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Jonckheere Trend Test under Indeterminacy with Applications

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Abstract. The classical Jonckheere trend test is a non-parametric statistical tool usually employed to compare the medians of multiple independent groups, especially when there is a natural ordering or trend among the groups. This paper aims to develop a more comprehensive and adaptable version of the Jonckheere trend test, called the neutrosophic Jonckheere trend test (NJT), which can be used to analyze different types of uncertainty data. This paper discusses neutrosophic hypotheses and decision rules pertaining to the NJT test. Furthermore, the practical uses of the NJT test have been discussed in the context of real-world applications with COVID-19 data. Lastly, a simulation study is carried out to evaluate the effectiveness of the proposed test in terms of Type I error and test power. The results validate that the proposed test is more effective and adaptable than the existing test in uncertain environments.

1. Introduction

Classical statistical tests often assume that data conform to a specific distribution, such as the normal distribution. However, this assumption may not hold true in many real-world situations, which can make it difficult to conduct parametric tests. Non-parametric tests provide a useful alternative to analyzing data when normality assumptions are violated, rendering them more versatile and applicable across various fields. The Jonckheere trend test is one of the most commonly used non-parametric tests for testing differences in medians in cases where there is an anticipated order to group medians.

To perform the Jonckheere trend test, it is imperative to gather sample elements from each group randomly and independently. It is also critical to ensure that samples within each group are independent of each other. Additionally, it is necessary to assume that groups' distributions have similar shapes and variability. The test involves ordering the observations in ascending order

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across all groups and finding the sum of signs for each group. The differences between these sums of signs can be used to test whether there is a significant difference in medians between the groups. One of the main advantages of the Jonckheere trend test over other non-parametric tests, such as the Mann-Whitney U test or Kruskal-Wallis test, is that it takes into account the expected order of the group medians. This makes it particularly useful in situations where there is a natural ordering to the groups, such as in medical studies where treatments may have a graded response. In this light, this test was first introduced by Jonckheere and Terpstra [1,2]. Vock and Balakrishnan [3] suggested an extension to the Jonckheere trend test used to identify a perfect ranking in balanced ranked set sampling. Ali et al. [4] proposed a non-parametric test called the Jonckheere trend test for ordered medians and discussed its application in medical research. Murakami and Lee [5] investigated the unbiasedness and biasedness of two statistical tests, Jonckheere trend and Kruskal-Wallis under different sample sizes and distributions. Joutard [6] proposed large deviation approximations for Mann-Whitney and Jonckheere trend statistics in nonparametric statistics. Magel [7] suggested an extension to the Jonckheere trend test for ranked-set sample data to improve its sensitivity and performance in identifying variations among treatment groups. A collection of articles and books discussing the Jonckheere trend test can be found in [8–11]. As mentioned above, the Jonckheere trend test has been considered and analyzed under classical statistics; however, experimenters are sometimes confronted with indeterminate data under uncertain conditions. Therefore, it is, therefore, necessary to look for an appropriate generalization of this test so that we may use indeterminate data in an uncertain environment.

Smarandache introduced neutrosophic statistics (NS) [12]. Aslam [13] provided an explanation of the distinctions among fuzzy statistics, NS, and classical statistics. According to Smarandache [14], NS encompasses a wider scope compared to interval statistics. It embraces various forms of indeterminacy such as uncommon sample sizes, neutrosophic random variables, and neutrosophic probability distributions. The neutrosophic sign test was applied to COVID-19 data by Sherwani, et al. [15]. The neutrosophic Kruskal Wallis H test was used to analyze COVID-19 data by Sherwani, et al. [16]. Aslam and Aldosari [17] discussed alloy melting point data using the Mann-Whitney test. Miari, et al. [18] suggested single-valued neutrosophic Kruskal-Wallis and Mann Whitney tests. Aslam [19] discussed the neutrosophic ANOVA method. Ullah, et al. [20] presented a comprehensive method for calculating the neutrosophic k-factor analysis of variance. AlAita and Talebi [21] provided an exact neutrosophic analysis of the missing value issue in an augmented randomized complete block design. AlAita and Aslam [22] introduced the application of neutrosophic analysis of covariance in three different types of experimental designs: neutrosophic completely random designs, neutrosophic randomized complete block designs, and neutrosophic split-plot designs. AlAita, et al [23] presented neutrosophic statistical analysis for split-plot designs. Aslam and Albassam [24] suggested post-hoc multiple comparison tests under NS. Salama, et al. [25] discussed simple linear regression and correlation. Nagarajan, et al [26] suggested neutrosophic multiple regression. Alomair and Shahzad [27] introduced a method known

as neutrosophic mean estimation to estimate both sensitive and non-sensitive variables using robust Hartley-Ross-Type estimators. Aslam and Saleem [28] presented a discussion that focused on the application of the neutrosophic test of linearity. Aslam [29] used analysis of means under neutrosophic statistics to examine wind power data. In recent years, numerous neutrosophic statistical studies have been discussed [30–32].

