1,0.2,0.3)} \right) \right\}. \] So the neutrosophic nano topology \( N = \{0_N, 1_N, N_B\} \) where the neutrosophic closed sets are \( N^C = \{0_N, 1_N, N_B, N^C_B\} \). Let \( Q_1 = \left\{ \left(\frac{p\subscript{1}}{(0.2,0.1,0.3)}, \frac{p\subscript{2}}{(0.3,0.1,0.2)}, \frac{p\subscript{3}}{(0.1,0.2,0.3)} \right) \right\} \), then \( Q_1 \) is N\textsubscript{\textbullet}RW-closed but it is not an N\textsubscript{\textbullet}N-closed set in U. \( Q_2 = \left\{ \left(\frac{p\subscript{1}}{(0.2,0.3,0.5)}, \frac{p\subscript{2}}{(0.3,0.3,0.5)}, \frac{p\subscript{3}}{(0.2,0.3,0.5)} \right) \right\} \), \( Q_2 \) is N\textsubscript{\textbullet}RW-closed but it is neither N\textsubscript{\textbullet}Regular-closed nor N\textsubscript{\textbullet}\pi-closed set and \( Q_3 = \left\{ \left(\frac{p\subscript{1}}{(0.1,0.3,0.6)}, \frac{p\subscript{2}}{(0.2,0.6,0.6)}, \frac{p\subscript{3}}{(0.1,0.2,0.6)} \right) \right\} \), then \( Q_3 \) is N\textsubscript{\textbullet}RW-closed but not N\textsubscript{\textbullet}W-closed set.

**Proposition 3.5:** (i) Every N\textsubscript{\textbullet}RW-closed set is N\textsubscript{\textbullet}RG-closed.

(ii) Every N\textsubscript{\textbullet}RW-closed set is N\textsubscript{\textbullet}GPR-closed.

(iii) Every N\textsubscript{\textbullet}RW-closed set is N\textsubscript{\textbullet}RGW-closed.

**Proof:** Follows from [4].

The converse of the Proposition 3.4 need not be true.

**Example 3.6:** Let U = \{p\subscript{1}, p\subscript{2}, p\subscript{3}, p\subscript{4}, p\subscript{5}\} be the universe set and the equivalence relation U \setminus R = \{\{p\subscript{1}, p\subscript{3}\}, \{p\subscript{2}\}, \{p\subscript{4}, p\subscript{5}\} \}. Let 
\[ S = \left\{ \left(\frac{p\subscript{1}}{(0.4,0.3,0.5)}, \frac{p\subscript{2}}{(0.5,0.3,0.5)}, \frac{p\subscript{3}}{(0.6,0.3,0.1)}, \frac{p\subscript{4}}{(0.5,0.3,0.1)} \right) \right\} \] be a neutrosophic nano subset of U. \[ N(S) = \left\{ \left(\frac{p\subscript{1}}{(0.5,0.3,0.2)}, \frac{p\subscript{2}}{(0.5,0.3,0.5)}, \frac{p\subscript{3}}{(0.6,0.3,0.1)} \right) \right\}. \]
\[
\left\{ \left( \frac{p_1}{0.4,0.3,0.0}, \frac{p_2}{0.5,0.3,0.1} \right) \right\} \text{ and } B(S) = \left\{ \left( \frac{p_1}{0.4,0.3,0.0}, \frac{p_2}{0.5,0.3,0.1} \right) \right\} \text{. The neutrosophic nano topology } \tau_N = \{0_N, 1_N, N, \overline{N}, B\} \text{. Let } R_1 = \left\{ \left( \frac{p_1}{0.3,0.3,0.7} \right), \left( \frac{p_2}{0.2,0.3,0.6} \right), \left( \frac{p_3}{0.2,0.3,0.5} \right), \left( \frac{p_4}{0.1,0.2,0.7} \right), \left( \frac{p_5}{0.1,0.3,0.8} \right) \right\} \text{. Then } R_1 \text{ is both } N \cap \text{GPR-closed and } N \cap \text{RWG-closed but it is not an } N \cap \text{RW-closed.}
\]

* In example 3.4, let \( R_2 = \left\{ \left( \frac{p_1}{0.3,0.7,0.5} \right), \left( \frac{p_2}{0.3,0.4,0.6} \right), \left( \frac{p_3}{0.2,0.5,0.5} \right), \left( \frac{p_4}{0.1,0.5,0.6} \right), \left( \frac{p_5}{0.1,0.6,0.7} \right) \right\} \), then \( R_2 \) is \( N \cap \text{RG-closed but not an } N \cap \text{RW-closed.}

**Proposition 3.7**: The finite union of \( N \cap \text{RW} \) –closed subsets of \( U \) is also an \( N \cap \text{RW} \) –closed subset of \( U \).

**Proof**: Assume that \( P \) and \( Q \) are \( N \cap \text{RW} \) –closed sets in \( U \). Let \( R \) be an \( N \cap \text{RSO} \) set in \( X \) such that \( P \cup Q \subseteq R \). Then \( P \subseteq R \) and \( Q \subseteq R \). Since \( P \) and \( Q \) are \( N \cap \text{RW} \) –closed sets, \( N \cap \text{Cl}(P) \subseteq R \) and \( N \cap \text{Cl}(Q) \subseteq R \). Then \( N \cap \text{Cl}(P \cup Q) = N \cap \text{Cl}(P) \cup N \cap \text{Cl}(Q) \subseteq R \). Hence \( P \cup Q \) is an \( N \cap \text{RW} \) –closed set in \( U \).

**Remark 3.8**: The intersection of two \( N \cap \text{RW} \)-closed sets in \((U, \tau_N(S))\) need not be an \( N \cap \text{RW} \)-closed set in \( U \).

