

10-7-2021

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Recommended Citation

Bhatta G. R., Vadiraja; K. J. Manasa; Gautham Shenoy B.; Prasanna Poojary; and B. J. Chaithra. "Introduction to NeutroNearings." *Neutrosophic Sets and Systems* 46, 1 (). https://digitalrepository.unm.edu/nss_journal/vol46/iss1/30

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Introduction to NeutroNearings

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Abstract. Algebraic concepts and structures are enriched with the special types of operations and axioms known as NeutroOperations and NeutroAxioms. Various types of NeutroAlgebras are studied using several such defined concepts. The objective of this paper is to introduce the concept of NeutroNearings. Several interesting results and examples of NeutroNearings, NeutroSubRings, NeutroQuotientNearings and NeutroNearingHomomorphisms are presented.

Keywords: Nearring; NeutroRings; NeutroNearing; Neutrosophy.

1. Introduction

The NeutroDefined and AntiDefined Laws, as well as the NeutroAxioms and AntiAxioms was first time introduced in 2019 by Smarandache [3, 5]. This concept has given birth to new fields of research called NeutroStructures and AntiStructures. For basic and recent results on Neutrosophy, NeutroAlgebraic structures and AntiAlgebraic structures we refer [4–8].

In [2], Agboola formally presented the notion of NeutroGroups. In this he showed that in general, Lagrange's theorem and first isomorphism theorem of the classical groups do not hold

in the NeutroGroups. Also in [1], Agboola studied NeutroRing, NeutroSubring, NeutroQuotientRings and he proved the 1st isomorphism theorem of the classical rings for this class of NeutroRing.

The present paper will be concerned with the introduction of NeutroNerrings.

2. Preliminaries

Definition 2.1. [7]

- (i) A classical operation is an operation well defined for all the set's elements while a NeutroOperation is an operation partially well defined, partially indeterminate, and partially outer defined on the given set. An AntiOperation is an operation that is outer defined for all the set's elements.
- (ii) A NeutroAlgebra is an algebra that has at least one NeutroOperation or one NeutroAxiom (axiom that is true for some elements, indeterminate for other elements, and false for other elements), and no AntiOperation or AntiAxiom. An AntiAlgebra is an algebra endowed with at least one AntiOperation or at least one AntiAxiom.

Definition 2.2. A NeutroGroup is a nonempty set G with binary operation $*$ satisfying following conditions:

- (i) The $*$ is **NeutroAssociative** if there exists atleast one triplet $(a, b, c) \in G$ such that

$$a * (b * c) = (a * b) * c$$

and there exists atleast one triplet $(x, y, z) \in G$ such that

$$x * (y * z) \neq (x * y) * z$$

- (ii) There exists a **NeutroNeutral element** in G if at least one of the below statements occurs:

- There exists at least one element x that has no unit-element.
- There exists at least one element $b \in G$ that has at least two distinct unit-elements $e_1, e_2 \in G, e_1 \neq e_2$ such that:

$$b * e_1 = e_1 * b = b$$

$$b * e_2 = e_2 * b = b$$

- There exists at least two different elements $r, s \in G, r \neq s$, such that they have different unit elements $e_r, e_s \in G, e_r \neq e_s$, with $e_r * r = r * e_r = r$ and $e_s * s = s * e_s = s$

- (iii) There exists a **NeuroInverse element** in G if there is an element $a \in G$ that has an inverse $b \in G$ with respect to a unit element $e \in G$ that is

$$b * a = a * b = e$$

or there exists atleast one element $b \in G$ that has two or more inverses $c, d \in G$ with respect to some unit element $u \in G$ that is

$$b * c = c * b = u$$

$$b * d = d * b = u$$

In addition, if $*$ is **NeuroCommutative** that is there exists atleast a duplet $(a, b) \in G$ such that

$$a * b = b * a$$

and there exists atleast a duplet $(c, d) \in G$ such that

$$c * d \neq d * c$$

then $(G, *)$ is called a NeuroCommutative group or NeuroAbelian group.

If condition (i) is satisfied, then $(G, *)$ is called a NeuroSemiGroup and if conditions (i) and (ii) are satisfied, then $(G, *)$ is called a NeuroMonoid [1].

