

10-7-2021

Pentapartitioned Neutrosophic Pythagorean Resolvable and Irresolvable Spaces

R. Radha

A. Stanis Arul Mary

Said Broumi

Follow this and additional works at: https://digitalrepository.unm.edu/nss_journal

Recommended Citation

Radha, R.; A. Stanis Arul Mary; and Said Broumi. "Pentapartitioned Neutrosophic Pythagorean Resolvable and Irresolvable Spaces." *Neutrosophic Sets and Systems* 46, 1 (). https://digitalrepository.unm.edu/nss_journal/vol46/iss1/15

This Article is brought to you for free and open access by UNM Digital Repository. It has been accepted for inclusion in Neutrosophic Sets and Systems by an authorized editor of UNM Digital Repository. For more information, please contact disc@unm.edu.



Pentapartitioned Neutrosophic Pythagorean Resolvable and Irresolvable Spaces

R. Radha ^{1,*}, A. Stanis Arul Mary ² and Said Broumi ³

- ¹ Research Scholar, Department of Mathematics, Nirmala College for Women, Coimbatore, India (TN);
radharmat2020@gmail.com
- ² Assistant Professor, Department of Mathematics, Nirmala College for Women, Coimbatore, India (TN);
stanisarulmary@gmail.com
- ³ Laboratory of Image Processing, Faculty of Science, Ben M ' Sik , University Hassan II, Casablanca, Morocco;
broumisaid78@gmail.com

* Correspondence: radharmat2020@gmail.com

Abstract: In this paper, the concepts of Pentapartitioned Neutrosophic Pythagorean resolvable , Pentapartitioned Neutrosophic Pythagorean irresolvable, Pentapartitioned Neutrosophic Pythagorean open hereditarily irresolvable and maximally Pentapartitioned Neutrosophic Pythagorean irresolvable spaces are introduced. Also we investigated several properties of the Pentapartitioned Neutrosophic Pythagorean open hereditarily irresolvable spaces besides giving characterization of these spaces by means of somewhat Pentapartitioned Neutrosophic Pythagorean continuous functions and somewhat Pentapartitioned Neutrosophic Pythagorean open functions.

Keywords: Pentapartitioned neutrosophic pythagorean resolvable, pentapartitioned neutrosophic pythagorean open hereditarily irresolvable, somewhat pentapartitioned neutrosophic pythagorean irresolvable, pentapartitioned neutrosophic pythagorean continuous and open functions.

1. Introduction

Zadeh[16] introduced the important and useful concept of a fuzzy set which has invaded almost all branches of mathematics. The speculation of fuzzy topological space was studied and developed by C.L. Chang [4]. The paper of Chang sealed the approach for the following tremendous growth of the various fuzzy topological ideas. Since then a lot of attention has been paid to generalize the fundamental ideas of general topology in fuzzy setting and therefore a contemporary theory of fuzzy topology has been developed. Atanassov and plenty of researchers [1] worked on intuitionistic fuzzy sets within the literature. Florentin Smarandache [13] introduced the idea of Neutrosophic set in 1995 that provides the information of neutral thought by introducing the new issue referred to as uncertainty within the set. Thus neutrosophic set was framed and it includes the parts of truth membership function(T), indeterminacy membership function(I), and falsity membership function(F) severally. Neutrosophic sets deals with non normal interval of]-0 1+[. Pentapartitioned neutrosophic set and its properties were introduced by Rama Malik and Surpati Pramanik [12]. In this case, indeterminacy is divided into three components: contradiction, ignorance, and an unknown

membership function. The concept of Pentapartitioned neutrosophic pythagorean sets was initiated by R. Radha and A. Stanis Arul Mary . The concept of neutrosophic fuzzy resolvable spaces and irresolvable spaces was introduced by M. Caldas et.al[3]. Now we extend the concepts to pentapartitioned neutrosophic pythagorean sets.

In this Paper we initiated the new concept of Pentapartitioned neutrosophic pythagorean resolvable, pentapartitioned neutrosophic pythagorean open hereditarily irresolvable, somewhat pentapartitioned neutrosophic pythagorean irresolvable, pentapartitioned neutrosophic pythagorean continuous and open functions and discussed some of its properties.

