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Neutrosophic Matrix Games to Solve Project Management Conflicts

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Abstract. A project is an effort made by a group of actors during a certain period to obtain a product, service, or result. Project management is the application of knowledge, skills, tools, and techniques to plan and execute activities to achieve the project requirements. Project management implies the need for negotiation between the parties because there may be a contradiction between the parties involved, that is, e.g. between the one who provides the knowledge or goods and the client who receives the final product or knowledge. That is why we propose a neutrosophic solution to manage the contradictions in the negotiation and execution of any project. The advantages offered by this solution are that the data can be entered in the form of linguistic terms, in addition to explicitly including the indeterminacy that exists in the modeling of this type of activity. This methodology is useful in negotiating qualitative content between parties.

Keywords: Single-valued triangular neutrosophic number, matrix games, neutrosophic games, project management, negotiation.

1 Introduction

In general, problems related to conflicts of interest or decision-making are characterized by the existence of a group of individuals who have to face a situation that may have more than one outcome concerning the specific personal preferences of each of the individuals. In addition, each individual controls some of the variables that determine the final result, although he does not control the totality. These situations are called games. Thus, a game can include situations as diverse as a game of cards, obtaining a contract by certain companies, or negotiating international agreements among some countries, [1].

The beginning of the mathematical theory that studies conflicts of interest, called Game Theory, was established in 1944, following the publication of the book “Game Theory and Economic Behavior” by John von Neumann and Oskar Morgensen, although there were already records of previous works at the beginning of the twentieth century, [2]. Since then, Game Theory has evolved widely and has seen how its models have been applied, especially to economics and politics as well as other social sciences such as philosophy or psychology, since its models fit the study of human behavior.

It is not that Game Theory can cover any problems related to decision-making or conflicts of interest; in general, it must be assumed that there is a specific number of players, that all possible outcomes of the game are known and determined, that each player has a preference among the different outcomes that can be expressed in terms of a utility function, and that each player's goal is to maximize the utility gained after the outcome of the game.

The problem for each player is to determine the strategy to follow so that their partial influence on the game is as beneficial as possible. Given this situation, the first classification between cooperative and non-cooperative games is presented, [1].

Non-cooperative game theory deals with the behavior of game agents in situations where each player's choice of optimal strategy depends on his forecast of opponents' choices and seeks to maximize his profit by ignoring the choice made by others.

If there can be communication between the players to negotiate or establish agreements that allow coalitions to be formed, then the situation is framed within the so-called cooperative games. In these situations, it is considered as basic information the profits that each coalition can obtain by coordinating the strategies of its members, independently of the actions of the rest of the agents in the game. Thus, the agreements among the members of each coalition are aimed at coordinating their actions or redistributing the payments or obtained profits.

Conflict situations modeled by game theory present uncertainty, which is why mathematical models have emerged that incorporate this component to the solution [3, 4]. For example, there are fuzzy cooperative games that appear in [5]. These solutions are obtained from applying the Zadeh extension principle to deterministic solutions of cooperative games, not only to Shapley's value but also to the Core. There are solutions to cooperative and non-cooperative games that use fuzzy theories, such as fuzzy sets or intuitionistic fuzzy sets, e.g., [6]. However, these solutions do not explicitly consider the indetermination as a result of lack of knowledge, hidden information, contradictions of interests between agents, and inconsistencies, among other reasons. That is why in this paper we introduce a problem-solving method with the help of neutrosophic theory. Neutrosophy is the branch of philosophy that studies all related to neutralities, where lack of information, contradictions, paradoxes, ambiguity, and so on are modeled [7, 8].

Neutrosophy applications to game theory can be found in [9-12] for non-cooperative games in matrix form. The paper in [13] proposes neutrosophic off-uninorms as an alternative aggregator to arithmetic addition in the definition of Shapley value in cooperative games. In [14], neutrosophic matrix games are used in the resolution of political conflicts.

A project is a plan that consists of a set of activities that are interrelated and coordinated. It is a temporary effort that is made to create a unique product, service, or result. From these concepts, it is evident that the reason for a project is to achieve specific results or goals within limits imposed by a budget, previously established qualities, and a previously defined period. There are multiple types of projects, one of which considers them productive and public.

A productive project seeks to generate economic profitability and obtain profits in money. On the other hand, public projects are those that seek to achieve an impact on the quality of life of the population, which are not necessarily expressed in money. Specifically, a scientific project is a set of plans, ideas, and actions that must be developed in a coordinated way to achieve a goal. This is called a project, such that scientific is an adjective that mentions its link to science (the grouping of methods, procedures, and techniques to generate objective knowledge).