Definition 2.3. Let R be a nonempty set and let $+, \cdot : R \times R \rightarrow R$ be binary operations of ordinary addition and multiplication on R . Then \cdot is both left and right NeuroDistributive over $+$ that is there exists atleast a triplet $(a, b, c) \in R$ and atleast a triplet $(d, e, f) \in R$ such that

$$a.(b + c) = a.b + a.c$$

$$d.(e + f) \neq d.e + d.f$$

then \cdot is **left NeuroDistributive** over $+$ on a set R .

Suppose if there exists atleast a triplet $(p, q, r) \in R$ and atleast a triplet $(x, y, z) \in R$ such that

$$(p + q).r = p.r + q.r$$

$$(x + y).z \neq x.z + y.z$$

then the binary operation \cdot is said to be **right NeuroDistributive** over $+$ on a set R .

A **right Nearring** is a set N together with two binary operations $+$ and \cdot such that:

- (1) $(N, +)$ is a group (not necessarily abelian)
- (2) (N, \cdot) is a semigroup

(3) For all $n_1, n_2, n_3 \in N : (n_1 + n_2) \cdot n_3 = n_1 \cdot n_3 + n_2 \cdot n_3$ (right distribution law).

If $n_1 \cdot (n_2 + n_3) = n_1 \cdot n_2 + n_1 \cdot n_3$ instead of condition (3) then set N is a **left Nearring**.

NeutroNearing and their properties

A NeutroNearing is a Nearing that has either a Neutro-operation or a Neutro-axiom. In this paper we define NeutroNearing as below.

Definition 2.4. Let N be a nonempty set and let $+, \cdot : N \times N \rightarrow N$ be binary operations of ordinary addition and multiplication on N . The triple $(N, +, \cdot)$ is called a **left NeutroNearing** if the following conditions are satisfied:

- (i) $(N, +)$ is a NeutroGroup (not necessarily abelian)
- (ii) (N, \cdot) is a NeutroSemiGroup
- (iii) the left NeutroDistributive law holds in N : that is there exists atleast one triplet $(a, b, c) \in N$ and atleast one triplet $(d, e, f) \in N$ such that:
 - $a \cdot (b + c) = a \cdot b + a \cdot c$
 - $d \cdot (e + f) \neq d \cdot e + d \cdot f$

Remark 2.5. If right NeutroDistributive law holds in N : that is there exists atleast one triplet $(p, q, r) \in N$ and atleast one triplet $(s, t, u) \in N$ such that:

- $(p + q) \cdot r = p \cdot r + q \cdot r$
- $(s + t) \cdot u \neq s \cdot u + t \cdot u$

then N is called **right NeutroNearing**.

Example 2.6. Let $X = \mathbb{Z}_{12}$ and let \oplus and \odot be two binary operations on X defined by $x \oplus y = x + 2y$ and $x \odot y = x + 4y$ for all $x, y \in X$ where “+” is addition modulo 12. Then (X, \oplus, \odot) is a NeutroNearing.

Example 2.7. Let $X = \{a, b, c\}$ with “+” and “.” be binary operations defined on X as shown in the Cayley tables below:

+	a	b	c
a	c	c	b
b	c	b	c
c	c	c	b

.	a	b	c
a	b	a	a
b	a	c	a
c	a	a	b

It is clear from the table that :

$$(a + b) + c = a + (b + c) = b,$$

$$(c + a) + b = c, \text{ but } c + (a + b) = b \neq c$$

This shows that $(X, +)$ is a NeutroSemiGroup.

Next, let N_x and I_x represent additive neutral and additive inverse element respectively with respect to any element $x \in X$. Then

$$N_b = b$$

$$I_b = b$$

$$N_a \text{ does not exist,}$$

$$I_b \text{ does not exist.}$$

Hence, $(X, +)$ is a NeutroGroup.

Next, consider

$$b(cb) = (bc)b$$

$$a(bc) = b \text{ but } (ab)c = a \neq b$$

This shows that (X, \cdot) is NeutroAssociative.

Lastly, consider

$$b.(b + b) = b.b + b.b = c,$$

$$a.(b + c) = a, \text{ but } a.b + a.c = c \neq a$$

This shows that “ \cdot ” is left distributive over “ $+$ ”. Hence, $(X, +, \cdot)$ is a left NeutroNearing.

Note 2.8. Every NeutroRing is a NeutroNearing.

