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An Innovative Approach on Yao's Three-Way Decision Model Using Intuitionistic Fuzzy Sets for Medical Diagnosis

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Abstract: In the realm of medical diagnosis, intuitionistic fuzzy data serves as a valuable tool for representing information that is uncertain and imprecise. Nevertheless, decision-making based on this kind of knowledge can be quite challenging due to the inherent vagueness of the data. To address this issue, we employ power aggregation operators, which prove effective in combining several sources of data, such as expert thoughts and patient information. This allows for a more correct diagnosis; a particularly crucial aspect of medical practice where precise and timely diagnoses can significantly impact medication policy and patient results. In our research, we introduce a novel methodology to the three-way decision idea. Initially, we revamp the three-way decision model using rough set theory and incorporate interval-valued classes to handle intuitionistic fuzzy data. Secondly, we explore the use of intuitionistic fuzzy power weighted and intuitionistic fuzzy power weighted geometric aggregation operators to consolidate attribute values within the data system. Furthermore, we present a case study in the medical field to exhibit the validity and efficiency of our offered technique. This innovative method enables us to classify participants into three distinct zones based on their symptoms. The manuscript concludes with a summary of key points provided by the authors.

Keywords: Intuitionistic Fuzzy Sets; Aggregation Operators; Information System; Three-Way Decision; Medical Diagnosis; Decision Making; Optimization.

1. Introduction

Medical conditions can manifest with a variety of symptoms, which can complicate the process of accurate diagnosis. Some typical symptoms include elevated body temperature, fatigue, coughing, nausea, diarrhea, and skin eruptions. The identification of medical illnesses typically necessitates the involvement of healthcare professionals, who rely on a combination of patient history, physical assessments, laboratory tests, medical imaging, and other diagnostic measures to establish an analysis. This decision-making [1] process in medical diagnosis is influenced by numerous features, involving the patient's age, medical history, and genetic susceptibility. Physicians may also utilize diagnostic algorithms or decision trees to aid in the assessment of intricate medical conditions. In recent times, advancements in medical technology have considerably enhanced the precision and speed of medical diagnoses. This includes the integration of new imaging methods, genetic testing, and decision-making tools based on artificial intelligence [2-5].

The rough set model is a mathematical framework designed to systematically address situations involving incomplete and uncertain information. Zdzislaw Pawlak [6] first introduced this theory in the early 1980s to handle vague and doubtful information. In the context of medical diagnosis, a rough set model proves valuable in determining the presence or absence of specific diseases or conditions, even when dealing with incomplete or uncertain data sources [7], such as symptoms, medical history, test results, and other related information [8]. The core principle following the rough set model revolves around the segmentation of information into subsets, primarily driven by their attributes, such as symptoms or test outcomes. This partitioning procedure aids in the discernment of the critical features or factors most closely linked to a specific ailment or medical condition. Once the data is organized into these distinct subsets, rough set theory can be leveraged to unveil rules or shapes that facilitate making predictions about whether a given patient is afflicted by an actual disease or condition. A multitude of scientists have contributed to the advancement of innovative algorithms for disease diagnosis using this methodology. For instance, El-Bably and colleagues [8-10] started the concept of soft, rough approximation and implemented it in the realm of medical difficulty diagnosis. Hosny et al. [11] expanded the application of rough sets by introducing the maximal right neighborhood system and exploring its uses in the field of medicine. Additionally, Al-Shami et al. [12] defined maximal rough neighborhoods and employed this approach for the diagnosis of medical conditions.

Atanassov [13] established the theory of an intuitionistic fuzzy set (IFS), which represents an expansion of the conventional fuzzy set (FS). Within the realm of IFS, one can express both the degree of membership and non-membership of an element within a universal set. IFSs hold significant importance in the field of medicine, particularly in the context of disease identification and problem-solving. Researchers have extensively investigated the utilization of IFS in medical diagnosis, especially in scenarios characterized by substantial hesitation and flexibility in symptoms and test outcomes. The intuitionistic fuzzy set offers a valuable tool for capturing and conveying this uncertainty, thus enhancing the precision of diagnostic knowledge. For instance, in the context of diagnosing complex conditions such as cancer, Intuitionistic Fuzzy Sets (IFS) can portray the degree of confidence or vagueness related to the diagnosis, considering a wide range of diagnostic criteria like blood test results, imaging studies, and biopsy conclusions. This approach ultimately leads to more precise and dependable diagnoses while also facilitating the creation of personalized treatment plans. Jiang et al. [14] employed IFS in medical image fusion, utilizing entropy measures, whereas Mehmood et al. [15, 16] extended the concept of intuitionistic fuzzy sets and applied these principles to the field of medical diagnosis. De et al. [17] also delved into the application of IFS in medical diagnosis, much like the work of Davvaz et al. [18]. Szmidt et al. [19] investigated the utilization of IFS in intelligent data analysis for medical diagnosis. In the decision-making process involving IFS, aggregation operators play a crucial role in computing attribute values. As a result, several experts have proposed a variety of aggregation operators for this purpose. For instance, Xu et al. [20] developed and implemented power aggregation operators for IFS in Multi-Attribute Decision Making (MADM). In 2006, Xu et al. [21] introduced geometric aggregation operators tailored for IFS. Mehmood et al. [22] presented similarity measures and power aggregation operators based on Intuitionistic Hesitant Fuzzy Sets (IHFS). More recently, Senapati et al. [23] and Garg et al. [24] have explored novel operators in this context.