We have considered the classic Jonckheere trend test and raised some ambiguities in data, such as indeterminacy and imprecise test statistic results. The probable inadequacy of the classic approach to dealing with such complexity motivated us to utilize NS. In this paper, the indeterminacies are tackled by examining the Jonckheere trend test in the NS framework, which provides additional information about uncertainty levels. This is the first time the Jonckheere trend test has been discussed in a neutrosophic environment.

In the following section, we delve into some fundamental NS-related concepts. Section 3 provides an overview of the proposed Jonckheere trend test methodology. Section 4 presents and explains a numerical example and simulation study. In Section 5, we introduce the discussion, followed by a presentation of future research directions in Section 6. Finally, we present our conclusions in Section 7.

2. Preliminaries

The following offers some basic concepts regarding neutrosophic random variables that will be useful in the subsequent sections.

The $X_N \in [X_L, X_U]$ is a neutrosophic random variable with indeterminacy interval, I_N, and is written as $X_N = X_L + X_U I_N$, where X_L is determinate part and $X_U I_N$ is indeterminate part, where $I_N \in [I_L, I_U]$ is measure of uncertainty. Clearly, X_N is reduced to the classical random variable at $I_L = 0$.

Assume that we have a population of size N with indeterminate observations, and take a neutrosophic random sample of size n from it. The neutrosophic population median is denoted by M_N and defined as the value that splits the population into two equal halves. In other words,

If n is odd, $M_N(x) = x_{N\left(\frac{n+1}{2}\right)}$.

If n is even, $M_N(x) = \frac{1}{2}(x_{N\left(\frac{n}{2}\right)} + x_{N\left(\frac{n}{2}+1\right)})$.

3. Computational method of the Jonckheere trend test under uncertainty

In neutrosophic statistics, neutrosophic nonparametric tests refer to methods of statistical analysis that require no particular distribution (especially when data are not normally distributed). The neutrosophic nonparametric tests can be used as an alternative to neutrosophic parametric tests, such as the neutrosophic T-test or NANOVA, if the underlying neutrosophic data fits certain criteria and assumptions. According to the literature on neutrosophic nonparametric tests, many statistical neutrosophic tests have been studied (e.g., neutrosophic Kruskal-Wallis H test [16], neutrosophic sign test [15], etc.). In this section, we delve into the application of the neutrosophic Jonckheere

trend test as a substitute for the neutrosophic Kruskal-Wallis H test in cases where the medians of each group conform to an anticipated pattern.

3.1. *Neutrosophic hypotheses*

Under neutrosophic statistics, the neutrosophic Jonckheere trend test is used to test the null hypothesis that all k samples have equal medians against the alternative hypothesis that at least one population differs from the rest. In other words; Suppose M_{Ni}, $i = 1, 2, \ldots, k$ represents the population median for the ith population, the neutrosophic null, and alternative hypotheses are formulated as follows;

$$H_{N0}: M_{N1} = M_{N2} = \cdots = M_{Nk},$$
$$H_{N1}: M_{N1} \leq M_{N2} \leq \ldots \leq M_{Nk}, \text{ with at least a strict inequality.}$$

3.2. *Neutrosophic Jonckheere trend test statistic*

The following steps are taken to perform the test:

Step 1: Organize the sample data in the anticipated sequence of group medians. In other words, the first column should consist of the data points with the lowest expected median, followed by the second column, which contains the data elements with the second-lowest expected median, and so on.

Step 2: The JT_N-test is computed under H_{N0}.

$$U_{Nij} = \sum_{s<r} \sum_{ij} I\left(X_{Nis} < X_{Njr}\right) + 0.5I\left(X_{Nis} = X_{Njr}\right); U_{Nij} \in [U_{Lij},\ U_{Uij}],$$

where i, j are observations in groups s and r respectively. The neutrosophic standardized test statistic is $JT_N = |Z_N|$, where

$$Z_N = \frac{U_{Nij} - \mu_{NU}}{\sigma_{NU}}; Z_N \in [Z_L,\ Z_U].$$

Assuming that there are no ties, the neutrosophic mean and the variance are, respectively

$$\mu_{NU} = \frac{N_N^2 - \sum_{i=1}^{k} N_{Ni}^2}{4}; \mu_{NU} \in [\mu_{LU},\ \mu_{UU}], \text{ and}$$
$$\sigma_{NU}^2 = \frac{N_N^2 (2N_N + 3) - \sum_{i=1}^{k} N_{Ni}^2 (2N_{Ni} + 3)}{72}; \sigma_{NU}^2 \in [\sigma_{LU}^2,\ \sigma_{UU}^2],$$

where N_N is neutrosophic total sample size and N_{Ni} is neutrosophic sample number in each group.