2. Preliminaries

2.1 Definition [13]

Let X be a universe. A Neutrosophic set A on X can be defined as follows:

$$A = \{ \langle x, T_A(x), I_A(x), F_A(x) \rangle : x \in X \}$$

Where $T_A, I_A, F_A: U \rightarrow [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

Here, $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of indeterminacy and $F_A(x)$ is the degree of non-membership.

2.2 Definition [7]

Let X be a universe. A Pentapartitioned neutrosophic pythagorean [PNP] set A with T, F, C and U as dependent neutrosophic components and I as independent component for A on X is an object of the form

$$A = \{ \langle x, T_A, C_A, I_A, U_A, F_A \rangle : x \in X \}$$

Where $T_A + F_A \leq 1, C_A + U_A \leq 1$ and

$$(T_A)^2 + (C_A)^2 + (I_A)^2 + (U_A)^2 + (F_A)^2 \leq 3$$

Here, $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership, $F_A(x)$ is the false membership and $I_A(x)$ is an unknown membership.

2.3 Definition [12]

Let P be a non-empty set. A Pentapartitioned neutrosophic set A over P characterizes each element p in P a truth -membership function T_A , a contradiction membership function C_A , an ignorance membership function G_A , unknown membership function U_A and a false membership function F_A , such that for each p in P

$$T_A + C_A + G_A + U_A + F_A \leq 5.$$

2.4 Definition [7]

The complement of a pentapartitioned neutrosophic pythagorean set A on R is denoted by A^c or A^* and is defined as

$$A^c = \{ \langle x, F_A(x), U_A(x), 1 - G_A(x), C_A(x), T_A(x) \rangle : x \in X \}$$

2.5 Definition [7]

Let $A = \langle x, T_A(x), C_A(x), G_A(x), U_A(x), F_A(x) \rangle$ and $B = \langle x, T_B(x), C_B(x), G_B(x), U_B(x), F_B(x) \rangle$ are pentapartitioned neutrosophic pythagorean sets. Then

$$A \cup B = \langle x, \max(T_A(x), T_B(x)), \max(C_A(x), C_B(x)), \min(G_A(x), G_B(x)), \min(U_A(x), U_B(x)), \min(F_A(x), F_B(x)), \rangle$$

$$A \cap B = \langle x, \min(T_A(x), T_B(x)), \min(C_A(x), C_B(x)), \max(G_A(x), G_B(x)), \max(U_A(x), U_B(x)), \max(F_A(x), F_B(x)) \rangle$$

2.6 Definition[7]

A PNP topology τ on a nonempty set R is a family of a PNP sets in R satisfying the following axioms

- 1) $0, 1 \in \tau$
- 2) $R_1 \cap R_2 \in \tau$ for any $R_1, R_2 \in \tau$
- 3) $\cup R_i \in \tau$ for any $R_i: i \in I \subseteq \tau$

The complement R^* of PNP open set (PNPOS, in short) in PNP topological space [PNPTS] (R, τ) , is called a PNP closed set [PNPCS].

2.7 Definition [7]

Let (R, τ) be a PNPTS and L be a PNPTS in R. Then the PNP interior and PNP Closure of R denoted by

$$Cl(L) = \cap \{K: K \text{ is a PNPCS in } R \text{ and } L \subseteq K\}.$$

$$Int(L) = \cup \{G: G \text{ is a PNPOS in } R \text{ and } G \subseteq L\}.$$

3. Pentapartitioned Neutrosophic Pythagorean Resolvable and Irresolvable Spaces

3.1 Definition

A Pentapartitioned neutrosophic pythagorean (PNP) set P in Pentapartitioned neutrosophic pythagorean topological space (PNPTS) (R, τ) is called pentapartitioned neutrosophic pythagorean dense if there exists no pentapartitioned neutrosophic pythagorean closed set Q in (R, τ) such that

$$P \subset Q \subset 1_R$$

Note: If P is a PNP open set, then the complement of PNP set P is a PNP closed set and it is denoted by P^* .

3.2 Example

Let $R = \{ e, f \}$ and define the pentapartitioned neutrosophic pythagorean set P as

$$P = \left\{ \begin{array}{l} \{e, 0.4, 0.5, 0.7, 0.2, 0.3\} \\ \{f, 0.5, 0.3, 0.6, 0.1, 0.2\} \end{array} \right\}$$

Then $\tau = \{0_R, 1_R, P\}$ is a pentapartitioned neutrosophic pythagorean topology on R. Hence P is a PNP dense set in (R, τ) .

