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Abstract: One of the highest causes of death in many countries in this modern era is cardiovascular diseases. There are a few symptoms that linked significantly to the cardiovascular diseases. The symptoms and diseases relationship can be represented by neutrosophic set values. In general, neutrosophic set gives remarkable contribution in denoising, clustering, segmentation, and classification in handling data of many real applications including in medical field. This study aims to analyse the cardiovascular disease risks by a new distance-based similarity measure motivated from intuitionistic fuzzy set theory. The proof for all the properties is presented clearly. Then, a case study is conducted by using the data on the severity level of the six symptoms found in two different patients. The neutrosophic data are analysed to determine the patients' possibility of having any one or combination of the three types of cardiovascular diseases. A comparative study involving three common distance measures is conducted. The results show that the similarity indexes for all measures of both patients are less than 0.5. This situation can further conclude as both patients are possibly not suffering from cardiovascular diseases.

Keywords: Cardiovascular Disease, Distance Based, Neutrosophic Set, Similarity Measure

1. Introduction

In human body, the cardiovascular system consisting of heart and circulatory system has its crucial role in protection, regulation and transport of nutrients and oxygen to all the tissues of the body [1]. The blood flows through a network of blood vessels in response to the heart pumps and produces a pressure gradient [2]. Unfortunately, diseases related to the cardiovascular system have been proven to be one of the main causes of death in the whole world. Few of the cardiovascular diseases are coronary artery disease, heart failure, congenital heart disease and heart attack [3]. The diseases occur due to factors such as genetic disposition, systolic blood pressure, cholesterol, diabetes, body mass index, depression, and unhealthy diet. Due to the severity cause of these diseases, it is important to make correct diagnosis and to provide appropriate treatment to the patients.

Invasive methods are commonly used in medical diagnosis to identify vascular health conditions. The diagnosis result is used as a guideline by the doctor to provide the appropriate treatment. Alternatively, in the past few decades, many studies used non-invasive method with the purpose of reducing the patients' health risk and clinical utility cost. Mathematical model developed by the physical law of fluid dynamics has the capability in understanding the blood flow behaviour in vascular system [2, 4, 5, 6]. Another aspect in making the correct diagnosis relates to the sufficient patients' information of their medical condition. There is an elegant branch of mathematics which gives us the ability to reduce the possibility in making inaccurate diagnosis despite of the incompleteness or uncertain information. The area uses fuzzy mathematics concept in defining the set theory where it evolves from fuzzy set to many more advance sets e.g intuitionistic fuzzy set and neutrosophic set. The studies on the development of more advance neutrosophic sets and its applications as well in medical or clinical diagnosis are found in [7-10].

Distance and similarity measures are important in various scientific research fields such as decision making, pattern identification, and market forecasting. Lots of studies have been done by adopting fuzzy sets [11], intuitionistic fuzzy sets (IFS) [12,13], and neutrosophic sets [8,14]. The use of similarity measures has significant role in data clustering process and a work of [15] had proven that single-valued neutrosophic set (SVNS) clustering algorithm improved the accuracy in representing the indeterminate or inconsistent information. The most widely used distance measures are Hamming distance and Euclidean distance. [16] introduced a new similarity measure in a real-life decision-making problem and proved its ability to handle multiple existing criteria of incomplete or inconsistent information. Several new similarity measures of the neutrosophic sets with exponential functions in the truth, indeterminacy and falsity memberships were produced by [17]. They concluded that the existing measures failed in some circumstances, while the proposed measures classify them more appropriate and precisely.

To date, the application of neutrosophic set theory is significantly found in decision making studies. The major advantage of the set is its ability to handle uncertainty and incompleteness of the data. Neutrosophic set is an important set for denoising, clustering, segmentation, and classification of real data in many areas which includes medical field. For effective diagnosis systems, neutrosophic set have been integrated with the clustering techniques to reduce ambiguity for competent diagnosis. [3] proposed the neutrosophic clinical decision-making system using explainable artificial intelligence approaches for the proper diagnose of cardiovascular disease risk. Then, [18] extended the same approach to help physicians in early diagnosis, identifying the type of treatment and diagnosis. [7] focused on heart disease diagnosis problem as an application of neutrosophic refined set using distance measure while [8] analysed the medical diagnosis for rough neutrosophic set using Dice and Cosine similarity measures. Meanwhile, [9] created a new model based on Neutrosophic Cognitive Map that integrates diagnosis, treatment, and prognosis processes for supporting clinical decision-making for the treatment of cardiovascular diseases during pregnancy. [10] proposed a novel framework based on Internet of Thing (IoT) and computer supported diagnosis to identify and control heart failure infected patients. They obtained preciseness of diagnosis with vague information and suggested the neutrosophic multi criteria decision making technique in guiding the physician to identify whether a patient is suffering from heart failure. A new distance-based similarity measure has been proposed by [19] for refined neutrosophic sets and they applied the findings in medical diagnosis of few diseases with a common set of symptoms. Further, [20] developed a new hybrid distance-based similarity measure for refined neutrosophic sets. Another approach introduced by [21] is a new parametric divergence measure for neutrosophic sets used not only in medical diagnosis but also in pattern recognition problem, and multi criteria decision making problem.

