2024

Associated A Nexus with A Treesoft Sets and Vice Versa

Akbar Rezae

Karim Ghadimi

Florentin Smarandache
University of New Mexico, smarand@unm.edu

By submitting this extended abstract to AIMC55 I confirm that (i) I and any other coauthor(s) are responsible for its content and its originality; (ii) any possible coauthors agreed to its submission to AIMC55.

ASSOCIATED A NEXUS WITH A TREESOFT SETS AND VICE VERSA

AKBAR REZAE[1], KARIM GHADIMI[2,*] AND FLORENTIN SMARANDACHE[3]

[1,2] *Department of Mathematics, Payame Noor University, P.O.Box 19395-4697, Tehran, Iran*
[1] *rezaei@pnu.ac.ir*, [2] *kghadimi@pnu.ac.ir;*
[3] *University of New Mexico, 705 Gurley Ave., Gallup, NM 87301, USA*
smarand@unm.edu

Abstract. We recall the definitions of a nexus and a TreeSoft Set, and investigate the relation between them. We associated a nexus with a TreeSoft Set induced by a tree graph and vice versa.

1. Introduction

The *nexus* was defined by using the notion of the *address set* by Nooshin [5] in 1984. Brillouin's [2] investigated the concept of *nexus algebras* as an abstract algebraic structure in 2009. The soft set was introduced by Molodtsov [3] in 1999. The *MultiSoft Set* was introduced by Alkhazaleh et al. [1] in 2010. To get more realistic results, in 2022 Smarandache [4] defined the concept of *TreeSoft Set* as a generalization of soft sets and MultiSoft Sets. In this note, we investigate the relation between a nexus and TreeSoft Set induced by a tree graph, and associated a TreeSoft Set with a given nexus.

2010 *Mathematics Subject Classification.* Primary 20N20; Secondary 20N25, 03E72, 08A72.

Key words and phrases. Nexus, Soft set, MultiSoft Set, TreeSoft Set.

[*] Speaker.

Hereafter, let $\mathbb{N}^*$ be the set of non-negative integers, $\mathbb{N}$ the set of neutral numbers, U a universe of discourse, $P(U)$ the power set of U, H a non-empty subset of U, $P(H)$ the power set of H, A a set of attributes (parameters, factors, etc.), and N denotes a nexus.

2. Preliminaries

Definition 2.1. ([5]) a) A non-empty *address* is of the form $(a_1, a_2, \cdots, a_n, 0, 0, \cdots)$ (briefly, $(a_1, a_2, \cdots, a_n)$), where $a_i, n \in \mathbb{N}$. The sequence of zero is called the empty address and denoted by $()$.

b) A *nexus* N, is a non-empty set of addresses with the properties: $(a_1, a_2, \cdots, a_{n-1}, a_n) \in N$ implies $(a_1, a_2, \cdots, a_{n-1}, t) \in N$, for all $0 \leqslant t \leqslant a_n$ and for an infinite nexus $\{a_i\}_{i=1}^{\infty} \in N, a_i \in \mathbb{N}$ implies $(a_1, \cdots, a_{n-1}, t) \in N$, for $0 \leqslant t \leqslant a_n$.

c) Let $a \in N$. The level of a is said to be:

(i) n, if $a = (a_1, a_2, \ldots, a_n)$, for some $a_k \in \mathbb{N}$,
(ii) ∞, if a is an infinite sequence of N,
(iii) 0, if $a = ()$.

Definition 2.2. ([3]) The pair (F, U), where $F : A \to P(U)$ is called a *soft set* over U.

Definition 2.3. ([4]) Let $A = \{A_1, A_2, \cdots, A_n\}$, for some $n \in \mathbb{N}$, where $A_1, A_2, \cdots, A_n$ are considered attributes of first level (since they have one-digit indexes). Each attribute A_i, $1 \leq i \leq n$, is formed by sub-attributes $A_1 = \{A_{1,1}, A_{1,2}, \cdots\}$, $A_2 = \{A_{2,1}, A_{2,2}, \cdots\}$,$\cdots$, $A_n = \{A_{n,1}, A_{n,2}, \cdots\}$, where the above $A_{i,j}$ are sub-attributes (or attributes of second level) (since they have two-digit indexes). Again, each sub-attribute $A_{i,j}$ is formed by sub-sub-attributes (or attributes of third level): $A_{i,j,k}$. And so on, as much refinement as needed into each application, up to sub-sub- $\cdots$ sub-attributes (or attributes of m-level) (or having m digits into the indexes): $A_{i_1,i_2,\cdots,i_m}$. Therefore, a graph-tree is formed, that we denote as $Tree(A)$, whose root is A (considered of level zero), then nodes of level 1, level 2, up to level m. We call leaves of the graph-tree, all terminal nodes (nodes that have no descendants). Then the *TreeSoft Set* is:

$$F : P(Tree(A)) \to P(H)$$

$Tree(A)$ is the set of all nodes and leaves (from level 1 to level m) of the graph-tree, and $P(Tree(A))$ is the powerset of the $Tree(A)$. All node sets of the TreeSoft Set of level m are:

$$Tree(A) = \{A_{i_1} : i_1 \in \mathbb{N}\} \bigcup \{A_{i_1,i_2} : i_1, i_2 \in \mathbb{N}\} \bigcup \cdots \bigcup \{A_{i_1,i_2,\cdots,i_m} : i_1, i_2, \cdots, i_m \in \mathbb{N}\}.$$

The first set is formed by the nodes of level 1, second set by the nodes of level 2, third set by the nodes of level 3, and so on, the last set is formed by the nodes of level m. If the graph-tree has only two levels ($m = 2$), then the TreeSoft Set is reduced to a MultiSoft Set [1].

