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On β generalized α closed sets in Neutrosophic Topological spaces

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Abstract: In this paper a new concept of neutrosophic closed sets called neutrosophic β generalized α closed set is introduced and their properties are thoroughly studied and analyzed and also discuss their relationship between basic closed sets in neutrosophic topological spaces. Some new interesting theorems are presented using newly introduced set.

Keywords: Neutrosophic β generalized α closed sets.

1. Introduction

The concept of intuitionistic fuzzy sets introduced by Atanassov(1), intuitionistic fuzzy topological space by Coker(2), after that Floretin Smarandache(3) in 1999 extended the neutrosophic sets, neutrosophic topological spaces by A. A. Salama and S. A. Alblowi(9). Further the basic sets like neutrosophic open sets(N-OS), neutrosophic semi open sets(N-SOS), neutrosophic pre open sets(N-POS), neutrosophic α open sets(N- α OS), neutrosophic regular open sets(N-ROS), neutrosophic β open sets(N- β os), neutrosophic b open sets(N-bOS) are introduced in neutrosophic topological spaces and their properties are studied by various authors(8,10).

The main aim of this paper is to analyze a new concept of neutrosophic closed sets called neutrosophic β generalized α closed sets also specialized some of their basic properties with examples.

2. Preliminaries:

Here in this paper (X, τ) is the neutrosophic topological space. Also the neutrosophic interior is denoted by $N\text{-Int}(A)$, neutrosophic closure is denoted by $N\text{-Cl}(A)$ and the complement of a neutrosophic set A is denoted by $N\text{-C}(A)$ and the empty and whole sets are denoted by 0 and 1 respectively.

Definition 2.1: Let X be a non-empty fixed set. A neutrosophic set (NS) A is an object having the form $A = \{ \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X \}$ where $\mu_A(x)$ represent the degree of membership, $\sigma_A(x)$ represent degree of indeterminacy and $\nu_A(x)$ represent the degree of nonmembership

Nonmembership respectively of each element $x \in X$ to the set A .

A Neutrosophic set $A = \{ \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X \}$ can be identified as an ordered triple $\langle \mu_A, \sigma_A, \nu_A \rangle$ in $]0, 1+[$ on X .

Definition 2.2: Let $A = \langle \mu_A, \sigma_A, \nu_A \rangle$ be a NS on X , then the complement Neutrosophic-C(A) may be defined as

1. Neutrosophic -C (A) = $\{ \langle x, (1-\mu_A(x)), (1-\nu_A(x)) \rangle : x \in X \}$
2. Neutrosophic -C (A) = $\{ \langle x, \nu_A(x), \sigma_A(x), \mu_A(x) \rangle : x \in X \}$
3. Neutrosophic -C (A) = $\{ \langle x, \nu_A(x), (1-\sigma_A(x)), \mu_A(x) \rangle : x \in X \}$

Definition 2.3: For any two neutrosophic sets $A = \{ \langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X \}$ and

$B = \{ \langle x, \mu_B(x), \sigma_B(x), \nu_B(x) \rangle : x \in X \}$ is

1. $(A \subseteq B) \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x)$ and $\nu_A(x) \geq \nu_B(x) \quad \forall x \in X$
2. $(A \subseteq B) \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \geq \sigma_B(x)$ and $\nu_A(x) \geq \nu_B(x) \quad \forall x \in X$
3. $(A \cap B) \Leftrightarrow \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ and $\nu_A(x) \vee \nu_B(x)$
4. $(A \cap B) \Leftrightarrow \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ and $\nu_A(x) \vee \nu_B(x)$
5. $(A \cup B) \Leftrightarrow \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ and $\nu_A(x) \wedge \nu_B(x)$
6. $(A \cup B) \Leftrightarrow \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ and $\nu_A(x) \wedge \nu_B(x)$

Definition 2.4: A neutrosophic topology (NT) on a non-empty set X is a family τ of neutrosophic subsets in X satisfies the following axioms:

$$(NT_1) 0_N, 1_N \in \tau$$

$$(NT_2) G_1 \cap G_2 \in \tau \text{ for any } G_1, G_2 \in \tau$$

$$(NT_3) \cup G_i \in \tau \quad \forall \{G_i : i \in J\} \subseteq \tau$$

In this case the pair (X, τ) is a neutrosophic topological space (NTS) and any neutrosophic set in τ is known as a neutrosophic open set (N-OS) in X . A neutrosophic set A is a neutrosophic closed set (NCS) if and only if its complement $N-C(A)$ is a neutrosophic open set in X .

