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**Abstract:** This paper demonstrates a concept of split domination in neutrosophic graphs.Minimal split domination, lower and upper split dominations in neutrosophic graphs are discussed. Theorems are derived for minimal split domination in neutrosophic graphs with suitable examples.

**Keywords:** Neutrosophic graph, Domination in neutrosophic graph, Split domination, Cardinality, Split domination number, Isolated vertex.

# 1 Introduction

A mathematical frame work to describe the phenomena of uncertainly in real life situation is first suggested by L.A.Zadeh in 1965[26]. Rosenfeld[16] introduced the notion of fuzzy graphs and several fuzzy analogs of graph theoretic concepts such as path, cycle and connectedness. The study of dominating sets in graphs was begun by Orge and Berge. Many authors discussed the concept of various dominations in graph, fuzzy and intuitionistic fuzzy graphs in [2,7,6,9,10,11,14,15, 18,19,24]. Q.M.Mahyoub and N.D.Soner[8] initiate the split dominating set and split domination number in fuzzy graphs. Also, the split domination number and its properties in Intuitionistic Fuzzy Graphs (IFGS) were studied by [11]. Neutrosophic set proposed by Smarandache[1] is powerful tool for dealing incomplete and indeterminate problems in the real world. It is the generalization of fuzzy sets [3] and Intuitionistic fuzzy sets [4,5]. Fuzzy graph and Intuitionistic approaches are failed in some applications when indeterminacy occurs. So Smarandache defined four main categories of Neutrosophic graphs in [20,21,22,23]. M.Mullai [27] introduced the concept of domination in neutrosophic graphs. By considering the existing split dominating sets, in this proposed work, the split domination in neutrosophic graph is developed with suitable examples to know the advantages of neutrosophic split domination in real world applications than other existing split dominations.

# 2 Preliminaries

**Definition 2.1.** [12].

An intuitionisic fuzzy graph is of the form G=(V, E) where
(1) V = $\{v_1, v_2, .........., v_n\}$ such that $\mu_1$: $V \rightarrow [0,1]$ and $\gamma_1$: $V \rightarrow [0,1]$ denote the degree of membership and non-membership of the element $v_1 \in$ V, respectively, and 0 $\leq \mu_1(v_i)$+$\gamma_1(v_i)$ $\leq$1, for every $v_i \in$ V,( i=1,2,3,.......,n),
(2) E $\subseteq$ V$\times$ V, where $\mu_2$ : V $\times V \rightarrow [0,1]$ and $\gamma_2$: V$\times V \rightarrow [0,1]$ are such that
$\mu_2$ $(v_i, v_j) \leq$ min[$\mu_1(v_i)$,$\mu_1(v_j)$] and $\gamma_2(v_i, v_j) \geq$ max[$\gamma_1(v_i)$,$\gamma_1(v_j)$] and
0$\leq$ $\mu_2(v_i,v_j)$+$\gamma_2(v_i,v_j)$$\leq$ 1, for every $(v_i,v_j)$ $\in$E, (i,j=1,2,.........,n).

**Definition 2.2.** [4].

Let G =(V,E) be an intuitionistic fuzzy graph (IFG). Then the cardinality of G is defined to be
$|G| =| \sum\limits_{v_i \in V} \frac{1+\mu_1(v_i)-\gamma_1(v_i)}{2} + \sum\limits_{v_i,v_j \in V} \frac{1+\mu_2(v_i,v_j)-\gamma_2(v_i,v_j)}{2} |$.

**Definition 2.3.** [6].

A vertex u $\in V$ of an IFG G = (V,E) is said to be an isolated vertex if $\mu_2(u,v) = 0$ and $\gamma_2(u,v) = 0$ for all v $\in$ V. That is N(u)= $\emptyset$. Thus, an isolated vertex does not dominate any other vertex in G.