3.3 Definition

A PNPTS (R, τ) is called PNP resolvable if there exists a PNP dense set P in (R, τ) such that $PNPCI(P^*) = 1_R$. Otherwise (R, τ) is called PNP irresolvable.

3.4 Example

Let $R = \{e, f\}$ and define the pentapartitioned neutrosophic pythagorean set P, Q and R as

$$P = \left\{ \begin{matrix} \{e, 0.3, 0.4, 0.3, 0.3, 0.1\} \\ \{f, 0.4, 0.2, 0.6, 0.5, 0.3\} \end{matrix} \right\}$$

$$Q = \left\{ \begin{matrix} \{e, 0.4, 0.2, 0.7, 0.1, 0.3\} \\ \{f, 0.6, 0.1, 0.3, 0.2, 0.2\} \end{matrix} \right\} \text{ and}$$

$$R = \left\{ \begin{matrix} \{e, 0.1, 0.2, 0.4, 0.3, 0.4\} \\ \{f, 0.5, 0.4, 0.3, 0.2, 0.1\} \end{matrix} \right\} .$$

It can be seen that $\tau = \{0_R, 1_R, P\}$ is a pentapartitioned neutrosophic pythagorean topology on R. Then (R, τ) is a pentapartitioned neutrosophic topological space. Now $PNPInt(Q) = 0_R$, $PNPInt(R) = 0_R$, $PNPCI(Q) = 1_R$ and $PNPCI(R) = 1_R$. Thus P and Q are PNP dense sets in (R, τ) such that $PNPCI(Q^*) = 1_R$ and $PNPCI(R^*) = 1_R$. Hence the PNP topological space (R, τ) is PNP resolvable.

3.5 Example

Let $R = \{e, f\}$ and define the pentapartitioned neutrosophic pythagorean set P, Q and R as

$$P = \left\{ \begin{matrix} \{e, 0.2, 0.3, 0.5, 0.4, 0.5\} \\ \{f, 0.1, 0.2, 0.5, 0.5, 0.3\} \end{matrix} \right\}$$

$$Q = \left\{ \begin{matrix} \{e, 0.4, 0.5, 0.5, 0.4, 0.3\} \\ \{f, 0.5, 0.4, 0.4, 0.3, 0.2\} \end{matrix} \right\} \text{ and}$$

$$R = \left\{ \begin{matrix} \{e, 0.3, 0.4, 0.4, 0.3, 0.2\} \\ \{f, 0.2, 0.3, 0.5, 0.2, 0.1\} \end{matrix} \right\} .$$

It can be seen that $\tau = \{0_R, 1_R, P\}$ is a pentapartitioned neutrosophic pythagorean topology on R. Then (R, τ) is a pentapartitioned neutrosophic topological space. Now $PNPInt(Q) = A$, $PNPInt(R) = A$, $PNPCI(Q) = 1_R$ and $PNPCI(R) = 1_R$. Thus P and Q are PNP dense sets in (R, τ) such that $PNPCI(Q^*) = P^*$ and $PNPCI(R^*) = P^*$. Hence the PNP topological space (R, τ) is PNP irresolvable.

3.6 Theorem

A PNPTS (R, τ) is a PNP resolvable space iff (R, τ) has a pair of PNP dense set K_1 and K_2 such that $K_1 \subseteq K_2^*$.

Proof

Let (R, τ) be a PNPTS and (R, τ) be PNP resolvable space. Suppose that for all PNP dense sets K_i and K_j , we have $K_i \not\subseteq K_j^*$. Then $K_i \supset K_j^*$. Then $PNPCI(K_i) \supset PNPCI(K_j^*)$ which implies that $1_R \supset PNPCI(K_j^*)$. Then $PNPCI(K_j^*) \neq 1_R$. Also $K_j \supset K_i^*$, then $PNPCI(K_j) \supset PNPCI(K_i^*)$ which implies that

$1_R \supset \text{PNPCI}(K_i^*)$. Therefore $\text{PNPCI}(K_i^*) \neq 1_R$. Hence $\text{PNPCI}(K_i^*) = 1_R$, but $\text{PNPCI}(K_i) \neq 1_R$ for all PNP set K_i in (R, τ) which is a contradiction. Hence (R, τ) has a pair of PNP dense set K_1 and K_2 such that $K_1 \subseteq K_2^*$.