**Example 3.9**: Let \( U = \{p_1, p_2, p_3, p_4, p_5\} \) be the universe set and the equivalence relation \( U \setminus R = \{\{p_1, p_2\}, \{p_3, p_4\}\} \). Let \( S = \{\left( \frac{p_1}{0.4,0.3,0.4} \right), \left( \frac{p_2}{0.5,0.3,0.5} \right), \left( \frac{p_3}{0.5,0.3,0.5} \right), \left( \frac{p_4}{0.6,0.3,0.1} \right), \left( \frac{p_5}{0.5,0.3,0.1} \right) \} \) be a neutrosophic nano subset of \( U \cap (S) = \{\left( \frac{p_1, p_3}{0.5,0.3,0.5} \right), \left( \frac{p_4, p_5}{0.6,0.3,0.1} \right) \} \), \( N(S) = \{\left( \frac{p_1, p_3}{0.4,0.3,0.4} \right), \left( \frac{p_2}{0.5,0.3,0.5} \right), \left( \frac{p_5}{0.5,0.3,0.1} \right) \} \) and \( B(S) = \{\left( \frac{p_1, p_3}{0.4,0.3,0.4} \right), \left( \frac{p_2}{0.5,0.3,0.5} \right), \left( \frac{p_5}{0.5,0.3,0.1} \right) \} \). The neutrosophic nano topology \( \tau_N = \{0_N, 1_N, N, \overline{N}, B\} \).

\( R_1 = \left\{ \left( \frac{p_1}{0.3,0.3,0.7} \right), \left( \frac{p_2}{0.2,0.3,0.6} \right), \left( \frac{p_3}{0.2,0.3,0.5} \right), \left( \frac{p_4}{0.1,0.2,0.7} \right), \left( \frac{p_5}{0.1,0.3,0.8} \right) \right\} \). Then \( R_1 \) and \( R_2 \) are \( N \cap \text{RW} \)-closed sets but \( R_1 \cap R_2 \) is not an \( N \cap \text{RW} \)-closed set.

**Proposition 3.10**: If a subset \( A \) of \( U \) is \( N \cap \text{RW} \) –closed set in \( U \), then \( N \cap \text{Cl}(A) \setminus A \) does not contain any non-empty neutrosophic nano regular semi-open set in \( U \).

**Proof**: Suppose that \( A \) is an \( N \cap \text{RW} \) –closed set in \( U \). We shall prove by contradiction. Let \( R \) be an \( N \cap \text{RSO} \) set such that \( N \cap \text{Cl}(A) \setminus A \supseteq R \) which implies \( R \subseteq U \setminus A \) i.e., \( A \subseteq U \setminus R \). Since \( R \) is \( N \cap \text{RSO} \), \( U \setminus R \) is also \( N \cap \text{RSO} \) set in \( U \). Since \( A \) is an \( N \cap \text{RW} \) –closed set, \( N \cap \text{Cl}(A) \subseteq U \setminus R \). So \( R \subseteq U \setminus N \cap \text{Cl}(A) \) also \( R \subseteq N \cap \text{Cl}(A) \) implies \( R = \phi \). Hence \( N \cap \text{Cl}(A) \setminus A \) does not contain any non-empty \( N \cap \text{RSO} \) set in \( U \).

The converse of the Proposition 3.10 need not be true as shown in the following example.

**Example 3.11**: In example 3.9, in the neutrosophic nano topological space \((U, \tau_N(S))\), let \( A = \{\left( \frac{p_1}{0.3,0.2,0.5} \right), \left( \frac{p_2}{0.3,0.2,0.6} \right), \left( \frac{p_3}{0.2,0.3,0.5} \right), \left( \frac{p_4}{0.1,0.2,0.7} \right), \left( \frac{p_5}{0.1,0.3,0.8} \right) \} \), then \( N \cap \text{Cl}(A) \setminus A \) does not contain any non-empty \( N \cap \text{RSO} \) set, but \( A \) is not an \( N \cap \text{RW} \)-closed set in \( U \).
Corollary 3.12: If a subset $A$ of $U$ is $N_{NW}$-closed set in $U$, then $N_{NC}(A) \setminus A$ does not contain any non-empty neutrosophic nano regular-open set in $U$.

**Proof:** Follows from the Proposition 3.10 and the fact that every $N_{RO}$ set is $N_{RSO}$ in $U$.

Proposition 3.13: If $A$ is $N_{RO}$ and $N_{NW}$-closed, then $A$ is $N_{RC}$ set and hence $N_{clop}$.

**Proof:** Suppose $A$ is $N_{RO}$ and $N_{NW}$-closed. As every $N_{RO}$ set is $N_{RSO}$ and $A \subseteq A$, we have $N_{NC}(A) \subseteq A$. Also $A \subseteq N_{NC}(A)$, thus $N_{NC}(A) = A$. Hence $A$ is a $N_{clop}$ set. Since $A$ is $N_{RO}$ it is $N_{SO}$ set. Now $N_{NC}(N_{NS}(A)) = N_{NC}(A) = A$. Therefore $A$ is $N_{RC}$ and Neutrosophic nano clopen.

Proposition 3.14: If $A$ is an $N_{NW}$-closed subset of $U$ such that $A \subseteq B \subseteq N_{NC}(A)$, then $B$ is an $N_{NW}$-closed set in $U$.

**Proof:** Let $A$ be an $N_{NW}$-closed subset of $U$ such that $A \subseteq B \subseteq N_{NC}(A)$. Let $R$ be $N_{RSO}$ set of $U$ such that $B \subseteq R$. Then $A \subseteq R$. Since $A$ is $N_{NW}$-closed set, we have $N_{NC}(A) \subseteq R$ and $N_{NC}(B) \subseteq N_{NC}(N_{NC}(A)) \subseteq R$. Therefore $B$ is also an $N_{NW}$-closed set in $U$.

The following example shows that the converse of the Proposition 3.13 need not be true.

Example 3.15: Let $U = \{n_1, n_2, n_3\}$ be the universe set and the equivalence relation $U \setminus R = \{\{n_1, n_3\}, \{n_2\}\}$.