**Definition 2.4.** [6].

Let G= (V, E) be an IFG and let u,v $\in$ V, we say that u dominates v in G if there exists a strong arc between them. A subset D $\subseteq$ V is said to be dominating set in G if for every v $\in$ V-D, there exists u $\in$ D dominates v.

**Definition 2.5.** [4].

A dominating set D of IFG is said to be minimal dominating set if no proper subset S of D is a dominating set. Minimum cardinality among all minimal dominating sets is called the inituitionistic fuzzy dominating number, and is denoted by $\gamma_{if}$(G).

**Definition 2.6.** [12].

A dominating set D of a intuitionistic fuzzy graph G = (V,E) is a split dominating set if the induced fuzzy subgraph H = ($< V - D >$,$V\prime$, $E\prime$) is disconnected. The minimum fuzzy cardinality of a split dominating set is called a split domination number and id denotedby $\gamma_s(G)$.

**Definition 2.7.** [12].

A split dominating set D of a intuitionistic fuzzy graph G is said to be a minimal split dominating set if no proper subset of D is a split dominating set of G with $|D'| < 1$.

**Definition 2.8.** [12].

Minimum cardinality among all minimal split dominating set is called lower split domination number of IFG of G and is denoted by $D_S(G)$.

**Definition 2.9.** [12].

Maximum cardinality among all minimal split dominating set is called upper split domination number of IFG of G and is denoted by $D_S(G)$.

**Definition 2.10.** [16].

Let X be a space of points(objects) with generic elements in X denoted by X, then the neutrosophic sets A (NS A) is an object having the form
A = $\{< x : T_A(x), I_A(x), F_A(x) >, x \in X\}$,
where the functions T, I, F: X→ $[0^-, 1^+]$ define respectively the truth membership function, indeterminacy membership function , and a falsity membership function of the element x ∈ X to the set A with the condition
$0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$,
the functions $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $[0^-, 1^+]$.

**Definition 2.11.** [16].

Let X be a space of points (objects) with generic elements in X denoted by X. A single valued neutrosophic set A (SVNS A) is characterized by truth membership function $T_A(X)$, an indeterminacy membership function $I_A(X)$ and a falsity membership function $F_A(X)$. For each point x in X, $T_A(X), I_A(X)$ and $F_A(X) \in [0, 1]$. A SVNS A can be written as
A =$\{< x : T_A(x), I_A(x), F_A(x) >, x \in X\}$.

**Definition 2.12.** [16].

Let A = $(T_A, I_A, F_A)$ and B = $(T_B, I_B, F_B)$ be single valued neutrosophic sets on a set X. If A = $(T_A, I_A, F_A)$ is a single valued neutrosophic relation on a set X, then A = ( $T_A, I_A, F_A$) is called a single valued neutrosophic relation on B = $(T_B, I_B, F_B)$, if $T_B(x, y) \leq min(T_A(x), T_A(y))$, $I_B(x, y) \geq max(I_A(x), I_A(y))$, $F_B(x, y) \geq max(F_A(x), F_A(y))$ for all x,y in X.
A Single valued neutrosophic relation A on X is called symmetric if,
$T_A(x, y) = T_A(y, x)$, $I_A(x, y) = I_A(y, x)$, $F_A(x, y) = F_A(y, x)$ , and
$T_B(x, y) = T_B(y, x)$, $I_B(x, y) = I_B(y, x)$ ,$F_B(x, y) = F_B(y, x)$, for all x,y in X.