Based on the related studies stated above, there are less studies focus on the neutrosophic sets with new distance formula modified from IFS. Thus, our present study extended [13] to present the new distance-based similarity measure in analysing the risk of the cardiovascular disease. This new measure is improvised to fulfil the gap within the indeterminate relationship. In this present study,

we have compared our new formula with the existing normalized Hamming distance, extended Hausdorff distance and normalised Euclidean distance measures. The study presents significant result on the medical diagnosis of three cardiovascular diseases for two patients. The description of the diseases together with the major factors linked to the diseases are well defined in Section 2. The section also provides the definitions of SVNS that contains three of membership functions (MFs) that are truth (T), indeterminacy (I), and falsity (F) together with several distance measures which being used in the subsequent section. By adopting the distance formula presented in [13] into NS domain, we derive the new formula to obtain the distance measure on SVNSs. Section 3 proves that the new formula satisfies all the four properties of distance measure. By means of the distance and similarity measures, Section 4 uses the clinical data in [3] regarding the symptoms shown on the two patients to relate them with the three cardiovascular diseases.

2. Preliminaries

This section introduced some preliminary notions which will be applied in the final analysis.

2.1. Single Valued Neutrosophic Set

A neutrosophic set which can be used in real scientific and engineering applications is known as Single valued neutrosophic set (SVNS).

Definition 2.1.1 [22]. Let X be a space of points (objects) with a generic element in X denoted by x. A single valued neutrosophic set A in X is characterized by a truth membership function, $T_A(x)$, an indeterminacy membership function, $I_A(x)$, and a falsity membership function $F_A(x)$. Here $T_A(x), I_A(x), F_A(x)$ are real subsets of [0,1].

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\}$$

2.2. Distance-based Similarity Measure of Neutrosophic set

Definition 2.2.1 [23]. Normalized Hamming distance measure $d_{NS}^{NH}(A,B)$ operator between neutrosophic set A and B is defined as follows:

$$d_{NS}^{NH}(A,B) = \frac{1}{3n}\sum_{i=1}^{n}(|T_A(x_i) - T_B(x_i)| + |I_A(x_i) - I_B(x_i)| + |F_A(x_i) - F_B(x_i)|)$$

Definition 2.2.2 [23]. Normalized Euclidean distance measure $d_{NS}^{NE}(A,B)$ operator between neutrosophic set A and B is defined as follows:

$$d_{NS}^{NE}(A,B) = \sqrt{\frac{1}{3n}\sum_{i=1}^{n}((T_A(x_i) - T_B(x_i))^2 + (I_A(x_i) - I_B(x_i))^2 + (F_A(x_i) - F_B(x_i))^2)}$$

Definition 2.2.3 [23]. An extended Hausdorff Distance $d_{NS}^{EH}(A,B)$ operator between neutrosophic set A and B is defined as follows:

$$d_{NS}^{EH}(A,B) = \frac{1}{n}\sum_{i=1}^{n} max\{|T_A(x_i) - T_B(x_i)|, |I_A(x_i) - I_B(x_i)|, |F_A(x_i) - F_B(x_i)|\}$$

Definition 2.2.4 [23]. Let A, B be two neutrosophic sets in X. The similarity measure between the neutrosophic sets A and B can be evaluate from distance measures, as follows:

$$S_N(A,B) = 1 - d_{NS}(A,B)$$

where $d_{NS}(A,B)$ is represent the distance measure between neutrosophic set A and B for all $x_i \in X$.

