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Abstract. The concept of neutrosophic normed spaces was introduced by Murat Kirisci* and Necip Simsek [3]. In this paper, by using the compact operator $\psi(z_j)$ and the concept of statistical convergence. we introduce some neutrosophic statistical convergence of sequence spaces defined by compact operator and study of neutrosophic norm and derive the relation between statistical convergence in NNS with the help of compact operator. We also focus on some topological and algebraic properties of these convergent sequence spaces.

Keywords: NNS; t-norm; t-conorm; Statistical convergence; compact linear operator; Bounded linear operator; neutrosophic set.

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1. **Introduction**

The notion of fuzzy set was presented by Zadeh [4], since then several kinds of research have appeared and fuzzication of many classical theories has also been made. Fuzzy sets hypothesis is an amazing handset for demonstrating vulnerability and dubiousness in different problems and issues emerging in field of science and designing. fuzzy topology is perhaps the absolute significant and generally utilized outfits and it ends up being valuable for managing such circumstances where utilization of old style speculations breakdown. [13] he used the concept of fuzzy sets and examine the fuzzy real valued p-absolutely summable multiple sequences in probabilistic normed spaces. Atanassov [29] generalized the fuzzy sets theory and studied the concepts of intuitionistic fuzzy sets (IFS). In 2004, [6] Park researched the idea of (IFS), further Saadati and Park analyzed this concept in the norm. The idea of $IFNS$ and the thought of statistical convergence is a valuable utilitarian apparatus for contemplating the convergence issues of mathematical problems (double sequence) herewith the idea of thickness. [14]. The idea of neutrosophic sets (NS) was introduced by Smarandache [23]. This set is an expansion of IFS regardless, if the summation of neutrosophic segments is < 1, or > 1, or $= 1$. For the situation when the aggregate of the components is 1 ($as\ in\ IFS$), in the wake of satisfying the condition by applying the neutrosophic set operators, different outcomes can be acquired by applying the intuitionistic fuzzy (IF) operators, since the IF operators disregard the ($indeterminacy$), while the NS operators taken into cognizance of the indeterminacy at a similar level as ($truth - membership$) and ($falsehood - nonmembersip$) are taken. [26] using the idea of neutrosophic sets defined the notion of Neutrosophic Bipolar Vague Soft Set and Its Application to Decision Making Problems. Further, Smarandache [9, 10, 12, 35] investigated neutroalgebra which is generalization of partial algebra , neutroalgebraic structures and antialgebraic structures. NS is likewise more adaptable and effective in light of the fact that it handles, aside from autonomous (free) components, additionally partially independent and dependent components,while IFS can't manage these cases. Moreover [38] define neutrosophic fuzzy matrices and Some Algebraic Operations. Smarandache [1, 34] analyzed the conflict between neutrosophic rationale, intuitionistic fuzzy rationale, and the comparing NS and IFS. [11] defined Neutrosophic simply soft open set in neutrosophic soft topological space. Moreover, Bera and Mahapatra [24] introduced the neutrosophic soft linear space. Bera and Mahapatra [25] studied convexity, metric, Cauchy sequence, and neutrosophic soft norm linear space (NSNLS). [15] A lot of developments have been made in this areas after the work of Das, S and Pramanik, S defined the Generalized neutrosophic b-open sets in neutrosophic topological space. Further, [21] examine Neutrosophic Multiset Topological Space. [17] another term find Statistically pre-Cauchy fuzzy real-valued sequences defined by Orlicz function. Later on, the concepts of statistical convergence of double sequences have been analyzed in IFNS by Mursaleen and Mohiuddin [14]. Quite recently, Kirisci and Simsek [3] introduced the notion of NNS and statistical convergence. Further [8] examine On almost statistical convergence of new type of generalized difference sequence of fuzzy numbers. Since NNS is a natural generalization of IFNS and statistical convergence. [19] defined a new concept On pointwise statistical convergence of order alpha of sequences of fuzzy mappings. In this paper we aim to define novel statistical convergence of sequence spaces. sequence spaces using neutrosophic norm and using compact operator as a tool and discussed their topological and algebraic properties. We mention the following notions that will be put to use in the paper further.

2. Preliminaries

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Definition 2.1. A sequence $z = (z_j)$ is called a δ -convergent to the number ξ for each $\varepsilon > 0$, the set $Y(\epsilon)$ has δ-density zero, where

$$Y_\epsilon(F) = \{j \in \mathbb{N} : |z_j - \xi| \geq \epsilon\} \tag{1}$$

we write $S_\delta - \lim z = \xi$ or $z_j \to \xi(S_\delta)$.

