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n-ary Fuzzy Hypersoft Expert Sets

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Abstract. In 2018, Smarandache introduced the concept of hypersoft set by replacing the approximate function of the Molodtsov's soft sets with the multi-argument approximate function. Moreover, the fuzzy hybrid model of hypersoft set was developed and thus the theory of fuzzy hypersoft set was initiated. This chapter is devoted to introduce the concept of n-ary fuzzy hypersoft set extending the fuzzy hypersoft set with multiple set of universes (or n-dimension universal sets), the concept of fuzzy hypersoft expert set that presents the opinions of all experts in one fuzzy hypersoft set model without any operations, and the concept of n-ary fuzzy hypersoft expert set that exhibits the opinions of all experts in one n-ary fuzzy hypersoft set model without any operations. Apparently, the n-ary fuzzy hypersoft expert sets include both n-ary fuzzy hypersoft sets and fuzzy hypersoft expert set. Some basic operations of each of these extended fuzzy hypersoft sets are derived and their structural properties are investigated. Finally, an application of ternary fuzzy hypersoft expert set (i.e., n=3) in real-life problem are given.

Keywords: Hypersoft set; fuzzy hypersoft set; n-ary fuzzy hypersoft set; fuzzy hypersoft expert set; n-ary fuzzy hypersoft expert set

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1. **Introduction**

Many fields deal with uncertain data that cannot be successfully modeled by ordinary mathematics. Fuzzy sets [40], intuitionistic fuzzy sets [10] and neutrosophic sets [38] are well-known and often useful approaches for describing uncertainty. Many generalized types of these uncertain sets were proposed (see [2, 5, 17–19, 33]), and are currently being studied on new extended types. In 1999, Molodtsov [28] developed soft sets as a new mathematical model for dealing with uncertainty-based parametric data. Moreover, many researchers studied basic operations of the soft sets [6, 11, 16, 20, 26]. In the last decade, it was discussed the extended types of soft sets such as fuzzy soft sets [12, 25], intuitionistic fuzzy soft sets [13], neutrosophic soft sets [23, 24] and N-soft sets [15]. In [14, 21, 22, 29–32], the theoretical

aspects on these hybrid models of soft sets were studied. A soft set can be considered as is a subset of parameterized family of a universal set. Açkgöz and Taş [3] initiated the theory of binary soft set based on two universal sets and a parameter set and emphasized that it can be adapted for n-dimension universal sets. Alkhazaleh and Salleh [7] proposed the idea of soft expert set, an extension of soft set, containing more than one expert opinion. A few years later, they generalized the soft expert set to fuzzy soft expert set, and argued that these sets are more effective and useful than soft expert set [8]. The approximate function in the structure of a soft set is defined from a parameter set to the power set of a universal set. In 2018, Smarandache [37] proposed defining the approximate function of a soft set from the cartesian product of n different sets of parameters to the power set of a universal set. Thus, Smarandache [37] conceptualized hypersoft set as a generalization of soft set, and then presented fuzzy hypersoft set sets as a fuzzy hybrid model of hypersoft sets. Abbas et al. [1] presented some basic operations like complement, union, intersection, difference of (fuzzy) hypersoft sets. Saeed et al. [34] studied of the fundamentals of hypersoft set theory. UrRahman et al. [39] developed a conceptual framework of convexity and concavity on the hypersoft sets. In [27, 35, 36], the authors proposed the extensions of hypersoft sets to make them more functional in various directions. In recent years, the research on the hypersoft sets and extensions have been progressing actively and rapidly.
This chapter aims to propose new extensions of fuzzy hypersoft sets called n-ary fuzzy hypersoft set, fuzzy hypersoft expert set and n-ary fuzzy hypersoft expert set. Simply, n-ary fuzzy hypersoft set is a fuzzy hypersoft set over the multiple set of universes, fuzzy hypersoft expert set is a fuzzy hypersoft set containing the opinions of experts, and n-ary fuzzy hypersoft expert set is a fuzzy hypersoft set over the multiple set of universes and contains the opinions of experts. Moreover, it intends to present the operations of complement, intersection and union on the n-ary fuzzy hypersoft sets, fuzzy hypersoft expert sets and n-ary fuzzy hypersoft expert sets. Also, the solution of a problem under the ternary fuzzy hypersoft expert set environment from the real world scene is addressed. This chapter organized as follows: Section 2 presents some fundamental concepts of fuzzy sets, soft sets, hypersoft sets, and fuzzy hypersoft sets. Sections 3, 4 and 5 are devoted to the theories of n-ary fuzzy hypersoft sets, fuzzy hypersoft expert sets and n-ary fuzzy hypersoft expert sets, respectively. Section 6 presents an real-life application of n-ary fuzzy hypersoft expert sets. The last section is the conclusions.

2. **Preliminaries**

In this section, some basic notions related to the fuzzy sets, soft sets, binary soft sets, soft expert sets, hypersoft sets, fuzzy hypersoft sets and fuzzy hypersoft set operations are recalled.

2.1. *Fuzzy Sets*

Definition 2.1. ([40]) Let A be a nonempty finite set. A fuzzy set $\mathcal{F}$ in A is defined as

$$\mathcal{F} = \{ \,^{(\mu_{\mathcal{F}}(a))}a : a \in A\} \tag{1}$$

where $\mu_{\mathcal{F}} : A \to [0,1]$ is called a membership function for $\mathcal{F}$ and $\mu_{\mathcal{F}}(a)$ represents the membership degree of a in $\mathcal{F}$. The set of all fuzzy sets in A is dented by $\mathfrak{F}(A)$.

Example 2.2. Let $A = \{a_1, a_2, a_3, a_4, a_5\}$ be the set of houses. According to the membership "cheap", one can create the fuzzy set

$$\mathcal{F} = \{\ {}^{0.2}a_1,\ {}^{0.6}a_2,\ {}^{0}a_3,\ {}^{1}a_1,\ {}^{0.3}a_1\}.$$

Definition 2.3. ([40]) Let F be a fuzzy set in A.

- **(a):** If $\mu_{\mathcal{F}}(a) = 0$ for all $a \in A$ then it is called null (empty) fuzzy set and denoted by $\widehat{\emptyset}$.
- **(b):** If $\mu_{\mathcal{F}}(a) = 1$ for all $a \in A$ then it is called absolute (universal) fuzzy set and denoted by $\widehat{A}$.

Definition 2.4. ([40]) Let $\mathcal{F}$ and $\mathcal{G}$ be two fuzzy sets in A. Then, we have the following operational laws.

- **(a):** $\mathcal{F}$ is a fuzzy subset of $\mathcal{G}$ if $\mu_{\mathcal{F}}(a) \leq \mu_{\mathcal{G}}(a)$ for all $a \in A$, and denoted by $\mathcal{F} \subseteq_f \mathcal{G}$.
- **(b):** The fuzzy sets $\mathcal{F}$ and $\mathcal{G}$ are equal if $\mu_{\mathcal{F}}(a) = \mu_{\mathcal{G}}(a)$ for all $a \in A$, and denoted by $\mathcal{F} = \mathcal{G}$.
- **(c):** The complement of $\mathcal{F}$ is denoted and defined by $\mathcal{F}^r$, where $\mu_{\mathcal{F}^r}(a) = 1 - \mu_{\mathcal{F}}(a)$ for all $a \in A$.
- **(d):** The intersection $\mathcal{F}$ and $\mathcal{G}$ is denoted and defined $\mathcal{F} \cap_f \mathcal{G}$, where $\mu_{(\mathcal{F} \cap_f \mathcal{G})}(a) = \min\{\mu_{\mathcal{F}}(a), \mu_{\mathcal{G}}(a)\} = \mu_{\mathcal{F}}(a) \wedge \mu_{\mathcal{G}}(a)$ for all $a \in A$.
- **(e):** The union $\mathcal{F}$ and $\mathcal{G}$ is denoted and defined $\mathcal{F} \cup_f \mathcal{G}$, where $\mu_{(\mathcal{F} \cup_f \mathcal{G})}(a) = \max\{\mu_{\mathcal{F}}(a), \mu_{\mathcal{G}}(a)\} = \mu_{\mathcal{F}}(a) \vee \mu_{\mathcal{G}}(a)$ for all $a \in A$.

2.2. *Soft Sets*

Let A be a universal set, and the power set of A is denoted by $P(A)$.

Definition 2.5. ([28]) Let X be a set of parameters and $Y \subseteq X$. A soft set $(\mathcal{S}, Y)$ over A is defined as

$$(\mathcal{S}, Y) = \{\ (x, \mathcal{S}(x)) : x \in Y \text{ and } \mathcal{S}(x) \in P(A)\} \tag{2}$$

where $\mathcal{S} : Y \to P(A)$.