A project is not free of contradictions among the agents' points of view and interests that make it up, including customers and/or users. That is why this paper aims to propose a neutrosophic matrix solution to the resolution of conflicts among the parts of a project. The advantage of applying this theory is that it allows us the use of natural language as input data for modeling problems and includes indeterminacy as part of the model. Some neutrosophic approaches to decision-making applied in project management can be read in [15-17].

This methodology is useful when it is necessary to establish a negotiation based on qualitative elements, for example in a scientific project it is not possible to establish a monetary value to the negotiations based on the knowledge that must be contributed for the success of this task. Another variant may be the decision that must be made on financial payoffs, but that decision-makers want to make based on qualitative payoffs. According to the authors ' knowledge, it is the first time that neutrosophic matrix games are used for the resolution of conflicts among parts of a project.

This paper is divided as follows; section 2 contains the main concepts of non-cooperative game theory in matrix form and neutrosophic sets. Section 3 contains the proposed model and one illustrative example. Finally, the paper finishes with the conclusions.

2 Preliminary concepts

In this section, we describe the main concepts necessary to comprehend the method proposed in this paper. The first subsection contains the basic concepts of matrix games. The second subsection describes the concepts of Neutrosophy [18].

2.1 Matrix games

Definition 1 ([1]): A game consists of a nonempty set of players, denoted by $\mathrm{N} = \{1, 2, \cdots, \mathrm{n}\}$, a set of moves (or pure strategies) available to those players, denoted by $\mathrm{A} = \{\mathrm{A}_1, \mathrm{A}_2, \cdots, \mathrm{A}_\mathrm{p}\}$, and a specification of rewards for each combination of strategies. In this case, where two players are considered, the rewards of the players are represented using a payoff matrix, one player selects the row and the other one the column. The element of the i-th row and the j-th column contains the utility obtained by player I (by rows) when applying the i-th strategy ($i \in \{1, 2, \cdots, p\}, p \geq 1$) when player II (by columns) applies the jth strategy ($j \in \{1, 2, \cdots, q\}, q \geq 1$) and also the utility obtained by player II. Let us call $\mathrm{u}_\mathrm{I}\colon \mathrm{A} \times \mathrm{B} \to \mathbb{R}$ the payoff function for player I, A is the set of strategies of player I and B is the set of strategies of player II, whereas $\mathrm{u}_\mathrm{II}\colon \mathrm{A} \times \mathrm{B} \to \mathbb{R}$ is the payoff function for player II.

Let us denote by S the Cartesian set of strategies sets $\mathrm{A} \times \mathrm{B}$.

The mixed strategies are defined as pure strategies, each of them is associated with one probability.

Definition 2 ([1]): The *mixed strategies* in the game of two players I and II, with strategies $A = \{A_1, A_2, \cdots, A_p\}$ for player I and $B = \{B_1, B_2, \cdots, B_q\}$ for player II, are defined as the vectors $x = (x_1, x_2, \cdots, x_p) \in [0,1]^p$ and $y = (y_1, y_2, \cdots, y_q) \in [0,1]^q$, such that $\sum_{i=1}^{p} x_i = \sum_{j=1}^{q} y_i = 1$.

In this case, to calculate the payoff, the average is used according to the following formulas:

$E_I(x, y) = \sum_{i=1}^{p} \sum_{j=1}^{q} x_i u_I(A_i, B_j) y_j$ and $E_{II}(x, y) = \sum_{i=1}^{p} \sum_{j=1}^{q} x_i u_{II}(A_i, B_j) y_j$.

Nash equilibrium in game theory is a solution concept for games with two or more players, which assumes that ([19]):

- Each player knows and has adopted their best strategy, and
- Everyone knows each other's strategies.

Consequently, each player wins nothing by modifying their strategy while the others keep theirs. Thus, each player executes the best "move" he can, given the other players' moves.

Often it is overlooked the fact that in a game, Nash equilibrium will be adopted only under certain conditions:

1. All players seek to maximize their expected payoffs according to the profits that describe the game.
2. Players execute their strategies without mistakes.
3. Players have enough intelligence to deduce their balances and those of others.
4. Players assume that changing their strategy will not lead to deviations in the strategies of others.
5. There is common knowledge of both, rules and assumptions of rationality.

