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A Novel Approach to Neutrosophic Hypersoft Graphs with Properties

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Abstract. Neutrosophic hypersoft set is the combination of neutrosophic set and hypersoft set. It resolves the limitations of intuitionistic fuzzy sets and soft sets for the consideration of the degree of indeterminacy and multi-argument approximate function respectively. In this research article, a novel framework i.e. neutrosophic hypersoft graph, is formulated for handling neutrosophic hypersoft information by combining the concept of neutrosophic hypersoft sets with graph theory. Firstly, some of essential and fundamental notions of neutrosophic hypersoft graph are characterized with the help of numerical examples and graphical representation. Secondly, some set theoretic operations i.e. union, intersection and complement, are investigated with illustrative examples and pictorial depiction.

Keywords: Neutrosophic Set; Soft set; Hypersoft set; Neutrosophic soft graph; Neutrosophic hypersoft set; Neutrosophic hypersoft graph.

1. Introduction

In different mathematical disciplines, fuzzy sets theory (FS-Theory) [1] and intuitionistic fuzzy set theory (IFS-Theory) [2] are considered apt mathematical modes to tackle several intricate problems involving various uncertainties. The former emphasizes on a certain object's degree of true belongingness from the initial sample space, while the latter emphasizes degree of true membership and degree of non-membership with the state of their interdependence. These theories portray some kind of inadequacy in terms of providing due status to a degree of indeterminacy. The implementation of neutrosophic set theory (NS-Theory) [3, 4] overcomes this impediment by taking into account not only the proper status of degree of indeterminacy but also the state of dependence. This theory is more adaptable and suitable for dealing with inconsistent data. Wang et el [5] conceptualized single-valued neutrosophic set in which truth membership degree, indeterminacy degree and falsity degree are restricted within unit closed interval. Many researchers [6]- [14] have been drawn to NS-Theory for further application in statistics, topological spaces, and the construction of some neutrosophic-like blended structures with other existing models for useful applications in decision making. Edalatpanah [15] studied a system of neutrosophic linear equations (SNLE) based on the embedding approach. He used (α, β, γ)-cut for transformation of SNLE into a crisp linear system. Kumar et al. [16] exhibited a novel linear programming approach for finding the neutrosophic shortest path problem (NSSPP) considering Gaussian valued neutrosophic number.

FS-Theory, IFS-Theory and NS-Theory have some kind of complexities which restrain them to solve problem involving uncertainty professionally. The reason for these hurdles is, possibly, the inadequacy of the parametrization tool. It demands a mathematical tool free of all such impediments to tackle such issues. This scantiness is resolved with the development of soft set theory (SS-Theory) [17] which is a new parameterized family of subsets of the universe of discourse. The researchers [18]- [27] studied and investigated some elementary properties, operations, laws and hybrids of SS-Theory with applications in decision making. The gluing concept of NS-Theory and SS-Theory, is studied in [28] to make the NS-Theory adequate with parameterized tool. In many real life situations, distinct attributes are further partitioned in disjoint attribute-valued sets but existing SS-Theory is insufficient for dealing with such kind of attribute-valued sets. Hypersoft set theory (HS-Theory) [29] is developed to make the SST in line with attribute-valued sets to tackle real life scenarios. HS-Theory is an extension of SS-Theory as it transforms the single argument function into a multi-argument function. Certain elementary properties, aggregation operations, laws, relations and functions of HS-Theory, are investigated by [30]- [32] for proper understanding and further utilization in different fields. The applications of HS-Theory in decision making is studied by [33]- [37] and the intermingling study of HS-Theory with complex sets, convex and concave sets is studied by [38,39]. Deli [40] characterized hybrid set structures under uncertainly parameterized hypersoft sets with theory and applications. Gayen et al. [41] analyzed some essential aspects of plithogenic hypersoft algebraic structures. They also investigated the notions and basic properties of plithogenic hypersoft subgroups ie plithogenic fuzzy hypersoft subgroup, plithogenic intuitionistic fuzzy hypersoft subgroup, plithogenic neutrosophic hypersoft subgroup. Saeed et al. [42,43] discussed decision making techniques for neutrosophic hypersoft mapping and complex multi-fuzzy hypersoft set. Rahman et al. [44–46] studied decision making applications based on neutrosophic parameterized hypersoft Set, fuzzy parameterized hypersoft set and rough hypersoft set. Ihsan et al. [47] investigated hypersoft expert set with application in decision making for the best

selection of product. The gluing concept of graph theory with uncertain environments like fuzzy, intuitionistic fuzzy, neutrosophic, fuzzy soft, intuitionistic fuzzy soft and neutrosophic soft set, is discussed and characterized by the authors [48]- [54]. Inspiring from above literature in general, and from [55], [56] in specific, new notions of neutrosophic hypersoft graph are conceptualized along with some elementary types, essential properties, aggregation operations and generalized typical results. The rest of the paper is organized as:
In section 2, some basic definitions and terminologies are presented. In section 3, the elementary notions of neutrosophic hypersoft graphs are discussed with properties and results. In section 4, some set theoretic operations of neutrosophic hypersoft graphs are presented with examples. In section 5, paper is summarized with future directions.

2. Preliminaries

Here some essential terms and definitions are recalled from existing literature.

Definition 2.1. [3]
A neutrosophic set $\mathcal{K}$ defined as $\mathcal{K} = \{(k, <\mathcal{M}_K(k), \mathcal{I}_K(k), \mathcal{N}_K(k)>)|k \in \mathcal{Z}\}$ such that $\mathcal{M}_K(k) : \mathcal{Z} \rightarrow ^{-}]0, 1[^{+}$, $\mathcal{I}_K(k) : \mathcal{Z} \rightarrow ^{-}]0, 1[^{+}$ and $\mathcal{N}_K(k) : \mathcal{Z} \rightarrow ^{-}]0, 1[^{+}$ where $\mathcal{M}_K(k)$ stands for membership, $\mathcal{N}_K(k)$ stands for non-membership and $\mathcal{I}_K(k)$ stands for indeterminacy under condition $^{-}0 \leq \mathcal{M}_K(k) + \mathcal{I}_K(k) + \mathcal{N}_K(k) \leq 3^{+}$.

Definition 2.2. [17]
A pair $(\Psi_M, \mathcal{W})$ is said to be soft set over $\mathcal{Z}$ (universe of discourse), where $\Psi_M : \mathcal{W} \rightarrow \mathcal{P}(\mathcal{Z})$ and $\mathcal{W}$ is a subset of set of attributes $\mathcal{X}$.

For more detail on soft set, see [18, 19].

Definition 2.3. [29]
A pair $(\xi_H, \mathcal{R})$ is said to be hypersoft set over $\mathcal{Z}$, where $\xi_H : \mathcal{R} \rightarrow \mathcal{P}(\mathcal{Z})$ and $\mathcal{R} = \mathcal{R}_1 \times \mathcal{R}_2 \times \mathcal{R}_3 \times ... \times \mathcal{R}_n$, $\mathcal{R}_i$ are disjoint attribute-valued sets corresponding to distinct attributes r_i respectively for $1 \leq i \leq n$.

Definition 2.4. [29]
A pair $(\zeta_N, \mathcal{U})$ is said to be neutrosophic hypersoft set over $\mathcal{Z}$ if $\zeta_N : \mathcal{U} \rightarrow \mathcal{P}(\mathcal{Z})$, where $\mathcal{P}(\mathcal{Z})$ is a collection of all neutrosophic subsets and $\mathcal{U} = \mathcal{U}_1 \times \mathcal{U}_2 \times \mathcal{U}_3 \times ... \times \mathcal{U}_n$, $\mathcal{U}_i$ are disjoint attribute-valued sets corresponding to distinct attributes u_i respectively for $1 \leq i \leq n$.

For more definitions and operations of hypersoft set, see [30–32].

Definition 2.5. [56]
Let $\mathcal{Q}$ and $\mathfrak{G}^{\star} = (\mathcal{V}, \mathcal{E})$ be a set of parameters and a simple graph respectively with $\mathcal{V}$ as set

of vertices and $\mathcal{E}$ as set of edges. Let $\mathcal{N}(\mathcal{V})$ be the set of all neutrosophic set in $\mathcal{V}$. By a neutrosophic soft graph (NS-Graph), we mean a 4-tuple $\mathfrak{G} = (\mathfrak{G}^\star, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ where $\mathbb{F} : \mathcal{Q} \to \mathcal{N}(\mathcal{V}), \mathbb{G} : \mathcal{Q} \to \mathcal{N}(\mathcal{V} \times \mathcal{V})$ given by

$$\mathbb{F}(\theta) = \mathbb{F}_\theta = \{\langle \nu, \mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}_\theta}(\nu) \rangle, \nu \in \mathcal{V}\}$$

and

$$\mathbb{G}(\theta) = \mathbb{G}_\theta = \{\langle (\nu, \mu), \mathcal{T}_{\mathbb{F}_\theta}(\nu, \mu), \mathcal{I}_{\mathbb{F}_\theta}(\nu, \mu), \mathcal{F}_{\mathbb{F}_\theta}(\nu, \mu) \rangle, (\nu, \mu) \in \mathcal{V} \times \mathcal{V}\}$$

are neutrosophic sets over $\mathcal{V}$ and $\mathcal{V} \times \mathcal{V}$ respectively with

$$\mathcal{T}_{\mathbb{F}_\theta}(\nu, \mu) \leq \min \{\mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{T}_{\mathbb{F}_\theta}(\mu)\}$$

$$\mathcal{I}_{\mathbb{F}_\theta}(\nu, \mu) \leq \min \{\mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}_\theta}(\mu)\}$$

$$\mathcal{F}_{\mathbb{F}_\theta}(\nu, \mu) \geq \max \{\mathcal{F}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}_\theta}(\mu)\}$$

for all $(\nu, \mu) \in (\mathcal{V} \times \mathcal{V})$ and $\theta \in \mathcal{Q}$.

3. Neutrosophic Hypersoft Graphs

In this section, notions of neutrosophic hypersoft graph are characterized with some properties and examples.