**Definition 2.13.** [16].

A single valued neutrosophic graph (SVN- graph) with underlying set V is defined to be a pair G = (A,B) where,
(1) The functions $T_A : V \to [0, 1]$, $I_A : V \to [0, 1]$, $F_A : V \to [0, 1]$ denote the degree of truth membership, degree of indeterminacy membership and degree of falsity membership of the element $v_i \in V$, respectively and
$0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$, for all $v_i \in V$, (i = 1,2,3,......,n)
(2) The functions $T_B : E \subseteq V \times V \to [0, 1]$, $I_B : E \subseteq V \times V \to [0, 1]$ and $F_B : E \subseteq V \times V \to [0, 1]$ are defined by $T_B(\{v_i, v_j\}) \leq min(T_A(v_i), T_A(v_j))$, $I_B(\{v_i, v_j\}) \geq max(I_A(v_i), I_A(v_j))$ and $F_B(\{v_i, v_j\}) \geq max(F_A(v_i), F_A(v_j))$.
denote the degree of truth membership, degree of indeterminacy membership and degree of falsity membership of the edge $(v_i, v_j) \in$ E respectively, where
$0 \leq T_B(\{v_i, v_j\}) + I_B(\{v_i, v_j\}) + F_B(\{v_i, v_j\}) \leq 3$ for all $\{v_i, v_j\} \in E$,( i,j = 1,,2,....,n).

**Definition 2.14.** [16].

Let G = (A,B) be a single valued neutrosophic graph on the vertex set V and x,y ∈ V. x dominates y in G if $T_A(x, y) = min\{T_B(x), T_B(y)\}$,
$I_A(x, y) = min\{I_B(x), I_B(y)\}$ and $F_A(x, y) = min\{F_B(x), F_B(y)\}$. A subset $D^N$ of V is called a dominating set in G if for every vertex v ∈ V-$D^N$ there exists u ∈ $D^N$ such that u dominates v.

**Definition 2.15.** [16].

The minimum cardinality of a dominating set in a neutrosophic graph G is called the domination number of G and is denoted by $\gamma^N$(G) (or) $\gamma^N$.

**Definition 2.16.** [16].

Let G be a neutrosophic graph. A dominating set $D^N$ of G is said to be a minimal dominating set if no proper subset of $D^N$ is a dominating set of G

**Definition 2.17.** [16].

A vertex x of a neutrosophic graph G is said to be an isolated vertex if,
$T_B(x,y) < min\{T_B(x), T_B(y)\}$
$I_B(x,y) < max\{I_B(x), I_B(y)\}$ and
$F_B(x,y) < max\{F_B(x), F_B(y)\}$, for all y $\in$ V-$\{x\}$,
(ie)N(x) = $\emptyset$.

**Definition 2.18.** [16].

A set of vertices $D^N$ of a neutrosophic graph G is said to be independent
$T_A(xy) < min\{T_A(x), T_A(y)\}$,
$I_A(xy) < max\{I_A(x), I_A(y)\}$ and
$F_A(xy) < max\{F_A(x), F_A(y)\}$, for all x,y $\in D^N$.

# 3 Split domination in neutrosophic graphs

**Definition 3.1.** A dominating set $D^N$ of a neutrosophic graph G = (A,B) is a split dominating set if the induced neutrosophic subgraph H = ($< V - D^N >$,$V'$, $E'$) is disconnected. The minimum neutrosophic cardinality of a split dominating set is called a split domination number and id denoted by $\gamma_s^N(G)$.

**Example 3.2.** Here strong arcs $e_1, e_2, e_4$ and $e_6$.
$[ie, T(v_1,v_3) > T^\infty(v_1,v_3), T(v_1,v_2) > T^\infty(v_1,v_2), T(v_2,v_4) > T^\infty(v_2,v_4)$ and $T(v_3,v_4) > T^\infty(v_3,v_4)]$.
Dominating set in neutrosophic is $D^N$ = $[v_2, v_3, v_5]$, V-$D^N$ = $\{v_1, v_4\}$.