Definition 2.6: A neutrosophic set A of a NTS X is said to be

- (i) A neutrosophic pre-open set (NP-OS) if $A \subseteq NInt(NCl(A))$
- (ii) A neutrosophic semi-open set (NS-OS) if $A \subseteq NCl(NInt(A))$
- (iii) A neutrosophic α -open set ($N\alpha$ -OS) if $A \subseteq NInt(NCl(NInt(A)))$
- (iv) A neutrosophic β -open set ($N\beta$ -OS) if $A \subseteq N-cl(N-int(N-cl(A)))$.
- (v) A neutrosophic regular open set (N-ROS) if $N-int(N-cl(A)) = A$,
- (vi) A neutrosophic b open set (N-bOS) if $A \subseteq N-int(N-cl(A)) \cup N-cl(N-int(A))$

Definition 2.7: A neutrosophic set A of a NTS X is said to be

- (i) A neutrosophic pre-closed set (NP-CS) if $NCl(NInt(A)) \subseteq A$
- (ii) A neutrosophic semi-closed set (NS-CS) if $NInt(NCl(A)) \subseteq A$
- (iii) A neutrosophic α -closed set ($N\alpha$ -CS) if $NCl(NInt(NCl(A))) \subseteq A$
- (iv) A neutrosophic β -closed set ($N\beta$ -CS) if $Nint(Ncl(Nint(A))) \subseteq A$
- (v) A neutrosophic regular closed set (N-RCS) if $N-cl(N-int(A)) = A$,
- (vi) A neutrosophic b closed set (N-bCS) if $N-int(N-cl(A)) \cap N-cl(N-int(A)) \subseteq A$

Definition 2.8:

Consider a NS A in NTS. The Neutrosophic beta interior & Neutrosophic beta closure of A are defined as

$$N\beta int(A) = \cup \{G, G \text{ is a } N\text{-}\beta\text{OS in } X \text{ and } G \subseteq A\}$$

$$N\beta cl(A) = \cap \{G, G \text{ is a } N\text{-}\beta\text{OS in } X \text{ and } A \subseteq G\}$$

Remark 2.9 :

Consider a NS A in NTS, then

$$(1) N\beta cl(A) = A \cap Nint(Ncl(Nint(A))),$$

$$(2) N\beta int(A) = A \cap Ncl(Nint(Ncl(A))).$$

Definition 2.10:

Consider a NS A in NTS. Then it is a Neutrosophic generalized beta closed set ($N\text{-}G\beta CS$) if $N\beta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is a NOS.

3. Neutrosophic β generalized α closed sets in Topological spaces

In this section we have introduced Neutrosophic β generalized α closed sets and studied some of their properties.

Definition 3.1: An Neutrosophic set A in an NTS (X, τ) is said to be an neutrosophic β generalized α closed set ($N\beta G\alpha CS$) if $N\beta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an $N\text{-}\alpha OS$ in (X, τ) .

The family of all $N\beta G\alpha CS$ s of an NTS (X, τ) is denoted by $N\beta G\alpha C(X)$.

Example 3.2: Let $X = \{a, b\}$ and $G_1 = \langle x, (0.5_a, 0.5_b), (0.5_a, 0.5_b), (0.5_a, 0.5_b) \rangle$ and $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_d), (0.6_a, 0.7_b) \rangle$. Then $\tau = \{0, G_1, G_2, 1\}$ is an NT on X . Here

Let $S = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b), (0.7_a, 0.8_b) \rangle$ then S is called an $N\beta G\alpha CS$ in X .