The three-way decision (TWD) concept represents a notable extension of the RS theory, initially introduced by Yao [25, 26]. In the realm of medical diagnosis, a three-way decision entails the assessment of three potential outcomes: positive, negative, or inconclusive. In the case of a Positive Outcome, when a medical diagnosis yields a positive result, it confirms that the patient indeed has the specific condition or disease under examination. This necessitates treatment for the diagnosed ailment, with healthcare professionals closely monitoring the patient's progress. Conversely, a Negative Outcome in a medical diagnosis indicates that the patient does not have the particular condition or disease under investigation. In such instances, the patient may not need any medication, and healthcare providers may need to explore other potential causes for the patient's symptoms. An

Inconclusive Outcome arises when the test results do not provide sufficient clarity to decide whether the patient possesses the condition or disease under scrutiny. In such situations, additional tests or evaluations may be necessary to succeed at a more definitive diagnosis. Lately, Li et al. [27, 28] applied TWD procedures to improve decision-making in medical diagnosis. Hu et al. [29, 30] introduced the notion of a lattice model for medical diagnosis, incorporating TWD. Jia and Fan [31] devised TWD models for multi-criteria situations, while Ye et al. [32] integrated the TWD concept into the emerging field of fuzzy information systems. In a similar vein, numerous scholars have explored this field, proposing innovative approaches across various extensions of fuzzy sets [33-35].

In our exploration of the literature, we discovered that TWD models prove highly beneficial for medical problem diagnosis. The fusion of IFS and TWD, as described in reference [36], yields a robust framework for addressing situations characterized by vagueness and ambiguity. It should be highlighted that aggregating the outcomes of numerous participants through TWD poses a significant challenge. Researchers have traditionally employed conventional methods to compute alternatives for TWD, as evidenced by references [37-40]. In the existing TWD model, as outlined in references [25, 37], an external concept becomes necessary for determining equivalence classes. Additionally, a threshold is employed to categorize the alternatives into three distinct regions.

The primary objective behind creating this piece of work is to create an innovative algorithm for the TWD by utilizing aggregation operators and enhancing the TWD decision process through interval-valued equivalence classes for Interval-Valued Fuzzy Sets (IVFS). This approach aims to address the existing deficiencies and challenges in TWD computation. The following is a depiction of the key contribution made by this analysis.

i. Establish the notion of intervals to represent the degrees of membership in Interval Fuzzy Sets (IFS) by utilizing the step size function.
ii. Create equivalence classes by leveraging intervals and refer to them as interval-valued classes.
iii. To address concerns related to computational efficiency and timesaving, we introduce the IFPWA and IFPWG aggregation operators specifically tailored for the TWD theory.
iv. Present an algorithm designed for the classification of diverse patients and the diagnosis of diseases using multiple symptom criteria.

The rest of the article follows this structure: Section 2 presents an overview of essential concepts, including IFS, power aggregation operators, and Three-way Decisions in Section 3, we define membership grade intervals using the step size function and create equivalence classes. We then proceed to adapt the TWD for IFS based on these intervals. Section 4 encompasses the development of a well-defined algorithm, including a flow chart, and an in-depth, step-by-step explanation of the approach. Section 5 delves into a case study where we apply the offered methodology to diagnose a medical issue and classify alternatives using power aggregation operators for IFS. We also extensively discuss the benefits and benefits of the suggested models. Finally, Section 6 encapsulates the authors' conclusion and their plan.

2. Preliminaries

In this section, we take a closer look at several fundamental concepts within intuitionistic fuzzy sets (IFS) and explore some notions about power aggregation operators.

2.1 Intuitionistic Fuzzy Sets and Aggregation Operators

Atanassov [13] proposed the theory of IFS as an expansion of FS. While FS offers the membership grade (MG) of an element within a specific set [0, 1], IFS simultaneously provides both MG and non-membership grade (NMG).

