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New Generalized Closed Set in Neutrosophic Topological Spaces.

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Abstract: The main intention of this paper is to develop the idea of Neutrosophic Semi-generalized pre-closed set in neutrosophic topological space. We also study relations and some properties between the existing Neutrosophic closed set. The examples are provided wherever necessary. Besides, we discuss some applications of Neutrosophic Semi-generalized pre closed set.

Keywords: \mathfrak{N} sgp-closed set, \mathfrak{N} sgp-open set, \mathfrak{N} -sgp- $T_{1/2}$, \mathfrak{N} -pc- $T_{1/2}$, \mathfrak{N} -open set, \mathfrak{N} -closed set.

1. Introduction

Fuzzy set theory is introduced and studied as a mathematical tool for dealing with uncertainties where each element had a degree of membership, truth(t), by Zadeh[14]. The falsehood (f), the degree of non-membership, was introduced by Atanassov [2] in an intuitionistic fuzzy set. Coker [3] developed intuitionistic fuzzy topology. Neutrality (i), the degree of indeterminacy, as an independent concept, was introduced by Smarandache[8,9,10]. He also defined the neutrosophic set on three components (t, f, i) = (truth, falsehood, indeterminacy). Salama et.al. [6,7] converted Neutrosophic crisp set in to neutrosophic topological spaces. This opened a wide range of investigation in terms of neutrosophic topology and its application in decision making problems. A.A. Salama et al in [7] introduced neutrosophic closed sets and continuous functions. R. Dhavaseelan et al [4] introduced generalized neutrosophic closed sets. Neutrosophic semi-open, pre-open, α -open and semipro-open are presented in [11]. In [13]authors discussed properties of Generalized pre-closed sets in neutrosophic topological space(NTS in short).

This paper is devoted to the study new generalized closed set in Neutrosophic topology called Neutrosophic semi-generalized pre closed set. The basic properties are discussed and compared the new set with existing neutrosophic closed sets. As its applications, we have defined as \mathfrak{N} eutrosophic-sgp- $T_{1/2}$ and as \mathfrak{N} eutrosophic-pc- $T_{1/2}$.

2. Preliminaries

Definition: 2.1[8,9]: Let S_1 be a non-empty fixed set. A neutrosophic set (in short NS) Λ is an object such that $\Lambda = \{(x, \mu_\Lambda(x), \sigma_\Lambda(x), \gamma_\Lambda(x)) : x \in S_1\}$ wherein $\mu_\Lambda(x)$, $\sigma_\Lambda(x)$ and $\gamma_\Lambda(x)$ which represents the degree of membership function (viz $\mu_\Lambda(x)$), the degree of indeterminacy (viz $\sigma_\Lambda(x)$) as well as the degree of non-membership (viz $\gamma_\Lambda(x)$) respectively of each element $x \in S_1$ to the set Λ .

Remark: 2.2[8,9]: (i) An N -set $\Lambda = \{ \langle x, \mu_\Lambda(x), \sigma_\Lambda(x), \Gamma_\Lambda(x) \rangle : x \in S_1 \}$ can be identified to an ordered triple $\langle \mu_\Lambda, \sigma_\Lambda, \Gamma_\Lambda \rangle$ in $]0^-, 1^+[$ on S_1 .

(ii) In this paper, we use the symbol $\Lambda = \langle \mu_\Lambda, \sigma_\Lambda, \Gamma_\Lambda \rangle$ for the N -set $\Lambda = \{ \langle x, \mu_\Lambda(x), \sigma_\Lambda(x), \Gamma_\Lambda(x) \rangle : x \in S_1 \}$.

Definition: 2.3[8,9]: Let $S_1 \neq \emptyset$ and the N -sets Λ and Γ be defined as

$$\Lambda = \{ \langle x, \mu_\Lambda(x), \sigma_\Lambda(x), \Gamma_\Lambda(x) \rangle : x \in S_1 \}, \Gamma = \{ \langle x, \mu_\Gamma(x), \sigma_\Gamma(x), \Gamma_\Gamma(x) \rangle : x \in S_1 \}.$$

