

12-11-2021

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Şahin, Memet; Abdullah Kargin; and Merve Sena Uz. "Generalized Euclid Measures Based on Generalized Set Valued Neutrosophic Quadruple Numbers and Multi Criteria Decision Making Applications." *Neutrosophic Sets and Systems* 47, 1 (2021). https://digitalrepository.unm.edu/nss_journal/vol47/iss1/37

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Generalized Euclid Measures Based on Generalized Set Valued Neutrosophic Quadruple Numbers and Multi Criteria Decision Making Applications

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Abstract: In this article, we define firstly generalized Euclid distance measure and generalized Euclid similarity measure based on generalized set valued neutrosophic quadruple numbers. Also, we show that generalized Euclid distance measure and generalized Euclid similarity measure satisfy the distance measure conditions and similarity measure conditions, respectively. Furthermore, we define a score function for generalized Euclid similarity measure. In addition, we generalize an algorithm, for single valued neutrosophic set, based on generalized Euclid similarity measure and generalized set valued neutrosophic quadruple numbers. Also, we give a multi criteria decision making applications for this generalized algorithm. This application based on patients, diseases, drugs and this application is different from previous applications because of generalized Euclid similarity measure and generalized set valued neutrosophic quadruple numbers. Furthermore, this application has different result according to some previous similarity measure. Thus, this application can be used for covid-19 treatment due to structure of generalized Euclid similarity measure and generalized set valued neutrosophic quadruple numbers

Keywords: Distance measure, similarity measure, Euclid measures, generalized set valued neutrosophic quadruple numbers, generalized Euclid distance measure, generalized Euclid similarity measure

1 Introduction

Many uncertainties arise in daily life. Most of the time, Aristotle logic (classical logic) is insufficient to explain these uncertainties mathematically. Fuzzy logic [1] and intuitionistic fuzzy logic [2] were defined to deal with uncertainties. However, in these structures, membership functions were defined as dependent on each other. Finally, neutrosophic logic and sets [3], (T, I, F) membership functions are independent of each other, were defined by Smarandache. Thus, uncertainties are taken into calculations more precisely. Due to this advantage, many studies have been carried out in both algebra and application areas by using neutrosophic sets [4-11]. In particular, decision making applications have found more application areas with the definition of neutrosophic sets and more precise results have been obtained. Therefore, many decision-making applications have been obtained. Recently, Hashmi et al. studied multi-criteria decision-making in medical diagnosis for m-Polar neutrosophic topology [12]; Khalil et al. introduced decision making applications for the single-valued neutrosophic fuzzy set and the soft set [13]; Olgun et al. studied neutrosophic logic on the decision tree [14]. Also, similarity measures defined for neutrosophic sets have an important place in these applications, and these similarity measures have been used in many studies [15-18]. Recently, Mukherjee et al. obtained several similarity measures for neutrosophic soft sets [19]; Saqlain et al. studied tangent similarity measure of single valued neutrosophic hypersoft sets [20]; Şahin and Kargin introduced decision making applications in professional proficiencies based on new similarity measure for single valued neutrosophic sets [21]; Saqlain et al. studied distance and similarity measures for neutrosophic HyperSoft Set with construction of NHSS-TOPSIS and applications [46]. Also, hybrid of neutrosophic numbers

and methods have very important place in decision making applications [40-43]. Recently, Abdel-Monem and Gawad studied a hybrid model using MCDM Methods and bipolar neutrosophic sets for select optimal wind turbine: case study in Egypt [44]; Fahmi obtained group decision based on trapezoidal neutrosophic Dombi fuzzy hybrid operator [45].

Neutrosophic quadruple sets [22], which are a generalized form of neutrosophic sets, were defined by Smarandache in 2015. Unlike neutrosophic sets, neutrosophic quadruple set contain a known part and an unknown part. However, known membership functions (T, I, F) are located in sets of neutrosophic quadruple set. A neutrosophic quadruple set is denoted by

$$\{(k, IT, mI, nF): k, l, m, n \in \mathbb{R} \text{ or } \mathbb{C}\}$$

Here, k is referred to as the known part, (IT, mI, nF) as the unknown part. With the help of this definition, many algebraic structures are reconsidered in the neutrosophic quadruple theory [23-32].

Also, set valued neutrosophic quadruple sets [33] and generalized set valued neutrosophic quadruple sets [34] have been defined in order to use neutrosophic quadruple sets in application studies. A generalized set valued neutrosophic quadruple set denoted by

$$G_{S_i} = \{(K_{S_i}, L_{S_i}T_{S_i}, M_{S_i}I_{S_i}, N_{S_i}F_{S_i}): K_{S_i}, L_{S_i}, M_{S_i}, N_{S_i} \in P(X); i = 1, 2, 3, \dots, n\}.$$

Where T_i , I_i and F_i have their usual neutrosophic logic; K_{S_i} is called the known part and $(L_{S_i}T_{S_i}, M_{S_i}I_{S_i}, N_{S_i}F_{S_i})$ is called the unknown part. Thanks to this definition, neutrosophic quadruple sets have become available in the field of application. Most importantly, this definition, which has a more general structure than neutrosophic sets, will find more application areas and will give more objective results to many problems with the help of the known part and unknown part. Recently, Kandasamy et al. studied neutrosophic quadruple algebraic codes over \mathbb{Z}_2 [35]; Ma et al. obtained neutrosophic quadruple rings [36]; Mohseni et al. introduced commutative neutrosophic quadruple ideals [37]; Rezaei et al. studied neutrosophic quadruple a-ideals [38]; Kargin et al. obtained generalized Hamming similarity measure based on neutrosophic quadruple numbers [39]; Şahin et al. studied Hausdorff Measures on generalized set valued neutrosophic quadruple numbers [40].

While the treatment of many diseases is known in the field of medicine, there are still diseases whose treatment is not fully known. It is also clear that new drugs can be found for the treatment of unknown diseases based on the treatment of known diseases. However, treating patients struggling with more than one disease can become even more complex. Because, in addition to the unknown treatment, they will have separate medications for other diseases. It is clear that in solving such problems there is a need for a structure in which the known part is the unknown part and (T, I, F) known neutrosophic membership functions. Since each known disease will have separate medications and it will be investigated which results (true, indeterminate, false) these drugs will give in unknown diseases, a structure such as $(L_{S_i}T_{S_i}, M_{S_i}I_{S_i}, N_{S_i}F_{S_i})$, containing both cluster and T, I, F, will be needed. For this reason, using generalized Euclid measures based on generalized set valued neutrosophic quadruple numbers in solving such problems can give better results. So, we give a multi criteria decision making application based on generalized algorithm for solving such problems. Also, this application is different from previous applications because of generalized Euclid similarity measure and generalized set valued neutrosophic quadruple numbers. Furthermore, this application has different result according to some previous similarity measure. Thus, this application can be used for covid-19 treatment due to structure of generalized Euclid similarity measure and generalized set valued neutrosophic quadruple numbers.

In this paper; in 2. Section, we give some information for neutrosophic sets, some similarity measures, generalized set valued neutrosophic quadruple sets and numbers, In 3. Section, we define firstly generalized Euclid distance measure and generalized Euclid similarity measure based on generalized set valued neutrosophic quadruple numbers. Also, we show that generalized Euclid distance measure and generalized Euclid similarity measure satisfy the distance measure conditions and similarity measure conditions, respectively. Furthermore, we define a score function for generalized Euclid similarity measure. Then we give examples for generalized Euclid distance measure, generalized Euclid similarity measure and score function. In 4. Section, we define a generalized algorithm and multi criteria decision making application based on generalized Euclid similarity measure and generalized set valued neutrosophic quadruple numbers. In 5. Section, we give conclusions.

2 Preliminaries

In this chapter, we give some information for neutrosophic sets, some similarity measures, generalized set valued neutrosophic quadruple sets and numbers.

Definition 2.1: [3] Let E be the universal set. For $\forall x \in E$,

$$0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$$

by the help of the functions $T_A: E \rightarrow]0, 1^+[$, $I_A: E \rightarrow]0, 1^+[$ and $F_A: E \rightarrow]0, 1^+[$ a neutrosophic set A on E is defined by

$$A = \{\langle x, T_A(x), I_A(x), F_A(x) \rangle: x \in E\}.$$

Here, $T_A(x)$, $I_A(x)$ and $F_A(x)$ are the degrees of truth, indeterminacy and falsity of $x \in E$ respectively. Also, for $\varepsilon > 0$, $0^- = 0 - \varepsilon$ and $1^+ = 1 + \varepsilon$.

Definition 2.2: [4] Let E be the universal set. For $\forall x \in E$,

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$$

using the functions $T_A: E \rightarrow [0,1]$, $I_A: E \rightarrow [0,1]$ and $F_A: E \rightarrow [0,1]$, a single-valued neutrosophic set A on E is defined by

$$A = \{\langle x, T_A(x), I_A(x), F_A(x) \rangle: x \in E\}.$$

Here, $T_A(x)$, $I_A(x)$ and $F_A(x)$ are the degrees of truth, indeterminacy and falsity of $x \in E$ respectively.

Also, a single valued neutrosophic number is denoted by

$$B = \langle T_B(x), I_B(x), F_B(x) \rangle.$$

Definition 2.3: [17] Let E be an universal set,

$$A_1 = \{x_i, \langle T_{A_1}(x_i), I_{A_1}(x_i), F_{A_1}(x_i) \rangle: x_i \in E\} \text{ and}$$

$A_2 = \{x_i, \langle T_{A_2}(x_i), I_{A_2}(x_i), F_{A_2}(x_i) \rangle: x_i \in E\}$ be two single – valued neutrosophic sets. The Euclid similarity measure between A_1 and A_2 , is defined by

$$S_E(A_1, A_2) = 1 - \sqrt{\frac{\sum_{i=1}^n \frac{(T_{A_1}(x_i) - T_{A_2}(x_i))^2 + (I_{A_1}(x_i) - I_{A_2}(x_i))^2 + (F_{A_1}(x_i) - F_{A_2}(x_i))^2}{3}}{n}}.$$

Also, The Euclid distance measure between A_1 and A_2 , is defined by

$$d_E(A_1, A_2) = \sqrt{\frac{\sum_{i=1}^n \frac{(T_{A_1}(x_i) - T_{A_2}(x_i))^2 + (I_{A_1}(x_i) - I_{A_2}(x_i))^2 + (F_{A_1}(x_i) - F_{A_2}(x_i))^2}{3}}{n}}.$$

Definition 2.4: [17] Let E be an universal set,

$$A_1 = \{x_i, \langle T_{A_1}(x_i), I_{A_1}(x_i), F_{A_1}(x_i) \rangle: x_i \in E\} \text{ and}$$

$A_2 = \{x_i, \langle T_{A_2}(x_i), I_{A_2}(x_i), F_{A_2}(x_i) \rangle: x_i \in E\}$ be two single – valued neutrosophic sets. The Hamming similarity measure between A_1 and A_2 , is defined by

$$S_{hd} = (A_1, A_2) = 1 - \frac{1}{3} \left[\sum_{i=1}^n |T_{A_1}(x_j) - T_{A_2}(x_j)| + |I_{A_1}(x_j) - I_{A_2}(x_j)| + |F_{A_1}(x_j) - F_{A_2}(x_j)| \right]$$

Theorem 2.5: [17] Let X_1, X_2 and X_3 be three single – valued neutrosophic sets, d_E be a distance measure. Then the following properties hold.

