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# On The Characterization Of Maximal and Minimal Ideals In Several Neutrosophic Rings

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**Abstract:** If  $R(I)$  is a neutrosophic ring, then every subset of  $R(I)$  has the form  $M = P + SI$ , where  $P, S$  are subsets of the classical ring  $R$ . The objective of this paper is to determine the necessary and sufficient condition on classical subsets  $P$  and  $S$  which makes  $M$  an ideal in  $R(I)$ . The main result is proving that every classical ideal in a neutrosophic ring must be an AH-ideal and determining the form of maximal and minimal ideals in  $R(I)$ . Also, a similar discussion of the case of refined neutrosophic rings will be presented.

**Keywords:** Neutrosophic ring, refined neutrosophic ring, maximal ideal, minimal ideal, AH-ideal.

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## 1. Introduction

Neutrosophy is a generalized view on intuitionistic fuzzy logic, it is considered as a new generalization of fuzzy ideas. The concept of neutrosophic set was built over the idea of dividing logical degrees into truth, falsity, and indeterminacy. This concept has an interesting effect in the study of optimization [16], computer science [18,22], decision making [15], and medical studies [19,21]. More applications of neutrosophy in many areas can be found in [33,34,35,36,37].

In the field of pure mathematics, we find many applications such as neutrosophic spaces [9,11,30], modules [4], groups [26], and rings [3,28,29].

The concept of neutrosophic ring was proposed by Smarandache and Kandasamy in [24], where they defined neutrosophic ring, neutrosophic ideal and neutrosophic isomorphism. Recently, many interesting results about neutrosophic rings were discussed [1,,3,20,14].

A neutrosophic ideal is an ideal by classical meaning i.e. it is a subset  $N$  from  $R(I)$  with the following properties:  $(N,+)$  is a subgroup of  $(R(I), +)$ , and  $r \cdot x \in N$  for all  $x \in N$  and  $r \in R(I)$ .

AH-ideals are subsets  $N = P + QI$ , where  $P, Q$  are two classical ideals in the classical ring  $R$  [1].

In [1], we find that AH-ideals are not supposed to be neutrosophic ideals, the converse is still unknown. A general study of AH-ideals and their relationships with Kothe's conjecture can be found in [31].

Through the first section of this paper, we present a characterization theorem of classical neutrosophic ideals in a neutrosophic ring  $R(I)$ . We prove that each neutrosophic ideal must be an AH-ideal. In addition, we determine the necessary and sufficient condition for any subset  $M = P + SI$  to be a neutrosophic ideal only using classical properties of  $P$  and  $S$ .

On the other hand, Agboola et.al presented a generalization of neutrosophic sets by splitting the degree of indeterminacy  $I$  into two degrees of indeterminacy  $I_1, I_2$ . This idea was used widely in algebra by studying refined neutrosophic rings [6,7], and  $n$ -refined neutrosophic rings and modules [12,13,25].

AH-ideals in refined neutrosophic rings were defined in [2], as subsets with form  $(P, QI_1, SI_2)$ , where  $P, Q, S$  are classical ideals in the ring  $R$ . According to [2], refined neutrosophic AH-ideals are not supposed to be ideals by classical meaning. In the second section of this work, we prove a characterization theorem of refined neutrosophic subsets to be classical refined ideals by depending on classical properties of  $P, Q, S$  only. This theorem ensures that each refined neutrosophic classical ideal must be a refined neutrosophic AH-ideal.

The main results of this work is to describe the structure of all non trivial maximal or minimal ideals in neutrosophic and refined neutrosophic rings.

This work is an extension of efforts to classify maximal and minimal ideals in neutrosophic rings in [38].

All rings through this paper are considered with unity 1.

### **Motivation**

Our motivation is to close an important research gap by determining all maximal and minimal ideals and their forms in neutrosophic rings, and refined neutrosophic rings.

### **2.Preliminaries**

#### **Definition 2.1: [24]**

Let  $R$  be a ring,  $I$  be the indeterminacy with property  $I^2 = I$ , then the neutrosophic ring is defined as follows:

$$R(I) = \{a + bI; a, b \in R\}.$$

Neutrosophic ring can be considered as an extension of classical ring by adding an indeterminacy element to R.

**Definition 2.2 : [24]**

Let R(I) be a neutrosophic ring, it is called commutative if and only if  $xy = yx \forall x, y \in R(I)$ .