Conversely, suppose that the PNP topological space (R, τ) has a pair of PNP dense set K_1 and K_2 such that $K_1 \subseteq K_2^*$. Suppose that (R, τ) is a PNP irresolvable space, Then for all PNP dense sets K_1 and K_2 in (R, τ) , we have $\text{PNPCI}(K_1^*) \neq 1_R$. Then $\text{PNPCI}(K_2^*) \neq 1_R$ implies that there exists a PNP closed set L in (R, τ) such that $K_2^* \subset L \subset 1_R$. Then $K_1 \subset K_2^* \subset L \subset 1_R$ implies that $K_1 \subset L \subset 1_R$. But this is a contradiction. Hence (R, τ) is a PNP resolvable space.

3.7 Theorem

If (R, τ) is a PNP irresolvable space iff $\text{PNInt}(P) \neq 0$ for all PNP dense set P in (R, τ) .

Proof

Since (R, τ) is PNP irresolvable space for all PNP dense set P in (R, τ) , $\text{PNPCI}(P^*) \neq 1_R$. Then $(\text{PNInt}(P))^* \neq 1_R$ which implies $\text{PNInt}(P) \neq 0_R$.

Conversely $\text{PNInt}(P) \neq 0_R$, for all PNP dense set P in (R, τ) . Suppose that (R, τ) is PNP resolvable. Then there exists a PNP dense set P in (R, τ) such that $\text{PNPCI}(P^*) = 1_R$. This implies that $(\text{PNInt}(P))^* = 1_R$ which again implies $\text{PNInt}(P) = 0_R$. But this is a contradiction. Hence (R, τ) is PNP resolvable space.

3.8 Definition

A PNP topological space (R, τ) is called a PNP submaximal space if for each PNP set P in (R, τ) , $\text{PNPCI}(P) = 1_R$.

3.9 Proposition

If the PNP topological space (R, τ) is PNP submaximal, then (R, τ) is PNP irresolvable.

Proof. Let (R, τ) be a PNP submaximal space. Assume that (R, τ) is a PNP resolvable space. Let P be a PNP dense set in (R, τ) . Then $\text{PNPCI}(P^*) = 1_R$. Hence $(\text{PNInt}(P))^* = 1_R$ which implies that $\text{PNInt}(P) = 0_R$. Then $P \notin \tau$. This is a contradiction. Hence (R, τ) is PNP irresolvable space.

The converse of the above theorem is not true, which can be shown by the following example. See example 3.5.

3.10 Definition

A PNP topological space (R, τ) is called a maximal PNP irresolvable space if (R, τ) is PNP irresolvable and every PNP dense set P of (R, τ) is PNP open.

3.11 Example

Let $R = \{e, f\}$ and define the pentapartitioned neutrosophic pythagorean set Q and R as

$$Q = \left\{ \begin{array}{l} \{e, 0.3, 0.4, 0.3, 0.3, 0.1\} \\ \{f, 0.4, 0.2, 0.6, 0.5, 0.3\} \end{array} \right\} \text{ and}$$

$$R = \left\{ \begin{array}{l} \{e, 0.1, 0.2, 0.4, 0.3, 0.4\} \\ \{f, 0.5, 0.4, 0.3, 0.2, 0.1\} \end{array} \right\} .$$

It can be seen that $\tau = \{0_R, 1_R, P\}$ is a pentapartitioned neutrosophic pythagorean topology on R. Then (R, τ) is a pentapartitioned neutrosophic topological space. Now $\text{PNPInt}(Q^*) = 0_R, \text{PNPInt}(R^*) = 0_R, \text{PNPCL}(Q) = 1_R$ and $\text{PNPCL}(R) = 1_R$. Thus P and Q are PNP dense sets in (R, τ) such that $\text{PNPCL}(Q^*) = Q^*$ and $\text{PNPCL}(R^*) = R^*$. Thus (R, τ) is PNP irresolvable and every PNP dense set of (R, τ) is PNP open. Therefore PNP topological space (R, τ) is maximally PNP irresolvable.