Notation: Let N be a NeutroNearing $d \in N$ is called NeutroDistributive if there exist atleast two pairs (n_1, n_2) and $(m_1, m_2) \in N$ such that $(n_1 + n_2)d = n_1d + n_2d$ and $(m_1 + m_2)d \neq m_1d + m_2d$. Let $N_d = \{d \in N | d \text{ is NeutroDistributive}\}$.

Remark 2.9. Let $(N, +, \cdot)$ be left NeutroNearing

- (i) If $(N, +)$ is NeutroAbelian, then N is a NeutroAbelian NeutroNearing.
- (ii) If (N, \cdot) is NeutroCommutative then N is a NeutroCommutative NeutroNearing.
- (iii) If $N = N_d$ then N is said to be NeutroDistributive.
- (iv) If (N^*, \cdot) where $N^* = N \setminus \{0\}$ is a NeutroGroup then N is called NeutroNearfield.

Theorem 2.10. Let $(N_i, +, \cdot), i = 1, 2, \dots, n$ be a family of NeutroNearings. Then

- (1) $N = \cap_{i=1}^n N_i$ is a NeutroNearing.

(2) $N = \prod_{i=1}^n N_i$ is a NeutroNearing.

(1) *Proof.* Obvious \square

(2) *Proof.* Proof is by induction on n .

For $n = 1$ result is trivial. Let $n = 2$. Consider $N = N_1 \times N_2$ then is closed with respect to coordinate wise addition and coordinate wise multiplication. Note that there exist $n_1 \in N_1$ such that $n_1 + e_1 = n_1$ and there exist $n_2 \in N_2$ such that $n_2 + e_2 = n_2$.

Also, there doesnot exist additive identity for $n'_1 \in N_1$ and $n'_2 \in N_2$.

But $(n_1, n_2) \in N$ such that $(n_1, n_2) + (e_1, e_2) = (n_1, n_2)$ and there doesnot exist additive identity for $(n'_1, n'_2) \in N$.

Similarly one can observe the existence of NeutroAdditive inverse in N .

Since $(N_1, +)$ and $(N_2, +)$ are NeutroAssociative, there exist $a_1, b_1, c_1, a'_1, b'_1, c'_1 \in N_1$ and $a_2, b_2, c_2, a'_2, b'_2, c'_2 \in N_2$ such that $a_1 + (b_1 + c_1) = (a_1 + b_1) + c_1$

$$a_2 + (b_2 + c_2) = (a_2 + b_2) + c_2$$

$$a'_1 + (b'_1 + c'_1) \neq (a'_1 + b'_1) + c'_1$$

$$a'_2 + (b'_2 + c'_2) \neq (a'_2 + b'_2) + c'_2$$

Now, $(a_1, a_2), (b_1, b_2), (c_1, c_2), (a'_1, a'_2), (b'_1, b'_2), (c'_1, c'_2) \in N$ such that

$$(a_1, a_2) + [(b_1, b_2) + (c_1, c_2)] = (a_1, a_2) + [(b_1 + c_1, b_2 + c_2)] = (a_1 + (b_1 + c_1), a_2 + (b_2 + c_2))$$

$$= ((a_1 + b_1) + c_1, (a_2 + b_2) + c_2) = (a_1 + b_1, a_2 + b_2) + (c_1, c_2)$$

$$= [(a_1, a_2) + (b_1, b_2)] + (c_1, c_2)$$

$$\text{and } (a'_1, a'_2) + [(b'_1, b'_2) + (c'_1, c'_2)] \neq [(a'_1, a'_2) + (b'_1, b'_2)] + (c'_1, c'_2)$$

Similarly we prove (N, \cdot) is NeutroAssociative.

Further there exist $x_1, y_1, z_1, x'_1, y'_1, z'_1 \in N_1$ and $x_2, y_2, z_2, x'_2, y'_2, z'_2 \in N_2$ such that

$$x_1 \cdot (y_1 + z_1) = x_1 \cdot y_1 + x_1 \cdot z_1$$

$$x_2 \cdot (y_2 + z_2) = x_2 \cdot y_2 + x_2 \cdot z_2$$

$$x'_1 \cdot (y'_1 + z'_1) \neq x'_1 \cdot y'_1 + x'_1 \cdot z'_1$$

$$x'_2 \cdot (y'_2 + z'_2) \neq x'_2 \cdot y'_2 + x'_2 \cdot z'_2$$

But then, $(x_1, x_2), (y_1, y_2), (z_1, z_2), (x'_1, x'_2), (y'_1, y'_2), (z'_1, z'_2) \in N$ such that

