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# Neutrosophic Tri-Topological Space

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**Abstract:** In this article, we present the notion of neutrosophic tri-topological space as a generalization of neutrosophic bi-topological space. Besides, we study the different types of open sets and closed sets namely neutrosophic tri-open sets, neutrosophic tri-closed sets, neutrosophic tri-semi-open sets, neutrosophic tri-pre-closed sets, etc. via neutrosophic tri-topological spaces. Further, we investigate several properties, and prove some propositions, theorems on neutrosophic tri-topological spaces.

**Keywords:** Tri-open set; Tri-closed set; Tri-semi-open set; Tri-pre-open set; Neutrosophic crisp tri-topology; Neutrosophic tri-topology.

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## 1. Introduction

The concept of Neutrosophic Set (NS) was grounded by Smarandache [1] by extending the concept of Fuzzy Set [2] and intuitionistic FS [3]. The notion of Neutrosophic Topological Space (NTS) was developed by Salama and Alblowi [4] in 2012. Afterwards, Arokiarani et al. [5] studied the neutrosophic semi-open functions and established a relation between them. Iswaraya and Bageerathi [6] introduced the notion of neutrosophic semi-closed set and neutrosophic semi-open set via NTSs. Later on, Dhavaseelan, and Jafari [7] introduced the generalized neutrosophic closed sets. Thereafter, Pushpalatha and Nandhini [8] studied the neutrosophic generalized closed sets in NTS. Shanthi et al. [9] introduced the concept of neutrosophic generalized semi closed sets in NTS. Ebenanjar et al. [10] presented the neutrosophic  $b$ -open sets in NTS. Maheswari et al. [11] introduced the concept of neutrosophic generalized  $b$ -closed sets in NTS. Afterwards, the concept of generalized neutrosophic  $b$ -open set via NTS was introduced by Das and Pramanik [12] in 2020. Thereafter, the concept of neutrosophic  $\Phi$ -open sets and neutrosophic  $\Phi$ -continuous functions was presented by Das and Pramanik [13].

The notion of neutrosophic crisp topology on neutrosophic crisp set was introduced by Salama and Alblowi [14]. Later on, the notion of neutrosophic crisp tri-topological space was introduced by Al-Hamido and Gharibah [15] in 2018.

In 1963, Kelly [16] introduced the notion of bi-topological space. Thereafter, the concept of neutrosophic bi-topological space was presented by Ozturk and Ozkan [17] in 2019. Later on, Das and Tripathy [18] introduced the pairwise neutrosophic  $b$ -open sets via neutrosophic bi-topological spaces. Recently, Tripathy and Das [19] studied the concept of pairwise neutrosophic  $b$ -continuous functions via neutrosophic bi-topological spaces.

So, we received enough motivation to do research on neutrosophic tri-topological space to extend the concept of neutrosophic bi-topological space.

In this study, we procure the notion of neutrosophic tri-topological space as a generalization of the neutrosophic bi-topological space. Besides, we introduce the different types of open sets and closed sets namely, neutrosophic tri-open sets, neutrosophic tri-closed sets, neutrosophic tri-semi-open sets, neutrosophic tri-pre-closed sets, etc. via neutrosophic tri-topological spaces. Further, we investigate several properties of these kinds of neutrosophic tri-open sets.

**Research Gap:** No investigation on neutrosophic tri-topological space has been reported in the recent literature.

**Motivation:** To reduce the research gap, we present the notion and different properties of neutrosophic tri-topological space.

The remaining part of this article is divided into the following sections:

Section-2 is on preliminaries and definitions. In this section, we give some definitions and theorems, which are relevant to this article. In section-3, we present the notion of neutrosophic tri-topology and neutrosophic tri-topological space and also we give proofs of some theorems on neutrosophic tri-topological space. In section-4, we give the concluding remarks of the work done in the present article.

Throughout this article, we use the following short terms for the clarity of the presentation.

Short Terms	
Neutrosophic Set	NS
Neutrosophic Topology	NT
Neutrosophic Topological Space	NTS
Neutrosophic Open Set	N-O-S
Neutrosophic Closed Set	N-O-S
Neutrosophic Semi-Open	NSO
Neutrosophic Pre-Open	NPO
Neutrosophic Bi-Topological Space	NBTS
Neutrosophic Tri-Topological Space	N-Tri-TS

Neutrosophic Tri-Open Set	N-tri-OS
Neutrosophic Tri-Closed Set	N-tri-CS

**2. Some Relevant Definitions:**

**Definition 2.1.**[1] A neutrosophic set  $L$  over a universe of discourse  $\Psi$  is defined as follows:

$$L = \{(n, T_L(n), I_L(n), F_L(n)) : n \in \Psi\},$$

where  $T_L(n), I_L(n), F_L(n) (\in ]0, 1^+])$  are respectively denotes the truth, indeterminacy and falsity membership values of  $n \in \Psi$ , and so  $0 \leq T_L(n) + I_L(n) + F_L(n) \leq 3^+$  for all  $n \in \Psi$ .

**Definition 2.2.**[1] The neutrosophic null set ( $0_N$ ) and neutrosophic whole set ( $1_N$ ) over a universe of discourse  $\Psi$  are defined as follows:

(i)  $0_N = \{(n, 0, 0, 1) : n \in \Psi\};$

(ii)  $1_N = \{(n, 1, 0, 0) : n \in \Psi\}.$

Obviously,  $0_N \subseteq 1_N$ .

**Definition 2.3.**[20] Assume that  $\Psi$  be a universe of discourse. Then, a neutrosophic crisp set  $Q$  is defined by  $Q = \{Q_1, Q_2, Q_3\}$ , where  $Q_i (i=1,2,3)$  is a subset of  $\Psi$  such that  $Q_i \cap Q_j = \emptyset (i, j = 1,2,3 \text{ and } i \neq j)$

**Definition 2.4.**[4] Assume that  $\Psi$  be a universe of discourse, and  $\tau$  be a set of some NSs over  $\Psi$ . Then,  $\tau$  is called a Neutrosophic Topology (NT) on  $\Psi$  if the following axioms hold:

(i)  $0_N, 1_N \in \tau;$

(ii)  $X_1, X_2 \in \tau \Rightarrow X_1 \cap X_2 \in \tau;$

(iii)  $\{X_i : i \in \Delta\} \subseteq \tau \Rightarrow \cup X_i \in \tau.$

The pair  $(\Psi, \tau)$  is said to be an NTS. If  $X \in \tau$ , then  $X$  is called a neutrosophic-open-set (N-O-S) and its complement  $X^c$  is called a neutrosophic-closed-set (N-C-S).

