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Abstract. In this study we introduce the concept of rough bipolar interval neutrosophic sets which is a combination of rough sets and bipolar interval neutrosophi sets. Also we define union, complement, intersection and some interesting properties of this set.

Keywords: Rough sets, Bipolar neutrosophic sets, interval neutrosophic sets , rough bipolar neutrosophic sets.

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1. **Introduction**

The notion of fuzzy set theory studied by Zadeh [9] in 1965 to deal with uncertainty. This theory has been applied in many real life applications to handle uncertainty. After Zadeh [10] introduced interval valued fuzzy sets. Attanasov [1] extended the fuzzy sets to intutionistic fuzzy set. In 1998, Smarandache [7] studied the concept of neutrosohic set. Lee [6] introduced the concept of bipolar fuzzy sets, as an extension of fuzzy sets. In bipolar fuzzy sets the degree of membership is extended from $[0, 1]$to $[-1, 1]$. In a bipolar fuzzy set, if the degree of membership of an element is zero, then we say the element is unrelated to the corresponding property, the membership degree (0,1] of an element specifies that the element somewhat satisfies the property, and the membership degree [-1,0) of an element implies that the element somewhat satisfies the implicit counter property. In 2014, Broumi et al. [2] , [3] presented the concept of rough neutrosophic set to deal indeterminacy in more flexible way. The rough set theory familiarized by Pawlak is an excellent mathematical tool for the analysis of uncertain, inconsistency and vague description of objects. Deli et al. [4] defined bipolar neutrosophic set and showed numerical example for multi-criteria decision making problem. Gong et al. [5]

introduced interval valued rough fuzzy set. Subha et al. [8] applied interval valued rough fuzzy sets in many real life applications.

2. Preliminaries

For basic concepts related to this paper refer [1], [2], [3], [4], [5], [6], [7], [9] , [10].

3. Rough Bipolar Interval Neutrosophic sets

Let H be the universe and R be an equivalence relation on H. Let I be a bipolar interval neutrosophic set in H. Then the lower and upper approximation of I in (H, R) is defined by
$L(I) = \{\langle x, L(a_I^p), L(b_I^p), L(c_I^p), L(a_I^n), L(b_I^n), L(c_I^n)\rangle, x \in H\}$
$U(I) = \{\langle x, U(a_I^p), U(b_I^p), U(c_I^p), U(a_I^n), U(b_I^n), U(c_I^n)\rangle, x \in H\}$
where
$L(a_I^p)(x) = \bigwedge_{z\in[x]_R} a^p(z)$, $L(b_I^p)(x) = \bigvee_{z\in[x]_R} b^p(z)$, $L(c_I^p)(x) = \bigvee_{z\in[x]_R} c^p(z)$
$L(a_I^n)(x) = \bigwedge_{z\in[x]_R} a^n(z)$, $L(b_I^n)(x) = \bigvee_{z\in[x]_R} b^n(z)$, $L(c_I^n)(x) = \bigvee_{z\in[x]_R} c^n(z)$
$U(a_I^p)(x) = \bigvee_{z\in[x]_R} a^p(z)$, $U(b_I^p)(x) = \bigwedge_{z\in[x]_R} b^p(z)$, $U(c_I^p)(x) = \bigwedge_{z\in[x]_R} c^p(z)$
$U(a_I^n)(x) = \bigvee_{z\in[x]_R} a^n(z)$, $U(b_I^n)(x) = \bigwedge_{z\in[x]_R} b^n(z)$, $U(c_I^n)(x) = \bigwedge_{z\in[x]_R} c^n(z)$ for all $x \in H$.
Then $R(I) = (L(I), U(I))$ is called a rough bipolar interval neutrosophic set in (H, R). Here $L(I)$ and $U(I)$ are also bipolar interval neutrosophic sets.