Example 2.6. Let $A = \{a_1, a_2, a_3, a_4, a_5\}$ be the set of suite rooms. Also, $X = \{x_1 = \textit{cheap},\ x_2 = \textit{modern},\ x_3 = \textit{beautiful}\}$ is the set of parameters, which describe the attractiveness of the suite rooms, and $Y = X$. Then, one can create the soft set

$$(\mathcal{S}, Y) = \{(x_1, \{a_1, a_4, a_5\}), (x_2, \{a_4, a_5\}), (x_3, \{a_1, a_2, a_3, a_4\})\}.$$

Definition 2.7. ([3]) Let A_1 and A_2 be two universal sets such that $A_1 \cap A_2 = \emptyset$, and $P(A_1)$ $P(A_2)$ are power sets of A_1 and A_2, respectively. Also, let X be a set of parameters and $Y \subseteq X$. A binary soft set $(\mathcal{S}_2, Y)$ over $\mathfrak{A} = \{A_1, A_2\}$, is defined as

$$(\mathcal{S}_2, Y) = \{\ (x, \mathcal{S}_2(x)) : x \in Y \text{ and } \mathcal{S}_2(x) \in P(A_1) \times P(A_2)\} \tag{3}$$

Example 2.8. Let $A_1 = \{a_1^1, a_2^1, a_3^1, a_4^1, a_5^1\}$ and $A_2 = \{a_1^2, a_2^2, a_3^2, a_4^2\}$ be the sets of suite rooms and king rooms. Also, $X = \{x_1 = cheap,\ x_2 = modern,\ x_3 = beautiful\}$ is the set of parameters, which describe the attractiveness of the rooms, and $Y = X$. Then, one can create the binary soft set

$$(\mathcal{S}_2, Y) = \{(x_1, (\{\{a_1^1, a_4^1, a_5^1\}, \{a_3^2, a_4^2\}\})), (x_2, (\{a_4^1, a_5^1\}, \{a_1^2, a_2^2, a_4^2\})), (x_3, (\{a_1^1, a_2^1, a_3^1, a_4^1\}, \{a_4^2, a_5^2\}))\}.$$

Definition 2.9. ([7]) Let X be a set of parameters, $\mathcal{E}$ be a set of experts and $\mathcal{O}$ be a set of opinion. Also, let $\mathcal{P} = X \times \mathcal{E} \times \mathcal{O}$ and $\mathcal{Q} \subseteq \mathcal{P}$. A soft expert set $(\mathcal{S}, \mathcal{Q})$ over A is defined as

$$(\mathcal{S}, \mathcal{Q}) = \{\ ((x, e, o), \mathcal{S}((x, e, o))) : (x, e, o) \in \mathcal{Q} \subseteq X \times \mathcal{E} \times \mathcal{O} \text{ and } \mathcal{S}(x) \in P(A)\} \tag{4}$$

where $\mathcal{S} : \mathcal{Q} \to P(A)$.

Example 2.10. Let $A = \{a_1, a_2, a_3, a_4, a_5\}$ be the set of suite rooms. Also, $X = \{x_1 = cheap,\ x_2 = modern,\ x_3 = beautiful\}$ is the set of parameters, which describe the attractiveness of the suite rooms, and $\mathcal{E} = \{e_1, e_2\}$ is the set of experts and $\mathcal{O} = \{o_1 = agree(1)\ o_2 = disagree(1)\}$ is the set of opinions. For $Q = \{(x_1, e_1, 1), (x_1, e_2, 1), (x_1, e_2, 1), (x_2, e_1, 1), (x_1, e_1, 0), (x_2, e_2, 0)\} \subseteq X \times \mathcal{E} \times \mathcal{O}$, one can create the soft expert set

$$(\mathcal{S}, \mathcal{Q}) = \left\{ \begin{array}{l} ((x_1, e_1, 1), \{a_1, a_2\}), ((x_1, e_2, 1), \{a_4, a_5\}), \\ ((x_1, e_2, 1), \emptyset), ((x_2, e_1, 1), \{a_1, a_3, a_4, a_5\}), \\ ((x_1, e_1, 0), \{a_3, a_4, a_5\}), ((x_2, e_2, 0), \{a_1\}) \end{array} \right\}.$$

2.3. *Hypersoft Sets*

Throughout this chapter, $X_1, X_2, ..., X_m$ are the pairwise disjoint sets of parameters (i.e., $X_i \cap X_{i'} = \emptyset$ for each $i, i' \in I = \{1, 2, ..., m\}$ and $i \neq i'$), and $\mathbf{X} = \prod\limits_{i \in I} X_i = X_1 \times X_2 \times ..., X_m$. Generally, the parameters are attributes, characteristics, properties of the objects.

Definition 2.11. ([37]) Let Y_i be the nonempty subset of X_i for each $i \in I = \{1, 2, ..., m\}$ and $\mathbf{Y} = \prod\limits_{i \in I} Y_i = Y_1 \times Y_2 \times ..., Y_m$. Then, the pair $(\mathcal{H}, \mathbf{Y})$ is called a hypersoft set over A, where $\mathcal{H}$ is mapping given by

$$\mathcal{H} : \mathbf{Y} \to P(A) \tag{5}$$

Also, x^i is an element of Y_i and $(x^i)_{i \in I} = (x^1, x^2, ..., x^m)$ is an element of $\mathbf{Y} = Y_1 \times Y_2 \times ..., Y_m$.
Note 1. In this chapter, we use the notation $\mathbf{x^I} = (x^i)_{i \in I}$.

Example 2.12. Assume that a person wants to buy a car and, for this purpose, visits to a car showroom where cars of the same segment are exhibited. Let $A = \{a_1, a_2, a_3\}$ be a universe containing cars in the same segment. The characteristics or attributes of these cars must be analyzed so that a decision can be made. The pairwise disjoint sets of attributes (parameters) are X_1, X_2 and X_3 and describe image-prestige, performance and economy, respectively. These sets are $X_1 = \{x_1^1 = safe,\ x_2^1 = comfortable,\ x_3^1 = design - aesthetic\}$ $X_2 = \{x_1^2 = engine\ power,\ x_2^2 = torque\}$, and

$X_3 = \{x_1^3 = fuel\ consumption,\ x_2^3 = tax,\ x_3^3 = sale\ price\}$. He/she determines the attributes (parameters) to be used in evaluating the cars as $Y_1 = X_1$, $Y_2 = X_2$ and $Y_3 = \{x_1^3, x_3^3\} \subseteq X_3$ (i.e., $\mathbf{Y} = Y_1 \times Y_2 \times Y_3$). As a result of the evaluation, it is created the following hypersoft set.

$$(\mathcal{H}, \mathbf{Y}) = \left\{ \begin{array}{l} ((x_1^1, x_1^2, x_1^3), \{a_1, a_2\}), \\ ((x_1^1, x_1^2, x_3^3), \{a_3\}), \\ ((x_1^1, x_2^2, x_1^3), \{a_1, a_3\}), \\ ((x_1^1, x_2^2, x_3^3), \emptyset), \\ ((x_2^1, x_1^2, x_1^3), \{a_2, a_3\}), \\ ((x_2^1, x_1^2, x_3^3), \{a_1\}), \\ ((x_2^1, x_2^2, x_1^3), \{a_2\}), \\ ((x_2^1, x_2^2, x_3^3), \{a_3\}), \\ ((x_3^1, x_1^2, x_1^3), \emptyset), \\ ((x_3^1, x_1^2, x_3^3), A), \\ ((x_3^1, x_2^2, x_1^3), \{a_2\}), \\ ((x_3^1, x_2^2, x_3^3), \{a_1\}) \end{array} \right\}.$$

2.4. *Fuzzy Hypersoft Sets*

Definition 2.13. ([37]) Let Y_i be the nonempty subset of X_i for each $i \in I = \{1, 2, ..., m\}$ and $\mathbf{Y} = \prod_{i \in I} Y_i = Y_1 \times Y_2 \times ..., Y_m$. Also, let $\mathfrak{F}(A)$ be the set of all fuzzy sets in A. Then, the pair $(\widetilde{\mathcal{H}}, \mathbf{Y})$ is called a fuzzy hypersoft set over A, where $\widetilde{\mathcal{H}}$ is mapping given by

$$\widetilde{\mathcal{H}} : \mathbf{Y} \to \mathfrak{F}(A) \tag{5}$$

Note 2. The collection of all fuzzy hypersoft set over the universal set A for $\mathbf{X}$ is denoted by $\mathfrak{C}\langle A, \mathbf{X}\rangle$.

Example 2.14. Consider the problem in Example 2.12. As a result of the evaluation under the fuzzy environment, it is created the following fuzzy hypersoft set.

$$(\mathcal{H}, \mathbf{Y}) = \left\{ \begin{array}{l} ((x_1^1, x_1^2, x_1^3), \{^{(0.4)}a_1^1, ^{(0.3)}a_2^1, ^{(0.6)}a_3^1\}), \\ ((x_1^1, x_1^2, x_3^3), \{^{(0.5)}a_1^1, ^{(0.6)}a_2^1, ^{(0.6)}a_3^1\}), \\ ((x_1^1, x_2^2, x_1^3), \{^{(0.3)}a_1^1, ^{(0.3)}a_2^1, ^{(0.2)}a_3^1\}), \\ ((x_1^1, x_2^2, x_3^3), \{^{(0.1)}a_1^1, ^{(0.4)}a_2^1, ^{(0)}a_3^1\}), \\ ((x_2^1, x_1^2, x_1^3), \{^{(0)}a_1^1, ^{(0)}a_2^1, ^{(0)}a_3^1\}), \\ ((x_2^1, x_1^2, x_3^3), \{^{(1)}a_1^1, ^{(1)}a_2^1, ^{(1)}a_3^1\}), \\ ((x_2^1, x_2^2, x_1^3), \{^{(0.5)}a_1^1, ^{(0.7)}a_2^1, ^{(0.4)}a_3^1\}), \\ ((x_2^1, x_2^2, x_3^3), \{^{(0.2)}a_1^1, ^{(0.1)}a_2^1, ^{(0.6)}a_3^1\}), \\ ((x_3^1, x_1^2, x_1^3), \{^{(0.8)}a_1^1, ^{(0.6)}a_2^1, ^{(0.4)}a_3^1\}), \\ ((x_3^1, x_1^2, x_3^3), \{^{(0.2)}a_1^1, ^{(0.4)}a_2^1, ^{(0.6)}a_3^1\}), \\ ((x_3^1, x_2^2, x_1^3), \{^{(0.9)}a_1^1, ^{(0.3)}a_2^1, ^{(1)}a_3^1\}), \\ ((x_3^1, x_2^2, x_3^3), \{^{(0.4)}a_1^1, ^{(0.7)}a_2^1, ^{(0.5)}a_3^1\}) \end{array} \right\}.$$

Definition 2.15. ([1]) Let $(\widetilde{\mathcal{H}}, \mathbf{Y}) \in \mathfrak{C}\langle A, \mathbf{X}\rangle$.