Proposition 3.3: Every Neutrosophic-CS is an Neutrosophic $\beta G\alpha CS$ in (X, τ) but reverse process is not true in general.

Proof: Let S be an $N\text{-}CS$ in X . Let we take $S \subseteq U$ where U is said to be an $N\text{-}\alpha OS$ in X . As $N\beta cl(S) \subseteq N\text{-}cl(S) = S \subseteq U$ by hypothesis, we have $N\beta cl(S) \subseteq U$. Thus S is an $N\beta G\alpha CS$ in (X, τ) .

Example 3.4: Let $X = \{a, b\}$, $G_1 = \langle x, (0.5_a, 0.5_b), (0.5_a, 0.5_b), (0.5_a, 0.5_b) \rangle$ and $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_d), (0.6_a, 0.7_b) \rangle$. Then $\tau = \{0, G_1, G_2, 1\}$ is an NT on X .

Here we take $S = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b), (0.7_a, 0.8_b) \rangle$ is an (N)- $\beta G\alpha CS$ but it not an (N)-CS in (X, τ) since $N-cl(S) = G_1^c \neq S$.

Proposition 3.5: Every Neutrosophic -SCS is an Neutrosophic - $\beta G\alpha CS$ in (X, τ) but reverse process is not true in general.

Proof: Let A be an N-SCS in X . Let we take $A \subseteq U$ and U is said to be an N- αOS in X . As $N\text{-}\beta\text{closure}(A) \subseteq N\text{-semi closure}(A) = A \subseteq U$ by hypothesis, we have $N\text{-}\beta\text{closure}(A) \subseteq U$. Then A is called an N- $\beta G\alpha CS$ in (X, τ) .

Example 3.6: Let $X = \{a, b\}$, $G_1 = \langle x, (0.5_a, 0.5_b), (0.5_a, 0.5_b), (0.5_a, 0.5_b) \rangle$ and $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b), (0.6_a, 0.7_b) \rangle$. Then $\tau = \{0, G_1, G_2, 1\}$ is an NT on X .

Here we take a point $S = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b), (0.7_a, 0.8_b) \rangle$ which satisfy N- $\beta G\alpha CS$ but does not satisfy N-SCS in (X, τ) since $N\text{-int}(N\text{-cl}(S)) = N\text{-int}(G_1^c) = G_1 \not\subseteq S$.

Proposition 3.7: Every Neutrosophic-PCS is an Neutrosophic - $\beta G\alpha CS$ in (X, τ) but reverse process is not true in general.

Proof: Let A be an N-PCS in X . Let we take $A \subseteq U$ and U is said to be an N- αOS in X . As $N\text{-}\beta\text{closure}(A) \subseteq N\text{-Pre closure}(A) = A \subseteq U$ by hypothesis, we have $N\text{-}\beta\text{closure}(A) \subseteq U$. Then A is called an N- $\beta G\alpha CS$ in (X, τ) .

Example 3.8: Let $X = \{a, b\}$, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b), (0.5_a, 0.4_b) \rangle$ and $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b), (0.6_a, 0.7_b) \rangle$. Then $\tau = \{0, G_1, G_2, 1\}$ is an NT on X . Here we take $A = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b), (0.6_a, 0.7_b) \rangle$ which satisfy N- $\beta G\alpha CS$ but does not satisfy N-PCS in (X, τ) as $N\text{-cl}(N\text{-int}(A)) = N\text{-cl}(G_2) = G_1^c \not\subseteq A$.

Proposition 3.9: Every Neutrosophic- αCS is an Neutrosophic- $\beta G\alpha CS$ in (X, τ) but not conversely in general.

Proof: Let A be an N- αCS in X . Let as assume $A \subseteq U$ and U is said to be an N- αOS in X . As $N\text{-}\beta cl(A) \subseteq N\text{-}\alpha cl(A) = A \subseteq U$ by hypothesis, we have $N\text{-}\beta cl(A) \subseteq U$. Then A is an N- $\beta G\alpha CS$ in (X, τ) .