**Definition 2.5.**[17] Assume that  $(\Psi, \tau_1)$  and  $(\Psi, \tau_2)$  be any two different NTSs. Then, we can call the triplet  $(\Psi, \tau_1, \tau_2)$  as a Neutrosophic Bi-Topological Space (NBTS).

**Definition 2.6.**[17] Assume that  $(\Psi, \tau_1, \tau_2)$  be an NBTS. Then, a neutrosophic subset  $X$  of  $\Psi$  is said to be a pairwise neutrosophic open set in  $(\Psi, \tau_1, \tau_2)$  if there exists an N-O-S  $T_1$  in  $(\Psi, \tau_1)$  and an N-O-S  $T_2$  in  $(\Psi, \tau_2)$  such that  $X = T_1 \cup T_2$ .

**Theorem 3.1.**[18] Let  $(\Psi, \tau_1, \tau_2)$  be an NBTS. Then, a neutrosophic subset  $X$  of  $\Psi$  is called as

(i)  $\tau_{ij}$ -neutrosophic-semi-open if and only if  $X \subseteq N_{cl}^i N_{int}^j(X);$

(ii)  $\tau_{ij}$ -neutrosophic-pre-open if and only if  $X \subseteq N_{int}^j N_{cl}^i(X);$

(iii)  $\tau_{ij}$ -neutrosophic- $b$ -open if and only if  $X \subseteq N_{cl}^i N_{int}^j(X) \cup N_{int}^j N_{cl}^i(X).$

**Theorem 3.2.**[18] Assume that  $(\Psi, \tau_1, \tau_2)$  be an NBTS. Then, every  $\tau_{ij}$ -neutrosophic-pre-open ( $\tau_{ij}$ -neutrosophic-semi-open) set is a  $\tau_{ij}$ -neutrosophic- $b$ -open.

**Definition 2.7.**[18] Assume that  $(\Psi, \tau_1, \tau_2)$  be an NBTS. Then  $X$ , an NS over  $\Psi$  is said to be a

(i) pairwise  $\tau_{ij}$ -neutrosophic-semi-open set (pairwise  $\tau_{ij}$ -neutrosophic-pre-open set) in an NBTS  $(\Psi, \tau_1, \tau_2)$  if and only if  $X = T \cup K$ , where  $T$  is a  $\tau_{ij}$ -neutrosophic-semi-open set ( $\tau_{ij}$ -neutrosophic-pre-open set) and  $K$  is a  $\tau_{ij}$ -neutrosophic-semi-open set ( $\tau_{ij}$ -neutrosophic-pre-open set) in  $(\Psi, \tau_1, \tau_2)$ .

(ii) pairwise  $\tau_{ij}$ -neutrosophic- $b$ -open set in an NBTS  $(\Psi, \tau_1, \tau_2)$  if  $X = T \cup K$ , where  $T$  is a  $\tau_{ij}$ -neutrosophic- $b$ -open set and  $K$  is a  $\tau_{ij}$ -neutrosophic- $b$ -open set in  $(\Psi, \tau_1, \tau_2)$ .

**Theorem 3.3.**[18] Assume that  $(\Psi, \tau_1, \tau_2)$  be an NBTS. If  $X$  is a pairwise  $\tau_{ij}$ -neutrosophic-semi open (pairwise  $\tau_{ij}$ -neutrosophic-pre-open) set, then it is also a pairwise  $\tau_{ij}$ -neutrosophic- $b$ -open set.

**Theorem 3.4.**[18] In an NBTS  $(\Psi, \tau_1, \tau_2)$ , the union of any two pairwise  $\tau_{ij}$ -neutrosophic- $b$ -open sets is a pairwise  $\tau_{ij}$ -neutrosophic- $b$ -open set.

**Definition 2.8.**[15] Assume that  $\tau_1, \tau_2$  and  $\tau_3$  be any three neutrosophic crisp topology on a universe of discourse  $W$ . Then,  $(W, \tau_1, \tau_2, \tau_3)$  is called a neutrosophic crisp tri-topological space.

### 3. Neutrosophic Tri-Topological Space:

In this section, we introduce the notion of neutrosophic tri-topological space, and formulate several results on it.

**Definition 3.1.** Suppose that  $(\Psi, \tau_1)$ ,  $(\Psi, \tau_2)$  and  $(\Psi, \tau_3)$  be any three different NTSs. Then, the structure  $(\Psi, \tau_1, \tau_2, \tau_3)$  is called a neutrosophic tri-topological space (N-Tri-TS).

**Example 3.1.** Let  $\Psi = \{u, v, w\}$  be a universe of discourse. Let  $X_1, X_2, Y_1, Y_2, Y_3, Z_1$  and  $Z_2$  be seven NSs over  $\Psi$  such that:

$$X_1 = \{(u, 0.9, 0.3, 0.7), (v, 0.5, 0.6, 0.8), (w, 0.3, 0.5, 0.2)\};$$

$$X_2 = \{(u, 0.7, 0.4, 0.7), (v, 0.5, 0.9, 0.9), (w, 0.1, 0.8, 0.4)\};$$

$$Y_1 = \{(u, 0.9, 0.3, 0.7), (v, 0.5, 0.6, 0.8), (w, 0.3, 0.5, 0.2)\};$$

$$Y_2 = \{(u, 1.0, 0.2, 0.5), (v, 0.9, 0.5, 0.8), (w, 0.5, 0.2, 0.1)\};$$

$$Y_3 = \{(u, 1.0, 0.2, 0.3), (v, 1.0, 0.1, 0.8), (w, 0.9, 0.1, 0.1)\};$$

$$Z_1 = \{(u, 0.5, 0.2, 0.5), (v, 0.9, 0.4, 0.3), (w, 0.7, 0.2, 0.5)\};$$

$$Z_2 = \{(u, 0.4, 0.5, 0.6), (v, 0.5, 0.6, 0.4), (w, 0.7, 0.6, 0.6)\};$$

Then, clearly  $\tau_1 = \{0_N, 1_N, X_1, X_2\}$ ,  $\tau_2 = \{0_N, 1_N, Y_1, Y_2, Y_3\}$  and  $\tau_3 = \{0_N, 1_N, Z_1, Z_2\}$  are three different NTs on  $\Psi$ . So,  $(\Psi, \tau_1, \tau_2, \tau_3)$  is a neutrosophic tri-topological space.