Step 3: Calculation of the p_N-value at the level $\alpha = 0.05$.
When dealing with a significant amount of data, Z tends to have an approximate normal distribution. By making this assumption, it becomes feasible to calculate the 1-tailed p-value as

$$p_N = \int_u^{\infty} f_U(u_N)\, du_N, (u_N > 0)$$

$$= \frac{1}{2}\left(\int_{z}^{\infty} f_Z(z_N)\, dz_N + \int_{-\infty}^{-z} f_Z(z_N)\, dz_N\right), (z_N > 0)$$
$$= \frac{1}{2}\left(1 - \text{erf}\left(\frac{z_N}{\sqrt{2}}\right)\right), (z_N > 0)$$
$$= 1 - \frac{1}{2}\,\text{erfc}\left(-\frac{JT_N}{\sqrt{2}}\right), (JT_N = |z_N|)\,.$$

Smarandache [12] outlined that the neutrosophic decision rule can be summarized as follows:

If $min\{p_N{-}value\} > \alpha$, then we accept the null hypothesis H_{N0} at the level α.
If $max\{p_N{-}value\} \leq \alpha$, then we reject the null hypothesis H_{N0} at the level α.
If $min\{p_N{-}value\} < \alpha < max\{p_N{-}value\}$, then there is indeterminacy.
Thus $\frac{\alpha - min\{p_N-value\}}{max\{p_N-value\}-min\{p_N-value\}}$ represents the chance to reject H_{N0} at the level α, and $\frac{max\{p_N-value\}-\alpha}{max\{p_N-value\}-min\{p_N-value\}}$ represents the chance to accept H_{N0} at the level α.

4. Application of the neutrosophic Jonckheere trend test

This section aims to evaluate the numerical performance of the proposed NS in handling the Jonckheere trend test. To achieve this, a set of real neutrosophic data with uncertain observations for the NJT test will be analyzed. In addition, a simulation study will be conducted for further evaluation. In order to assess the effectiveness of the NJT test, the JT_N-test is calculated and compared with the existing test under classical statistics in terms of uncertainty.

4.1. *Real data example*

To apply the proposed neutrosophic Jonckheere trend test, daily ICU occupancy data representing Corona-positive patients from Pakistan have been analyzed. In this study, the hypothesis being tested is whether there is a statistically significant difference in ICU occupancy of COVID-19 patients according to their age group. The data shown in Table 1 is uncertainty data [16]. This study uses the neutrosophic Jonckheere trend test to test the null hypothesis that there are no differences in the daily occupancy of COVID-19 patients from different age groups in Pakistan for December 2020. There are three categories of age groups in COVID-19 for daily ICU occupancy (35 years and below, 35 to 55 years, and 55 years and above).

As previously mentioned, the proposed test is a generalization of JT test. The neutrosophic logic literature has indicated that a method based on data in an indeterminate interval is more effective and suitable for use in uncertainty than determining values under classical statistics. The neutrosophic form of the JT_N-test is $JT_N{=}JT_L{+}JT_UI_N; I_N{\in}[I_L, I_U]$, where the first part JT_L is known as the determined part and presents the value of JT-test under CS. The second part $JT_UI_N; I_N{\in}[I_L, I_U]$ is known as the indeterminate part. Note that the JT_N-test reduces to JT-test under classical statistics if $I_N{=}\,0$. This means that the NS approach provides the JT_N-test values in an interval with the measure of indeterminacy which is more general and includes the determinate

TABLE 1. Daily ICU occupancy data representing COVID-19 positive patients from Pakistan.

Day	**Age**		
	35 and below	**35-55**	**55 and above**
1	[460, 465]	[359, 361]	[443, 450]
2	[427, 429]	[352, 365]	[421, 426]
3	[407, 410]	[445, 455]	[436, 450]
4	[378, 380]	410	[376, 385]
5	[364, 368]	[458, 464]	458
6	[345, 349]	[410, 415]	[408, 420]
7	[342, 346]	[463, 470]	[422, 425]
8	345	[580, 584]	[431, 440]
9	[313, 318]	[432, 440]	[459, 462]
10	[277, 280]	379	369
11	[268, 271]	370	360
12	[259, 262]	[584, 589]	[431, 445]
13	[256, 260]	[410, 416]	[403, 415]
14	251	[587, 590]	[436, 445]
15	249	415	376
16	[233, 227]	[419, 422]	370
17	[209, 211]	357	443
15	[187, 191]	[467, 472]	445
19	[173, 175]	[415, 418]	[355, 365]
20	168	358	450

part of the CS. As an example, the neutrosophic form of the JT_N-test for groups in Table 2 is $4.116 - 4.198I_N;\ I_N \in [0,\ 0.020]$. It means that the proposed JT_N-test ranges between 4.116 and 4.198 with the degree of indeterminacy 0.020. After evaluating the comparisons, it can be inferred that the proposed test conducted under NS provides more comprehensive information compared to the current test carried out under CS.