4 PNP open hereditarily Irresolvable space

4.1 Definition

A PNP topological space (R, τ) is said to be PNP open hereditarily irresolvable if $\text{PNPInt}(\text{PNPCL}(P)) \neq 0_R$ and $\text{PNPInt}(P) \neq 0_R$, for any PNP set P in (R, τ) .

4.2 Example

Let $R = \{e, f\}$ and define the pentapartitioned neutrosophic pythagorean set Q as

$$P = \left\{ \begin{array}{l} \{e, 0.2, 0.1, 0.5, 0.4, 0.5\} \\ \{f, 0.1, 0.2, 0.6, 0.5, 0.4\} \end{array} \right\}$$

It can be seen that $\tau = \{0_R, 1_R, P\}$ is a pentapartitioned neutrosophic pythagorean topology on R. Then (R, τ) is a pentapartitioned neutrosophic topological space. Now $\text{PNPInt}(P) = P \neq 0_R$ and $\text{PNPInt}(\text{PNPCL}(P)) = \text{PNPInt}(P^*) = P \neq 0_R$. Thus (R, τ) is PNP open hereditarily irresolvable space.

4.3 Theorem

Let (R, τ) be a PNP topological space. If (R, τ) is PNP open hereditarily irresolvable, then (R, τ) is PNP irresolvable.

Proof

Let P be a PNP dense set in (R, τ) . Then $\text{PNPCL}(P) = 1_R$ which implies that $\text{PNPInt}(\text{PNPCL}(P)) = 1_R \neq 0_R$. Since (R, τ) is PNP open hereditarily irresolvable, we have $\text{PNPInt}(P) \neq 0_R$. Therefore by theorem 3.7, $\text{PNPInt}(P) \neq 0_R$ for all PNP dense set in (R, τ) implies that (R, τ) is PNP irresolvable.

The converse of the above theorem is not true. See Example 4.4.

□

4.4 Example

Let $R = \{e, f\}$ and define the pentapartitioned neutrosophic pythagorean set P, Q, R and S as

$$P = \left\{ \begin{array}{l} \{e, 0.1, 0.5, 0.5, 0.2, 0.6\} \\ \{f, 0.2, 0.3, 0.6, 0.3, 0.3\} \end{array} \right\}$$

$$Q = \left\{ \begin{array}{l} \{e, 0.4, 0.5, 0.1, 0.2, 0.4\} \\ \{f, 0.3, 0.2, 0.7, 0.2, 0.1\} \end{array} \right\}$$

$$R = \left\{ \begin{array}{l} \{e, 0.4, 0.5, 0.1, 0.2, 0.4\} \\ \{f, 0.3, 0.3, 0.6, 0.2, 0.1\} \end{array} \right\} \text{ and}$$

$$S = \left\{ \begin{array}{l} \{e, 0.2, 0.1, 0.7, 0.6, 0.2\} \\ \{f, 0.1, 0.2, 0.6, 0.5, 0.4\} \end{array} \right\} .$$

It can be seen that $\tau = \{0_R, 1_R, P, Q, R\}$ is a pentapartitioned neutrosophic pythagorean topology on R . Then (R, τ) is a pentapartitioned neutrosophic topological space. Now $\text{PNPCI}(P) = 1_R$, $\text{PNPCI}(Q) = 1_R$, $\text{PNPCI}(R) = 1_R$ and $\text{PNPCI}(S) = 1_R$. Thus P, Q, R and S are PNP dense sets in (R, τ) such that $\text{PNPCI}(P^*) = P^*$, $\text{PNPCI}(Q^*) = Q^*$ and $\text{PNPCI}(R^*) = R^*$ and $\text{PNPCI}(S^*) = P^*$. Hence the PNP topological space (R, τ) is PNP irresolvable. But $\text{PNPInt}(\text{PNPCI}(S^*)) = \text{PNPInt}(P^*) = 0_R$. Therefore (R, τ) is not a PNP open hereditarily irresolvable space.

4.5 Theorem

Let (R, τ) be a PNP open hereditarily irresolvable. Then $\text{PNPInt}(P) \not\subseteq \text{PNPInt}(Q)^*$ for any two PNP dense sets P and Q in (R, τ) .

Proof.