**Definition 3.2.** Assume that  $(\Psi, \tau_1, \tau_2, \tau_3)$  be an N-Tri-TS. Then  $R$ , an NS over  $\Psi$  is called a neutrosophic tri-open set (N-tri-OS) if  $R \in \tau_1 \cup \tau_2 \cup \tau_3$ . A neutrosophic set  $R$  is called a neutrosophic tri-closed set (N-Tri-CS) if and only if  $R^c$  is an N-Tri-OS in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

**Remark 3.1.** The collection of all neutrosophic tri-open sets and neutrosophic tri-closed sets in  $(\Psi, \tau_1, \tau_2, \tau_3)$  are denoted by N-Tri-O( $\Psi$ ) and N-Tri-C( $\Psi$ ) respectively.

**Theorem 3.1.** Every N-O-S in  $(\Psi, \tau_i)$ ,  $i=1,2,3$  is a neutrosophic tri-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

**Proof.** Assume that  $W$  be an N-O-S in  $(\Psi, \tau_i)$ ,  $i=1,2,3$ . Therefore,  $W \in \tau_i$ ,  $i=1,2,3$ . This implies,  $W \in \cup_{i \in \{1,2,3\}} \tau_i$ . Hence,  $W$  is a neutrosophic tri-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ . Therefore, every N-O-S in  $(\Psi, \tau_i)$ ,  $i=1,2,3$  is a neutrosophic tri-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

**Example 3.2.** Let us consider an N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$  as shown in Example 3.1. Then, the neutrosophic open sets  $X_1, X_2$  (in  $(\Psi, \tau_1)$ ),  $Y_1, Y_2, Y_3$  (in  $(\Psi, \tau_2)$ ),  $Z_1, Z_2$  (in  $(\Psi, \tau_3)$ ) are neutrosophic tri-open sets in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

**Remark 3.2.** In an N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$ , the union of two neutrosophic tri-open sets may not be a neutrosophic tri-open set. This follows from the following example.

**Example 3.3.** Consider the N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$  which has been shown in Example 3.1. Then, clearly  $Y_3$  and  $Z_1$  are two neutrosophic tri-open sets in  $(\Psi, \tau_1, \tau_2, \tau_3)$ . But their union  $Y_3 \cup Z_1 = \{(u, 1.0, 0.2, 0.3)$ ,

$(v,1.0,0.1,0.3), (w,0.9,0.1,0.1)$  is not a neutrosophic tri-open set, because  $Y_3 \cup Z_1 \notin \bigcup_{i \in \{1,2,3\}} \tau_i$ . Hence, the union of two neutrosophic tri-open sets may not be a neutrosophic tri-open set.

**Remark 3.3.** In an N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$ , the intersection of two neutrosophic tri-open sets may not be a neutrosophic tri-open set. This follows from the following example.

**Example 3.4.** Consider the N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$  which is shown in Example 3.1. Then, clearly  $X_1$  and  $Z_2$  are two neutrosophic tri-open sets in  $(\Psi, \tau_1, \tau_2, \tau_3)$ . But their union  $X_1 \cap Z_2 = \{(u,0.4,0.5,0.7), (v,0.5,0.6,0.8), (w,0.3,0.6,0.6)\}$  is not a neutrosophic tri-open set, because  $X_1 \cap Z_2 \notin \bigcup_{i \in \{1,2,3\}} \tau_i$ . Hence, the intersection of two neutrosophic tri-open sets may not be a neutrosophic tri-open set.

**Theorem 3.2.** Every N-C-S in  $(\Psi, \tau_i)$  ( $i=1,2,3$ ) is a neutrosophic tri-closed set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

**Proof.** Assume that  $W$  be an N-C-S in  $(\Psi, \tau_i)$ . So  $W^c$  is an N-O-S in  $(\Psi, \tau_i)$  ( $i=1,2,3$ ). Therefore,  $W^c \in \tau_i$ ,  $i=1,2,3$ . This implies,  $W^c \in \bigcup_{i \in \{1,2,3\}} \tau_i$ . This implies,  $W^c$  is a neutrosophic tri-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ . Therefore,  $W$  is a neutrosophic tri-closed set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ . Hence, every N-C-S in  $(\Psi, \tau_i)$ ,  $i=1,2,3$  is a neutrosophic tri-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

**Example 3.5.** Suppose that  $(\Psi, \tau_1, \tau_2, \tau_3)$  be an N-Tri-TS as shown in Example 3.1. Clearly,  $X_1^c = \{(u,0.1,0.7,0.3), (v,0.5,0.4,0.2), (w,0.7,0.5,0.8)\}$ ,  $X_2^c = \{(u,0.3,0.6,0.3), (v,0.5,0.1,0.1), (w,0.9,0.2,0.6)\}$  are NCSs in  $(\Psi, \tau_1)$ ,  $Y_1^c = \{(u,0.1,0.7,0.3), (v,0.1,0.5,0.2), (w,0.5,0.8,0.9)\}$ ,  $Y_2^c = \{(u,0.0,0.8,0.5), (v,0.1,0.5,0.2), (w,0.5,0.8,0.9)\}$ ,  $Y_3^c = \{(u,0.0,0.8,0.7), (v,0.0,0.9,0.2), (w,0.1,0.9,0.9)\}$  are NCSs in  $(\Psi, \tau_2)$ , and  $Z_1^c = \{(u,0.5,0.8,0.5), (v,0.1,0.6,0.7), (w,0.3,0.8,0.5)\}$ ,  $Z_2^c = \{(u,0.6,0.5,0.4), (v,0.5,0.4,0.6), (w,0.3,0.4,0.4)\}$  are NCSs in  $(\Psi, \tau_3)$ . Therefore,  $X_1, X_2$  are NOSs in  $(\Psi, \tau_1)$ ,  $Y_1, Y_2, Y_3$  are NOSs in  $(\Psi, \tau_2)$ , and  $Z_1, Z_2$  are NOSs in  $(\Psi, \tau_3)$ . By Example 3.2,  $X_1, X_2, Y_1, Y_2, Y_3, Z_1$  and  $Z_2$  are neutrosophic tri-open sets in  $(\Psi, \tau_1, \tau_2, \tau_3)$ . Therefore, by definition 3.2,  $X_1^c, X_2^c, Y_1^c, Y_2^c, Y_3^c, Z_1^c$  and  $Z_2^c$  are neutrosophic tri-closed sets in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

**Definition 3.3.** In an N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$ , an NS  $G$  over  $\Psi$  is called a neutrosophic tri-semi-open set if  $G$  is a neutrosophic semi open set in at least one of three NTSs  $(\Psi, \tau_1)$ ,  $(\Psi, \tau_2)$ , and  $(\Psi, \tau_3)$ .