TABLE 2. Neutrosophic Jonckheere trend test for COVID 19 data

Observed JT_N Statistic U_N	[898.000, 904.000]
μ_{NU}	[595.500, 595.500]
σ_{NU}	[73.488, 73.484]
JT_N-test$= \lvert Z_N \rvert$	[4.116, 4.198]
$p_N - value$ (1-tailed)	[0.000, 0.000]

4.2. *Simulation study*

The following simulation study compares the proposed test with the existing test under classic statistics using neutrosophic statistics. To do so, the empirical neutrosophic type I error and the power of the test for neutrosophic treatment effects at a given significant nominal level were calculated. The simulation was conducted on a given number of groups. The data are generated from the neutrosophic normal distribution, neutrosophic gamma distribution, and neutrosophic t distribution which has a non-centrality parameter (neutrosophic noncentral t-distribution). Then, by assuming significant nominal levels at 0.05 and 0.01, the MC method was employed to compute both the empirical neutrosophic type I error and the power of the test. The simulation was replicated 10000 times.

To calculate the neutrosophic empirical Type I error rate and the test power for an MC experiment, the following steps need to be completed:

MC simulation to compute $\alpha_{\text{Empirical}}$

Step 1: The random sample $x_{N1}^{(i)}, x_{N2}^{(i)}, ..., x_{Nn}^{(i)}$ is generated from a neutrosophic continuous distribution under H_{N0}, $i= 1, 2, \ldots, 10000$.

Step 2: The JT_N-test is computed under H_{N0}.

Step 3: The results are recorded by assigning a value of $I_{Ni}= 1$ when the H_{N0} is rejected, and $I_{Ni}= 0$ otherwise.

Step 4: The ratio $\frac{1}{10000}\sum_{i=1}^{10000} I_{Ni}$ is computed and take it as $\alpha_{\text{Empirical}}$.

MC simulation to compute $Power_{Empirical}$

Step 1: The random sample $x_{N1}^{(i)}, x_{N2}^{(i)}, ..., x_{Nn}^{(i)}$ is generated from a neutrosophic continuous distribution under H_{N1}, $i = 1, 2, \ldots, 10000$.

Step 2: The JT_N-test is computed under H_{N1}.

Step 3: The results are recorded by assigning a value of $I_{Ni} = 1$ when the H_{N1} is rejected, and $I_{Ni} = 0$ otherwise.

Step 4: The ratio $\frac{1}{10000}\sum_{i=1}^{10000} I_{Ni}$ is computed and take it as $Power_{Empirical}$.

5. Discussion

This study enhanced the analysis of uncertain and indeterminate data within NS for a Jonckheere trend test. The study established the fundamental analytical requirements for the Jonckheere trend test in a neutrosophic context, which can also apply to classical statistics. Subsequently, the proposed method was evaluated using COVID-19 data and verified through a simulation study to determine its accuracy. This test has been evaluated using empirical type I error and test power. Results indicated that the neutrosophic approach outperformed the existing approach, with better compatibility between empirical and nominal α values. Furthermore, the proposed test exhibited

TABLE 3. Simulation results for the NJT test in neutrosophic normal distribution with means $\delta = (\mu_{N1}, \mu_{N2}\ ,\ \mu_{N3},\ \mu_{N4})$ and $\sigma_N^2 = 1$ at the nominal levels of 0.01 and 0.05.

Sample Size	α	$\alpha_{Empirical}$	$Power_{Empirical}$					
			$\delta_1 = (\mathbf{0,0,0,0.1})$	$\delta_2 = (\mathbf{0.1,0.1,0.3,0.4})$	$\delta_3 = (\mathbf{0.1,0.2,0.4,0.5})$	$\delta_4 = (\mathbf{0,0.2,0.4,0.6})$	$\delta_5 = (\mathbf{0,0.3,0.5,0.7})$	$\delta_6 = (\mathbf{0,0.2,0.8,0.9})$
(5,5,5,5)	0.01	[0.0088, 0.0097]	[0.0129, 0.0139]	[0.0271, 0.0296]	[0.0394, 0.0432]	[0.0655, 0.0760]	[0.0820, 0.0939]	[0.1626, 0.1884]
	0.05	[0.0504, 0.0502]	[0.0631, 0.0647]	[0.1219, 0.1268]	[0.1494, 0.1585]	[0.2173, 0.2292]	[0.2600, 0.2850]	[0.4121, 0.4517]
(20,20,20,20)	0.01	[0.0111, 0.0100]	[0.0194, 0.0197]	[0.0881, 0.0977]	[0.1403, 0.1609]	[0.2922, 0.3346]	[0.3891, 0.4462]	[0.7346, 0.7982]
	0.05	[0.0492, 0.0501]	[0.0826, 0.0871]	[0.2520, 0.2780]	[0.3531, 0.3820]	[0.5638, 0.6085]	[0.6604, 0.7152]	[0.9034, 0.9353]
(4,6,7,5)	0.01	[0.0088, 0.0093]	[0.0128, 0.0146]	[0.0266, 0.0308]	[0.0371, 0.0416]	[0.0577, 0.0666]	[0.0709, 0.0799]	[0.1614, 0.1849]
	0.05	[0.0451, 0.0462]	[0.0588, 0.0598]	[0.1173, 0.1229]	[0.1413, 0.1499]	[0.1949, 0.2135]	[0.2244, 0.2472]	[0.3918, 0.4292]
(14,11,17,12)	0.01	[0.0106, 0.0097]	[0.0170, 0.0175]	[0.0566, 0.0640]	[0.0879, 0.1020]	[0.1890, 0.2183]	[0.2454, 0.2809]	[0.5183, 0.5830]
	0.05	[0.0479, 0.0494]	[0.0686, 0.0707]	[0.2014, 0.2172]	[0.2560, 0.2843]	[0.4169, 0.4571]	[0.5070, 0.5569]	[0.7734, 0.8210]