Definition 1: [13] An IFS T on set E is represented using the two mappings $l(e)$ and $m(e)$. Mathematically, this representation is expressed through the following structure:

$$T = \langle e, l_T(e), m_T(e)) | e \in E \rangle,$$

Where, $l_T(e): E \to [0,1]$ and $m_T(e): E \to [0,1]$ signifies the MG and NMG including the condition $0 \le l(e) + m(e) \le 1$, for each $e \in E$.

Definition 2: let $T = (l_T, m_T)$ be an IFN, then the score function and accuracy function are stated and represented as:

$$S(T) = l_T - m_T, \qquad S(T) \in [-1,1];$$
$$H(T) = l_T + m_T, \qquad H(T) \in [0,1].$$

Definition 3: Suppose $T_1 = (l_1, m_1)$, $T_2 = (l_2, m_2)$ be intuitionistic fuzzy sets (IFSs), then some basic operations are described as below:

i. $T_1 \oplus T_2 = (\{l_1 + l_2 - l_1 l_2\}, \{m_1 m_2\});$
ii. $T_1 \otimes T_2 = (\{l_1 l_2\}, \{m_1 + m_2 - m_1 m_2);$
iii. $\lambda T_1 = \left(1 - (1-l)^{\lambda}, m^{\lambda}\right), \lambda > 0;$
iv. $T_1^{\lambda} = \left((l)^{\lambda}, 1 - (1-m)^{\lambda}\right), \lambda > 0;$
v. $T_1^{c} = (m_1, l_1).$

Definition 4: [21] Suppose that $T_j = \left(l_j, m_j\right)$ is a collection of IFS and the $k_j = (k_1, k_2, \dots, k_m)^T$ is weight vector for T_j, and $\sum_{j=1}^{m} k_j = 1$. Then $IFPWP_k$ an operator is a mapping $IFPWP_k: T^m \to T$ where

$$IFPWP_k(T_1, T_2, \dots, T_m) = \frac{\oplus_{j=1}^{m}(k_j(1 + J(T_j)T_j)}{\sum_{j=1}^{m} k_j(1 + J(T_j))} =$$
$$\left(1 - \prod_{j=1}^{m}(1 - (l_j)^{\frac{k_j((1+J(T_j))}{\sum_{j=1}^{m} k_j(1+J(T_j))}}, \prod_{j=1}^{m}(m_j)^{\frac{k_j(1+J(T_j))}{\sum_{j=1}^{m} k_j(1+TJT_j))}}\right),$$

Where,

$$J(T_j) = \textstyle\sum_{\substack{i=1\\ i\ne j}}^{m} k_j Sup(T_j, T_i),$$
$$Sup(T_j, T_i) = 1 - d(T_j, T_i),$$
$$d(T_j, T_i) = \frac{1}{l}\textstyle\sum_{\substack{i=1\\ i\ne j}}^{l}(|l_i - l_j| + |m_i - m_j|).$$

Definition 5: For IFSs $T_i = (l_i, m_i)$ with k_j such that $k_j > 0$ and $\sum_{j=1}^{m} k_j = 1$. A mapping $IFPOWP_k$: $T^m \to T$, is stated as:

$$IFPOWP_k(T_1, T_2, \dots, T_m) = \frac{\oplus_{j=1}^{m}(k_j(1 + J(T_{\sigma(j)})T_{\sigma(j)})}{\sum_{j=1}^{m} k_j(1 + J(T_{\sigma(j)}))}$$
$$= \left(1 - \prod_{j=1}^{m}(1 - (l_{\sigma(j)})^{\frac{(k_j(1+J(T_{\sigma(j)}))}{\sum_{j=1}^{m} k_j(1+J(T_{\sigma(j)}))}}, \prod_{j=1}^{m}(m_{\sigma(j)})^{\frac{(k_j(1+J(T_{\sigma(j)}))}{\sum_{j=1}^{m} k_j(1+J(T_{\sigma(j)}))}}\right).$$

Definition 6: For a set of IFS $T_j = \left(l_j, m_j\right)$ and the weights k_j for T_j, and $\sum_{j=1}^{m} k_j = 1$. Then $IFPWG_k$ an operator is a mapping $IFPWG_k: T^m \to T$.

$$IFPWG_k(T_1, T_2, \dots, T_m) = \frac{\otimes_{j=1}^{m}(k_j(1 + J(T_j)T_j)}{\sum_{j=1}^{m} k_j(1 + J(T_j))} =$$
$$\left(\prod_{j=1}^{m}(l_j)^{\frac{k_j\left(1+J(T_j)\right)}{\sum_{j=1}^{m} k_j(1+J(T_j))}}, 1 - \prod_{j=1}^{m}(1 - (m_j)^{\frac{k_j((1+J(T_j))}{\sum_{j=1}^{m} k_j(1+J(T_j))}}\right).$$

Where, $J(T_j) = \sum_{\substack{i=1\\ i\ne j}}^{m} k_j Sup(T_j, T_i)$.