Example 3.1. Let $H = \{i, j, k, l, m\}$ be the universe. Let I be the bipolar interval neutrosophic set defined by,
$i = ([0.60, 0.70], [0.40, 0.50], [0.10, 0.20], [-0.90, -0.80], [-0.70, -0.60], [-0.30, -0.20])$
$j = ([0.40, 0.50], [0.10, 0.20], [0.01, 0.20], [-0.70, -0.50], [-0.40, -0.30], [-0.80, -0.70])$
$k = ([0.50, 0.40], [0.10, 0.30], [0.60, 0.70], [-0.60, -0.50], [-0.30, -0.20], [-0.70, -0.60])$
$l = ([0.65, 0.75], [0.58, 0.68], [0.51, 0.61], [-0.85, -0.75], [-0.81, -0.71], [-0.68, -0.58])$
$m = ([0.81, 0.91], [0.62, 0.72], [0.34, 0.44], [-0.85, -0.75], [-0.65, -0.55], [-0.30, -0.20])$
Then the equivalence classes of H are defined by $\{\{i, j, m\}, \{k, l\}\}$. The lower approximation of I is
$L(a_I^p)(x) = \{(i, [.40, .50]), (j, [.40, .50]), (k, [.50, .40]), (l, [.50, .40]), (m, [.40, .50])\}$
$L(b_I^p)(x) = \{(i, [.62, .72]), (j, [.62, .72]), (k, [.58, .68]), (l, [.58, .48]), (m, [.62, .72])\}$
$L(c_I^p)(x) = \{(i, [.34, .44]), (j, [.34, .44]), (k, [.60, .70]), (l, [.60, .70]), (m, [.34, .44])\}$
$L(a_I^n)(x) =$
$\{(i, [-.90, -.80]), (j, [-.90, -.80]), (k, [-.85, -.75]), (l, [-.85, -.75]), (m, [-.90, -.80])\}$
$L(b_I^n)(x) =$
$\{(i, [-.40, -.30]), (j, [-.40, -.30]), (k, [-.30, -.20]), (l, [-.30, -.20]), (m, [-.40, -.30])\}$
$L(c_I^n)(x) =$

$\{(i,[-.30,-.20]),(j,[-.30,-.20]),(k,[-.68,-.58]),(l,[-.68,-.58]),(m,[-.30,-.20])\}$

Also

$U(a_I^p)(x) = \{(i,[.81,.91]),(j,[.81,.91]),(k,[.65,.75]),(l,[.65,.75]),(m,[.81,.91])\}$

$U(b_I^p)(x) = \{(i,[.10,.20]),(j,[.10,.20]),(k,[.10,.30]),(l,[.10,.30]),(m,[.10,.20])\}$

$U(c_I^p)(x) = \{(i,[.01,.20]),(j,[.01,.20]),(k,[.51,.61]),(l,[.51,.61]),(m,[.01,.20])\}$

$U(a_I^n)(x) =$
$\{(i,[-.70,-.50]),(j,[-.70,-.50]),(k,[-.60,-.50]),(l,[-.60,-.50]),(m,[-.70,-.50])\}$

$U(b_I^n)(x) =$
$\{(i,[-.70,-.60]),(j,[-.70,-.60]),(k,[-.81,-.71]),(l,[-.81,-.71]),(m,[-.70,-.60])\}$

$U(c_I^n)(x) =$
$\{(i,[-.80,-.70]),(j,[-.80,-.70]),(k,[-.70,-.60]),(l,[-.70,-.60]),(m,[-.80,-.70])\}$

Example 3.2. Let $H = \{p,q,r,s,t\}$ be the universe. Let j be the bipolar interval neutrosophic set defined by,

$p = ([0.50,0.60],[0.20,0.30],[0.10,0.20],[-0.80,-0.70],[-0.70,-0.50],[-0.30,-0.20])$

$q = ([0.30,0.50],[0.10,0.20],[0.03,0.40],[-0.70,-0.60],[-0.50,-0.40],[-0.90,-0.80])$

$r = ([0.40,0.30],[0.01,0.30],[0.50,0.40],[-0.70,-0.50],[-0.80,-0.60],[-0.70,-0.50])$

$s = ([0.71,0.81],[0.52,0.62],[0.44,0.54],[-0.75,-0.65],[-0.45,-0.35],[-0.14,-0.01])$

$t = ([0.50,0.60],[0.40,0.50],[0.60,0.80],[-0.85,-0.75],[-0.71,-0.61],[-0.58,-0.40])$

Then the equivalence classes of H are defined by $\{\{p,q,t\},\{r,s\}\}$. The lower approximation of I is

$L(a_I^p)(x) = \{(p,[.30,.50]),(q,[.30,.50]),(r,[.40,.30]),(s,[.40,.30]),(t,[.30,.50])\}$

$L(b_I^p)(x) = \{(p,[.40,.50]),(q,[.40,.50]),(r,[.52,.62]),(s,[.52,.62]),(t,[.40,.50])\}$