Let $S = \left\{ \left( \frac{x_1}{0.1,0.2,0.3}, \frac{x_2}{0.2,0.3,0.4} \right), \left( \frac{x_3}{0.1,0.6,0.4} \right) \right\}$ be a neutrosophic nano subset of $U$. Then $N(S) = \left\{ \left( \frac{x_1}{0.1,0.6,0.3}, \frac{x_2}{0.2,0.3,0.4} \right), \left( \frac{x_3}{0.1,0.2,0.4} \right) \right\}$, $N(S) = \left\{ \left( \frac{x_1}{0.1,0.2,0.4}, \frac{x_2}{0.2,0.3,0.4} \right) \right\}$ and $B(S) = \left\{ \left( \frac{x_1}{0.1,0.6,0.3}, \frac{x_2}{0.2,0.3,0.4} \right) \right\}$.

So the neutrosophic nano topology $\tau_N = \{0_N, 1_N, N_C, B_C\}$ and the neutrosophic closed sets are $\tau_N = \{0_N, 1_N, N_C, B_C\}$.

Let $A = \left\{ \left( \frac{x_1}{0.1,0.3,0.6}, \frac{x_2}{0.2,0.6,0.6} \right), \left( \frac{x_3}{0.1,0.2,0.6} \right) \right\}$ and $B = \left\{ \left( \frac{x_1}{0.2,0.3,0.3}, \frac{x_2}{0.3,0.6,0.5} \right), \left( \frac{x_3}{0.2,0.3,0.3} \right) \right\}$. Then $A$ and $B$ are $N_{NW}$-closed sets in $(U, \tau_N(S))$, but $A \subseteq B$ is not a subset of $N_{NC}(A)$.

Proposition 3.16: Let $A$ be an $N_{NW}$-closed in $(U, \tau_N(S))$. Then $A$ is $N_{NW}$-closed if and only if $N_{NC}(A) \setminus A$ is $N_{RSO}$.

**Proof:** Let $A$ be an $N_{NW}$-closed in $(U, \tau_N(S))$. Then $N_{NC}(A) \setminus A = \emptyset$ which is $N_{RSO}$.

Conversely, suppose $N_{NC}(A) \setminus A$ is $N_{RSO}$ in $U$. By hypothesis, $A$ is $N_{NW}$-closed implies $N_{NC}(A) \setminus A$ does not contain any non-empty $N_{RSO}$ in $U$. Then $N_{NC}(A) \setminus A = \emptyset$ which implies that $A$ is $N_{NW}$-closed in $U$.

Proposition 3.17: If $A$ is $N_{RO}$ and $N_{RG}$ closed, then $A$ is $N_{NW}$-closed in $U$.

**Proof:** Let $A$ be an $N_{RO}$ and $N_{RG}$-closed. Let $Q$ be any $N_{RSO}$ set in $U$ such that $A \subseteq R$. Since $A$ is $N_{RO}$ and $N_{RG}$ we have $N_{NC}(A) \subseteq A \subseteq R$. Therefore $A$ is $N_{NW}$-closed.
Proposition 3.18: If a subset A of a neutrosophic nano topological space U is both \(N_{NSO}\) and \(N_{NRW}\)-closed, then it is \(N_{NS}\)-closed.

Proof: Suppose A be a subset of a neutrosophic nano topological space U is both \(N_{NSO}\) and \(N_{NRW}\)-closed. Then \(A \subset A\) and \(N_{NC}(A) \subset A\) which implies A is \(N_{NS}\)-closed.

Remark 3.19: The concept of \(N_{NS}\)-RW-closed set is independent with the concepts of (i) \(N_{NS}\)-semi –closed (ii) \(N_{NS}\)-preclosed (iii) \(N_{NS}\)-closed (iv) \(N_{NS}\)-WG - closed sets which is shown by the following example.

Example 3.20: Let \(U = \{n_1, n_2, n_3\}\) be the universe set. \(U \cap R = \{(n_1), (n_2, n_3)\}\) be an equivalence relation.

Let \(S = \{\left(\frac{n_1}{(0,1,0,2,0,3)}, \frac{n_2}{(0,3,0,4,0,5)}, \frac{n_3}{(0,6,0,4,0,1)}\right)\}\) be a neutrosophic nano subset of U. Then \(N(S) = \{\left(\frac{n_1,n_3}{(0,1,0,2,0,3)}, \frac{n_2}{(0,6,0,4,0,1)}\right)\}\), \(N(S) = \{\left(\frac{n_1,n_3}{(0,1,0,2,0,3)}, \frac{n_2}{(0,3,0,4,0,5)}\right)\}\) and \(B(S) = \{\left(\frac{n_1,n_3}{(0,1,0,2,0,3)}, \frac{n_2}{(0,1,0,4,0,6)}\right)\}\) . So the neutrosophic nano topology \(T_N = \{0,1, N, \overline{N}, B\}\) . In the neutrosophic nano topology \((U, T_N(S))\),

- Let \(A = \{\left(\frac{n_1}{(0,2,0,5,0,3)}, \frac{n_2}{(0,1,0,5,0,6)}, \frac{n_3}{(0,1,0,4,0,7)}\right)\}\) and \(B = \{\left(\frac{n_1}{(0,2,0,5,0,3)}, \frac{n_2}{(0,1,0,4,0,7)}, \frac{n_3}{(0,4,0,5,0,4)}\right)\}\), then A is \(N_{NS}\)-semi-closed but not an \(N_{NS}\)-RW-closed and B is \(N_{NS}\)-RW-closed but it is not an \(N_{NS}\)-semi-closed.

- Let \(C = \{\left(\frac{n_1}{(0,1,0,2,0,4)}, \frac{n_2}{(0,3,0,3,0,5)}, \frac{n_3}{(0,1,0,3,0,5)}\right)\}\) and \(D = \{\left(\frac{n_1}{(0,1,0,2,0,4)}, \frac{n_2}{(0,1,0,4,0,7)}, \frac{n_3}{(0,3,0,4,0,4)}\right)\}\), then C is both \(N_{NS}\)-pre-closed and \(N_{NS}\)-WG-closed but not an \(N_{NS}\)-RW-closed and D is \(N_{NS}\)-RW-closed but it is neither \(N_{NS}\)-pre-closed nor an \(N_{NS}\)-WG-closed sets.