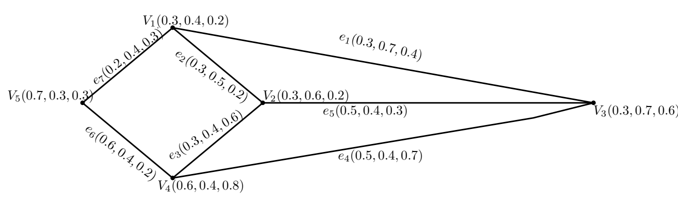


Figure 1: Neutrosophic Graphs

For every, v $\in$ V-$D^N$, there exists u $\in D^N$ and V-$D^N$ is a induced neutrosophic subgraph and it is disconnected. (i.e)There exists two isolated vertices $v_1$ and $v_4$. The minimum cardinality of a split dominating set is called split domination number $\gamma_s^N(G)$ = 1.35

**Theorem 3.3.** *For any neutrosophic graph G = (A,B),*
*(i)W($D^N$) $\geq \delta(G)$*
*(ii) W($D^N$) $\leq \Delta(G)$, where W($D^N$) is a weight of a split dominating set.*

**Proof:**

*Consider Fig.1,*
*The strong arcs are $e_1, e_2, e_4$ and $e_6$. The minimum degree $\delta$ is*
$\delta_T(G)$ = $min\{d_T(v_i)/v_i \in V\}$ = *0.6*

$\delta_I(G) = min\{d_I(v_i)/v_i \in V\}$ *= 0.8*
$\delta_F(G) = min\{d_F(v_i)/v_i \in V\}$ *= 0.8*
*The minimum degree of G is* $\delta(G) = min\{d_T(v_i), d_I(v_i), d_F(v_i)/v_i \in V\}$ *= (0.6,0.8,0.8)*
*The maximum T degree is*
$\Delta_T(G) = max\{d_T(v_i)/v_i \in V\}$ *= 0.9*
*The maximum I degree is*
$\Delta_I(G) = max\{d_I(v_i)/v_i \in V\}$ *= 1.6*
*The maximum F degree is*
$\Delta_F(G) = max\{d_F(v_i)/v_i \in V\}$ *= 1.6*
*The maximum degree of G is*
$\Delta(G) = max\{d_T(v_i), d_I(v_i), d_F(v_i)/v_i \in V\}$ *= (0.9,1.6,1.6)*
$W_T(D) = W_T\{v_2, v_3, v_5\} = W_T(v_2) + W_T(v_3) + W_T(v_5)$ *= 0.6*
$W_I(D) = W_I\{v_2, v_3, v_5\} = W_I(v_2) + W_I(v_3) + W_I(v_5)$ *= 1.2*
$W_F(D) = W_F\{v_2, v_3, v_5\} = W_F(v_2) + W_F(v_3) + W_F(v_5)$ *= 1.2*
*W(*$D^N$*) = (0.6,1.2,1.2)*
*Also,* $\delta(G)$ *= (0.6,0.8,0.8) and* $\Delta(G)$ *= (0.9,1.6,1.6)*
*Therefore, W(*$D^N$*)*$\geq \delta(G)$ *and W(*$D^N$*)*$\leq \Delta(G)$.
*Hence the proof.*

**Theorem 3.4.** *A dominating set* $D^N$ *of a neutrosophic graph G is a split dominating set if and only if there exists two neutrosophic vertices u,v* $\in$ *V-*$D^N$ *such that every u-v path contains an neutrosophic vertex of* $D^N$.
**Proof:**
*Let* $D^N$ *be a split dominating set of a neutrosophic graph G. Then induced neutrosophic subgraph* $< V - D^N >$ *is disconnected. Hence, there exist two vertices u,v* $\in$ *V-*$D^N$ *such that every u-v path contains a neutrosophic vertex of* $D^N$.
*Let* $D^N$ *be a dominating set. Then induced subgraph V-*$D^N$ *of a neutrosophic graph G is connected (or) disconnected.*
*If it is connected, then there exist two vertices u,v in V-*$D^N$ *such that some u-v path does not contain a neutrosophic vertex of* $D^N$*, which is a contradiction. Hence V-*$D^N$ *is disconnected, which implies* $D^N$ *is a neutrosophic split dominating set of G.*