Definition 3 ([19]): Given a rectangular game $(N, A \times B, u_I \times u_{II})$, $\tau \in S$ is said to be a Nash equilibrium in pure strategies if for each player in N the following conditions are satisfied:

$u_I(\tau) \geq u_I(\tau/A_j)$ and $u_{II}(\tau) \geq u_{II}(\tau/B_j)$ $\forall A_j \in A$ and $\forall B_j \in B$.

This means that if each player changes their strategy for any other, while the other players keep the strategy given by profile τ, then the payoff of the player who changes his or her strategy does not improve.

Definition 4 ([19]): A mixed strategy profile X is said to be a Nash equilibrium of mixed strategy if for each player $k \in \{I, II\}$ the following property is fulfilled:

$$E_k(X) \geq E_k(X/X_k)\ \forall X_k.$$

Where $E_k(X)$ is the expected payoff (or average payoff) that player I will get by always playing the mixed strategy profile X. Intuitively, a mixed strategy profile is a Nash equilibrium if on average no player can improve their payoff by changing their mixed strategies when the rest of the players stick to the current strategy.

2.2 Basic concepts on Neutrosophy

Definition 5 ([7]): The *Neutrosophic set* N is characterized by three membership functions, which are the truth-membership function T_A, indeterminacy-membership function I_A, and falsity-membership function F_A, where U is the Universe of Discourse and $\forall x \in U$, $T_A(x), I_A(x), F_A(x) \subseteq]^-0, 1^+[$, and $^-0 \leq \inf T_A(x) + \inf I_A(x) + \inf F_A(x) \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

See that according to Definition 5, $T_A(x), I_A(x), F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$ and hence, $T_A(x), I_A(x), F_A(x)$ can be subintervals of $[0, 1]$.

Definition 6 ([7]): The *Single-Valued Neutrosophic Set* (SVNS) N over U is $A = \{< x;\ T_A(x), I_A(x), F_A(x) > : x \in U\}$, where $T_A: U \to [0,1]$, $I_A: U \to [0,1]$, and $F_A: U \to [0,1]$, $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

The *Single-Valued Neutrosophic Number* (SVNN) is represented by $N = (t, i, f)$, such that $0 \leq t, i, f \leq 1$ and $0 \leq t + i + f \leq 3$.

Definition 7 ([20-24]): The *single-valued triangular neutrosophic number* $\tilde{a} = \langle (a_1, a_2. a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, is a neutrosophic set on $\mathbb{R}$, whose truth, indeterminacy and falsity membership functions are defined as follows, respectively:

$$T_{\tilde{a}}(x) = \begin{cases} \alpha_{\tilde{a}}\left(\frac{x-a_1}{a_2-a_1}\right), & a_1 \leq x \leq a_2 \\ \alpha_{\tilde{a}}, & x = a_2 \\ \alpha_{\tilde{a}}\left(\frac{a_3-x}{a_3-a_2}\right), & a_2 < x \leq a_3 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{\left(a_2 - x + \beta_{\tilde{a}}(x - a_1)\right)}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \beta_{\tilde{a}}, & x = a_2 \\ \frac{\left(x - a_2 + \beta_{\tilde{a}}(a_3 - x)\right)}{a_3 - a_2}, & a_2 < x \le a_3 \\ 1, & \text{otherwise} \end{cases} \quad (2)$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{\left(a_2 - x + \gamma_{\tilde{a}}(x - a_1)\right)}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \gamma_{\tilde{a}}, & x = a_2 \\ \frac{\left(x - a_2 + \gamma_{\tilde{a}}(a_3 - x)\right)}{a_3 - a_2}, & a_2 < x \le a_3 \\ 1, & \text{otherwise} \end{cases} \quad (3)$$

Where $\alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \in [0, 1]$, $a_1, a_2, a_3 \in \mathbb{R}$ and $a_1 \le a_2 \le a_3$.

Definition 8 ([20-22, 25]): Given $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$ and $\tilde{b} = \langle (b_1, b_2, b_3); \alpha_{\tilde{b}}, \beta_{\tilde{b}}, \gamma_{\tilde{b}} \rangle$, two single-valued triangular neutrosophic numbers and λ any non-null number in the real line. Then, the following operations are defined:

Addition: $\tilde{a} + \tilde{b} = \langle (a_1 + b_1, a_2 + b_2, a_3 + b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$

Subtraction: $\tilde{a} - \tilde{b} = \langle (a_1 - b_3, a_2 - b_2, a_3 - b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle$

Inversion: $\tilde{a}^{-1} = \langle (a_3^{-1}, a_2^{-1}, a_1^{-1}); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$, where $a_1, a_2, a_3 \neq 0$.