$$(x_1, x_2) \cdot [(y_1, y_2) + (z_1, z_2)] = (x_1, x_2) \cdot (y_1 + z_1, y_2 + z_2)$$

$$= (x_1(y_1 + z_1), x_2(y_2 + z_2)) = (x_1 \cdot y_1 + x_1 \cdot z_1, x_2 \cdot y_2 + x_2 \cdot z_2)$$

$$= (x_1 \cdot y_1, x_2 \cdot y_2) + (x_1 \cdot z_1, x_2 \cdot z_2) = (x_1, x_2) \cdot (y_1, y_2) + (x_1, x_2) \cdot (z_1, z_2)$$

$$\text{and } (x'_1, x'_2) \cdot [(y'_1, y'_2) + (z'_1, z'_2)] \neq (x'_1, x'_2) \cdot (y'_1, y'_2) + (x'_1, x'_2) \cdot (z'_1, z'_2)$$

$\therefore N$ is a NeutroNearing for $n = 2$.

Let $n > 2$. Assume the result for $n - 1$. Note that $M = \prod_{i=1}^{n-1} N_i$ forms a NeutroNearing with respect to coordinate wise addition and coordinate wise multiplication.

But then, $N = M \times N_n$ forms a NeutroNearing with respect to coordinate wise addition and coordinate wise multiplication. \square

Definition 2.11. Let $(N, +, \cdot)$ be a NeutroNearing. A nonempty subset S of N is called a **NeutroNearSubring** of N if $(S, +, \cdot)$ is also a NeutroNearing. The only trivial NeutroNearSubring of N is N .

Theorem 2.12. Let $(N, +, \cdot)$ be a NeutroNearing and let $\{S_i\}, i = 1, 2, \dots, n$ be a family of NeutroNearSubrings of N . Then

- (1) $S = \bigcap_{i=1}^n S_i$ is a NeutroNearSubring of N .
- (2) $S = \prod_{i=1}^n S_i$ is a NeutroNearSubring of N .

Proof. Both result follows directly from Theorem 2.10. \square

Definition 2.13. Let $(N, +, \cdot)$ be a NeutroNearing. A nonempty subset I of N is called a **left NeutroNearIdeal** of N if the following conditions hold:

- (1) I is a NeutroNearSubring of N .
- (2) There exist $x \in I$ such that $xr \in I, \forall r \in N$.

Definition 2.14. Let $(N, +, \cdot)$ be a NeutroNearing. A nonempty subset I of N is called a **right NeutroNearIdeal** of N if the following conditions hold:

- (1) I is a NeutroNearSubring of N .
- (2) There exist $x \in I$ such that $rx \in I, \forall r \in N$

Definition 2.15. Let $(N, +, \cdot)$ be a NeutroNearing. A nonempty subset I of N is called a **NeutroNearIdeal** of N if the following condition hold:

- (1) I is a NeutroNearSubring of N .
- (2) There exist $x \in I$ such that $xr, rx \in I, \forall r \in N$

Theorem 2.16. Let $(N, +, \cdot)$ be a NeutroNearing and let $\{I_i\}, i = 1, 2, \dots, n$ be a family of NeutroNearIdeals of N . Then

- (1) $I = \bigcap_{i=1}^n I_i$ is a NeutroNearIdeal of N .
- (2) $I = \sum_{i=1}^n I_i$ is NeutroNearIdeal of N .

- (1) *Proof.* Since each $I_i, 1 \leq i \leq n$ is a NeutroNearSubring of N , it follows from Theorem 1.8 that $I = \bigcap_{i=1}^n I_i$ is a NeutroNearSubring of N .

Note that there exist $x_i \in I_i$ such that $x_i r, r x_i \in I_i, \forall r \in N$ and $\forall i, 1 \leq i \leq n$.

Let $y = x_1^2 x_2^2 \cdots x_n^2$. Then $y \in I_i, \forall i, 1 \leq i \leq n$.

For any $r \in N$ $ry, yr \in I_i, \forall i, 1 \leq i \leq n$

$\therefore y \in I$ with $ry, yr \in I$. \square

(2) Obivous.