Definition 3.1. Let $\mathfrak{G}^\star = (\mathcal{V}, \mathcal{E})$ be a simple graph with $\mathcal{V}$ as set of vertices and $\mathcal{E}$ as set of edges and $\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3, ..., \mathcal{Q}_n$ are disjoint attribute-valued sets corresponding to distinct attributes $\alpha_1, \alpha_2, \alpha_3, ..., \alpha_n$ with $\mathcal{Q} = \mathcal{Q}_1 \times \mathcal{Q}_2 \times \mathcal{Q}_3 \times ... \times \mathcal{Q}_n$. Let $\mathcal{N}(\mathcal{V})$ be the set of all neutrosophic set in $\mathcal{V}$. By a neutrosophic hypersoft graph (NHS-Graph), we mean a 4-tuple $\mathfrak{G} = (\mathfrak{G}^\star, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ where $\mathbb{F} : \mathcal{Q} \to \mathcal{N}(\mathcal{V}), \mathbb{G} : \mathcal{Q} \to \mathcal{N}(\mathcal{V} \times \mathcal{V})$ given by

$$\mathbb{F}(\theta) = \mathbb{F}_\theta = \{\langle \nu, \mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}_\theta}(\nu) \rangle, \nu \in \mathcal{V}\}$$

and

$$\mathbb{G}(\theta) = \mathbb{G}_\theta = \{\langle (\nu, \mu), \mathcal{T}_{\mathbb{F}_\theta}(\nu, \mu), \mathcal{I}_{\mathbb{F}_\theta}(\nu, \mu), \mathcal{F}_{\mathbb{F}_\theta}(\nu, \mu) \rangle, (\nu, \mu) \in \mathcal{V} \times \mathcal{V}\}$$

are neutrosophic sets over $\mathcal{V}$ and $\mathcal{V} \times \mathcal{V}$ with

$$\mathcal{T}_{\mathbb{F}_\theta}(\nu, \mu) \leq \min \{\mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{T}_{\mathbb{F}_\theta}(\mu)\}$$

$$\mathcal{I}_{\mathbb{F}_\theta}(\nu, \mu) \geq \min \{\mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}_\theta}(\mu)\}$$

$$\mathcal{F}_{\mathbb{F}_\theta}(\nu, \mu) \geq \max \{\mathcal{F}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}_\theta}(\mu)\}$$

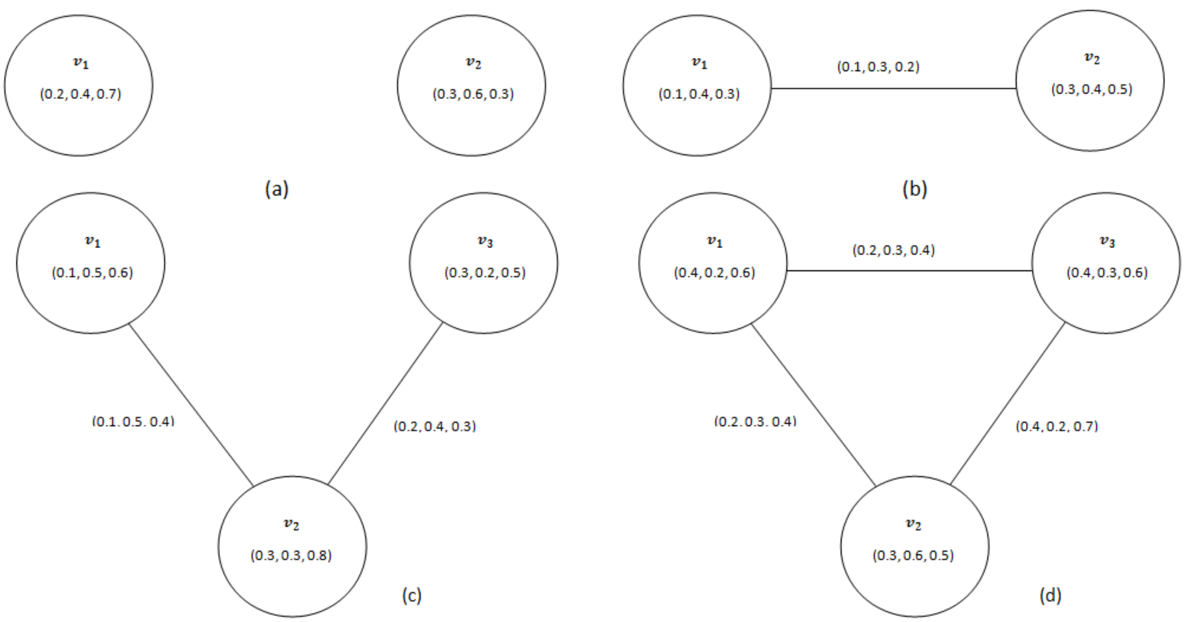
for all $(\nu, \mu) \in (\mathcal{V} \times \mathcal{V})$ and $\theta \in \mathcal{Q}$.

Note: The collection of all neutrosophic hypersoft graphs is denoted by Ω_{NHSG}.

TABLE 1. Tabular Representation of NHS-Graph $\mathfrak{G} = (\mathfrak{G}^{\star}, \mathcal{Q}, \mathbb{F}, \mathbb{G})$

$\mathbb{F}$	ν_1	ν_2	ν_3
θ_1	$(0.2, 0.4, 0.7)$	$(0.3, 0.6, 0.3)$	$(0, 0, 1)$
θ_2	$(0.1, 0.4, 0.3)$	$(0.3, 0.4, 0.5)$	$(0, 0, 1)$
θ_3	$(0.1, 0.5, 0.6)$	$(0.3, 0.3, 0.8)$	$(0.3, 0.2, 0.5)$
θ_4	$(0.4, 0.2, 0.6)$	$(0.3, 0.6, 0.5)$	$(0.4, 0.3, 0.6)$
$\mathbb{G}$	(ν_1, ν_2)	(ν_2, ν_3)	(ν_1, ν_3)
θ_1	$(0, 0, 1)$	$(0, 0, 1)$	$(0, 0, 1)$
θ_2	$(0.1, 0.3, 0.2)$	$(0, 0, 1)$	$(0, 0, 1)$
θ_3	$(0.1, 0.5, 0.4)$	$(0.2, 0.4, 0.3)$	$(0, 0, 1)$
θ_4	$(0.2, 0.3, 0.4)$	$(0.2, 0.5, 0.3)$	$(0.4, 0.2, 0.7)$

FIGURE 1. Graphical Representation of TABLE 1 with (a) $\mathcal{N}(\theta_1)$, (b) $\mathcal{N}(\theta_2)$, (c) $\mathcal{N}(\theta_3)$ and (d) $\mathcal{N}(\theta_4)$



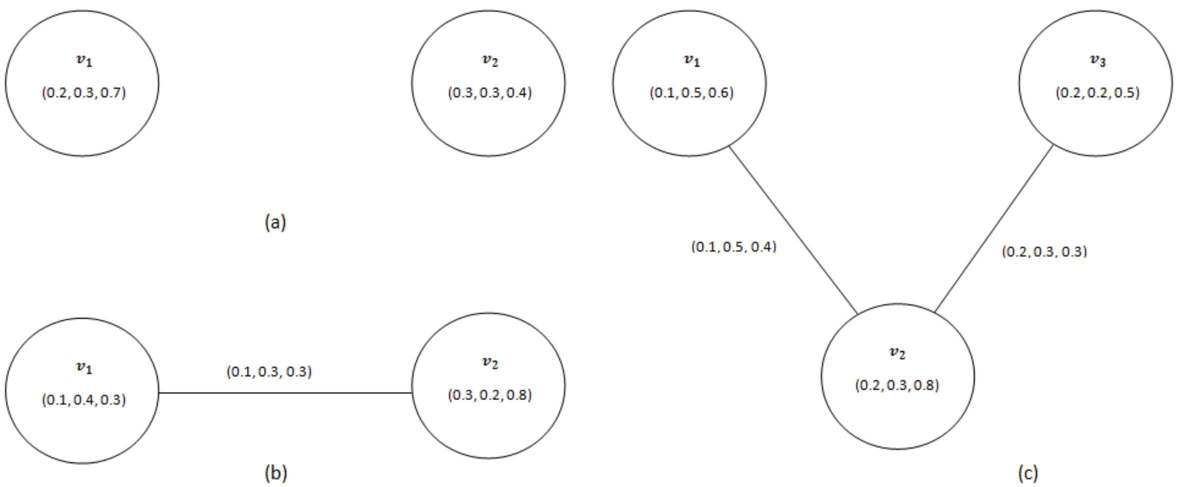
Example 3.2. Let $\mathfrak{G}^{\star} = (\mathcal{V}, \mathcal{E})$ be a simple graph with $\mathcal{V} = \{\nu_1, \nu_2, \nu_3\}$ and $\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3$ are disjoint attribute-valued sets corresponding to distinct attributes $\alpha_1, \alpha_2, \alpha_3$ where $\mathcal{Q}_1 = \{\alpha_{11}, \alpha_{12}\}$, $\mathcal{Q}_2 = \{\alpha_{21}, \alpha_{22}\}$ and $\mathcal{Q}_3 = \{\alpha_{31}\}$. $\mathcal{Q} = \mathcal{Q}_1 \times \mathcal{Q}_2 \times \mathcal{Q}_3 = \{\theta_1, \theta_2, \theta_3, \theta_4\}$ where each θ_i is a 3-tuple element of $\mathcal{Q}$ and $\mathcal{T}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{I}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{F}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 1$ for all $(\nu_i, \nu_j) \in \mathcal{V} \times \mathcal{V} \setminus \{(\nu_1, \nu_2), (\nu_2, \nu_3), (\nu_1, \nu_3)\}$. The tabular and graphical representation of NHS-Graph $\mathfrak{G} = (\mathfrak{G}^{\star}, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ are given in TABLE 1 and FIGURE 1 respectively.

Definition 3.3. A neutrosophic hypersoft graph $\mathfrak{G} = \left(\mathfrak{G}^{\star}, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ is called a neutrosophic hypersoft subgraph of $\mathfrak{G} = (\mathfrak{G}^{\star}, \mathcal{A}, \mathbb{F}, \mathbb{G})$ if

TABLE 2. Tabular Representation of NHS-subgraph $\mathfrak{G} = \left(\mathfrak{G}^\star, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$

$\mathbb{F}$	ν_1	ν_2	ν_3
θ_1	$(0.2, 0.3, 0.7)$	$(0.3, 0.3, 0.4)$	$(0, 0, 1)$
θ_2	$(0.1, 0.4, 0.3)$	$(0.3, 0.2, 0.8)$	$(0, 0, 1)$
θ_3	$(0.1, 0.5, 0.6)$	$(0.2, 0.3, 0.8)$	$(0.2, 0.2, 0.5)$
$\mathbb{G}$	(ν_1, ν_2)	(ν_2, ν_3)	(ν_1, ν_3)
θ_1	$(0, 0, 1)$	$(0, 0, 1)$	$(0, 0, 1)$
θ_2	$(0.1, 0.3, 0.3)$	$(0, 0, 1)$	$(0, 0, 1)$
θ_3	$(0.1, 0.5, 0.4)$	$(0.2, 0.3, 0.3)$	$(0, 0, 1)$

FIGURE 2. Graphical Representation of TABLE 2 with (a) $\mathcal{N}(\theta_1)$, (b) $\mathcal{N}(\theta_2)$ and (c) $\mathcal{N}(\theta_3)$



(1) $\mathcal{Q}^1 \subseteq \mathcal{Q}$

(2) $\mathbb{F}^1_\theta \subseteq f$ which means $\mathcal{T}_{\mathbb{F}^1_\theta}(\nu) \leq \mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}^1_\theta}(\nu) \leq \mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}^1_\theta}(\nu) \geq \mathcal{F}_{\mathbb{F}_\theta}(\nu)$.