Multiplication by a scalar number:

$$\lambda\tilde{a} = \begin{cases} \langle (\lambda a_1, \lambda a_2, \lambda a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, & \lambda > 0 \\ \langle (\lambda a_3, \lambda a_2, \lambda a_1); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle, & \lambda < 0 \end{cases}$$

Division of two triangular neutrosophic numbers:

$$\frac{\tilde{a}}{\tilde{b}} = \begin{cases} \langle \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 > 0 \ and \ b_3 > 0 \\ \langle \left(\frac{a_3}{b_3}, \frac{a_2}{b_2}, \frac{a_1}{b_1}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \ and \ b_3 > 0 \\ \langle \left(\frac{a_3}{b_1}, \frac{a_2}{b_2}, \frac{a_1}{b_3}\right); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, a_3 < 0 \ and \ b_3 < 0 \end{cases}$$

Multiplication of two triangular neutrosophic numbers:

$$\tilde{a}\tilde{b} = \begin{cases} \langle (a_1 b_1, a_2 b_2, a_3 b_3); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, & a_3 > 0 \ and \ b_3 > 0 \\ \langle (a_1 b_3, a_2 b_2, a_3 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, & a_3 < 0 \ and \ b_3 > 0 \\ \langle (a_3 b_3, a_2 b_2, a_1 b_1); \alpha_{\tilde{a}} \wedge \alpha_{\tilde{b}}, \beta_{\tilde{a}} \vee \beta_{\tilde{b}}, \gamma_{\tilde{a}} \vee \gamma_{\tilde{b}} \rangle, & a_3 < 0 \ and \ b_3 < 0 \end{cases}$$

Where, $\wedge$ is a t-norm and $\vee$ is a t-conorm, [26].

Let $\tilde{a} = \langle (a_1, a_2, a_3); \alpha_{\tilde{a}}, \beta_{\tilde{a}}, \gamma_{\tilde{a}} \rangle$ be a single-valued triangular neutrosophic number, then,

$$S(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} - \gamma_{\tilde{a}}) \quad (4)$$

$$\mathbf{A}(\tilde{a}) = \frac{1}{8}[a_1 + a_2 + a_3](2 + \alpha_{\tilde{a}} - \beta_{\tilde{a}} + \gamma_{\tilde{a}}) \quad (5)$$

They are called the score and accuracy degrees of ã, respectively.

Definition 9 ([14, 20-22, 27]): Let ã and $\tilde{b}$ be two SVTNNs. Let us define the order relation denoted by $\preccurlyeq$, such that $\tilde{a} \preccurlyeq \tilde{b}$ if and only if $\mathbf{A}(\tilde{a}) \le \mathbf{A}(\tilde{b})$.

Let $\{\tilde{A}_1, \tilde{A}_2, \cdots, \tilde{A}_n\}$ be a set of n SVTNNs, where $\tilde{A}_j = \langle (a_j, b_j, c_j); \alpha_{\tilde{a}_j}, \beta_{\tilde{a}_j}, \gamma_{\tilde{a}_j} \rangle$ (j = 1, 2, …, n), then the *weighted mean of the SVTNNs* is calculated with the following Equation:

$$\tilde{A} = \sum_{j=1}^{n} \lambda_j \tilde{A}_j \quad (6)$$

Where λ_j is the weight of A_j, $\lambda_j \in [0, 1]$ and $\sum_{j=1}^{n} \lambda_j = 1$.

3 Methodology

In this methodology, some particularities are taken into account. The first one is that there are supposed to be pair-wise negotiations among the agents involved in the implementation of the project. Each of these negotiations is modeled using a matrix or bimatrix. According to the results of each of the two parts of the project, they decide the extent that these two parts contribute to the project and how the profits obtained will be divided as fairly as possible. For example, in the construction of a public social work, one construction company is hired, which in turn subcontracts two others. However, these two companies must decide whether it is in their best interest to work together or one of them prefers to work with a third one. This situation is best explained in [28], where it is resolved with the help of the prisoner's dilemma model game.