$L(c_I^p)(x) = \{(p,[.60,.80]),(q,[.60,.80]),(r,[.50,.54]),(s,[.50,.54]),(t,[.60,.80])\}$

$L(a_I^n)(x) =$
$\{(p,[-.85,-.75]),(q,[-.84,-.75]),(r,[-.75,-.65]),(s,[-.75,-.65]),(t,[-.85,-.75])\}$

$L(b_I^n)(x) =$
$\{(p,[-.50,-.40]),(q,[-.50,-.40]),(r,[-.45,-.35]),(s,[-.45,-.35]),(m,[-.50,-.40])\}$

$L(c_I^n)(x) =$
$\{(p,[-.30,-.20]),(q,[-.30,-.20]),(r,[-.14,-.01]),(s,[-.14,-.01]),(t,[-.30,-.20])\}$

Also

$U(a_I^p)(x) = \{(p,[.50,.60]),(q,[.50,.60]),(r,[.71,.81]),(s,[.71,.81]),(t,[.50,.60])\}$

$U(b_I^p)(x) = \{(p,[.10,.20]),(q,[.10,.20]),(r,[.01,.30]),(s,[.01,.30]),(t,[.10,.20])\}$

$U(c_I^p)(x) = \{(p,[.03,.20]),(q,[.03,.20]),(r,[.44,.40]),(s,[.44,.40]),(t,[.03,.20])\}$

$U(a_I^n)(x) =$
$\{(p,[-.70,-.60]),(q,[-.70,-.60]),(r,[-.70,-.50]),(s,[-.70,-.50]),(t,[-.70,-.60])\}$

$U(b_I^n)(x) =$
$\{(p, [-.71, -.61]), (q, [-.71, -.61]), (r, [-.80, -.60]), (s, [-.80, -.60]), (t, [-.71, -.61])\}$
$U(c_I^n)(x) =$
$\{(p, [-.90, -.80]), (q, [-.90, -.80]), (r, [-.70, -.50]), (s, [-.70, -.50]), (t, [-.90, -.80])\}$

Definition 3.3. Let $R(I)$ and $R(J)$ be two rough bipolar interval neutrosophic sets. Then for all $l \in H$ $R(I) \subseteq R(J) \Leftrightarrow L(a_I^p)(l) \leq U(a_J^p)(l)$, $L(b_I^p)(l) \leq U(b_J^p)(l)$, $L(c_I^p)(l) \geq U(b_J^p)(l)$ and $L(a_I^n)(l) \geq U(a_J^n)(l)$, $L(b_I^n)(l) \geq U(b_J^n)(l)$, $L(c_I^n)(l) \leq U(b_J^n)(l)$.

Definition 3.4. Union of two rough bipolar interval neutrosophic sets $R(I)$ and $R(J)$, is defined as

$$L(I) \cup L(J)(l) = max\left(L(a_I^p)(l), L(a_J^p)(l)\right), \frac{L(b_I^p)(l)+L(b_J^p)(l)}{2}, min\left(L(c_I^p)(l), L(c_J^p)(l)\right),$$
$$min\left(L(a_I^n)(l), L(a_J^n)(l)\right), \frac{L(b_I^n)(l)+L(b_J^n)(l)}{2}, max\left(L(c_I^n)(l), L(c_J^n)(l)\right)$$
$$U(I) \cup U(J)(l) = max\left(U(a_I^p)(l), U(a_J^p)(l)\right), \frac{U(b_I^p)(l)+U(b_J^p)(l)}{2}, min\left(U(c_I^p)(l), U(c_J^p)(l)\right),$$
$$min\left(U(a_I^n)(l), U(a_J^n)(l)\right), \frac{U(b_I^n)(l)+U(b_J^n)(l)}{2}, max\left(U(c_I^n)(l), U(c_J^n)(l)\right)$$

for every $l \in H$.