- i. $0 \leq d_E(X_1, X_2) \leq 1$
- ii. $X_1 = X_2$ if and only if $d_E(X_1, X_2) = 0$
- iii. $d_E(X_1, X_2) = d_E(X_2, X_1)$
- iv. If $X_1 \subseteq X_2 \subseteq X_3$, then $d_E(X_1, X_2) \leq d_E(X_1, X_3)$ and $d_E(X_2, X_3) \leq d_E(X_1, X_3)$.

Theorem 2.6: [17] Let X_1, X_2 and X_3 be three single – valued neutrosophic sets, S be a similarity measure. Then the following properties hold.

- i. $0 \leq S(A_1, A_2) \leq 1$
- ii. $S(A_1, A_2) = 1 \Leftrightarrow A_1 = A_2$
- iii. $S(A_1, A_2) = S(A_2, A_1)$
- iv. If $A_1 \subseteq A_2 \subseteq A_3 \in E$, then $(A_1, A_3) \leq S(A_1, A_2)$ and $S(A_1, A_3) \leq S(A_2, A_3)$.

Definition 2.7: [5] Let $A_1 = \langle T_1, I_1, F_1 \rangle$ and $A_2 = \langle T_2, I_2, F_2 \rangle$ be two single-valued neutrosophic numbers. Let's define the measure of similarity between A_1 and A_2 as follows

$$S_N(A_1, A_2) = 1 - 2/3 \cdot \left\{ \frac{\min\{\sqrt{3(T_1-T_2)^2 + (I_1-I_2)^2}, |2(T_1-T_2)-(I_1-I_2)|/3\}}{\{\max\{\sqrt{3(T_1-T_2)^2 + (I_1-I_2)^2}, |2(T_1-T_2)-(I_1-I_2)|/3\}/2\}+1} \right. \\ + \frac{\min\{\sqrt{3(T_1-T_2)^2 + (F_1-F_2)^2}, |2(T_1-T_2)-(F_1-F_2)|/3\}}{\{\max\{\sqrt{3(T_1-T_2)^2 + (F_1-F_2)^2}, |2(T_1-T_2)-(F_1-F_2)|/3\}/2\}+1} \\ \left. + \frac{\min\{\sqrt{2(T_1-T_2)^2 + (I_1-I_2)^2 + (F_1-F_2)^2}, |3(T_1-T_2)-(I_1-I_2)-(F_1-F_2)|/5\}}{\{\max\{\sqrt{2(T_1-T_2)^2 + (I_1-I_2)^2 + (F_1-F_2)^2}, |3(T_1-T_2)-(I_1-I_2)-(F_1-F_2)|/5\}/2\}+1} \right\}$$

Definition 2.8: [34] Let X be a set and $P(X)$ be power set of X . A generalized set – valued neutrosophic quadruple set is a set of the form

$$G_{S_i} = \{(A_{S_i}, B_{S_i}T_{S_i}, C_{S_i}I_{S_i}, D_{S_i}F_{S_i}) : A_{S_i}, B_{S_i}, C_{S_i}, D_{S_i} \in P(X); i = 1, 2, 3, \dots, n\}.$$

Where T_i, I_i and F_i have their usual neutrosophic logic means and generalized set – valued neutrosophic quadruple number defined by

$$G_{N_i} = (A_{S_i}, B_{S_i}T_{S_i}, C_{S_i}I_{S_i}, D_{S_i}F_{S_i}).$$

As in neutrosophic quadruple number, for a generalized set – valued neutrosophic quadruple number $(A_{S_i}, B_{S_i}T_{S_i}, C_{S_i}I_{S_i}, D_{S_i}F_{S_i})$ representing any entity which may be a number, an idea, an object, etc.; A_{S_i} is called the known part and $(B_{S_i}T_{S_i}, C_{S_i}I_{S_i}, D_{S_i}F_{S_i})$ is called the unknown part.

Definition 2.9: [34] Let $G_{N_i} = (A_{S_i}, B_{S_i}T_{S_i}, C_{S_i}I_{S_i}, D_{S_i}F_{S_i})$ and $G_{N_j} = (A_{S_j}, B_{S_j}T_{S_j}, C_{S_j}I_{S_j}, D_{S_j}F_{S_j})$ be two generalized set – valued neutrosophic quadruple numbers. $A_{S_i} \subseteq A_{S_j}, A_{S_i} \subseteq A_{S_j}, A_{S_i} \subseteq A_{S_j}, A_{S_i} \subseteq A_{S_j}$ and $T_{S_i} \leq T_{S_j}, I_{S_i} \leq I_{S_j}, F_{S_i} \leq F_{S_j}$ if and only if we say G_{N_i} is a subset of G_{N_j} and denote it by $G_{N_i} \subseteq G_{N_j}$.

3 Generalized Euclid Measures Based on Generalized Set Valued Neutrosophic Quadruple Numbers

In this chapter, we define firstly generalized Euclid distance measure and generalized Euclid similarity measure based on generalized set-valued neutrosophic quadruple numbers.

Also, in this paper we assume that $T, I, F \in [0, 1]$ as single valued neutrosophic numbers.

Definition 3.1: Let $X \neq \emptyset$ be a non-empty set and $P(X)$ be the power set of X .

Let $G_{N_i^1} = (A_{S_i^1}, B_{S_i^1}T_{S_i^1}, C_{S_i^1}I_{S_i^1}, D_{S_i^1}F_{S_i^1})$ and $G_{N_i^2} = (A_{S_i^2}, B_{S_i^2}T_{S_i^2}, C_{S_i^2}I_{S_i^2}, D_{S_i^2}F_{S_i^2})$ be two generalized set-valued neutrosophic quadruple numbers.

Define a function $d_E: G_{N_i^1} \times G_{N_i^2} \rightarrow [0, 1]$ such that

$$d_G(G_{N_i^1}, G_{N_i^2}) = \frac{1}{2} \left[\frac{\sqrt{(T_{S_i^1} - T_{S_i^2})^2} + \sqrt{(I_{S_i^1} - I_{S_i^2})^2} + \sqrt{(F_{S_i^1} - F_{S_i^2})^2}}{3} \right. \\ \left. + \sqrt{\frac{s(A_{S_i^1} \setminus A_{S_i^2}) + s(A_{S_i^2} \setminus A_{S_i^1})}{\max\{s(A_{S_i^1} \cup A_{S_i^2}), 1\}} + \frac{s(B_{S_i^1} \setminus B_{S_i^2}) + s(B_{S_i^2} \setminus B_{S_i^1})}{\max\{s(B_{S_i^1} \cup B_{S_i^2}), 1\}} + \frac{s(C_{S_i^1} \setminus C_{S_i^2}) + s(C_{S_i^2} \setminus C_{S_i^1})}{\max\{s(C_{S_i^1} \cup C_{S_i^2}), 1\}} + \frac{s(D_{S_i^1} \setminus D_{S_i^2}) + s(D_{S_i^2} \setminus D_{S_i^1})}{\max\{s(D_{S_i^1} \cup D_{S_i^2}), 1\}}} \right]$$

Then, $d_G(G_{N_i^1}, G_{N_i^2})$ is called generalized Euclid distance measure for generalized set valued neutrosophic quadruple numbers.

Where, $s(A)$ is the number of element of set A . Also, we generalize Euclid distance measure in Definition 2.3.

Theorem 3.2: Let

$$G_{N_i^1} = (A_{S_i^1}, B_{S_i^1}T_{S_i^1}, C_{S_i^1}I_{S_i^1}, D_{S_i^1}F_{S_i^1}),$$

$$G_{N_i^2} = (A_{S_i^2}, B_{S_i^2}T_{S_i^2}, C_{S_i^2}I_{S_i^2}, D_{S_i^2}F_{S_i^2}),$$

$$G_{N_i^3} = (A_{S_i^3}, B_{S_i^3}T_{S_i^3}, C_{S_i^3}I_{S_i^3}, D_{S_i^3}F_{S_i^3})$$

be three generalized set valued neutrosophic quadruple numbers. The generalized Euclid distance measure in Definition 3.1 satisfies the following conditions.

$$i) d_G(G_{N_i^1}, G_{N_i^2}) \in [0, 1]$$

$$ii) d_G(G_{N_i^1}, G_{N_i^2}) = 0 \Leftrightarrow G_{N_i^1} = G_{N_i^2}$$

$$iii) d_G(G_{N_i^1}, G_{N_i^2}) = d_G(G_{N_i^2}, G_{N_i^1})$$

$$iv) \text{ If } G_{N_i^1} \subset G_{N_i^2} \subset G_{N_i^3}, \text{ then}$$

$$d_G(G_{N_i^1}, G_{N_i^2}) \leq d_G(G_{N_i^1}, G_{N_i^3}) \text{ and } d_G(G_{N_i^2}, G_{N_i^3}) \leq d_G(G_{N_i^1}, G_{N_i^3}).$$

Proof:

i) We assume that $G_{N_i^1} = G_{N_i^2}$. From Definition 2.9, we obtain that

$$A_{S_i^1} = A_{S_i^2}, B_{S_i^1} = B_{S_i^2}, C_{S_i^1} = C_{S_i^2}, D_{S_i^1} = D_{S_i^2}.$$

Thus,

$$\begin{aligned} d_G(G_{N_i^1}, G_{N_i^1}) &= \frac{1}{2} \left[\sqrt{\frac{(T_{S_i^1} - T_{S_i^1})^2 + (I_{S_i^1} - I_{S_i^1})^2 + (F_{S_i^1} - F_{S_i^1})^2}{3}} \right. \\ &\quad \left. + \sqrt{\frac{\frac{s(A_{S_i^1} \setminus A_{S_i^1}) + s(A_{S_i^1} \setminus A_{S_i^1})}{\max\{s(A_{S_i^1} \cup A_{S_i^1}), 1\}} + \frac{s(B_{S_i^1} \setminus B_{S_i^1}) + s(B_{S_i^1} \setminus B_{S_i^1})}{\max\{s(B_{S_i^1} \cup B_{S_i^1}), 1\}} + \frac{s(C_{S_i^1} \setminus C_{S_i^1}) + s(C_{S_i^1} \setminus C_{S_i^1})}{\max\{s(C_{S_i^1} \cup C_{S_i^1}), 1\}} + \frac{s(D_{S_i^1} \setminus D_{S_i^1}) + s(D_{S_i^1} \setminus D_{S_i^1})}{\max\{s(D_{S_i^1} \cup D_{S_i^1}), 1\}}}{2}} \right] \\ &= \frac{1}{2} \left[\frac{0+0+0}{3} + \frac{\sqrt{0+0+0}}{2} \right] = \frac{1}{2} \cdot 0 = 0. \end{aligned} \quad (1)$$

We assume that

$$A_{S_i^1} \neq A_{S_i^2}, B_{S_i^1} \neq B_{S_i^2}, C_{S_i^1} \neq C_{S_i^2}, D_{S_i^1} \neq D_{S_i^2}.$$

In this case,

$$d_G(G_{N_i^1}, G_{N_i^2}) > 0. \quad (2)$$

We assume that

$$G_{N_i^1} \neq \emptyset \text{ and } G_{N_i^2} = (\emptyset, \emptyset T_{S_i^1}, \emptyset I_{S_i^1}, \emptyset F_{S_i^1}).$$