(3) $\mathbb{G}^1_\theta \subseteq g$ which means $\mathcal{T}_{\mathbb{G}^1_\theta}(\nu) \leq \mathcal{T}_{\mathbb{G}_\theta}(\nu), \mathcal{I}_{\mathbb{G}^1_\theta}(\nu) \leq \mathcal{I}_{\mathbb{G}_\theta}(\nu), \mathcal{F}_{\mathbb{G}^1_\theta}(\nu) \geq \mathcal{F}_{\mathbb{G}_\theta}(\nu)$.

for all $\theta \in \mathcal{Q}^1$ and $\mathcal{Q}^1 = \mathcal{Q}_1 \times \mathcal{Q}_2 \times ... \times \mathcal{Q}_n$ where $\mathcal{Q}_1, \mathcal{Q}_2, ..., \mathcal{Q}_n$ are disjoint attribute-valued sets corresponding to distinct attributes $\alpha_1, \alpha_2, ..., \alpha_n$ respectively.

Example 3.4. Let $\mathfrak{G}^\star = (\mathcal{V}, E)$ be a simple graph with $\mathcal{V} = \{\nu_1, \nu_2, \nu_3\}$ and $\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3$ are disjoint attribute-valued sets corresponding to disjoint attributes $\alpha_1, \alpha_2, \alpha_3$ where $\mathcal{Q}_1 = \{\alpha_{11}, \alpha_{12}\}$, $\mathcal{Q}_2 = \{\alpha_{21}\}$ and $\mathcal{Q}_3 = \{\alpha_{31}\}$. $\mathcal{Q} = \mathcal{Q}_1 \times \mathcal{Q}_2 \times \mathcal{Q}_3 = \{\theta_1, \theta_2, \theta_3\}$ where each θ_i is a 3-tuple element of $\mathcal{Q}$. The tabular and graphical representation of NHS-subgraph $\mathfrak{G} = \left(\mathfrak{G}^\star, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ are given in TABLE 2 and FIGURE 2 respectively. In this graph, $\mathcal{T}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{I}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{F}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 1$ for all $(\nu_i, \nu_j) \in \mathcal{V} \times \mathcal{V} \setminus \{(\nu_1, \nu_2), (\nu_2, \nu_3), (\nu_1, \nu_3)\}$ and for all $\theta \in \mathcal{Q}$.

Definition 3.5. A neutrosophic hypersoft subgraph $\mathfrak{G} = \left(\mathfrak{G}^{\star}, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ is called a neutrosophic hypersoft spanning subgraph of $\mathfrak{G} = (\mathfrak{G}^{\star}, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ if $\mathbb{F}^1_{\theta}(\nu) = \mathbb{F}(\nu)$ for all $\nu \in \mathcal{V}, e \in \mathcal{Q}$ where $\mathcal{Q}^1 = \mathcal{Q}_1 \times \mathcal{Q}_2 \times ... \times \mathcal{Q}_n$ and $\mathcal{Q}_1, \mathcal{Q}_2, ..., \mathcal{Q}_n$ are disjoint attribute-valued sets corresponding to disjoint attributes $\alpha_1, \alpha_2, ..., \alpha_n$ respectively.

Definition 3.6. A strong neutrosophic hypersoft subgraph $\mathfrak{G} = (\mathfrak{G}^{\star}, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ is a neutrosophic hypersoft subgraph with condition that $\mathbb{G}_{\theta}(\nu, \mu) = \mathbb{F}_{\theta}(\nu) \cap \mathbb{F}_{\theta}(\mu)$ for $x, y \in \mathcal{V}$ and $e \in \mathcal{Q}$ such that $\mathcal{Q} = \mathcal{Q}_1 \times \mathcal{Q}_2 ... \times \mathcal{Q}_n$ and $\mathcal{Q}_1, \mathcal{Q}_2, ..., \mathcal{Q}_n$ are disjoint attribute-valued sets corresponding to disjoint attributes $\alpha_1, \alpha_2, ..., \alpha_n$ respectively.

4. **Set Theoretic Operations of NHS-Graphs**

In this section, some theoretic operations (i.e. union, intersection and complement) of neutrosophic hypersoft graph (NHS-Graphs) are investigated with suitable examples and results.

Definition 4.1. The union of two NHS-Graphs $\mathfrak{G}_1 = \left(\mathfrak{G}^{\star}_1, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$, $\mathfrak{G}_2 = \left(\mathfrak{G}^{\star}_2, \mathcal{Q}^2, \mathbb{F}^2, \mathbb{G}^2\right)$, denoted by $\mathfrak{G}_1 \cup \mathfrak{G}_2$, is a NHS-Graph $\mathfrak{G} = (\mathfrak{G}^{\star}, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ such that $\mathcal{Q} = \mathcal{Q}^1 \cup \mathcal{Q}^2$. In this graph, the neutrosophic components for $\mathbb{F}$ are given as follows:

$$\mathcal{T}_{\mathbb{F}_{\theta}}(\nu) = \begin{cases} \mathcal{T}_{\mathbb{F}^1_{\theta}}(\nu) \begin{cases} if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_1 - \mathcal{V}_2\, or \\ if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_1 \cap \mathcal{V}_2\, or \\ if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_1 - \mathcal{V}_2 \end{cases} \\ \mathcal{T}_{\mathbb{F}^2_{\theta}}(\nu) \begin{cases} if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1\, and\, \nu \in \mathcal{V}_2 - \mathcal{V}_1\, or \\ if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1\, and\, \nu \in \mathcal{V}_2 \cap \mathcal{V}_1\, or \\ if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_2 - \mathcal{V}_1 \end{cases} \\ \max\left\{\mathcal{T}_{\mathbb{F}^1_{\theta}}(\nu), \mathcal{T}_{\mathbb{F}^2_{\theta}}(\nu)\right\} \left\{if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_1 \cap \mathcal{V}_2\right. \\ 0,\, otherwise \end{cases}$$

and

$$\mathcal{I}_{\mathbb{F}_{\theta}}(\nu) = \begin{cases} \mathcal{I}_{\mathbb{F}^1_{\theta}}(\nu) \begin{cases} if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_1 - \mathcal{V}_2\, or \\ if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_1 \cap \mathcal{V}_2\, or \\ if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_1 - \mathcal{V}_2 \end{cases} \\ \mathcal{I}_{\mathbb{F}^2_{\theta}}(\nu) \begin{cases} if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1\, and\, \nu \in \mathcal{V}_2 - \mathcal{V}_1\, or \\ if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1\, and\, \nu \in \mathcal{V}_2 \cap \mathcal{V}_1\, or \\ if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_2 - \mathcal{V}_1 \end{cases} \\ \max\left\{\mathcal{I}_{\mathbb{F}^1_{\theta}}(\nu), \mathcal{I}_{\mathbb{F}^2_{\theta}}(\nu)\right\} \left\{if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_1 \cap \mathcal{V}_2\right. \\ 0,\, otherwise \end{cases}$$

and

$$\mathcal{F}_{\mathbb{F}_\theta}(\nu) = \begin{cases} \mathcal{F}_{\mathbb{F}^1_\theta}(\nu) \begin{cases} if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_1 - \mathcal{V}_2\, or \\ if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_1 \cap \mathcal{V}_2\, or \\ if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_1 - \mathcal{V}_2 \end{cases} \\ \mathcal{F}_{\mathbb{F}^2_\theta}(\nu) \begin{cases} if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1\, and\, \nu \in \mathcal{V}_2 - \mathcal{V}_1\, or \\ if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1\, and\, \nu \in \mathcal{V}_2 \cap \mathcal{V}_1\, or \\ if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_2 - \mathcal{V}_1 \end{cases} \\ \min\left\{\mathcal{F}_{\mathbb{F}^1_\theta}(\nu), \mathcal{F}_{\mathbb{F}^2_\theta}(\nu)\right\} \left\{ if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, \nu \in \mathcal{V}_1 \cap \mathcal{V}_2 \right. \\ 0,\ otherwise \end{cases}.$$

Also the neutrosophic components for $\mathbb{G}$ are given as follows:

$$\mathcal{T}_{\mathbb{G}_\theta}(\nu) = \begin{cases} \mathcal{T}_{\mathbb{G}^1_\theta}(\nu) \begin{cases} if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2\, and\, (\nu,\mu) \in (\mathcal{V}_1 \times \mathcal{V}_1) - (\mathcal{V}_2 \times \mathcal{V}_2)\ or \\ if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2\, and\, (\nu,\mu) \in (\mathcal{V}_1 \times \mathcal{V}_1) \cap (\mathcal{V}_2 \times \mathcal{V}_2)\ or \\ if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, (\nu,\mu) \in (\mathcal{V}_1 \times \mathcal{V}_1) - (\mathcal{V}_2 \times \mathcal{V}_2) \end{cases} \\ \mathcal{T}_{\mathbb{G}^2_\theta}(\nu) \begin{cases} if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1\, and\, (\nu,\mu) \in (\mathcal{V}_2 \times \mathcal{V}_2) - (\mathcal{V}_1 \times \mathcal{V}_1)\ or \\ if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1\, and\, (\nu,\mu) \in (\mathcal{V}_2 \times \mathcal{V}_2) \cap (\mathcal{V}_1 \times \mathcal{V}_1)\ or \\ if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, (\nu,\mu) \in (\mathcal{V}_2 \times \mathcal{V}_2) - (\mathcal{V}_1 \times \mathcal{V}_1) \end{cases} \\ \max\left\{\mathcal{T}_{\mathbb{G}^1_\theta}(\nu), \mathcal{T}_{\mathbb{G}^2_\theta}(\nu)\right\} \left\{ if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, (\nu,\mu) \in (\mathcal{V}_1 \times \mathcal{V}_1) \cap (\mathcal{V}_2 \times \mathcal{V}_2) \right. \\ 0,\ otherwise \end{cases}$$

