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Abstract

We introduce for the first time the concepts of Neutrosophic TwoFold Algebra with its corresponding Neutrosophic TwoFold Law, and their derivatives of Fuzzy (and all fuzzy-extensions) Two Fold Algebras and Laws. A practical application and a numerical example are also presented.

Keywords: Classical Algebra, Hybrid Algebra, Neutrosophic Algebra, Neutrosophic TwoFold Algebra, Neutrosophic TwoFold Law, Multiple Concentrations, Immiscible Liquids, Spaces Composed of Heterogeneous Subspaces.

1 | Introduction

Applications of the TwoFold Algebra in general occur in chemistry for the mixtures of liquids of various concentrations, and of immiscible liquids as well, and in any field whose space (set) is made up of heterogeneous subspaces (subsets) of elements.

2 | Neutrosophic TwoFold Algebra

This is called a TwoFold Algebra because it has two types of algebras:

(i) The first algebra is with respect to the elements x belonging to a set A (classical type algebra);

(ii) and the second algebra is with respect to the neutrosophic components (t, i, f) of the elements.

This is a hybrid structure, because a classical algebraic operation is inter-related with a fuzzy (or fuzzy-extensions) operation.

In the following we use, as fuzzy-extensions, the neutrosophic set / logic / probability.

3 | Definition of Neutrosophic TwoFold Algebra

Let U be a universe of disclosure and a non-empty neutrosophic set $A \subset U$,

$A_{(T,I,F)} = \{x(T_A(x), I_A(x), F_A(x)), (T_A(x), I_A(x), F_A(x)) \in [0,1]^3, x \in U\},$

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where $T_A(x), I_A(x), F_A(x)$ are degrees of truth-membership, indeterminacy-membership and falsehood-membership of the generic element x with respect to the set A.

4 | Definition of Neutrosophic TwoFold Law

In consequence, we define the Neutrosophic TwoFold Law Δ as follows:

$\Delta: A_{(T,I,F)} \times A_{(T,I,F)} \rightarrow A_{(T,I,F)}$

$x_{1_{(t_1,i_1,f_1)}} \,\Delta\, x_{2_{(t_2,i_2,f_2)}} = (x_1 \# x_2)_{(t_1,i_1,f_1)*(t_2,i_2,f_2)}$

The law Δ is formed by two *sub-laws* # and $*$ respectively, that may be totally dependent, partially dependent and partially independent, or totally independent of each other, in function of the application they are used to.

5 | The Most General Form of Neutrosophic TwoFold Law

It is defined as below:

$g: A_{(T,I,F)} \times A_{(T,I,F)} \rightarrow A_{(T,I,F)}$

$$g(x_1,t_1,i_1,f_1,x_2,t_2,i_2,f_2) = h(x_1,t_1,i_1,f_1,x_2,t_2,i_2,f_2)_{(T(x_1,t_1,i_1,f_1,x_2,t_2,i_2,f_2),I(x_1,t_1,i_1,f_1,x_2,t_2,i_2,f_2),F(x_1,t_1,i_1,f_1,x_2,t_2,i_2,f_2))}$$

where all five functions g, h, T, I, F are functions of eight variables $x_1,t_1,i_1,f_1,x_2,t_2,i_2,f_2$.

6 | Practical Application of General Neutrosophic TwoFold Algebra

Applications of the TwoFold Algebra in general occur in chemistry for the mixtures of liquids of various concentrations, and of immiscible liquids as well, and in any field whose space (set) is made up of heterogeneous subspaces (subsets) of elements.

Let's consider: 2 liters of concentrated liquid of three substances that do not mixt with each other (immiscible): $2_{(0.6,0.1,0.3)} = (1.2, 0.2, 0.6)$, because 0.6 of 2 = 60% of 2 = 0.6 × 2 = 1.2 liters, 0.1 of 2 = 0.1 × 2 = 0.2 liters, and 0.3 of 2 = 0.3 × 2 = 0.6 liters, with 1.2 + 0.2 + 0.6 = 2.

Therefore, the space (quantity) of 2 liters is formed of three heterogeneous sub-spaces (sub-quantities) of 1.2, 0.2, and respectively 0.6 liters. Similarly, the space (quantity) of 3 liters of concentrated liquid is formed of three heterogeneous sub-spaces (sub-quantities) of 1.5, 1.2, and 0.3 liters, $3_{(0.5,0.4,0.1)} = (1.5, 1.2, 0.3)$, where 1.5 + 1.2 + 0.3 = 3.