Definition 7: For IFSs $T_i = (l_i, m_i)$ with their weights k_j such that $k_j > 0$ and $\sum_{j=1}^{m} k_j = 1$. A mapping $IFPOWG_k$: $T^m \to T$, is expressed as:

$$IFPOWG_k(T_1, T_2, \dots, T_m) = \frac{\otimes_{j=1}^{m}(k_j(1 + J(T_{\sigma(j)})T_{\sigma(j)})}{\sum_{j=1}^{m} k_j(1 + J(T_{\sigma(j)}))}$$
$$= \left(\prod_{j=1}^{m}(l_{\sigma(j)})^{\frac{(k_j(1+J(T_{\sigma(j)}))}{\sum_{j=1}^{m} k_j(1+J(T_{\sigma(j)}))}}, 1 - \prod_{j=1}^{m}(1 - (m_{\sigma(j)})^{\frac{(k_j(1+J(T_{\sigma(j)}))}{\sum_{j=1}^{m} k_j(1+J(T_{\sigma(j)}))}}\right).$$

2.2 Three-Way Decision Model Based on Rough Sets

Rough sets [6] constitute a mathematical theory that was formulated by Zdzislaw Pawlak, a Polish computer scientist, during the early 1980s. This notion deals with a structured methodology for addressing uncertainty and handling incomplete information within the realm of data analysis. The theory of three-way decision (TWD) [25] is an expansion of a rough set model, designed to accommodate the notion of "don't know" or "undetermined" elements in decision-making processes. In the traditional two-way decision-making framework, data is typically categorized into two sets: one that fulfills specific conditions and another that does not. The primary constraints associated with this approach are detailed below.

Definition 8: [38] Let $\mathcal{S} = (E, At, V_a, f)$ be an information system (IS), where $E = \{P_1, P_2, \dots P_m\}$ is the universe of discourse. $At = \{c_1, c_2, \dots, c_l\}$ is the set of the attributes, $V_a = \cup_{c \in Pt} \mathbb{V}_c$ is the range of values, $\mathbb{V}_c$ represents the value under attribute c, and $f = E \times At \to V_a$ is an information mapping function.

Furthermore, within these IS, two different categories of attributes exist condition attributes (C) and decision attributes (D), which collectively form the set of attributes denoted as $At = C \cup D$. These information systems are occasionally referred to as decision IS. To collect the parts in E based on the features in Pt, equivalence classes are constructed in rough sets (RSs). In this context, equivalence classes of the relations $BND(C)$ and $BND(D)$ will be called condition and decision classes, respectively.

Definition 9: Let $\mathcal{S} = (E, At, V_a, f)$ be an IS, and B is, $B \subseteq At$, an equivalence relation R is defined as:

$$R_B = \left\{ \left(P_{c_i}, P_{c_j}\right) \in E \times E \,|\, for\ all\ c \in B\ \left(P_{c_i} = P_{c_j}\right) \right\}.$$

With this relationship, the equivalence class of an element P_B in set E is established as follows,

$$\left[P_{c_i}\right]_B = \left\{ P_{c_j} \in E \mid \left(P_{c_i}, P_{c_i}\right) \in R_B \right\}$$

The main objective of an equivalence relation is to show the inability to distinguish objects. Using the equivalence relation denoted as R_B, the IS can be separated into three different segments by approximation classes.

Definition 10: Pawlak [6] started the concept of approximation classes for the approximation space $Appr(E, R)$ of E, defined for all $U \subseteq E$ as follows:

$$\underline{Appr}(U) = \{P \in E \mid [P]_B \subseteq U\},$$

$$\overline{Appr}(U) = \{P \in E \mid [P]_B \cap U \neq \emptyset\},$$

These categories are referred to as the lower approximation class denoted as $\underline{Appr}(U)$ and the upper approximation class indicated as $\overline{Appr}(U)$ with $[P]_B$ representing the equivalence class of P.

Definition 11: Using the classification of approximations, three distinct regions are defined in the following manner.

$$POS(U) = \underline{Appr}(U),$$

$$NEG(U) = E - \overline{Appr}(U),$$

$$BND(U) = \overline{Appr}(U) - \underline{Appr}(U).$$

Definition 12: Let $\mathcal{S} = (E, At, V_a, f)$ be an IS and a subset of attributes $B \subseteq At$, then the decision rules of $U \subseteq E$ and $z \in U$ are designed as:

($\mathcal{A}$) If $q \vDash Des([P]_B)\ for\ [P]_B \in POS(U)$, then accept q,

($\mathcal{R}$) If $q \vDash Des([P]_B)\ for\ [P]_B \in NEG(U)$, then reject q,

($\mathcal{N}$) If $q \vDash Des([P]_B)\ for\ [P]_B \in BND(U)$, then neither accept nor reject q.