and

$$\mathcal{I}_{\mathbb{G}_\theta}(\nu) = \begin{cases} \mathcal{I}_{\mathbb{G}^1_\theta}(\nu) \begin{cases} if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2\, and\, (\nu,\mu) \in (\mathcal{V}_1 \times \mathcal{V}_1) - (\mathcal{V}_2 \times \mathcal{V}_2)\ or \\ if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2\, and\, (\nu,\mu) \in (\mathcal{V}_1 \times \mathcal{V}_1) \cap (\mathcal{V}_2 \times \mathcal{V}_2)\ or \\ if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, (\nu,\mu) \in (\mathcal{V}_1 \times \mathcal{V}_1) - (\mathcal{V}_2 \times \mathcal{V}_2) \end{cases} \\ \mathcal{I}_{\mathbb{G}^2_\theta}(\nu) \begin{cases} if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1\, and\, (\nu,\mu) \in (\mathcal{V}_2 \times \mathcal{V}_2) - (\mathcal{V}_1 \times \mathcal{V}_1)\ or \\ if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1\, and\, (\nu,\mu) \in (\mathcal{V}_2 \times \mathcal{V}_2) \cap (\mathcal{V}_1 \times \mathcal{V}_1)\ or \\ if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, (\nu,\mu) \in (\mathcal{V}_2 \times \mathcal{V}_2) - (\mathcal{V}_1 \times \mathcal{V}_1) \end{cases} \\ \max\left\{\mathcal{I}_{\mathbb{G}^1_\theta}(\nu), \mathcal{I}_{\mathbb{G}^2_\theta}(\nu)\right\} \left\{ if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, (\nu,\mu) \in (\mathcal{V}_1 \times \mathcal{V}_1) \cap (\mathcal{V}_2 \times \mathcal{V}_2) \right. \\ 0,\ otherwise \end{cases}$$

and

$$\mathcal{F}_{\mathbb{G}_\theta}(\nu) = \begin{cases} \mathcal{F}_{\mathbb{G}^1_\theta}(\nu) \begin{cases} if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2\, and\, (\nu,\mu) \in (\mathcal{V}_1 \times \mathcal{V}_1) - (\mathcal{V}_2 \times \mathcal{V}_2)\ or \\ if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2\, and\, (\nu,\mu) \in (\mathcal{V}_1 \times \mathcal{V}_1) \cap (\mathcal{V}_2 \times \mathcal{V}_2)\ or \\ if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, (\nu,\mu) \in (\mathcal{V}_1 \times \mathcal{V}_1) - (\mathcal{V}_2 \times \mathcal{V}_2) \end{cases} \\ \mathcal{F}_{\mathbb{G}^2_\theta}(\nu) \begin{cases} if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1\, and\, (\nu,\mu) \in (\mathcal{V}_2 \times \mathcal{V}_2) - (\mathcal{V}_1 \times \mathcal{V}_1)\ or \\ if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1\, and\, (\nu,\mu) \in (\mathcal{V}_2 \times \mathcal{V}_2) \cap (\mathcal{V}_1 \times \mathcal{V}_1)\ or \\ if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, (\nu,\mu) \in (\mathcal{V}_2 \times \mathcal{V}_2) - (\mathcal{V}_1 \times \mathcal{V}_1) \end{cases} \\ \min\left\{\mathcal{F}_{\mathbb{G}^1_\theta}(\nu), \mathcal{F}_{\mathbb{G}^2_\theta}(\nu)\right\} \left\{ if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2\, and\, (\nu,\mu) \in (\mathcal{V}_1 \times \mathcal{V}_1) \cap (\mathcal{V}_2 \times \mathcal{V}_2) \right. \\ 0,\ otherwise \end{cases}.$$

Theorem 4.2. *If* $\mathfrak{G}_1, \mathfrak{G}_2 \in \Omega_{NHSG}$ *then* $\mathfrak{G}_1 \cup \mathfrak{G}_2 \in \Omega_{NHSG}$.

Proof. Consider two NHS-Graphs $\mathfrak{G}_1 = \left(\mathfrak{G}_1^\star, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ and $\mathfrak{G}_2 = \left(\mathfrak{G}_2^\star, \mathcal{Q}^2, \mathbb{F}^2, \mathbb{G}^2\right)$ as defined in 3.1. Let $\mathfrak{G} = (\mathfrak{G}^\star, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ be the union of NHS-Graphs $\mathfrak{G}_1$ and $\mathfrak{G}_2$ where $\mathcal{Q} = \mathcal{Q}^1 \cup \mathcal{Q}^2$. Now let $\theta \in \mathcal{Q}^1 - \mathcal{Q}^2$ and $(\nu, \mu) \in (\mathcal{V}_1 \times \mathcal{V}_1) - (\mathcal{V}_2 \times \mathcal{V}_2)$, then

$$\begin{aligned} \mathcal{T}_{\mathbb{G}_\theta}(\nu, \mu) = \mathcal{T}_{\mathbb{G}^1_\theta}(\nu, \mu) &\leq \min\left\{\mathcal{T}_{\mathbb{F}^1_\theta}(\nu), \mathcal{T}_{\mathbb{F}^1_\theta}(\mu)\right\} \\ &= \min\left\{\mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{T}_{\mathbb{F}_\theta}(\mu)\right\} \end{aligned}$$

so

$$\mathcal{T}_{\mathbb{G}_\theta}(\nu, \mu) \leq \min\left\{\mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{T}_{\mathbb{F}_\theta}(\mu)\right\}.$$

Also

$$\begin{aligned} \mathcal{I}_{\mathbb{G}_\theta}(\nu, \mu) = \mathcal{I}_{\mathbb{G}^1_\theta}(\nu, \mu) &\leq \min\left\{\mathcal{I}_{\mathbb{F}^1_\theta}(\nu), \mathcal{I}_{\mathbb{F}^1_\theta}(\mu)\right\} \\ &= \min\left\{\mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}_\theta}(\mu)\right\} \end{aligned}$$

so

$$\mathcal{I}_{\mathbb{G}_\theta}(\nu, \mu) \leq \min\left\{\mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}_\theta}(\mu)\right\}.$$

Now

$$\begin{aligned} \mathcal{F}_{\mathbb{G}_\theta}(\nu, \mu) = \mathcal{F}_{\mathbb{G}^1_\theta}(\nu, \mu) &\geq \max\left\{\mathcal{F}_{\mathbb{F}^1_\theta}(\nu), \mathcal{F}_{\mathbb{F}^1_\theta}(\mu)\right\} \\ &= \max\left\{\mathcal{F}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}_\theta}(\mu)\right\} \end{aligned}$$

so

$$\mathcal{F}_{\mathbb{G}_\theta}(\nu, \mu) \geq \max\left\{\mathcal{F}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}_\theta}(\mu)\right\}.$$

Similar results are obtained when $\theta \in \mathcal{Q}^1 - \mathcal{Q}^2$ and $(\nu, \mu) \in (\mathcal{V}_1 \times \mathcal{V}_1) \cap (\mathcal{V}_2 \times \mathcal{V}_2)$ or $\theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2$ and $(\nu, \mu) \in (\mathcal{V}_1 \times \mathcal{V}_1) - (\mathcal{V}_2 \times \mathcal{V}_2)$.
Now if $\theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2$ and $(\nu, \mu) \in (\mathcal{V}_1 \times \mathcal{V}_1) \cap (\mathcal{V}_2 \times \mathcal{V}_2)$ then

$$\begin{aligned} \mathcal{T}_{\mathbb{G}_\theta}(\nu, \mu) &= \max\left\{\mathcal{T}_{\mathbb{G}^1_\theta}(\nu, \mu), \mathcal{T}_{\mathbb{G}^2_\theta}(\nu, \mu)\right\} \\ &\leq \max\left\{\min\left\{\mathcal{T}_{\mathbb{F}^1_\theta}(\nu), \mathcal{T}_{\mathbb{F}^1_\theta}(\mu)\right\}, \min\left\{\mathcal{T}_{\mathbb{F}^2_\theta}(\nu), \mathcal{T}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\} \\ &\leq \min\left\{\max\left\{\mathcal{T}_{\mathbb{F}^1_\theta}(\nu), \mathcal{T}_{\mathbb{F}^1_\theta}(\mu)\right\}, \max\left\{\mathcal{T}_{\mathbb{F}^2_\theta}(\nu), \mathcal{T}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\} \\ &= \min\left\{\mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{T}_{\mathbb{F}_\theta}(\mu)\right\}. \end{aligned}$$

Also

$$\begin{aligned} \mathcal{I}_{\mathbb{G}_\theta}(\nu, \mu) &= \max\left\{\mathcal{I}_{\mathbb{G}^1_\theta}(\nu, \mu), \mathcal{I}_{\mathbb{G}^2_\theta}(\nu, \mu)\right\} \\ &\leq \max\left\{\min\left\{\mathcal{I}_{\mathbb{F}^1_\theta}(\nu), \mathcal{I}_{\mathbb{F}^1_\theta}(\mu)\right\}, \min\left\{\mathcal{I}_{\mathbb{F}^2_\theta}(\nu), \mathcal{I}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\} \\ &\leq \min\left\{\max\left\{\mathcal{I}_{\mathbb{F}^1_\theta}(\nu), \mathcal{I}_{\mathbb{F}^1_\theta}(\mu)\right\}, \max\left\{\mathcal{I}_{\mathbb{F}^2_\theta}(\nu), \mathcal{I}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\} \\ &= \min\left\{\mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}_\theta}(\mu)\right\}. \end{aligned}$$

In the same way

$$\mathcal{F}_{\mathbb{G}_\theta}(\nu, \mu) = \min\left\{\mathcal{F}_{\mathbb{G}^1_\theta}(\nu, \mu), \mathcal{F}_{\mathbb{G}^2_\theta}(\nu, \mu)\right\}$$

TABLE 3. Tabular Representation of NHS-Graph $\mathfrak{G}_1 = \left(\mathfrak{G}_1^{\star}, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ according to Example 4.3