- I. $\Lambda \subseteq \Gamma$ iff $\mu_\Lambda(x) \leq \mu_\Gamma(x)$, $\sigma_\Lambda(x) \leq \sigma_\Gamma(x)$ and $\Gamma_\Lambda(x) \geq \Gamma_\Gamma(x)$ for all $x \in S_1$;
- II. $\Lambda = \Gamma$ iff $\Lambda \subseteq \Gamma$ and $\Gamma \subseteq \Lambda$;
- III. $\bar{\Lambda} = \{ \langle x, \Gamma_\Lambda(x), \sigma_\Lambda(x), \mu_\Lambda(x) \rangle : x \in S_1 \}$; [Complement of Λ]
- IV. $\Lambda \cap \Gamma = \{ \langle x, \mu_\Lambda(x) \wedge \mu_\Gamma(x), \sigma_\Lambda(x) \wedge \sigma_\Gamma(x), \Gamma_\Lambda(x) \vee \Gamma_\Gamma(x) \rangle : x \in S_1 \}$;
- V. $\Lambda \cup \Gamma = \{ \langle x, \mu_\Lambda(x) \vee \mu_\Gamma(x), \sigma_\Lambda(x) \vee \sigma_\Gamma(x), \Gamma_\Lambda(x) \wedge \Gamma_\Gamma(x) \rangle : x \in S_1 \}$;
- VI. $|\Lambda = \{ \langle x, \mu_\Lambda(x), \sigma_\Lambda(x), 1 - \mu_\Lambda(x) \rangle : x \in S_1 \}$;
- VII. $\langle \rangle \Lambda = \{ \langle x, 1 - \Gamma_\Lambda(x), \sigma_\Lambda(x), \Gamma_\Lambda(x) \rangle : x \in S_1 \}$.

Definition: 2.4[9,10]: Let $\{ \Lambda_i : i \in J \}$ be an arbitrary family of N -sets in S_1 . Thereupon

- I. $\cap \Lambda_i = \{ \langle p, \mu_{\cap \Lambda_i}(p), \sigma_{\cap \Lambda_i}(p), \Gamma_{\cap \Lambda_i}(p) \rangle : p \in S_1 \}$;
- II. $\cup \Lambda_i = \{ \langle p, \mu_{\cup \Lambda_i}(p), \sigma_{\cup \Lambda_i}(p), \Gamma_{\cup \Lambda_i}(p) \rangle : p \in S_1 \}$.

The main theme is to construct the tools for developing NTS, so we establish the neutrosophic sets 0_{\aleph} along with 1_{\aleph} in X as follows:

Definition: 2.5[9,10]: $0_{\aleph} = \{ \langle q, 0, 0, 1 \rangle : q \in X \}$ and $1_{\aleph} = \{ \langle q, 1, 1, 0 \rangle : q \in X \}$.

Definition: 2.6[7]: A neutrosophic topology (in short, $\aleph T$) $S_1 \neq \emptyset$ is a family ξ_1 of N -sets in S_1 satisfying the laws given below:

- I. $0_{\aleph}, 1_{\aleph} \in \xi_1$,
- II. $W_1 \cap W_2 \in \xi_1$ being $W_1, W_2 \in \xi_1$,
- III. $\cup W_i \in \xi_1$ for arbitrary family $\{ W_i | i \in \Lambda \} \subseteq \xi_1$.

In this case the ordered pair (S_1, ξ_1) or simply S_1 is termed as NTS and each NS in ξ_1 is named as neutrosophic open set (in short, $\aleph OS$) . The complement $\bar{\Lambda}$ of an \aleph -open set Λ in S_1 is known as neutrosophic closed set (briefly, $\aleph CS$) in S_1 .

Definition: 2.7[7,8]: Let Λ be an NS in an NTS S_1 . Thereupon

$\aleph int(\Lambda) = \cup \{ G | G \text{ is an } \aleph OS \text{ in } S_1 \text{ and } G \subseteq \Lambda \}$ is termed as neutrosophic interior (in brief $\aleph int$) of Λ ;

$\mathfrak{N}cl(\Lambda) = \cap \{G \mid G \text{ is an } \mathfrak{N}CS \text{ in } S_1 \text{ and } G \supseteq \Lambda\}$ is termed as neutrosophic closure (shortly $\mathfrak{N}cl$) of Λ .