3. A Novel Three-Way Decision Model Based on Interval-Valued Classes

In this portion, we introduce an innovative approach to model TWD by creating intervals. These intervals lead to the creation of unique sets of interval-valued equivalence classes, which, in turn, are used to categorize participants into three different regions: POS (positive), NEG (negative), and BND (boundary), facilitating their classification.

To transform the information system into a discrete form, we replace traditional equivalence classes with interval-valued equivalence classes, guided by the step size function. This function assists in dividing the alternatives into intervals, and its definition is as follows.

Definition 13: For the collection of IFNs (T_i), the intervals ($\mathcal{I}_m$) for approximation classes established on MGs are defined and denoted as:

$$\mathcal{I}_m = [Mim(l_i), Mim(l_i) + h]$$

When the step size function (h) is established for the MGs of IFNs, it is defined by:

$$h = \frac{Max(l_i) - Mim(l_i)}{m}$$

Where m is the number of intervals $\mathcal{I}_m$ which we required.

According to the parental concept of TWD by Yao [25], by using the equivalence classes, we can provide the approximation classes. Continually, by the definition of intervals $\mathcal{I}_n$ in definition 13, nth interval-valued equivalence classes $[P]_B$ for the participants are developed as below:

Definition 14: The design of interval-valued equivalence classes $[P]_B$ for the alternatives P_i is structured such that:

$$[P]_B = \{P: P_i \in \mathcal{I}_m\}$$

Definition 15: The approximation classes within the approximation space $Appr\ (E, R)$ for all $U \subseteq E$, as defined by:

$$\underline{Appr}(U) = \{P \in E | [P]_B \subseteq U\},$$
$$\overline{Appr}(U) = \{P \in E | [P]_B \cap\ U \neq \emptyset\}.$$

Definition 16: Using the approximation classes outlined in definition 15, we can introduce three distinct regions as below.

$$POS(U) = \underline{Appr}(U),$$
$$NEG(U) = E - \overline{Appr}(U),$$
$$BND(U) = \overline{Appr}(U) - \underline{Appr}(U).$$

Definition 17: The three kinds of decision rules ($\mathcal{A}2 - \mathcal{N}2$) of $U \subseteq E$ for an IS $\mathcal{S} = (E, At, V_a, f)$ are described as:

($\mathcal{A}2$) If $q \vDash Des([P]_B) for\ [P]_B \in POS(U)$, then accept q,

($\mathcal{R}2$) If $q \vDash Des([P]_B) for\ [P]_B \in NEG(U)$, then reject q,

($\mathcal{N}2$) If $q \vDash Des([P]_B) for\ [P]_B \in BND(U)$, then neither accept nor reject q.

4. An Algorithm for the Proposed Model

This portion delves into the detailed application of $IFPWP_k$ and $IFPWG_k$ aggregation operators under IF information for three-way decision-making. We plan five stages for choosing the TWD rules for distinct partakers. Let $\mathrm{E} = \{P_1, P_2 \dots P_n\}$ represent the set of participants and consider $U = \{Yes, No\}$ denote the set of states indicating the decisions of participants, where $U \subseteq E$. The flowchart of the TWD model is displayed in Figure 1.

Step 1. Assess the data system with conditional and decision attributes using an intuitionistic fuzzy approach.

Step 2. For participants $P_i (i = 1,2, \dots, m)$ aggregate all the IF attributes $P_{ij} (j = 1,2, \dots, l)$ into a general result P_i utilizing $IFPWA_{\mathrm{k}}$ and $IFPWG_k$operators as below:

$$IFPWA_k(T_1, T_2, \dots, T_m) = \frac{\oplus_{j=1}^{m}(k_j(1 + J(T_j)T_j)}{\sum_{j=1}^{m} k_j(1 + J(T_j))} =$$
$$\left(1 - \prod_{j=1}^{m}(1 - (l_j)^{\frac{k_j((1+J(T_j))}{\sum_{j=1}^{m} k_j(1+J(T_j))}}, \prod_{j=1}^{m}(m_j)^{\frac{k_j(1+J(T_j))}{\sum_{j=1}^{m} k_j(1+J(T_j))}}\right),$$