Thus,

$$\begin{aligned} d_G(G_{N_i^1}, G_{N_i^2}) &= \frac{1}{2} \left[\sqrt{\frac{(T_{S_i^1} - T_{S_i^2})^2 + (I_{S_i^1} - I_{S_i^2})^2 + (F_{S_i^1} - F_{S_i^2})^2}{3}} \right. \\ &\quad \left. + \sqrt{\frac{\frac{s(A_{S_i^1} \setminus \emptyset) + s(\emptyset \setminus A_{S_i^1})}{\max\{s(A_{S_i^1} \cup \emptyset), 1\}} + \frac{s(B_{S_i^1} \setminus \emptyset) + s(\emptyset \setminus B_{S_i^1})}{\max\{s(B_{S_i^1} \cup \emptyset), 1\}} + \frac{s(C_{S_i^1} \setminus \emptyset) + s(\emptyset \setminus C_{S_i^1})}{\max\{s(C_{S_i^1} \cup \emptyset), 1\}} + \frac{s(D_{S_i^1} \setminus \emptyset) + s(\emptyset \setminus D_{S_i^1})}{\max\{s(D_{S_i^1} \cup \emptyset), 1\}}}{2}} \right] \\ &= \frac{1}{2} \left[\frac{1+1+1}{3} + \frac{\sqrt{1+1+1}}{2} \right] \\ &= \frac{1}{2} \cdot [1 + 1] = 1 \end{aligned} \quad (3)$$

Hence, from (1), (2) and (3) we obtain

$$0 \leq d_G(G_{N_i^1}, G_{N_i^2}) \leq 1.$$

ii) \Rightarrow : We assume that

$$d_G(G_{N_i^1}, G_{N_i^2}) = \frac{1}{2} \left[\frac{\sqrt{(T_{S_i^1} - T_{S_i^2})^2} + \sqrt{(I_{S_i^1} - I_{S_i^2})^2} + \sqrt{(F_{S_i^1} - F_{S_i^2})^2}}{3} \right. \\ \left. + \frac{\frac{s(A_{S_i^1} \setminus A_{S_i^2}) + s(A_{S_i^2} \setminus A_{S_i^1})}{\max\{s(A_{S_i^1} \cup A_{S_i^2}), 1\}} + \frac{s(B_{S_i^1} \setminus B_{S_i^2}) + s(B_{S_i^2} \setminus B_{S_i^1})}{\max\{s(B_{S_i^1} \cup B_{S_i^2}), 1\}} + \frac{s(C_{S_i^1} \setminus C_{S_i^2}) + s(C_{S_i^2} \setminus C_{S_i^1})}{\max\{s(C_{S_i^1} \cup C_{S_i^2}), 1\}} + \frac{s(D_{S_i^1} \setminus D_{S_i^2}) + s(D_{S_i^2} \setminus D_{S_i^1})}{\max\{s(D_{S_i^1} \cup D_{S_i^2}), 1\}}}{2} \right] \\ = 0.$$

Thus,

$$\frac{\sqrt{(T_{S_i^1} - T_{S_i^2})^2} + \sqrt{(I_{S_i^1} - I_{S_i^2})^2} + \sqrt{(F_{S_i^1} - F_{S_i^2})^2}}{3} = 0 \quad (4)$$

and

$$\frac{\frac{s(A_{S_i^1} \setminus A_{S_i^2}) + s(A_{S_i^2} \setminus A_{S_i^1})}{\max\{s(A_{S_i^1} \cup A_{S_i^2}), 1\}} + \frac{s(B_{S_i^1} \setminus B_{S_i^2}) + s(B_{S_i^2} \setminus B_{S_i^1})}{\max\{s(B_{S_i^1} \cup B_{S_i^2}), 1\}} + \frac{s(C_{S_i^1} \setminus C_{S_i^2}) + s(C_{S_i^2} \setminus C_{S_i^1})}{\max\{s(C_{S_i^1} \cup C_{S_i^2}), 1\}} + \frac{s(D_{S_i^1} \setminus D_{S_i^2}) + s(D_{S_i^2} \setminus D_{S_i^1})}{\max\{s(D_{S_i^1} \cup D_{S_i^2}), 1\}}}{2} = 0 \quad (5)$$

From (4), we obtain

$$\sqrt{(T_{S_i^1} - T_{S_i^2})^2} + \sqrt{(I_{S_i^1} - I_{S_i^2})^2} + \sqrt{(F_{S_i^1} - F_{S_i^2})^2} = 0.$$

Hence, we obtain that

$$\sqrt{(T_{S_i^1} - T_{S_i^2})^2} = 0 \text{ and } T_{S_i^1} = T_{S_i^2} \\ \sqrt{(I_{S_i^1} - I_{S_i^2})^2} = 0 \text{ and } I_{S_i^1} = I_{S_i^2} \\ \sqrt{(F_{S_i^1} - F_{S_i^2})^2} = 0 \text{ and } F_{S_i^1} = F_{S_i^2} \quad (6)$$

Also, From (5), we obtain

$$\frac{s(A_{S_i^1} \setminus A_{S_i^2}) + s(A_{S_i^2} \setminus A_{S_i^1})}{\max\{s(A_{S_i^1} \cup A_{S_i^2}), 1\}} + \frac{s(B_{S_i^1} \setminus B_{S_i^2}) + s(B_{S_i^2} \setminus B_{S_i^1})}{\max\{s(B_{S_i^1} \cup B_{S_i^2}), 1\}} \\ + \frac{s(C_{S_i^1} \setminus C_{S_i^2}) + s(C_{S_i^2} \setminus C_{S_i^1})}{\max\{s(C_{S_i^1} \cup C_{S_i^2}), 1\}} + \frac{s(D_{S_i^1} \setminus D_{S_i^2}) + s(D_{S_i^2} \setminus D_{S_i^1})}{\max\{s(D_{S_i^1} \cup D_{S_i^2}), 1\}}$$

$$= 0.$$

Hence,

$$\begin{aligned} s(A_{S_i^1} \setminus A_{S_i^2}) + s(A_{S_i^2} \setminus A_{S_i^1}) &= 0 \\ s(B_{S_i^1} \setminus B_{S_i^2}) + s(B_{S_i^2} \setminus B_{S_i^1}) &= 0 \\ s(C_{S_i^1} \setminus C_{S_i^2}) + s(C_{S_i^2} \setminus C_{S_i^1}) &= 0 \\ s(D_{S_i^1} \setminus D_{S_i^2}) + s(D_{S_i^2} \setminus D_{S_i^1}) &= 0 \end{aligned} \quad (7)$$

From (7), we obtain

$$A_{S_i^1} = A_{S_i^2}, B_{S_i^1} = B_{S_i^2}, C_{S_i^1} = C_{S_i^2} \text{ and } D_{S_i^1} = D_{S_i^2} \quad (8)$$

Therefore, from (6), (9) and Definition 2.9 we obtain

$$G_{N_i^1} = G_{N_i^2}$$

\Leftarrow : We assume that $G_{N_i^1} = G_{N_i^2}$. It is clear that from (1),

$$d_G(G_{N_i^1}, G_{N_i^1}) = 0.$$

Hence, we obtain

$$d_G(G_{N_i^1}, G_{N_i^2}) = 0 \Leftrightarrow G_{N_i^1} = G_{N_i^2}.$$

iii)

$$\begin{aligned} d_G(G_{N_i^1}, G_{N_i^2}) &= \frac{1}{2} \left[\frac{\sqrt{(T_{S_i^1} - T_{S_i^2})^2} + \sqrt{(I_{S_i^1} - I_{S_i^2})^2} + \sqrt{(F_{S_i^1} - F_{S_i^2})^2}}{3} \right. \\ &\quad + \sqrt{\frac{s(A_{S_i^1} \setminus A_{S_i^2}) + s(A_{S_i^2} \setminus A_{S_i^1})}{\max\{s(A_{S_i^1} \cup A_{S_i^2}), 1\}} + \frac{s(B_{S_i^1} \setminus B_{S_i^2}) + s(B_{S_i^2} \setminus B_{S_i^1})}{\max\{s(B_{S_i^1} \cup B_{S_i^2}), 1\}} + \frac{s(C_{S_i^1} \setminus C_{S_i^2}) + s(C_{S_i^2} \setminus C_{S_i^1})}{\max\{s(C_{S_i^1} \cup C_{S_i^2}), 1\}} + \frac{s(D_{S_i^1} \setminus D_{S_i^2}) + s(D_{S_i^2} \setminus D_{S_i^1})}{\max\{s(D_{S_i^1} \cup D_{S_i^2}), 1\}}} \\ &\quad \left. + \frac{\sqrt{(T_{S_i^2} - T_{S_i^1})^2} + \sqrt{(I_{S_i^2} - I_{S_i^1})^2} + \sqrt{(F_{S_i^2} - F_{S_i^1})^2}}{3} \right. \\ &\quad + \sqrt{\frac{s(A_{S_i^2} \setminus A_{S_i^1}) + s(A_{S_i^1} \setminus A_{S_i^2})}{\max\{s(A_{S_i^2} \cup A_{S_i^1}), 1\}} + \frac{s(B_{S_i^2} \setminus B_{S_i^1}) + s(B_{S_i^1} \setminus B_{S_i^2})}{\max\{s(B_{S_i^2} \cup B_{S_i^1}), 1\}} + \frac{s(C_{S_i^2} \setminus C_{S_i^1}) + s(C_{S_i^1} \setminus C_{S_i^2})}{\max\{s(C_{S_i^2} \cup C_{S_i^1}), 1\}} + \frac{s(D_{S_i^2} \setminus D_{S_i^1}) + s(D_{S_i^1} \setminus D_{S_i^2})}{\max\{s(D_{S_i^2} \cup D_{S_i^1}), 1\}}} \\ &\quad \left. \right] \\ &= d_G(G_{N_i^2}, G_{N_i^1}) \end{aligned}$$

iv) We assume that

$$G_{N_i^1} \subseteq G_{N_i^2} \subseteq G_{N_i^3}.$$

From Definition 2.9, we obtain

$$\begin{aligned} s(A_{S_i^1}) &\leq s(A_{S_i^2}) \leq s(A_{S_i^3}) \\ s(B_{S_i^1}) &\leq s(B_{S_i^2}) \leq s(B_{S_i^3}) \\ s(C_{S_i^1}) &\leq s(C_{S_i^2}) \leq s(C_{S_i^3}) \\ s(D_{S_i^1}) &\leq s(D_{S_i^2}) \leq s(D_{S_i^3}) \end{aligned} \quad (9)$$

$$\begin{aligned} s(A_{S_i^1} \setminus A_{S_i^2}) &= s(A_{S_i^1} \setminus A_{S_i^3}) = s(A_{S_i^2} \setminus A_{S_i^3}) = \emptyset \\ s(B_{S_i^1} \setminus B_{S_i^2}) &= s(B_{S_i^1} \setminus B_{S_i^3}) = s(B_{S_i^2} \setminus B_{S_i^3}) = \emptyset \\ s(C_{S_i^1} \setminus C_{S_i^2}) &= s(C_{S_i^1} \setminus C_{S_i^3}) = s(C_{S_i^2} \setminus C_{S_i^3}) = \emptyset \\ s(D_{S_i^1} \setminus D_{S_i^2}) &= s(D_{S_i^1} \setminus D_{S_i^3}) = s(D_{S_i^2} \setminus D_{S_i^3}) = \emptyset \end{aligned} \quad (10)$$