Definition 2.2. [14] A sequence $z = (z_j)$ is called statistically Cauchy sequences if $\exists$, a number $\mathcal{K} = \mathcal{K}(\epsilon)$ such that for each $\epsilon > 0$

$$\lim_{j\to\infty} \frac{1}{j}|\{i \leq j : |z_i - (z_{\mathcal{N}})| \geq \epsilon\}| = 0. \tag{2}$$

Definition 2.3. [7] Let $X \neq \varnothing$, the $\mathcal{W}$ intuitionistic fuzzy set, $\mathcal{W} \subset X$ is defined by

$$\mathcal{W} = \{< z, \mathbf{T}(z),\ \mathbf{F}(z) >: z \in X\}, \tag{3}$$

where $\mathbf{T}(z), \mathbf{F}(z) : X \to [0,1]$, $\mathbf{T}(z) =$ (Truth) and $\mathbf{F}(z) =$ (Falsehood) respectively.

$$0 \leq \mathbf{T}(z) + \mathbf{F}(z) \leq 1$$

Definition 2.4. [31] Let $X \neq \phi$ and $\mathcal{W} \subset X$ Then,

$$\mathcal{W}_{NS} = \{< z, \mathbf{T}(z), \mathbf{I}(z), \mathbf{F}(z) >: z \in X\},$$

where $\mathbf{T}(z), \mathbf{I}(z), \mathbf{F}(z) : X \to [0,1]$, $\mathbf{T}(z) =$ Truth, $\mathbf{I}(z) =$ Indeterminacy, and $\mathbf{F}(z) =$ Falsehood respectively.

$$0 \leq \mathbf{T}(z) + \mathbf{I}(z) + \mathbf{F}(z) \leq 3.$$

The components of neutrosophic are $\mathbf{T}(z), \mathbf{I}(z)$ and $\mathbf{F}(z)$ independent of each other.

Definition 2.5. [1] Suppose $X \neq \phi$, Q and R are neutrosophic sets in X. Then,

(a) $Q \subset R \iff \mathbf{T}_Q(z) \leq \mathbf{T}_R(z), \mathbf{I}_Q(z) \leq \mathbf{I}_R(z), \mathbf{F}_Q(z) \geq \mathbf{F}_R(z)\ \forall\ z \in X$

(b) $Q = R \iff \mathbf{T}_Q(z) = \mathbf{T}_R(z), \mathbf{I}_Q(z) = \mathbf{I}_R(z), \mathbf{F}_Q(z) = \mathbf{F}_R(z)\ \forall\ z \in X$

(c) $Q \cap R = \{\langle z, \min(\mathbf{T}_Q(z), \mathbf{T}_R(z)), \min(\mathbf{I}_Q(z), \mathbf{I}_R(z)), \min(\mathbf{F}_Q(z), \mathbf{F}_R(z))\rangle \mid z \in X\}$

(d) $Q \cup R = \{\langle z, \max(\mathbf{T}_Q(z), \mathbf{T}_R(z, \max(\mathbf{I}_Q(z), \mathbf{I}_R(z)), \max(\mathbf{F}_Q(z), \mathbf{F}_R(z))\rangle, z \in X\}$

(e) $Q^c = \{\langle z, \mathbf{F}_Q(z), 1 - \mathbf{I}_R(z)), \mathbf{T}_Q(z)\rangle \mid z \in X\}$

(f) $Q \backslash R = \{\langle z, \mathbf{T}_Q(z) \min \mathbf{F}_R(z), \mathbf{I}_Q(z) \min 1 - \mathbf{I}_R(z), \mathbf{F}_Q(z) \max \mathbf{T}_R(z)\rangle \mid z \in X\}$.

Definition 2.6. [40] Consider a binary operation $\diamond$ on the interval.if satisfying following axioms,

$$\diamond : [0,1]^2 \longrightarrow [0,1]$$

(1) $\hat{\alpha} \diamond \hat{\beta} = \hat{\beta} \diamond \hat{\alpha}$

(2) $(\hat{\alpha} \diamond \hat{\beta}) \diamond \hat{\lambda} = \hat{\alpha} \diamond (\hat{\beta} \diamond \hat{\lambda})$

(3) $\hat{\alpha} \diamond 1 = \hat{\alpha}\ \forall\ \hat{\alpha} \in [0,1]$,

(4) $\hat{\alpha} \diamond \hat{\beta} \leq \hat{\lambda} \diamond \hat{v}$ whenever $\hat{\alpha} \leq \hat{\lambda}$ and $\hat{\beta} \leq \hat{v}$ for each $\hat{\alpha}, \hat{\beta}, \hat{\lambda}, \hat{v} \in [0,1]$.

is called a continuous t-norm.
where $\wedge = \min$ and $\vee = \max$.

Example 2.7. For $\hat{\alpha}, \hat{\beta} \in [0,1]$, define $\hat{\alpha} \diamond \hat{\beta} = \hat{\alpha}\hat{\beta}$ or $\hat{\alpha} \diamond \hat{\beta} = \wedge\{\hat{\alpha}, \hat{\beta}\}$. then $\diamond$ is continuous t-norm.

Definition 2.8. [40] Consider a binary operation $\star$ on the interval.if satisfying following axioms,

$$\star : [0,1]^2 \longrightarrow [0,1]$$

(1) $\hat{\alpha} \star \hat{\beta} = \hat{\beta} \star \hat{\alpha}$
(2) $(\hat{\alpha} \star \hat{\beta}) \star \hat{\lambda} = \hat{\alpha} \star (\hat{\beta} \star \hat{\lambda})$
(3) $\hat{\alpha} \star 0 = \hat{\alpha} \ \forall \ \hat{\alpha} \in [0,1]$,
(4) $\hat{\alpha} \star \hat{\beta} \leq \hat{\lambda} \star \hat{\upsilon}$ whenever $\hat{\alpha} \leq \hat{\lambda}$ and $\hat{\beta} \leq \hat{\upsilon}$ for each $\hat{\alpha}, \hat{\beta}, \hat{\lambda}, \hat{\upsilon} \in [0,1]$.
is called a continuous t-conorm.
where $\wedge = \min$ and $\vee = \max$.