Proposition 2.2.1: The distance measures for neutrosophic set $d_{NS}(A,B)$and similarity measure for neutrosophic set $S_N(A,B)$ satisfies the following properties:

(C1) $0 \leq d_{NS}(A,B) \leq 1; 0 \leq S_N(A,B) \leq 1;$

(C2) $d_{NS}(A,B) = 0$ if and only if $A = B$; $S_N(A,B) = 1$ if and only if for $A = B$;

(C3) $d_{NS}(A,B) = d_{NS}(B,A)$; $S_N(A,B) = S_N(B,A)$;

(C4) $d_{NS}(A,C) \leq d_{NS}(A,B)$ and $d_{NS}(A,C) \leq d_{NS}(B,C)$ if C is neutrosophic set in X and $A \subseteq B \subseteq C$; $S_N(A,C) \leq S_N(A,B)$ and $S_N(A,C) \leq S_N(B,C)$ if C is neutrosophic set in X and $A \subseteq B \subseteq C$.

All the proof of the proposition are shown in [20-21].

2.3. Distance Measure on Intuitionistic Fuzzy set

Definition 2.3.1 [13]. Let $X = \{x_1, x_2, \ldots., x_n\}$ be the universe of discourse. Let $A = \{x_i, T_A(x_i), F_A(x_i)) : x_i \in X\}$ and $B = \{x_i, T_B(x_i), F_B(x_i) : x_i \in X\}$ be two intuitionistic fuzzy sets. Then, the distance measure between A and B can be defined as:

$$d_{IFS}(A,B) = \frac{2}{n}\sum_{i=1}^{n} \frac{sin\left\{\frac{\pi}{6}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{6}|F_A(x_i) - F_B(x_i)|\right\}}{1 + sin\left\{\frac{\pi}{6}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{6}|F_A(x_i) - F_B(x_i)|\right\}}$$

Proposition 2.3.1: The distance measures for intuitionistic fuzzy set $d_{IFS}(A,B)$ satisfies the following properties:

(C1) $0 \leq d_{IFS}(A,B) \leq 1$;

(C2) $d_{IFS}(A,B) = 0$ if and only if $A = B$;

(C3) $d_{IFS}(A,B) = d_{IFS}(B,A)$;

(C4) $d_{IFS}(A,C) \leq d_{IFS}(A,B)$ and $d_{IFS}(A,C) \leq d_{IFS}(B,C)$ if C is intuitionistic fuzzy set in X and $A \subseteq B \subseteq C$.

All the proof of the proposition are shown in [13].

2.4. Major factors of cardiovascular diseases

There is several factors for cardiovascular diseases found in [25]-[27] but this present study only focuses on the six highly significant factors which are:

i. **Cholesterol** - a fatty substance found in all cells in the human body and the bloodstream. The human body needs cholesterol to make hormones, vitamin D, and substances that help you digest food. Cholesterol is usually produced in the liver, but it can be found in a variety of animal -based foods. It can affect your health if you have too high cholesterol levels.

ii. **Depression** - a emotional disorder that can affect a person's daily life. It may be described as prolonged sadness, fatigue, irritability, and loss of interest in daily activities.

iii. **Diabetes** - a disease that occurs when too high blood glucose (blood sugar) in human body caused by the body not being able to produce enough insulin. Insulin is a hormone made by the pancreas that controls the balance of glucose in the body by helping the movement of glucose from the blood into the cells to be used for energy.

iv. **Blood pressure** - the pressure exerted by the blood on the arteries when blood is pumped by the heart throughout the human body. High blood pressure is a silent disease that can lead to complications and even death if left untreated.

v. **Body mass index** - a measure to access person's weight versus height. A high BMI can be an indicator of high body fatness. Being overweight exposes a person to diseases such as heart disease, stroke, diabetes, and high blood pressure.

vi. **Unhealthy diet –** fail to deliver human body with the proper quantities and varieties of nutrients for optimum health, especially when the diet contains high calories and less fruits and vegetables.

2.5. Cardiovascular diseases (CVDs)

Many types of cardiovascular diseases (CVDs) as stated in [24]-[26] that cause the death. This study considers only three types of CVDs, as follows:

i. **Heart attack** - when a blood clot blocks the flow of blood through the blood vessels that feed the heart, perhaps harming or ruining part of the heart muscle. A heart attack can be caused by atherosclerosis
ii. **Heart failure** - heart disease's most frequent complications. When your heart blood pumping ability is not enough to supply blood to comply your body's needs, hence heart failure occurs. Heart failure can be caused by various forms of heart disease, including high blood pressure, heart defects, cardiovascular disease, diabetes, vascular heart disease, heart infections or heart muscle disease.
iii. **Congenital heart disease** - malformations of heart structure existing at birth.