- In example 3.8, in the topological space \((U, T_N(S))\), \(E = \{\left(\frac{n_1}{(0,6,0,3,0,3)}, \frac{n_2}{(0,5,0,3,0,3)}, \frac{n_3}{(0,3,0,3,0,3)}, \frac{n_4}{(0,3,0,3,0,3)}, \frac{n_5}{(0,4,0,4,0,1)}\right)\}\) and \(F = \{\left(\frac{n_1}{(0,3,0,3,0,7)}, \frac{n_2}{(0,2,0,3,0,6)}, \frac{n_3}{(0,2,0,3,0,5)}, \frac{n_4}{(0,1,0,2,0,7)}, \frac{n_5}{(0,1,0,3,0,8)}\right)\}\) E is \(N_{NS}\)-RW-closed set but not an \(N_{NS}\)-\(\alpha\)-closed set and F is \(N_{NS}\)-\(\alpha\)-closed but it is not an \(N_{NS}\)-RW-closed set.

Proposition 3.21: If an \(N_{NS}\) subset A is both \(N_{NS}\)-open and \(N_{NS}\)-closed in \((U, T_N(S))\), then it is \(N_{NS}\)-RW-closed in U.

Proof: Let A be \(N_{NS}\)-open and \(N_{NS}\)-closed in U. Let A \(\subset U\) and U be an \(N_{NS}\)-RO in U. Now, A \(\subset A\). By hypothesis, \(N_{NC}(A) \subset U\). Thus A is \(N_{NS}\)-RW-closed.

Remark 3.22: If A is both \(N_{NS}\)-open and \(N_{NS}\)-RW-closed in U, then A need not be \(N_{NS}\)-closed in general which is shown in the following example.

Example 3.23: In example 3.8, the \(N_{NS}\)-open set B is \(N_{NS}\)-RW-closed but it is not an \(N_{NS}\)-closed set.

The above discussions are implicated in the following diagram.
1. \(N_{\text{NRW}}\)-closed  
2. \(N_{\text{NR}}\)-closed  
3. \(N_{\text{NR}}\)-closed  
4. \(N_{\text{N}}\pi\)-closed  
5. \(N_{\text{RSC}}\)-closed  
6. \(N_{\text{NRW}}\)-closed  
7. \(N_{\text{GRP}}\)-closed  
8. \(N_{\text{semi}}\)-closed  
9. \(N_{\text{pre}}\)-closed  
10. \(N_{\text{NRW}}\)-closed  
11. \(N_{\text{NRW}}\)-closed.

**Proposition 3.24:** If a subset \(A\) of a neutrosophic nano topological space \(U\) is both \(N_{\text{NR}}\)-open and \(N_{\text{NRW}}\)-closed, then it is \(N_{\text{NRW}}\)-closed.

**Proof:** Suppose a subset \(A\) of \(U\) is both \(N_{\text{NR}}\)-open and \(N_{\text{NRW}}\)-closed. Let \(A \subset U\) and \(U\) is \(N_{\text{NR}}\)-SO. Then \(N_{\text{R}} \text{Cl}(N_{\text{R}} \text{Int}(A)) = A \subset A\), since \(A\) is \(N_{\text{NR}}\)-open. Hence \(N_{\text{R}} \text{Cl}(A) \subset U\) implies that \(A\) is an \(N_{\text{NRW}}\)-closed in \(U\).

**Definition 3.25:** A neutrosophic nano subset \(A\) of a neutrosophic nano topological space \((U, \tau_{\text{N}}(S))\) is called an \(N_{\text{NRW}}\)-open if and only if its complement \(A^c\) is \(N_{\text{NRW}}\)-closed.

**Proposition 3.26:** An \(N_{\text{NR}}\) set \(A\) of a topological space \((U, \tau_{\text{N}}(S))\) is \(N_{\text{NRW}}\)-open if \(F \subset N_{\text{NR}} \text{Int}(A)\) whenever \(F\) is \(N_{\text{NR}}\)-SO and \(F \subset A\).

**Proof:** Follows from the definition 3.1.

**Proposition 3.27:** Let \(A\) be an \(N_{\text{NRW}}\)-open set of neutrosophic nano topological space \((U, \tau_{\text{N}}(S))\) and \(N_{\text{NR}} \text{Int}(A) \subset B \subset A\). Then \(B\) is \(N_{\text{NRW}}\)-open.

**Proof:** Suppose that \(A\) is an \(N_{\text{NRW}}\)-open in \(U\) and \(N_{\text{NR}} \text{Int}(A) \subset B \subset A\) implies \(A^c \subset B^c \subset N_{\text{NR}} \text{Cl}(A^c)\). Since \(A^c\) is \(N_{\text{NRW}}\)-closed, by Proposition 3.14, \(B^c\) is \(N_{\text{NRW}}\)-closed. Hence \(B\) is \(N_{\text{NRW}}\)-open.

**Proposition 3.28:** Let \((U, \tau_{\text{N}}(S))\) be a neutrosophic nano topological space and \(N_{\text{NR}}\text{SO}(X)\) and \(N_{\text{R}}\text{C}(X)\) be the family of all \(N_{\text{NR}}\text{SO}\) sets and \(N_{\text{R}}\text{C}\) sets respectively. Then \(N_{\text{R}}\text{SO}(X) \subset N_{\text{R}}\text{C}(X)\) if and only if every
neutrosophic nano set of \( U \) is \( N_nRW \)-closed.

**Proof: Necessity:** Suppose that \( N_nRSO(X) \subseteq N_nC(X) \) and let \( A \) be an \( N_n \)-set of \( U \) such that \( A \subseteq R \in N_nRSO(X) \). Then \( N_nCl(A) \subseteq N_nCl(R) = R \), by hypothesis. Hence \( N_nCl(A) \subseteq R \) when \( A \subseteq R \) and \( R \) is \( N_nRSO \) which implies that \( A \) is \( N_nRW \)-closed.