**Example 3.6.** Consider the N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$  which is shown in Example 3.1. Then,  $W = \{(u,1.0,0.2,0.5), (v,0.7,0.5,0.7), (w,0.9,0.8,0.3)\}$  is a neutrosophic tri-semi-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ , because  $W$  is a neutrosophic semi open set in  $(\Psi, \tau_1)$ .

**Definition 3.4.** In an N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$ , a neutrosophic set  $G$  over  $\Psi$  is called a neutrosophic tri-pre-open set if  $G$  is a neutrosophic pre-open set in at least one of three NTSs  $(\Psi, \tau_1)$ ,  $(\Psi, \tau_2)$  and  $(\Psi, \tau_3)$ .

**Example 3.7.** Consider the N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$  which is shown in Example 3.1. Then,  $W = \{(u,0.5,0.3,0.2), (v,0.6,0.3,0.2), (w,0.8,0.4,0.2)\}$  is a neutrosophic tri-pre-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ , because  $W$  is a neutrosophic pre-open set in  $(\Psi, \tau_1)$ .

**Definition 3.5.** In an N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$ , an NS  $G$  over  $\Psi$  is called a neutrosophic tri- $b$ -open set if  $G$  is a neutrosophic  $b$ -open set in at least one of three NTSs  $(\Psi, \tau_1)$ ,  $(\Psi, \tau_2)$ , and  $(\Psi, \tau_3)$ .

**Example 3.8.** Consider the N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$  which is shown in Example 3.1. Then,  $W = \{(u,0.9,0.3,0.6), (v,0.8,0.6,0.4), (w,0.9,0.9,0.9)\}$  is a neutrosophic tri- $b$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ , because  $W$  is a neutrosophic  $b$ -open set in  $(\Psi, \tau_1)$ .

**Remark 3.4.** Assume that  $(\Psi, \tau_1, \tau_2, \tau_3)$  be an N-Tri-TS. Let  $\tau_{1,2,3} = \tau_1 \cup \tau_2 \cup \tau_3$ . Then,  $\tau_{1,2,3}$  may not be a neutrosophic topology on  $\Psi$  in general. This follows from the following example.

**Example 3.9.** Suppose that  $(\Psi, \tau_1, \tau_2, \tau_3)$  is a neutrosophic tri-topological space, where  $\tau_1 = \{0_N, 1_N, \{(u,0.6,0.3,0.6), (v,0.5,0.4,0.5), (w,0.8,0.5,0.8)\}, \{(u,0.7,0.1,0.4), (v,0.9,0.3,0.3), (w,0.9,0.1,0.5)\}\}$ ,  $\tau_2 = \{0_N, 1_N, \{(u,1.0,0.3,0.8), (v,0.8,0.4,0.7), (w,0.8,0.6,0.8)\}, \{(u,0.8,0.4,0.9), (v,0.5,0.5,1.0), (w,0.5,0.8,1.0)\}\}$ ,  $\tau_3 = \{0_N, 1_N, \{(u,0.5,0.2,0.5), (v,0.9,0.4,0.3), (w,0.7,0.2,0.5)\}, \{(u,0.4,0.5,0.6), (v,0.5,0.6,0.4), (w,0.7,0.6,0.6)\}\}$  are three different NTs on  $\Psi$ . Clearly,  $\{(u,0.7,0.1,0.4), (v,0.9,0.3,0.3), (w,0.9,0.1,0.5)\}$  and  $\{(u,0.8,0.4,0.9), (v,0.5,0.5,1.0), (w,0.5,0.8,1.0)\} \in \tau_{1,2,3}$ , but their intersection  $\{(u,0.7,0.4,0.9), (v,0.5,0.5,1.0), (w,0.5,0.8,1.0)\} \notin \tau_{1,2,3}$ . Hence,  $\tau_{1,2,3}$  does not form a topology on  $\Psi$ .

**Definition 3.6.** Assume that  $(\Psi, \tau_1, \tau_2, \tau_3)$  is an N-Tri-TS. Then  $R$ , an NS over  $\Psi$  is said to be a neutrosophic tri- $t$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$  if and only if there exist neutrosophic open sets  $R_1$  in  $\tau_1$ ,  $R_2$  in  $\tau_2$ , and  $R_3$  in  $\tau_3$  such that  $R = R_1 \cup R_2 \cup R_3$ .

**Example 3.10.** Consider the N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$  as shown in Example 3.7. Then,  $W = \{(u,1.0,0.2,0.5), (v,0.9,0.4,0.3), (w,0.8,0.2,0.5)\}$  is a neutrosophic tri- $t$ -open set, since there exist NOSs  $R_1 = \{(u,0.6,0.3,0.6), (v,0.5,0.4,0.5), (w,0.8,0.5,0.8)\}$  in  $\tau_1$ ,  $R_2 = \{(u,1.0,0.3,0.8), (v,0.8,0.4,0.7), (w,0.8,0.6,0.8)\}$  in  $\tau_2$ , and  $R_3 = \{(u,0.5,0.2,0.5), (v,0.9,0.4,0.3), (w,0.7,0.2,0.5)\}$  in  $\tau_3$  such that  $W = R_1 \cup R_2 \cup R_3$ .

**Remark 3.5.** In an N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$ , an NS  $G$  is called a neutrosophic tri- $t$ -closed set if and only if  $G^c$  is a neutrosophic tri- $t$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

**Theorem 3.3.** Assume that  $(\Psi, \tau_1, \tau_2, \tau_3)$  be an N-Tri-TS.

- (i) The neutrosophic null set ( $0_N$ ) and the neutrosophic whole set ( $1_N$ ) are always a neutrosophic tri- $t$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ ;
- (ii) Every NOS in  $(\Psi, \tau_1)$ ,  $(\Psi, \tau_2)$  and  $(\Psi, \tau_3)$  are neutrosophic tri- $t$ -open sets in  $(\Psi, \tau_1, \tau_2, \tau_3)$ ;
- (iii) Every NCS in  $(\Psi, \tau_1)$ ,  $(\Psi, \tau_2)$  and  $(\Psi, \tau_3)$  are neutrosophic tri- $t$ -closed sets in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

**Proof.** (i) We can write the neutrosophic null set ( $0_N$ ) as  $0_N = W \cup M \cup N$ , where  $W = 0_N, M = 0_N, N = 0_N$  are NOSs in  $(\Psi, \tau_1)$ ,  $(\Psi, \tau_2)$  and  $(\Psi, \tau_3)$  respectively. Hence,  $0_N$  is a neutrosophic tri- $t$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

Similarly, we can write the neutrosophic whole set ( $1_N$ ) as  $1_N = W \cup M \cup N$ , where  $W = 1_N, M = 1_N, N = 1_N$  are NOSs in  $(\Psi, \tau_1)$ ,  $(\Psi, \tau_2)$  and  $(\Psi, \tau_3)$  respectively. Hence,  $1_N$  is a neutrosophic tri- $t$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