Example 3.10: Let $X = \{a, b\}$, $G_1 = \langle x, (0.5_a, 0.5_b), (0.5_a, 0.5_b), (0.5_a, 0.5_b) \rangle$ and $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b), (0.6_a, 0.7_b) \rangle$. Then $\tau = \{0, G_1, G_2, 1\}$ is an IFT on X . Here we take $S = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b), (0.7_a, 0.8_b) \rangle$ which satisfy $N\text{-}\beta G\alpha CS$ but does not satisfy $N\text{-}\alpha CS$ in (X, τ) since $N\text{-cl}(N\text{-int}(N\text{-cl}(S))) = N\text{-cl}(N\text{-int}(G_1^c)) = N\text{-cl}(G_1) = G_1^c \not\subseteq S$.

Proposition 3.11: Every Neutrosophic-bCS is an Neutrosophic- $\beta G\alpha CS$ in (X, τ) but reverse process is not true in general.

Proof: Let A be an NbCS in X . Let as assume $A \subseteq U$ and U is said to be an $N\text{-}\alpha OS$ in X . As $N\text{-}\beta cl(A) \subseteq N\text{-}bcl(A) = A \subseteq U$ by hypothesis, we have $N\text{-}\beta cl(A) \subseteq U$. Then A is an $N\text{-}\beta G\alpha CS$ in (X, τ) .

Example 3.12: Let $X = \{a, b\}$, $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b), (0.5_a, 0.7_b) \rangle$ and $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b), (0.6_a, 0.7_b) \rangle$. Then $\tau = \{0, G_1, G_2, 1\}$ is an NT on X . Here we assume $A = \langle x, (0.4_a, 0.6_b), (0.4_a, 0.4_b), (0.4_a, 0.4_b) \rangle$ which satisfy $N\text{-}\beta G\alpha CS$ but does not satisfy $N\text{-}bCS$ in (X, τ) since $N\text{-int}(N\text{-cl}(A)) \cap N\text{-cl}(N\text{-int}(A)) = G_1 \cap G_1^c = G_1 \not\subseteq A$.

Proposition 3.13: Every Neutrosophic-RCS is an Neutrosophic- $\beta G\alpha CS$ in (X, τ) but reverse process is not true in general.

Proof: Let A be an N-RCS in X . Then A is an N-CS as every N-RCS is an N-CS, A is an $N\text{-}\beta G\alpha CS$ in (X, τ) .

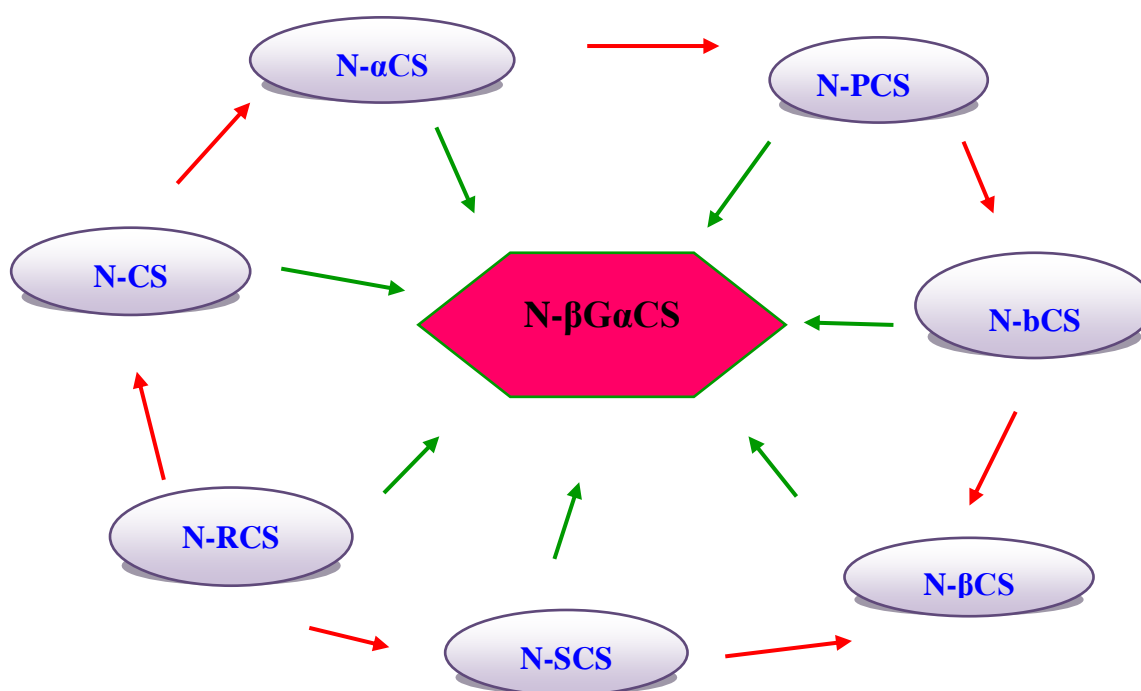
Example 3.14: Let $X = \{a, b\}$, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b), (0.5_a, 0.4_b) \rangle$ and $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b), (0.6_a, 0.7_b) \rangle$. Then $\tau = \{0, G_1, G_2, 1\}$ is an NT on X . Here we take $A = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b) \rangle$ which satisfy $N\text{-}\beta G\alpha CS$ but does not satisfy $N\text{-}RCS$ in (X, τ) as $N\text{-cl}(N\text{-int}(A)) = N\text{-cl}(G_2) = G_1^c \neq A$.