and

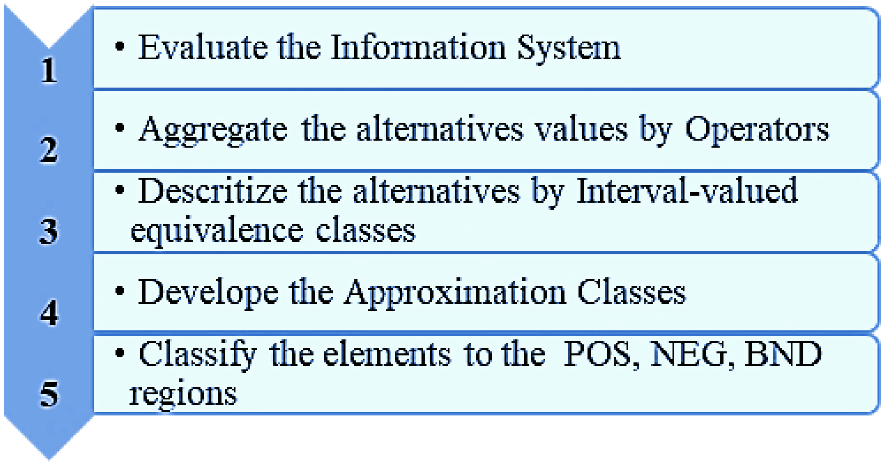

Figure 1. Five steps flow chart of interval-valued TWD model.

$$IFPWG_k(T_1, T_2, \dots, T_m) = \frac{\otimes_{j=1}^{m} (k_j(1+J(T_j)T_j)}{\sum_{j=1}^{m} k_j(1+J(T_j))} = \left(\prod_{j=1}^{m} (l_j)^{\frac{k_j(1+J(T_j))}{\sum_{j=1}^{m} k_j(1+J(T_j))}}, 1-\prod_{j=1}^{m} (1-(m_j)^{\frac{k_j((1+J(T_j))}{\sum_{j=1}^{m} k_j(1+J(T_j))}}\right),$$

Step 3. Determine the interval-valued equivalence classes based on the interval specified in Definition 13.

Step 4. Discretize the upper approximation class and lower approximation class defined in Definition 15.

Step 5. Categorize the options into the POS, NEG, and BND regions based on their approximate classes.

5. Mathematical Model

Now, we offer an illustrative case that serves as a practical example for making decisions regarding the investigation of medical issues, with a focus on confirming or ruling out diseases in patients.

5.1 Explanation of the Problem

Medical diagnosis involves identifying the specific illness or condition that matches a person's symptoms. Healthcare professionals strive to make precise determinations by evaluating a patient's symptoms. It's a process where doctors select a particular disease based on the symptoms exhibited by an individual. The use of IFRS aids healthcare experts in handling complex linguistic concepts and minimizes inaccuracies. The effectiveness of IFRS in medical diagnosis is demonstrated in references [17, 28]. Figure 2 provides a visual representation of the medical diagnosis procedure.

Suppose that a collection of alternatives denoted as P_i (where $i = 1,2,\dots,15$), participates in the investigation to diagnose the disease "Coronavirus."

Additionally, let I be the group of conditional attributes, specifically $B = \{B_1\ (Chest\ paim), B_2\ (Fever), B_3(Fatigue), B_4(Cough),\}$. Furthermore, the set U, represented as $U = \{P_1, P_2, P_4, P_{15}, P_{11}\}$, indicates the decision attributes that provide the concept of "Yes" for the disease. Let the experts diagnose the disease for all participants and classify their decisions using the weighty vector $k = \{0.2, 0.3, 0.4, 0.1\}$. We will now employ a step-by-step algorithm to provide a detailed explanation of this medical condition.

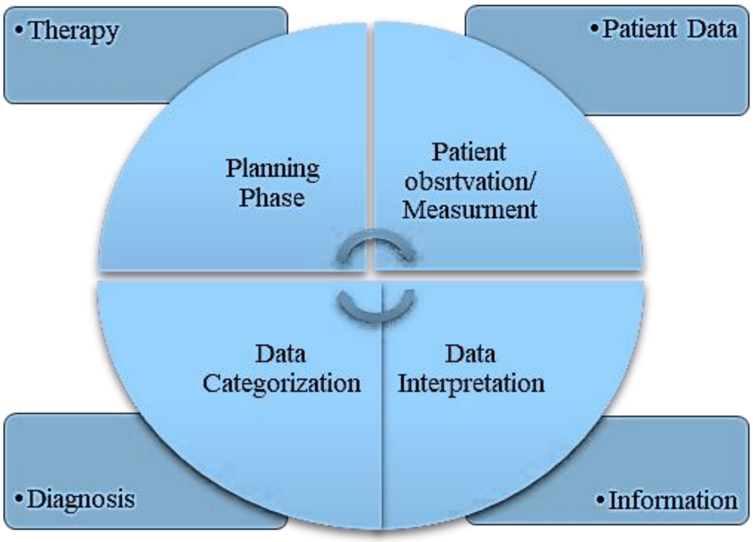

Figure 2. Medical diagnosis diagram.