**Sufficiency:** Assume that every neutrosophic nano set of \( U \) is \( N_nRW \)-closed. Let \( R \in N_nRSO(X) \). Then since \( R \subseteq R \) and \( R \) is \( N_nRW \)-closed, \( N_nCl(R) \subseteq R \) then \( R \in N_nCl(X) \). Therefore \( N_nRSO(X) \subseteq N_nCl(X) \).

**Definition 3.29:** A neutrosophic nano topological space \((U, \tau_n(S))\) is called as \( N_nRW \)-connected if there is no proper \( N_n \)-subset of \( U \) which is both \( N_nRW \)-open \( N_nRW \)-closed.

**Proposition 3.30:** Every \( N_nRW \)-connected space is \( N_n \)-connected.

**Proof:** Let \((U, \tau_n(S))\) be an \( N_nRW \)-connected and suppose that \((U, \tau_n(S))\) is not \( N_n \)-connected. Then there exists a proper \( N_n \)-set \( A \) \((A \neq 0_n, A \neq 1_n)\) such that \( A \) is both \( N_n \)-open and \( N_n \)-closed set. Since every \( N_n \)-open and \( N_n \)-closed set is \( N_nRW \)-open and \( N_nRW \)-closed, \((U, \tau_n(S))\) is not an \( N_nRW \)-connected which is a contradiction. This shows that \( U \) is \( N_n \)-connected.

**Proposition 3.31:** A \( N_nT \) space is \( N_nRW \)-connected if and only if there exists no non-zero \( N_nRW \)-open sets \( A \) and \( B \) in \( X \) such that \( A = B^C \).

**Proof:** Suppose that \( A \) and \( B \) are \( N_nRW \)-open sets such that \( A \neq 0_n \neq B \) and \( A = B^C \). Since \( B = A^C \), \( A \) is \( N_nRW \)-closed set and \( B \neq 0_n \) implies \( B^C \neq 1_n \), i.e., \( A \neq 1_n \). Hence there exists a proper \( N_n \)-set \( A \) which is both \( N_nRW \)-open and \( N_nRW \)-closed which is a contradiction to the fact that \( U \) is \( N_nRW \)-connected.

**Sufficiency:** Let \((U, \tau_n(S))\) be an \( N_nTS \) and \( A \) is both \( N_nRW \)-open and \( N_nRW \)-closed set in \( U \) such that \( 0_n \neq A \neq 1_n \). Take \( B = A^C \) implies that \( B \) is \( N_nRW \)-open and \( A \neq 1_n \) \( \Rightarrow B = A^C \neq 0_n \) which is a contradiction. Hence there is no proper \( N_n \)-subset of \( U \) which is both \( N_nRW \)-open and \( N_nRW \)-closed. Therefore \( N_nTS \) \((U, \tau_n(S))\) is \( N_nRW \)-connected.

**Definition 3.32:** A neutrosophic nano topological space \((U,\tau_n(S))\) is said to be an \( N_nRWT_{1/2} \)-space if every \( N_nRW \)-closed set in \( U \) is \( N_n \)-closed in \( U \).

**Proposition 3.33:** A neutrosophic nano topological space \((U,\tau_n(S))\) is \( N_nRWT_{1/2} \) space, then the following statements are equivalent:

(i) \( U \) is \( N_nRW \)-connected
   (ii) \( U \) is \( N_n \)-connected.

**Proof:** (i) \( \Rightarrow \) (ii): Follows from the Proposition 3.29. 

(ii) \( \Rightarrow \) (i): Assume that \( U \) is \( N_nRWT_{1/2} \)-space, and \( N_n \)-connected. Suppose that \( U \) is not an \( N_nRW \)-connected, then there exists a proper \( N_n \)-set \( A \) which is both \( N_nRW \)-open and \( N_nRW \)-closed. Since \((U,\tau_n(S))\) is \( N_nRWT_{1/2} \), \( A \) is both \( N_n \)-open and \( N_n \)-closed which is a contradiction to the fact that \( U \) is \( N_n \)-connected. This shows that \( U \) is \( N_nRW \)-connected.
4. \textbf{N}_\text{N}RW-\text{CONTINUOUS FUNCTIONS}

\textbf{Definition 4.1:} (i) A function \( f: (U, \tau(S)) \rightarrow (V, \tau(T)) \) is said to be a neutrosophic nano RW-continuous (\textbf{In short N}_\text{N}RW-\text{continuous}) if the inverse image of \( \text{N}_\text{N} \)-closed set of \( V \) is \( \text{N}_\text{N}RW \)-closed in \((U, \tau(S))\).

(ii) A function \( f: (U, \tau(S)) \rightarrow (V, \tau(T)) \) is said to be a neutrosophic nano RW-irresolute (\textbf{In short N}_\text{N}RW-\text{irresolute}) if the inverse image of \( \text{N}_\text{N}RW \)-closed set of \( V \) is \( \text{N}_\text{N}RW \)-closed in \((U, \tau(S))\).

\textbf{Proposition 4.2:} A mapping \( f: (U, \tau(S)) \rightarrow (V, \tau(T)) \) is \( \text{N}_\text{N}RW \)-continuous if and only if the inverse image of every \( \text{N}_\text{N} \)-open set of \( V \) is \( \text{N}_\text{N}RW \)-open in \( U \).

\textbf{Proof:} It is obvious because \( f^{-1}(A^c) = [f^{-1}(A)]^c \) for every \( \text{N}_\text{N} \)-set \( A \) of \( V \).

\textbf{Proposition 4.3:} If \( f: (U, \tau(S)) \rightarrow (V, \tau(T)) \) is \( \text{N}_\text{N}RW \)-continuous, then \( f(\text{N}_\text{N}RWCl(A)) \subseteq \text{N}_\text{N}Cl(f(A)) \) for every \( \text{N}_\text{N} \)-set \( A \) of \( U \).