$$\begin{aligned} s(A_{S_i^2} \setminus A_{S_i^1}) &\leq s(A_{S_i^3} \setminus A_{S_i^1}), \\ s(B_{S_i^2} \setminus B_{S_i^1}) &\leq s(B_{S_i^3} \setminus B_{S_i^1}), \\ s(C_{S_i^2} \setminus C_{S_i^1}) &\leq s(C_{S_i^3} \setminus C_{S_i^1}), \\ s(D_{S_i^2} \setminus D_{S_i^1}) &\leq s(D_{S_i^3} \setminus D_{S_i^1}), \\ s(A_{S_i^3} \setminus A_{S_i^2}) &\leq s(A_{S_i^3} \setminus A_{S_i^1}), \\ s(B_{S_i^3} \setminus B_{S_i^2}) &\leq s(B_{S_i^3} \setminus B_{S_i^1}), \\ s(C_{S_i^3} \setminus C_{S_i^2}) &\leq s(C_{S_i^3} \setminus C_{S_i^1}), \\ s(D_{S_i^3} \setminus D_{S_i^2}) &\leq s(D_{S_i^3} \setminus D_{S_i^1}) \end{aligned} \quad (11)$$

$$\begin{aligned} \max\{s(A_{S_i^1} \cup A_{S_i^2}), 1\} &= \max\{s(A_{S_i^2}), 1\}, \\ \max\{s(B_{S_i^1} \cup B_{S_i^2}), 1\} &= \max\{s(B_{S_i^2}), 1\}, \\ \max\{s(C_{S_i^1} \cup C_{S_i^2}), 1\} &= \max\{s(C_{S_i^2}), 1\}, \\ \max\{s(D_{S_i^1} \cup D_{S_i^2}), 1\} &= \max\{s(D_{S_i^2}), 1\}, \\ \max\{s(A_{S_i^2} \cup A_{S_i^3}), 1\} &= \max\{s(A_{S_i^3}), 1\}, \\ \max\{s(B_{S_i^2} \cup B_{S_i^3}), 1\} &= \max\{s(B_{S_i^3}), 1\}, \end{aligned}$$

$$\begin{aligned}
\max\{s(C_{S_i^2} \cup C_{S_i^3}), 1\} &= \max\{s(C_{S_i^3}), 1\}, \\
\max\{s(D_{S_i^2} \cup D_{S_i^3}), 1\} &= \max\{s(D_{S_i^3}), 1\}, \\
\max\{s(A_{S_i^1} \cup A_{S_i^3}), 1\} &= \max\{s(A_{S_i^3}), 1\}, \\
\max\{s(B_{S_i^1} \cup B_{S_i^3}), 1\} &= \max\{s(B_{S_i^3}), 1\}, \\
\max\{s(C_{S_i^1} \cup C_{S_i^3}), 1\} &= \max\{s(C_{S_i^3}), 1\}, \\
\max\{s(D_{S_i^1} \cup D_{S_i^3}), 1\} &= \max\{s(D_{S_i^3}), 1\}
\end{aligned} \tag{12}$$

$$\begin{aligned}
\sqrt{(T_{S_i^1} - T_{S_i^2})^2} + \sqrt{(I_{S_i^1} - I_{S_i^2})^2} + \sqrt{(F_{S_i^1} - F_{S_i^2})^2} &\leq \sqrt{(T_{S_i^1} - T_{S_i^3})^2} + \sqrt{(I_{S_i^1} - I_{S_i^3})^2} + \sqrt{(F_{S_i^1} - F_{S_i^3})^2} \\
\sqrt{(T_{S_i^2} - T_{S_i^3})^2} + \sqrt{(I_{S_i^2} - I_{S_i^3})^2} + \sqrt{(F_{S_i^2} - F_{S_i^3})^2} &\leq \sqrt{(T_{S_i^1} - T_{S_i^3})^2} + \sqrt{(I_{S_i^1} - I_{S_i^3})^2} + \sqrt{(F_{S_i^1} - F_{S_i^3})^2}
\end{aligned} \tag{13}$$

Hence, from (9), (10), (11), (12), (13);

$$\begin{aligned}
d_G(G_{N_i^1}, G_{N_i^2}) &= \frac{1}{2} \left[\frac{\sqrt{(T_{S_i^1} - T_{S_i^2})^2} + \sqrt{(I_{S_i^1} - I_{S_i^2})^2} + \sqrt{(F_{S_i^1} - F_{S_i^2})^2}}{3} \right. \\
&\quad + \sqrt{\frac{\frac{s(A_{S_i^1} \setminus A_{S_i^2}) + s(A_{S_i^2} \setminus A_{S_i^1})}{\max\{s(A_{S_i^1} \cup A_{S_i^2}), 1\}} + \frac{s(B_{S_i^1} \setminus B_{S_i^2}) + s(B_{S_i^2} \setminus B_{S_i^1})}{\max\{s(B_{S_i^1} \cup B_{S_i^2}), 1\}} + \frac{s(C_{S_i^1} \setminus C_{S_i^2}) + s(C_{S_i^2} \setminus C_{S_i^1})}{\max\{s(C_{S_i^1} \cup C_{S_i^2}), 1\}} + \frac{s(D_{S_i^1} \setminus D_{S_i^2}) + s(D_{S_i^2} \setminus D_{S_i^1})}{\max\{s(D_{S_i^1} \cup D_{S_i^2}), 1\}}}{2}} \\
&\leq \frac{1}{2} \left[\frac{\sqrt{(T_{S_i^1} - T_{S_i^3})^2} + \sqrt{(I_{S_i^1} - I_{S_i^3})^2} + \sqrt{(F_{S_i^1} - F_{S_i^3})^2}}{3} \right. \\
&\quad + \sqrt{\frac{\frac{s(A_{S_i^1} \setminus A_{S_i^3}) + s(A_{S_i^3} \setminus A_{S_i^1})}{\max\{s(A_{S_i^1} \cup A_{S_i^3}), 1\}} + \frac{s(B_{S_i^1} \setminus B_{S_i^3}) + s(B_{S_i^3} \setminus B_{S_i^1})}{\max\{s(B_{S_i^1} \cup B_{S_i^3}), 1\}} + \frac{s(C_{S_i^1} \setminus C_{S_i^3}) + s(C_{S_i^3} \setminus C_{S_i^1})}{\max\{s(C_{S_i^1} \cup C_{S_i^3}), 1\}} + \frac{s(D_{S_i^1} \setminus D_{S_i^3}) + s(D_{S_i^3} \setminus D_{S_i^1})}{\max\{s(D_{S_i^1} \cup D_{S_i^3}), 1\}}}{2}} \\
&= d_G(G_{N_i^1}, G_{N_i^3}).
\end{aligned}$$

Therefore,

$$d_G(G_{N_i^1}, G_{N_i^2}) \leq d_G(G_{N_i^1}, G_{N_i^3})$$

Also, from (9), (10), (11), (12), (13); we obtain

$$d_G(G_{N_i^2}, G_{N_i^3}) \leq d_G(G_{N_i^1}, G_{N_i^3})$$

Definition 3.3: Let $X \neq \emptyset$ be a non-empty set and $P(X)$ be the power set of X .

Let $G_{N_i^1} = (A_{S_i^1}, B_{S_i^1}T_{S_i^1}, C_{S_i^1}I_{S_i^1}, D_{S_i^1}F_{S_i^1})$ and $G_{N_i^2} = (A_{S_i^2}, B_{S_i^2}T_{S_i^2}, C_{S_i^2}I_{S_i^2}, D_{S_i^2}F_{S_i^2})$ be two generalized set-valued neutrosophic quadruple numbers.

Define a function $S_E: G_{N_i^1} \times G_{N_i^2} \rightarrow [0,1]$ such that

$$S_G(G_{N_i^1}, G_{N_i^2}) = 1 - \frac{1}{2} \left[\frac{\sqrt{(T_{S_i^1} - T_{S_i^2})^2} + \sqrt{(I_{S_i^1} - I_{S_i^2})^2} + \sqrt{(F_{S_i^1} - F_{S_i^2})^2}}{3} \right. \\ \left. + \frac{\sqrt{\frac{s(A_{S_i^1} \setminus A_{S_i^2}) + s(A_{S_i^2} \setminus A_{S_i^1})}{\max\{s(A_{S_i^1} \cup A_{S_i^2}), 1\}} + \frac{s(B_{S_i^1} \setminus B_{S_i^2}) + s(B_{S_i^2} \setminus B_{S_i^1})}{\max\{s(B_{S_i^1} \cup B_{S_i^2}), 1\}} + \frac{s(C_{S_i^1} \setminus C_{S_i^2}) + s(C_{S_i^2} \setminus C_{S_i^1})}{\max\{s(C_{S_i^1} \cup C_{S_i^2}), 1\}} + \frac{s(D_{S_i^1} \setminus D_{S_i^2}) + s(D_{S_i^2} \setminus D_{S_i^1})}{\max\{s(D_{S_i^1} \cup D_{S_i^2}), 1\}}}}{2} \right]$$

Then, $S_G(G_{N_i^1}, G_{N_i^2})$ is called generalized Euclid similarity measure for generalized set valued neutrosophic quadruple numbers.

Where, $s(A)$ is the number of element of set A . Also, we generalize Euclid similarity measure in Definition 2.3.

Corollary 3.4: Let $S_G(G_{N_i^1}, G_{N_i^2})$ be Euclid similarity measure for generalized set valued neutrosophic quadruple numbers in Definition 3.3 and $d_G(G_{N_i^1}, G_{N_i^2})$ be Euclid distance measure for generalized set valued neutrosophic quadruple numbers in Definition 3.1. Then,

$$S_G(G_{N_i^1}, G_{N_i^2}) = 1 - d_G(G_{N_i^1}, G_{N_i^2})$$

Theorem 3.5: Let

$$G_{N_i^1} = (A_{S_i^1}, B_{S_i^1}T_{S_i^1}, C_{S_i^1}I_{S_i^1}, D_{S_i^1}F_{S_i^1}),$$

$$G_{N_i^2} = (A_{S_i^2}, B_{S_i^2}T_{S_i^2}, C_{S_i^2}I_{S_i^2}, D_{S_i^2}F_{S_i^2}),$$

$$G_{N_i^3} = (A_{S_i^3}, B_{S_i^3}T_{S_i^3}, C_{S_i^3}I_{S_i^3}, D_{S_i^3}F_{S_i^3})$$

be three generalized set valued neutrosophic quadruple numbers. the generalized Euclid similarity measure in Definition 3.3 satisfies the following conditions.

$$i) S_G(G_{N_i^1}, G_{N_i^2}) \in [0,1]$$

$$ii) S_G(G_{N_i^1}, G_{N_i^2}) = 1 \Leftrightarrow G_{N_i^1} = G_{N_i^2}$$

$$iii) S_G(G_{N_i^1}, G_{N_i^2}) = S_G(G_{N_i^2}, G_{N_i^1})$$

$$iv) \text{ If } G_{N_i^1} \subset G_{N_i^2} \subset G_{N_i^3}, \text{ then}$$

$$S_G(G_{N_i^1}, G_{N_i^3}) \leq S_G(G_{N_i^1}, G_{N_i^2}) \text{ and } S_G(G_{N_i^1}, G_{N_i^3}) \leq S_G(G_{N_i^2}, G_{N_i^3}).$$

Proof: From Corollary 3.4 and Theorem 3.2, it is clear that $S_G(G_{N_i^1}, G_{N_i^2})$ satisfies the conditions of Theorem 3.5.