Example 2.9. Consider $\hat{\alpha}, \hat{\beta} \in [0,1]$. define $\hat{\alpha} \star \hat{\beta} = \wedge\{\hat{\alpha} + \hat{\beta}, 1\}$ or $\hat{\alpha} \star \hat{\beta} = \vee\{\hat{\alpha}, \hat{\beta}\}$. Then $\star$ is continuous t-conorm

Definition 2.10. [3] Take X as a vector space and $\mathcal{M} = \{< x, \mathbf{T}(z), \mathbf{I}(z), \mathbf{F}(z) >: z \in X\}$ be a normed space such that $\mathbf{T}(z), \mathbf{I}(z), \mathbf{F}(z) : X \times R^+ \to [0,1]$. Assume $\diamond$ show the continuous t-norm and $\star$ be a continuous t-conorm respectively, then $\mathcal{V} = (X, \mathcal{M}, \diamond, \star)$ is called (NNS). if the subsequent terms holds; $\forall \ z, a \in X$ and $s, k \ > 0$

(i) $0 \leq \mathbf{T}(z,s) \leq 1$, $0 \leq \mathbf{I}(z,s) \leq 1$, $0 \leq \mathbf{F}(z,s) \leq 1$, $s \in R^+$,

(ii) $\mathbf{T}(z,s) + \mathbf{I}(z,s) + \mathbf{F}(z,s) \leq 3$, for $s \in R^+$,

(iii) $\mathbf{T}(z,s) = 1$ for $s > 0$ iff $z = 0$

(iv) $\mathbf{T}(\alpha x, s) = \mathbf{T}(z, \frac{s}{|\alpha|})$,

(v) $\mathbf{T}(z,s) \diamond \mathbf{T}(a,s) \leq \mathbf{T}(z + a, s + k)$,

(vi) $\mathbf{T}(z, \diamond)$ is continuous non-decreasing function

(vii) $\lim_{s\to\infty} \mathbf{T}(z,s) = 1$

(viii) $\mathbf{I}(z,s) = 0$ for $s > 0$ iff $z = 0$

(ix) $\mathbf{I}(\alpha z, s) = \mathbf{I}(y, \frac{s}{|\alpha|})$, for each $\alpha \neq 0$,

(x) $\mathbf{I}(z,s) \star \mathbf{I}(a,k) \geq \mathbf{I}(z+a, s+k)$,

(xi) $\mathbf{I}(z,\star)$ is continuous non-increasing function,

(xii) $\lim_{s\to\infty} \mathbf{I}(z,s) = 0$,

(xiii) $\mathbf{F}(z,s) = 0$ for $s > 0$ iff $z = 0$

(xiv) $\mathbf{F}(\alpha z, s) = \mathbf{F}(z, \frac{s}{|\alpha|})$, for each $\alpha \neq 0$,

(xv) $\mathbf{F}(z,s) \star \mathbf{F}(z,k) \geq \mathbf{F}(z+a, s+k)$,

(xvi) $\mathbf{F}(z,.)$ is continuous non-increasing function,

(xvii) $\lim_{s\to\infty} \mathbf{F}(z,s) = 0$,

(xviii) If $s \leq 0$, then $\mathbf{T}(z,s) = 0$, $\mathbf{I}(z,s) = 1$, $\mathbf{F}(z,s) = 1$.

In this case $\mathcal{M} = (\mathbf{T}, \mathbf{I}, \mathbf{F})$ is said to be neutrosophic normed (NNS).

Example 2.11. [3] Suppose $(X, \|\ .\ \|)$ be a NNS. Give the operations as TC $z \diamond a = z + a - za$ and TN $z \star a = \min(z,a)$. For $s > ||z||$,

$$\mathbf{T}_0(z,s) = \frac{s}{s + ||z||},\ \mathbf{I}_0(z,s) = \frac{||z||}{s + ||z||},\ \mathbf{F}_0(z,s) = \frac{||z||}{s}. \tag{4}$$

for all $z, a \in [0,1]$ and $s > 0$. If we take $s \leq ||z||$, and let $\mathbf{T}_0,\ \mathbf{I}_0\ and\ \mathbf{F}_0$ be neutrosophic sets on $X \times (0,\infty)$ then

$$\mathbf{T}_0(z,s) = 0, \mathbf{I}_0(z,s) = 1 \text{ and } \mathbf{F}_0(z,s) = 1.$$

Then, $(X, \mathcal{M}, \diamond, \star,)$ is neutrosophic normed space such that $\mathcal{M} : X \times R^+ \to [0,1]$.

3. Main Results

In this article, we examine the algebriac and topological properties on the space of neutrosophic statistical convegence given by :

Definition 3.1. [3] Suppose $(X, \mathcal{M}, \star, \diamond)$ be a NNS and $z = (z_j)$ is called a Cauchy sequence with respect to $\mathcal{M}$, if for each $\epsilon > 0$ and $s > 0$ $\exists$, $u \in \mathbb{N}$ such that $\mathbf{T}(z_j - a_k, s) > 1 - \epsilon$, $\mathbf{I}(z_j - a_k, s) < \epsilon$ and $\mathbf{F}(z_j - a_k, s) < \epsilon$ for all $j, k \geq u$.