Proposition 3.15: Every Neutrosophic- βCS is an Neutrosophic- $\beta G\alpha CS$ in (X, τ) but reverse process is not true in general.

Proof: Let A be an $N\text{-}\beta CS$ in X . Let as assume $A \subseteq U$ and U is said to be an $N\text{-}\alpha OS$ in X . Now $N\text{-}\beta cl(A) = A \subseteq U$, by hypothesis. Therefore we have $N\text{-}\beta cl(A) \subseteq U$. Hence A is an $N\text{-}\beta G\alpha CS$ in (X, τ) .

Example 3.16: Let $X = \{a, b\}$, $G_1 = \langle x, (0.5_a, 0.3_b), (0.5_a, 0.7_b), (0.5_a, 0.7_b) \rangle$ and $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b), (0.6_a, 0.7_b) \rangle$. Then $\tau = \{0, G_1, G_2, 1\}$ is an NT on X . Here $A = \langle x, (0.4_a, 0.6_b), (0.4_a, 0.4_b), (0.4_a, 0.4_b) \rangle$ which satisfy $N-\beta G\alpha CS$ but does not satisfy $N-\beta CS$ in (X, τ) as $N-int(N-cl(N-int(A))) = N-int(N-cl(G_2)) = N-int(G_1^c) = G_1 \not\subseteq A$.

In the following diagram, we have provided the relation between various types of neutrosophic closedness.



Remark 3.17: The union of any two $N-\beta G\alpha CS$ s is not an $N-\beta G\alpha CS$ in general as seen in the following example.

Example 3.18: Let us assume $X = \{a, b\}$, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b), (0.5_a, 0.4_b) \rangle$, $G_2 = \langle x, (0.2_a, 0.3_b), (0.8_a, 0.7_b), (0.8_a, 0.7_b) \rangle$ and $G_3 = \langle x, (0.6_a, 0.7_b), (0.4_a, 0.3_b), (0.4_a, 0.3_b) \rangle$. Then $\tau = \{0, G_1, G_2, G_3, 1\}$ is an Neutrosophic Topology on X .

The NSs $A = \langle x, (0.1_a, 0.5_b), (0.9_a, 0.5_b), (0.9_a, 0.5_b) \rangle$, $B = \langle x, (0.5_a, 0.2_b), (0.5_a, 0.8_b), (0.5_a, 0.8_b) \rangle$ are $N\text{-}\beta\text{G}\alpha\text{CS}$ s in (X, τ) . But $A \cup B$ is not an $N\text{-}\beta\text{G}\alpha\text{CS}$ as $A \cup B = \langle x, (0.5_a, 0.5_b), (0.5_a, 0.5_b), (0.5_a, 0.5_b) \rangle \subseteq G_1$ but $N\text{-}\beta\text{cl}(A \cup B) = \langle x, (0.6_a, 0.7_b), (0.4_a, 0.3_b), (0.4_a, 0.3_b) \rangle \not\subseteq G_1$.