**Definition 2.3: [24]**

Let R(I) be a neutrosophic ring, a non-empty subset P of R(I) is called a neutrosophic ideal if

(a) P is a neutrosophic subring of R(I)

(b)  $rx \in P$  for every  $x \in P$  and  $r \in R(I)$ .

**Definition 2.4: [1]**

Let R(I) be a neutrosophic ring and  $P = P_0 + P_1I = \{a_0 + a_1I; a_0 \in P_0, a_1 \in P_1\}$ .

(a) We say that P is an AH-ideal if  $P_0, P_1$  are ideals in the ring R.

(b) We say that P is an AHS-ideal if  $P_0 = P_1$ .

(c) The AH-ideal P is called null if  $P_0, P_1 \in \{R, O\}$ .

**Theorem 2.5: [1]**

Let R(I) be a neutrosophic ring and  $P = P_0 + P_1I$  be an AH-ideal, then P is not a neutrosophic ideal in general by the classical meaning.

**Definition 2.6: [6]**

The element I can be split into two indeterminacies  $I_1, I_2$  with conditions:

$$I_1^2 = I_1, I_2^2 = I_2, I_1I_2 = I_2I_1 = I_1.$$

**Definition 2.7: [6]**

If X is a set then  $X(I_1, I_2) = \{(a, bI_1, cI_2); a, b, c \in X\}$  is called the refined neutrosophic set generated by X,  $I_1, I_2$ .

**Definition 2.8: [2]**

Let  $(R, +, \times)$  be a ring,  $(R(I_1, I_2), +, \times)$  is called a 2-refined neutrosophic ring generated by R,  $I_1, I_2$ .

**Example 2.9: [6]**

The refined neutrosophic ring of integers is  $Z(I_1, I_2) = \{(a, bI_1, cI_2); a, b, c \in Z\}$ .

**Definition 2.10: [2]**

Let  $(R(I_1, I_2), +, \cdot)$  be a refined neutrosophic ring and  $P_0, P_1, P_2$  be ideals in the ring  $R$  then the set  $P = (P_0, P_1 I_1, P_2 I_2) = \{(a, b I_1, c I_2) : a \in P_0, b \in P_1, c \in P_2\}$  is called a refined neutrosophic AH-ideal.

If  $P_0 = P_1 = P_2$  then  $P$  is called a refined neutrosophic AHS-ideal.

### 3. Ideals in Neutrosophic rings

#### Remark 3.1:

Since every neutrosophic ring  $R(I)$  can be understood as  $R(I) = R + RI = (a + bI; a, b \in R)$ ,

Then each subset of  $R(I)$  has the form  $M = P + SI$ ;  $P, S$  are two subsets of  $R$ . We call  $P$  the real part,  $S$  the neutrosophic part of  $M$ .

An important question arises here. This question is:

When  $M$  is a neutrosophic ideal of  $R(I)$ ? In other words, what conditions on the real part  $P$  and neutrosophic part  $S$  which make  $M$  be an ideal?.

The following theorem clarifies the necessary and sufficient condition to answer the previous question.

#### Theorem 3.2:

Let  $R(I)$  be a neutrosophic ring,  $M = P + SI$  be any subset of  $R(I)$ , then

$M$  is a neutrosophic ideal if and only if the following conditions are true:

- (a)  $P$  is an ideal on  $R$ .
- (b)  $P$  is contained in  $S$ .
- (c)  $S$  is an ideal of  $R$ .

Proof:

Firstly, we assume that (a),(b), and (c) are true, we have:

$(M, +)$  is a subgroup of  $(R(I), +)$ , that is because if  $a + bI, c + dI \in M; a, c \in P, b, d \in S$ , we find

$$(a + bI) - (c + dI) = (a - c) + (b - d)I \in M; a - c \in P, b - d \in S.$$

Now, suppose that  $a + bI \in M$  and  $r = m + nI \in R(I)$ , we have

$$r \cdot (a + bI) = m \cdot a + I[m \cdot b + n \cdot b + n \cdot a], \text{ by the assumption, we regard that } m \cdot b + n \cdot b \in S, \text{ and } n \cdot a \in$$

$$P \leq S, \text{ thus } r \cdot (a + bI) = m \cdot a + I[m \cdot b + n \cdot b + n \cdot a] \in P + SI = M, \text{ which means that } M \text{ is a}$$

neutrosophic ideal of  $R(I)$ .