Definition 3.2. Let X be a NNS, the sequence $z = (z_j)$ in X is called convergent at $\xi \in X$ iff $\exists$, $\mathring{N} \in \mathbb{N}$, with respect NN $(\mathbf{T}, \mathbf{I}, \mathbf{F})$ if for every $\epsilon > 0$, $s > 0$

$$\mathbf{T}(\psi(z_j) - \xi, s) > 1 - \epsilon, \mathbf{I}(\psi(z_j) - \xi, s) < \epsilon \text{ and } \mathbf{F}(\psi(z_j) - \xi, s) < \epsilon \tag{5}$$

for all $j \geq \mathring{N}$, i.e.,

$$\lim_{j\to\infty} \mathbf{T}(\psi(z_j) - \xi, s) = 1,\ \lim_{j\to\infty} \mathbf{I}(\psi(z_j) - \xi, s) = 0 \text{ and } \lim_{j\to\infty} \mathbf{F}(\psi(z_j) - \xi, s) = 0.$$

In such case, we denote $\mathcal{M} - \lim z_j = \xi$.

Definition 3.3. (See [2]).Suppose A and B be two normed linear spaces (NLS) and $\psi : \mathcal{D} \to B$ be a linear operator, where $\mathcal{D}(\psi) \subset A$ Then, the operator ψ is called a bounded, if $\exists$, a +ve real no c' such that

$$||\psi z|| \leq c'||z||,\ \forall z \in \mathcal{D}(\psi).$$

The set of all bounded linear operators $\mathbf{B}(A, B)$ [2] is a (NLS) normed by

$$||\psi|| = \sup_{z\in A,\ ||z||=1} ||\psi z||$$

and $\mathbf{B}(A, B)$ is a Banach space if B is a Banach space.

Definition 3.4. Let us Consider two NLS A and B

$$\psi : A \to B$$

is called a compact linear operator, if

(i) ψ is linear

(ii) ψ maps every bounded sequence (z_j) in A on to a sequence $\psi(z_j)$ in B which has a convergent subsequence.

The set of all compact linear operators $\mathcal{C}(A, B)$ is a closed subspace of $\mathbf{B}(A, B)$ and $\mathcal{C}(A, B)$ is Banach space, if B is a Banach space.

Inspire by this [5], we proposed the previous sequence spaces with the assist of compact operator in NNS:

$$\mathcal{S}^{N}_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi) = \big\{(z_j) \in \ell_\infty : \ \exists\, k \in \mathbb{N}, \forall\, j \geq k, \mathbf{T}(\psi(z_j) - \xi, s) > 1 - \epsilon \ \text{ or } \ \mathbf{I}(\psi(z_j) - \xi, s) \geq \epsilon \ \mathbf{F}(\psi(z_j) - \xi, s) \geq \epsilon\big\}.$$

$$S^{N}_{0(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi) = \big\{(z_j) \in \ell_\infty : \ \exists\, k \in \mathbb{N}, \forall\, j \geq k, \mathbf{T}(\psi(z_j) - \xi, s) > 1 - \epsilon \text{ or } \mathbf{I}(\psi(z_j) - \xi, s) \geq \epsilon \ \mathbf{F}(\psi(z_j) - \xi, s) \geq \epsilon\big\}.$$

Definition 3.5. [5] Let $\mathcal{V} = (X, M, \diamond, \star)$ is a NNS. For $s > 0$. we define a open ball (OB) $\mathcal{B}(r, z, s)(\psi)$ with centre $z \in X$ and raduis $0 < r < 1$, there exist $k \in \mathbb{N}$ such that for all $j \geq k$

$$\mathcal{B}(z, r, s)(\psi) = \{(a_j) \in \ell_\infty : \mathbf{T}(\psi(z_j) - \psi(a_k), s) < 1 - \epsilon \text{ or } \mathbf{I}(\psi(z_j) - \psi(a_k), s) < \epsilon, \ \mathbf{F}(\psi(z_j) - \psi(a_k), s) < \epsilon\}.$$

Presently, we are prepared to state and demonstrate our primary outcomes. This hypothesis depends on the linearity of new characterize sequence spaces which is expressed as follows.

Theorem 3.6. *$\mathcal{S}^{N}_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$ and $S^{N}_{0(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$ are linear spaces.*