3. A Novel Distance Measure on Neutrosophic Sets

3.1. Distance-based Similarity Measure

Definition 3.1.1: Let $X = \{x_1, x_2, \dots., x_n\}$ be the universe of discourse. Let $A = \{x_i, T_A(x_i), I_A(x_i), F_A(x_i)): x_i \in X\}$ and $B = \{x_i, T_B(x_i), I_B(x_i), F_B(x_i)): x_i \in X\}$ be two neutrosophic sets. Then, by Definition 2.3.1, a new distance measure can be defined as:

$$d_{New}^{N}(A,B) =$$

$$\frac{2}{n}\sum_{i=1}^{n} \frac{sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\}}{1 + sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\}} \quad (1)$$

where $\frac{\pi}{6}$ is replaced by the factor $\frac{\pi}{10}$ to fulfil the proof of Proposition 3.1.1.

Proposition 3.1.1: The distance measures $d_{New}^{N}(A,B)$ for neutrosophic sets A and B comply with the following properties:

(C1) $0 \le d_{New}^{N}(A,B) \le 1$;
(C2) $d_{New}^{N}(A,B) = 0$ if and only if $A = B$;
(C3) $d_{New}^{N}(A,B) = d_{New}^{N}(B,A)$;
(C4) $d_{New}^{N}(A,C) \le d_{New}^{N}(A,B)$ and $d_{New}^{N}(A,C) \le d_{New}^{N}(B,C)$ if C is neutrosophic set in X and $A \subseteq B \subseteq C$.

The new distance measure satisfies all the properties, and the proofs are given below. The degree of truth, indeterminacy, and falsity membership for neutrosophic set maybe in decreasing or increasing order.

Proof:

(C1) $0 \le d_{New}^{N}(A,B) \le 1$.

As we know the degree of truth, indeterminacy, and falsity membership for neutrosophic set is $0 \le T_A(x), I_A(x), F_A(x) \le 1$. This implies for $A = \{x_i, T_A(x_i), I_A(x_i), F_A(x_i)): x_i \in X\}$ and $B = \{x_i, T_B(x_i), I_B(x_i), F_B(x_i)): x_i \in X\}$.

$$0 \le |T_A(x_i) - T_B(x_i)| \le 1, 0 \le |I_A(x_i) - I_B(x_i)| \le 1, \text{and } 0 \le |F_A(x_i) - F_B(x_i)| \le 1$$
$$\Rightarrow 0 \le sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} \le \frac{1}{3}, 0 \le sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} \le \frac{1}{3},$$

and

$$0 \le sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\} \le \frac{1}{3}$$
$$\Rightarrow 0 \le sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\} \le 1 \quad (2)$$
$$\Rightarrow 0 \le 1 + sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\} \le 2 \quad (3)$$

Therefore, from equation (2) and equation (3)

$$\Rightarrow 0 \le 2.\frac{sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\}}{1 + sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\}} \le 1$$
$$\Rightarrow 0 \le \frac{2}{n}\sum_{i=1}^{n} \frac{sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\}}{1 + sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\}} \le 1$$

$$\Rightarrow 0 \le d_{New}^{N}(A, B) \le 1.$$

(C2) $d_{New}^{N}(A, B) = 0$ if and only if $A = B$.

If $A = B$, then $T_A(x_i) = T_B(x_i), I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$ which states that $|T_A(x_i) - T_B(x_i)| = 0, |I_A(x_i) - I_B(x_i)| = 0$, and $|F_A(x_i) - F_B(x_i)| = 0$. Hence,
$sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} = 0, sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} = 0$, and $sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\} = 0$.
Thus, $d_{New}^{N}(A, B) = 0$.

Conversely,

$$d_{New}^{N}(A, B) = 0$$
$$\frac{2}{n}\sum_{i=1}^{n} \frac{sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\}}{1 + sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\}} = 0$$
$$\frac{sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\}}{1 + sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\}} = 0$$
$$sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\} = 0,$$

For this reason
$sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} = 0, sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} = 0$, and $sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\} = 0$
$|T_A(x_i) - T_B(x_i)| = 0, |I_A(x_i) - I_B(x_i)| = 0$, and $|F_A(x_i) - F_B(x_i)| = 0$
$T_A(x_i) = T_B(x_i), I_A(x_i) = I_B(x_i)$, and $F_A(x_i) = F_B(x_i)$
$\Rightarrow A = B$
Hence $d_{New}^{N}(A, B) = 0$ if and only if $A = B$.