Conversely, we suppose that  $M = P + SI$  is a neutrosophic ideal of  $R(I)$ . Let  $a, c$  be two arbitrary elements in  $P$ , and  $b, d$  be two arbitrary elements in  $S$ , we have  $a + bI, c + dI \in M$ , by using the assumption we have  $M$  as an ideal, hence  $(a + bI) - (c + dI) = (a - c) + (b - d)I \in M = P + SI$ , thus

$a - c \in P$ , and  $b - d \in S$ , thus  $(P, +), (S, +)$  are two subgroups of  $(R, +)$ .

For every  $r \in R$ , we have  $r = r + 0I \in R(I)$ , and  $r \cdot (a + bI) = r \cdot a + r \cdot bI \in M = P + SI$ , thus  $r \cdot a \in P, r \cdot b \in S$ , this means that  $P, S$  are ideals in the classical ring  $R$ .

Now, we prove that  $P$  is contained in  $S$ . We have  $(1 - I) \in R(I)$ , that is because  $R(I)$  has a unity 1. On the other hand, we can write  $(1 - I)(a + bI) = (a - aI) \in M = P + SI$ , and that is because  $M$  is an ideal of  $R(I)$ , hence  $-a \in S$ , thus  $a \in S$ , by regarding that  $a$  is an arbitrary element of  $P$ , we get that  $P \leq S$ .

The previous theorem ensures that each ideal is an AH-ideal, since  $P, S$  are supposed to be classical ideals of  $R$ .

**Example 3.3:**

Let  $R = Z$  be the ring of integers,  $R(I) = Z(I) = \{a + bI; a, b \in Z\}$  be the corresponding neutrosophic ring, we have:

- (a)  $P = \langle 2 \rangle, Q = \langle 4 \rangle, S = \langle 3 \rangle$ , are three ideals of  $R$ , with  $Q \leq P$ .
- (b)  $M = Q + PI = \{4m + 2nI; m, n \in Z\}$  is an ideal of  $R(I)$ .
- (c)  $N = P + SI = \{2m + 3nI; m, n \in Z\}$  is not a neutrosophic ideal, that is because  $P$  is not contained in  $S$ .

**Example 3.4:**

Let  $R = Z_8$  be the ring of integers modulo 8.  $R(I) = \{a + bI; a, b \in Z_8\}$ , be the corresponding neutrosophic ring. Consider the set  $M = \{0, 4, 2I, 4I, 6I, 4 + 2I, 4 + 6I, 4 + 4I\}$ . We have  $M$  as an ideal of  $R(I)$ , that is because  $M = \langle 4 \rangle + \langle 2 \rangle I$  and  $\langle 4 \rangle \leq \langle 2 \rangle$ .

**Theorem 3.5:**

The following theorem determines the form of maximal ideals in  $R(I)$ .

Let  $R(I)$  be a neutrosophic ring,  $M = P + SI$  be an ideal of  $R(I)$ , then  $M$  is maximal if and only if  $P$  is maximal in  $R$  with  $S = R$  or  $M = R(I)$ .

Proof:

Suppose that  $M$  is maximal of  $R(I)$ , let  $N = V + WI$  be any ideal of  $R(I)$  with the property  $M \leq N$ , then  $P \leq V$  and  $S \leq W$ , by the assumption of the maximality of  $M$ , we find that  $N = M$  or  $N = R(I)$ , this implies that  $(V = P \text{ with } W = R) \text{ or } (V = W = R)$ , which means that  $P$  is maximal in  $R$  or  $P = R$ . On the other hand  $P \leq S$  and  $P$  is maximal, thus  $S = P$  or  $S = R$ . Since  $P + SI \leq P + RI$ , hence the only non trivial maximal ideal is  $M = P + RI$ , with  $P$  as a maximal ideal in  $R$ .

The converse is clear.

**Theorem 3.6:**

The following theorem describes minimal ideals in  $R(I)$ .

Let  $R(I)$  be a neutrosophic ring,  $M = P + SI$  be an ideal of  $R(I)$ , then  $M$  is minimal if and only if  $S$  is minimal in  $R$  and  $P = \{0\}$ .