(a): If $\widetilde{\mathcal{H}}(\mathbf{x^I}) = \widehat{\emptyset}$ for each $\mathbf{x^I} \in \mathbf{Y}$ then it is said to be a relative null fuzzy hypersoft set (with respect to $\mathbf{Y}$), denoted by $\widehat{\emptyset}_{\mathbf{Y}}$. If $\mathbf{Y} = \mathbf{X}$ then it is called a null fuzzy hypersoft set and denoted by $\widehat{\emptyset}_{\mathbf{X}}$.

(b): If $\widetilde{\mathcal{H}}(\mathbf{x^I}) = \widehat{A}$ for each $\mathbf{x^I} \in \mathbf{Y}$ then it is said to be a relative whole fuzzy hypersoft set (with respect to $\mathbf{Y}$), denoted by $\widehat{A}_{\mathbf{Y}}$. If $\mathbf{Y} = \mathbf{X}$ then it is called an absolute fuzzy hypersoft set and denoted by $\widehat{A}_{\mathbf{X}}$.

Note 3. $\mathbf{Y} \subseteq \mathbf{Z}$ (i.e., $(Y_1 \times Y_2 \times ... \times Y_m) \subseteq (Z_1 \times Z_2 \times ... \times Z_m)$) iff $Y_i \subseteq Z_i$ for all $i \in I$.

Definition 2.16. ([1]) Let $(\widetilde{\mathcal{H}}, \mathbf{Y}), (\widetilde{\mathcal{K}}, \mathbf{Z}) \in \mathfrak{C}\langle A, \mathbf{X}\rangle$.

(a): $(\widetilde{\mathcal{H}}, \mathbf{Y})$ is called a fuzzy hypersoft subset of $(\widetilde{\mathcal{K}}, \mathbf{Z})$, denoted by $(\widetilde{\mathcal{H}}, \mathbf{Y}) \sqsubseteq (\widetilde{\mathcal{K}}, \mathbf{Z})$, if $\mathbf{Y} \subseteq \mathbf{Z}$ and $\widetilde{\mathcal{H}}(\mathbf{x^I}) \subseteq_f \widetilde{\mathcal{K}}(\mathbf{x^I})$ for each $\mathbf{x^I} \in \mathbf{Y}$.

(b): The fuzzy hypersoft sets $(\widetilde{\mathcal{H}}, \mathbf{Y})$ and $(\widetilde{\mathcal{K}}, \mathbf{Z})$ are called equal, denoted by $(\widetilde{\mathcal{H}}, \mathbf{Y}) = (\widetilde{\mathcal{K}}, \mathbf{Z})$, if $(\widetilde{\mathcal{H}}, \mathbf{Y}) \sqsubseteq (\widetilde{\mathcal{K}}, \mathbf{Z})$ and $(\widetilde{\mathcal{K}}, \mathbf{Z}) \sqsubseteq (\widetilde{\mathcal{H}}, \mathbf{Y})$.

Definition 2.17. ([1]) Let $(\widetilde{\mathcal{H}}, \mathbf{Y}) \in \mathfrak{C}\langle A, \mathbf{X}\rangle$. Then, the relative complement of fuzzy hypersoft set $(\widetilde{\mathcal{H}}, \mathbf{Y})$, denoted by $(\widetilde{\mathcal{H}}, \mathbf{Y})^r$, is defined as

$$(\widetilde{\mathcal{H}}, \mathbf{Y})^r = (\widetilde{\mathcal{H}}^r, \mathbf{Y}), \tag{5}$$

where $\widetilde{\mathcal{H}}^r(\mathbf{x^I})$ is the fuzzy complement of $\widetilde{\mathcal{H}}(\mathbf{x^I})$ for each $\mathbf{x^I} \in \mathbf{Y}$.

Note 4. It is clear that $\mathbf{T} = \mathbf{Y} \cap \mathbf{Z} = (Y_1 \times Y_2 \times ... \times Y_m) \cap (Z_1 \times Z_2 \times ... \times Z_m) = (Y_1 \cap Z_1) \times (Y_2 \cap Z_2) \times ... \times (Y_m \cap Z_m)$ and $\mathbf{T} = \mathbf{Y} \cup \mathbf{Z} = (Y_1 \times Y_2 \times ... \times Y_m) \cup (Z_1 \times Z_2 \times ... \times Z_m) = (Y_1 \cup Z_1) \times (Y_2 \cup Z_2) \times ... \times (Y_m \cup Z_m)$. If $Y_i \cap Z_i = \emptyset$ for some $i \in I$ then $\mathbf{T} = \mathbf{Y} \cap \mathbf{Z} = \emptyset$. From now on, we assume that $\mathbf{T} = \mathbf{Y} \cap \mathbf{Z} \neq \emptyset$. (Similarly, $\mathfrak{S} = \mathfrak{Q} \cap \mathfrak{R} \neq \emptyset$ in Sections 4 and 5).

Definition 2.18. ([1]) Let $(\widetilde{\mathcal{H}}, \mathbf{Y}), (\widetilde{\mathcal{K}}, \mathbf{Z}) \in \mathfrak{C}\langle A, \mathbf{X}\rangle$. Then, the restricted intersection of fuzzy hypersoft sets $(\widetilde{\mathcal{H}}, \mathbf{Y})$ and $(\widetilde{\mathcal{K}}, \mathbf{Z})$ is denoted and defined by $(\widetilde{\mathcal{L}}, \mathbf{T}) = (\widetilde{\mathcal{H}}, \mathbf{Y}) \Cap (\widetilde{\mathcal{K}}, \mathbf{Z})$ where $\mathbf{T} = \mathbf{Y} \cap \mathbf{Z}$ and

$$\widetilde{\mathcal{L}}(\mathbf{x^I}) = \widetilde{\mathcal{H}}(\mathbf{x^I}) \cap_f \widetilde{\mathcal{K}}(\mathbf{x^I}) \tag{6}$$

for each $\mathbf{x^I} \in \mathbf{T}$.

Definition 2.19. ([1]) Let $(\widetilde{\mathcal{H}}, \mathbf{Y}), (\widetilde{\mathcal{K}}, \mathbf{Z}) \in \mathfrak{C}\langle A, \mathbf{X}\rangle$. Then, the extended intersection of fuzzy hypersoft sets $(\widetilde{\mathcal{H}}, \mathbf{Y})$ and $(\widetilde{\mathcal{K}}, \mathbf{Z})$ is denoted and defined by $(\widetilde{\mathcal{L}}, \mathbf{T}) = (\widetilde{\mathcal{H}}, \mathbf{Y}) \sqcap (\widetilde{\mathcal{K}}, \mathbf{Z})$ where $\mathbf{T} = \mathbf{Y} \cup \mathbf{Z}$ and

$$\widetilde{\mathcal{L}}(\mathbf{x^I}) = \begin{cases} \widetilde{\mathcal{H}}(\mathbf{x^I}), & \text{if } \mathbf{x^I} \in \mathbf{Y}, \\ \widetilde{\mathcal{K}}(\mathbf{x^I}), & \text{if } \mathbf{x^I} \in \mathbf{Z}, \\ \widetilde{\mathcal{H}}(\mathbf{x^I}) \cap_f \widetilde{\mathcal{K}}(\mathbf{x^I}), & \text{if } \mathbf{x^I} \in \mathbf{Y} \cap \mathbf{Z}, \end{cases} \tag{7}$$

for each $\mathbf{x^I} \in \mathbf{T}$.

Definition 2.20. ([1]) Let $(\widetilde{\mathcal{H}}, \mathbf{Y}), (\widetilde{\mathcal{K}}, \mathbf{Z}) \in \mathfrak{C}\langle A, \mathbf{X}\rangle$. Then, the restricted union of fuzzy hypersoft sets $(\widetilde{\mathcal{H}}, \mathbf{Y})$ and $(\widetilde{\mathcal{K}}, \mathbf{Z})$ is denoted and defined by $(\widetilde{\mathcal{L}}, \mathbf{T}) = (\widetilde{\mathcal{H}}, \mathbf{Y}) \Cup (\widetilde{\mathcal{K}}, \mathbf{Z})$ where $\mathbf{T} = \mathbf{Y} \cap \mathbf{Z}$ and

$$\widetilde{\mathcal{L}}(\mathbf{x^I}) = \widetilde{\mathcal{H}}(\mathbf{x^I}) \cup_f \widetilde{\mathcal{K}}(\mathbf{x^I}) \tag{8}$$

for each $\mathbf{x^I} \in \mathbf{T}$.

Definition 2.21. ([1]) Let $(\widetilde{\mathcal{H}}, \mathbf{Y}), (\widetilde{\mathcal{K}}, \mathbf{Z}) \in \mathfrak{C}\langle A, \mathbf{X}\rangle$. Then, the extended union of fuzzy hypersoft sets $(\widetilde{\mathcal{H}}, \mathbf{Y})$ and $(\widetilde{\mathcal{K}}, \mathbf{Z})$ is denoted and defined by $(\widetilde{\mathcal{L}}, \mathbf{T}) = (\widetilde{\mathcal{H}}, \mathbf{Y}) \sqcup (\widetilde{\mathcal{K}}, \mathbf{Z})$ where $\mathbf{T} = \mathbf{X} \cup \mathbf{Y}$ and

$$\widetilde{\mathcal{L}}(\mathbf{x^I}) = \begin{cases} \widetilde{\mathcal{H}}(\mathbf{x^I}), & \text{if } \ \mathbf{x^I} \in \mathbf{Y}, \\ \widetilde{\mathcal{K}}(\mathbf{x^I}), & \text{if } \ \mathbf{x^I} \in \mathbf{Z}, \\ \widetilde{\mathcal{H}}(\mathbf{x^I}) \cup_f \widetilde{\mathcal{K}}(\mathbf{x^I}), & \text{if } \ \mathbf{x^I} \in \mathbf{Y} \cap \mathbf{Z}, \end{cases} \tag{9}$$

for each $\mathbf{x^I} \in \mathbf{T}$.