Proof. We shall prove the result for $\mathcal{S}^{N}_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$ the proof for the different space will take accordingly, let $z = (z_j), a = (a_j) \in \mathcal{S}^{N}_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$ and $\hbar, \eta$ be scalars then for a given $\epsilon > 0$, we have

$$\mathcal{W}_1 = \Big\{j \in \mathbb{N} : \mathbf{T}\Big(\psi(z_j) - \xi_1, \frac{s}{2|\hbar|}\Big) \leq 1 - \epsilon \text{ or } \mathbf{I}\Big(\psi(z_j) - \xi_1, \frac{s}{2|\hbar|}\Big) \geq \epsilon, \ \mathbf{F}\Big(z_j - \xi_1, \frac{s}{2|\hbar|}\Big) \geq \epsilon\Big\}$$

$$\mathcal{W}_2 = \Big\{j \in \mathbb{N} : \mathbf{T}\Big(\psi(a_j) - \xi_2, \frac{s}{2|\eta|}\Big) \leq 1 - \epsilon \text{ or } \mathbf{I}\Big(\psi(a_j) - \xi_2, \frac{s}{2|\eta|}\Big) \geq \epsilon, \mathbf{F}\Big(\psi(a_j) - \xi_2, \frac{s}{2|\eta|}\Big) \geqslant \epsilon\Big\}$$

$$\mathcal{W}^c_1 = \Big\{j \in \mathbb{N} : \mathbf{T}\Big(\psi(z_j) - \xi_1, \frac{s}{2|\hbar|}\Big) > 1 - \epsilon \text{ or } \mathbf{I}\Big(\psi(z_j) - \xi_1, \frac{s}{2|\hbar|}\Big) < \epsilon, \mathbf{F}(\psi(z_j) - \xi_1, \frac{s}{2|\hbar|}) < \epsilon\Big\}$$

$$\mathcal{W}^c_2 = \Big\{j \in \mathbb{N} : \mathbf{T}\Big(\psi(a_j) - \xi_2, \frac{s}{2|\eta|}\Big) > 1 - \epsilon \text{ or } \mathbf{I}\Big(\psi(a_j) - \xi_2, \frac{s}{2|\eta|}\Big) < \epsilon, \mathbf{F}\Big(\psi(a_j) - \xi_2, \frac{s}{2|\eta|}) < \epsilon\Big\}$$

Define the set $\mathcal{W}_3 = \mathcal{W}_1 \cup \mathcal{W}_2$ *so that* $\mathcal{W}_3 \in \mathcal{M}$. It follow that $\mathcal{W}_3^c \neq \phi$. we shall show that for each $(z_j), (a_j) \in \mathcal{S}^N_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$.

$$\mathcal{W}_3^c \subset \Big\{ j \in \mathbb{N} : \ \mathbf{T}\Big((\hbar\psi(z_j) + \eta\psi(a_j)) - (\hbar\xi_1 + \eta\xi_2), s\Big) > 1 - \epsilon \ or \ \mathbf{I}\Big(\hbar\psi(z_j) + \eta\psi(a_j)) - (\hbar\xi_1 + \eta\xi_2), s\Big) < \epsilon, \ \mathbf{F}\Big((\hbar\psi(z_j) + \eta\psi(a_j)) - (\hbar\xi_1 + \eta\xi_2), s\Big) < \epsilon \Big\}$$

let $m \in \mathcal{W}_3^c$. In this case,

$$\mathbf{T}\Big(\psi(z_m) - \xi_1, \tfrac{s}{2|\hbar|}\Big) > 1 - \epsilon \ or \ \mathbf{I}\Big(\psi(z_m) - \xi_1, \tfrac{s}{2|\hbar|}\Big) < \epsilon, \mathbf{F}\Big(\psi(z_m) - \xi_1, \tfrac{s}{2|\hbar|}\Big) < \epsilon$$

and

$$\mathbf{T}\Big(\psi(a_m) - \xi_2, \tfrac{s}{2|\eta|}\Big) > 1 - \epsilon \ or \ \mathbf{I}\Big(\psi(a_m) - \xi_2, \tfrac{s}{2|\eta|}\Big) < \epsilon, \mathbf{F}\Big(\psi(a_m) - \xi_2, \tfrac{s}{2|\eta|}\Big) < \epsilon.$$

We have

$$\mathbf{T}\Big((\hbar\psi(z_m) + \eta\psi(a_m)) - (\hbar\xi_1 + \eta\xi_2), s\Big)$$

$$\geq \mathbf{T}\Big(\hbar\psi(z_m) - \hbar\xi_1, \frac{t}{2}\Big) \diamond \mathbf{T}\Big(\eta\psi(z_m) - \eta\xi_2, \frac{t}{2}\Big)$$

$$= \mathbf{T}\Big(\psi(z_m) - \xi_1, \frac{s}{2|\hbar|}\Big) \star \mathbf{T}\Big(\psi(z_m) - \xi_2, \frac{s}{2|\eta|}\Big)$$

$$> (1 - \epsilon) \diamond (1 - \epsilon) = 1 - \epsilon.$$

In similar way,

$$\mathbf{I}\Big((\hbar\psi(z_m) + \eta\psi(a_m)) - (\hbar\xi_1 + \eta\xi_2), s\Big)$$

$$\leq \mathbf{I}\Big(\hbar\psi(z_m) - \hbar\xi_1, \frac{s}{2}\Big) \star \mathbf{I}\Big(\eta\psi(z_m) - \eta\xi_2, \frac{s}{2}\Big)$$

$$= \mathbf{I}\Big(\psi(z_m) - \xi_1, \frac{s}{2|\hbar|}\Big) \star \mathbf{I}\Big(\psi(z_m) - \xi_2, \frac{s}{2|\eta|}\Big)$$

$$< \epsilon \star \epsilon < \epsilon.$$

and,

$$\mathbf{F}\Big((\hbar\psi(z_m) + \eta\psi(a_m)) - (\hbar\xi_1 + \eta\xi_2), s\Big)$$