$\mathbb{F}$	ν_1	ν_2	ν_3
θ_1	$(0.2, 0.3, 0.4)$	$(0.3, 0.6, 0.8)$	$(0.3, 0.4, 0.5)$
θ_2	$(0.2, 0.4, 0.8)$	$(0.2, 0.3, 0.4)$	$(0.5, 0.7, 0.8)$
θ_3	$(0.6, 0.7, 0.8)$	$(0.4, 0.5, 0.7)$	$(0.7, 0.9, 0.9)$
$\mathbb{G}$	(ν_1, ν_2)	(ν_2, ν_3)	(ν_1, ν_3)
θ_1	$(0.2, 0.3, 0.6)$	$(0.2, 0.4, 0.9)$	$(0.2, 0.3, 0.8)$
θ_2	$(0.2, 0.3, 0.9)$	$(0.2, 0.2, 0.9)$	$(0.2, 0.3, 0.8)$
θ_3	$(0, 0, 1)$	$(0.3, 0.4, 0.9)$	$(0.2, 0.4, 0.9)$

$$\geq \min\left\{\max\left\{\mathcal{F}_{\mathbb{F}^1_\theta}(\nu), \mathcal{F}_{\mathbb{F}^1_\theta}(\mu)\right\}, \max\left\{\mathcal{F}_{\mathbb{F}^2_\theta}(\nu), \mathcal{F}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\}$$

$$\geq \max\left\{\min\left\{\mathcal{F}_{\mathbb{F}^1_\theta}(\nu), \mathcal{F}_{\mathbb{F}^1_\theta}(\mu)\right\}, \min\left\{\mathcal{F}_{\mathbb{F}^2_\theta}(\nu), \mathcal{F}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\}$$

$$= \max\left\{\mathcal{F}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}_\theta}(\mu)\right\}.$$

Hence the union $\mathfrak{G} = \mathfrak{G}_1 \cup \mathfrak{G}_2$ is NHS-Graphs. □

Example 4.3. Let $\mathfrak{G}_1 = \left(\mathfrak{G}_1^{\star}, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ be a neutrosophic hypersoft graph where $\mathfrak{G}_1^{\star} = (\mathcal{V}_1, \mathcal{E}_1)$ with $\mathcal{V}_1 = \{\nu_1, \nu_2, \nu_3\}$ and $\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3$ are disjoint attribute-valued sets corresponding to distinct attributes $\alpha_1, \alpha_2, \alpha_3$ where $\mathcal{Q}_1 = \{\alpha_{11}\}$, $\mathcal{Q}_2 = \{\alpha_{21}\}$ and $\mathcal{Q}_3 = \{\alpha_{31}, \alpha_{32}, \alpha_{33}\}$. $\mathcal{Q}^1 = \mathcal{Q}_1 \times \mathcal{Q}_2 \times \mathcal{Q}_3 = \{\theta_1, \theta_2, \theta_3\}$ where each θ_i is a 3-tuple element of $\mathcal{Q}^1$ and $\mathcal{T}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{I}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{F}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 1$ for all $(\nu_i, \nu_j) \in \mathcal{V}_1 \times \mathcal{V}_1 \backslash \{(\nu_1, \nu_2), (\nu_2, \nu_3), (\nu_1, \nu_3)\}$. Its tabular representation is given in TABLE 3. Also let $\mathfrak{G}_2 = \left(\mathfrak{G}_2^{\star}, \mathcal{Q}^2, \mathbb{F}^2, \mathbb{G}^2\right)$ be a neutrosophic hypersoft graph where $\mathfrak{G}_2^{\star} = (\mathcal{V}_2, \mathcal{E}_2)$ with $\mathcal{V}_2 = \{\nu_3, \nu_4, \nu_5\}$ and $\mathcal{Q}_3, \mathcal{Q}_4, \mathcal{Q}_5$ are disjoint attribute-valued sets corresponding to distinct attributes $\alpha_3, \alpha_4, \alpha_5$ where $\mathcal{Q}_3 = \{\alpha_{31}, \alpha_{32}\}$, $\mathcal{Q}_4 = \{\alpha_{41}\}$, $\mathcal{Q}_5 = \{\alpha_{51}\}$. $\mathcal{Q}^2 = \mathcal{Q}_3 \times \mathcal{Q}_4 \times \mathcal{Q}_5 = \{\theta_2, \theta_4\}$ where each θ_i is a 3-tuple element of $\mathcal{Q}^2$ and $\mathcal{T}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{I}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{F}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 1$ for all $(\nu_i, \nu_j) \in \mathcal{V}_2 \times \mathcal{V}_2 \backslash \{(\nu_3, \nu_4), (\nu_4, \nu_5), (\nu_3, \nu_5)\}$. Its tabular representation is given in TABLE 4.
Now Let $\mathfrak{G} = (\mathfrak{G}^{\star}, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ be the union of two neutrosophic hypersoft graphs $\mathfrak{G}_1 = \left(\mathfrak{G}_1^{\star}, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ and $\mathfrak{G}_2 = \left(\mathfrak{G}_2^{\star}, \mathcal{Q}^2, \mathbb{F}^2, \mathbb{G}^2\right)$ where $\mathcal{Q} = \mathcal{Q}^1 \cup \mathcal{Q}^2$ and $\mathcal{T}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{I}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{F}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 1$ for all $(\nu_i, \nu_j) \in \mathcal{V} \times \mathcal{V} \backslash \{(\nu_1, \nu_2), (\nu_1, \nu_3), (\nu_2, \nu_3), (\nu_3, \nu_4), (\nu_3, \nu_5), (\nu_4, \nu_5)\}$. Its (union of these two graphs) tabular representation is given in TABLE 5.

Definition 4.4. The intersection of two NHS-Graphs $\mathfrak{G}_1 = \left(\mathfrak{G}_1^{\star}, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$, $\mathfrak{G}_2 = \left(\mathfrak{G}_2^{\star}, \mathcal{Q}^2, \mathbb{F}^2, \mathbb{G}^2\right)$, denoted by $\mathfrak{G}_1 \cap \mathfrak{G}_2$, is a NHS-Graph $\mathfrak{G} = (\mathfrak{G}^{\star}, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ such that $\mathcal{Q} =$

FIGURE 3. Graphical Representation of TABLE 3 with (a) $\mathcal{N}(\theta_1)$, (b) $\mathcal{N}(\theta_2)$ and (c) $\mathcal{N}(\theta_3)$

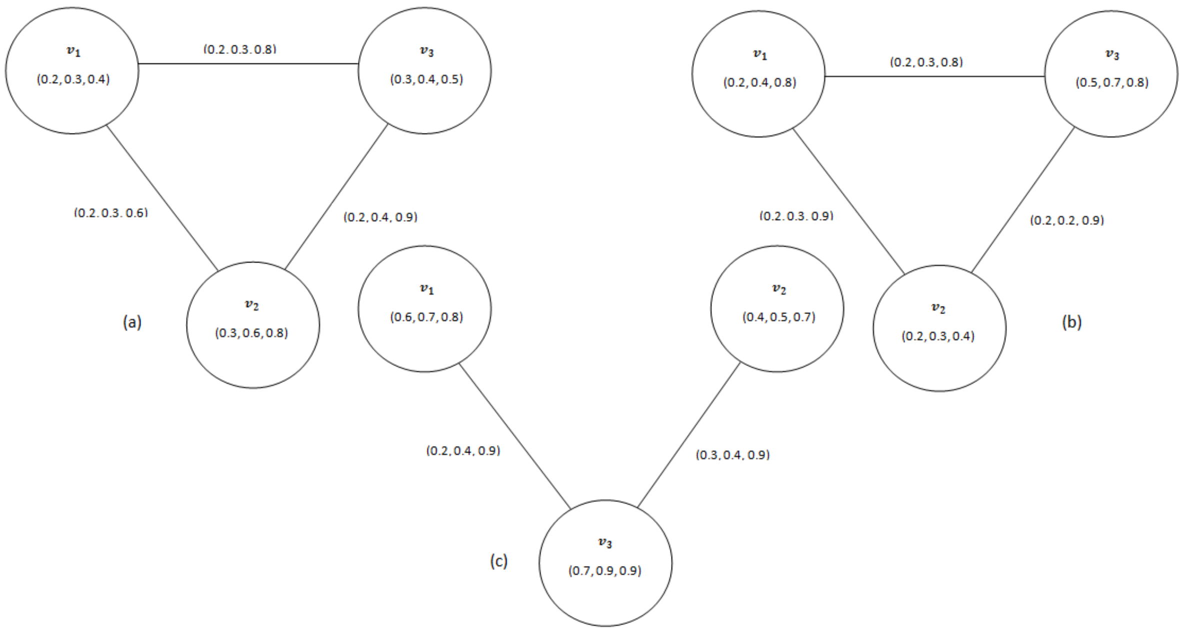


TABLE 4. Tabular Representation of NHS-Graph $\mathfrak{G}_2 = \left(\mathfrak{G}_2^\star, \mathcal{Q}^2, \mathbb{F}^2, \mathbb{G}^2\right)$ according to Example 4.3

$\mathbb{F}$	ν_3	ν_4	ν_5
θ_2	$(0.3, 0.4, 0.5)$	$(0.2, 0.3, 0.5)$	$(0.5, 0.7, 0.8)$
θ_4	$(0.6, 0.8, 0.9)$	$(0.4, 0.7, 0.9)$	$(0.4, 0.5, 0.6)$
$\mathbb{G}$	(ν_3, ν_4)	(ν_4, ν_5)	(ν_3, ν_5)
θ_2	$(0.2, 0.3, 0.9)$	$(0.3, 0.4, 0.9)$	$(0, 0, 1)$
θ_4	$(0.2, 0.2, 0.9)$	$(0.3, 0.3, 0.9)$	$(0.3, 0.4, 0.9)$

TABLE 5. Tabular Representation of $\mathfrak{G} = \mathfrak{G}_1 \cup \mathfrak{G}_2$