3. n-ary Fuzzy Hypersoft Sets

In this section, we introduce the notion of n-ary fuzzy hypersoft set and derive its fundamental operations.

Let $\{A_j : j \in J = \{1, 2, ..., n\}\}$ be a collection of universal sets such that $A_j \cap A_{j'} = \emptyset$ for each $j, j' \in J = \{1, 2, ..., n\}$ and $j \neq j'$. Also, let $\mathfrak{F}(\mathfrak{A}) = \prod_{j \in J} \mathfrak{F}(A_j) = \mathfrak{F}(A_1) \times \mathfrak{F}(A_2) \times ... \times \mathfrak{F}(A_n)$, where $\mathfrak{F}(A_j)$ denotes the set of all fuzzy sets in A_j.

Definition 3.1. A pair $(\widetilde{\mathcal{H}}_n, \mathbf{Y})$ is said to be an n-ary fuzzy hypersoft set over $\mathfrak{A} = \{A_1, A_2, ..., A_n\}$, where $\widetilde{\mathcal{H}}_n$ is mapping given by

$$\widetilde{\mathcal{H}}_n : \mathbf{Y} \to \mathfrak{F}(\mathfrak{A}). \tag{10}$$

Simply, an n-ary fuzzy hypersoft set is described as the following:

$$\begin{aligned} (\widetilde{\mathcal{H}}_n, \mathbf{Y}) &= \{(\mathbf{x^I}, \widetilde{\mathcal{H}}_n(\mathbf{x^I})) : \mathbf{x^I} \in \mathbf{Y} \text{ and } \widetilde{\mathcal{H}}_n(\mathbf{x^I}) \in \mathfrak{A}\} \\ &= \left\{ \left(\mathbf{x^I}, \begin{pmatrix} \{^{(\mu_{\widetilde{\mathcal{H}}_n(\mathbf{x^I})}(a^1))}a^1 : a^1 \in A_1\}, \\ \{^{(\mu_{\widetilde{\mathcal{H}}_n(\mathbf{x^I})}(a^2))}a^2 : a^2 \in A_2\}, \\ . \\ . \\ . \\ \{^{(\mu_{\widetilde{\mathcal{H}}_n(\mathbf{x^I})}(a^n))}a^n : a^n \in A_n\} \end{pmatrix} \right) : \mathbf{x^I} \in \mathbf{Y} \right\}, \end{aligned}$$

where $\widetilde{\widetilde{\mathcal{H}}}_{n(j)}(\mathbf{x^I}) = \{^{(\mu_{\widetilde{\mathcal{H}}_n(\mathbf{x^I})}(a^j))}a^j : a^j \in A_j\}$ for $j = 1, 2, ..., n$ and it is termed to be an A_j-part of $\widetilde{\mathcal{H}}_n(\mathbf{x^I})$.

Especially, if $n = 2, 3, 4$ and 5 then it is called a binary fuzzy hypersoft set, ternary fuzzy hypersoft set, quaternary fuzzy hypersoft set, quinary fuzzy hypersoft set, respectively.

Note 5. The set of all n-ary fuzzy hypersoft sets over $\mathfrak{A} = \{A_1, A_2, ..., A_n\}$ for $\mathbf{X}$ is denoted by $\mathfrak{C}_{N_n}\langle\mathfrak{A}, \mathbf{X}\rangle$.

Example 3.2. We consider the problem in Examples 2.12 and 2.14. However, he/she aims to determine the optimal car in each segment by evaluating cars in different segments simultaneously. Assume that $A_1 = \{a^1_1, a^1_2, a^1_3\}$, $A_2 = \{a^2_1, a^2_2\}$ and $A_3 = \{a^3_1, a^3_2, a^3_3\}$ are sets of cars in the B-segment (Supermini family), C-segment (Small family) and D-segment(Large family), respectively. Considering the parameter subsets $Y_1 = X_1$, $Y_2 = X_2$ and $Y_3 = \{x^3_1, x^3_3\} \subseteq X_3$, he/she evaluates the cars in different segments, and thus constructs the following ternary fuzzy hypersoft set.

$$(\widetilde{\mathcal{H}}_3, \mathbf{Y}) = \left\{ \begin{array}{l}
((x^1_1, x^2_1, x^3_1), (\{^{(0.4)}a^1_1, ^{(0.3)}a^1_2, ^{(0.6)}a^1_3\}, \{^{(0.3)}a^2_1, ^{(0.7)}a^2_2\}, \{^{(0.5)}a^3_1, ^{(0.5)}a^3_2, ^{(0.4)}a^3_3\})), \\
((x^1_1, x^2_1, x^3_3), (\{^{(0.5)}a^1_1, ^{(0.6)}a^1_2, ^{(0.6)}a^1_3\}, \{^{(0.7)}a^2_1, ^{(0.7)}a^2_2\}, \{^{(0.3)}a^3_1, ^{(0.4)}a^3_2, ^{(0.5)}a^3_3\})), \\
((x^1_1, x^2_2, x^3_1), (\{^{(0.3)}a^1_1, ^{(0.3)}a^1_2, ^{(0.2)}a^1_3\}, \{^{(0.8)}a^2_1, ^{(0.7)}a^2_2\}, \{^{(0.6)}a^3_1, ^{(0.3)}a^3_2, ^{(0.4)}a^3_3\})), \\
((x^1_1, x^2_2, x^3_3), (\{^{(0.1)}a^1_1, ^{(0.4)}a^1_2, ^{(0)}a^1_3\}, \{^{(0.2)}a^2_1, ^{(0.4)}a^2_2\}, \{^{(0.6)}a^3_1, ^{(0.4)}a^3_2, ^{(0.4)}a^3_3\})), \\
((x^1_2, x^2_1, x^3_1), (\{^{(0)}a^1_1, ^{(0)}a^1_2, ^{(0)}a^1_3\}, \{^{(1)}a^2_1, ^{(0)}a^2_2\}, \{^{(0.2)}a^3_1, ^{(0.6)}a^3_2, ^{(0.8)}a^3_3\})), \\
((x^1_2, x^2_1, x^3_3), (\{^{(1)}a^1_1, ^{(1)}a^1_2, ^{(1)}a^1_3\}, \{^{(0.3)}a^2_1, ^{(0.7)}a^2_2\}, \{^{(0.5)}a^3_1, ^{(0.4)}a^3_2, ^{(0.3)}a^3_3\})), \\
((x^1_2, x^2_2, x^3_1), (\{^{(0.5)}a^1_1, ^{(0.7)}a^1_2, ^{(0.4)}a^1_3\}, \{^{(0.1)}a^2_1, ^{(0.5)}a^2_2\}, \{^{(0.2)}a^3_1, ^{(0.4)}a^3_2, ^{(0.6)}a^3_3\})), \\
((x^1_2, x^2_2, x^3_3), (\{^{(0.2)}a^1_1, ^{(0.1)}a^1_2, ^{(0.6)}a^1_3\}, \{^{(0.4)}a^2_1, ^{(0.6)}a^2_2\}, \{^{(0.8)}a^3_1, ^{(0.4)}a^3_2, ^{(0.1)}a^3_3\})), \\
((x^1_3, x^2_1, x^3_1), (\{^{(0.8)}a^1_1, ^{(0.6)}a^1_2, ^{(0.4)}a^1_3\}, \{^{(0.6)}a^2_1, ^{(0.3)}a^2_2\}, \{^{(0)}a^3_1, ^{(0.2)}a^3_2, ^{(0)}a^3_3\})), \\
((x^1_3, x^2_1, x^3_3), (\{^{(0.2)}a^1_1, ^{(0.4)}a^1_2, ^{(0.6)}a^1_3\}\{^{(0.7)}a^2_1, ^{(0.4)}a^2_2\}, \{^{(0.4)}a^3_1, ^{(0.6)}a^3_2, ^{(0.3)}a^3_3\})), \\
((x^1_3, x^2_2, x^3_1), (\{^{(0.9)}a^1_1, ^{(0.3)}a^1_2, ^{(1)}a^1_3\}, \{^{(0.2)}a^2_1, ^{(0.2)}a^2_2\}, \{^{(0.4)}a^3_1, ^{(0.2)}a^3_2, ^{(0.4)}a^3_3\})), \\
((x^1_3, x^2_2, x^3_3), (\{^{(0.4)}a^1_1, ^{(0.7)}a^1_2, ^{(0.5)}a^1_3\}, \{^{(0.6)}a^2_1, ^{(0.8)}a^2_2\}, \{^{(0.8)}a^3_1, ^{(0.8)}a^3_2, ^{(0.5)}a^3_3\}))
\end{array} \right\}.$$

Definition 3.3. Let $(\widetilde{\mathcal{H}}_n, \mathbf{Y}) \in \mathfrak{C}_{N_n}\langle\mathfrak{A}, \mathbf{X}\rangle$.