3. Main theorem and examples

Theorem 3.1. *Let $F : P(Tree(A)) \to P(H)$ be a TreeSoft Set. Then there is a Nexus N associated with this TreeSoft Set and vice versa.*

We can () associated with A, addresses of level 1 associated with attribute A_i, for $1 \le i \le n$, addresses of level 2 associated with attribute $A_{i,j}$, for $1 \le i, j \le n$, and so on, addresses of level n associated with attribute $A_{i_1,i_2,\cdots,i_n}$, for $1 \le i_1, \cdots, i_n \le n$, and vice versa.
The following example illustrated the Theorem 3.1.

Example 3.2. Consider a nexus:
$N = \{(), (1), (2), (3), (1, 1), (1, 1, 1), (1, 2), (1, 2, 1), (2, 1), (2, 3), (2, 3, 1), (3, 1)\}$
with the following diagram:

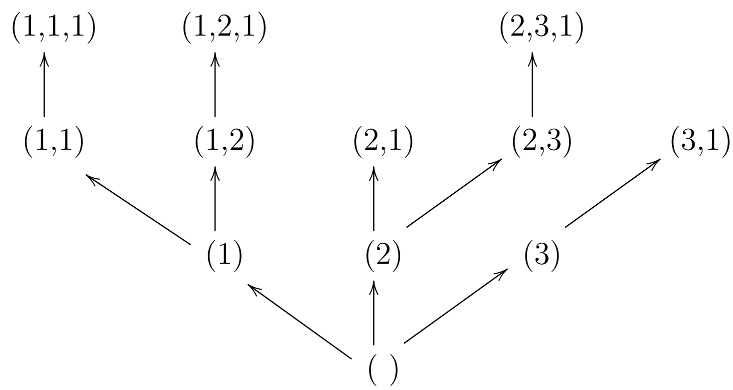

Fig. 3. Diagram of nexus N.

We put:
Level 0 (the root) is the node: ():=Attributes,
Level 1 is formed by the nodes: (1):=Size, (2):=Location, (3):=Weather,
Level 2 is formed by the nodes: (1,1):=Small, (1,2):=Big, (2,1):=Kerman, (2,3):=Isfahan, (3,1):=Warm,
Level 3 is formed by the nodes: (1,1,1):=Only room, (1,2,1):=Villa, (2,3,1):=Kashan.
Let $H := \{h_1, h_2, \cdots, h_{10}\}$ and the set of Attributes: $A := \{A_1, A_2, A_3\}$, where A_1 = Size, A_2 = Location, A_3 =Weather.
Then
$A_1 = \{A_{1,1}, A_{1,2}\} = \{\text{Small}, \text{Big}\}$ as Sizes,
$A_2 = \{A_{2,1}, A_{2,2}\} = \{\text{Kerman}, \text{Isfahan}\}$ as Iranian province,
$A_3 = \{A_{3,1}\} = \{\text{Warm}\}$ as Weather,

$A_{1,1} = \{A_{1,1,1}\} = \{\text{Only room}\}$ as Small size,
$A_{1,2} = \{A_{1,2,1}\} = \{\text{Villa}\}$ as Big size,
$A_{2,2} = \{A_{2,3,1}\} = \{\text{Kashan}\}$ as Isfahan cities.

The diagram of Tree(A) associate with the nexus N is:

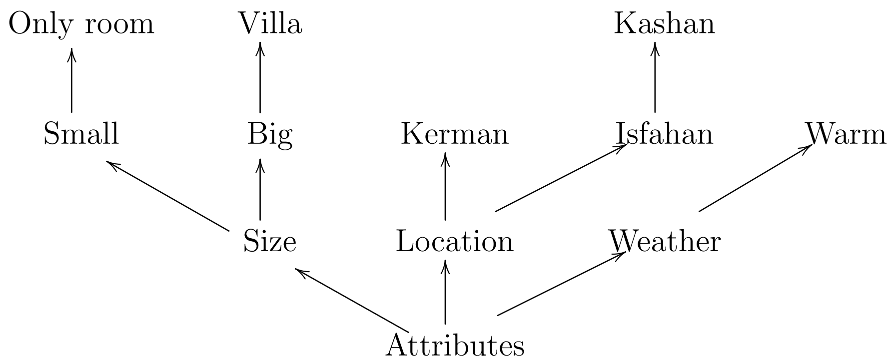

Fig. 3. Diagram of Tree(A).

Acknowledgement

The authors wish to dedicate this work to children and their families who fight against cancer.

References

1. S. Alkhazaleh, A. R. Salleh, N. Hassan and A. G. Ahmad, *Multisoft Sets*, Proc. 2^{nd} Int. Con. Math. Sci., 910–917, Kuala Lumpur, Malaysia, 2010.
2. M. Bolourian, *Theory of plenices*, Ph. D. Thesis, University of Surrey, Guildford, UK, 2009.
3. D. Molodtsov, *Soft Set Theory First Results*, Computer Math. Applic. 37 (1999), 19–31.
4. F. Smarandache, *Practical Applications of IndetermSoft Set and IndetermHyperSoft Set and Introduction to TreeSoft Set as an extension of the MultiSoft Set*, Neutrosophic Sets Syst., 51 (2022), 939–947.
5. H. Nooshin, *Formex configuration processing in structural engineering*, Elsevier Applied Science Publishers: London, UK, 1984.