Remark 3.19: The intersection of any two $N\text{-}\beta\text{G}\alpha\text{CS}$ s is not an $N\text{-}\beta\text{G}\alpha\text{CS}$ in general as seen in the following example.

Example 3.20: Let $X = \{a, b\}$, $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b), (0.5_a, 0.4_b) \rangle$, $G_2 = \langle x, (0.2_a, 0.3_b), (0.8_a, 0.7_b), (0.8_a, 0.7_b) \rangle$ and $G_3 = \langle x, (0.6_a, 0.7_b), (0.4_a, 0.3_b), (0.4_a, 0.3_b) \rangle$. Then $\tau = \{0, G_1, G_2, G_3, 1\}$ is an NT on X . The IFSs $A = \langle x, (0.5_a, 0.8_b), (0.5_a, 0.2_b), (0.5_a, 0.2_b) \rangle$, $B = \langle x, (0.8_a, 0.6_b), (0.2_a, 0.4_b) \rangle$ are $N\text{-}\beta\text{G}\alpha\text{CS}$ s in (X, τ) . But $A \cap B$ is not an $N\text{-}\beta\text{G}\alpha\text{CS}$ as $A \cap B = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b), (0.5_a, 0.4_b) \rangle \subseteq G_1$ but $N\text{-}\beta\text{cl}(A \cap B) = \langle x, (0.6_a, 0.7_b), (0.4_a, 0.3_b), (0.4_a, 0.3_b) \rangle \not\subseteq G_1$.

Proposition 3.21: Let (X, τ) be an NTS. Then for every $A \in N\text{-}\beta\text{G}\alpha\text{C}(X)$ and for every $B \in \text{NS}(X)$, $A \subseteq B \subseteq N\text{-}\beta\text{cl}(A) \Rightarrow B \in N\text{-}\beta\text{G}\alpha\text{C}(X)$.

Proof: Let $B \subseteq U$ and also U be an $N\text{-}\alpha$ open set in X . Then since $A \subseteq B$, $A \subseteq U$. By hypothesis $B \subseteq N\text{-}\beta\text{cl}(A)$. Therefore $N\text{-}\beta\text{cl}(B) \subseteq N\text{-}\beta\text{cl}(N\text{-}\beta\text{cl}(A)) = N\text{-}\beta\text{cl}(A) \subseteq U$, since A is an $N\text{-}\beta\text{G}\alpha\text{CS}$ in X . Hence $B \in N\text{-}\beta\text{G}\alpha\text{C}(X)$.

Proposition 3.22: If R is an $N\text{-}\alpha\text{OS}$ and an $N\text{-}\beta\text{G}\alpha\text{CS}$ in (X, τ) , then R is an $N\text{-}\beta\text{CS}$ in (X, τ) .

Proof: Since $R \subseteq R$ and R is an $N\text{-}\alpha\text{OS}$ in X , by hypothesis $N\text{-}\beta\text{cl}(R) \subseteq R$. But $R \subseteq N\text{-}\beta\text{cl}(R)$. Therefore $N\text{-}\beta\text{cl}(R) = R$. Then R is an $N\text{-}\beta\text{CS}$ in (X, τ) .

Proposition 3.23: Let $H \subseteq R \subseteq X$ where R is said to be an $N\text{-}\alpha\text{OS}$ and it is an $N\text{-}\beta\text{G}\alpha\text{CS}$ in X . Then H is an $N\text{-}\beta\text{G}\alpha\text{CS}$ in R if and only if H is an $N\text{-}\beta\text{G}\alpha\text{CS}$ in X .