Step 1: Table 1 represents the IF data of patients.

Table 1. An information Table of alternative conditions.

At	B_1	B_2	B_3	B_4	D
P_1	(0.1,0.3)	(0.4,0.5)	(0.1,0.5)	(0.1,0.5)	Yes
P_2	(0.4,0.5)	(0.5,0.4)	(0.5,0.3)	(0.2,0.6)	Yes
P_3	(0.2,0.3)	(0.2,0.4)	(0.6,0.2)	(0.4,0.5)	No
P_4	(0.4,0.2)	(0.1,0.2)	(0.7,0.4)	(0.3,0.1)	Yes
P_5	(0.5,0.3)	(0.5,0.2)	(0.3,0.2)	(0.4,0.2)	No
P_6	(0.6,0.2)	(0.7,0.1)	(0.4,0.1)	(0.4,0.4)	No
P_7	(0.7,0.1)	(0.2,0.2)	(0.5,0.2)	(0.5,0.2)	No
P_8	(0.3,0.4)	(0.3,0.3)	(0.6,0.2)	(0.2,0.3)	No
P_9	(0.4,0.2)	(0.5,0.2)	(0.7,0.2)	(0.3,0.5)	No
P_{10}	(0.5,0.2)	(0.8,0.1)	(0.2,0.3)	(0.4,0.3)	No
P_{11}	(0.6,0.2)	(0.9,0.1)	(0.5,0.3)	(0.5,0.4)	Yes
P_{12}	(0.8,0.1)	(0.0,0.9)	(0.6,0.4)	(0.2,0.2)	No
P_{13}	(0.9,0.1)	(0.3,0.2)	(0.4,0.3)	(0.4,0.3)	No
P_{14}	(0.1,0.2)	(0.2,0.2)	(0.6,0.3)	(0.3,0.4)	No
P_{15}	(0.8,0.1)	(0.1,0.3)	(0.3,0.4)	(0.4,0.2)	Yes

Step 2. For participants P_i calculate all the conditional attributes information applying $IFPWA_k$ and $IFPWG_k$ operators in the following:

$$IFPWA_k(T_1, T_2, \dots, T_m) = \frac{\oplus_{i=1}^{m}(k_j(1+J(T_i)T_i)}{\sum_{i=1}^{m} k_j(1+J(T_i))} =$$

$$\left(1 - \prod_{i=1}^{m}(1-(l_i)^{\frac{k_j((1+J(T_i))}{\sum_{i=1}^{m} k_j(1+J(T_i))}}, \prod_{i=1}^{m}(m_i)^{\frac{k_j(1+J(T_i))}{\sum_{i=1}^{m} k_j(1+J(T_i))}}\right),$$

and

$$IFPWG_k(T_1, T_2, \dots, T_m) = \frac{\otimes_{i=1}^{m}(k_j(1+J(T_i)T_i)}{\sum_{i=1}^{m} k_j(1+J(T_i))} =$$

$$\left(\prod_{i=1}^{m}(l_i)^{\frac{k_j(1+J(T_i))}{\sum_{j=1}^{m}k_j(1+J(T_i))}}, 1-\prod_{i=1}^{m}(1-(m_i)^{\frac{k_j((1+J(T_i))}{\sum_{i=1}^{m}k_j(1+J(T_i))}}\right).$$

The results are presented in Table 2.

Table 2. Aggregated results of all alternatives.

At	$IFPWA_k$	$IFPWG_k$
P_1	(0.203, 0.458)	(0.151, 0.470)
P_2	(0.466, 0.374)	(0.450, 0.393)
P_3	(0.429, 0.281)	(0.347, 0.306)
P_4	(0.500, 0.261)	(0.333, 0.292)
P_5	(0.409, 0.214)	(0.389, 0.217)
P_6	(0.545, 0.124)	(0.507, 0.142)
P_7	(0.471, 0.177)	(0.401, 0.183)
P_8	(0.452, 0.261)	(0.400, 0.275)
P_9	(0.581, 0.213)	(0.541, 0.226)
P_{10}	(0.523, 0.201)	(0.372, 0.227)
P_{11}	(0.703, 0.205)	(0.615, 0.236)
P_{12}	(0.507, 0.384)	(0.00, 0.618)
P_{13}	(0.535, 0.220)	(0.420, 0.239)
P_{14}	(0.411, 0.252)	(0.302, 0.262)
P_{15}	(0.395, 0.276)	(0.259, 0.312)

Step 3. Determine the interval-based equivalence classes applying the prescribed method with a step size of $n = 5$ as illustrated in Table 3.