\textbf{Proof:} Let \( A \) be an \( \text{N}_\text{N} \)-set of \( U \). Then \( \text{N}_\text{N}Cl(f(A)) \) is an \( \text{N}_\text{N} \)-closed set of \( V \). Since \( f \) is an \( \text{N}_\text{N}RW \)-continuous function, \( f^{-1}(\text{N}_\text{N}Cl(f(A))) \) is \( \text{N}_\text{N}RW \)-closed in \( U \). Clearly \( A \subseteq f^{-1}(\text{N}_\text{N}Cl(f(A))) \). Therefore \( \text{N}_\text{N}RWCl(A) \subseteq \text{N}_\text{N}RWCl \left(f^{-1}(\text{N}_\text{N}Cl(f(A)))\right) = f^{-1}(\text{N}_\text{N}Cl(f(A))) \). Hence \( f(\text{N}_\text{N}RWCl(A)) \subseteq \text{N}_\text{N}Cl(f(A)) \) for every \( \text{N}_\text{N} \)-set \( A \) of \( U \).

\textbf{Proposition 4.4:} (i) Every \( \text{N}_\text{N} \)-continuous map is \( \text{N}_\text{N}RW \)-continuous.

(ii) Every \( \text{N}_\text{N} \)-regular continuous map is \( \text{N}_\text{N}RW \)-continuous.

(iii) Every \( \text{N}_\text{N} \)-\( \pi \)-continuous set is \( \text{N}_\text{N}RW \)-continuous.

(iv) Every \( \text{N}_\text{N}W \)-continuous map is \( \text{N}_\text{N}RW \)-continuous.

(v) Every \( \text{N}_\text{N}RW \)-irresolute map is \( \text{N}_\text{N}RW \)-continuous.

\textbf{Proof:} Obvious.

\textbf{Remark 4.4:} The following example makes clear that the converse of the Proposition 4.4 may not be true.

\textbf{Example 4.5:} Let \( U = \{n_1, n_2, n_3\} = V \) be the universe sets. \( U \setminus R_1 = \{(n_1), (n_2, n_3)\} \) and \( U \setminus R_2 = \{(n_1, n_3), (n_2)\} \) be equivalence relations. Let \( S_1 = \{\left(\frac{n_1}{(0.3, 0.4, 0.3)}, \frac{n_2}{(0.6, 0.3, 0.1)}\right), \left(\frac{n_3}{(0.2, 0.6, 0.2)}\right)\} \) and \( S_2 = \{\left(\frac{n_1}{(0.1, 0.2, 0.3)}, \frac{n_2}{(0.2, 0.3, 0.4)}\right), \left(\frac{n_3}{(0.1, 0.6, 0.4)}\right)\} \) be neutrosophic nano subsets of \( U \). Then \( \tau(S_1) = [0, \overline{N}(S_1), N(S_1), B(S_1), 1_N] \) and \( \tau(S_2) = [0, \overline{N}(S_2), N(S_2), B(S_2), 1_N] \) be the neutrosophic nano topologies on \( U \) and \( V \) respectively. Define an identity map \( f: (U, \tau(S_1)) \rightarrow (V, \tau(S_2)) \). Then \( f \) is \( \text{N}_\text{N}RW \)-continuous but is neither \( \text{N}_\text{N} \)-continuous nor \( \text{N}_\text{N}W \)-continuous. Similarly it’s not an \( \text{N}_\text{N}R \)-continuous, \( \text{N}_\text{N} \pi \)-continuous and \( \text{N}_\text{N}RW \)-irresolute.

\textbf{Proposition 4.6:} (i) Every \( \text{N}_\text{N}RW \)-continuous map is \( \text{N}_\text{N}RG \)-continuous.

(ii) Every \( \text{N}_\text{N}RW \)-continuous map is \( \text{N}_\text{N}GPR \)-continuous.
(iii) Every $N_{N\text{RW}}$-continuous map is $N_{N\text{RWG}}$-continuous.

**Proposition 4.7:** If $f: (U, \tau_N(S)) \to (V, \tau_N(T))$ is $N_{N\text{RW}}$-continuous and $g: (V, \tau_N(T)) \to (W, \tau_N(R))$ is $N_{N\text{N}}$-continuous. Then $g \circ f: (U, \tau_N(S)) \to (W, \tau_N(R))$ is $N_{N\text{RW}}$-continuous.

**Proof:** Let $A$ be an $N_{N}$-closed in $W$. Then $g^{-1}(A)$ is $N_{N}$-closed in $V$, because $g$ is $N_{N}$-continuous. Therefore $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is $N_{N\text{RW}}$-closed in $U$. Hence $g \circ f$ is $N_{N\text{RW}}$-continuous.

**Proposition 4.8:** If $f: (U, \tau_N(S)) \to (V, \tau_N(T))$ is $N_{N\text{RW}}$-continuous and $g: (V, \tau_N(T)) \to (W, \tau_N(R))$ is $N_{N\text{RG}}$-continuous and $(V, \tau_N(T))$ is N-T clopen, then $f \circ g: (U, \tau_N(S)) \to (W, \tau_N(R))$ is $N_{N\text{RW}}$-continuous.

**Proof:** Let $A$ be an $N_{N}$-closed set in $W$, then $g^{-1}(A)$ is $N_{N\text{RG}}$-closed in $V$. Since $V$ is N-T clopen then $g^{-1}(A)$ is $N_{N}$-closed in $V$. Hence, $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is $N_{N\text{RW}}$-closed in $U$. Hence $g \circ f$ is $N_{N\text{RW}}$-continuous.

**Proposition 4.9:** If $f: (U, \tau_N(S)) \to (V, \tau_N(T))$ is $N_{N\text{RG}}$-irresolute and $g: (V, \tau_N(V)) \to (W, \tau_N(R))$ is $N_{N\text{RW}}$-continuous, then $g \circ f: (U, \tau_N(S)) \to (W, \tau_N(R))$ is $N_{N\text{RG}}$-continuous.