$\mathbb{F}$	ν_1	ν_2	ν_3	ν_4	ν_5	
θ_1	$(0.2, 0.3, 0.4)$	$(0.3, 0.4, 0.5)$	$(0.3, 0.6, 0.8)$	$(0, 0, 1)$	$(0, 0, 1)$	
θ_2	$(0.2, 0.4, 0.8)$	$(0.2, 0.3, 0.4)$	$(0.3, 0.5, 0.5)$	$(0.2, 0.3, 0.4)$	$(0.5, 0.7, 0.8)$	
θ_3	$(0.6, 0.7, 0.8)$	$(0.4, 0.5, 0.7)$	$(0.7, 0.9, 0.9)$	$(0, 0, 1)$	$(0, 0, 1)$	
θ_4	$(0, 0, 1)$	$(0, 0, 1)$	$(0.6, 0.8, 0.9)$	$(0.4, 0.7, 0.9)$	$(0.4, 0.5, 0.6)$	
$\mathbb{G}$	(ν_1, ν_2)	(ν_1, ν_3)	(ν_2, ν_3)	(ν_3, ν_4)	(ν_3, ν_5)	(ν_4, ν_5)
θ_1	$(0.2, 0.3, 0.8)$	$(0.2, 0.3, 0.9)$	$(0.2, 0.4, 0.9)$	$(0, 0, 1)$	$(0, 0, 1)$	$(0, 0, 1)$
θ_2	$(0.2, 0.3, 0.8)$	$(0.2, 0.3, 0.9)$	$(0.2, 0.2, 0.9)$	$(0.2, 0.3, 0.9)$	$(0.3, 0.4, 0.9)$	$(0, 0, 1)$
θ_3	$(0.2, 0.4, 0.9)$	$(0, 0, 1)$	$(0.3, 0.4, 0.9)$	$(0, 0, 1)$	$(0, 0, 1)$	$(0, 0, 1)$
θ_4	$(0, 0, 1)$	$(0, 0, 1)$	$(0, 0, 1)$	$(0.2, 0.2, 0.9)$	$(0.3, 0.3, 0.9)$	$(0.3, 0.4, 0.9)$

FIGURE 4. Graphical Representation of TABLE 4 with (a) $\mathcal{N}(\theta_2)$ and (b) $\mathcal{N}(\theta_4)$

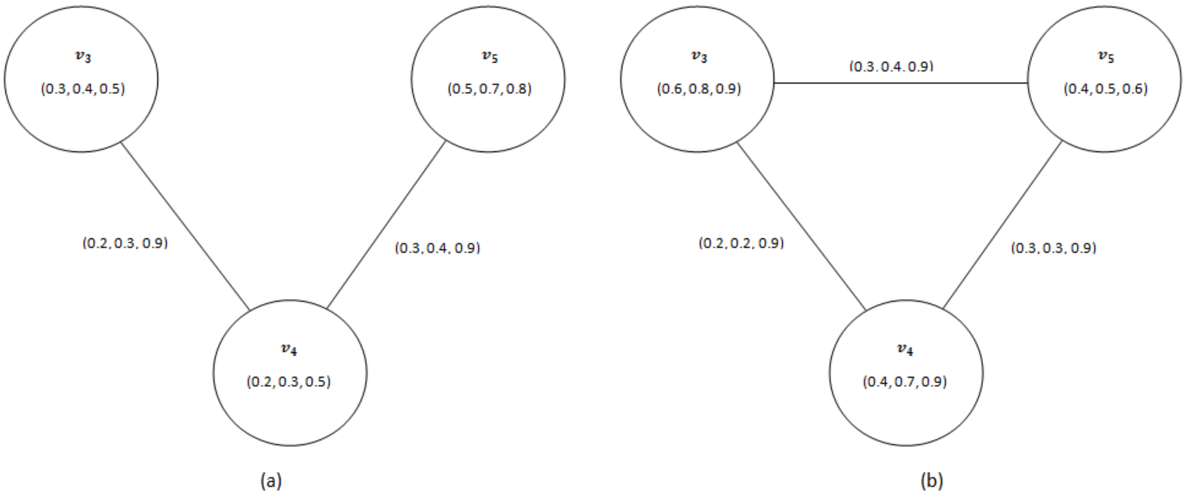
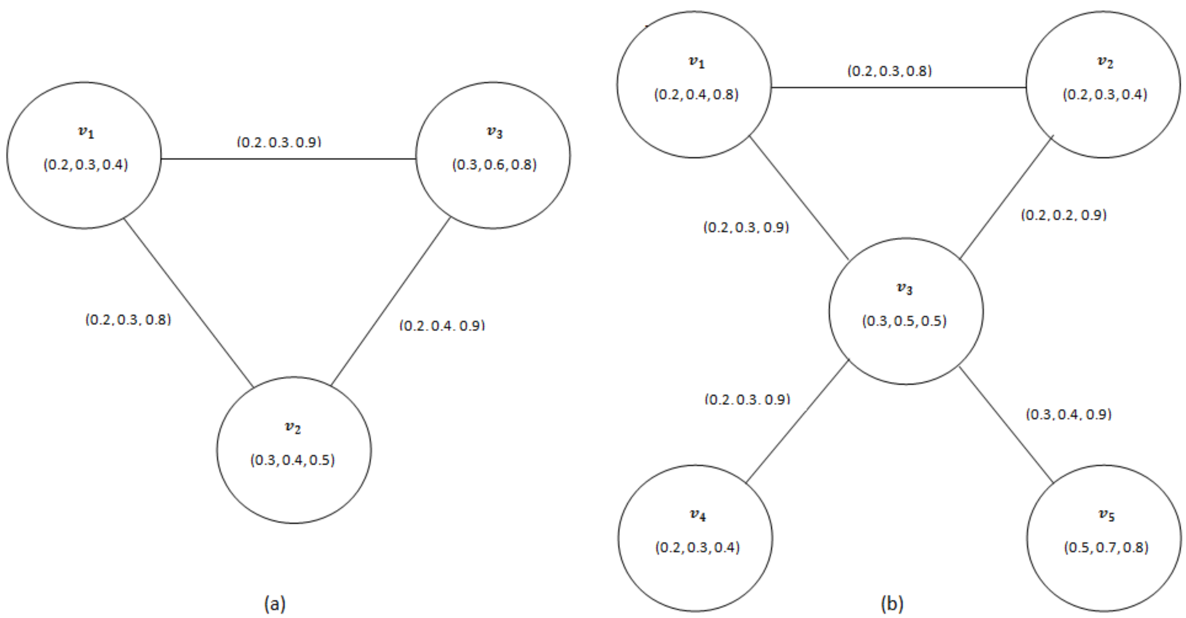


FIGURE 5. Graphical Representation of TABLE 5 with (a) $\mathcal{N}(\theta_1)$ and (b) $\mathcal{N}(\theta_2)$



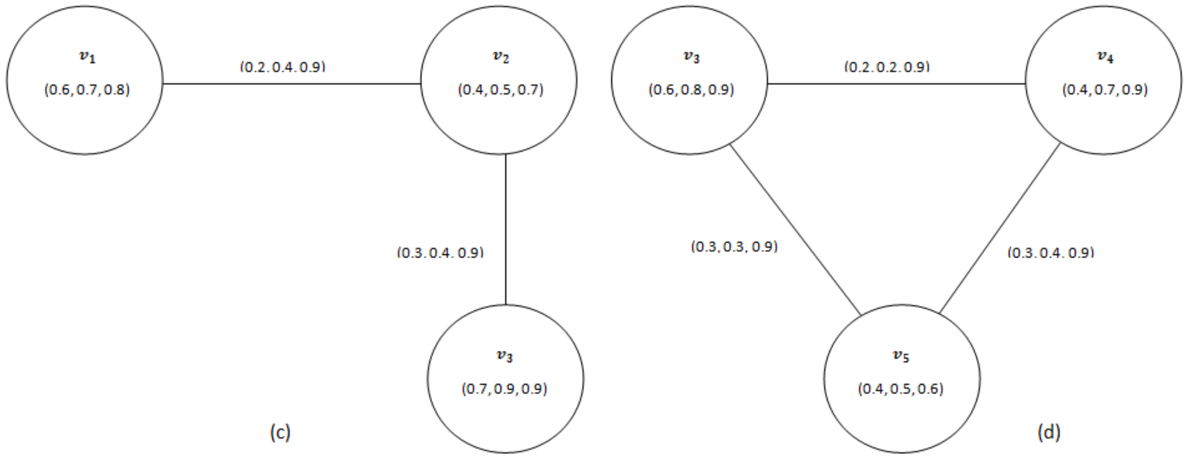
$\mathcal{Q}^1 \cap \mathcal{Q}^2, \mathcal{V} = \mathcal{V}_1 \cap \mathcal{V}_2$. In this graph, the neutrosophic components for $\mathbb{F}$ are given as follows:

$$\mathcal{T}_{\mathbb{F}_\theta} = \begin{cases} \mathcal{T}^1_{\mathbb{F}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2 \\ \mathcal{T}^2_{\mathbb{F}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1 \\ \min\left\{\mathcal{T}^1_{\mathbb{F}_\theta}(\nu), \mathcal{T}^2_{\mathbb{F}_\theta}(\nu)\right\}\, if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2 \end{cases},$$

and

$$\mathcal{I}_{\mathbb{F}_\theta} = \begin{cases} \mathcal{I}^1_{\mathbb{F}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2 \\ \mathcal{I}^2_{\mathbb{F}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1 \\ \min\left\{\mathcal{I}^1_{\mathbb{F}_\theta}(\nu), \mathcal{I}^2_{\mathbb{F}_\theta}(\nu)\right\}\, if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2 \end{cases},$$

FIGURE 6. Graphical Representation of TABLE 5 with (c) $\mathcal{N}(\theta_3)$ and (d) $\mathcal{N}(\theta_4)$



and

$$\mathcal{F}_{\mathbb{F}_\theta} = \begin{cases} \mathcal{F}^1_{\mathbb{F}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2 \\ \mathcal{F}^2_{\mathbb{F}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1 \\ \max\left\{\mathcal{F}^1_{\mathbb{F}_\theta}(\nu), \mathcal{F}^2_{\mathbb{F}_\theta}(\nu)\right\}\, if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2 \end{cases}.$$

The neutrosophic components for $\mathbb{G}$ are given as follows:

$$\mathcal{T}_{\mathbb{G}_\theta} = \begin{cases} \mathcal{T}^1_{\mathbb{G}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2 \\ \mathcal{T}^2_{\mathbb{G}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1 \\ \min\left\{\mathcal{T}^1_{\mathbb{G}_\theta}(\nu), \mathcal{T}^2_{\mathbb{G}_\theta}(\nu)\right\}\, if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2 \end{cases},$$

and

$$\mathcal{I}_{\mathbb{G}_\theta} = \begin{cases} \mathcal{I}^1_{\mathbb{G}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2 \\ \mathcal{I}^2_{\mathbb{G}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1 \\ \min\left\{\mathcal{I}^1_{\mathbb{G}_\theta}(\nu), \mathcal{I}^2_{\mathbb{G}_\theta}(\nu)\right\}\, if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2 \end{cases},$$

and

$$\mathcal{F}_{\mathbb{G}_\theta} = \begin{cases} \mathcal{F}^1_{\mathbb{G}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^1 - \mathcal{Q}^2 \\ \mathcal{F}^2_{\mathbb{G}_\theta}(\nu)\, if\, \theta \in \mathcal{Q}^2 - \mathcal{Q}^1 \\ \max\left\{\mathcal{F}^1_{\mathbb{G}_\theta}(\nu), \mathcal{F}^2_{\mathbb{G}_\theta}(\nu)\right\}\, if\, \theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2 \end{cases}.$$