Let P and Q be any two PNP dense sets in (R, τ) . Then $\text{PNPCI}(P) = 1_R$ and $\text{PNPCI}(Q) = 1_R$ implies that $\text{PNPInt}(\text{PNPCI}(P)) \neq 0_R$ and $\text{PNPInt}(\text{PNPCI}(Q)) \neq 0_R$. Since (R, τ) is PNP open hereditarily irresolvable, $\text{PNPInt}(P) \neq 0_R$ and $\text{PNPInt}(Q) \neq 0_R$. Hence by theorem 3.6, $P \not\subseteq Q^*$. Therefore $\text{PNPInt}(P) \subseteq P \not\subseteq Q^* \subseteq (\text{PNPInt}(Q))^*$. Hence we have $\text{PNPInt}(P) \subseteq (\text{PNPInt}(Q))^*$ for any two PNP dense sets P and Q in (R, τ) .

4.6 Theorem

Let (R, τ) be a PNP topological space. If (R, τ) is PNP open hereditarily irresolvable, then $\text{PNPInt}(P) = 0_R$ for any nonzero PNP dense set P in (R, τ) which implies that $\text{PNPInt}(\text{PNPCI}(P)) = 0_R$.

Proof:

Let P be a PNP set in (R, τ) such that $\text{PNPInt}(P) = 0_R$. We claim that $\text{PNPInt}(\text{PNPCI}(P)) = 0_R$. Suppose that $\text{PNPInt}(\text{PNPCI}(P)) \neq 0_R$. Since (R, τ) is PNP open hereditarily irresolvable, we have $\text{PNPInt}(P) \neq 0_R$ which is a contradiction to $\text{PNPInt}(P) = 0_R$. Hence $\text{PNPInt}(\text{PNPCI}(P)) = 0_R$.

4.7 Theorem

Let (R, τ) be a PNP topological space. If (R, τ) is PNP open hereditarily irresolvable, then $\text{PNPCI}(P) = 1_R$ for any nonzero PNP dense set P in (R, τ) which implies that $\text{PNPCI}(\text{PNPInt}(P)) = 0_R$.

Proof

Let P be a PNP set in (R, τ) such that $\text{PNPInt}(P) = 0_R$. Then we have $(\text{PNPInt}(P))^* = 0_R$ which implies that $\text{PNPInt}(P^*) = 0_R$. Since (R, τ) is PNP open hereditarily irresolvable by theorem 4.6. We have $\text{PNPInt}(\text{PNPCI}(P^*)) = 0_R$. Therefore $(\text{PNPCI}(\text{PNPInt}(P)))^* = 0_R$ implies that $\text{PNPCI}(\text{PNPInt}(P)) = 1_R$.

5 Somewhat PNP Continuous and PNP Somewhat PNP open

5.1 Definition

Let (R, τ) and (M, σ) be any two PNP topological spaces. A function $f: (R, \tau) \rightarrow (M, \sigma)$ is called somewhat PNP continuous if for a $P \in \sigma$ and $f^{-1}(P) \neq 0_R$, there exists a $Q \in \tau$ such that $Q \neq 0_R$ and $Q \subseteq f^{-1}(P)$.

5.2 Definition

Let (R, τ) and (M, σ) be any two PNP topological spaces. A function $f: (R, \tau) \rightarrow (M, \sigma)$ is called somewhat PNP open if for a $P \in \sigma$ and $P \neq 0_R$, there exists a $Q \in \tau$ such that $Q \neq 0_R$ and $Q \subseteq f(P)$.

5.3 Theorem

Let (R, τ) and (M, σ) be any two PNP topological spaces. A function $f: (R, \tau) \rightarrow (M, \sigma)$ is called somewhat PNP continuous and injective. If $\text{PNPInt}(P) = 0_R$ for any non-zero PNP set P in (R, τ) , then $\text{PNPInt}(f(P)) = 0_M$ in (M, σ) .

Proof

Let P be a non-zero PNP set in (R, τ) such that $\text{PNPInt}(P) = 0_R$. Now we prove that $\text{PNPInt}(f(P)) = 0_M$. Suppose that $\text{PNPInt}(f(P)) \neq 0_M$ in (M, σ) . Then there exists a nonzero PNP set Q in (M, σ) such that $Q \subseteq f(P)$. Thus, we have $f^{-1}(Q) \subseteq f^{-1}(f(P))$. Since f is somewhat PNP continuous, there exists a $S \in \tau$ such that $S \neq 0_R$ and $S \subseteq f^{-1}(Q)$. Hence $S \subseteq f^{-1}(Q) \subseteq P$ which implies that $\text{PNPInt}(P) \neq 0_R$. This is a contradiction. Hence $\text{PNPInt}(f(P)) = 0_M$ in (M, σ) .