(ii) Suppose that  $W$  be an NOS in  $(\Psi, \tau_1)$ . Now, we can write  $W = W \cup 0_N \cup 0_N$ . Therefore, there exist NOSs  $W, 0_N, 0_N$  in  $(\Psi, \tau_1)$ ,  $(\Psi, \tau_2)$  and  $(\Psi, \tau_3)$  respectively such that  $W = W \cup 0_N \cup 0_N$ . Hence,  $W$  is a neutrosophic tri- $t$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

Suppose that  $W$  be an NOS in  $(\Psi, \tau_2)$ . Now, we can write  $W = 0_N \cup W \cup 0_N$ . Therefore, there exist NOSs  $0_N, W, 0_N$  in  $(\Psi, \tau_1)$ ,  $(\Psi, \tau_2)$  and  $(\Psi, \tau_3)$  respectively such that  $W = 0_N \cup W \cup 0_N$ . Hence,  $W$  is a neutrosophic tri- $t$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

Suppose that  $W$  be an NOS in  $(\Psi, \tau_3)$ . Now, we can write  $W = 0_N \cup 0_N \cup W$ . Therefore, there exist NOSs  $0_N, 0_N, W$  in  $(\Psi, \tau_1)$ ,  $(\Psi, \tau_2)$ , and  $(\Psi, \tau_3)$  respectively such that  $W = 0_N \cup 0_N \cup W$ . Hence,  $W$  is a neutrosophic tri- $t$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

(iii) Suppose that  $W$  be an NCS in  $(\Psi, \tau_1)$ . So  $W^c$  is an NOS in  $(\Psi, \tau_1)$ . By the second part of this theorem,  $W^c$  is a neutrosophic tri- $t$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ . Hence,  $W$  is a neutrosophic tri- $t$ -closed set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

Suppose that  $W$  be an NCS in  $(\Psi, \tau_2)$ . So  $W^c$  is an NOS in  $(\Psi, \tau_2)$ . By the second part of this theorem,  $W^c$  is a neutrosophic tri- $t$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ . Hence,  $W$  is a neutrosophic tri- $t$ -closed set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

Suppose that  $W$  be an NCS in  $(\Psi, \tau_3)$ . So  $W^c$  is an NOS in  $(\Psi, \tau_3)$ . By the second part of this theorem,  $W^c$  is a neutrosophic tri- $t$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ . Hence,  $W$  is a neutrosophic tri- $t$ -closed set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

**Theorem 3.4.** In an N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$ , the union of two neutrosophic tri- $t$ -open sets is a neutrosophic tri- $t$ -open set.

**Proof.** Assume that  $X$  and  $Y$  are any two neutrosophic tri- $t$ -open sets in an N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$ . So there exist NOSs  $X_1, Y_1$  in  $(\Psi, \tau_1)$ ,  $X_2, Y_2$  in  $(\Psi, \tau_2)$ , and  $X_3, Y_3$  in  $(\Psi, \tau_3)$ , such that  $X = X_1 \cup X_2 \cup X_3$  and  $Y = Y_1 \cup Y_2 \cup Y_3$ . Now  $X \cup Y = (X_1 \cup X_2 \cup X_3) \cup (Y_1 \cup Y_2 \cup Y_3) = (X_1 \cup Y_1) \cup (X_2 \cup Y_2) \cup (X_3 \cup Y_3)$ . Since  $X_1, Y_1$  are NOSs in  $(\Psi, \tau_1)$ , so  $X_1 \cup Y_1$  is an NOS in  $(\Psi, \tau_1)$ . Since  $X_2, Y_2$  are NOSs in  $(\Psi, \tau_2)$ , so  $X_2 \cup Y_2$  is an NOS in  $(\Psi, \tau_2)$ . Since  $X_3, Y_3$  are NOSs in  $(\Psi, \tau_3)$ , so  $X_3 \cup Y_3$  is an NOS in  $(\Psi, \tau_3)$ . Therefore,  $X \cup Y$  is a neutrosophic tri- $t$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

**Remark 3.6.** In an N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$ , the intersection of any two neutrosophic tri- $t$ -open sets may not be a neutrosophic tri- $t$ -open set.

**Definition 3.7.** Assume that  $(\Psi, \tau_1, \tau_2, \tau_3)$  be an N-Tri-TS. Then  $Q$ , an NS over  $\Psi$  is said to be a neutrosophic tri- $t$ -semi-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$  if and only if there exists a neutrosophic semi open sets  $Q_1$  in  $(W, \tau_1)$ ,  $Q_2$  in  $(W, \tau_2)$ , and  $Q_3$  in  $(W, \tau_3)$  such that  $Q = Q_1 \cup Q_2 \cup Q_3$ .

**Theorem 3.5.** In an N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$ , every neutrosophic tri-semi-open set is a neutrosophic tri- $t$ -semi-open set.

**Proof.** Assume that  $X$  be a neutrosophic tri-semi-open set in an N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$ . So,  $X$  must be an NSO set in at least one of the NTS  $(\Psi, \tau_1)$ ,  $(\Psi, \tau_2)$ ,  $(\Psi, \tau_3)$ . So, there will be seven cases.

Case-1:  $X$  is an NSO set in  $(\Psi, \tau_1)$ ;

Case-2:  $X$  is an NSO set in  $(\Psi, \tau_2)$ ;

Case-3:  $X$  is an NSO set in  $(\Psi, \tau_3)$ ;

Case-4:  $X$  is an NSO set in  $(\Psi, \tau_1)$ , and  $(\Psi, \tau_2)$ ;

Case-5:  $X$  is an NSO set in  $(\Psi, \tau_1)$ , and  $(\Psi, \tau_3)$ ;

Case-6:  $X$  is an NSO set in  $(\Psi, \tau_2)$ , and  $(\Psi, \tau_3)$ ;

Case-7:  $X$  is an NSO set in  $(\Psi, \tau_1)$ ,  $(\Psi, \tau_2)$ , and  $(\Psi, \tau_3)$ .