Definition 2.17. Let $(N, +, \cdot)$ be a NeutroNearing and let I be a NeutroNearIdeal of N . The set N/I is defined by

$$N/I = \{x + I : x \in N\}$$

for $x + I, y + I \in N/I$ with at least a pair $(x, y) \in N$, let \oplus and \odot be binary operations on N/I defined as follows:

$$\begin{aligned}(x + I) \oplus (y + I) &= (x + y) + I \\ (x + I) \odot (y + I) &= xy + I\end{aligned}$$

Then it can be shown that the tripple $(N/I, \oplus, \odot)$ is a NeutroNearing which we call Neutro-QuotientNearing.

Theorem 2.18. Let I be a NeutroNearIdeal of the NeutroNearing $(N, +, \cdot)$. Suppose N is NeutroCommutative NeutroNearing with Neutro unity then so is N/I .

Proof. There exist $a, b, c, d \in N$ such that $ab = ba$ and $cd \neq dc$.

But then $a + I, b + I, c + I, d + I \in N/I$ such that $(a + I)(b + I) = ab + I = ba + I = (b + I)(a + I)$ and $(c + I)(d + I) = cd + I \neq dc + I = (d + I)(c + I)$

Let e_y be a Neutro unity of N . Then there exist $y \in N$ such that $ye_y = e_y y = y$

But then $y + I \in N/I$ such that $(y + I)(e_y + I) = ye_y + I = y + I = (e_y + I)(y + I)$

$\therefore N/I$ is NeutroCommutative NeutroNearing with Neutro unity $e_y + I$. \square

Definition 2.19. Let $(N, +, \cdot)$ and (S, \oplus, \odot) be any two NeutroNearings. The mapping $\phi : N \rightarrow S$ is called a NeutroNearingHomomorphism if ϕ preserves the binary operations of N and S that is if for at least a pair $(x, y) \in N$, we have:

$$\begin{aligned}\phi(x + y) &= \phi(x) \oplus \phi(y) \\ \phi(x \cdot y) &= \phi(x) \odot \phi(y)\end{aligned}$$

The kernel of ϕ denoted by $\ker\phi$ is defined as $\ker\phi = \{x \in N : \phi(x) = e_N\}$

Where $e_N \in N$ is a neutral element for at least one $n \in N$. The image of ϕ denoted by $Im\phi$ is defined as

$$Im\phi = \{y \in S : y = \phi(x)\}$$

for at least one $x \in N$. If in addition ϕ is a NeutroBijection, then ϕ is called a NeutroNearingIsomorphism and we write $N \cong S$. NeutroNearingEpimorphism, NeutroNearingMonomorphism, NeutroNearingEndomorphism and NeutroNearingAutomorphism are defined similarly.

Theorem 2.20. *Let R and S be two NeutroNearings. Let $N_x = e_R$ for at least one $x \in R$ and let $N_y = e_S$ for at least one $y \in S$. Suppose that $\phi : R \rightarrow S$ is a NeutroNearHomomorphism. Then:*

- (1) $\phi(e_R)$ is not necessarily equals e_S .
- (2) $\text{Ker } \phi$ is a NeutroNearSubring of R .
- (3) $\text{Im } \phi$ is not necessarily a Neutro Near Subring of S .
- (4) ϕ is NeutroInjective if and only if $\text{Ker } \phi = \{e_R\}$ for at least one $e_R \in R$.

Theorem 2.21. *Let I be a NeutroNearIdeal of a NeutroNearing $(N, +, \cdot)$. Then the mapping $\phi : N \rightarrow N/I$ defined by $\phi(x) = x + I$ is a NeutroNearingIsomorphism with $\text{Ker } \phi = I$*

Proof. For atleast one pair x, y in N ,

$$\phi(x + y) = (x + y) + I = (x + I) + (y + I) = \phi(x) + \phi(y)$$

$$\text{and } \phi(xy) = xy + I = (x + I)(y + I) = \phi(x)\phi(y)$$

$$\text{Ker } \phi = \{x \in N | \phi(x) = e_{N/I}\}, \text{ where } e_{N/I} \in N/I \text{ such that } N_r = e_{N/I} \text{ for atleast one } r \in N/I$$

$$\cdot = \{x \in N | x + I = e_{N/I}\} = \{x \in N | x \in I\} = I \quad \square$$

