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Abstract

In this paper, Coulomb's Law is extended from two stationary charged particles, on linear trajectory, to: many charged particles, or quantum level particles, or objects on arbitrary motions (with velocity, acceleration, time delay), on non-linear trajectories, at even superluminal and instantaneous speeds.

Keywords: Charged Particle, (One) Action at a Distance, MultiAction at a Distance, MultiNonLocality, Superluminal Speed, Superluminal Physics, Instantaneous Speed, Instantaneous Physics, Electromagnetism, Vacuum Polarization Effect, Maxwell Equations, Gauss's Law, Moving Charges, Time Dependance, Point Charge, Length Charge, Volume Charge, Law of Superposition, Theorem of Superposition, Superposition Principle, Linear Circuit, NonLinear Circuit, Norton Circuit, Thevenin Circuit, Power Dissipation, Permittivity.

1 | The Coulomb's Law in Scalar Form

The Coulomb's Law in scalar form is the following:

$$|F_{12}| = k_e \cdot \frac{|q_1| \cdot |q_2|}{r_{12}^2} = \frac{|q_1| \cdot |q_2|}{4\pi\varepsilon_0 \cdot r_{12}^2}$$

where:

$F \equiv F_{12}$ = electrostatic (attractive or repulsive) force between q_1 and q_2;

k_e = Coulomb constant (electrostatic constant);

$k_e = \frac{1}{4\pi\varepsilon_0}$, where ε_0 = electric constant (vacuum electric permittivity).

Permittivity of a material, under an electric field, is its ability to store electrical potential energy.

$k_e \approx 8.99 \times 10^9 \text{N m}^2/\text{C}^2$;

q_1, q_2 = point (particle) charges;

$r \equiv r_{12}$ = distance between q_1 and q_2;

and | | means absolute value.

Coulomb's Law states that the absolute value of the electrostatic (attractive or repulsive) force between two charged particles (points) is directly proportional to the product of the magnitudes of their charges, and inverse proportional to the square of the distance between them. This is similar to Newton's Law of Universal Gravitation, with the distinction that under gravitational forces things only attract (do not repel) each other, but the inverse-square law acts identically. Opposite charges attract each other, while like charges repel each other. The law was published by the French physicist Charles-Augustin de Coulomb in 1785.

Action at a Distance between charged particles occurs in the Coulomb's Law.

Coulomb's Law led to the development of Electromagnetism.

2 | The Coulomb's Law in Vector Form

When considering the directions of the force, Coulomb's law can be expressed in vector form:

$$F_{12} = k_e \frac{q_1 \cdot q_2}{r^2} \cdot \hat{r}_{12}$$

where $\hat{r}_{12}$ is the unit vector from point charge q_1 to point charge q_2.

Coulomb: "l'action répulsive que les deux balles électrifiées de la même nature d'électricité exercent l'une sur l'autre, suit la raison inverse du carré des distances." [1].

3 | Extensions of Coulomb's Law

The electrostatic force between stationary charged particles may be affected by considering complex scenarios and factors:

- Instead of stationary, taking charges in arbitrary motion – complicated by velocity, acceleration, and time-delay effects;

- Forces may not even lie along the line joining the two charges;

- More than two charges in a dynamic interplay of electric forces;

- Nonlinear trajectories the charges are moving on;

- Charged objects instead of particles (points);

- Particles at superluminal or instantaneous speeds;

- Quantum level particles;

- Interreactions of the electrical fields created by all charged particles/objects.

3.1 | Coulomb's Law in a Medium

In a medium with a dielectric constant (ε_r), the force is reduced by a factor equal to the dielectric constant of the medium.

$$F = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \cdot \frac{|q_1| \cdot |q_2|}{r^2},$$

where ε_0 is the permittivity of free space, and ε_r is the relative permittivity (dielectric constant) of the medium.

3.2 | Superposition Law

The Superposition Law allows us to find the force on a charge due to multiple other charges. Imagine three charged particles (see Figure 1 below). Coulomb's Law gives the force between any two of them.

Superposition Law says the total force on a single charge is the vector sum of the individual forces due to each of the other charges, treated as if they were alone.

Having multiple charges, the total force F_i on a particular charge q_i is the vector sum of the forces exerted by all other charges.

$$F_i = k_e \sum_{j \neq i} \frac{q_i q_j}{r_{ij}^2} \hat{r}_{ij},$$

where

r_{ij} is the distance between charges q_i and q_j,

and $\hat{r}_{ij}$ is the unit vector from q_i to q_j.

The Law of Superposition is an extension of Coulomb's Law from two to three or more point charges, for linear bilateral networks.

On a single point charge many forces act from the other point charges: therefore, one has some multi-action at a distance.