TABLE 4. Simulation results for the NJT test in neutrosophic gamma distribution with shape parameter $\alpha = (\alpha_{N1},\ \alpha_{N2},\ \alpha_{N3},\ \alpha_{N4})$ and inverse scale parameter $\beta_N = 1$ at the nominal levels of 0.01 and 0.05.

Sample Size	α	$\alpha_{Empirical}$	$Power_{Empirical}$					
			$\alpha_1 = (\mathbf{0,0,0,0.1})$	$\alpha_2 = (\mathbf{0.1,0.1,0.3,0.4})$	$\alpha_3 = (\mathbf{0.1,0.2,0.4,0.5})$	$\alpha_4 = (\mathbf{0,0.2,0.4,0.6})$	$\alpha_5 = (\mathbf{0,0.3,0.5,0.7})$	$\alpha_6 = (\mathbf{0,0.2,0.8,0.9})$
(5,5,5,5)	0.01	[0.0081, 0.0097]	[0.0137, 0.0134]	[0.0281, 0.0320]	[0.0360, 0.0444]	[0.0601, 0.0783]	[0.0725, 0.0928]	[0.1376, 0.1706]
	0.05	[0.0481, 0.0507]	[0.0650, 0.0677]	[0.1217, 0.1372]	[0.1456, 0.1639]	[0.2139, 0.2529]	[0.2452, 0.2851]	[0.3683, 0.4366]
(20,20,20,20)	0.01	[0.0123, 0.0099]	[0.0206, 0.0264]	[0.0874, 0.1155]	[0.1281, 0.1726]	[0.2732, 0.3652]	[0.3604, 0.4617]	[0.6565, 0.7616]
	0.05	[0.0520, 0.0502]	[0.0871, 0.0962]	[0.2526, 0.3014]	[0.3353, 0.4032]	[0.5482, 0.6506]	[0.6239, 0.7206]	[0.8701, 0.9230]
(4,6,7,5)	0.01	[0.0091, 0.0099]	[0.0109, 0.0124]	[0.0266, 0.0312]	[0.0322, 0.0434]	[0.0563, 0.0704]	[0.0642, 0.0827]	[0.1310, 0.1661]
	0.05	[0.0427, 0.0480]	[0.0598, 0.0646]	[0.1105, 0.1264]	[0.1314, 0.1531]	[0.1979, 0.2262]	[0.2199, 0.2532]	[0.3506, 0.4104]
(14,11,17,12)	0.01	[0.0090, 0.0101]	[0.0146, 0.0171]	[0.0614, 0.0776]	[0.0865, 0.1113]	[0.1720, 0.2301]	[0.2237, 0.2889]	[0.4546, 0.5548]
	0.05	[0.0487, 0.0503]	[0.0729, 0.0805]	[0.1950, 0.2274]	[0.2511, 0.2992]	[0.3965, 0.4781]	[0.4802, 0.5609]	[0.7179, 0.8000]

TABLE 5. Simulation results for the NJT test in neutrosophic t distribution with non-centrality parameter (ncp) ranging from 1 to 1.9 at the nominal levels of 0.01 and 0.05.