Example 3.6: Let

$$X_1 = (\{\omega_4, \omega_1\}, \{\omega_7, \omega_6\}(0,3), \{\omega_8, \omega_9\}(0,4), \{\omega_{10}\}(0,1))$$

$$X_2 = (\{\omega_1, \omega_2\}, \{\omega_6\}(1), \emptyset(0), \emptyset(0))$$

be two generalized set valued neutrosophic quadruple numbers, $S_G(G_{N_i^1}, G_{N_i^2})$ be Euclid similarity measure for generalized set valued neutrosophic quadruple numbers in Definition 3.3 and $d_G(G_{N_i^1}, G_{N_i^2})$ be Euclid distance measure for generalized set valued neutrosophic quadruple numbers in Definition 3.1. Then,

$$d_G(X_1, X_2) = \frac{1}{2} \left[\frac{\sqrt{(0,3-1)^2} + \sqrt{(0,4-0)^2} + \sqrt{(0,1-0)^2}}{3} + \sqrt{\frac{s(\{\omega_4, \omega_1\} \setminus \{\omega_1, \omega_2\}) + s(\{\omega_1, \omega_2\} \setminus \{\omega_4, \omega_1\})}{\max\{s(\{\omega_4, \omega_1\}), 1\}} + \frac{s(\{\omega_7, \omega_6\} \setminus \{\omega_6\}) + s(\{\omega_6\} \setminus \{\omega_7, \omega_6\})}{\max\{s(\{\omega_7, \omega_6\}), 1\}} + \frac{s(\{\omega_8, \omega_9\} \setminus \emptyset) + s(\emptyset \setminus \{\omega_8, \omega_9\})}{\max\{s(\{\omega_8, \omega_9\}), 1\}} + \frac{s(\{\omega_{10}\} \setminus \emptyset) + s(\emptyset \setminus \{\omega_{10}\})}{\max\{s(\{\omega_{10}\}), 1\}}} \right]$$

$$= \frac{1}{2} \left[\left(\frac{(0,7) + (0,4) + (0,1)}{3} \right) + \left(\frac{\frac{1+1}{3} + \frac{1+0}{2} + \frac{2+0}{2} + \frac{1+0}{1}}{2} \right) \right]$$

$$= \frac{1}{2} \left[\frac{1,2}{3} + \frac{\sqrt{\frac{21}{12}}}{2} \right]$$

$$= 0.53$$

and

$$S_G(X_1, X_2) = 1 - d_G(X_1, X_2) = 1 - 0,53 = 0,47.$$

Definition 3.7: (Score Function) Let

$$X_1 = (A_1, B_1T_1, C_1I_1, D_1F_1)$$

$$X_2 = (A_2, B_2T_2, C_2I_2, D_2F_2)$$

be two generalized set valued neutrosophic number, X_i be a generalized set valued neutrosophic number representing a sample and S_G be Euclid similarity measure for generalized set valued neutrosophic quadruple numbers in Definition 3.3. It is unclear which number is more similar to the sample in case

$$S_G(X_i, X_1) = S_G(X_i, X_2)$$

In these cases, we will define a score function to determine which number is more similar to the sample.

a)

If $s(A_1) + s(B_1) > s(A_2) + s(B_2)$, then choose X_1 .

If $s(A_1) + s(B_1) < s(A_2) + s(B_2)$, then choose X_2 .

b) We assume that $s(A_1) + s(B_1) = s(A_2) + s(B_2)$.

If $T_1 > T_2$, then choose X_1 .

If $T_1 < T_2$, then choose X_2 .

c) We assume that $s(A_1) + s(B_1) = s(A_2) + s(B_2)$ and $T_1 = T_2$.

If $F_1 > F_2$, then choose X_2 .

If $F_1 < F_2$, then choose X_1 .

d) We assume that $s(A_1) + s(B_1) = s(A_2) + s(B_2)$, $T_1 = T_2$ and $F_1 = F_2$.

If $I_1 > I_2$, then choose X_2 .

If $I_1 < I_2$, then choose X_1 .

e) We assume that $s(A_1) + s(B_1) = s(A_2) + s(B_2)$, $T_1 = T_2$ and $I_1 = I_2$.

If $F_1 > F_2$, then choose X_2 .

If $F_1 < F_2$, then choose X_1 .

f) We assume that $s(A_1) + s(B_1) = s(A_2) + s(B_2)$, $T_1 = T_2$, $I_1 = I_2$ and $F_1 = F_2$.

If $s(C_1) + s(D_1) > s(C_2) + s(D_2)$, then choose X_2 .

If $s(C_1) + s(D_1) < s(C_2) + s(D_2)$, then choose X_1 .

g) We assume that $s(A_1) + s(B_1) = s(A_2) + s(B_2)$, $T_1 = T_2$, $I_1 = I_2$ and $F_1 = F_2$ and $s(C_1) + s(D_1) = s(C_2) + s(D_2)$.

Then, we choose X_1 or X_2 .

Example 3.8: Let

$$X_1 = (\{\omega_4, \omega_1, \omega_{12}, \omega_{13}\}, \{\omega_7, \omega_6\}(0,3), \{\omega_8, \omega_9, \omega_{14}\}(0,2), \{\omega_{15}, \omega_{16}\}(0,1))$$

$$X_2 = (\{\omega_2, \omega_3, \omega_4, \omega_{12}\}, \{\omega_8, \omega_6\}(0,4), \{\omega_9, \omega_{15}\}(0,3), \{\omega_8, \omega_{10}, \omega_{16}\}(0,1))$$

be two generalized set valued neutrosophic quadruple numbers and

$X = (\{\omega_1, \omega_2\}, \{\omega_6\}(1), \emptyset(0), \emptyset(0))$ be a generalized set valued neutrosophic number representing a sample. We choose X_1 or X_2 according to S_G in Definition 3.3.

$$S_G(X_1, X) = 1 - \frac{1}{2} \left[\frac{\sqrt{(0,3-1)^2} + \sqrt{(0,2-0)^2} + \sqrt{(0,1-0)^2}}{3} + \sqrt{\frac{s(\{\omega_4, \omega_1, \omega_{12}, \omega_{13}\} \setminus \{\omega_1, \omega_2\}) + s(\{\omega_1, \omega_2\} \setminus \{\omega_4, \omega_1, \omega_{12}, \omega_{13}\})}{\max\{s(\{\omega_4, \omega_1, \omega_{12}, \omega_{13}\}), 1\}} + \frac{s(\{\omega_7, \omega_6\} \setminus \{\omega_6\}) + s(\{\omega_6\} \setminus \{\omega_7, \omega_6\})}{\max\{s(\{\omega_7, \omega_6\}), 1\}} + \frac{s(\{\omega_8, \omega_9, \omega_{14}\} \setminus \emptyset) + s(\emptyset \setminus \{\omega_8, \omega_9, \omega_{14}\})}{\max\{s(\{\omega_8, \omega_9, \omega_{14}\}), 1\}} + \frac{s(\{\omega_{15}, \omega_{16}\} \setminus \emptyset) + s(\emptyset \setminus \{\omega_{15}, \omega_{16}\})}{\max\{s(\{\omega_{15}, \omega_{16}\}), 1\}} \right]}$$

$$= 1 - 0,62 = 0,38.$$

$$S_G(X_2, X) = 1 - \frac{1}{2} \left[\frac{\sqrt{(0,4-1)^2} + \sqrt{(0,3-0)^2} + \sqrt{(0,1-0)^2}}{3} + \sqrt{\frac{s(\{\omega_2, \omega_3, \omega_4, \omega_{12}\} \setminus \{\omega_1, \omega_2\}) + s(\{\omega_1, \omega_2\} \setminus \{\omega_2, \omega_3, \omega_4, \omega_{12}\})}{\max\{s(\{\omega_2, \omega_3, \omega_4, \omega_{12}, \omega_1\}), 1\}} + \frac{s(\{\omega_8, \omega_6\} \setminus \{\omega_6\}) + s(\{\omega_6\} \setminus \{\omega_8, \omega_6\})}{\max\{s(\{\omega_8, \omega_6\}), 1\}} + \frac{s(\{\omega_9, \omega_{15}\} \setminus \{\emptyset\}) + s(\{\emptyset\} \setminus \{\omega_9, \omega_{15}\})}{\max\{s(\{\omega_9, \omega_{15}\}), 1\}} + \frac{s(\{\omega_8, \omega_{10}, \omega_{16}\} \setminus \{\emptyset\}) + s(\{\emptyset\} \setminus \{\omega_8, \omega_{10}, \omega_{16}\})}{\max\{s(\{\omega_8, \omega_{10}, \omega_{16}\}), 1\}}} \right]$$

$$= 1 - 0,62 = 0,38.$$

Thus,

$$S_G(X_1, X) = S_G(X_2, X).$$

In this case, by the Score Function in Definition 3.7,

$$s(A_1) + s(B_1) = s(\{\omega_4, \omega_1, \omega_{12}, \omega_{13}\}) + s(\{\omega_7, \omega_6\}) = 6$$

and

$$s(A_2) + s(B_2) = s(\{\omega_2, \omega_3, \omega_4, \omega_{12}\}) + s(\{\omega_8, \omega_6\}) = 6.$$

As $s(A_1) + s(B_1) = s(A_1) + s(B_1)$, we compare T_1 with T_2 .

Since,

$$T_1 = 0.3 \text{ and } T_2 = 0.4$$

by the Score Function, we choose X_2 .

4 MULTI CRITERIA DECISION MAKING APPLICATIONS WITH GENERALIZED SET VALUED NEUTROSOPHIC QUADRUPLE NUMBERS AND GENERALIZED EUCLID SIMILARITY MEASURE

In this section, we use the generalized algorithm used in [39, 40] using neutrosophic sets and give it again for the generalized set valued neutrosophic quadruple numbers. We will also use the generalized Euclid similarity measure (in Definition 3.3) in this algorithm.

Using this algorithm [39, 40], we will give an example of individuals with more than one disease to determine which of the known disease medications will be good for their unknown disease. We compared the results we obtained in this example with the results obtained in neutrosophic numbers and showed that we obtained different results.

This example will be especially useful for healthcare professionals in determining which drugs to use in corona (covid-19) treatment of an individual with various diseases.

4.1 Multi Critarias Decision Making Algorithm with Generalized Set Valued Neutrosophic Quadruple Numbers and Generalized Euclid Similarity Measure

Step 1: Let $H = \{h_1, h_2, \dots, h_n\}$ be set of criterias.

Step 2: Let $W = \{w_1, w_2, \dots, w_n\}$ be weighted value set of criterias such that

$$w_1 \text{ is weighted value of } h_1,$$

w_2 is weighted value of h_2 ,

.

.

.

w_n is weighted value of h_n

Also, $\sum_{i=1}^n w_i = 1$ and $w_i \in \mathbb{R}^+$.

Step 3:

Let us express an ideal object K that we can compare as a generalized set-valued neutrophic quadruple set

$$K = \{h_1: (P(A), P(A)T_{1i}, \emptyset I_{1i}, \emptyset F_{1i}), h_2: (P(B), P(B)T_{2i}, \emptyset I_{2i}, \emptyset F_{2i}), \dots, \\ h_n: (P(Y), P(Y)T_{ni}, \emptyset I_{ni}, \emptyset F_{ni})\}$$

Where,

$(P(A)T_{1i}, \emptyset I_{1i}, \emptyset F_{1i})$ is ideal set for h_1 ,

$(P(A)T_{2i}, \emptyset I_{2i}, \emptyset F_{2i})$ is ideal set for h_2 ,

.