Theorem 4.5. *If $\mathfrak{G}_1, \mathfrak{G}_2 \in \Omega_{NHSG}$ then $\mathfrak{G}_1 \cap \mathfrak{G}_2 \in \Omega_{NHSG}$.*

Proof. Consider two NHS-Graphs $\mathfrak{G}_1 = \left(\mathfrak{G}^\star_1, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ and $\mathfrak{G}_2 = \left(\mathfrak{G}^\star_2, \mathcal{Q}^2, \mathbb{F}^2, \mathbb{G}^2\right)$ as defined in 3.1. Let $\mathfrak{G} = (\mathfrak{G}^\star, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ be the intersection of NHS-Graphs $\mathfrak{G}_1$ and $\mathfrak{G}_2$ where $\mathcal{Q} = \mathcal{Q}^1 \cup \mathcal{Q}^2$ and $\mathcal{V} = \mathcal{V}_1 \cap \mathcal{V}_2$. Let $\theta \in \mathcal{Q}^1 - \mathcal{Q}^2$ then

$$\mathcal{T}_{\mathbb{G}_\theta}(\nu, \mu) = \mathcal{T}_{\mathbb{G}^1_\theta}(\nu, \mu)$$

$$\leq \min\left\{\mathcal{T}_{\mathbb{F}^1_\theta}(\nu), \mathcal{T}_{\mathbb{F}^1_\theta}(\mu)\right\}$$

$$= \min\left\{\mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{T}_{\mathbb{F}_\theta}(\mu)\right\}$$

so

$$\mathcal{T}_{\mathbb{G}_\theta}(\nu,\mu) \leq \min\left\{\mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{T}_{\mathbb{F}_\theta}(\mu)\right\}$$

Also

$$\mathcal{I}_{\mathbb{G}_\theta}(\nu,\mu) = \mathcal{I}_{\mathbb{G}^1_\theta}(\nu,\mu)$$
$$\leq \min\left\{\mathcal{I}_{\mathbb{F}^1_\theta}(\nu), \mathcal{I}_{\mathbb{F}^1_\theta}(\mu)\right\}$$
$$= \min\left\{\mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}_\theta}(\mu)\right\}$$

so

$$\mathcal{I}_{\mathbb{G}_\theta}(\nu,\mu) \leq \min\left\{\mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}_\theta}(\mu)\right\}$$

Now

$$\mathcal{F}_{\mathbb{G}_\theta}(\nu,\mu) = \mathcal{F}_{\mathbb{G}^1_\theta}(\nu,\mu)$$
$$\geq \max\left\{\mathcal{F}_{\mathbb{F}^1_\theta}(\nu), \mathcal{F}_{\mathbb{F}^1_\theta}(\mu)\right\}$$
$$= \max\left\{\mathcal{F}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}_\theta}(\mu)\right\}$$

so

$$\mathcal{F}_{\mathbb{G}_\theta}(\nu,\mu) \geq \max\left\{\mathcal{F}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}_\theta}(\mu)\right\}$$

Similar results are obtained when $\theta \in \mathcal{Q}^2 - \mathcal{Q}^1$

Now if $\theta \in \mathcal{Q}^1 \cap \mathcal{Q}^2$ then

$$\mathcal{T}_{\mathbb{G}_\theta}(\nu,\mu) = \min\left\{\mathcal{T}_{\mathbb{G}^1_\theta}(\nu,\mu), \mathcal{T}_{\mathbb{G}^2_\theta}(\nu,\mu)\right\}$$
$$\leq \min\left\{\min\left\{\mathcal{T}_{\mathbb{F}^1_\theta}(\nu), \mathcal{T}_{\mathbb{F}^1_\theta}(\mu)\right\}, \min\left\{\mathcal{T}_{\mathbb{F}^2_\theta}(\nu), \mathcal{T}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\}$$
$$\leq \min\left\{\min\left\{\mathcal{T}_{\mathbb{F}^1_\theta}(\nu), \mathcal{T}_{\mathbb{F}^2_\theta}(\mu)\right\}, \min\left\{\mathcal{T}_{\mathbb{F}^1_\theta}(\nu), \mathcal{T}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\}$$
$$= \min\left\{\mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{T}_{\mathbb{F}_\theta}(\mu)\right\}$$

Also

$$\mathcal{I}_{\mathbb{G}_\theta}(\nu,\mu) = \min\left\{\mathcal{I}_{\mathbb{G}^1_\theta}(\nu,\mu), \mathcal{I}_{\mathbb{G}^2_\theta}(\nu,\mu)\right\}$$
$$\leq \min\left\{\min\left\{\mathcal{I}_{\mathbb{F}^1_\theta}(\nu), \mathcal{I}_{\mathbb{F}^1_\theta}(\mu)\right\}, \min\left\{\mathcal{I}_{\mathbb{F}^2_\theta}(\nu), \mathcal{I}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\}$$
$$\leq \min\left\{\min\left\{\mathcal{I}_{\mathbb{F}^1_\theta}(\nu), \mathcal{I}_{\mathbb{F}^2_\theta}(\mu)\right\}, \min\left\{\mathcal{I}_{\mathbb{F}^1_\theta}(\nu), \mathcal{I}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\}$$
$$= \min\left\{\mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}_\theta}(\mu)\right\}$$

In the same way

$$\mathcal{F}_{\mathbb{G}_\theta}(\nu,\mu) = \max\left\{\mathcal{F}_{\mathbb{G}^1_\theta}(\nu,\mu), \mathcal{F}_{\mathbb{G}^2_\theta}(\nu,\mu)\right\}$$
$$\geq \max\left\{\max\left\{\mathcal{F}_{\mathbb{F}^1_\theta}(\nu), \mathcal{F}_{\mathbb{F}^1_\theta}(\mu)\right\}, \max\left\{\mathcal{F}_{\mathbb{F}^2_\theta}(\nu), \mathcal{F}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\}$$
$$\geq \max\left\{\max\left\{\mathcal{F}_{\mathbb{F}^1_\theta}(\nu), \mathcal{F}_{\mathbb{F}^2_\theta}(\mu)\right\}, \max\left\{\mathcal{F}_{\mathbb{F}^1_\theta}(\nu), \mathcal{F}_{\mathbb{F}^2_\theta}(\mu)\right\}\right\}$$
$$= \max\left\{\mathcal{F}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}_\theta}(\mu)\right\}$$

Hence the intersection $\mathfrak{G} = \mathfrak{G}_1 \cap \mathfrak{G}_2$ is NHS-Graphs. □

TABLE 6. Tabular Representation of NHS-Graph $\mathfrak{G}_1 = \left(\mathfrak{G}_1^\star, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ according to Example 4.6

$\mathbb{F}$	ν_1	ν_2	ν_3
θ_1	$(0.2, 0.3, 0.4)$	$(0.3, 0.5, 0.6)$	$(0.2, 0.6, 0.8)$
θ_2	$(0.3, 0.4, 0.8)$	$(0.5, 0.7, 0.8)$	$(0.4, 0.5, 0.7)$
$\mathbb{G}$	(ν_1, ν_2)	(ν_2, ν_3)	(ν_1, ν_3)
θ_1	$(0.2, 0.2, 0.7)$	$(0.2, 0.4, 0.9)$	$(0.2, 0.2, 0.9)$
θ_2	$(0.3, 0.4, 0.8)$	$(0.4, 0.5, 0.9)$	$(0.3, 0.4, 0.8)$

TABLE 7. Tabular Representation of NHS-Graph $\mathfrak{G}_2 = \left(\mathfrak{G}_2^\star, \mathcal{Q}^2, \mathbb{F}^2, \mathbb{G}^2\right)$ according to Example 4.6

$\mathbb{F}$	ν_2	ν_3	ν_4
θ_2	$(0.4, 0.6, 0.7)$	$(0.5, 0.6, 0.9)$	$(0.3, 0.5, 0.7)$
θ_3	$(0.3, 0.5, 0.6)$	$(0.2, 0.6, 0.8)$	$(0.2, 0.3, 0.7)$
$\mathbb{G}$	(ν_2, ν_3)	(ν_3, ν_4)	(ν_2, ν_4)
θ_2	$(0.2, 0.2, 0.7)$	$(0.2, 0.4, 0.9)$	$(0.2, 0.2, 0.9)$
θ_3	$(0.3, 0.4, 0.8)$	$(0.4, 0.5, 0.9)$	$(0.3, 0.4, 0.8)$

Example 4.6. Let $\mathfrak{G}_1 = \left(\mathfrak{G}_1^\star, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ be a neutrosophic hypersoft graph where $\mathfrak{G}_1^\star = (\mathcal{V}_1, \mathcal{E}_1)$ with $\mathcal{V}_1 = \{\nu_1, \nu_2, \nu_3\}$ and $\mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3$ are disjoint attribute-valued sets corresponding to distinct attributes $\alpha_1, \alpha_2, \alpha_3$ where $\mathcal{Q}_1 = \{\alpha_{11}\}$, $\mathcal{Q}_2 = \{\alpha_{21}\}$ and $\mathcal{Q}_3 = \{\alpha_{31}, \alpha_{32}\}$. $\mathcal{Q}^1 = \mathcal{Q}_1 \times \mathcal{Q}_2 \times \mathcal{Q}_3 = \{\theta_1, \theta_2\}$ where each θ_i is a 3-tuple element of $\mathcal{Q}^1$ and $\mathcal{T}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{I}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{F}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 1$ for all $(\nu_i, \nu_j) \in \mathcal{V}_1 \times \mathcal{V}_1 \backslash \{(\nu_1, \nu_2), (\nu_2, \nu_3), (\nu_1, \nu_3)\}$. Its tabular and graphical representation are given in TABLE 6 and FIGURE 7 respectively. Also let $\mathfrak{G}_2 = \left(\mathfrak{G}_2^\star, \mathcal{Q}^2, \mathbb{F}^2, \mathbb{G}^2\right)$ be a neutrosophic hypersoft graph where $\mathfrak{G}_2^\star = (\mathcal{V}_2, \mathcal{E}_2)$ with $\mathcal{V}_2 = \{\nu_2, \nu_3, \nu_4\}$ and $\mathcal{Q}_2, \mathcal{Q}_3, \mathcal{Q}_4$ are disjoint attribute-valued sets corresponding to distinct attributes $\alpha_2, \alpha_3, \alpha_4$ where $\mathcal{Q}_2 = \{\alpha_{21}\}$, $\mathcal{Q}_3 = \{\alpha_{31}, \alpha_{32}\}$, $\mathcal{Q}_4 = \{\alpha_{41}\}$. $\mathcal{Q}^2 = \mathcal{Q}_2 \times \mathcal{Q}_3 \times \mathcal{Q}_4 = \{\theta_2, \theta_3\}$ where each θ_i is a 3-tuple element of $\mathcal{Q}^2$ and $\mathcal{T}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{I}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 0, \mathcal{F}_{\mathbb{G}_\theta}(\nu_i, \nu_j) = 1$ for all $(\nu_i, \nu_j) \in \mathcal{V}_2 \times \mathcal{V}_2 \backslash \{(\nu_2, \nu_3), (\nu_3, \nu_4), (\nu_2, \nu_4)\}$. Its tabular and graphical representation are given in TABLE 7 and FIGURE 8 respectively.
Now Let $\mathfrak{G} = (\mathfrak{G}^\star, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ be the intersection of two neutrosophic hypersoft graphs $\mathfrak{G}_1 = \left(\mathfrak{G}_1^\star, \mathcal{Q}^1, \mathbb{F}^1, \mathbb{G}^1\right)$ and $\mathfrak{G}_2 = \left(\mathfrak{G}_2^\star, \mathcal{Q}^2, \mathbb{F}^2, \mathbb{G}^2\right)$ where $\mathcal{Q} = \mathcal{Q}^1 \cap \mathcal{Q}^2$.Its (intersection of these two NHS-graphs) tabular and graphical representation are given in TABLE 8 and FIGURE 9 respectively.