(C3) $d_{New}^{N}(A, B) = d_{New}^{N}(B, A)$.
It is obvious that $T_A(x_i) - T_B(x_i) \neq T_B(x_i) - T_A(x_i)$, $I_A(x_i) - I_B(x_i) \neq I_B(x_i) - I_A(x_i)$, and $F_A(x_i) - F_B(x_i) \neq F_B(x_i) - F_A(x_i)$.

But, $|T_A(x_i) - T_B(x_i)| = |T_B(x_i) - T_A(x_i)|$, $|I_A(x_i) - I_B(x_i)| = |I_B(x_i) - I_A(x_i)|$, and $|F_A(x_i) - F_B(x_i)| = |F_B(x_i) - F_A(x_i)|$.

Hence,

$$d^N_{New}(A,B)$$
$$= \frac{2}{n}\sum_{i=1}^{n} \frac{sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\}}{1 + sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\}}$$
$$= \frac{2}{n}\sum_{i=1}^{n} \frac{sin\left\{\frac{\pi}{10}|T_B(x_i) - T_A(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_B(x_i) - I_A(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_B(x_i) - F_A(x_i)|\right\}}{1 + sin\left\{\frac{\pi}{10}|T_B(x_i) - T_A(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_B(x_i) - I_A(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_B(x_i) - F_A(x_i)|\right\}}$$
$$= d^N_{New}(B,A).$$

(C4) $d^N_{New}(A,C) \le d^N_{New}(A,B)$ and $d^N_{New}(A,C) \le d^N_{New}(B,C)$ if C is a neutrosophic set in X and $A \subseteq B \subseteq C$.

Consider $C = \{x_i, T_C(x_i), I_C(x_i), F_C(x_i)) : x_i \in X\}$ is a neutrosophic set in X and let $A \subseteq B \subseteq C$.

This implies that $T_A(x) \le T_B(x) \le T_C(x), I_A(x) \le I_B(x) \le I_C(x), F_A(x) \le F_B(x) \le F_C(x)$ for every $x_i \in X$. Then, we will have the following relations:

$a) |T_A(x_i) - T_C(x_i)| \le |T_A(x_i) - T_B(x_i)|$, and $|T_A(x_i) - T_C(x_i)| \le |T_B(x_i) - T_C(x_i)|$

$b) |I_A(x_i) - I_C(x_i)| \le |I_A(x_i) - I_B(x_i)|$, and $|I_A(x_i) - I_C(x_i)| \le |I_B(x_i) - I_C(x_i)|$

$c) |F_A(x_i) - F_C(x_i)| \le |F_A(x_i) - F_B(x_i)|$, and $|F_A(x_i) - F_C(x_i)| \le |F_B(x_i) - F_C(x_i)|$

Then,

$$sin\left\{\frac{\pi}{10}|T_A(x_i) - T_C(x_i)|\right\} \le sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} \text{ and}$$
$$sin\left\{\frac{\pi}{10}|T_A(x_i) - T_C(x_i)|\right\} \le sin\left\{\frac{\pi}{10}|T_B(x_i) - T_C(x_i)|\right\}$$

Similarly,

$$sin\left\{\frac{\pi}{10}|I_A(x_i) - I_C(x_i)|\right\} \le sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} \text{ and}$$
$$sin\left\{\frac{\pi}{10}|I_A(x_i) - I_C(x_i)|\right\} \le sin\left\{\frac{\pi}{10}|I_B(x_i) - I_C(x_i)|\right\}$$
$$sin\left\{\frac{\pi}{10}|F_A(x_i) - F_C(x_i)|\right\} \le sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\} \text{ and}$$
$$sin\left\{\frac{\pi}{10}|F_A(x_i) - F_C(x_i)|\right\} \le sin\left\{\frac{\pi}{10}|F_B(x_i) - F_C(x_i)|\right\}$$

Then,

$$sin\left\{\frac{\pi}{10}|T_A(x_i) - T_C(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_C(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_C(x_i)|\right\} \le$$
$$sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\}.$$

and

$$sin\left\{\frac{\pi}{10}|T_A(x_i) - T_C(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_C(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_C(x_i)|\right\} \le$$
$$sin\left\{\frac{\pi}{10}|T_B(x_i) - T_C(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_B(x_i) - I_C(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_B(x_i) - F_C(x_i)|\right\}.$$