.

.

$(P(A)T_{ni}, \emptyset I_{ni}, \emptyset F_{ni})$ is ideal set for h_n ,

Step 4: Let $X = \{X_1, X_2, \dots, X_n\}$ be the set of objects that we will choose the best according to their ideal object similarity values. Now, we give each object as a generalized set valued neutrophic quadruple set.

$$X_1 = \{h_1: (A_{11}, A_{12}T_{11}, A_{13}I_{11}, A_{14}F_{11}), h_2: (B_{11}, B_{12}T_{12}, B_{13}I_{12}, B_{14}F_{12}), \dots, \\ h_n: (Y_{11}, Y_{12}T_{1n}, Y_{13}I_{1n}, Y_{14}F_{1n})\}$$

$$X_2 = \{h_1: (A_{21}, A_{22}T_{21}, A_{23}I_{21}, A_{24}F_{21}), h_2: (B_{21}, B_{22}T_{22}, B_{23}I_{22}, B_{24}F_{22}), \dots, \\ h_n: (Y_{21}, Y_{22}T_{2n}, Y_{23}I_{2n}, Y_{24}F_{2n})\}$$

.

.

.

$$X_i = \{h_1: (A_{i1}, A_{i2}T_{i1}, A_{i3}I_{i1}, A_{i4}F_{i1}), h_2: (B_{i1}, B_{i2}T_{i2}, B_{i3}I_{i2}, B_{i4}F_{i2}), \dots, \\ h_n: (Y_{i1}, Y_{i2}T_{in}, Y_{i3}I_{in}, Y_{i4}F_{in})\}, \quad i = 1, 2, \dots, n.$$

Where,

$$A_{11}, A_{12}, A_{13}, A_{14}, A_{21}, A_{22}, A_{23}, A_{24}, \dots, A_{i1}, A_{i2}, A_{i3}, A_{i4} \in P(A)$$

$$B_{11}, B_{12}, B_{13}, B_{14}, B_{21}, B_{22}, B_{23}, B_{24}, \dots, B_{i1}, B_{i2}, B_{i3}, B_{i4} \in P(B)$$

.

.

.

$$Y_{11}, Y_{12}, Y_{13}, Y_{14}, Y_{21}, Y_{22}, Y_{23}, Y_{24}, \dots, Y_{i1}, Y_{i2}, Y_{i3}, Y_{i4} \in P(Y).$$

Step 5: Let's show the objects in step 4 in Table 1.

Table 1. Table of Objects

	h_1	h_2	...	h_n
X_1	$(A_{11}, A_{12}T_{11}, A_{13}I_{11}, A_{14}F_{11})$	$(B_{11}, B_{12}T_{12}, B_{13}I_{12}, B_{14}F_{12})$...	$(Y_{11}, Y_{12}T_{1n}, Y_{13}I_{1n}, Y_{14}F_{1n})$
X_2	$(A_{21}, A_{22}T_{21}, A_{23}I_{21}, A_{24}F_{21})$	$(B_{21}, B_{22}T_{22}, B_{23}I_{22}, B_{24}F_{22})$...	$(Y_{21}, Y_{22}T_{2n}, Y_{23}I_{2n}, Y_{24}F_{2n})$
.
.
.
X_i	$(A_{i1}, A_{i2}T_{i1}, A_{i3}I_{i1}, A_{i4}F_{i1})$	$(B_{i1}, B_{i2}T_{i2}, B_{i3}I_{i2}, B_{i4}F_{i2})$...	$(Y_{i1}, Y_{i2}T_{in}, Y_{i3}I_{in}, Y_{i4}F_{in})$

Step 6: We find the similarity value of the criteria value of each object in Table 1 and the criteria values of the ideal object with the generalized Euclidean similarity measure. Thus, we obtain Table 2.

Table 2. Criteria Similarity Table

	h_1	h_2	...	h_n
X_1	$S_G(K(h_1), X_1(h_1))$	$S_G(K(h_2), X_1(h_2))$...	$S_G(K(h_n), X_1(h_n))$
X_2	$S_G(K(h_1), X_2(h_1))$	$S_G(K(h_2), X_2(h_2))$...	$S_G(K(h_n), X_2(h_n))$
.
.
.
X_i	$S_G(K(h_1), X_i(h_1))$	$S_G(K(h_2), X_i(h_2))$...	$S_G(K(h_n), X_i(h_n))$

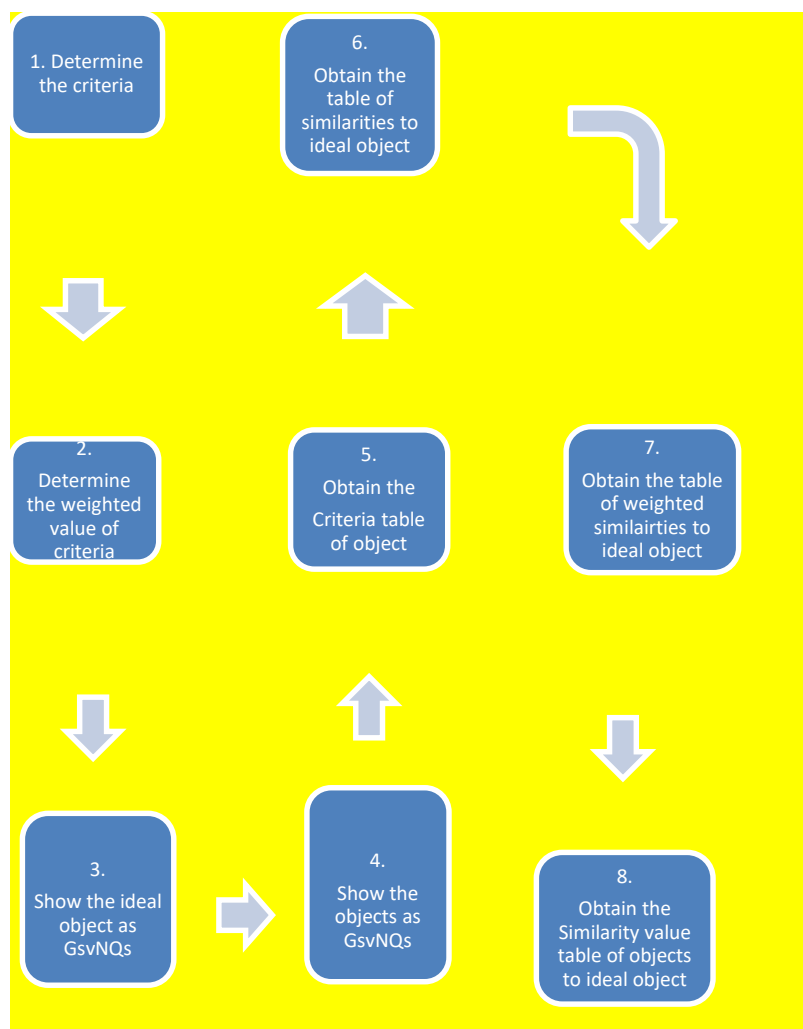
Step 7: Each criteria similarity value in Table 2 is multiplied by its own criteria weight value, and by adding the weighted similarity values for each object, the ideal object similarity values are obtained. Thus, we obtain Table 3.

Where, $i = 1, 2, \dots, n$ and $S_{Gi}(K, X_i) = \sum_{k=1}^n w_k \cdot S_G(K(h_k), X_i(h_k))$.

Table 3. Weighted Similarity Table of Objects with Ideal Object

	$w_1 h_1$	$w_2 h_2$...	$w_n h_n$	$\sum_{k=1}^n w_k \cdot S_G(K(h_k), X_i(h_k))$
X_1	$w_1 \cdot S_G(K(h_1), X_1(h_1))$	$w_2 \cdot S_G(K(h_2), X_1(h_2))$...	$w_n \cdot S_G(K(h_n), X_1(h_n))$	$S_{G1}(K, X_1)$
X_2	$w_1 \cdot S_G(K(h_1), X_2(h_1))$	$w_2 \cdot S_G(K(h_2), X_2(h_2))$...	$w_n \cdot S_G(K(h_n), X_2(h_n))$	$S_{G2}(K, X_2)$
.
.
.
X_i	$w_1 \cdot S_G(K(h_1), X_i(h_1))$	$w_2 \cdot S_G(K(h_2), X_i(h_2))$...	$w_n \cdot S_G(K(h_n), X_i(h_n))$	$S_{Gi}(K, X_i)$

According to the values of S_{Gi} in Table 3, the objects closest to the ideal object are determined.



Graph 1: Diagram of the algorithm. [39]

4.2 Multi Criteria Decision Making Applications with Generalized Set Valued Neutrosophic Quadruple Numbers and Generalized Euclid Similarity Measure

In this section, we give an application of individuals with more than one disease to determine which of the known disease medications be good for their unknown disease using to algorithm in 4.2.

In this application, we find out which drugs used in 10 patients with 4 different known diseases are the most ideal treatment for an unknown disease. Where, diseases are taken as criteria and patients as objects for algorithm 4.2. It is clear that in solving such problems there is a need for a structure in which the known part is the unknown part and (T, I, F) known neutrosophic membership functions. Since each known disease will have separate medications and it will be investigated which results (true, indeterminate, false) these drugs will give in unknown diseases, a structure such as $(L_{s_i}T_{s_i}, M_{s_i}I_{s_i}, N_{s_i}F_{s_i})$, containing both cluster and T, I, F , will be needed.

Step 1: Let $H = \{h_1, h_2, \dots, h_n\}$ be set of diseases.

Step 2: Let $W = \{w_1, w_2, \dots, w_n\}$ be weighted value set of diseases such that

$$w_1 = 0.2 \text{ is weighted value of } h_1,$$

$w_2 = 0.3$ is weighted value of h_2 ,

$w_3 = 0.4$ is weighted value of h_3 ,

$w_4 = 0.1$ is weighted value of h_4 ,

Step 3: We choose the ideal patient K such that

$$K = \{h_1: (\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9\}, \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9\}1, \emptyset, \emptyset), \\ h_2: (\{b_1, b_2, b_3, b_4, b_5, b_6, b_8, b_9\}, \{b_1, b_2, b_3, b_4, b_5, b_6, b_8, b_9\}1, \emptyset, \emptyset), \\ h_3: (\{c_1, c_2, c_3, c_5, c_6, c_7, c_8, c_9, c_{10}\}, \{c_1, c_2, c_3, c_5, c_6, c_7, c_8, c_9, c_{10}\}1, \emptyset, \emptyset), \\ h_4: (\{d_1, d_2, d_4, d_6, d_7, d_8, d_9, d_{10}\}, \{d_1, d_2, d_4, d_6, d_7, d_8, d_9, d_{10}\}1, \emptyset, \emptyset)\}.$$

Since K is ideal patient, the truth set of the known part of the criteria and the unknown part must be equal, and the truth value of the unknown part must be 1. Also, other sets must be empty and other values must be 0. For example, at ideal patient K;

for h_1 ,

the set $\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9\}$ is regarded as a set of drugs that are good for disease h_1 and $(\{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9\}1, \emptyset, \emptyset)$ is regarded as a set of drugs that are good for unknown disease. Also, this applies to h_1 and other diseases.