Definition: 2.8[4]: Let X be a nonempty set. Whenever r, t, s be real standard or non standard subsets of $]0^-, 1^+[$ then the neutrosophic set $x_{r,t,s}$ is termed as neutrosophic point (in short NP) in X given by $x_{r,t,s}(x_p) = \begin{cases} (r, t, s), & \text{if } x = x_p \\ (0, 0, 1), & \text{if } x \neq x_p \end{cases}$ for $x_p \in X$ is termed as the support of $x_{r,t,s}$, wherein r indicates the degree of membership value, t indicates the degree of indeterminacy along with s as the degree of non-membership value of $x_{r,t,s}$.

Definition 2.9[11]: For an $\mathfrak{N}S D$ in an NTS (X, T) . We have,

- (i) \mathfrak{N} eutrosophic semiopen set ($\mathfrak{N}SOS$) if $D \subseteq \mathfrak{N}cl(\mathfrak{N}int(D))$.
- (ii) \mathfrak{N} eutrosophic preopen set ($\mathfrak{N}POS$) if $D \subseteq \mathfrak{N}int(\mathfrak{N}cl(D))$.
- (iii) \mathfrak{N} eutrosophic α -open set ($\mathfrak{N}\alpha OS$) if $D \subseteq \mathfrak{N}int(\mathfrak{N}cl(\mathfrak{N}int(D)))$.
- (iv) \mathfrak{N} eutrosophic semi-preopen ($\mathfrak{N}SPOS$) if $D \subseteq \mathfrak{N}cl(\mathfrak{N}int(\mathfrak{N}cl(D)))$.

The complement of D is an $\mathfrak{N}SOS$, $\mathfrak{N}POS$, $\mathfrak{N}\alpha OS$, $\mathfrak{N}SPOS$ is called respectively as $\mathfrak{N}SCS$, $\mathfrak{N}PCS$, $\mathfrak{N}\alpha CS$, $\mathfrak{N}SPCS$

Definition 2.10[13]: Let (X, T) be an NTS then neutrosophic pre-closure of D (in short, $p\mathfrak{N}Cl(D)$) is defined as

- (i) $p\mathfrak{N}Cl(D) = \cap \{K : K \text{ is an NPC in } T, D \subseteq K\}$.
- (ii) $p\mathfrak{N}Int(D) = \cup \{Q : Q \text{ is an NPO in } T, D \subseteq Q\}$.

Definition 2.11 [13]: An NS is said to be a neutrosophic generalized pre-closed set (GNPCS in short) in (X, T) if $p\mathfrak{N}Cl(R) \subseteq Q$ whenever $R \subseteq Q$ and Q is a NOS in (X, T) .

Definition 2.12 [13]: An NTS (X, S) is named as \mathfrak{N} eutrosophic-gp- $T_{1/2}$ ($\mathfrak{N}gp-T_{1/2}$ in short) space if every GNPCS in X is a $\mathfrak{N}PCS$.

3. Neutrosophic Semi-Generalized-Pre-Closed Sets.

Definition 3.1: An NS μ of NTS (X, S) is termed as Neutrosophic Semigeneralized pre-closed set ($\mathfrak{N}sgp-CS$ in short) if $p\mathfrak{N}Cl(\mu) \subseteq \eta$ whenever $\mu \subseteq \eta$ and η is $\mathfrak{N}SOS$ in X .

Definition 3.2: Let (X, S) be a NTS and η be an NS in X . Then the neutrosophic semigeneralized pre-closure and neutrosophic semigeneralized pre-interior of η are denoted and defined by,

$$\mathfrak{N}sgpCl(\eta) = \cap \{ \lambda : \lambda \text{ is a } \mathfrak{N}sgp-CS \text{ in } X \text{ and } \eta \subseteq \lambda \}$$

$$\mathfrak{N}sgpInt(\eta) = \cup \{ \lambda : \lambda \text{ is a } \mathfrak{N}sgp-OS \text{ in } X \text{ and } \eta \supseteq \lambda \}$$

Proposition 3.3: Consider (X, S) be any NTS and A and B be neutrosophic sets in (X, S) . Then the $\mathfrak{N}sg$ -closure and $\mathfrak{N}sg$ -interior operator satisfy the following properties