Hence,

$$\frac{2}{n}\sum_{i=1}^{n} \frac{sin\left\{\frac{\pi}{10}|T_A(x_i) - T_C(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_C(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_C(x_i)|\right\}}{1 + sin\left\{\frac{\pi}{10}|T_A(x_i) - T_C(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_C(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_C(x_i)|\right\}}$$
$$\le \frac{2}{n}\sum_{i=1}^{n} \frac{sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\}}{1 + sin\left\{\frac{\pi}{10}|T_A(x_i) - T_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|I_A(x_i) - I_B(x_i)|\right\} + sin\left\{\frac{\pi}{10}|F_A(x_i) - F_B(x_i)|\right\}}$$

and

$$\frac{2}{n}\sum_{i=1}^{n}\frac{\sin\left\{\frac{\pi}{10}|T_A(x_i)-T_C(x_i)|\right\}+\sin\left\{\frac{\pi}{10}|I_A(x_i)-I_C(x_i)|\right\}+\sin\left\{\frac{\pi}{10}|F_A(x_i)-F_C(x_i)|\right\}}{1+\sin\left\{\frac{\pi}{10}|T_A(x_i)-T_C(x_i)|\right\}+\sin\left\{\frac{\pi}{10}|I_A(x_i)-I_C(x_i)|\right\}+\sin\left\{\frac{\pi}{10}|F_A(x_i)-F_C(x_i)|\right\}}$$
$$\leq\frac{2}{n}\sum_{i=1}^{n}\frac{\sin\left\{\frac{\pi}{10}|T_B(x_i)-T_C(x_i)|\right\}+\sin\left\{\frac{\pi}{10}|I_B(x_i)-I_C(x_i)|\right\}+\sin\left\{\frac{\pi}{10}|F_B(x_i)-F_C(x_i)|\right\}}{1+\sin\left\{\frac{\pi}{10}|T_B(x_i)-T_C(x_i)|\right\}+\sin\left\{\frac{\pi}{10}|I_B(x_i)-I_C(x_i)|\right\}+\sin\left\{\frac{\pi}{10}|F_B(x_i)-F_C(x_i)|\right\}}$$

$\Longrightarrow d_{New}^{N}(A,C)\leq d_{New}^{N}(A,B)$and $d_{New}^{N}(A,C)\leq d_{New}^{N}(B,C)$.

The proof is completed. ■

Example 3.1.1: Let $A=\{x_1,(0.7,0.6,0.2):x_1\in X\}$, $B=\{x_1,(0.8,0.2,0.9):x_1\in X\}$, and $C=\{x_1,(0.4,0.5,0.6):x_1\in X\}$ be a three neutrosophic sets in *X.* Then, by using a new distance-similarity measure as equation (1), the Proposition 3.1.1 is satisfied.

(C1) $0\leq d_{New}^{N}(A,B)\leq 1$;

$$d_{New}^{N}(A,B)=2\left(\frac{\sin\left\{\frac{\pi}{10}|0.7-0.8|\right\}+\sin\left\{\frac{\pi}{10}|0.6-0.2|\right\}+\sin\left\{\frac{\pi}{10}|0.2-0.9|\right\}}{1+\sin\left\{\frac{\pi}{10}|0.7-0.8|\right\}+\sin\left\{\frac{\pi}{10}|0.6-0.2|\right\}+\sin\left\{\frac{\pi}{10}|0.2-0.9|\right\}}\right)=0.5452\ \in[0,1].$$

(C2) $d_{New}^{N}(A,B)=0$ if and only if $A = B$;

$$\text{If } A=B,\ d_{New}^{N}(A,A)=2\left(\frac{\sin\left\{\frac{\pi}{10}|0.7-0.7|\right\}+\sin\left\{\frac{\pi}{10}|0.6-0.6|\right\}+\sin\left\{\frac{\pi}{10}|0.2-0.2|\right\}}{1+\sin\left\{\frac{\pi}{10}|0.7-0.7|\right\}+\sin\left\{\frac{\pi}{10}|0.6-0.6|\right\}+\sin\left\{\frac{\pi}{10}|0.2-0.2|\right\}}\right)=0.$$

(C3) $d_{New}^{N}(A,B)=d_{New}^{N}(B,A)$;

It is obviously that:

$|0.7-0.8|=|0.8-0.7|$, $|0.6-0.2|=|0.2-0.6|$ and $|0.2-0.9|=|0.9-0.2|$.

Then, $d_{New}^{N}(A,B)=d_{New}^{N}(B,A)=0.5452$.