Sample Size and df	α	$\alpha_{Empirical}$	$Power_{Empirical}$					
			$ncp_1 = (\mathbf{1,1,1,1.1})$	$ncp_2 = (\mathbf{1.1,1.1,1.3,1.4})$	$ncp_3 = (\mathbf{1.1,1.2,1.4,1.5})$	$ncp_4 = (\mathbf{1,1.2,1.4,1.6})$	$ncp_5 = (\mathbf{1,1.3,1.5,1.7})$	$ncp_6 = (\mathbf{1,1.2,1.8,1.9})$
(5,5,5,5) and $df = \mathbf{3}$	0.01	[0.0112, 0.0107]	[0.0105, 0.0117]	[0.0255, 0.0271]	[0.0315, 0.0330]	[0.0543, 0.0580]	[0.0677, 0.0690]	[0.1265, 0.1376]
	0.05	[0.0523, 0.0511]	[0.0626, 0.0636]	[0.1069, 0.1103]	[0.1280, 0.1326]	[0.1873, 0.1938]	[0.2187, 0.2243]	[0.3468, 0.3622]
(20,20,20,20) and $df = \mathbf{10}$	0.01	[0.0108, 0.0099]	[0.0186, 0.0199]	[0.0855, 0.0925]	[0.1294, 0.1479]	[0.2718, 0.3081]	[0.3621, 0.4071]	[0.6839, 0.7314]
	0.05	[0.0475, 0.0495]	[0.0821, 0.0873]	[0.2379, 0.2516]	[0.3313, 0.3531]	[0.5413, 0.5753]	[0.6253, 0.6689]	[0.8819, 0.9087]
(4,6,7,5) and $df = \mathbf{3}$	0.01	[0.0086, 0.0091]	[0.0112, 0.0119]	[0.0265, 0.0275]	[0.0319, 0.0323]	[0.0485, 0.0538]	[0.0595, 0.0622]	[0.1179, 0.1294]
	0.05	[0.0435, 0.0442]	[0.0545, 0.0590]	[0.1050, 0.1121]	[0.1178, 0.1271]	[0.1668, 0.1780]	[0.1934, 0.2017]	[0.3145, 0.3300]
(14,11,17,12) and $df = \mathbf{10}$	0.01	[0.0098, 0.0101]	[0.0156, 0.0163]	[0.0540, 0.0584]	[0.0838, 0.0957]	[0.1710, 0.1906]	[0.2248, 0.2557]	[0.4741, 0.5262]
	0.05	[0.0518, 0.0509]	[0.0732, 0.0780]	[0.1848, 0.1953]	[0.2477, 0.2685]	[0.3930, 0.4302]	[0.4700, 0.5131]	[0.7408, 0.7823]

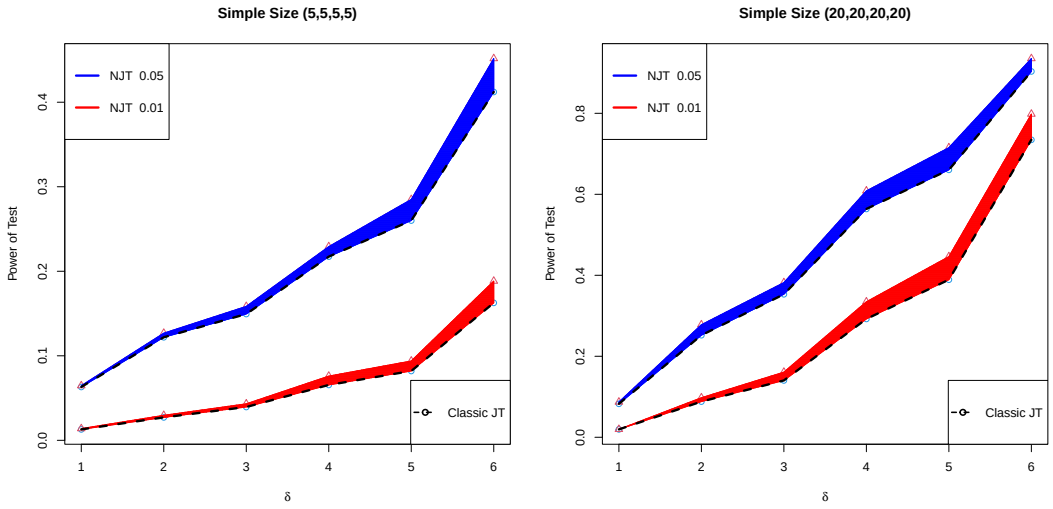

FIGURE 1. Power curves for the NJT test in neutrosophic normal distribution with sample sizes of $(5,5,5,5)$ and $(20,20,20,20)$ at the nominal levels of 0.01 and 0.05.

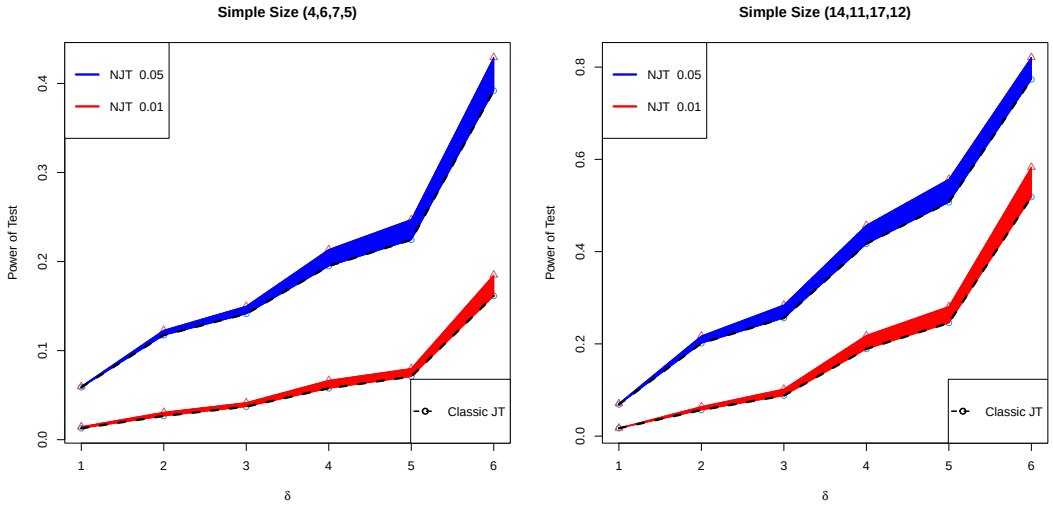

FIGURE 2. Power curves for the NJT test in neutrosophic normal distribution with sample sizes of $(4,6,7,5)$ and $(14,11,17,12)$ at the nominal levels of 0.01 and 0.05.

higher testing power than the existing test, as demonstrated by Tables 3-5 and Figures 1-6. The JT_N-test offers superior flexibility, applicability, and information than the current JT-test in uncertain environments.