- **(a):** If $\widetilde{\mathcal{H}}_n(\mathbf{x^I}) = (\widehat{\emptyset}, \widehat{\emptyset}, ..., \widehat{\emptyset})$ (i.e., $\widetilde{\widetilde{\mathcal{H}}}_{n(j)}(\mathbf{x^I}) = \widehat{\emptyset} \ \ \forall j \in J$) for each $\mathbf{x^I} \in \mathbf{Y}$ then it is said to be a relative null n-ary fuzzy hypersoft set (with respect to $\mathbf{Y}$), denoted by $\widehat{\emptyset}^{N_n}_{\mathbf{Y}}$. If $\mathbf{Y} = \mathbf{X}$ then it is called a null n-ary fuzzy hypersoft set and denoted by $\widehat{\emptyset}^{N_n}_{\mathbf{X}}$.
- **(b):** If $\widetilde{\mathcal{H}}_n(\mathbf{x^I}) = (\widehat{A}_1, \widehat{A}_2, ..., \widehat{A}_n)$ (i.e., $\widetilde{\widetilde{\mathcal{H}}}_j(\mathbf{x^I}) = \widehat{A}_j \ \ \forall j \in J$) for each $\mathbf{x^I} \in \mathbf{Y}$ then it is called a relative whole n-ary fuzzy hypersoft set (with respect to $\mathbf{Y}$), denoted by $\widehat{\mathfrak{A}}^{N_n}_{\mathbf{Y}}$. If $\mathbf{Y} = \mathbf{X}$ then it is said to be an absolute n-ary fuzzy hypersoft set and denoted by $\widehat{\mathfrak{A}}^{N_n}_{\mathbf{X}}$.

Definition 3.4. Let $(\widetilde{\mathcal{H}}_n, \mathbf{Y}), (\widetilde{\mathcal{K}}_n, \mathbf{Z}) \in \mathfrak{C}_{N_n}\langle\mathfrak{A}, \mathbf{X}\rangle$.

- **(a):** $(\widetilde{\mathcal{H}}_n, \mathbf{Y})$ is termed a fuzzy hypersoft subset of $(\widetilde{\mathcal{K}}_n, \mathbf{Z})$, denoted by $(\widetilde{\mathcal{H}}_n, \mathbf{Y}) \sqsubseteq_{N_n} (\widetilde{\mathcal{K}}_n, \mathbf{Z})$, if $\mathbf{Y} \subseteq \mathbf{Z}$ and

$$\widetilde{\mathcal{H}}_{n(j)}(\mathbf{x^I}) \subseteq_f \widetilde{\mathcal{H}}_{n(j)}(\mathbf{x^I}) \ \ \forall j \in J \tag{11}$$

for each $\mathbf{x^I} \in \mathbf{Y}$.

(b): The n-ary fuzzy hypersoft sets $(\widetilde{\mathcal{H}}_n, \mathbf{Y})$ and $(\widetilde{\mathcal{K}}_n, \mathbf{Z})$ are called equal, denoted by $(\widetilde{\mathcal{H}}_n, \mathbf{Y}) =_{N_n} (\widetilde{\mathcal{K}}_n, \mathbf{Z})$, if $(\widetilde{\mathcal{H}}_n, \mathbf{Y}) \sqsubseteq_{N_n} (\widetilde{\mathcal{K}}_n, \mathbf{Z})$ and $(\widetilde{\mathcal{K}}_n, \mathbf{Z}) \sqsubseteq_{N_n} (\widetilde{\mathcal{H}}_n, \mathbf{Y})$.

Example 3.5. Consider the ternary fuzzy hypersoft set $(\widetilde{\mathcal{H}}_3, \mathbf{Y})$ in Example 3.2. Also, we assume that the disjoint parameter subsets $Z_1 = \{x_1^1, x_3^1\} \subseteq X_1$, $Z_2 = X_2$, $Z_3 = \{x_1^3, x_3^3\} \subseteq X_3$ (i.e., $\mathbf{Z} = Z_1 \times Z_2 \times Z_3$) and

$$(\widetilde{\mathcal{K}}_3, \mathbf{Z}) = \left\{ \begin{array}{l} ((x_1^1, x_1^2, x_1^3), (\{^{(0.4)}a_1^1, ^{(0.1)}a_2^1, ^{(0.4)}a_3^1\}, \{^{(0.1)}a_1^2, ^{(0.6)}a_2^2\}, \{^{(0.5)}a_1^3, ^{(0.3)}a_2^3, ^{(0.1)}a_3^3\})), \\ ((x_1^1, x_1^2, x_3^3), (\{^{(0.2)}a_1^1, ^{(0)}a_2^1, ^{(0.1)}a_3^1\}, \{^{(0.1)}a_1^2, ^{(0.4)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.1)}a_2^3, ^{(0.1)}a_3^3\})), \\ ((x_1^1, x_2^2, x_1^3), (\{^{(0.2)}a_1^1, ^{(0.3)}a_2^1, ^{(0.2)}a_3^1\}, \{^{(0.8)}a_1^2, ^{(0.7)}a_2^2\}, \{^{(0.2)}a_1^3, ^{(0.2)}a_2^3, ^{(0.1)}a_3^3\})), \\ ((x_1^1, x_2^2, x_3^3), (\{^{(0.1)}a_1^1, ^{(0.3)}a_2^1, ^{(0)}a_3^1\}, \{^{(0.1)}a_1^2, ^{(0.3)}a_2^2\}, \{^{(0.2)}a_1^3, ^{(0.1)}a_2^3, ^{(0.1)}a_3^3\})), \\ ((x_3^1, x_1^2, x_1^3), (\{^{(0.4)}a_1^1, ^{(0.4)}a_2^1, ^{(0.4)}a_3^1\}, \{^{(0.3)}a_1^2, ^{(0.3)}a_2^2\}, \{^{(0)}a_1^3, ^{(0.2)}a_2^3, ^{(0)}a_3^3\})), \\ ((x_3^1, x_1^2, x_3^3), (\{^{(0.1)}a_1^1, ^{(0.1)}a_2^1, ^{(0.1)}a_3^1\}\{^{(0.3)}a_1^2, ^{(0.4)}a_2^2\}, \{^{(0.3)}a_1^3, ^{(0.2)}a_2^3, ^{(0.2)}a_3^3\})), \\ ((x_3^1, x_2^2, x_1^3), (\{^{(0.3)}a_1^1, ^{(0.3)}a_2^1, ^{(1)}a_3^1\}, \{^{(0.1)}a_1^2, ^{(0.1)}a_2^2\}, \{^{(0.3)}a_1^3, ^{(0.1)}a_2^3, ^{(0.1)}a_3^3\})), \\ ((x_3^1, x_2^2, x_3^3), (\{^{(0.2)}a_1^1, ^{(0.2)}a_2^1, ^{(0.2)}a_3^1\}, \{^{(0.6)}a_1^2, ^{(0.8)}a_2^2\}, \{^{(1)}a_1^3, ^{(1)}a_2^3, ^{(0.5)}a_3^3\})) \end{array} \right\}.$$

Then, we have $\mathbf{Z} \subseteq \mathbf{Y}$ (by considering Note 3) but $(\widetilde{\mathcal{K}}_3, \mathbf{Z})$ is not a ternary fuzzy hypersoft subset $(\widetilde{\mathcal{H}}_3, \mathbf{Y})$ since $\widetilde{\mathcal{K}}_{3(3)}((x_3^1, x_2^2, x_3^3)) \nsubseteq_f \widetilde{\mathcal{H}}_{3(3)}((x_3^1, x_2^2, x_3^3))$. If we take $\widetilde{\mathcal{K}}_{3(3)}((x_3^1, x_2^2, x_3^3)) \subseteq_f \{^{(0.8)}a_1^3, ^{(0.8)}a_2^3, ^{(0.5)}a_3^3\}$ then we can say that $(\widetilde{\mathcal{K}}_3, \mathbf{Z})$ is an n-ary fuzzy hypersoft subset $(\widetilde{\mathcal{H}}_3, \mathbf{Y})$ (i.e., $(\widetilde{\mathcal{H}}_3, \mathbf{Y}) \sqsubseteq_{N_3} (\widetilde{\mathcal{K}}_3, \mathbf{Z})$).

Definition 3.6. Let $(\widetilde{\mathcal{H}}_n, \mathbf{Y}) \in \mathfrak{C}_{N_n}\langle \mathfrak{A}, \mathbf{X} \rangle$. Then, the relative complement of n-ary fuzzy hypersoft set $(\widetilde{\mathcal{H}}_n, \mathbf{Y})$, denoted by $(\widetilde{\mathcal{H}}_n, \mathbf{Y})^{r_{N_n}}$, is defined as

$$(\widetilde{\mathcal{H}}_n, \mathbf{Y})^{r_{N_n}} = (\widetilde{\mathcal{H}}_n^r, \mathbf{Y}), \tag{12}$$

where $\widetilde{\mathcal{H}}_{n(j)}^r(\mathbf{x}^\mathbf{I})$ is the fuzzy complement of $\widetilde{\mathcal{H}}_{n(j)}(\mathbf{x}^\mathbf{I})$ $(\forall j \in J)$ for each $\mathbf{x}^\mathbf{I} \in \mathbf{Y}$.