$$\leq \mathbf{F}\Big(\hbar\psi(z_m) - \hbar\xi_1, \frac{s}{2}\Big) \star \mathbf{F}\Big(\eta\psi(z_m) - \eta\xi_2, \frac{s}{2}\Big)$$

$$= \mathbf{F}\Big(\psi(z_m) - \xi_1, \frac{s}{2|\hbar|}\Big) \star \mathbf{F}\Big(\psi(z_m) - \xi_2, \frac{s}{2|\eta|}\Big)$$

$$< \epsilon \star \epsilon < \epsilon.$$

This implies that,

$$\mathcal{W}_3^c \subset \Big\{ j \in \mathbb{N} : \begin{array}{l} \mathbf{T}\Big((\hbar\psi(z_j) + \eta\psi(a_j)) - (\hbar\xi_1 + \eta\xi_2), s\Big) > 1 - \epsilon \ or \ \mathbf{I}\Big(\hbar\psi(z_j) + \eta\psi(a_j)) - (\hbar\xi_1 + \eta\xi_2), s\Big) < \epsilon, \\ \mathbf{F}\Big((\hbar\psi(z_j) + \eta\psi(a_j)) - (\hbar\xi_1 + \eta\xi_2), s\Big) < \epsilon \Big\}. \end{array}$$

Hence $\mathcal{S}^N_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$ is a linear space. □

Remark 3.7. $\mathcal{S}^N_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$ is an NNS.

Define $\mathcal{T}^N_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi) = \{W \subset \mathcal{S}^N_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$: for every $z \in W$, $\exists$, $s > 0$ and $r \in (0,1)$ such that $\mathcal{B}(z, r, s)(\psi) \subset W\}$.

Theorem 3.8. *The topology $\mathcal{T}^N_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$ on $\mathcal{S}^N_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$ is first countable.*

Proof. $\{B_x(\frac{1}{n}, \frac{1}{n}) : n = 1, 2, 3, ...\}$ is a local base at x, $\mathcal{T}^N_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$ on $\mathcal{S}^N_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$ is first countable. $\mathcal{S}^N_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$ is Hausdorff. □

Theorem 3.9. *$\mathcal{S}^N_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$ is an NNS and $\mathcal{T}^N_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$ is a topology on $\mathcal{S}^N_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$ Then a sequence $(z_j) \in \mathcal{S}^N_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi), (z_j) \to z$,if and only if $\mathbf{T}((\psi(z_j) - z), t) \to 1, \mathbf{I}((\psi(z_j) - z), s) \to 1$ and $\mathbf{F}((\psi(z_j) - z), s) \to 1$ as $j \to \infty$.*

Proof. Fix $s_0 > 0$. suppose $\psi(z_j) \to z$. Then for $r \in (0,1)$ $\exists$, $n_0 \in \mathbb{N}$ such that $(\psi(z_j)) \in B_z(r,s)$ for all $j \geq n_0$,

$$B_z(r,s) = \Big\{j \in \mathbb{N} : \mathbf{T}(\psi(z_j) - z, s) \ or \ \mathbf{I}(\psi(z_j) - z, s) < r, \mathbf{F}(\psi(z_j) - z, s) < r\Big\}$$

Such that, $B_z^c(r,s)$ Then,

$$1 - \mathbf{T}((\psi(z_j) - z), s) < r \ and \ \mathbf{I}((\psi(z_j) - z), s) < r, \mathbf{F}((\psi(z_j) - z), s) < r.$$

Hence,

$$\mathbf{T}((\psi(z_j) - z), s) \to 1 \ and \ \mathbf{I}((\psi(z_j - z), s) \to 0, \mathbf{F}((\psi(z_j - z), s) \to 0 \ as \ j \to \infty.$$

$\Longleftarrow$

if for every $s > 0$, $\mathbf{T}((\psi(z_j) - z), s) \to 1$ and $\mathbf{I}((\psi(z_j) - z), s) \to 0$, $\mathbf{F}((\psi(x_k) - x), s) \to 0$ as $j \to \infty$ then for $r \in (0,1)$ $\exists, n_0 \in \mathbb{N}$ such that $1 - \mathbf{T}((\psi(z_j) - z), s) < r$ and $\mathbf{I}((\psi(z_j) - z), s) < r$, $\mathbf{F}((\psi(z_j) - z), s) < r$ for all $j \geq n_0$.

It follows that $\mathbf{T}((\psi(z_j) - z), s) > 1 - r$ and $\mathbf{I}((\psi(z_j) - z), s) < r$, $\mathbf{F}((\psi(z_j) - z), s) < r$ for all $j \geq n_0$.Thus $(\psi(z_j)) \in B_z^c(r,s)$ for all $j \geq n_0$ and hence $\to \psi(z_j)$ □

Theorem 3.10. *A sequence $z = (z_k) \in \mathcal{S}^N_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$ is $\mathcal{M}$-convergent iff for each $\epsilon > 0$ and $s > 0$ $\exists$, a number $\mathcal{K} = \mathcal{K}(z, \epsilon, t)$ such that*

$$\Big\{j \in \mathbb{N} : \mathbf{T}\Big((\psi(z_j) - \ell), \frac{s}{2}\Big) > 1 - \epsilon \ or \ \mathbf{I}\Big(\psi((z_j) - \ell), \frac{s}{2}\Big) < \epsilon \ or \ \mathbf{F}\Big((\psi(z_j) - \ell), \frac{s}{2}\Big) < \epsilon\Big\}$$