6. Future research directions

This research paper focuses on the Jonckheere trend test and its application in uncertain environments. However, this test can also be adapted to address more complicated scenarios, such as missing data. Missing cells can create various situations that require further investigation. Future

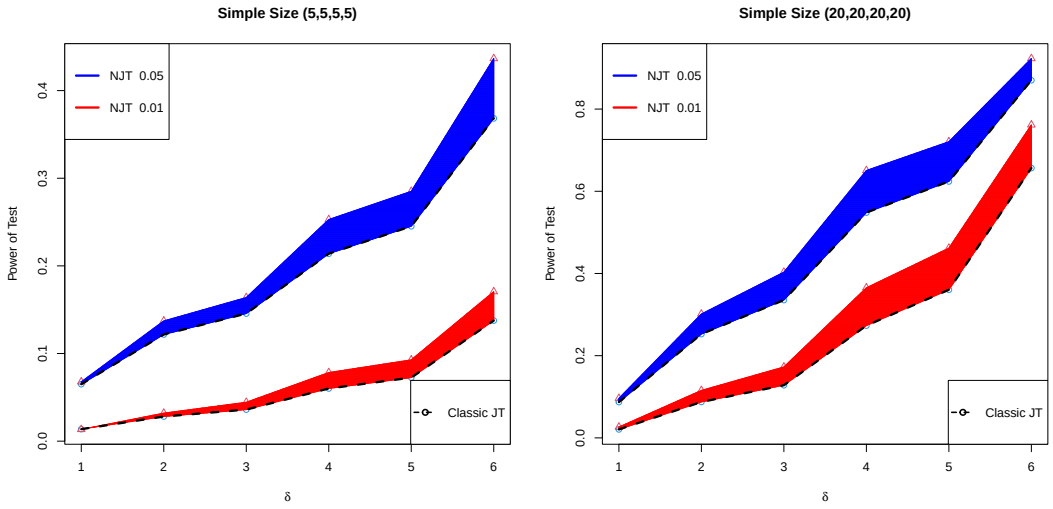

FIGURE 3. Power curves for the NJT test in neutrosophic gamma distribution with sample sizes of $(5, 5, 5, 5)$ and $(20, 20, 20, 20)$ at the nominal levels of 0.01 and 0.05.

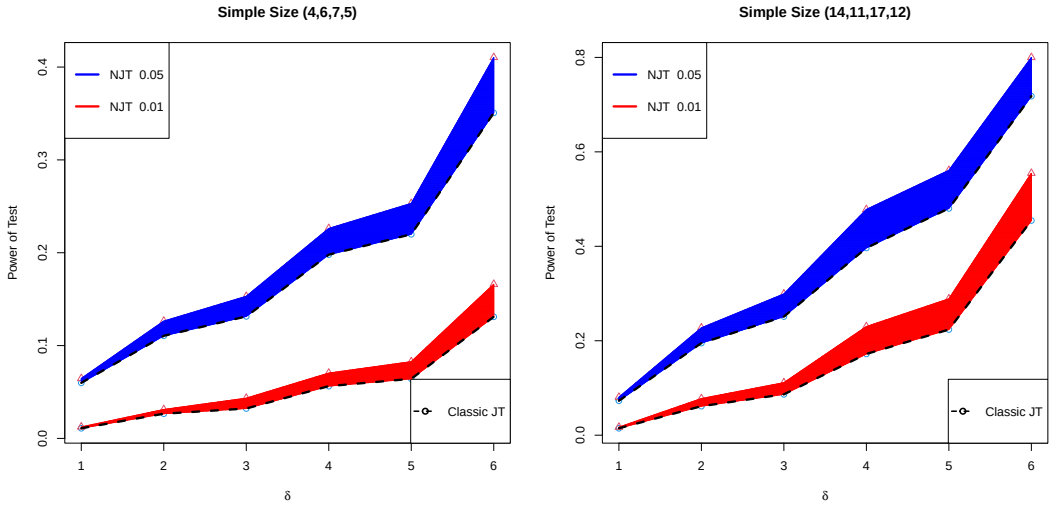

FIGURE 4. Power curves for the NJT test in neutrosophic gamma distribution with sample sizes of (4, 6, 7, 5) and (14, 11, 17, 12) at nominal levels of 0.01 and 0.05.

studies could explore this issue and expand upon the results to incorporate missing values into the test. The presence of missing data can introduce bias, making it crucial to select the most effective method for minimizing bias and obtaining accurate outcomes.