Example of Superposition Law of Three Vectorial Like Forces

$\overrightarrow{F_1}, \overrightarrow{F_2}$, and $\vec{F}$ of the positively charged particles $P_1(+), P_2(+), P_3(+)$ that repel each other two by two.
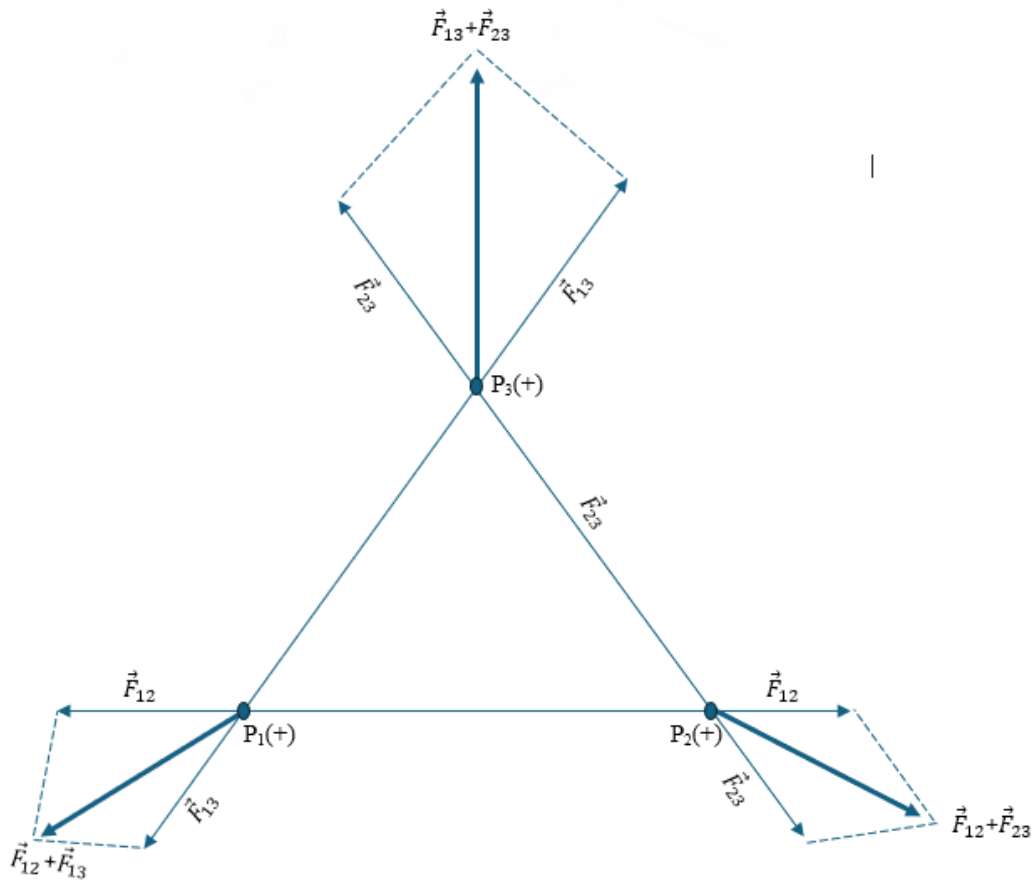

Figure 1. Superposition law of three vectorial like forces.

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The Law of Superposition, used in Geology to calculate the relative age of rock strata or layers, states that the layers are superimposed, one on top of another: the oldest layer at the bottom, and the youngest layer at the top.

The Theorem of Superposition, in Physics, is referring to a linear, bilateral network, with two or more sources, and it states that the response of each element is the sum of all sources' responses.

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The Theorem of Superposition doesn't apply to nonlinear circuits and, as a consequence, nor to the power dissipation that is a nonlinear function.

In circuit analysis, the superposition theorem converts a complex circuit to an equivalent Norton or Thevenin circuit.

The power dissipation means that, as result of primary action, an electrical or electronic device produces heat – and, as such, it loses power in components.

3.3 | Object Charge Distributions

From Coulomb's Law, that only applies to point charges, one extends to Object Charges. In our reality, charges can be spread out over a region (object). The concept is extended by dividing the charge distribution into tiny pieces, approximating each piece as a point charge, and summing the forces due to each tiny piece. This gives us the net force due to the entire charge distribution.

One talks about charge density.

For extended charge distributions, Coulomb's Law is integrated over the volume, or surface, or length of the charge distribution to find the net electrostatic force or potential.

$$F = k_e \int_V \frac{\rho(r')dV'}{|r-r'|^2} \hat{r}$$

where $\rho(r')$ is the charge density at point r'.

3.4 | Moving Charges and Time Dependance

Similarly, from Coulomb's law, that only applies to static situations, an extension to a more general approach is to describe the electric field created by a single charge. This field extends a force on any other charge placed within it.

When charges are moving or accelerating, things get more complex. The force depends on their velocities, accelerations, and even the concept of force needs adjustments due to time delays in the interaction. This realm is described by a more sophisticated theory: Maxwell's Equations. They encompass Coulomb's law but go beyond to describe electric and magnetic fields arising from moving charges.

And similarly, Gauss's Law.

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Maxwell's (or Maxwell-Heaviside) Equations describe how the charges and currents and charges of the fields generate electric and magnetic fields. The mathematician and physicist James Clerk Maxwell published them in 1861-1862.

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Gauss's Law presents the connection between electric charges and electric field: the electric field points towards negative charges and away from positive charges. The electric field outflown through a closed surface is proportional to the enclosed charge.

3.5 | Effects of Superluminal and Instantaneous Motions

At superluminal and instantaneous speeds, effects can alter the force calculations. Coulomb's Law is adjusted to include these superluminal and instantaneous corrections, especially in the context of electromagnetic fields as described by the theory of electromagnetism.

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Electromagnetism deals with the electromagnetic force that acts between particles that are charged electrically. The electromagnetic force is a combination of all electrical and magnetic forces. By introducing current in a conductor, one creates a magnetic field.

3.6 | Quantum Electrodynamic Corrections

The interactions between charges are mediated by photons in quantum electrodynamics (QED), and the force between charges can include quantum corrections, such as vacuum polarization effects.

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Vacuum Polarization (or self-energy of the gauge boson), in quantum electrodynamics, describes how the virtual electron-positron pairs, produced in a background electromagnetic field, change the distribution of currents and charges in the original electromagnetic field (EM field).

4 | Conclusion

This is a review paper assembling many types of extensions of the Coulomb's law.

We introduce for the first time the MultiAction at a Distance (giving birth to MultiNonLocality) – as generalization of the (One) Action (Only) at a Distance – which occur in the Newton's Universal Law of Gravitation and in the Law of Superposition. We also extend Coulomb's Law to the superluminal and instantaneous speeds.

Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

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