5.4 Theorem

Let (R, τ) and (M, σ) be any two PNP topological spaces. A function $f: (R, \tau) \rightarrow (M, \sigma)$ is called somewhat PNP continuous, injective and $\text{PNPInt}(\text{PNPCL}(P)) = 0_R$ for any non-zero PNP set P in (R, τ) , then $\text{PNPInt}(\text{PNPCL}(f(P))) = 0_M$ in (M, σ) .

Proof

Let P be a non-zero PNP set in (R, τ) such that $\text{PNPInt}(\text{PNPCL}(P)) = 0_R$. Now we claim that $(\text{PNPCL}(f(P))) = 0_M$. Suppose that $\text{PNPInt}(\text{PNPCL}(f(P))) \neq 0_M$ in (M, σ) . Then $\text{PNPCL}(f(P)) \neq 0_M$ and $\text{PNPCL}(f(P))^* \neq 0_M$. Now $\text{PNPCL}(f(P))^* \neq 0_M \in M$. Since f is somewhat PNP continuous, there exists a $Q \in \tau$ such that $Q \neq 0_R$ and $Q \subseteq f^{-1}((\text{PNPCL}(f(P)))^*)$. Observe that $Q \subseteq f^{-1}((\text{PNPCL}(f(P)))^*)$ which implies that $f^{-1}(\text{PNPCL}(f(P))) \subseteq Q^*$.

Since f is injective, thus $P \subseteq f^{-1}(f(P)) \subseteq f^{-1}(\text{PNPCL}(f(P))) \subseteq Q^*$ which implies that $P \subseteq Q^*$. Therefore $Q \subseteq P^*$. This implies that $\text{PNPInt}(P^*) \neq 0_R$. Let $\text{PNPInt}(P^*) = S \neq 0_R$. Then we have $\text{PNPCL}(\text{PNPInt}(P^*)) = \text{PNPCL}(S) \neq 1_R$ which implies that $\text{PNPInt}(\text{PNPCL}(P)) \neq 0_R$. This is a contradiction. Hence $\text{PNPInt}(\text{PNPCL}(f(P))) = 0_M$ in (M, σ) .

5.5 Theorem

Let (R, τ) and (M, σ) be any two PNP topological spaces. If the function $f: (R, \tau) \rightarrow (M, \sigma)$ is somewhat PNP open and $\text{PNPInt}((P)) = 0_R$ for any non-zero PNP set P in (M, σ) , then $\text{PNPInt}(f^{-1}(P)) = 0_R$ in (R, τ) .

Proof

Let P be a non-zero PNP set in (M, σ) such that $\text{PNPInt}(P) = 0_R$. Now we claim that $\text{PNPInt}(f^{-1}(P)) = 0_R$ in (R, τ) . Suppose that $\text{PNPInt}(f^{-1}(P)) \neq 0_R$ in (R, τ) . Then there exists a non-zero PNP open set Q in (R, τ) such that $Q \subseteq f^{-1}(P)$. Thus we have $f(Q) \subseteq f(f^{-1}(P)) \subseteq P$. This implies that $f(Q) \subseteq P$. Since f is somewhat PNP open, there exists a $S \in \tau$ such that $S \neq 0_R$ and $S \subseteq f(Q)$. Therefore $S \subseteq f(Q) \subseteq P$ which implies that $S \subseteq A$. Hence $\text{PNPInt}((P)) \neq 0_R$ which is a contradiction. Hence $\text{PNPInt}(f^{-1}(P)) = 0_R$ in (R, τ) .

5.6 Theorem

Let (R, τ) and (M, σ) be any two PNP topological spaces Let (R, τ) be a PNP open hereditarily irresolvable space. If the function $f: (R, \tau) \rightarrow (M, \sigma)$ is somewhat PNP open, somewhat PNP continuous and a bijective function, then (M, σ) is a PNP open hereditarily irresolvable space.