In case-1, we can express,  $X = X \cup 0_N \cup 0_N$ , that is  $X$  is the union of NSO sets  $X$  (in  $(W, \tau_1)$ ),  $0_N$  (in  $(W, \tau_2)$ ), and  $0_N$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ -semi-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

In case-2, we can express,  $X = 0_N \cup X \cup 0_N$ , that is  $X$  is the union of NSO sets  $0_N$  (in  $(W, \tau_1)$ ),  $X$  (in  $(W, \tau_2)$ ), and  $0_N$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ -semi-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

In case-3, we can express,  $X = 0_N \cup 0_N \cup X$ , that is  $X$  is the union of NSO sets  $0_N$  (in  $(W, \tau_1)$ ),  $0_N$  (in  $(W, \tau_2)$ ), and  $X$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ -semi-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

In case-4, we can express,  $X = X \cup X \cup 0_N$ , that is  $X$  is the union of NSO sets  $X$  (in  $(W, \tau_1)$ ),  $X$  (in  $(W, \tau_2)$ ), and  $0_N$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ -semi-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

In case-5, we can express,  $X = X \cup 0_N \cup X$ , that is  $X$  is the union of NSO sets  $X$  (in  $(W, \tau_1)$ ),  $0_N$  (in  $(W, \tau_2)$ ), and  $X$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ -semi-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .



In case-6, we can express,  $X = 0_N \cup X \cup X$ , that is  $X$  is the union of NSO sets  $0_N$  (in  $(W, \tau_1)$ ),  $X$  (in  $(W, \tau_2)$ ), and  $X$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ -semi-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

In case-7, we can express,  $X = X \cup X \cup X$ , that is  $X$  is the union of NSO sets  $X$  (in  $(W, \tau_1)$ ),  $X$  (in  $(W, \tau_2)$ ), and  $X$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ -semi-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

Hence, every neutrosophic tri-semi-open set is a neutrosophic tri- $t$ -semi-open set.

**Definition 3.8.** Assume that  $(\Psi, \tau_1, \tau_2, \tau_3)$  be an N-Tri-TS. Then  $Q$ , an NS over  $\Psi$  is said to be a neutrosophic tri- $t$ -pre-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$  iff there exist a neutrosophic-pre-open sets  $Q_1$  in  $\tau_1$ ,  $Q_2$  in  $\tau_2$ , and  $Q_3$  in  $\tau_3$  such that  $Q = Q_1 \cup Q_2 \cup Q_3$ .

**Theorem 3.6.** In an N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$ , every neutrosophic tri-pre-open set is a neutrosophic tri- $t$ -pre-open set.

**Proof.** Assume that  $X$  is a neutrosophic tri-pre-open set in an N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$ . So,  $X$  must be an NPO set in at least one of the NTS  $(\Psi, \tau_1)$ ,  $(\Psi, \tau_2)$ ,  $(\Psi, \tau_3)$ . So, there will be seven cases.

Case-1:  $X$  is an NPO set in  $(\Psi, \tau_1)$ ;

Case-2:  $X$  is an NPO set in  $(\Psi, \tau_2)$ ;

Case-3:  $X$  is an NPO set in  $(\Psi, \tau_3)$ ;

Case-4:  $X$  is an NPO set in  $(\Psi, \tau_1)$ , and  $(\Psi, \tau_2)$ ;

Case-5:  $X$  is an NPO set in  $(\Psi, \tau_1)$ , and  $(\Psi, \tau_3)$ ;

Case-6:  $X$  is an NPO set in  $(\Psi, \tau_2)$ , and  $(\Psi, \tau_3)$ ;

Case-7:  $X$  is an NPO set in  $(\Psi, \tau_1)$ ,  $(\Psi, \tau_2)$ , and  $(\Psi, \tau_3)$ .

In case-1, we can express,  $X = X \cup 0_N \cup 0_N$ , that is  $X$  is the union of NPO sets  $X$  (in  $(W, \tau_1)$ ),  $0_N$  (in  $(W, \tau_2)$ ), and  $0_N$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ -pre-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

In case-2, we can express,  $X = 0_N \cup X \cup 0_N$ , that is  $X$  is the union of NPO sets  $0_N$  (in  $(W, \tau_1)$ ),  $X$  (in  $(W, \tau_2)$ ), and  $0_N$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ -pre-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

In case-3, we can express,  $X = 0_N \cup 0_N \cup X$ , that is  $X$  is the union of NPO sets  $0_N$  (in  $(W, \tau_1)$ ),  $0_N$  (in  $(W, \tau_2)$ ), and  $X$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ -pre-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

In case-4, we can express,  $X = X \cup X \cup 0_N$ , that is  $X$  is the union of NPO sets  $X$  (in  $(W, \tau_1)$ ),  $X$  (in  $(W, \tau_2)$ ), and  $0_N$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ -pre-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

In case-5, we can express,  $X = X \cup 0_N \cup X$ , that is  $X$  is the union of NPO sets  $X$  (in  $(W, \tau_1)$ ),  $0_N$  (in  $(W, \tau_2)$ ), and  $X$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ -pre-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

In case-6, we can express,  $X = 0_N \cup X \cup X$ , that is  $X$  is the union of NPO sets  $0_N$  (in  $(W, \tau_1)$ ),  $X$  (in  $(W, \tau_2)$ ), and  $X$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ -pre-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

In case-7, we can express,  $X = X \cup X \cup X$ , that is  $X$  is the union of NPO sets  $X$  (in  $(W, \tau_1)$ ),  $X$  (in  $(W, \tau_2)$ ), and  $X$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ -pre-open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

Hence, every neutrosophic tri-pre-open set is a neutrosophic tri- $t$ -pre-open set.

**Definition 3.9.** Assume that  $(\Psi, \tau_1, \tau_2, \tau_3)$  is an N-Tri-TS. Then  $Q$ , an NS over  $\Psi$  is said to be a neutrosophic tri- $t$ - $b$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$  if and only if there exist three neutrosophic- $b$ -open sets, namely  $Q_1$  in  $\tau_1$ ,  $Q_2$  in  $\tau_2$ , and  $Q_3$  in  $\tau_3$  such that  $Q = Q_1 \cup Q_2 \cup Q_3$ .

**Theorem 3.7.** In an N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$ , every neutrosophic tri- $b$ -open set is a neutrosophic tri- $t$ - $b$ -open set.

**Proof.** Assume that  $X$  be a neutrosophic tri- $b$ -open set in an N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$ . So,  $X$  must be a neutrosophic  $b$ -open set in at least one of the NTS  $(\Psi, \tau_1), (\Psi, \tau_2), (\Psi, \tau_3)$ . So there will be seven cases.