(C4) $d_{New}^{N}(A,C)\leq d_{New}^{N}(A,B)$ and $d_{New}^{N}(A,C)\leq d_{New}^{N}(B,C)$ if C is a neutrosophic set in X and $A\subseteq B\subseteq C$.

$$d_{New}^{N}(A,C)=2\left(\frac{\sin\left\{\frac{\pi}{10}|0.7-0.4|\right\}+\sin\left\{\frac{\pi}{10}|0.6-0.5|\right\}+\sin\left\{\frac{\pi}{10}|0.2-0.6|\right\}}{1+\sin\left\{\frac{\pi}{10}|0.7-0.4|\right\}+\sin\left\{\frac{\pi}{10}|0.6-0.5|\right\}+\sin\left\{\frac{\pi}{10}|0.2-0.6|\right\}}\right)=0.4005$$
$$d_{New}^{N}(B,C)=2\left(\frac{\sin\left\{\frac{\pi}{10}|0.8-0.4|\right\}+\sin\left\{\frac{\pi}{10}|0.2-0.5|\right\}+\sin\left\{\frac{\pi}{10}|0.9-0.6|\right\}}{1+\sin\left\{\frac{\pi}{10}|0.8-0.4|\right\}+\sin\left\{\frac{\pi}{10}|0.2-0.5|\right\}+\sin\left\{\frac{\pi}{10}|0.9-0.6|\right\}}\right)=0.4770$$

As a result, $0.4005\leq 0.5452$ and $0.4005\leq 0.4770$.

Therefore, $d_{New}^{N}(A,C)\leq d_{New}^{N}(A,B)$ and $d_{New}^{N}(A,C)\leq d_{New}^{N}(B,C)$ if C is a neutrosophic set in X and $A\subseteq B\subseteq C$.

4. Methodology

There are three steps to complete the cardiovascular disease risk analysis using distance-based similarity measure of neutrosophic set. This study uses the Normalized Hamming distance, extended Hausdorff distance, Normalized Euclidean distance, and new distance measure of NS as defined in Section 3 for comparative analysis. The steps to complete the analysis are as follows:

Step 1: The extraction of data.

The data on the symptoms experienced by the patients are given in the form of neutrosophic sets values [3]. Further, the relationships between the symptoms and diseases are displayed in binary form and can easily being used as a reference in determining which cardiovascular diseases that put the patients at a high risk.

Step 2: Distance-based similarity measure

Determine the similarity measure of neutrosophic set for each patient by using four different distance measures. Neutrosophic set is used to determine the similarity measure of the relationship between symptoms and diseases, patients, and symptoms by using Definitions 2.2.1 – 2.2.4 and Definition 3.3.1.

Step 3: Discussion of a complete data analysis

Finally, based on the results in step 2, the whole data analysis can be discussed whether the patients' symptoms are close to the diseases. The conclusion can be made depend on the value of the similarity measures. The patient possibly suffers from the disease when the value of similarity measure is bigger than 0.5. Meanwhile, the patient may not possibly suffer from the disease when the value of similarity measure is less than 0.5.

5. Case Study: Implementation in Cardiovascular Disease Risks Analysis

This section discusses on the case study of two patients having five similar symptoms but different in body mass index. The patients' data on the severity degree of the symptoms together with the experts' consensus on the symptoms-cardiovascular diseases relationship are represented in SVNS. The data considers the degree of truth membership, indeterminacy membership and falsity membership for each element set. Let $P = \{p_1, p_2\}$ is a set of patients, $S = \{s_1, s_2, s_3, s_4, s_5, s_6\}$ is a set of symptoms. Table 1 shows data relationship between patients and symptoms as discussed in [3]. Besides, the relationship between cardiovascular diseases with symptoms is presented in Table 2.

Table 1. The relationship between patients and symptoms [3]

Symptom	Patient 1, p_1	Patient 1, p_2
Cholesterol, s_1	High (0.21, 0.81, 0.8)	High (0.21, 0.81, 0.8)
Depression, s_2	High (0.8, 0.33, 0.25)	High (0.8, 0.33, 0.25)
Diabetes, s_3	High (0.5, 0.58, 0.53)	High (0.5, 0.58, 0.53)
Blood Pressure, s_4	High (0.8, 0.4, 0.29)	High (0.8, 0.4, 0.29)
Body Mass Index, s_5	Med (0, 0.75, 1)	High (0.86, 0.57, 0.2)
Unhealthy Diet, s_6	Frequently (0.5,0.83,0.625)	Frequently (0.5,0.83,0.625)

Table 2. The relationship between cardiovascular disease with symptoms [3]