Proof:

Suppose that  $M$  is minimal of  $R(I)$ , let  $N = V + WI$  be any ideal of  $R(I)$  with the property  $N \leq M$ , then  $V \leq P$  and  $W \leq S$ , by the assumption of the minimality of  $M$ , we find that  $N = M$  or  $N = \{0\}$ , this implies that  $(V = P \text{ with } W = S) \text{ or } (W = N = \{0\})$ , which means that  $P, S$  are minimal in  $R$ . On the other hand  $P \leq S$  and  $S$  is minimal, thus  $S = P$  or  $P = \{0\}$ . Since  $SI$  is a sub-ideal of  $P+SI$ , hence  $P = \{0\}$ .

The converse is clear.

**Remark 3.7:**

According to Theorem 5.1 and Theorem 6.1, we get a full description of the structure of maximal and minimal ideals in the neutrosophic ring  $R(I)$ .

- (a) Non trivial Maximal ideals in  $R(I)$  has the form  $\{P+RI\}$ , where  $P$  is maximal in  $R$ .
- (b) Non trivial minimal ideals have the form  $\{\{0\}+SI\}$  where  $S$  is minimal in  $R$ .

**Example 3.8:**

Let  $Z(I)$  be the neutrosophic ring of integers, non trivial maximal ideals in  $Z(I)$  are  $\{\langle p \rangle + ZI\}$ , where  $p$  is any prime number.

**4. Ideals in refined neutrosophic rings**

**Remark 4.1:**

Since every refined neutrosophic ring  $R(I_1, I_2)$  can be understood as  $R(I_1, I_2) = (R, RI_1, RI_2) = \{(a, bI_1, cI_2); a, b, c \in R\}$ ,

Then each subset of  $R(I_1, I_2)$  has the form  $M = (P, QI_1, SI_2)$ ;  $P, Q, S$  are two subsets of  $R$ .

An important question arises here. This question is:

When  $M$  is a refined neutrosophic ideal of  $R(I_1, I_2)$ ? In other words, what conditions on  $P, Q, S$  which make  $M$  an ideal?.

The following theorem clarifies the necessary and sufficient condition to answer the previous question.

**Theorem 4.2:**

Consider the following:

$R(I_1, I_2) = \{(a, bI_1, cI_2); a, b, c \in R\}$  be a refined neutrosophic ring,  $M = (P, QI_1, SI_2)$  be a subset of  $R(I_1, I_2)$

.  $M$  is an ideal of  $R(I_1, I_2)$  if and only if

(a)  $P, Q, S$  are ideals on  $R$

(b)  $P \leq S \leq Q$ .

Proof:

Suppose that  $M$  is an ideal, then we have for every  $a, m \in P$  and  $b, n \in Q$  and  $c, t \in S$ ,

$x = (a, bI_1, cI_2), y = (m, nI_1, tI_2)$ , are two elements of  $R(I_1, I_2)$ .

$x - y = (a - m, [b - n]I_1, [c - t]I_2) \in M$ , thus  $a - m \in P, b - n \in Q, c - t \in$

$S$ , hence  $(P, +), (Q, +), (S, +)$  are subgroups of  $(R, +)$ .

For every  $r \in R$ , we have  $(r, 0, 0) \in R(I_1, I_2)$  and  $(r, 0, 0) \cdot (a, bI_1, cI_2) = (r \cdot a, r \cdot bI_1, r \cdot cI_2) \in M$ , thus  $r \cdot a \in P, r \cdot b \in Q, r \cdot c \in S$ , thus  $P, Q, S$  are ideals of  $R$ .

On the other hand, we have  $(1, 0, -I_2) \in R(I_1, I_2)$ , thus  $(1, 0, -I_2) \cdot (a, bI_1, cI_2) = (a, 0, -aI_2) \in M$ , hence  $-a \in S$  and  $P \leq S$ , that is because  $a$  is an arbitrary element in  $P$ .

Also,  $(1, -I_1, 0) \in R(I_1, I_2)$ , thus  $(1, -I_1, 0) \cdot (0, bI_1, cI_2) = (0, -cI_1, cI_2) \in M$ , hence  $-c \in Q$  and  $S \leq Q$ .

That is because  $c$  is an arbitrary element in  $S$ .

For the converse, we suppose that (a) and (b) are true, we have  $(M, +)$  as a subgroup of  $R(I_1, I_2)$ .