The second particularity of the methodology is that the model uses classical game theory in the case of quantitative input values are measured in price, cost, monetary gains, etc. in these cases the particular gains can be divided into the amount of money or other quantitative and divisible goods that each party will receive in the negotiation. The methodology we propose allows us to divide profits into qualitative terms, which serves to complement the quantitative negotiations mentioned above. An example is scientific projects, which on the one hand need investors who want to be part of the profits of the product obtained, which is done through financial negotiations. In these negotiations, we recommend that a classic solution be used. On the other hand, the scientific project also needs to agree on institutions and scientific personnel, whose profits cannot be measured in the amount of money, for this, it would be necessary to measure the profits qualitatively, preferably through the use of a linguistic scale. In this last sense, our proposal is useful, which will also take into account the imprecision that is typical of all negotiations.

As the third and final point of the methodology, it is assumed that each party will try to make as much profit as possible and will not cooperate with the others. The methodology will therefore be based on the theory of non-cooperative games.

The table listing the linguistic terms and the single-valued triangular neutrosophic numbers associated with them is summarized below:

Linguistic term	SVTNN
Very low (VL)	$\langle(0,0,1); 0.00, 1.00, 1.00\rangle$
Medium-low (ML)	$\langle(0,1,3); 0.17, 0.85, 0.83\rangle$
Low (L)	$\langle(1,3,5); 0.33, 0.75, 0.67\rangle$
Medium(M)	$\langle(3,5,7); 0.50, 0.50, 0.50\rangle$
High (H)	$\langle(5,7,9); 0.67, 0.25, 0.33\rangle$
Medium-high (MH)	$\langle(7,9,10); 0.83, 0.15, 0.17\rangle$
Very high (VH)	$\langle(9,10,10); 1.00, 0.00, 0.00\rangle$

Table 1: Scale of linguistic terms and neutrosophic triangular scale associated with them. Sources: [17, 29].

Therefore the methodology that is used is the following:

1. Define which two players are going to negotiate.
2. Define the type of solution:
   2.1. If the payoff of the negotiation has to be expressed in the form of money or other goods that can be expressed in monetary form, the traditional game theory is used, with quantitative input data.
   2.2. If on the other hand the payoff has or needs to be expressed qualitatively based on linguistic terms, go to the next step.
3. The possible strategies of players I and II are defined, which are the sets $\mathrm{A} = \{\mathrm{A}_1, \mathrm{A}_2, \cdots, \mathrm{A}_\mathrm{p}\}$ and $\mathrm{B} = \{\mathrm{B}_1, \mathrm{B}_2, \cdots, \mathrm{B}_\mathrm{q}\}$, respectively.
4. Players define what kind of game to utilize, whether it is a prisoner's dilemma, a chicken game, or other, see [28]. For example, the chicken game is based on strategies in which each party delays making concessions until the end of the negotiation period is imminent. Psychological pressure can force a negotiator to give in for avoiding a negative outcome. This can be a very dangerous tactic as if neither side gives in there will be a collision. The payoff bimatrix of both players is defined through the payoff functions, $\mathrm{u}_\mathrm{I}: \mathrm{A} \times \mathrm{B} \rightarrow \widetilde{\mathrm{A}}$ and $\mathrm{u}_{\mathrm{II}}: \mathrm{A} \times \mathrm{B} \rightarrow \widetilde{\mathrm{A}}$, such that $\widetilde{\mathrm{A}} = \{\widetilde{\mathrm{A}}_1, \widetilde{\mathrm{A}}_2, \cdots, \widetilde{\mathrm{A}}_7\}$ where each of the $\widetilde{\mathrm{A}}_\mathrm{l}$ is an SVTNN of Table 1, which is selected by its equivalent linguistic term. This bimatrix has the following form:

$$\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_p \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & \cdots & B_q \end{array} \\ \begin{pmatrix} (\tilde{t}_{11}, \tilde{s}_{11}) & (\tilde{t}_{12}, \tilde{s}_{12}) & \cdots & (\tilde{t}_{1q}, \tilde{s}_{1q}) \\ (\tilde{t}_{21}, \tilde{s}_{21}) & (\tilde{t}_{22}, \tilde{s}_{22}) & \cdots & (\tilde{t}_{2q}, \tilde{s}_{2q}) \\ \vdots & \vdots & \vdots & \vdots \\ (\tilde{t}_{p1}, \tilde{s}_{p1}) & (\tilde{t}_{p2}, \tilde{s}_{p2}) & \cdots & (\tilde{t}_{pq}, \tilde{s}_{pq}) \end{pmatrix} \end{array} \quad (7)$$

Where $\tilde{t}_{ij} = u_I(A_i, B_j)$ and $\tilde{s}_{ij} = u_{II}(A_i, B_j)$.