Example 3.7. The complement of the ternary fuzzy hypersoft set $(\widetilde{\mathcal{H}}_3, \mathbf{Y})$ in Example 3.2 is

$$(\widetilde{\mathcal{H}}_3, \mathbf{Y})^{r_{N_3}} = \left\{ \begin{array}{l} ((x_1^1, x_1^2, x_1^3), (\{^{(0.6)}a_1^1, ^{(0.7)}a_2^1, ^{(0.4)}a_3^1\}, \{^{(0.7)}a_1^2, ^{(0.3)}a_2^2\}, \{^{(0.5)}a_1^3, ^{(0.5)}a_2^3, ^{(0.6)}a_3^3\})), \\ ((x_1^1, x_1^2, x_3^3), (\{^{(0.5)}a_1^1, ^{(0.4)}a_2^1, ^{(0.4)}a_3^1\}, \{^{(0.3)}a_1^2, ^{(0.3)}a_2^2\}, \{^{(0.7)}a_1^3, ^{(0.6)}a_2^3, ^{(0.5)}a_3^3\})), \\ ((x_1^1, x_2^2, x_1^3), (\{^{(0.7)}a_1^1, ^{(0.7)}a_2^1, ^{(0.8)}a_3^1\}, \{^{(0.2)}a_1^2, ^{(0.3)}a_2^2\}, \{^{(0.4)}a_1^3, ^{(0.7)}a_2^3, ^{(0.6)}a_3^3\})), \\ ((x_1^1, x_2^2, x_3^3), (\{^{(0.9)}a_1^1, ^{(0.6)}a_2^1, ^{(1)}a_3^1\}, \{^{(0.8)}a_1^2, ^{(0.6)}a_2^2\}, \{^{(0.4)}a_1^3, ^{(0.6)}a_2^3, ^{(0.6)}a_3^3\})), \\ ((x_2^1, x_1^2, x_1^3), (\{^{(1)}a_1^1, ^{(1)}a_2^1, ^{(1)}a_3^1\}, \{^{(0)}a_1^2, ^{(1)}a_2^2\}, \{^{(0.8)}a_1^3, ^{(0.4)}a_2^3, ^{(0.2)}a_3^3\})), \\ ((x_2^1, x_1^2, x_3^3), (\{^{(0)}a_1^1, ^{(0)}a_2^1, ^{(0)}a_3^1\}, \{^{(0.7)}a_1^2, ^{(0.3)}a_2^2\}, \{^{(0.5)}a_1^3, ^{(0.6)}a_2^3, ^{(0.7)}a_3^3\})), \\ ((x_2^1, x_2^2, x_1^3), (\{^{(0.5)}a_1^1, ^{(0.3)}a_2^1, ^{(0.6)}a_3^1\}, \{^{(0.9)}a_1^2, ^{(0.5)}a_2^2\}, \{^{(0.8)}a_1^3, ^{(0.6)}a_2^3, ^{(0.4)}a_3^3\})), \\ ((x_2^1, x_2^2, x_3^3), (\{^{(0.8)}a_1^1, ^{(0.9)}a_2^1, ^{(0.4)}a_3^1\}, \{^{(0.6)}a_1^2, ^{(0.4)}a_2^2\}, \{^{(0.2)}a_1^3, ^{(0.6)}a_2^3, ^{(0.9)}a_3^3\})), \\ ((x_3^1, x_1^2, x_1^3), (\{^{(0.2)}a_1^1, ^{(0.4)}a_2^1, ^{(0.6)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.7)}a_2^2\}, \{^{(1)}a_1^3, ^{(0.8)}a_2^3, ^{(1)}a_3^3\})), \\ ((x_3^1, x_1^2, x_3^3), (\{^{(0.8)}a_1^1, ^{(0.6)}a_2^1, ^{(0.4)}a_3^1\}\{^{(0.3)}a_1^2, ^{(0.6)}a_2^2\}, \{^{(0.6)}a_1^3, ^{(0.4)}a_2^3, ^{(0.7)}a_3^3\})), \\ ((x_3^1, x_2^2, x_1^3), (\{^{(0.1)}a_1^1, ^{(0.7)}a_2^1, ^{(0)}a_3^1\}, \{^{(0.8)}a_1^2, ^{(0.8)}a_2^2\}, \{^{(0.6)}a_1^3, ^{(0.8)}a_2^3, ^{(0.6)}a_3^3\})), \\ ((x_3^1, x_2^2, x_3^3), (\{^{(0.6)}a_1^1, ^{(0.3)}a_2^1, ^{(0.5)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.2)}a_2^2\}, \{^{(0.2)}a_1^3, ^{(0.2)}a_2^3, ^{(0.5)}a_3^3\})) \end{array} \right\}.$$

Proposition 3.8. *Let $(\widetilde{\mathcal{H}}_n, \mathbf{Y}) \in \mathfrak{C}_{N_n}\langle \mathfrak{A}, \mathbf{X} \rangle$. Then, we have the following.*

(i): $((\widetilde{\mathcal{H}}_n, \mathbf{Y})^{r_{N_n}})^{r_{N_n}} =_{N_n} (\widetilde{\mathcal{H}}_n, \mathbf{Y})$.
(ii): $(\widehat{\mathfrak{A}}^{N_n}_{\mathbf{Y}})^{r_{N_n}} =_{N_n} \widehat{\emptyset}^{N_n}_{\mathbf{Y}}$.
(iii): $(\widehat{\emptyset}^{N_n}_{\mathbf{Y}})^{r_{N_n}} =_{N_n} \widehat{\mathfrak{A}}^{N_n}_{\mathbf{Y}}$.

Proof. The proofs are straightforward. □

Definition 3.9. Let $(\widetilde{\mathcal{H}}_n, \mathbf{Y}), (\widetilde{\mathcal{K}}_n, \mathbf{Z}) \in \mathfrak{C}_{N_n}\langle \mathfrak{A}, \mathbf{X}\rangle$. Then, the restricted intersection of n-ary fuzzy hypersoft sets $(\widetilde{\mathcal{H}}_n, \mathbf{Y})$ and $(\widetilde{\mathcal{K}}_n, \mathbf{Z})$ is denoted and defined by $(\widetilde{\mathcal{L}}_n, \mathbf{T}) = (\widetilde{\mathcal{H}}_n, \mathbf{Y}) \Cap_{N_n} (\widetilde{\mathcal{K}}_n, \mathbf{Z})$ where $\mathbf{T} = \mathbf{Y} \cap \mathbf{Z}$ and

$$\widetilde{\mathcal{K}}_{n(j)}(\mathbf{x}^{\mathbf{I}}) = \widetilde{\mathcal{H}}_{n(j)}(\mathbf{x}^{\mathbf{I}}) \cap_f \widetilde{\mathcal{K}}_{n(j)}(\mathbf{x}^{\mathbf{I}}) \quad \forall j \in J \tag{13}$$

for each $\mathbf{x}^{\mathbf{I}} \in \mathbf{T}$.

Definition 3.10. Let $(\widetilde{\mathcal{H}}_n, \mathbf{Y}), (\widetilde{\mathcal{K}}_n, \mathbf{Z}) \in \mathfrak{C}_{N_n}\langle \mathfrak{A}, \mathbf{X}\rangle$. Then, the extended intersection of n-ary fuzzy hypersoft sets $(\widetilde{\mathcal{H}}_n, \mathbf{Y})$ and $(\widetilde{\mathcal{K}}_n, \mathbf{Z})$ is denoted and defined by $(\widetilde{\mathcal{L}}_n, \mathbf{T}) = (\widetilde{\mathcal{H}}_n, \mathbf{Y}) \sqcap_{N_n} (\widetilde{\mathcal{K}}_n, \mathbf{Z})$ where $\mathbf{T} = \mathbf{Y} \cup \mathbf{Z}$ and

$$\widetilde{\mathcal{L}}_n(\mathbf{x}^{\mathbf{I}}) = \begin{cases} \widetilde{\mathcal{H}}_n(\mathbf{x}^{\mathbf{I}}), & \text{if} \ \ \mathbf{x}^{\mathbf{I}} \in \mathbf{Y}, \\ \widetilde{\mathcal{K}}_n(\mathbf{x}^{\mathbf{I}}), & \text{if} \ \ \mathbf{x}^{\mathbf{I}} \in \mathbf{Z}, \\ \widetilde{\mathcal{H}}_n(\mathbf{x}^{\mathbf{I}}) \cap_f \widetilde{\mathcal{K}}_n(\mathbf{x}^{\mathbf{I}}), & \text{if} \ \ \mathbf{x}^{\mathbf{I}} \in \mathbf{Y} \cap \mathbf{Z}, \end{cases} \tag{14}$$

for each $\mathbf{x}^{\mathbf{I}} \in \mathbf{T}$, where Eq. (13) is applied to obtain $\widetilde{\mathcal{H}}_n(\mathbf{x}^{\mathbf{I}}) \cap_f \widetilde{\mathcal{K}}_n(\mathbf{x}^{\mathbf{I}})$.