Let  $r = (m, nI_1, tI_2) \in R(I_1, I_2)$  and  $x = (a, bI_1, cI_2) \in M$ , we have

$r.x = (m.a, [m.b + n.a + n.b + n.c + t.b]I_1, [m.c + t.a + t.c]I_2)$ , it is clear that

$m.c + t.c \in S, t.a \in P \leq S$ , thus  $m.a + t.a + t.c \in S$ . Also,

$m.b + n.b + t.b \in Q$ , and  $n.a + n.c \in S \leq Q$ , thus  $m.b + n.a + n.b + n.c + t.b \in Q$ . This implies that

$r.x \in M$ , hence  $M$  is an ideal.

**Example 4.3:**

Let  $Z(I_1, I_2)$  be the refined neutrosophic ring of integers, we have

$(\langle 8 \rangle, \langle 2 \rangle I_1, \langle 4 \rangle I_2) = \{(8a, 2bI_1, 4cI_2); a, b, c \in Z\}$  is an ideal in  $Z(I_1, I_2)$ . That is because

$\langle 8 \rangle \leq \langle 4 \rangle \leq \langle 2 \rangle$ .

**Example 4.4:**

Let  $Z_{20}(I_1, I_2)$  be the refined neutrosophic ring of integers modulo 20, we have

$(0, \langle 5 \rangle I_1, \langle 10 \rangle I_2) =$

$\{(0,0,0), (0,5I_1, 0), (0,5I_1, 10I_2), (0,10I_1, 0), (0,10I_1, 10I_2), (0,15I_1, 0), (0,15I_1, 10I_2), (0,0,10I_2)\}$ .

**Theorem 4.5:**

Consider the following:

$R(I_1, I_2) = \{(a, bI_1, cI_2); a, b, c \in R\}$  be a refined neutrosophic ring,  $M = (P, QI_1, SI_2)$  be any non trivial maximal ideal of  $R(I_1, I_2)$

$M$  has the following form:

$(P, RI_1, RI_2)$ . Where  $P$  is any maximal ideal of  $R$ .

Proof:

We assume that  $M$  is a maximal ideal, and  $N = (X, YI_1, ZI_2)$  is an ideal of  $R(I_1, I_2)$  with  $M \leq N$ , hence

$M = N$  or  $N = R(I_1, I_2)$ , we have  $P = X, Q = Y, S = Z$ , or  $X = Y = Z = R$ . This implies that  $P, S, Q$  should be maximal; but we have that

$P \leq S \leq Q$ , hence  $(R = S, Q = R; P$  is maximal in  $R)$ .

The converse is clear.

**Theorem 4.6:**

Consider the following:

$R(I_1, I_2) = \{(a, bI_1, cI_2); a, b, c \in R\}$  be a refined neutrosophic ring,  $M = (P, QI_1, SI_2)$  be any non trivial minimal ideal of  $R(I_1, I_2)$

M has the following form:

$(0, PI_1, 0)$ . Where P is any minimal ideal of R.

Proof:

The proof is similar to Theorem 5.2.

#### Example 4.7:

(a) Consider  $Z_8(I_1, I_2)$  the refined neutrosophic ring of integers modulo 8, we have  $\langle 4 \rangle = \{0, 4\}$  is a minimal ideal of  $Z_8$ . Hence  $(0, \langle 4 \rangle I_1, 0) = \{(0, 0, 0), (0, 4I_1, 0)\}$  is a minimal ideal of  $Z_8(I_1, I_2)$ .

(b)  $\langle 2 \rangle = \{2, 4, 6, 0\}$  is maximal in  $Z_8$ . Hence  $(\langle 2 \rangle, Z_8I_1, Z_8I_2) = \{(a, bI_1, cI_2); a \in \langle 2 \rangle \text{ and } b, c \in Z_8\}$  is maximal in  $Z_8(I_1, I_2)$

#### 4. Conclusions

In this article, we have studied algebraic ideals in neutrosophic rings and refined neutrosophic rings, where we proved that every ideal in a neutrosophic or refined neutrosophic ring with unity must be an AH-ideal. Also, we have determined the structure of all maximal and minimal ideals in any neutrosophic ring and any refined neutrosophic ring with unity. In addition, many examples were built to clarify the validity of this work.

As a future research direction, we aim to classify neutrosophic factors and refined neutrosophic factors.

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