Proof. Suppose $\mathcal{M} - \lim x = \ell$. For a given $\epsilon > 0$, Let $s > 0$ such that $(1-\epsilon) \diamond (1-\epsilon) > 1 - s$ and $\epsilon \star \epsilon < s, \epsilon \star \epsilon < s$. Then
for every $z \in S^N_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$

$$\mathcal{W} = \Big\{j \in \mathbb{N} : \mathbf{T}\Big((\psi(z_j) - \ell), \frac{s}{2}\Big) \leq 1 - \epsilon \ or \ \mathbf{I}\Big((\psi(z_j) - \ell), \frac{s}{2}\Big) \geq \epsilon, \mathbf{F}((\psi(z_j) - \ell), \frac{s}{2}) \geq \epsilon\Big\}$$

which implies that

$$\mathcal{W}^c = \Big\{j \in \mathbb{N} : \mathbf{T}\Big((\psi(z_j) - \ell), \frac{s}{2}\Big) > 1 - \epsilon \ or \ \mathbf{I}\Big((\psi(z_j) - \ell), \frac{s}{2}\Big) < \epsilon, \mathbf{F}\Big((\psi(z_j) - \ell), \frac{s}{2}\Big) < \epsilon\Big\}$$

Conversely, take $N \in \mathcal{W}$. Then

$$\mathbf{T}\Big((\psi(z_j) - \ell), \frac{s}{2}\Big) > 1 - \epsilon \ or \ \mathbf{I}\Big((\psi(z_j) - \ell), \frac{s}{2}\Big) < \epsilon, \mathbf{F}\Big((\psi(z_j) - \ell), \frac{s}{2}\Big) < \epsilon.$$

Now we have to prove that $\exists$, a number $\mathcal{K} = \mathcal{K}(z, \epsilon, t)$ such that,

$$\Big\{j \in \mathbb{N} : \mathbf{T}(\psi(z_j) - \psi(z_N), s) \leq 1 - \gamma \ or \ \mathbf{I}(\psi(z_k) - \psi(z_N)), s) \geq \gamma, \mathbf{F}(\psi(z_j) - \psi(z_N), s) \geq \gamma\Big\}$$

for each $z \in S^N_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$

$$B = \Big\{ j \in \mathbb{N} : \mathbf{T}(\psi(z_j) - \psi(z_N), s) \leq 1 - \gamma \ or \ \mathbf{I}(\psi(z_j) - \psi(z_N), s) \geq \gamma, \mathbf{F}(\psi(z_j) - \psi(z_N), s) \geq \gamma \Big\}.$$

Now we will prove that $B \subset \mathcal{W}$. consider that $B \subseteq \mathcal{W}$ Then $\exists$, $n \in B$ and $n \notin \mathcal{W}$ so we get

$$\mathbf{T}(\psi(z_n) - \psi(x_N)), s) \leq 1 - s \ or \ \mathbf{I}(\psi(z_n - \ell), \frac{s}{2}) > 1 - \epsilon, \mathbf{F}\Big(\psi(z_n - \ell), \frac{s}{2}) > 1 - \epsilon$$

In a diffrent way $\mathbf{T}\Big((\psi(z_N) - \ell), \frac{s}{2}\Big) > 1 - \epsilon$ Therefore we have

$$1 - \gamma \geq \mathbf{T}(\psi(z_j) - \psi(z_N)), s) \geq \mathbf{T}\Big((\psi(z_n) - \ell), \frac{s}{2}\Big) \diamond \mathbf{T}\Big((\psi(z_N) - \ell), \frac{s}{2}\Big) \geq (1 - \epsilon) \diamond (1 - \epsilon) > 1 - \gamma,$$

which is impossible. At the same time,

$$\mathbf{I}(\psi(z_n) - \psi(z_N)), s) \geq \gamma \ or \ \mathbf{I}\Big((\psi(z_n) - \ell), \frac{s}{2}\Big) < \epsilon,$$

In a similar way,

$$\mathbf{I}\Big((\psi(z_N) - \ell), \frac{s}{2}\Big) < \epsilon,$$

$$\gamma \leq \mathbf{I}(\psi(z_n) - \psi(z_N), s) \leq \mathbf{I}\Big((\psi(z_n) - \ell), \frac{s}{2}\Big) \star \mathbf{I}\Big((\psi(z_N) - \ell), \frac{s}{2}\Big) \leq \epsilon \star \epsilon < \gamma,$$

which is impossible.At the same time,

$$\mathbf{F}(\psi(z_n) - (z_N)), s) \geq \gamma \ or \ \mathbf{F}\Big((\psi(x_n) - \ell), \frac{s}{2}\Big) < \epsilon,$$