Symptom	Heart Attack (D1)	Heart Failure(D2)	Congenital Heart Disease (D3)
Cholesterol, s_1	(1,0,0)	(0,0,1)	(0,0,1)
Depression, s_2	(1,0,0)	(0,0,1)	(0,0,1)
Diabetes, s_3	(1,0,0)	(1,0,0)	(1,0,0)
Blood Pressure, s_4	(1,0,0)	(1,0,0)	(0,0,1)
Body Mass Index, s_5	(1,0,0)	(1,0,0)	(0,0,1)
Unhealthy Diet, s_6	(1,0,0)	(0,0,1)	(0,0,1)

According to data collected in Table 1, the truth membership degree for cholesterol for both patients are 0.21, the indeterminacy membership degree for cholesterol for both patients is 0.81 and the falsity membership degree for cholesterol for both patients are 0.8. The same description

is indicated for each data. It is obvious that the SVNS value of body mass index (BMI) for the two patients is different. The BMI of patient 2 is categorized as high with the value of the truth membership degree is 0.86. As for patient 1, his BMI falls into the category of medium with 0.75 and 1 indicating the indeterminacy and falsity membership degrees respectively.

Table 3. The distance measure for neutrosophic set

Distance	Normalized Hamming		Extended Hausdorff		Normalized Euclidean	
	P1	**P2**	**P1**	**P2**	**P1**	**P2**
Heart Attack, D1	0.576944	0.474722	0.658333	0.586667	0.631358	0.526889
Heart Failure, D2	0.558611	0.456389	0.736667	0.6650	0.616668	0.509195
Congenital heart disease, D3	0.504167	0.586389	0.761667	0.7800	0.570819	0.623922

Table 4. The new distance measure for neutrosophic set

New distance measure	**P1**	**P2**
Heart Attack, D1	0.6718168	0.5924242
Heart Failure, D2	0.6683397	0.5889471
Congenital heart disease, D3	0.6297732	0.7032119

By using the data in Table 1 and Table 2, the three distance-based similarity measures discussed in Section 2 are calculated and the values are displayed in Table 3. Further, the new distance values obtained by the new formula in Section 3 are presented in Table 4. Then, the associated values of similarity for all the four distance measures are calculated and tabulated in Table 5 and 6 respectively.

Table 5. Similarity measure values of the three-distance measure of neutrosophic set

Similarity	Normalized Hamming		Extended Hausdorff		Normalized Euclidean	
	P1	**P2**	**P1**	**P2**	**P1**	**P2**
Heart Attack, D1	0.423056	**0.525278**	0.341667	**0.413333**	0.368642	**0.473111**
Heart Failure, D2	0.441389	**0.543611**	0.263333	**0.335000**	0.383332	**0.490805**
Congenital heart disease, D3	**0.495833**	0.413611	0.238333	0.220000	**0.429181**	0.376078

Table 6. The new similarity measure for neutrosophic set

New similarity measure	**P1**	**P2**
Heart Attack, D1	0.3281832	**0.4075758**
Heart Failure, D2	0.3316603	**0.4110529**
Congenital heart disease, D3	**0.3702268**	0.2967881

A lower distance or higher similarity measure value implies higher possibility of one patient having a particular disease. The slight difference of symptoms shown in both patients results to distinct conclusion on the type of cardiovascular diseases that they experience. Patient 1 has the highest severity degree of symptoms for congenital heart disease. Meanwhile, Patient 2 is more likely to be diagnosed of having heart attack and heart failure. Concurrently, it is apparent that most of the similarity measure values in Table 5 and 6 are less than 0.5. Hence, it is probable to conclude that both patients are possibly not suffering from any of the three cardiovascular diseases.

6. Conclusions

This research proposes a novel distance measure for single value neutrosophic set which results to the use of similarity measure. The effectiveness of the new developed measure formula is demonstrated by adopted it in the process of medical diagnosis. Its similarity values are found to be consistent with similarity measures of the three existing distance measures i.e Normalized Hamming distance, extended Hausdorff distance and Normalized Euclidean distance. The analyses show that the new distance-based similarity measure is well executed in the case of truth membership, indeterminacy membership and falsity membership functions. In the future study, it is recommended that one might consider additional significant symptoms and other distance or similarity measure to increase the accuracy level in diagnosing a patient with any cardiovascular diseases. Besides, it is also recommended to utilize new entropy-based similarity measures of SVNS [28] to overcome the restriction of the distance similarity measures. The neutrosophic set also can be extend to pythagorean neutrosophic multi set as this can provide many applications to multi attribute group decision making problems in medical diagnosis and many other real-life problems [29].

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