Example 3.5. Consider two rough bipolar interval neutrosophic sets as in Example 3.1 and 3.2 then

$$L(I) \cup L(J) = \begin{cases} i, [.40..50], [.51, .61], [.34, .44], [-.90, -.80], [-.45, -.35], [-.30, -.20] \\ j, [.40..50], [.51, .61], [.34, .44], [-.90, -.80], [-.45, -.35], [-.30, -.20] \\ k, [.50, .40], [.55, .65], [.50, .54], [-.85, -.75], [-.38, -.28], [-.14, -.01] \\ l, [.50, .40], [.55, .65], [.50, .54], [-.85, -.75], [-.38, -.28], [-.14, -.01] \\ m, [.40..50], [.51, .61], [.34, .44], [-.90, -.80], [-.45, -.35], [-.30, -.20] \end{cases}$$

also

$$U(I) \cup U(J) = \begin{cases} i, [.81, .91], [.10, .20], [.01, .20], [-.70, -.60], [-.71, -.61], [-.90, -.80] \\ j, [.81, .91], [.10, .20], [.01, .20], [-.70, -.60], [-.71, -.61], [-.90, -.80] \\ k, [.71, .81], [.16, .30], [.51, .61], [-.60, -.50], [-.81, -.66], [-.70, -.50] \\ l, [.71, .81], [.16, .30], [.51, .61], [-.60, -.50], [-.81, -.66], [-.70, -.50] \\ m, [.81, .91], [.10, .20], [.01, .20], [-.70, -.60], [-.71, -.61], [-.90, -.80] \end{cases}$$

Definition 3.6. Intersection of two rough bipolar interval neutrosophic sets I and J, is defined as

$$L(I) \cap L(J)(l) = min\left(L(a_I^p)(l), L(a_J^p)(l)\right), \frac{L(b_I^p)(l)+L(b_J^p)(l)}{2}, max\left(L(c_I^p)(l), L(c_J^p)(l)\right),$$
$$max\left(L(a_I^n)(l), L(a_J^n)(l)\right), \frac{L(b_I^n)(l)+L(b_J^n)(l)}{2}, min\left(L(c_I^n)(l), L(c_J^n)(l)\right)$$
$$U(I) \cap U(J)(l) = min\left(U(a_I^p)(l), U(a_J^p)(l)\right), \frac{U(b_I^p)(l)+U(b_J^p)(l)}{2}, max\left(U(c_I^p)(l), U(c_J^p)(l)\right),$$

$$max\left(U(a_I^n)(l), U(a_J^n)(l)\right), \frac{U(b_I^n)(l)+U(b_J^n)(l)}{2}, min\left(U(c_I^n)(l), U(c_J^n)(l)\right)$$

for every $l \in H$.

Example 3.7. Consider two rough bipolar interval neutrosophic sets as in Example 3.1 and 3.2 then

$$L(I) \cap L(J) = \begin{cases} i, [.30..50], [.51, .61], [.60, .80], [-.85, -.75], [-.45, -.35], [-.30, -.20] \\ j, [.30..50], [.51, .61], [.60, .80], [-.85, -.75], [-.45, -.35], [-.30, -.20] \\ k, [.40, .30], [.55, .65], [.60, .70], [-.75, -.65], [-.38, -.28], [-.68, -.58] \\ l, [.40, .30], [.55, .65], [.60, .70], [-.75, -.65], [-.38, -.28], [-.68, -.58] \\ m, [.30..50], [.51, .61], [.60, .80], [-.85, -.75], [-.45, -.35], [-.30, -.20] \end{cases}$$

also

$$U(I) \cap U(J) = \begin{cases} i, [.50, .60], [.10, .20], [.03, .20], [-.70, -.50], [-.71, -.61], [-.90, -.80] \\ j, [.50, .60], [.10, .20], [.03, .20], [-.70, -.50], [-.71, -.61], [-.90, -.80] \\ k, [.65, .75], [.14, .30], [.51, .61], [-.60, -.50], [-.81, -.66], [-.70, -.60] \\ l, [.65, .75], [.14, .30], [.51, .61], [-.60, -.50], [-.81, -.66], [-.70, -.60] \\ m, [.50, .60], [.10, .20], [.03, .20], [-.70, -.50], [-.71, -.61], [-.90, -.80] \end{cases}$$

Definition 3.8. The complement of a rough bipolar interval neutrosophic set $R(I)$ is defined as $R(I)^c = (L(I)^c, U(I)^c)$ where $L(I)^c$and $U(I)^c$ are the lower and upper approximations of $R(I)^c$.

$L(a_I^p)^c(l) = 1 - L(a_I^p)(l)$, $L(b_I^p)^c(l) = 1 - L(b_I^p)(l)$ and $L(c_I^p)^c(l) = 1 - L(c_I^p)(l)$

$L(a_I^n)^c(l) = -1 - L(a_I^p)(l)$, $L(b_I^n)^c(l) = -1 - L(b_I^p)(l)$ and $L(c_I^n)^c(l) = -1 - L(c_I^p)(l)$.