Example 3.11. Consider the ternary fuzzy hypersoft set $(\widetilde{\mathcal{H}}_3, \mathbf{Y})$ in Example 3.2. Also, we suppose that the disjoint parameter subsets $Z_1 = \{x^1_2\} \subseteq X_1$, $Z_2 = X_2$, $Z_3 = \{x^3_2, x^3_3\} \subseteq X_3$ (i.e., $\mathbf{Z} = Z_1 \times Z_2 \times Z_3$) and

$$(\widetilde{\mathcal{K}}_3, \mathbf{Z}) = \left\{ \begin{array}{l} ((x^1_2, x^2_1, x^3_2), (\{^{(0.4)}a^1_1, ^{(0.6)}a^1_2, ^{(0.5)}a^1_3\}, \{^{(0.2)}a^2_1, ^{(0.6)}a^2_2\}, \{^{(0.4)}a^3_1, ^{(0.5)}a^3_2, ^{(0.4)}a^3_3\})), \\ ((x^1_2, x^2_1, x^3_3), (\{^{(0.8)}a^1_1, ^{(0.4)}a^1_2, ^{(0.6)}a^1_3\}, \{^{(0.3)}a^2_1, ^{(0.7)}a^2_2\}, \{^{(0.6)}a^3_1, ^{(0.3)}a^3_2, ^{(0.5)}a^3_3\})), \\ ((x^1_2, x^2_2, x^3_2), (\{^{(0.4)}a^1_1, ^{(0.4)}a^1_2, ^{(0.2)}a^1_3\}, \{^{(0.5)}a^2_1, ^{(0.5)}a^2_2\}, \{^{(0.2)}a^3_1, ^{(0.4)}a^3_2, ^{(0.7)}a^3_3\})), \\ ((x^1_2, x^2_2, x^3_3), (\{^{(0.5)}a^1_1, ^{(0.4)}a^1_2, ^{(0.2)}a^1_3\}, \{^{(0.7)}a^2_1, ^{(0.7)}a^2_2\}, \{^{(0.2)}a^3_1, ^{(0.2)}a^3_2, ^{(0.1)}a^3_3\})), \end{array} \right\}.$$

Then, the restricted intersection and extended intersection of the ternary fuzzy hypersoft sets $(\widetilde{\mathcal{H}}_3, \mathbf{Y})$ and $(\widetilde{\mathcal{K}}_3, \mathbf{Z})$ are respectively

$$(\widetilde{\mathcal{H}}_3, \mathbf{Y}) \Cap_{N_3} (\widetilde{\mathcal{K}}_3, \mathbf{Z}) = \left\{ \begin{array}{l} ((x^1_2, x^2_1, x^3_3), (\{^{(0.8)}a^1_1, ^{(0.4)}a^1_2, ^{(0.6)}a^1_3\}, \{^{(0.3)}a^2_1, ^{(0.3)}a^2_2\}, \{^{(0.5)}a^3_1, ^{(0.3)}a^3_2, ^{(0.3)}a^3_3\})), \\ ((x^1_2, x^2_2, x^3_3), (\{^{(0.2)}a^1_1, ^{(0.1)}a^1_2, ^{(0.2)}a^1_3\}, \{^{(0.4)}a^2_1, ^{(0.6)}a^2_2\}, \{^{(0.2)}a^3_1, ^{(0.2)}a^3_2, ^{(0.1)}a^3_3\})) \end{array} \right\},$$

and

$$(\widetilde{\mathcal{H}}_3,\mathbf{Y})\sqcap_{N_3}(\widetilde{\mathcal{K}}_3,\mathbf{Z})=\left\{\begin{array}{l}((x_1^1,x_1^2,x_1^3),(\{^{(0.4)}a_1^1,^{(0.3)}a_2^1,^{(0.6)}a_3^1\},\{^{(0.3)}a_1^2,^{(0.7)}a_2^2\},\{^{(0.5)}a_1^3,^{(0.5)}a_2^3,^{(0.4)}a_3^3\})),\\((x_2^1,x_1^2,x_2^3),(\{^{(0.4)}a_1^1,^{(0.6)}a_2^1,^{(0.5)}a_3^1\},\{^{(0.2)}a_1^2,^{(0.6)}a_2^2\},\{^{(0.4)}a_1^3,^{(0.5)}a_2^3,^{(0.4)}a_3^3\})),\\((x_2^1,x_1^2,x_3^3),(\{^{(0.8)}a_1^1,^{(0.4)}a_2^1,^{(0.6)}a_3^1\},\{^{(0.3)}a_1^2,^{(0.3)}a_2^2\},\{^{(0.5)}a_1^3,^{(0.3)}a_2^3,^{(0.3)}a_3^3\})),\\((x_1^1,x_2^2,x_1^3),(\{^{(0.3)}a_1^1,^{(0.3)}a_2^1,^{(0.2)}a_3^1\},\{^{(0.8)}a_1^2,^{(0.7)}a_2^2\},\{^{(0.6)}a_1^3,^{(0.3)}a_2^3,^{(0.4)}a_3^3\})),\\((x_2^1,x_2^2,x_2^3),(\{^{(0.4)}a_1^1,^{(0.4)}a_2^1,^{(0.2)}a_3^1\},\{^{(0.5)}a_1^2,^{(0.5)}a_2^2\},\{^{(0.2)}a_1^3,^{(0.4)}a_2^3,^{(0.7)}a_3^3\})),\\((x_2^1,x_2^2,x_3^3),(\{^{(0.2)}a_1^1,^{(0.1)}a_2^1,^{(0.2)}a_3^1\},\{^{(0.4)}a_1^2,^{(0.6)}a_2^2\},\{^{(0.2)}a_1^3,^{(0.2)}a_2^3,^{(0.1)}a_3^3\})),\\((x_2^1,x_1^2,x_1^3),(\{^{(0)}a_1^1,^{(0)}a_2^1,^{(0)}a_3^1\},\{^{(1)}a_1^2,^{(0)}a_2^2\},\{^{(0.2)}a_1^3,^{(0.6)}a_2^3,^{(0.8)}a_3^3\})),\\((x_2^1,x_1^2,x_3^3),(\{^{(1)}a_1^1,^{(1)}a_2^1,^{(1)}a_3^1\},\{^{(0.3)}a_1^2,^{(0.7)}a_2^2\},\{^{(0.5)}a_1^3,^{(0.4)}a_2^3,^{(0.3)}a_3^3\})),\\((x_2^1,x_2^2,x_1^3),(\{^{(0.5)}a_1^1,^{(0.7)}a_2^1,^{(0.4)}a_3^1\},\{^{(0.1)}a_1^2,^{(0.5)}a_2^2\},\{^{(0.2)}a_1^3,^{(0.4)}a_2^3,^{(0.6)}a_3^3\})),\\((x_2^1,x_2^2,x_3^3),(\{^{(0.2)}a_1^1,^{(0.1)}a_2^1,^{(0.6)}a_3^1\},\{^{(0.4)}a_1^2,^{(0.6)}a_2^2\},\{^{(0.8)}a_1^3,^{(0.4)}a_2^3,^{(0.1)}a_3^3\})),\\((x_3^1,x_1^2,x_1^3),(\{^{(0.8)}a_1^1,^{(0.6)}a_2^1,^{(0.4)}a_3^1\},\{^{(0.6)}a_1^2,^{(0.3)}a_2^2\},\{^{(0)}a_1^3,^{(0.2)}a_2^3,^{(0)}a_3^3\})),\\((x_3^1,x_1^2,x_3^3),(\{^{(0.2)}a_1^1,^{(0.4)}a_2^1,^{(0.6)}a_3^1\}\{^{(0.7)}a_1^2,^{(0.4)}a_2^2\},\{^{(0.4)}a_1^3,^{(0.6)}a_2^3,^{(0.3)}a_3^3\})),\\((x_3^1,x_2^2,x_1^3),(\{^{(0.9)}a_1^1,^{(0.3)}a_2^1,^{(1)}a_3^1\},\{^{(0.2)}a_1^2,^{(0.2)}a_2^2\},\{^{(0.4)}a_1^3,^{(0.2)}a_2^3,^{(0.4)}a_3^3\})),\\((x_3^1,x_2^2,x_3^3),(\{^{(0.4)}a_1^1,^{(0.7)}a_2^1,^{(0.5)}a_3^1\},\{^{(0.6)}a_1^2,^{(0.8)}a_2^2\},\{^{(0.8)}a_1^3,^{(0.8)}a_2^3,^{(0.5)}a_3^3\}))\end{array}\right\}.$$

Definition 3.12. Let $(\widetilde{\mathcal{H}}_n,\mathbf{Y}),(\widetilde{\mathcal{K}}_n,\mathbf{Z})\in\mathfrak{C}_{N_n}\langle\mathfrak{A},\mathbf{X}\rangle$. Then, the restricted union of n-ary fuzzy hypersoft sets $(\widetilde{\mathcal{H}}_n,\mathbf{Y})$ and $(\widetilde{\mathcal{K}}_n,\mathbf{Z})$ is denoted and defined by $(\widetilde{\mathcal{L}}_n,\mathbf{T})=(\widetilde{\mathcal{H}}_n,\mathbf{Y})\Cup_{N_n}(\widetilde{\mathcal{K}}_n,\mathbf{Z})$ where $\mathbf{T}=\mathbf{Y}\cap\mathbf{Z}$ and

$$\widetilde{\mathcal{K}}_{n(j)}(\mathbf{x^I})=\widetilde{\mathcal{H}}_{n(j)}(\mathbf{x^I})\cup_f\widetilde{\mathcal{K}}_{n(j)}(\mathbf{x^I})\quad\forall j\in J \tag{15}$$

for each $\mathbf{x^I}\in\mathbf{T}$.

Definition 3.13. Let $(\widetilde{\mathcal{H}}_n,\mathbf{Y}),(\widetilde{\mathcal{K}}_n,\mathbf{Z})\in\mathfrak{C}_{N_n}\langle\mathfrak{A},\mathbf{X}\rangle$. Then, the extended union of n-ary fuzzy hypersoft sets $(\widetilde{\mathcal{H}}_n,\mathbf{Y})$ and $(\widetilde{\mathcal{K}}_n,\mathbf{Z})$ is denoted and defined by $(\widetilde{\mathcal{L}}_n,\mathbf{T})=(\widetilde{\mathcal{H}}_n,\mathbf{Y})\sqcup_{N_n}(\widetilde{\mathcal{K}}_n,\mathbf{Z})$ where $\mathbf{T}=\mathbf{X}\cup\mathbf{Y}$ and

$$\widetilde{\mathcal{L}}_n(\mathbf{x^I})=\begin{cases}\widetilde{\mathcal{H}}_n(\mathbf{x^I}), & \text{if}\ \ \mathbf{x^I}\in\mathbf{Y},\\ \widetilde{\mathcal{K}}_n(\mathbf{x^I}), & \text{if}\ \ \mathbf{x^I}\in\mathbf{Z},\\ \widetilde{\mathcal{H}}_n(\mathbf{x^I})\cup_f\widetilde{\mathcal{K}}_n(\mathbf{x^I}), & \text{if}\ \ \mathbf{x^I}\in\mathbf{Y}\cap\mathbf{Z},\end{cases} \tag{16}$$

for each $\mathbf{x^I}\in\mathbf{T}$, where where Eq. (15) is applied to obtain $\widetilde{\mathcal{H}}_n(\mathbf{x^I})\cup_f\widetilde{\mathcal{K}}_n(\mathbf{x^I})$.