- i. $\eta \subseteq \aleph \text{sgpCl}(\eta)$
- ii. $\aleph \text{sgpInt}(\eta) \subseteq \eta$
- iii. $\eta \subseteq \lambda \Rightarrow \aleph \text{sgpCl}(\eta) \subseteq \aleph \text{sgpCl}(\lambda)$
- iv. $\eta \subseteq \lambda \Rightarrow \aleph \text{sgpInt}(\eta) \subseteq \aleph \text{sgpInt}(\lambda)$
- v. $\aleph \text{sgpCl}(\eta \cup \lambda) = \aleph \text{sgpCl}(\eta) \cup \aleph \text{sgpCl}(\lambda)$
- vi. $\aleph \text{sgpInt}(\eta \cap \lambda) = \aleph \text{sgpInt}(\eta) \cap \aleph \text{sgpInt}(\lambda)$
- vii. $\aleph \text{sgpCl}(\bar{\eta}) = \aleph \text{sgpInt}(\eta)$
- viii. $\aleph \text{sgpInt}(\bar{\eta}) = \aleph \text{sgpCl}(\eta)$

Proposition 3.4: Each \aleph CS set is \aleph sgp-CS.

Proof: Let μ is \aleph CS such that $\mu \subseteq \eta$ and η is \aleph SOS in X . As μ is \aleph CS, $\mu = \aleph \text{cl}(\mu)$. Hence $\aleph \text{cl}(\mu) \subseteq \mu$. But $\mu \subseteq \aleph \text{cl}(\mu)$, therefore $\mu = \aleph \text{cl}(\mu)$ whenever $\mu \subseteq \eta$ and η is \aleph SOS in X . Therefore μ is \aleph sgp-CS.

Remark 3.5: The example makes clear that converse of the above proposition is not true.

Example 3.6: Let $X = \{a, b, c\}$. Define the neutrosophic sets A, B and C in X as follows $A = \langle x, (\frac{a}{0.6}, \frac{b}{0.6}, \frac{c}{0.6}), (\frac{a}{0.7}, \frac{b}{0.7}, \frac{c}{0.7}), (\frac{a}{0.3}, \frac{b}{0.3}, \frac{c}{0.3}) \rangle$, $B = \langle x, (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.5}), (\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}) \rangle$ and $C = \langle x, (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.6}), (\frac{a}{0.4}, \frac{b}{0.4}, \frac{c}{0.6}), (\frac{a}{0.5}, \frac{b}{0.5}, \frac{c}{0.5}) \rangle$. Then the families $\tau = \{0_N, 1_N, A, B\}$ is topology on X and the space (X, τ) is NTS. Then C is a \aleph sgp-CS but not a \aleph CS.

Proposition 3.7: Every $\aleph\alpha$ CS is \aleph sgp-CS.

Proof: Let W is \aleph CS such that $W \subseteq Q$ and Q is \aleph SOS in X . Since W is $\aleph\alpha$ CS, $W = \aleph \alpha \text{cl}(W)$. Hence $\aleph \alpha \text{cl}(W) \subseteq Q$. But $W \subseteq \aleph \alpha \text{cl}(W)$, therefore $W = \aleph \alpha \text{cl}(W)$ whenever $W \subseteq Q$ and Q is \aleph SOS in X . Therefore W is \aleph sgp-CS.

Remark 3.8: Converse of the above proposition is not true as seen below.

Example 3.9: Consider $X = \{\eta, \beta, \delta\}$. Define the neutrosophic sets P, Q and R in X as follows $P = \langle x, (\frac{\eta}{0.6}, \frac{\beta}{0.6}, \frac{\delta}{0.6}), (\frac{\eta}{0.7}, \frac{\beta}{0.7}, \frac{\delta}{0.7}), (\frac{\eta}{0.3}, \frac{\beta}{0.3}, \frac{\delta}{0.3}) \rangle$, $Q = \langle x, (\frac{\eta}{0.4}, \frac{\beta}{0.4}, \frac{\delta}{0.5}), (\frac{\eta}{0.4}, \frac{\beta}{0.4}, \frac{\delta}{0.5}), (\frac{\eta}{0.5}, \frac{\beta}{0.5}, \frac{\delta}{0.5}) \rangle$ and $R = \langle x, (\frac{\eta}{0.4}, \frac{\beta}{0.4}, \frac{\delta}{0.5}), (\frac{\eta}{0.4}, \frac{\beta}{0.4}, \frac{\delta}{0.5}), (\frac{\eta}{0.5}, \frac{\beta}{0.5}, \frac{\delta}{0.5}) \rangle$. Then the families $\tau = \{0_N, 1_N, P, Q\}$ is topology on X and the space (X, τ) is NTS. Then C is a \aleph sgp-CS but not a $\aleph\alpha$ CS.