Case-1:  $X$  is a neutrosophic  $b$ -open set in  $(\Psi, \tau_1)$ ;

Case-2:  $X$  is a neutrosophic  $b$ -open set in  $(\Psi, \tau_2)$ ;

Case-3:  $X$  is a neutrosophic  $b$ -open set in  $(\Psi, \tau_3)$ ;

Case-4:  $X$  is a neutrosophic  $b$ -open set in  $(\Psi, \tau_1)$ , and  $(\Psi, \tau_2)$ ;

Case-5:  $X$  is a neutrosophic  $b$ -open set in  $(\Psi, \tau_1)$ , and  $(\Psi, \tau_3)$ ;

Case-6:  $X$  is a neutrosophic  $b$ -open set in  $(\Psi, \tau_2)$ , and  $(\Psi, \tau_3)$ ;

Case-7:  $X$  is a neutrosophic  $b$ -open set in  $(\Psi, \tau_1), (\Psi, \tau_2)$ , and  $(\Psi, \tau_3)$ .

In case-1, we can express,  $X = X \cup 0_N \cup 0_N$ , that is  $X$  is the union of neutrosophic  $b$ -open sets  $X$  (in  $(W, \tau_1)$ ),  $0_N$  (in  $(W, \tau_2)$ ), and  $0_N$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ - $b$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

In case-2, we can express,  $X = 0_N \cup X \cup 0_N$ , that is  $X$  is the union of neutrosophic  $b$ -open sets  $0_N$  (in  $(W, \tau_1)$ ),  $X$  (in  $(W, \tau_2)$ ), and  $0_N$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ - $b$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

In case-3, we can express,  $X = 0_N \cup 0_N \cup X$ , that is  $X$  is the union of neutrosophic  $b$ -open sets  $0_N$  (in  $(W, \tau_1)$ ),  $0_N$  (in  $(W, \tau_2)$ ), and  $X$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ - $b$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

In case-4, we can express,  $X = X \cup X \cup 0_N$ , that is  $X$  is the union of neutrosophic  $b$ -open sets  $X$  (in  $(W, \tau_1)$ ),  $X$  (in  $(W, \tau_2)$ ), and  $0_N$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ - $b$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

In case-5, we can express,  $X = X \cup 0_N \cup X$ , that is  $X$  is the union of neutrosophic  $b$ -open sets  $X$  (in  $(W, \tau_1)$ ),  $0_N$  (in  $(W, \tau_2)$ ), and  $X$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ - $b$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

In case-6, we can express,  $X = 0_N \cup X \cup X$ , that is  $X$  is the union of neutrosophic  $b$ -open sets  $0_N$  (in  $(W, \tau_1)$ ),  $X$  (in  $(W, \tau_2)$ ), and  $X$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ - $b$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

In case-7, we can express,  $X = X \cup X \cup X$ , that is  $X$  is the union of neutrosophic  $b$ -open sets  $X$  (in  $(W, \tau_1)$ ),  $X$  (in  $(W, \tau_2)$ ), and  $X$  (in  $(W, \tau_3)$ ). Therefore,  $X$  is a neutrosophic tri- $t$ - $b$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

Hence, every neutrosophic tri- $b$ -open set is a neutrosophic tri- $t$ - $b$ -open set.

**Definition 3.10.** Assume that  $(\Psi, \tau_1, \tau_2, \tau_3)$  be an N-Tri-TS. Let  $X$  be an NS over  $\Psi$ . The neutrosophic tri- $t$ -interior ( $N$ -tri- $t_{int}$ ) and neutrosophic tri- $t$ -closure ( $N$ -tri- $t_{cl}$ ) of  $X$  is defined as follows:

$$N\text{-tri-}t_{int}(X) = \cup \{Y : Y \text{ is a neutrosophic tri-}t\text{-open set and } Y \subseteq X\};$$

$$N\text{-tri-}t_{cl}(X) = \cap \{Y : Y \text{ is a neutrosophic tri-}t\text{-closed set and } X \subseteq Y\}.$$

It is clearly observed that  $N$ -tri- $t_{int}(X)$  is the largest neutrosophic tri- $t$ -open set which is contained in  $X$  and  $N$ -tri- $t_{cl}(X)$  is the smallest neutrosophic tri- $t$ -closed set which contains  $X$ .

**Theorem 3.8.** Let  $(\Psi, \tau_1, \tau_2, \tau_3)$  be an N-Tri-TS. Let  $X$  and  $Y$  be two neutrosophic sets over  $\Psi$ . Then,

(i)  $N$ -tri- $t_{int}(X) \subseteq X$ ;

(ii)  $X \subseteq Y \Rightarrow N$ -tri- $t_{int}(X) \subseteq N$ -tri- $t_{int}(Y)$ ;

(iii) If  $X$  is a neutrosophic tri- $t$ -open set, then  $N$ -tri- $t_{int}(X) = X$ ;

(iv)  $N$ -tri- $t_{int}(0_N) = 0_N$ , and  $N$ -tri- $t_{int}(1_N) = 1_N$ .

**Proof.**

(i) From the definition 3.10, we see that  $N$ -tri- $t_{int}(X) = \cup \{B : B \text{ is a neutrosophic tri-}t\text{-open set and } B \subseteq X\}$ . Since  $B \subseteq X$ , so  $\cup \{B : B \text{ is a neutrosophic tri-}t\text{-open set and } B \subseteq X\} \subseteq X$ . Therefore,  $N$ -tri- $t_{int}(X) \subseteq X$ .

(ii) Suppose that  $X$  and  $Y$  are two neutrosophic sets over  $\Psi$  such that  $X \subseteq Y$ . Then,

$$N\text{-tri-}t_{int}(X)$$

$$\begin{aligned}
 &= \cup\{B: B \text{ is a neutrosophic tri-}t\text{-open set and } B \subseteq X\}; \\
 &\subseteq \cup\{B: B \text{ is a neutrosophic tri-}t\text{-open set and } B \subseteq Y\} \quad [\text{since } X \subseteq Y] \\
 &= N\text{-tri-}t_{int}(Y) \\
 &\Rightarrow N\text{-tri-}t_{int}(X) \subseteq N\text{-tri-}t_{int}(Y).
 \end{aligned}$$

Therefore,  $X \subseteq Y \Rightarrow N\text{-tri-}t_{int}(X) \subseteq N\text{-tri-}t_{int}(Y)$ .

(iii) Assume that  $X$  be a neutrosophic tri- $t$ -open set in an N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$ . Now,  $N\text{-tri-}t_{int}(X) = \cup\{B: B \text{ is a neutrosophic tri-}t\text{-open set and } B \subseteq X\}$ . Since  $X$  is a neutrosophic tri- $t$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ , so  $X$  is the largest neutrosophic tri- $t$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ , which is contained in  $X$ . Hence  $\cup\{B: B \text{ is a neutrosophic tri-}t\text{-open set and } B \subseteq X\} = X$ . Therefore,  $N\text{-tri-}t_{int}(X) = X$ .