**Proof:** Let $A$ be an $N_{N}$-closed set in $W$, then $g^{-1}(A)$ is $N_{N\text{RG}}$-closed in $V$, since $g$ is $N_{N\text{RW}}$-continuous. Every $N_{N\text{RW}}$-closed set is $N_{N\text{RG}}$-closed, $g^{-1}(V)$ is $N_{N\text{RG}}$-closed set in $V$. Then $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$ is $N_{N\text{RG}}$-closed in $U$, by hypothesis. Hence $g \circ f: (U, \tau_N(S)) \to (W, \tau_N(R))$ is $N_{N\text{RG}}$-continuous.

**Proposition 4.10:** If $f: (U, \tau_N(S)) \to (V, \tau_N(T))$ is $N_{N\text{RW}}$-continuous surjection and $U$ is $N_{N\text{RW}}$-connected then $V$ is $N_{N}$-connected.

**Proof:** Assume that $V$ is not an $N_{N}$-connected space. Then there exists a proper $N_{N}$-subset $F$ of $V$ which is both $N_{N}$-open and $N_{N}$-closed. Therefore, by hypothesis, $f^{-1}(F)$ is a proper $N_{N}$-set of $U$ which is both $N_{N\text{RW}}$-open and $N_{N\text{RW}}$-closed in $U$ implies that $U$ is not an $N_{N\text{RW}}$-connected which is a contradiction. This shows that $V$ is $N_{N}$-connected.

**Definition 4.11:** (i) A mapping $f: (U, \tau_N(S)) \to (V, \tau_N(T))$ is said to be $N_{N\text{RW}}$-open map if the image of every $N_{N}$-open set of $U$ is $N_{N\text{RW}}$-open set in $V$.

(ii) A mapping $f: (U, \tau_N(S)) \to (V, \tau_N(T))$ is said to be $N_{N\text{RW}}$-closed map if the image of every $N_{N}$-closed set of $U$ is $N_{N\text{RW}}$-closed set in $V$.

**Proposition 4.12:** A mapping $f: (U, \tau_N(S)) \to (V, \tau_N(T))$ is $N_{N\text{RW}}$-open if and only if for every $N_{N}$-set $A$ of $U$, $f(N_N\text{Int}(A)) \subseteq N_{N\text{RW Int}}(f(A))$.

**Proof:** **Necessity:** Let $f$ be an $N_{N\text{RW}}$-open map and $A$ is an $N_{N}$-open set in $U$, $N_N\text{Int}(A) \subseteq A$ which implies that $f(N_N\text{Int}(A)) \subseteq f(A)$. Since $f$ is an $N_{N\text{RW}}$-open mapping, $f(N_N\text{Int}(A))$ is $N_{N\text{RW}}$-open set in $V$ such that $f(N_N\text{Int}(A)) \subseteq f(A)$. Therefore $f(N_N\text{Int}(A)) \subseteq N_{N\text{RW Int}}(f(A))$.

**Sufficiency:** Suppose that $A$ is an $N_{N}$-open set of $U$. Then $f(A) = f(N_N\text{Int}(A)) \subseteq N_{N\text{RW Int}}(f(A))$. But $N_{N\text{RW Int}}(f(A)) \subseteq f(A)$. Consequently $f(A) = N_{N\text{RW Int}}(A)$ which implies that $f(A)$ is an $N_{N\text{RW}}$-open set of $V$ and hence $f$ is an $N_{N\text{RW}}$-open map.
Proposition 4.13: A mapping \( f: (U, \tau_S(S)) \to (V, \tau_T(T)) \) is \( \text{N}_{\text{N}-\text{RW}} \)-open if and only if for every neutrosophic nano set \( A \) of \( V \) and for each \( \text{N}_{\text{N}} \)-closed set \( B \) of \( U \) containing \( f^{-1}(A) \) there is a \( \text{N}_{\text{N}-\text{RW}} \)-closed set \( F \) of \( V \) such that \( A \subseteq F \) and \( f^{-1}(F) \subseteq B \).

Proof: Necessity: Suppose that \( f \) is \( \text{N}_{\text{N}-\text{RW}} \)-open map. Let \( A \) be a \( \text{N}_{\text{N}} \)-closed set of \( V \) and \( B \) be a \( \text{N}_{\text{N}} \)-set of \( U \) such that \( f^{-1}(A) \subseteq B \). Then \( F = f^{-1}(B^c) \) is a \( \text{N}_{\text{N}-\text{RW}} \)-closed set of \( V \) such that \( f^{-1}(F) \subseteq B \).

Sufficiency: Let \( F \) be \( \text{N}_{\text{N}} \text{-O} \) set of \( U \). Then \( f^{-1}(f(F))^c \subseteq F^c \) and \( F^c \) is \( \text{N}_{\text{N}} \)-set in \( X \). By hypothesis there is an \( \text{N}_{\text{N}-\text{RW}} \)-closed set \( G \) of \( V \) such that \( (f(F))^c \subseteq G \) and \( f^{-1}(G) \subseteq F^c \). Therefore \( F \subseteq (f^{-1}(G))^c \). Hence \( G^c \subseteq f^{-1}(F) \subseteq (f^{-1}(G))^c \subseteq G^c \). i.e., \( f(F) = G^c \) which is \( \text{N}_{\text{N}-\text{RW}} \)-open in \( V \) and thus \( f \) is \( \text{N}_{\text{N}-\text{RW}} \)-open map.

Proposition 4.14: If a mapping \( f: (U, \tau_S(S)) \to (V, \tau_T(T)) \) is \( \text{N}_{\text{N}-\text{RW}} \)-open, then \( \text{N}_{\text{N}} \text{Int}(f^{-1}(G)) \subseteq f^{-1}(\text{N}_{\text{N}-\text{RW}} \text{Int}(G)) \) for every neutrosophic nano set \( G \) of \( Y \).