Table 3. Interval-valued equivalence classes.

$IFPWA_k$	$[P_1] = \{P_1\}$ $[P_2] = \{P_2, P_3, P_4, P_5, P_7, P_8, P_{14}\}$ $[P_6] = \{P_6, P_9, P_{10}, P_{12}, P_{13}\}$ $[P_{11}] = \{P_{11}\}$ $[P_{15}] = \{P_{15}\}$
$IFPWG_k$	$[P_1] = \{P_1\}$ $[P_2] = \{P_2, P_5, P_7, P_8, P_{10}, P_{13}\}$ $[P_3] = \{P_3, P_4, P_{14}, P_{15}\}$ $[P_6] = \{P_6, P_9, P_{11}\}$ $[P_{12}] = \{P_{12}\}$

Step 4. Determine the lower approximation and upper approximation by Definition 15 for the given decision attributes $U = \{P_1, P_2, P_4, P_{15}, P_{11}\}$.

Table 4. Approximation classes.

IFPA	$\underline{Appr}(U) = \{P_1, P_{15}, P_{11}\}$ $\overline{Appr}(U) = \{P_1, P_{15}, P_3, P_5, P_7, P_8, P_4, P_{14}, P_2\}$
IFPG	$\underline{Appr}(U) = \{P_1\}$, $\overline{Appr}(U) =$ $\{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{13}, P_{14}\}$

Step 5. Finally, the classification of the elements for POS, NEG, and BND regions respectively represented in Table 5.

Table 5. Classification of alternatives accordingly.

IFPA	IFPG
$\mathbf{POS(U) = \{P_1, P_{15}, P_{11}\}}$ $\mathbf{NEG(U) = \{P_{12}, P_6, P_{13}, P_9, P_{10}\}}$ $\mathbf{BND(U) = \{P_2, P_3, P_4, P_8, P_7, P_{10}\}}$	$POS(U) = \{P_1\}$ $NEG(U) = \{P_{12}\}$ $BND(U) = \{P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{13}, P_{14}\}$

Thus, the findings indicate that individuals in the POS zone have tested positive for Coronavirus disease, those in the NEG regions are free from the virus, and individuals in the BND regions have not received confirmation. Additionally, we can categorize newcomers' choices based on the descriptions of previously tested elements.

5.2 Advantages Offered by the Proposed Model

The proposed approach offers several advantages as outlined below:

i. One of the key benefits of this approach is its broader applicability. It serves as a more generalized version of intuitionistic fuzzy sets. When the non-membership grades are set to zero, intuitionistic fuzzy sets transform into FSs.
ii. Power aggregation operators prove to be highly effective and straightforward tools for addressing decision-making problems in fuzzy environments. These operators facilitate the determination of attribute values for elements and can account for their significance when aggregating data.
iii. Many of the existing methods in the literature for TWD primarily adhere to conventional theories such as Yao's [37]. In contrast, our approach presents novel procedures for TWD, which encompass the development of power aggregation operators. Furthermore, we introduce interval-valued categories to categorize participants.
iv. In the context of medical diagnosis, especially in complex cases, such as the one presented here, accurate disease diagnosis is a critical concern for both experts and patients. To tackle this challenge, we have established an idea that accounts for distinct patient profiles and their disease attributes. Ultimately, decisions are made based on input from experts.

6. Conclusion and Future Work

In conclusion, it is imperative to categorize potential solutions and opt for the most practical choices. Decision-making can be quite challenging as it varies depending on the context. Therefore, it's crucial to weigh both the pros and cons of each option. Furthermore, effective decision-making is beneficial for your overall well-being and enhances the chances of identifying the most suitable choice. It's vital to determine the exact amount of essential information that decision-makers need. In the decision-making model, the most efficient strategy involves closely focusing on your objectives.

In the article, we initially explored the fundamental concept of three-way decisions introduced by Yao [25] and the utilization of power aggregation operators. We devised an innovative approach for discretizing the information table. For classifying participants, we employed interval-valued classes, creating three zones based on these classes. The use of aggregation operators is highly advantageous for consolidating results and combining attribute values into single values. Given the significance of these operators, we employed power aggregation operators. Furthermore, we developed an algorithm for disease identification utilizing the suggested method.

Moving forward, the outcomes of this study will be extended to encompass fuzzy and rough data [41- 44], and we will devise novel aggregation operators to address real-life issues.

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Author Contributions
All authors contributed equally to this research.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

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Conflict of interest

The authors declare that there is no conflict of interest in the research.

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