Proposition 3.10: Each \aleph PCS is \aleph sgp-CS.

Proof: Consider J is \aleph CS with $J \subseteq K$ and K is \aleph SOS in X . Since J is \aleph PCS, $J = p\aleph \text{Cl}(J)$. Hence $J \subseteq p\aleph \text{Cl}(J)$ whenever $J \subseteq K$ and K is \aleph SOS in X . Therefore J is \aleph sgp-CS.

Remark 3.11: The example shows that the reverse implication of above proposition is not possible.

Example 3.12: For $X = \{p, q, r\}$ the neutrosophic sets K, M and L in X are defined as $A = \langle x, (\frac{p}{0.6}, \frac{q}{0.6}, \frac{r}{0.6}), (\frac{p}{0.7}, \frac{q}{0.7}, \frac{r}{0.7}), (\frac{p}{0.3}, \frac{q}{0.3}, \frac{r}{0.3}) \rangle, M = \langle x, (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.5}), (\frac{p}{0.5}, \frac{q}{0.5}, \frac{r}{0.5}) \rangle$ and $L = \langle x, (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.4}, \frac{q}{0.4}, \frac{r}{0.6}), (\frac{p}{0.5}, \frac{q}{0.5}, \frac{r}{0.5}) \rangle$. Then the families $\tau = \{0_N, 1_N, K, M\}$ is topology on X and the space (X, τ) is NTS. Thereupon C is a \aleph sgp-Closed set but not a \aleph PCS

Proposition 3.13: Every is \aleph sgp-CS is GNPCS.

Proof: Let J is \aleph sgp-CS with $J \subseteq K$ and K is \aleph OS in X . Since each \aleph OS is \aleph SOS, K is \aleph SOS such that $J \subseteq K$. By definition 3.1, $p\aleph Cl(J) \subseteq K$ whenever $J \subseteq K$ and K is \aleph OS in X . Therefore J is GNPCS.

Proposition 3.14: Let D be a \aleph sgp-CS in an \aleph TS (X, S) and $D \subseteq E \subseteq p\aleph Cl(D)$. Thereupon E is \aleph sgp-CS in X .

Proof: Consider G be a \aleph SOS in X thereby $E \subseteq G$. Then $D \subseteq G$ and since D is \aleph sgp-CS, $p\aleph Cl(D) \subseteq G$. Now $E \subseteq p\aleph Cl(D)$ implies $\aleph cl(E) \subseteq p\aleph Cl(D) \subseteq G$. Consequently E is \aleph sgp-CS in X .

Definition 3.15: An NS B of a \aleph TS (X, S) is named as neutrosophic semi-generalized open set (\aleph sgp-OS in short) if and only if $B \subseteq p\aleph Cl(B)$.

Remark 3.16: For any two Neutrosophic Sets A and B of $NTS(X, S)$. Then

- i). A is a \aleph closed set iff $\aleph cl(A) = A$.
- ii). A is a \aleph open set iff $\aleph int(A) = A$.
- iii). $\aleph cl(\bar{A}) = \aleph int(\bar{A})$
- iv). $\aleph int(\bar{A}) = \aleph cl(\bar{A})$

Proposition 3.17: An NS F of a \aleph TS (X, S) is \aleph sgp-OS if $G \subseteq p\aleph Cl(F)$ whenever G is \aleph SCS and $G \subseteq F$.

Proof: Follows from definition 3.1 and remark 3.16.

Proposition 3.18: Let A be a \aleph sgp-OS in a \aleph TS (X, S) and $p\aleph Int(A) \subseteq B \subseteq A$. Then B is \aleph sgp-OS.

Proof: Suppose A is \aleph sgp-OS in X and $p\aleph Int(A) \subseteq B \subseteq A$ implies $\bar{A} \subseteq \bar{B} \subseteq (p\aleph Int(\bar{A}))$ implies $\bar{A} \subseteq \bar{B} \subseteq p\aleph Cl(\bar{A})$. Then B is \aleph sgp-OS.