Theorem 2.22. *Let $\phi : R \rightarrow S$ be a NeutroNearingHomomorphism and let $K = \text{Ker } \phi$. Then the mapping $\psi : R/K \rightarrow \text{Im } \phi$ defined by $\psi(x + K) = \phi(x)$ is a NeutroNearingIsomorphism.*

Proof. For atleast one pair $x, y \in R$

$$\psi((x + K) + (y + K)) = \psi((x + y) + K) = \phi(x + y) = \phi(x) + \phi(y)$$

$$= \psi(x + K) + \psi(y + K) \text{ and } \psi((x + K)(y + K)) = \psi((xy) + K) = \phi(xy) = \phi(x)\phi(y)$$

$$= \psi(x + K)\psi(y + K)$$

Also $\text{Ker } \psi = \{x + K \in R/K : \psi(x + K) = e_{\text{Im } \phi}\}$ where $e_{\text{Im } \phi} \in \text{Im } \phi$ such that $N_r = e_{\text{Im } \phi}$ for atleast $r \in \text{Im } \phi$.

$$= \{x + K \in R/K : \phi(x) = e_{\text{Im } \phi}\} = \{e_{R/K}\}$$

Thus ψ is a NeutroBijectiveNearingHomomorphism. \square

Note 2.23. *The above map ϕ is an epimorphism. So, we can treat ϕ as NeutroNearingEpimorphism.*

Theorem 2.24. *NeutroNearingIsomorphism of NeutroNearings is an equivalence relation.*

Proof. Define a relation \sim as follows:

For any two NeutroNearrings N and N' , we say $N \sim N'$ iff there exist a NeutroNearingIso-morphism between N and N' . Clearly \sim is reflexive.

Suppose NeutroNearrings N, N' are such that $N \sim N'$, let $f : N \rightarrow N'$ be NeutroNearingI-somorphism.

Then there exist $x, y \in N$ such that $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$.

Now, $f(x), f(y) \in N'$ and

$$f^{-1}(f(x) + f(y)) = f^{-1}(f(x + y)) = x + y = f^{-1}(f(x)) + f^{-1}(f(y))$$

$$f^{-1}(f(x)f(y)) = f^{-1}(f(xy)) = xy = f^{-1}(f(x))f^{-1}(f(y))$$

$\therefore N' \sim N$ and f^{-1} is a NeutroNearingIsomorphism.

Let $f : N \rightarrow N'$ and $g : N' \rightarrow N''$ be NeutroNearingIsomorphisms.

Then $g \circ f : N \rightarrow N''$ is bijective and there exist $x', y' \in N'$ such that $g(x' + y') = g(x') + g(y')$ and $g(x'y') = g(x')g(y')$

Now, $x' = f(x), y' = f(y)$ for some $x, y \in N$ with $f(x + y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$

Consider,

$$\begin{aligned} g \circ f(x + y) &= g(f(x + y)) = g(f(x) + f(y)) = g(x' + y') = g(x') + g(y') = g(f(x)) + g(f(y)) \\ &= g \circ f(x) + g \circ f(y) \end{aligned}$$

$$\text{and } g \circ f(xy) = g(f(xy)) = g(f(x)f(y)) = g(x'y') = g(x')g(y') = g(f(x))g(f(y))$$

$$= g \circ f(x)g \circ f(y)$$

$\therefore N \sim N'' \quad \square$

3. Conclusion

We have introduced in this paper the concept of NeutroNearrings by considering three NeutroAxioms(NeutroGroup(additive)), NeutroSemigroup(multiplicative) and left and right NeutroDistributive laws(multiplication over addition). Several interesting results and examples on NeutroNearrings, NeutroSubrings, NeutroQuotientNearrings and NeutroNearingHo-momorphisms are presented.

Funding: “Fourth author and fifth author received funding from VGST/KSTePS and DST, GoK, India”

Acknowledgments: Fourth author and fifth author thanks VGST/KSTePS and DST, GoK, India for sanctioning the project [VGST/GRD-897/2019-20/2020-21]. The authors are grateful to Prof. Dr. Florentin Smarandache from University of New Mexico for his valuable ideas and guidance.

Conflicts of Interest: “The authors declare no conflict of interest.”

Vadiraja Bhatta G. R., Manasa K. J., Gautham Shenoy B., Prasanna Poojary, Chaithra B. J., Introduction to NeutroNearrings

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Received: May 9, 2021. Accepted: October 5, 2021