Step 4: Let $X = \{X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}\}$ be set of patients such that

$$X_1 = \{h_1: (\{a_1, a_2, a_4, a_7\}, \{a_1, a_2\}(0.5), \{a_1, a_4\}(0.2), \{a_7\}(0.1)), \\ h_2: (\{b_3, b_4, b_5, b_6\}, \{b_4, b_5, b_6\}(0.6), \{b_4, b_6\}(0.1), \{b_4, b_5\}(0.1)), \\ h_3: (\{c_1, c_3, c_5, c_{10}\}, \{c_3\}(0.3), \{c_1, c_5\}(0.4), \{c_{10}\}(0.2)), \\ h_4: (\{d_1, d_2, d_8\}, \{d_1, d_8\}(0.4), \{d_8\}(0.1), \{d_2\}(0.2))\} \\ X_2 = \{h_1: (\{a_7, a_8, a_9\}, \{a_7\}(0.2), \{a_8, a_9\}(0.3), \{a_9\}(0.2)), \\ h_2: (\{b_1, b_3, b_8\}, \{b_1, b_3\}(0.5), \{b_3, b_8\}(0.2), \{b_8\}(0.1)), \\ h_3: (\{c_7, c_9, c_{10}\}, \{c_7\}(0.3), \{c_9\}(0.4), \{c_{10}\}(0.2)), \\ h_4: (\{d_6, d_8\}, \{d_8\}(0.4), \{d_6, d_8\}(0), \{d_6\}(0.6))\} \\ X_3 = \{h_1: (\{a_3, a_5, a_6\}, \{a_5, a_6\}(0.3), \{a_5\}(0.2), \{a_3\}(0.2)), \\ h_2: (\{b_1, b_2\}, \{b_1\}(0.7), \{b_1\}(0.1), \{b_2\}(0.1)), \\ h_3: (\{c_6, c_7, c_8\}, \{c_7\}(0.1), \{c_6, c_8\}(0.2), \{c_6\}(0.5)), \\ h_4: (\{d_7, d_{10}\}, \{d_{10}\}(0.4), \{d_7\}(0.2), \{d_7, d_{10}\}(0))\} \\ X_4 = \{h_1: (\{a_5, a_9\}, \{a_9\}(0.2), \{a_5\}(0.5), \{a_5\}(0.2)), \\ h_2: (\{b_3, b_6, b_9\}, \{b_3, b_9\}(0.1), \{b_6, b_9\}(0.2), \{b_3\}(0.5)), \\ h_3: (\{c_2, c_5, c_7, c_{10}\}, \{c_5, c_{10}\}(0.3), \{c_7, c_{10}\}(0.4), \{c_2\}(0.3)), \\ h_4: (\{d_2, d_8, d_9\}, \{d_8, d_9\}(0.6), \{d_2, d_9\}(0.1), \{d_2, d_8\}(0.1))\} \\ X_5 = \{h_1: (\{a_2, a_3, a_5, a_7, a_9\}, \{a_2, a_3, a_9\}(0.7), \{a_5, a_7, a_9\}(0.1), \{a_7\}(0.2)), \\ h_2: (\{b_2, b_3, b_5, b_8\}, \{b_2, b_5\}(0.4), \{b_3\}(0.2), \{b_8\}(0.3)), \\ h_3: (\{c_1, c_7, c_9, c_{10}\}, \{c_7, c_9, c_{10}\}(0.6), \{c_1\}(0.2), \{c_1, c_7\}(0.2)), \\ h_4: (\{d_2, d_4\}, \{d_2\}(0.3), \{d_4\}(0.3), \{d_4\}(0.2))\} \\ X_6 = \{h_1: (\{a_5, a_6, a_9\}, \{a_5, a_6\}(0.4), \{a_5\}(0.2), \{a_9\}(0.3)), \\ h_2: (\{b_3, b_4, b_8\}, \{b_8\}(0.2), \{b_3, b_8\}(0.1), \{b_4\}(0.5)), \\ h_3: (\{c_3, c_6, c_9\}, \{c_6, c_9\}(0.5), \{c_3\}(0.3), \{c_9\}(0.2)), \\ h_4: (\{d_1, d_4, d_7, d_9\}, \{d_1, d_4, d_9\}(0.6), \{d_1, d_9\}(0.1), \{d_7\}(0.2))\} \\ X_7 = \{h_1: (\{a_1, a_7\}, \{a_7\}(0.2), \{a_7\}(0.5), \{a_1\}(0.2)), \\ h_2: (\{b_2, b_4, b_6, b_8, b_9\}, \{b_4, b_6\}(0.2), \{b_3, b_8, b_9\}(0.3), \{b_2, b_6\}(0.4)), \\ h_3: (\{c_2, c_3, c_6, c_7\}, \{c_3, c_6, c_7\}(0.6), \{c_2\}(0.2), \{c_2, c_6, c_3\}(0.1)), \\ h_4: (\{d_4, d_8, d_{10}\}, \{d_8, d_{10}\}(0.7), \{d_4\}(0.1), \{d_4, d_{10}\}(0.1))\} \\ X_8 = \{h_1: (\{a_2, a_4, a_6, a_7, a_8\}, \{a_4, a_6, a_7, a_8\}(0.7), \{a_2, a_4, a_6, a_7\}(0.1), \{a_4, a_6, a_7\}(0.1)), \\ h_2: (\{b_2, b_3, b_5, b_6, b_9\}, \{b_2, b_3\}(0.3), \{b_3, b_5, b_6, b_9\}(0.1), \{b_2, b_5, b_7\}(0.5)),$$

$$\begin{aligned}
& h_3: (\{c_3, c_7, c_9\}, \{c_7, c_9\}(0.4), \{c_3, c_7\}(0.5), \{c_7\}(0.1)), \\
& h_4: (\{d_8, d_9, d_{10}\}, \{d_8, d_{10}\}(0.3), \{d_8\}(0.3), \{d_9\}(0.2)) \\
X_9 = & \{h_1: (\{a_6, a_7, a_8\}, \{a_7, a_8\}(0.5), \{a_6, a_7\}(0.2), \{a_6, a_8\}(0.3)), \\
& h_2: (\{b_3, b_4, b_6, b_8, b_9\}, \{b_4, b_6, b_8, b_9\}(0.7), \{b_3, b_4, b_6\}(0.1), \{b_3, b_4, b_6, b_8\}(0.1)), \\
& h_3: (\{c_3, c_6, c_7\}, \{c_6, c_7\}(0.4), \{c_3, c_6\}(0.3), \{c_6\}(0.1)), \\
& h_4: (\{d_4, d_7, d_{10}\}, \{d_7, d_{10}\}(0.5), \{d_4\}(0.3), \{d_4, d_{10}\}(0.2))\} \\
X_{10} = & \{h_1: (\{a_2, a_3, a_7, a_8, a_9\}, \{a_7, a_8, a_9\}(0.4), \{a_2, a_7, a_8, a_9\}(0.1), \{a_3\}(0.4)), \\
& h_2: (\{b_2, b_3, b_5, b_6, b_8, b_9\}, \{b_3, b_6, b_8, b_9\}(0.6), \{b_2, b_6, b_8, b_9\}(0.2), \{b_3, b_5\}(0.2)), \\
& h_3: (\{c_3, c_7, c_{10}\}, \{c_7, c_{10}\}(0.6), \{c_3, c_7\}(0.1), \{c_3\}(0.2)), \\
& h_4: (\{d_1, d_7, d_9, d_{10}\}, \{d_1, d_{10}\}(0.3), \{d_7, d_9, d_{10}\}(0.1), \{d_1, d_9\}(0.4))\}
\end{aligned}$$

Step 5: Let's show the diseases according to patients in step 4 in Table 4.

Table 4. Table of Diseases

	h_1	h_2	h_3	h_4
X_1	$(\{a_1, a_2, a_4, a_7\},$ $\{a_1, a_2\}(0.5),$ $\{a_1, a_4\}(0.2),$ $\{a_7\}(0.1))$	$(\{b_3, b_4, b_5, b_6\},$ $\{b_4, b_5, b_6\}(0.6),$ $\{b_4, b_6\}(0.1),$ $\{b_4, b_5\}(0.1))$	$(\{c_1, c_3, c_5, c_{10}\},$ $\{c_3\}(0.3),$ $\{c_1, c_5\}(0.4),$ $\{c_{10}\}(0.2))$	$(\{d_1, d_2, d_8\},$ $\{d_1, d_8\}(0.4),$ $\{d_8\}(0.1),$ $\{d_2\}(0.2))$
X_2	$(\{a_7, a_8, a_9\},$ $\{a_7\}(0.2),$ $\{a_8, a_9\}(0.3),$ $\{a_9\}(0.2))$	$(\{b_1, b_3, b_8\},$ $\{b_1, b_3\}(0.5),$ $\{b_3, b_8\}(0.2),$ $\{b_8\}(0.1))$	$(\{c_7, c_9, c_{10}\},$ $\{c_7\}(0.3),$ $\{c_9\}(0.4),$ $\{c_{10}\}(0.2))$	$(\{d_6, d_8\},$ $\{d_8\}(0.4),$ $\{d_6, d_8\}(0),$ $\{d_6\}(0.6))$
X_3	$(\{a_3, a_5, a_6\},$ $\{a_5, a_6\}(0.3),$ $\{a_5\}(0.2),$ $\{a_3\}(0.2))$	$(\{b_1, b_2\},$ $\{b_1\}(0.7),$ $\{b_1\}(0.1),$ $\{b_2\}(0.1))$	$(\{c_6, c_7, c_8\},$ $\{c_7\}(0.1),$ $\{c_6, c_8\}(0.2),$ $\{c_6\}(0.5))$	$(\{d_7, d_{10}\},$ $\{d_{10}\}(0.4),$ $\{d_7\}(0.2),$ $\{d_7, d_{10}\}(0))$
X_4	$(\{a_5, a_9\},$ $\{a_9\}(0.2),$ $\{a_5\}(0.5),$ $\{a_5\}(0.2))$	$(\{b_3, b_6, b_9\},$ $\{b_3, b_9\}(0.1),$ $\{b_6, b_9\}(0.2),$ $\{b_3\}(0.5))$	$(\{c_2, c_5, c_7, c_{10}\},$ $\{c_5, c_{10}\}(0.3),$ $\{c_7, c_{10}\}(0.4),$ $\{c_2\}(0.3))$	$(\{d_2, d_8, d_9\},$ $\{d_8, d_9\}(0.6),$ $\{d_2, d_9\}(0.1),$ $\{d_2, d_8\}(0.1))$