Therefore, both liquids, mixed together, give:

$$(x_1 + x_2)_{\left(\frac{x_1t_1+x_2t_2}{x_1+x_2}, \frac{x_1i_1+x_2i_2}{x_1+x_2}, \frac{x_1f_1+x_2f_2}{x_1+x_2}\right)}$$

where $x_1 = 2$ with $(t_1, i_1, f_1) = (0.6, 0.1, 0.3)$, and $x_2 = 3$ with $(t_2, i_2, f_2) = (0.5, 0.4, 0.1)$, whence $x_1 + x_2 = 2 + 3 = 5$ liters of concentrations: and $\left(\frac{x_1t_1+x_2t_2}{x_1+x_2}, \frac{x_1i_1+x_2i_2}{x_1+x_2}, \frac{x_1f_1+x_2f_2}{x_1+x_2}\right) = (2.7, 1.4, 0.9)$, therefore $5_{(2.7,1.4,0.9)}$, where $2.7 + 1.4 + 0.9 = 5$.

The space of 5 liters is formed by three heterogeneous sub-spaces of 2.7, 1.4, and 0.9 liters.

7 | Fuzzy and (Fuzzy-Extensions) TwoFold Algebra

Applications of the TwoFold Algebra in general occur in chemistry for the mixtures of liquids of various concentrations, and of immiscible liquids as well, and in any field whose space (set) is made up of heterogeneous subspaces (subsets) of elements.

The Neutrosophic TwoFold Algebra may be adjusted to any fuzzy-extensions theory. Let *U* be a universe of discourse, and let *A* be a non-empty set included in *U*.

(i) Fuzzy TwoFold Algebra

Let $A_{(T)} = \{x(T_A), T_A \in [0,1]; x \in U\}$, where T_A is the degree of truth-membership of the generic element x with respect to the set A.

The Fuzzy TwoFold Law is then defined as:

$\Delta: A_{(T)} \times A_{(T)} \to A_{(T)}$

${x_1}_{(t_1)} \Delta {x_2}_{(t_2)} = (x_1 \# x_2)_{(t_1 * t_2)}$

Similarly, the Fuzzy TwoFold Law Δ is formed by two (totally dependent, partially dependent and partially independent, or totally independent) sub-laws # and $*$.

(ii) Intuitionistic Fuzzy TwoFold Algebra

$A_{(T,F)} = \{x(T_A(x), F_A(x)), (T_A(x), F_A(x)) \in [0,1]^2, x \in U\}$, where $T_A(x)$ and $F_A(x)$ are degrees of truth-membership and falsehood-nonmembership of the generic element x with respect to the set A.

The Intuitionistic TwoFold Law is defined as:

$\Delta: A_{(T,F)} \times A_{(T,F)} \to A_{(T,F)}$

${x_1}_{(t_1,f_1)} \Delta {x_2}_{(t_2,f_2)} = (x_1 \# x_2)_{(t_1,f_1)*(t_2,f_2)}$

(iii) Similarly, for {any fuzzy-extensions [1] } TwoFold Algebra and Law such as:

Inconsistent Intuitionistic Fuzzy Set (or Picture Fuzzy Set, or Ternary Fuzzy Set) TwoFold Algebra and Law;

Pythagorean Fuzzy Set (Atanassov's Intuitionistic Fuzzy Set of second type) TwoFold Algebra and Law;

q-Rung Orthopair Fuzzy TwoFold Algebra and Law;

Spherical Fuzzy TwoFold Algebra and Law;

n-HyperSpherical Fuzzy TwoFold Algebra and Law;

Refined Neutrosophic and MultiNeutrosophic TwoFold Algebra and Law;

Plithogenic TwoFold Algebra and Law.

As future research it would be to develop some of the above new types of TwoFold Algebras and Laws, with their real-life applications.

8 | Single-Valued, Interval-Valued, and Subset-Value Fuzzy (and fuzzy-extensions) TwoFold Algebras and Laws

All previous (fuzzy or fuzzy-extensions) TwoFold Algebras and Laws are Single-Valued, meaning that their components (degrees of membership, indeterminacy, nonmembership, etc.) are single numbers included in [0, 1].

But, if the components are interval-valued, or most generally subset-valued, of [0, 1], they are called Interval-Valued (Fuzzy or fuzzy-extensions) TwoFold Algebras and Laws and respectively Subset-Valued (Fuzzy or fuzzy-extensions) TwoFold Algebras and Laws.

9 | Numerical Example of Neutrosophic TwoFold Algebra and Law

Let *U* be a neutrosophic universal set and *A* be a non-empty subset of *U*.