Proof

Let P be a non-zero PNP set in (M, σ) such that $\text{PNPInt}(P) = 0_R$. Now $\text{PNPInt}(P) = 0_R$ and f is somewhat PNP open which implies $\text{PNPInt}(f^{-1}(P)) = 0_R$ in (R, τ) by theorem 5.5. Since (R, τ) is a PNP open hereditarily irresolvable, we have Suppose that $\text{PNPInt}(\text{PNPCI}(f^{-1}(P))) = 0_R$ in (R, τ) by theorem 4.6. Since $\text{PNPInt}(\text{PNPCI}(f^{-1}(P))) = 0_R$ and f is somewhat PNP continuous by theorem 5.4, we have that $\text{PNPInt}(\text{PNPCI}(f(f^{-1}(P)))) = 0_R$. Since f is onto, thus $\text{PNPInt}(\text{PNPCI}(P)) = 0_R$. Hence, by theorem 4.6, (M, σ) is a PNP open hereditarily irresolvable space.

5. Conclusion

In this paper we have proposed Pentapartitioned neutrosophic pythagorean resolvable and irresolvable spaces and studied some of its properties. Furthermore we also characterized Pentapartitioned Neutrosophic Pythagorean open hereditarily spaces and open functions in Pentapartitioned neutrosophic pythagorean topological spaces. In the future work, we extend the concept to Pentapartitioned Pythagorean almost resolvable and irresolvable spaces.

Funding: "This research received no external funding"

Conflicts of Interest: "The authors declare no conflict of interest."

References

1. K. Atanassov, Intuitionistic Fuzzy Sets, *Fuzzy Sets and Systems*.1986, volume 20 87-96.
2. Broumi S, Smarandache F (2014) Rough Neutrosophic sets, *Ital J Pure Appl Math* ,2014, volume 32:493-502
3. M. Caldas, R. Dhavaseelan, M. Ganster and S. Jafari, Neutrosophic Resolvable and irresolvable Spaces, *New trends in Theory and Applications*, volume II .327-335
4. C.L. Chang Fuzzy topological spaces, *J. Math. Anal. Appl*, 1984, volume 24, 182-190.

5. Rajashi Chatterjee, P. Majumdar and S. K. Samanta, On some similarity measures and entropy on quadripartitioned single valued neutrosophic sets, *Journal of Intelligent and Fuzzy Systems* ,**2016**, volume 302475-2485.
6. R. Radha, A. Stanis Arul Mary. Pentapartitioned Neutrosophic pythagorean Soft set, *IRJMETs*, **2021** , Volume 3(2),905-914.
7. R. Radha, A. Stanis Arul Mary. Pentapartitioned Neutrosophic Pythagorean Set, *IRJASH*, **2021**, volume 3, 62-82.
8. R. Radha, A. Stanis Arul Mary. Heptapartitioned neutrosophic sets, *IRJCT*, **2021** ,volume 2,222-230.
9. R. Radha, A. Stanis Arul Mary, F. Smarandache. Quadripartitioned Neutrosophic Pythagorean soft set, *International journal of Neutrosophic Science*, **2021**, volume14(1),9-23.
10. R. Radha, A. Stanis Arul Mary, F. Smarandache. Neutrosophic Pythagorean soft set, *Neutrosophic sets and systems*, **2021**,vol 42,65-78.
11. R. Radha ,A.Stanis Arul Mary, Pentapartitioned neutrosophic pythagorean resolvable and irresolvable spaces(Communicated)
12. Rama Malik, Surapati Pramanik. Pentapartitioned Neutrosophic set and its properties, *Neutrosophic Sets and Systems*, **2020**, Vol 36,184-192,2020
13. F.Smarandache, A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic, *American Research Press, Rehoboth*.
14. S. Sharmila, I. Arockiarani, On Intuitionistic Fuzzy almost resolvable and irresolvable spaces, *Asian Journal of Applied Sciences*,**2015**, Volume 3, Issue 6,918-928
15. Wang H, Smarandache F, Zhang YQ, Sunderraman R ,Single valued neutrosophic sets, *Multispace Multistruct* ,**2010** ,volume 4:410-413.
16. L. Zadeh , Fuzzy sets, *Information and Control* **1965**, volume 8, 87-96.

Received: May 7, 2021. Accepted: October 1, 2021