X_5	$(\{a_2, a_3, a_5, a_7, a_9\},$ $\{a_2, a_3, a_9\}(0.7),$ $\{a_5, a_7, a_9\}(0.1),$ $\{a_7\}(0.2))$	$(\{b_2, b_3, b_5, b_8\},$ $\{b_2, b_5\}(0.4),$ $\{b_3\}(0.2),$ $\{b_8\}(0.3))$	$(\{c_1, c_7, c_9, c_{10}\},$ $\{c_7, c_9, c_{10}\}(0.6),$ $\{c_1\}(0.2),$ $\{c_1, c_7\}(0.2))$	$(\{d_2, d_4\},$ $\{d_2\}(0.3),$ $\{d_4\}(0.3),$ $\{d_4\}(0.2))$
X_6	$(\{a_5, a_6, a_9\},$ $\{a_5, a_6\}(0.4),$ $\{a_5\}(0.2),$ $\{a_9\}(0.3))$	$(\{b_3, b_4, b_8\},$ $\{b_8\}(0.2),$ $\{b_3, b_8\}(0.1),$ $\{b_4\}(0.5))$	$(\{c_3, c_6, c_9\},$ $\{c_6, c_9\}(0.5),$ $\{c_3\}(0.3),$ $\{c_9\}(0.2))$	$(\{d_1, d_4, d_7, d_9\},$ $\{d_1, d_4, d_9\}(0.6),$ $\{d_1, d_9\}(0.1),$ $\{d_7\}(0.2))$
X_7	$(\{a_1, a_7\},$ $\{a_7\}(0.2),$ $\{a_7\}(0.5),$ $\{a_1\}(0.2))$	$(\{b_2, b_4, b_6, b_8, b_9\},$ $\{b_4, b_6\}(0.2),$ $\{b_3, b_8, b_9\}(0.3),$ $\{b_2, b_6\}(0.4))$	$(\{c_2, c_3, c_6, c_7\},$ $\{c_3, c_6, c_7\}(0.6),$ $\{c_2\}(0.2),$ $\{c_2, c_6, c_3\}(0.1))$	$(\{d_4, d_8, d_{10}\},$ $\{d_8, d_{10}\}(0.7),$ $\{d_4\}(0.1),$ $\{d_4, d_{10}\}(0.1))$
X_8	$(\{a_2, a_4, a_6, a_7, a_8\},$ $\{a_4, a_6, a_7, a_8\}(0.7),$ $\{a_2, a_4, a_6, a_7\}(0.1),$ $\{a_4, a_6, a_7\}(0.1))$	$(\{b_2, b_3, b_5, b_6, b_9\},$ $\{b_2, b_3\}(0.3),$ $\{b_3, b_5, b_6, b_9\}(0.1),$ $\{b_2, b_5, b_7\}(0.5))$	$(\{c_3, c_7, c_9\},$ $\{c_7, c_9\}(0.4),$ $\{c_3, c_7\}(0.5),$ $\{c_7\}(0.1))$	$(\{d_8, d_9, d_{10}\},$ $\{d_8, d_{10}\}(0.3),$ $\{d_8\}(0.3),$ $\{d_9\}(0.2))$
X_9	$(\{a_6, a_7, a_8\},$ $\{a_7, a_8\}(0.5),$ $\{a_6, a_7\}(0.2),$ $\{a_6, a_8\}(0.3))$	$(\{b_3, b_4, b_6, b_8, b_9\},$ $\{b_4, b_6, b_8, b_9\}(0.7),$ $\{b_3, b_4, b_6\}(0.1),$ $\{b_3, b_4, b_6, b_8\}(0.1))$	$(\{c_3, c_6, c_7\},$ $\{c_6, c_7\}(0.4),$ $\{c_3, c_6\}(0.3),$ $\{c_6\}(0.1))$	$(\{d_4, d_7, d_{10}\},$ $\{d_7, d_{10}\}(0.5),$ $\{d_4\}(0.3),$ $\{d_4, d_{10}\}(0.2))$
X_{10}	$(\{a_2, a_3, a_7, a_8, a_9\},$ $\{a_7, a_8, a_9\}(0.4),$ $\{a_2, a_7, a_8, a_9\}(0.1),$ $\{a_3\}(0.4))$	$(\{b_2, b_3, b_5, b_6, b_8, b_9\},$ $\{b_3, b_6, b_8, b_9\}(0.6),$ $\{b_2, b_6, b_8, b_9\}(0.2),$ $\{b_3, b_5\}(0.2))$	$(\{c_3, c_7, c_{10}\},$ $\{c_7, c_{10}\}(0.6),$ $\{c_3, c_7\}(0.1),$ $\{c_3\}(0.2))$	$(\{d_1, d_7, d_9, d_{10}\},$ $\{d_1, d_{10}\}(0.3),$ $\{d_7, d_9, d_{10}\}(0.1),$ $\{d_1, d_9\}(0.4))$

Step 6: We obtain diseases similarity in Table 5.

Table 5. Diseases Similarity Table

	h_1	h_2	h_3	h_4
X_1	0.4102	0.4580	0.3193	0.3907
X_2	0.3119	0.4073	0.3119	0.3906

X_3	0.3526	0.4073	0.2619	0.3906
X_4	0.2712	0.2740	0.3102	0.4407
X_5	0.4590	0.3659	0.4179	0.3240
X_6	0.3526	0.2989	0.3693	0.4413
X_7	0.2712	0.3080	0.4345	0.4573
X_8	0.4836	0.3413	0.3360	0.3407
X_9	0.3693	0.4927	0.3693	0.3740
X_{10}	0.3757	0.4520	0.4193	0.3854

Step 7: We obtain weighted similarity of patients with ideal patient in Table 6.

Table 6. Weighted Similarity Table of Patients with Ideal Patient

	$(0, 2).h_1$	$(0, 3).h_2$	$(0, 4).h_3$	$(0, 1).h_4$	$\sum_{k=1}^4 w_k \cdot S_G(K(h_k), X_i(h_k))$
X_1	0,0820	0,1374	0,1277	0,0390	$S_{G1}(K, X_1) = 0,3861$
X_2	0,0623	0,1221	0,1247	0,0390	$S_{G2}(K, X_2) = 0,3481$
X_3	0,0705	0,1221	0,1047	0,0390	$S_{G3}(K, X_3) = 0,3363$
X_4	0,0542	0,0822	0,1240	0,0440	$S_{G4}(K, X_4) = 0,3044$
X_5	0,0918	0,1097	0,1671	0,0324	$S_{G5}(K, X_5) = 0,4010$
X_6	0,0705	0,0896	0,1477	0,0441	$S_{G6}(K, X_6) = 0,3519$
X_7	0,0542	0,0924	0,1738	0,0457	$S_{G7}(K, X_7) = 0,3661$
X_8	0,0967	0,1023	0,1344	0,0340	$S_{G8}(K, X_8) = 0,3674$
X_9	0,0738	0,1478	0,1477	0,0374	$S_{G9}(K, X_9) = 0,4067$

X_{10}	0,0751	0,1356	0,1677	0,0385	$S_{G10}(K, X_{10}) = 0,4169$
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According to the values in the Table 6, the patients with the best treatment are

$$X_{10}, X_9, X_5, X_1, X_8, X_7, X_6, X_2, X_3, X_4$$

respectively.

4.3 Comparison Analysis

In this section, we compare the results of the 4.2 Application with the results of some similarity measures previously defined for neutrosophic sets. However, since generalized set valued neutrosophic numbers are used in 5.2 Application, only the components in the unknown parts (T, I, F) of the generalized set valued neutrosophic numbers be taken for neutrosophic similarity measures in this comparison.

a)

In 4.2 Application,

if we use the Euclid similarity measure in Definition 2.3 [17], we obtain weighted similarity of patients with ideal patient in Table 7.

Table 7. Weighted Similarity Table of Patients with Ideal Patient according to Euclid similarity measure

X_1	0,7887
X_2	0,7476
X_3	0,7554
X_4	0,7017
X_5	0,8132
X_6	0,7609
X_7	0,7448
X_8	0,7598
X_9	0,8146
X_{10}	0,8141

According to the values in the Table 7, the patients with the best treatment are

$$X_9, X_{10}, X_5, X_1, X_6, X_8, X_3, X_2, X_7, X_4$$

respectively.

b) In 4.2 Application,

if we use the Hamming similarity measure in Definition 2.4 [17], we obtain weighted similarity of patients with ideal patient in Table 8.

Table 8. Weighted Similarity Table of Patients with Ideal Patient according to Hamming similarity measure

X_1	0,6832
X_2	0,6198
X_3	0,6364
X_4	0,5332
X_5	0,7032
X_6	0,6297
X_7	0,6399
X_8	0,6365
X_9	0,7164
X_{10}	0,7131

According to the values in the Table 8, the patients with the best treatment are

$$X_9, X_{10}, X_5, X_1, X_7, X_8, X_3, X_6, X_2, X_4$$

respectively.

c)) In 4.2 Application,

if we use the similarity measure in Definition 2.7 [5], we obtain weighted similarity of patients with ideal patient in Table 9.

Table 9. Weighted Similarity Table of Patients with Ideal Patient according to similarity measure in Definition 2.5

X_1	0,4238
X_2	0,3532
X_3	0,3822
X_4	0,2779
X_5	0,4691
X_6	0,3764
X_7	0,3974
X_8	0,3933
X_9	0,4747
X_{10}	0,4741

According to the values in the Table 9, the patients with the best treatment are

$$X_9, X_{10}, X_5, X_1, X_7, X_8, X_3, X_6, X_2, X_4$$

respectively.

We give Comparison of similarity measures in Table 10.

Table 10. Comparison of similarity measures

Similarity Measure	Result
Generalized Euclid Similarity Measure (proposed method)	$X_{10}, X_9, X_5, X_1, X_8, X_7, X_6, X_2, X_3, X_4$
Euclid Similarity Measure [17]	$X_9, X_{10}, X_5, X_1, X_6, X_8, X_3, X_2, X_7, X_4$
Hamming Similarity Measure [17]	$X_9, X_{10}, X_5, X_1, X_7, X_8, X_3, X_6, X_2, X_4$
Similarity Measure in Definition 2.5 [5]	$X_9, X_{10}, X_5, X_1, X_7, X_8, X_3, X_6, X_2, X_4$

From Table 10, we obtain different result from Euclid similarity measure [17], Hamming similarity measure [17] and similarity measure in Definition 2.7 [5].

5 Discussion and Conclusions

In this article, we define firstly generalized Euclid distance measure and generalized Euclid similarity measure based on generalized set valued neutrosophic quadruple numbers. Also, we show that generalized Euclid distance measure and generalized Euclid similarity measure satisfy the distance measure conditions and similarity measure conditions, respectively. Furthermore, we define a score function for generalized Euclid similarity measure.

In addition, we generalized algorithm, for single valued neutrosophic set, based on generalized Euclid similarity measure and generalized set valued neutrosophic quadruple numbers. Using this algorithm, we give an example of individuals with more than one disease to determine which of the known disease medications will be good for their unknown disease (for example covid-19). We compared the results we obtained in this example with the results obtained in neutrosophic numbers and showed that we obtained different results. For example, in Table 10, we obtain different result from Euclid similarity measure [17], Hamming similarity measure [17] and similarity measure in Definition 2.7 [5]. It is clear that in solving such problems there is a need for a structure in which the known part is the unknown part and (T, I, F) known neutrosophic membership functions. Since each known disease will have separate medications and it will be investigated which results (true, indeterminate, false) these drugs will give in unknown diseases, a structure such as $(L_{S_i}T_{S_i}, M_{S_i}I_{S_i}, N_{S_i}F_{S_i})$, containing both cluster and T, I, F, will be needed. For this reason, using generalized Euclid measures based on generalized set valued neutrosophic quadruple numbers in solving such problems can give better results.

Also, using the similarity measures and algorithm in this article, solutions can be found to other problems in the medical field. In addition, decision making applications can be obtained with the help of these similarity measures and algorithms for other branches of science.

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Received: Aug 3, 2021. Accepted: Dec 5, 2021