7. CONCLUSIONS

Using a neutrosophic approach is necessary when dealing with data uncertainties. However, traditional methods can be limited by errors and vagueness in the environment, making a neutrosophic

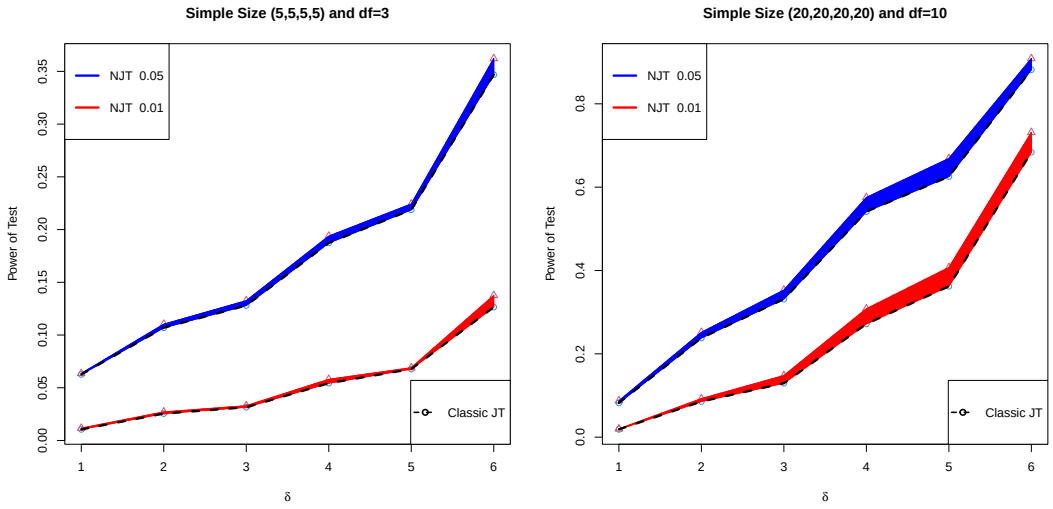

FIGURE 5. Power curves for the NJT test in neutrosophic noncentral t-distribution with sample sizes of $(5,5,5,5)$ and $(20,20,20,20)$ at the nominal levels of 0.01 and 0.05.

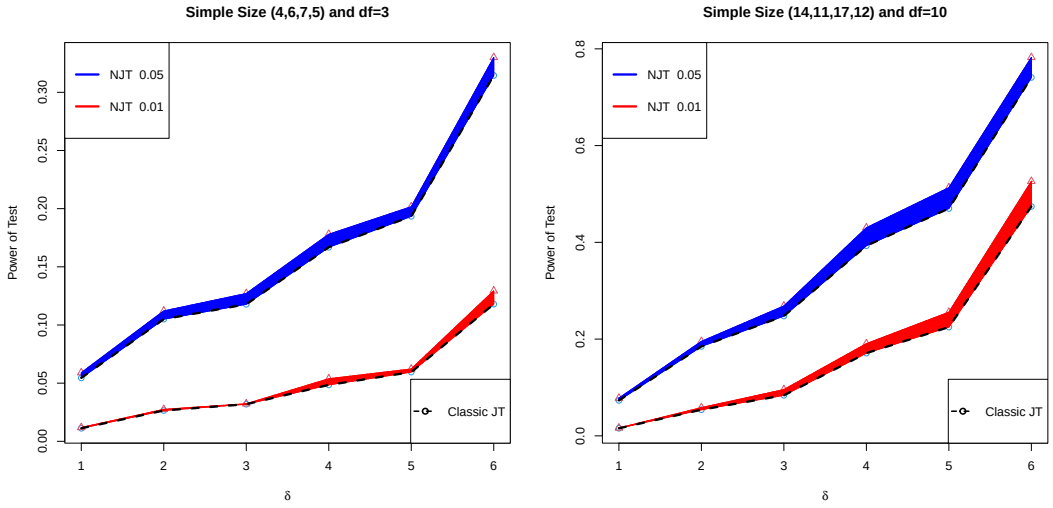

FIGURE 6. Power curves for the NJT test in neutrosophic noncentral t-distribution with sample sizes of $(4,6,7,5)$ and $(14,11,17,12)$ at the nominal levels of 0.01 and 0.05.

model a more effective option. It is worthwhile to note that this represents a methodological advancement, making the improvement significant. It was found that the proposed test significantly impacted the handling of uncertainties in this study. Importantly, this study also acknowledges computational progress. This was the first attempt to propose the neutrosophic Jonckheere trend test to address these issues while also considering the intrinsic indeterminacy of the data. As such, this study provides an appropriate framework for analyzing data under such conditions. The proposed neutrosophic test offers superior precision, flexibility, applicability, and information than existing tests in uncertain environments.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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