4. Applications of Neutrosophic Semi Generalized Pre Closed Set.

Definition 4.1: An \mathfrak{N} TS (X, S) is named as \mathfrak{N} eutrosophic-sgp- $T_{1/2}$ (in short \mathfrak{N} -sgp- $T_{1/2}$) space if every \mathfrak{N} sgp-CS in X is a \mathfrak{N} CS.

Definition 4.2: An \mathfrak{N} TS (X, S) is named as \mathfrak{N} eutrosophic-pc- $T_{1/2}$ (in short \mathfrak{N} -pc- $T_{1/2}$) space if every \mathfrak{N} sgp-CS in X is a \mathfrak{N} PCS.

Proposition 4.3: Each \mathfrak{N} -pc- $T_{1/2}$ space is \mathfrak{N} -sgp- $T_{1/2}$.

Proof: Consider X to be a \mathfrak{N} -pc- $T_{1/2}$ space and G be \mathfrak{N} sgp-CS in X . By assumption, G is \mathfrak{N} PCS in X . Since every \mathfrak{N} PCS is \mathfrak{N} sgp-CS, G is \mathfrak{N} sgp-CS in X . Hence, X is \mathfrak{N} -pc- $T_{1/2}$.

Proposition 4.5: Each \mathfrak{N} -sgp- $T_{1/2}$ is \mathfrak{N} gp $T_{1/2}$.

Proof: Consider X to be a \mathfrak{N} -sgp- $T_{1/2}$ space and Q be \mathfrak{N} GPCS in X . By assumption, Q is \mathfrak{N} sgp-CS in X . Since every \mathfrak{N} sgp-CS is \mathfrak{N} GPCS, Q is \mathfrak{N} GPCS in X . Hence, X is \mathfrak{N} gp- $T_{1/2}$.

Proposition 4.6: Let (X, S) be a \mathfrak{N} TS and \mathfrak{N} -sgp- $T_{1/2}$. Then the following statements hold.

(i) Any union of \mathfrak{N} sgp-CS is a \mathfrak{N} sgp-CS.

(ii) Any intersection of \mathfrak{N} sgp-CS is a \mathfrak{N} sgp-CS.

Proof:(i) Let $\{B_i\}_{i \in J}$ be a collection of \mathfrak{N} sgp-CS in a \mathfrak{N} -sgp- $T_{1/2}$ space (X, S) . Therefore every \mathfrak{N} sgp-CS is \mathfrak{N} CS. However, the union of \mathfrak{N} CS is a \mathfrak{N} CS. Hence the union of \mathfrak{N} sgp-CS is \mathfrak{N} sgp-CS in X .

(ii) It can be proved by taking complement in (i).

Proposition 4.7: An \mathfrak{N} TS X is an \mathfrak{N} -sgp- $T_{1/2}$ iff \mathfrak{N} sgp-OS = \mathfrak{N} POS.

Proof:(i) Consider K be a \mathfrak{N} sgp-OS in X , thereupon K^c is \mathfrak{N} sgp-CS in X . By presumption, K^c is an \mathfrak{N} PCS in X . Thus, K is \mathfrak{N} sgp-OS in X . Therefore, \mathfrak{N} sgp-OS = \mathfrak{N} POS.

(ii) Consider \mathfrak{N} sgp-CS in X . Then, K^c is \mathfrak{N} sgp-OS in X . By assumption, K^c is an \mathfrak{N} POS in X . Then, K is an \mathfrak{N} PCS in X . Thereupon, X is an \mathfrak{N} -sgp- $T_{1/2}$.

5. Conclusions

The class of neutrosophic semi-generalized pre closed sets in neutrosophic topological space is useful not only increase our understanding of some special features of the already known notions of neutrosophic topology but also useful in developing the neutrosophic multifunction theory in neutrosophic control theory as well as in neutrosophic economy. Some results have been proved to show that how far topological structures are preserved by the new neutrosophic set defined. We

have given examples where such properties fail to be preserved. Here we have presented the idea; still some more theoretical research is to be carried out to establish a general frame work for decision making and to define patterns for complex network conceiving and practical application.

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