Let $A_{(T,I,F)} = \{x_j(t_j, i_j, f_j), x_j \in U, t_j, i_j, f_j \in [0,1]\}$ be a neutrosophic set endowed with the Neutrosophic TwoFold Law defined as follows: $x_1(t_1, i_1, f_1)\, \Delta\, x_1(t_2, i_2, f_2) = (x_1 \cdot x_2)[(t_1, i_1, f_1) * (t_2, i_2, f_2)]$

where each sub-law is defined respectively as:

$$\cdot : A^2 \to A$$

and

$$*: \{(t, i, f); t, i, f \in [0,1]\}^2 \to \{(t, i, f); t, i, f \in [0,1]\}.$$

Let's assume the classical integer multiplication *modulo 3* as first classical sub-operation (sub-law), $x_1 \cdot x_2 = (x_1 \cdot x_2)(mod\ 3)$ and the neutrosophic sub-law be

$$(t_1, i_1, f_i) * (t_2, i_2, f_2) = \left(\frac{t_1+t_2}{2}, \frac{i_1+i_2}{2}, \frac{f_1+f_2}{2}\right)$$

or the averages of *t, i, f* neutrosophic components respectively.

Therefore, $x_1(t_1, i_1, f_i)\, \Delta\, x_1(t_2, i_2, f_2) = (x_1 \cdot x_2)(mod\ 3)_{\left(\frac{t_1+t_2}{2}, \frac{i_1+i_2}{2}, \frac{f_1+f_2}{2}\right)}$

Let $A = \{0(0.2,0.4,0.6), 1(0.8,0.2,0.0), 2(0.4,0.8,0.6)\}$ be a neutrosophic set.

Table 1. of the Neutrosophic TwoFold Law Δ.

Δ	0(0.2,0.4,0.6)	1(0.8,0.2,0.0)	2(0.4,0.8,0.6)
0(0.2,0.4,0.6)	0(0.2,0.4,0.6)	0(0.5,0.3,0.3)	0(0.3,0.6,0.6)
1(0.8,0.2,0.0)	0(0.5,0.3,0.3)	1(0.8,0.2,0.0)	2(0.6,0.5,0.3)
2(0.4,0.8,0.6)	0(0.3,0.6,0.6)	2(0.6,0.5,0.3)	1(0.4,0.8,0.6)

Let $A = \{0(0.2,0.4,0.6), 1(0.8,0.2,0.0), 2(0.4,0.8,0.6)\}$ be a neutrosophic set.

The Neutrosophic TwoFold Law Δ is partially inner defined, since for example

$1(0.8,0.2,0.0)\, \Delta\, 1(0.8,0.2,0.0) = 1(0.8,0.2,0.0) \in A$

and partially outer-defined, because for example

$2(0.4,0.8,0.6)\, \Delta\, 1(0.8,0.2,0.8) = 2(0.6,0.5,0.3) \notin A$, but $2_{(0.6,0.5,0.3)} \in U \backslash A$.

Anti-commutative:
because (according to Table 1) for any two elements

$a_1(t_1, i_1, f_i)$ and $a_2(t_2, i_2, f_2) \in A$, with $a_1 \neq a_2$, one has:

$a_1(t_1, i_1, f_i)\, \Delta\, a_2(t_2, i_2, f_2) = a_2(t_2, i_2, f_2)\, \Delta\, a_1(t_1, i_1, f_i) \notin A$

Anti-neutral element

Since there is no neutral element (see the above Table 1).

Anti-inverse element

And no element has an inverse, since there is no neutral element.

We got an unusual hybrid structure!

10 | Conclusions

In this paper we have founded a new type of hybrid algebra, called Neutrosophic TwoFold Algebra and its corresponding Neutrosophic TwoFold Law, by combining two algebras:

the first algebra is a classical algebra of the elements, and the second algebra is an algebra of the elements' components (the degrees of membership / indeterminacy / nonmembership).

Then we extended it to all Fuzzy (and fuzzy-extensions) sets. The TwoFold Algebras have applications in any field whose space is formed by heterogeneous sub-spaces.

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Author Contributaion

All authors contributed equally to this work.

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Data Availability

The datasets generated during and/or analyzed during the current study are not publicly available due to the privacy-preserving nature of the data but are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that there is no conflict of interest in the research.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

References

[1] Florentin Smarandache, Neutrosophic Set is a Generalization of Intuitionistic Fuzzy Set, Inconsistent Intuitionistic Fuzzy Set (Picture Fuzzy Set, Ternary Fuzzy Set), Pythagorean Fuzzy Set (Atanassov's Intuitionistic Fuzzy Set of second type), q-Rung Orthopair Fuzzy Set, Spherical Fuzzy Set, and n-HyperSpherical Fuzzy Set, while NeutroSophication is a Generalization of Regret Theory, Grey System Theory, and Three-Ways Decision (revisited), Journal of New Theory, 29 (2019) 01-35; arXiv, Cornell University, New York City, NY, USA, pp. 1-50, 17-29 November 2019, https://arxiv.org/ftp/arxiv/papers/1911/1911.07333.pdf ; The University of New Mexico, Albuquerque, USA, Digital Repository, pp. 1-50, https://digitalrepository.unm.edu/math_fsp/21.