Also, $U(a_I^p)^c(l) = 1 - U(a_I^p)(l)$, $U(b_I^p)^c(l) = 1 - U(b_I^p)(l)$ and $U(c_I^p)^c(l) = 1 - U(c_I^p)(l)$

$U(a_I^n)^c(l) = -1 - U(a_I^p)(l)$, $U(b_I^n)^c(l) = -1 - U(b_I^p)(l)$ and $U(c_I^n)^c(l) = -1 - U(c_I^p)(l)$.

for all $l \in H$.

Definition 3.9. If $R(I)$ and $R(J)$ are two rough bipolar interval neutrosophic sets in H, then

(1) $R(I) = R(J) \Leftrightarrow L(I) = L(J), U(I) = U(J)$
(2) $R(I) \subseteq R(J) \Leftrightarrow L(I) \subseteq L(J), U(I) \subseteq U(J)$
(3) $R(I) \cup R(J) \Leftrightarrow \langle L(I) \cup L(J), U(I) \cup U(J)\rangle$
(4) $R(I) \cap R(J) \Leftrightarrow \langle L(I) \cap L(J), U(I) \cap U(J)\rangle$
(5) $R(I) + R(J) \Leftrightarrow \langle L(I) + L(J), U(I) + U(J)\rangle$
(6) $R(I) \circ R(J) \Leftrightarrow \langle L(I) \circ L(J), U(I) \circ U(J)\rangle$

Proposition 3.10. *Let I and J are rough bipolar interval neutrosophic sets in (H, R) then*

(1) $I^c(I^c) = I$

(2) $L(I) \subseteq U(I)$

(3) $(L(I) \cup L(J))^c = L(I)^c \cap L(J)^c$

(4) $(L(I) \cap L(J))^c = L(I)^c \cup L(J)^c$

(5) $(U(I) \cup U(J))^c = U(I)^c \cap U(J)^c$

(6) $(U(I) \cap U(J))^c = U(I)^c \cup U(J)^c$

Proposition 3.11. *If $R(I)$ and $R(J)$ are rough bipolar interval neutrosophic sets then*

(1) $(R(I) \cup R(J))^c = (R(I))^c \cap (R(J))^c$

(2) $(R(I) \cap R(J))^c = (R(I))^c \cup (R(J))^c$

Proposition 3.12. *If I and J are bipolar interval neutrosophic sets such that $I \subseteq J$ implies $R(I) \subseteq R(J)$*

(1) $R(I \cup J) \supseteq R(I) \cup R(J)$

(2) $R(I \cap J) \supseteq R(I) \cap R(J)$

Proof : Let $l \in H$ **then**

$$\begin{aligned} L(a^n_{I\cup J})(l) &= \bigwedge_{z\in[l]_R} a^n_{I\cup J}(z) \\ &= \bigwedge_{z\in[l]_R} \max\{a^n_I, a^n_J\} \\ &\geq \max\left\{\bigwedge_{z\in[l]_R} a^n_I, \bigwedge_{z\in[l]_R} a^n_J\right\} \\ &= \max\{L(a^n_I), L(a^n_J)\} \\ &= L(a^n_I) \cup L(a^n_J) \end{aligned}$$

for all $l \in H$**.**

Similarly we can prove $L(b^n_{I\cup J})(l) \leq L(b^n_I) \cup L(b^n_J)$

$L(c^n_{I\cup J})(l) \leq L(c^n_I) \cup L(c^n_J)$

$L(a^p_{I\cup J})(l) \leq L(a^p_I) \cup L(a^p_J)$

$L(b^p_{I\cup J})(l) \geq L(b^p_I) \cup L(b^p_J)$

$L(c^p_{I\cup J})(l) \geq L(c^p_I) \cup L(c^p_J)$

Hence, $L(I \cup J) \supseteq L(I) \cup L(J)$

4. Conclusions

In this paper we introduce the notion of rough bipolar interval neutrosophic set. We also study some properties of this set and prove some propositions. The rough bipolar interval neutrosophic set is a combination of rough bipolar set and interval neutrosophic set. The proposed concept can be used in many aplications such as decision making problem, recognition pattern etc.

Conflicts of Interest: The authors declare that there is no conflict of interest regarding the publication of the paper.

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