5. The bimatrix in Equation 7 is converted into a crisp bimatrix as in Equation 8.

$$\begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_p \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & \cdots & B_q \end{array} \\ \begin{pmatrix} (t_{11}, s_{11}) & (t_{12}, s_{12}) & \cdots & (t_{1q}, s_{1q}) \\ (t_{21}, s_{21}) & (t_{22}, s_{22}) & \cdots & (t_{2q}, s_{2q}) \\ \vdots & \vdots & \vdots & \vdots \\ (t_{p1}, s_{p1}) & (t_{p2}, s_{p2}) & \cdots & (t_{pq}, s_{pq}) \end{pmatrix} \end{array} \quad (8)$$

Where, $t_{ij} = \mathbf{A}(\tilde{t}_{ij})$ and $s_{ij} = \mathbf{A}(\tilde{s}_{ij})$, using $\mathbf{A}(\cdot)$ which is the accuracy degree of Equation 5.

6. The Nash equilibrium point is calculated using Definition 3. This gives at least a pair of strategies, one for each player, which is selected as the most suitable for both cases.

To illustrate the usefulness of this methodology we will use an example.

Example 1: This example is based on the case presented in [28]. Suppose a project is at 70% of its execution and is in its critical phase of completion. The missing part needs certain results from a contracted Research and Development (R&D) firm to provide the scientific-technical know-how necessary to culminate with the remaining 30%. However, as usual, the expected scientific results are not necessarily obtained at a specific time. This situation creates a dispute between the contracting company and the contracted R&D Company.

1. The hiring company will be denoted as player I, while the R&D Company will be denoted as player II.
2. By the nature of the problem, it is determined that it is qualitative, which is why the proposed methodology is needed to seek a solution.
3. The strategies that will be taken into account are the following:
   For player I:
   A_1 = “Gives extension time”.
   A_2 = “Gives no extension time”.
   For player II:
   B_1 = “Non-overtime work (fixed speed)”.
   B_2 = “Overtime work (increase mobility)”.
   That is, for the contractor, there are two options, either it allows finishing the project in a long time, or it forces the R&D Company to comply with its commitment to finish in the established time. While the two strategies of the R&D Company are to maintain its same working speed or on the contrary increase the speed to achieve in the given time.
4. To predict what to do, player I decides that the best role model is chicken game.
   The bimatrix that was determined from the linguistic terms of Table 1 is as follows:

Player II (R&D enterprise)		Strategies that can be used by contractors and R&D enterprises, in chicken game theory.	
Non-overtime work (fixed speed)	Overtime work (increase mobility)		
(ML, H)	(M, M)	Gives extension time	Player I (Contractor)
(VL, VL)	(H, ML)	Gives no extension time	

Table 2: Bimatrix obtained to solve the problem posed in the example.

Note that the results are expressed in the form of linguistic terms, which is very convenient for analysts.

5. The bimatrix of Table 2 becomes a crisp bimatrix by applying the accuracy degree of Equation 5 on the SVTNN associated with the linguistic terms that appear in the bimatrix of Table 2. The following bimatrix is obtained as it is shown in Table 3.

Player II (R&D enterprise)		Strategies that can be used by contractors and R&D enterprises, in chicken game theory.	
Non-overtime work (fixed speed)	Overtime work (increase mobility)		
(1.0750, 7.2188)	(4.6875, 4.6875)	Gives extension time	Player I (Contractor)
(0.2500, 0.2500)	(7.2188, 1.0750)	Gives no extension time	

Table 3: Crisp bimatrix obtained to solve the problem posed in the example.

In the case of the chicken game the Nash equilibrium points are (A_1, B_2) and (A_2, B_1).

Note that if the calculations are made with mixed strategies, then we must use the crisp bimatrix in Table 3.

Conclusion

In this paper, we expose a methodology for resolving conflicts among the parties within the same project. Specifically, the use of linguistic terms is proposed to express the evaluation of payoffs in the bimatrix, which is useful when it comes to qualitative assessments. The calculations are made with the help of single-valued triangular neutrosophic numbers, which implicitly include the indeterminacy of any negotiation. The advantages of this methodology are that experts can easily express their evaluations, since the model supports linguistic terms, unlike using numerical values. Finally, different solutions or problem-solving models can be used [30]. We specifically propose the Nash equilibrium, the prisoner's dilemma, and the chicken games models. The use of the methodology was illustrated in the case of a hypothetical example. Future research works will include the extension of the methodology to cases involving more than two players, as well as the possible modeling of cooperative games.

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