**Theorem 3.5.** *For any neutrosophic graph G = (A,B),* $\gamma_S^N(G) \geq \beta(G)$*, where* $\beta(G)$ *is a fuzzy vertex covering number of G.*
**Proof:**
*Let S be a subset of A which is an independent set satisfying the condition* $T_B(u,v) < T_B^\infty(u,v)$ *,* $I_B(u,v) < I_B^\infty(u,v)$ *and* $F_B(u,v) < F_B^\infty(u,v)$ *for all u,v* $\in$ *S. If S is maximal independent set then for every vertex v* $\subset$ *V , S the set S* $\cup\{v\}$ *is not independent.*
*That is, there exists strong neighbor adjacent to every vertex in S.*
*Hence, the minimum cardinality of a split dominating set is greater than maximum cardinality of independent set* $\beta(G)$*, a vertex covering of neutrosophic G. Hence,* $\gamma_S^N(G) \geq \beta(G)$.

**Definition 3.6.** A split dominating set $D^N$ of a neutrosophic graph G is said to be a minimal split dominating set if no proper subset of $D^N$ is a split dominating set of G with $|D^{N'}| < 1$.

**Theorem 3.7.** *A split dominating set* $D^N$ *of neutrosophic graph G is minimal if and only if for each vertex v* $\in$ $D^N$ *one of the following conditions holds,*

*(i) there exists a vertex u $\in$ V-$D^N$ such that N(u)$\cup D^N$ = $\{v\}$.*
*(ii) v is an isolate in $< D^N >$.*
*(iii) $< V - (D^N)' >$is connected.*

**Proof:**

*Suppose that $D^N$ is minimal and there exists a vertex v $\in D^N$ such that v does not satisfy any of the above conditions.*
*(ie) by condition (i), there exists a vertex u $\in$ V-$D^N$ such that N(u)$\cup D^N \neq \{v\}$ and by condition(ii), v is not an isolate vertex of the induced subgraph $< D^N >$.*
*Let $(D^N)'$ = $D^N$-$\{V\}$, then $D^N$ is a split dominating set, which satisfies above two conditions.*
*Hence, the induced subgraph $< V - (D^N) >$ is disconnected. which contradicts the third condition.*
*This implies a vertex v is in $D^N$.*
*Therefore, $D^N$ is minimal split dominating set, which satisfies one of the above conditions.*

**Theorem 3.8.** *For any neutrosophic graph G = (A,B) with neutrosophic end vertex $\gamma_2^N(G) \geq \gamma^N(G)$, Furthermore, there exists a split dominating set of G containing some vertices adjacent to neutrosophic end vertices.*

**Proof:**

*Let v be a neutrosophic end vertex of neutrosophic graph G, then there exists neutrosophic cut vertex $'w'$ such that $T(u, v) > 0$ , $I(u, v) > 0$ and $F(u, v) > 0$.*
*Let $D^N$ be a dominating set of neutrosophic G. Suppose that w $\in D^N$, then $D^N$ is a split dominating set of G. Repeating this process for all such cut vertices adjacent to neutrosophic end vertices, we obtain a split dominating set of G containing some vertices adjacent to the end- vertices.*

**Definition 3.9.** Minimum cardinality among all minimal split dominating set is called lower split domination number of neutrosophic graph G and is denoted by $d_S^N(G)$.

**Definition 3.10.** Maximum cardinality among all minimal split dominating set is called upper split domination number of neutrosophic graph G and is denoted by $D_S^N(G)$.