Proof: Let \( G \) be neutrosophic nano set of \( V \). Then \( \text{N}_{\text{N}} \text{Int}(f^{-1}(G)) \) is \( \text{N}_{\text{N}} \)-set in \( U \). Since \( f \) is \( \text{N}_{\text{N}-\text{RW}} \)-open \( f \text{N}_{\text{N}} \text{Int}(f^{-1}(G)) \subseteq \text{N}_{\text{N}-\text{RW}} \text{Int}(f(f^{-1}(G))) \subseteq \text{N}_{\text{N}-\text{RW}} \text{Int}(G) \). Thus \( \text{N}_{\text{N}} \text{Int}(f^{-1}(G)) \subseteq f^{-1}(\text{N}_{\text{N}-\text{RW}} \text{Int}(G)) \).

Proposition 4.15: A mapping \( f: (U, \tau_S(S)) \to (V, \tau_T(T)) \) is \( \text{N}_{\text{N}-\text{RW}} \)-closed if and only if for every neutrosophic nano set \( A \) of \( V \) and for each \( \text{N}_{\text{N}} \)-set \( B \) of \( U \) containing \( f^{-1}(A) \) there is a \( \text{N}_{\text{N}-\text{RW}} \)-open set \( F \) of \( V \) such that \( A \subseteq F \) and \( f^{-1}(F) \subseteq B \).

Proof: Necessity: Suppose that \( f \) is \( \text{N}_{\text{N}} \)-almost irresolute and \( \text{N}_{\text{N}-\text{RW}} \)-closed map. Let \( A \) be a \( \text{N}_{\text{N}} \)-set of \( V \) and \( B \) be a \( \text{N}_{\text{N}} \)-set of \( U \) such that \( f^{-1}(A) \subseteq B \). Then \( F = V \setminus f^{-1}(B) \) is a \( \text{N}_{\text{N}-\text{RW}} \)-open set of \( V \) such that \( f^{-1}(F) \subseteq B \).

Sufficiency: Let \( F \) be a \( \text{N}_{\text{N}} \)-set of \( U \). Then \( f^{-1}(f(F))^c \subseteq F^c \) and \( F^c \) is \( \text{N}_{\text{N}} \)-set in \( X \). By hypothesis there is an \( \text{N}_{\text{N}-\text{RW}} \)-open set \( R \) of \( V \) such that \( (f(F))^c \subseteq R \) and \( f^{-1}(R) \subseteq F^c \). Therefore \( F \subseteq (f^{-1}(R))^c \). Hence \( R^c \subseteq f^{-1}(F) \subseteq (f^{-1}(R))^c \subseteq R^c \) i.e., \( f(F) = R^c \) which is \( \text{N}_{\text{N}-\text{RW}} \)-closed in \( V \). Thus \( f \) is \( \text{N}_{\text{N}-\text{RW}} \)-closed map.

Proposition 4.16: If \( f: (U, \tau_S(S)) \to (V, \tau_T(T)) \) is \( \text{NN} \)-almost irresolute and \( \text{N}_{\text{N}-\text{RW}} \)-closed map. If \( A \) is \( \text{N}_{\text{N}-\text{RW}} \)-closed set of \( U \), then \( f(A) \) is \( \text{N}_{\text{N}-\text{RW}} \)-closed in \( V \).

Proof: Let \( f(A) \subseteq R \) where \( R \) is an \( \text{N}_{\text{N}} \text{-RSO} \) set of \( V \), since \( f \) is an \( \text{N}_{\text{N}} \)-almost irresolute, \( f^{-1}(R) \) is an \( \text{N}_{\text{N}} \)-set of \( U \) such that \( A \subseteq f^{-1}(R) \). Since \( A \) is \( \text{N}_{\text{N}} \)-closed set of \( U \) which implies that \( \text{N}_{\text{N}} \text{Cl}(A) \subseteq f^{-1}(R) \Rightarrow f(\text{N}_{\text{N}} \text{Cl}(A)) \subseteq R \), i.e., \( \text{N}_{\text{N}} \text{Cl}(f(\text{N}_{\text{N}} \text{Cl}(A))) \subseteq R \). Therefore \( \text{N}_{\text{N}} \text{Cl}(f(A)) \subseteq R \) whenever \( f(A) \subseteq R \) where \( R \) is an \( \text{N}_{\text{N}} \text{-RSO} \) set of \( V \). Hence \( f(A) \) is an \( \text{N}_{\text{N}-\text{RW}} \)-closed set of \( V \).

Proposition 4.17: If \( f: (U, \tau_S(S)) \to (V, \tau_T(T)) \) is \( \text{N}_{\text{N}} \)-closed and \( g: (V, \tau_T(T)) \to (W, \tau_T(R)) \) is \( \text{N}_{\text{N}-\text{RW}} \)-closed then \( g \circ f: (U, \tau_S(S)) \to (W, \tau_T(R)) \) is \( \text{N}_{\text{N}-\text{RW}} \)-closed.

Proof: Let \( F \) be an \( \text{N}_{\text{N}} \)-closed set of neutrosophic nano topological space \( (U, N(S)) \). Then \( f(F) \) is an \( \text{N}_{\text{N}} \)-closed set of \( (V, \tau_T(T)) \). By hypothesis, \( g \circ f(F) = g(f(F)) \) is an \( \text{N}_{\text{N}-\text{RW}} \)-closed set in \( \text{N}_{\text{N}} \)-topological space \( W \). Thus \( g \circ f: (U, \tau_S(S)) \to (W, \tau_T(R)) \) is \( \text{N}_{\text{N}-\text{RW}} \)-closed.

Conclusions: In this article, the authors have introduced and studied the concepts such as, Neutrosophic nano RW-closed set, \( \text{N}_{\text{N}-\text{RW}} \)-open set, \( \text{N}_{\text{N}-\text{RW}} \text{-T}1/2 \) space, \( \text{N}_{\text{N}-\text{RW}} \)-connected space, \( \text{N}_{\text{N}-\text{RW}} \)-continuous, \( \text{N}_{\text{N}-\text{RW}} \)- irresolute, \( \text{N}_{\text{N}-\text{RW}} \)-open and \( \text{N}_{\text{N}} \)-closed maps. In future it can be extended to...
some new forms of continuous functions and homeomorphisms.

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