Proof: Necessity: Let as assume J be an $N\text{-}\alpha\text{OS}$ in X and $F \subseteq J$. Also let H be an $N\text{-}\beta\text{G}\alpha\text{CS}$ in R . Then clearly $H \subseteq R \cap J$ and $R \cap J$ is an $N\text{-}\alpha\text{OS}$ in A . Hence neutrosophic beta closure of H in R , $N\text{-}\beta\text{cl}_R(H) \subseteq R \cap J$ and by Proposition 3.22, R is an $N\text{-}\beta\text{CS}$. Therefore $N\text{-}\beta\text{cl}(R) = R$. Now neutrosophic beta closure of H in X ,

$N-\beta cl(H) \subseteq N-\beta cl(H) \cap N-\beta cl(R) = N-\beta cl(H) \cap R = N-\beta cl_R(H) \subseteq R \cap J \subseteq J$, that is $N-\beta cl(H) \subseteq R$, whenever $H \subseteq R$. Hence H is called $N-\beta G\alpha CS$ in X .

Sufficiency: Let V be an $N-\alpha OS$ in A , such that $F \subseteq V$. Since A is an $N-\alpha OS$ in X , V is an $N-\alpha OS$ in X . Therefore $N-\beta cl(F) \subseteq V$, as F is an $N-\beta G\alpha CS$ in X . Thus, $N-\beta cl_A(F) = N-\beta cl(F) \cap A \subseteq V \cap A \subseteq V$. Hence F is an $N-\beta G\alpha CS$ in A .

Proposition 3.24: An NS A is both an $N-OS$ and an $N-\beta G\alpha CS$ if and only if A is an $N-ROS$ in X .

Proof: Necessity: Let A be both an $N-OS$ and an $N-\beta G\alpha CS$ in X . Then A is an $N-\alpha OS$ and an $N-\beta G\alpha CS$. By Proposition 3.22, A is an $N-\beta CS$ and $N-int(N-cl(N-int(A))) \subseteq A$. Since A is an $N-OS$, $N-int(A) = A$. Therefore $N-int(N-cl(A)) \subseteq A$. Since A is an $N-OS$, it is an $N-POS$. Hence $A \subseteq N-int(N-cl(A))$. Therefore $A = N-int(N-cl(A))$ and A is an $N-ROS$ in X .

Sufficiency: Let A be an $N-ROS$ in X then $A = N-int(N-cl(A))$. Since every $N-ROS$ is an $N-OS$, A is an $N-OS$. We know $N-int(N-cl(N-int(A))) = N-int(N-cl(A)) = A \subseteq A$. Therefore A is an Neutrosophic β Closed set in X , and by Proposition 3.15, A is an $N-\beta G\alpha CS$ in X .

Proposition 3.25: Let (X, τ) be an NTS. Then $N-\beta C(X) = N-\beta G\alpha C(X)$ if every IFS in (X, τ) is an $N-\alpha OS$ in X .

Proof: Suppose that every NS in (X, τ) is an $N-\alpha OS$ in X . Let $A \in N-\beta G\alpha C(X)$. Then A is also an $N-\alpha OS$ by hypothesis. We know that A is an $N-\beta CS$. Therefore $A \in N-\beta C(X)$.

Hence $N-\beta G\alpha C(X) \subseteq N-\beta C(X) \longrightarrow (i)$

Let $A \in N-\beta C(X)$. Then by Proposition 3.15, A is an $N-\beta G\alpha CS$ and $A \in N-\beta G\alpha C(X)$. Hence $N-\beta C(X) \subseteq N-\beta G\alpha C(X) \xrightarrow{(ii)}$ From (i) and (ii) $N-\beta C(X) = N-\beta G\alpha C(X)$.

Proposition 3.26: Let R be an $N-\alpha OS$ and an $N-\beta G\alpha CS$ of (X, τ) . Then $R \cap F$ is an $N-\beta G\alpha CS$ of (X, τ) where F is an $N-CS$ of X .

Proof: Suppose that R is an $N-\alpha OS$ and an $N-\beta G\alpha CS$ of (X, τ) , then by Proposition 3.22, R is an $N-\beta CS$. But F is an $N-CS$ in X . Hence $R \cap F$ is an $N-\beta CS$ as every $N-CS$ is an $N-\beta CS$. Then $R \cap F$ is an $N-\beta G\alpha CS$ in X .

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