**Example 3.11.** Here, strong arcs are $(e_2, e_6, e_8, e_9, e_{10})$
Let the neutrosophic dominating sets are $D_1^N = \{v_1, v_2, v_4, v_5\}$; $D_2^N = \{v_1, v_3, v_4, v_6\}$;

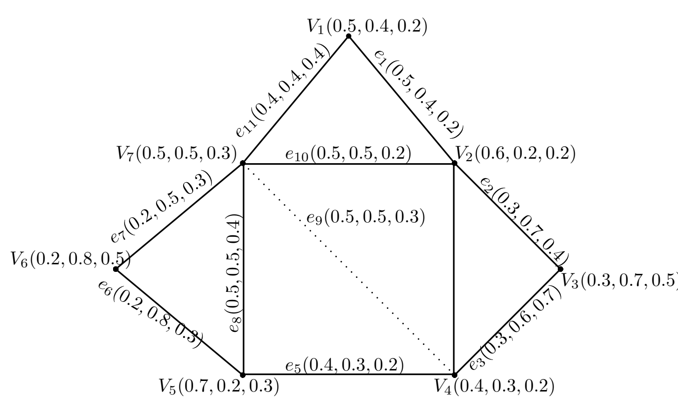


Figure 2: Neutrosophic minimal split dominating sets

$D_3^N = \{v_1, v_2, v_4, v_6\}$; $D_4^N = \{v_1, v_2, v_5, v_7\}$ and $D_5^N = \{v_1, v_3, v_5, v_7\}$.... etc.
Out of these neutrosophic dominating sets, minimal split dominating sets are
$D_1^N = \{v_1, v_2, v_4, v_5\}$, V-$D_1^N = \{v_3, v_6, v_7\}$

Therefore, $\gamma_S(D_1^N)$ = 2.55
$D_3^N = \{v_1, v_2, v_4, v_6\}$, V-$D_3^N = \{v_3, v_5, v_7\}$
Therefore, $\gamma_S(D_3^N)$ = 2
$D_5^N = \{v_1, v_3, v_5, v_7\}$, V-$D_5^N = \{v_2, v_4, v_6\}$
Therefore, $\gamma_S(D_5^N)$ = 2.1
Here upper split domination number of neutrosophic graph G is $D_S^N(G)$ = 2.55 and lower split domination number of G is $d_S^N(G)$ = 2.

**Theorem 3.12.** *For any neutrosophic graph G = (A,B)*
*(i)* $\gamma(G) \geq P/(\Delta_T(G) + 1)$
*(ii)* $\gamma(G) \geq P/(\Delta_I(G) + 1)$ *(iiI)* $\gamma(G) \geq P/(\Delta_F(G) + 1)$ *where* $\Delta_T(G)$ *is the maximum T -degree of G and* $\Delta_I(G)$ *is the maximum I - degree of G and* $\Delta_F(G)$ *is the maximum F- degree of G.*
**Proof:**
*(i) Let* $D^N$ *be a neutrosophic dominating set of G with* $|D^N|$ = $\gamma$ *and since every vertex in V-*$D^N$ *is adjacent to some vertices in* $D^N$*, we have*
$|V - D^N| \leq \sum_{i=1}^{r} d(v_i) \leq \gamma.\Delta_\mu(G)$
$\Rrightarrow p - \gamma(G) \leq \gamma.\Delta = \gamma(G).\Delta_\mu(G).$
$\Rrightarrow p \leq \gamma(G) + \gamma(G).\Delta_\mu(G) = \gamma(G).(1 + \Delta_\mu).$
$\Rrightarrow P/(\Delta_\mu(G) + 1) \leq \gamma(G).$
*Similarly,* $\gamma(G) \geq P/(\Delta_\gamma(G) + 1).$

**Theorem 3.13.** *For any neutrosophic graph G = (A,B)*
*(i)* $\gamma_S(G) \geq P.\Delta - T(G)/(\Delta_T(G) + 1)$
*(ii)* $\gamma_S(G) \geq P.\Delta_I(G)/(\Delta_I(G) + 1)$
*(ii)* $\gamma_S(G) \geq P.\Delta_F(G)/(\Delta_F(G) + 1)$
**Proof:**
*The proof is similar to the above theorem.*