Definition 4.7. The compliment $\overline{\mathfrak{G}} = \left(\overline{\mathfrak{G}^\star}, \overline{\mathcal{Q}}, \overline{\mathbb{F}}, \overline{\mathbb{G}}\right)$ of strong neutrosophic hypersoft subgraph $\mathfrak{G} = (\mathfrak{G}^\star, \mathcal{Q}, \mathbb{F}, \mathbb{G})$ with $\mathbb{G}_\theta(\nu, \mu) = \mathbb{F}_\theta(\nu) \cap \mathbb{F}_\theta(\mu)$ where

FIGURE 7. Graphical Representation of TABLE 6with (a) $\mathcal{N}(\theta_1)$ and (b) $\mathcal{N}(\theta_2)$

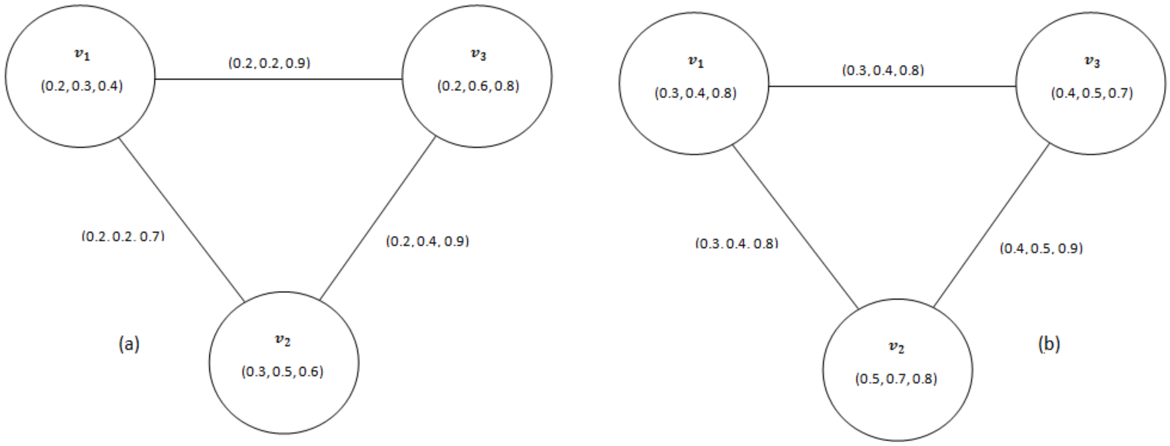


FIGURE 8. Graphical Representation of TABLE 7with (a) $\mathcal{N}(\theta_2)$ and (b) $\mathcal{N}(\theta_3)$

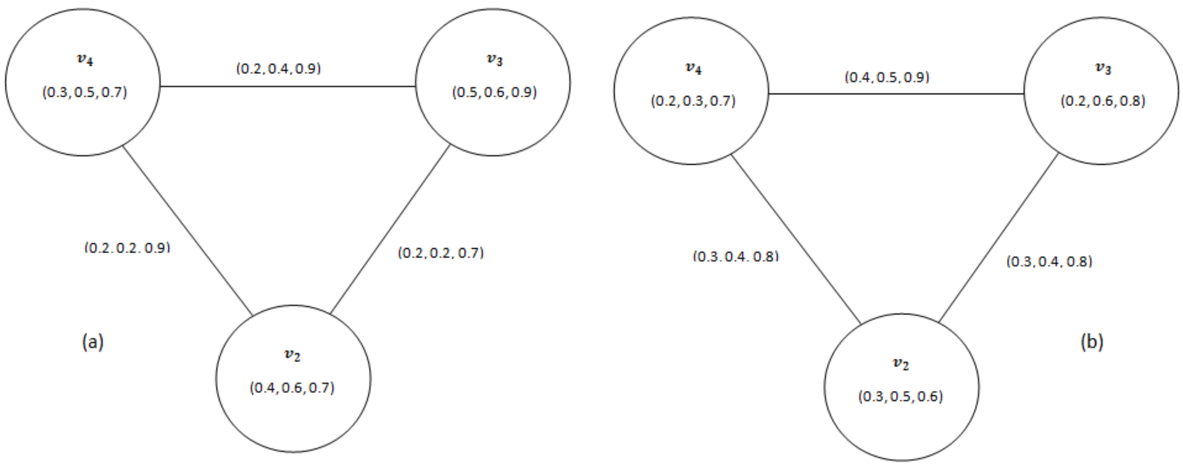


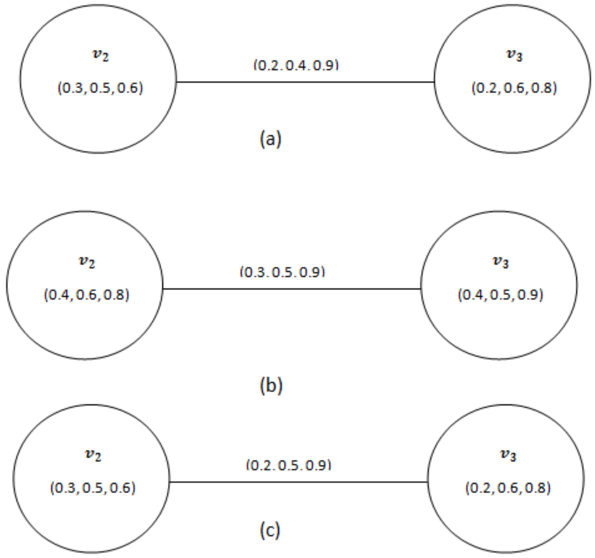
TABLE 8. Tabular Representation of NHS-Graph $\mathfrak{G} = \mathfrak{G}_1 \cap \mathfrak{G}_2$

$\mathbb{F}$	ν_2	ν_3
θ_1	$(0.3, 0.5, 0.6)$	$(0.2, 0.6, 0.8)$
θ_2	$(0.4, 0.6, 0.8)$	$(0.4, 0.5, 0.9)$
θ_3	$(0.3, 0.5, 0.6)$	$(0.2, 0.6, 0.8)$
$\mathbb{G}$	(ν_2, ν_3)	
θ_1	$(0.2, 0.4, 0.9)$	
θ_2	$(0.3, 0.5, 0.9)$	
θ_3	$(0.2, 0.5, 0.9)$	

(1) $\overline{\mathcal{Q}} = \mathcal{Q}$

(2) $\overline{\mathcal{T}_{\mathbb{F}_\theta}(\nu)} = \mathcal{T}_{\mathbb{F}_\theta}(\nu), \overline{\mathcal{I}_{\mathbb{F}_\theta}(\nu)} = \mathcal{I}_{\mathbb{F}_\theta}(\nu), \overline{\mathcal{F}_{\mathbb{F}_\theta}(\nu)} = \mathcal{F}_{\mathbb{F}_\theta}(\nu)$ for all $\nu \in \mathcal{V}$

FIGURE 9. Graphical Representation of TABLE 8 with (a) $\mathcal{N}(\theta_1)$, (b) $\mathcal{N}(\theta_2)$ and (c) $\mathcal{N}(\theta_3)$



(3) $$\overline{\mathcal{T}_{\mathbb{F}_\theta}(\nu,\mu)} = \begin{cases} \min\{\mathcal{T}_{\mathbb{F}_\theta}(\nu), \mathcal{T}_{\mathbb{F}_\theta}(\mu)\} & if\ \mathcal{T}_{\mathbb{G}_\theta}(\nu,\mu) = 0 \\ 0 & otherwise \end{cases}$$

$$\overline{\mathcal{I}_{\mathbb{F}_\theta}(\nu,\mu)} = \begin{cases} \min\{\mathcal{I}_{\mathbb{F}_\theta}(\nu), \mathcal{I}_{\mathbb{F}_\theta}(\mu)\} & if\ \mathcal{I}_{\mathbb{G}_\theta}(\nu,\mu) = 0 \\ 0 & otherwise \end{cases}$$

$$\overline{\mathcal{F}_{\mathbb{F}_\theta}(\nu,\mu)} = \begin{cases} \max\{\mathcal{F}_{\mathbb{F}_\theta}(\nu), \mathcal{F}_{\mathbb{F}_\theta}(\mu)\} & if\ \mathcal{F}_{\mathbb{G}_\theta}(\nu,\mu) = 0 \\ 0 & otherwise \end{cases}.$$

5. Conclusions

In this study, a gluing concept of neutrosophic hypersoft set and graph theory is characterized. Some of elementary properties, types, operations and results are generalized under neutrosophic hypersoft set environment. Future work may include the extension of this study for the following structures and fields:

- Interval valued neutrosophic hypersoft set
- Neutrosophic parameterized hypersoft set
- m-polar neutrosophic hypersoft set
- Decision making problems
- New kinds of graphs
- Energies of graph

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