(iv) We know that  $0_N$ , and  $1_N$  are neutrosophic tri- $t$ -open sets in  $(\Psi, \tau_1, \tau_2, \tau_3)$ , so by the third part of this theorem, we have  $N\text{-tri-}t_{int}(0_N) = 0_N, N\text{-tri-}t_{int}(1_N) = 1_N$ .

**Theorem 3.9.** Let  $(\Psi, \tau_1, \tau_2, \tau_3)$  be an N-Tri-TS. Let  $X$  and  $Y$  be two neutrosophic sets over  $\Psi$ . Then,

- (i)  $X \subseteq N\text{-tri-}t_{cl}(X)$ ;
- (ii)  $X \subseteq Y \Rightarrow N\text{-tri-}t_{cl}(X) \subseteq N\text{-tri-}t_{cl}(Y)$ ;
- (iii)  $X$  is a neutrosophic tri- $t$ -closed set iff  $N\text{-tri-}t_{cl}(X) = X$ ;
- (iv)  $N\text{-tri-}t_{cl}(0_N) = 0_N$ , and  $N\text{-tri-}t_{cl}(1_N) = 1_N$ ;

**Proof.** (i) From the definition 3.10, we see that  $N\text{-tri-}t_{cl}(X) = \cap\{B: B \text{ is a neutrosophic tri-}t\text{-closed set and } X \subseteq B\}$ . Since each  $X \subseteq B$ , so  $X \subseteq \cap\{B: B \text{ is a neutrosophic tri-}t\text{-closed set and } X \subseteq B\}$ . Therefore,  $X \subseteq N\text{-tri-}t_{cl}(X)$ .

(ii) Suppose that  $X$  and  $Y$  are two neutrosophic sets over  $\Psi$  such that  $X \subseteq Y$ . Then,  $N\text{-tri-}t_{cl}(X) = \cap\{B: B \text{ is a neutrosophic tri-}t\text{-closed set and } X \subseteq B\}$ .   
 $\subseteq \cap\{B: B \text{ is a neutrosophic tri-}t\text{-closed set and } Y \subseteq B\} \quad [\text{since } X \subseteq Y]$   
 $= N\text{-tri-}t_{cl}(Y)$ .

Therefore,  $X \subseteq Y \Rightarrow N\text{-tri-}t_{cl}(X) \subseteq N\text{-tri-}t_{cl}(Y)$ .

(iii) Assume that  $X$  be a neutrosophic tri- $t$ -closed set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ . Now,  $N\text{-tri-}t_{cl}(X) = \cap\{B: B \text{ is a neutrosophic tri-}t\text{-closed set and } X \subseteq B\}$ . Since  $X$  is a neutrosophic tri- $t$ -closed set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ , so  $X$  is the smallest neutrosophic tri- $t$ -closed set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ , which contains  $X$ . Therefore,  $\cap\{B: B \text{ is a neutrosophic tri-}t\text{-closed set and } X \subseteq B\} = X$ . Therefore,  $N\text{-tri-}t_{cl}(X) = X$ .

(iv) It is known that,  $0_N$  and  $1_N$  are neutrosophic tri- $t$ -closed sets in  $(\Psi, \tau_1, \tau_2, \tau_3)$ . So, by the third part of this theorem, we have  $N\text{-tri-}t_{cl}(0_N) = 0_N, N\text{-tri-}t_{cl}(1_N) = 1_N$ .

**Theorem 3.11.** Let  $(\Psi, \tau_1, \tau_2, \tau_3)$  be an N-Tri-TS. Let  $X$  be an NS over  $\Psi$ . Then,  $\tau_i\text{-}N_{int}(X) = N\text{-tri-}t_{int}(X)$ .

**Proof.** Assume that  $X$  be a neutrosophic subset of an N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$ . Now,  $\tau_i\text{-}N_{int}(X) = \cup\{Y: Y \text{ is an NOS in } (\Psi, \tau_i) \text{ and } Y \subseteq X\}$ . Since  $Y$  is an NOS in  $(\Psi, \tau_i)$ , so by second part of Theorem 3.3.,  $Y$  is a neutrosophic tri- $t$ -open set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

$$\begin{aligned}
 &\text{Therefore, } \tau_i\text{-}N_{int}(X) \\
 &= \cup\{Y: Y \text{ is an NOS in } (\Psi, \tau_i) \text{ and } Y \subseteq X\} \\
 &= \cup\{Y: Y \text{ is a neutrosophic tri-}t\text{-open set in } (\Psi, \tau_1, \tau_2, \tau_3), \text{ and } Y \subseteq X\} \\
 &= N\text{-tri-}t_{int}(X).
 \end{aligned}$$

Hence, in an N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$ ,  $\tau_i\text{-}N_{int}(X) = N\text{-tri-}t_{int}(X)$  for any neutrosophic set  $X$ .

**Theorem 3.12.** Let  $(\Psi, \tau_1, \tau_2, \tau_3)$  be an N-Tri-TS. Let  $X$  be an NS over  $\Psi$ . Then,  $\tau_i\text{-}N_{cl}(X) \subseteq N\text{-tri-}t_{cl}(X)$ .

**Proof.** Assume that  $X$  be a neutrosophic subset of an N-Tri-TS  $(\Psi, \tau_1, \tau_2, \tau_3)$ . Now,  $\tau_i\text{-}N_{cl}(X) = \cap\{Y: Y \text{ is an NCS in } (\Psi, \tau_i) \text{ and } X \subseteq Y\}$ . Since  $Y$  is an NCS in  $(\Psi, \tau_i)$ , so by third part of Theorem 3.3.,  $Y$  is a neutrosophic tri- $t$ -closed set in  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

Therefore,  $\tau_i\text{-}N_{cl}(X)$

$$= \cap\{Y: Y \text{ is an NCS in } (\Psi, \tau_i) \text{ and } X \subseteq Y\}$$

$$= \cap\{Y: Y \text{ is a neutrosophic tri-}t\text{-closed set in } (\Psi, \tau_1, \tau_2, \tau_3) \text{ and } X \subseteq Y\}$$

$$= N\text{-tri-}t_{cl}(X).$$

Hence,  $\tau_i\text{-}N_{cl}(X) = N\text{-tri-}t_{cl}(X)$ , for any neutrosophic subset  $X$  of  $(\Psi, \tau_1, \tau_2, \tau_3)$ .

#### 4. Conclusions

In this study, we introduce the notion neutrosophic tri-topological spaces. Also, we establish some of their basic properties. By defining neutrosophic tri-topology and neutrosophic tri-topological space, we present well described examples and proofs of some theorems on neutrosophic tri-topological spaces.

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