**Theorem 3.14.** *If neutrosophic graph G = (A,B) has one neutrosophic cut vertex v and at least two neutrosophic blocks* $H_1$ *and* $H_2$ *with v adjacent to all vertices of* $H_1$ *and* $H_2$*, then v is in every split dominating set of G.*
**Proof:**
*Let* $D^N$ *be a split dominating set of neutrosophic of G.*
*Suppose v is a cut vertex does not belong to* $D^N$*.*
*This implies v* $\in$ *V-*$D^N$*, then each of* $H_1$ *and* $H_2$ *contributes at least one vertex to* $D^N$ *say u and w, which is adjacent to v. If V-*$D^N$ *includes v then every V-*$D^N$ *is connected.*
*This implies,* $D^N$ *is not a split dominating set, which is a contradiction.*
*Hence, v is in every split dominating set of neutrosophic of G.*

**Example 3.15.** In fig.3, strong arcs are $e_1, e_3, e_5, e_6$ and $e_9$.
Let $H_1 = v_2, v_3, v_5, v_6$ and $H_2 = v_3, v_4, v_6$ be two blocks with one cut vertex $v_3$, which is adjacent to all vertices of $H_1$ and $H_2$.
Then $v_3$ is in every split dominating set of neutrosophic of G.

**Theorem 3.16.** *Let v be a neutrosophic cut vertex of neutrosophic graph G, if there is a block H in G such that v is the only cut vertex of H and v is adjacent to all vertices of H. Then there is a split dominating set of G*

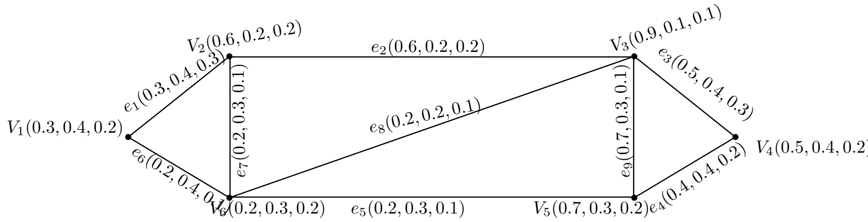


Figure 3: Neutrosophic split dominating sets

*containing v.*

**Proof:**

*If there exists two blocks in neutrosophic graph G, satisfying the given condition, then by above theorem, v is in every split dominating set of neutrosophic graph G.*

**Theorem 3.17.** *If $\gamma_S^N(G) \leq \gamma_D^N(G)$, then for any split dominating set $D^N$ of neutrosophic graph G, V-$D^N$ is a split dominating set of G.*

**Proof:**

*Since $D^N$ is minimal split dominating set in neutrosophic graph G, we know that $V - D^N$ is dominating set of G and $D^N$ is a split dominating set, since $< D^N >$ is disconnected.*

**Theorem 3.18.** *Let G be a neutrosophic graph such that both G and $\overline{G}$ are connected, then $\gamma_S(G) + \gamma_S(\overline{G}) \geq 2|V|$.*

**Proof:**

*We know that $\gamma_S(G) \geq \beta(G)$. Since both G and $\overline{G}$ are connected, $\Delta(G), \Delta(\overline{G}) > P$ this implies $\alpha_0(G), \alpha_0(\overline{G}) \leq 0$. Hence $\gamma_S(G) \geq |v|$.*
*Similarly, $\gamma_S(\overline{G}) \geq p$, which implies $\gamma_S(G) + \gamma_S(\overline{G}) \geq |v| + |v| \geq 2|v|$.*
*Hence the theorem*

# Conclusion

Neutrosophic set is the generalization of fuzzy set and intuitionistic fuzzy set. Neutrosophic models in real world applications are flexible and compatiblethan fuzzy and intuitionistic fuzzy models. In this proposed work, the definition of split domination number in a neutrosophic set is defined with suitable examples and some theorems in split domination in neutrosophic graph are developed. Also, the bound on split domination number related to the above concepts are studied.Neutrosophic split dominating set gives more efficient results than other existing split dominating sets. In future, the concept of split domination in neutrosophic graphs will be extented and applied to many real life situation problems.

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