In particular,

$$\mathbf{F}\Big(\psi(z_N) - \ell), \frac{s}{2}\Big) < \epsilon$$

$$\gamma \leq \mathbf{F}(\psi(z_n) - \psi(z_N), s) \leq \mathbf{F}\Big(\psi(z_n) - \ell), \frac{s}{2}\Big) \star \mathbf{F}\Big((\psi(z_N) - \ell), \frac{s}{2}\Big) \leq \epsilon \star \epsilon < \gamma,$$

which is impossible. Therefore $B \subset \mathcal{W}$. Hence $\mathcal{W} \in \mathcal{M}$ implies $B \in \mathcal{M}$. □

Definition 3.11. Let $\mathcal{V} = (X, M, \diamond, \star)$ is a NNS. For $0 < s < 1$ we fix a closed ball $\mathcal{B}[z, r, s]$ with centre $z \in X$ and raduis $r > 0$, there exist $k \in \mathbb{N}$ such that for all $j \geq k$

$$\mathcal{B}[z, r, s] = \{(a_k) \in \ell_\infty : \mathbf{T}(\psi(z_j) - \psi(a_k), s) \geq 1 - r \text{ or } \mathbf{I}(\psi(z_j) - \psi(a_k), s) \leq r, \ \mathbf{F}(\psi(z_j) - \psi(a_k), s) \leq r\}.$$

Lemma 3.12. *Every closed ball $\mathcal{B}[z, r, s](\psi)$ is a closed set (CS).*

Proof. Consider $a = (a_j) \in \ell_\infty$ such that $a \in \overline{\mathcal{B}[z,r,s](\psi)}$. Since X is first countable, $\exists$, $a = (a_j) \in \overline{\mathcal{B}[z,r,s](\psi)}$ such that $a_j \longrightarrow a$ as $j \longrightarrow \infty$. implies the set

$$X = \Big\{(a_j) \in \ell_\infty : \mathbf{T}(\psi(z_j) - \psi(a_j), s) \geq 1 - r \text{ or } \mathbf{I}(\psi(z_j) - \psi(a_j), s) \leq r,\ \mathbf{F}(\psi(z_j) - \psi(a_j), s) \leq r\Big\}.$$

Since $a_j \longrightarrow a$, $\mathbf{T}\big(\psi(a_j) - \psi(a), s\big) \longrightarrow 1$ and $\mathbf{I}\big(\psi(a_j) - \psi(a), s\big) \longrightarrow 0$, $\mathbf{F}\big(\psi(a_i) - \psi(a), s\big) \longrightarrow 0$, as $j \longrightarrow \infty$. for all s. For a given $\epsilon > 0$,

Hence for $j \in X$

$$\mathbf{T}\big(\psi(z) - \psi(a), s + \epsilon\big) \geq \lim_{j \longrightarrow \infty} \mathbf{T}\big(\psi(z) - \psi(a_j), s\big) \diamond \mathbf{T}\big(\psi(a_j) - \psi(a), \epsilon\big) \geq 1 \diamond (1 - r) = 1 - r$$

and

$$\mathbf{I}\big(\psi(z) - \psi(a), s + \epsilon\big) \leq \lim_{j \longrightarrow \infty} \mathbf{I}\big(\psi(z) - \psi(a_j), s\big) \star \mathbf{I}\big(\psi(a_j) - \psi(a), \epsilon\big) \leq 0 \star r = r.$$

$$\mathbf{F}\big(\psi(z) - \psi(a), s + \epsilon\big) \leq \lim_{j \longrightarrow \infty} \mathbf{F}\big(\psi(z) - \psi(a_j), s\big) \star \mathbf{F}\big(\psi(a_j) - \psi(a), \epsilon\big) \leq 0 \star r = r.$$

In particuler for $b \in \mathbb{N}$, take $\epsilon = \frac{1}{b}$. Then,

$$\mathbf{T}\big(\psi(z) - \psi(a), s\big) = \lim_{b \longrightarrow \infty} \mathbf{T}\Big(\psi(z) - \psi(a), s + \frac{1}{b}\Big) \geq 1 - r.$$

and

$$\mathbf{I}\big(\psi(z) - \psi(a), s\big) = \lim_{b \longrightarrow \infty} \mathbf{I}\Big(\psi(z) - \psi(a), r + \frac{1}{b}\Big) \leq r.$$

$$\mathbf{F}\big(\psi(z) - \psi(a), s\big) = \lim_{b \longrightarrow \infty} \mathbf{F}\Big(\psi(z) - \psi(a), s + \frac{1}{b}\Big) \leq r.$$

$\Longrightarrow$ The set

$$\Big\{(a_j) \in \ell_\infty : \mathbf{T}(\psi(z_j) - \psi(a_j), s) \geq 1 - r \text{ or } \mathbf{I}(\psi(z_j) - \psi(a_j), s) \leq r,\ \mathbf{F}(\psi(z_j) - \psi(a_j), s) \leq r\Big\}.$$

$\Longrightarrow$ $a \in B[z,r,s](\psi)$. Therefore $B[z,r,s](\psi)$ is a closed set. $\square$

4. Conclusions

In this review, we have studied the concept of statistical convergence using neutrosophic norm space with the help of compact operator, which has an important place in the literature. We have defined the compact operator form statistical convergence of sequence spaces $\mathcal{S}^N_{(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$ and $S^N_{0(\mathbf{T},\mathbf{I},\mathbf{F})}(\psi)$ and investigated basic properties. These are illustrated by proper examples.

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