Example 3.14. Consider the ternary fuzzy hypersoft sets $(\widetilde{\mathcal{H}}_3,\mathbf{Y})$ and $(\widetilde{\mathcal{K}}_3,\mathbf{Z})$ in Examples 3.2 and 3.11. Then, the restricted union and extended union of the ternary fuzzy hypersoft sets $(\widetilde{\mathcal{H}}_3,\mathbf{Y})$ and $(\widetilde{\mathcal{K}}_3,\mathbf{Z})$ are respectively

$$(\widetilde{\mathcal{H}}_3,\mathbf{Y})\Cup_{N_3}(\widetilde{\mathcal{K}}_3,\mathbf{Z})=\left\{\begin{array}{l}((x_2^1,x_1^2,x_3^3),(\{^{(1)}a_1^1,^{(1)}a_2^1,^{(1)}a_3^1\},\{^{(0.7)}a_1^2,^{(0.7)}a_2^2\},\{^{(0.6)}a_1^3,^{(0.4)}a_2^3,^{(0.5)}a_3^3\})),\\((x_2^1,x_2^2,x_3^3),(\{^{(0.5)}a_1^1,^{(0.4)}a_2^1,^{(0.6)}a_3^1\},\{^{(0.7)}a_1^2,^{(0.7)}a_2^2\},\{^{(0.8)}a_1^3,^{(0.4)}a_2^3,^{(0.1)}a_3^3\}))\end{array}\right\},$$

and

$$(\widetilde{\mathcal{H}}_3, \mathbf{Y}) \sqcup_{N_3} (\widetilde{\mathcal{K}}_3, \mathbf{Z}) = \left\{ \begin{array}{l}
((x_1^1, x_1^2, x_1^3), (\{^{(0.4)}a_1^1, ^{(0.3)}a_2^1, ^{(0.6)}a_3^1\}, \{^{(0.3)}a_1^2, ^{(0.7)}a_2^2\}, \{^{(0.5)}a_1^3, ^{(0.5)}a_2^3, ^{(0.4)}a_3^3\})), \\
((x_2^1, x_1^2, x_2^3), (\{^{(0.4)}a_1^1, ^{(0.6)}a_2^1, ^{(0.5)}a_3^1\}, \{^{(0.2)}a_1^2, ^{(0.6)}a_2^2\}, \{^{(0.4)}a_1^3, ^{(0.5)}a_2^3, ^{(0.4)}a_3^3\})), \\
((x_2^1, x_1^2, x_3^3), (\{^{(1)}a_1^1, ^{(1)}a_2^1, ^{(1)}a_3^1\}, \{^{(0.7)}a_1^2, ^{(0.7)}a_2^2\}, \{^{(0.6)}a_1^3, ^{(0.4)}a_2^3, ^{(0.5)}a_3^3\})), \\
((x_1^1, x_2^2, x_1^3), (\{^{(0.3)}a_1^1, ^{(0.3)}a_2^1, ^{(0.2)}a_3^1\}, \{^{(0.8)}a_1^2, ^{(0.7)}a_2^2\}, \{^{(0.6)}a_1^3, ^{(0.3)}a_2^3, ^{(0.4)}a_3^3\})), \\
((x_2^1, x_2^2, x_2^3), (\{^{(0.4)}a_1^1, ^{(0.4)}a_2^1, ^{(0.2)}a_3^1\}, \{^{(0.5)}a_1^2, ^{(0.5)}a_2^2\}, \{^{(0.2)}a_1^3, ^{(0.4)}a_2^3, ^{(0.7)}a_3^3\})), \\
((x_2^1, x_2^2, x_3^3), (\{^{(0.5)}a_1^1, ^{(0.4)}a_2^1, ^{(0.6)}a_3^1\}, \{^{(0.7)}a_1^2, ^{(0.7)}a_2^2\}, \{^{(0.8)}a_1^3, ^{(0.4)}a_2^3, ^{(0.1)}a_3^3\})), \\
((x_2^1, x_1^2, x_1^3), (\{^{(0)}a_1^1, ^{(0)}a_2^1, ^{(0)}a_3^1\}, \{^{(1)}a_1^2, ^{(0)}a_2^2\}, \{^{(0.2)}a_1^3, ^{(0.6)}a_2^3, ^{(0.8)}a_3^3\})), \\
((x_2^1, x_1^2, x_3^3), (\{^{(1)}a_1^1, ^{(1)}a_2^1, ^{(1)}a_3^1\}, \{^{(0.3)}a_1^2, ^{(0.7)}a_2^2\}, \{^{(0.5)}a_1^3, ^{(0.4)}a_2^3, ^{(0.3)}a_3^3\})), \\
((x_2^1, x_2^2, x_1^3), (\{^{(0.5)}a_1^1, ^{(0.7)}a_2^1, ^{(0.4)}a_3^1\}, \{^{(0.1)}a_1^2, ^{(0.5)}a_2^2\}, \{^{(0.2)}a_1^3, ^{(0.4)}a_2^3, ^{(0.6)}a_3^3\})), \\
((x_2^1, x_2^2, x_3^3), (\{^{(0.2)}a_1^1, ^{(0.1)}a_2^1, ^{(0.6)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.6)}a_2^2\}, \{^{(0.8)}a_1^3, ^{(0.4)}a_2^3, ^{(0.1)}a_3^3\})), \\
((x_3^1, x_1^2, x_1^3), (\{^{(0.8)}a_1^1, ^{(0.6)}a_2^1, ^{(0.4)}a_3^1\}, \{^{(0.6)}a_1^2, ^{(0.3)}a_2^2\}, \{^{(0)}a_1^3, ^{(0.2)}a_2^3, ^{(0)}a_3^3\})), \\
((x_3^1, x_1^2, x_3^3), (\{^{(0.2)}a_1^1, ^{(0.4)}a_2^1, ^{(0.6)}a_3^1\}\{^{(0.7)}a_1^2, ^{(0.4)}a_2^2\}, \{^{(0.4)}a_1^3, ^{(0.6)}a_2^3, ^{(0.3)}a_3^3\})), \\
((x_3^1, x_2^2, x_1^3), (\{^{(0.9)}a_1^1, ^{(0.3)}a_2^1, ^{(1)}a_3^1\}, \{^{(0.2)}a_1^2, ^{(0.2)}a_2^2\}, \{^{(0.4)}a_1^3, ^{(0.2)}a_2^3, ^{(0.4)}a_3^3\})), \\
((x_3^1, x_2^2, x_3^3), (\{^{(0.4)}a_1^1, ^{(0.7)}a_2^1, ^{(0.5)}a_3^1\}, \{^{(0.6)}a_1^2, ^{(0.8)}a_2^2\}, \{^{(0.8)}a_1^3, ^{(0.8)}a_2^3, ^{(0.5)}a_3^3\}))
\end{array} \right\}.$$

Proposition 3.15. *Let* $(\widetilde{\mathcal{H}}_n, \mathbf{Y}), (\widetilde{\mathcal{K}}_n, \mathbf{Z}), (\widetilde{\mathcal{L}}_n, \mathbf{T}) \in \mathfrak{C}_{N_n}\langle \mathfrak{A}, \mathbf{X} \rangle$. *Then, we have the following equalities.*

- **(i):** $(\widetilde{\mathcal{H}}_n, \mathbf{Y}) \diamond (\widetilde{\mathcal{K}}_n, \mathbf{Z}) =_{N_n} (\widetilde{\mathcal{K}}_n, \mathbf{Z}) \diamond (\widetilde{\mathcal{H}}, \mathbf{Y})$ *for each* $\diamond \in \{\Cap_{N_n}, \Cup_{N_n}\}$.
- **(ii):** $(\widetilde{\mathcal{H}}_n, \mathbf{Y}) \diamond ((\widetilde{\mathcal{K}}_n, \mathbf{Z}) \diamond (\widetilde{\mathcal{L}}_n, \mathbf{T})) =_{N_n} ((\widetilde{\mathcal{H}}_n, \mathbf{Y}) \diamond (\widetilde{\mathcal{K}}_n, \mathbf{Z})) \diamond (\widetilde{\mathcal{L}}_n, \mathbf{T})$ *for each* $\diamond \in \{\Cap_{N_n}, \Cup_{N_n}\}$.
- **(iii):** $(\widetilde{\mathcal{H}}_n, \mathbf{Y}) \diamond ((\widetilde{\mathcal{K}}_n, \mathbf{Z}) \circ (\widetilde{\mathcal{L}}_n, \mathbf{T})) =_{N_n} ((\widetilde{\mathcal{H}}_n, \mathbf{Y}) \diamond (\widetilde{\mathcal{K}}_n, \mathbf{Z})) \circ ((\widetilde{\mathcal{H}}_n, \mathbf{Y}) \diamond (\widetilde{\mathcal{L}}_n, \mathbf{T}))$ *for each* $\diamond, \circ \in \{\Cap_{N_n}, \Cup_{N_n}\}$.
- **(iv):** $((\widetilde{\mathcal{H}}_n, \mathbf{Y}) \diamond (\widetilde{\mathcal{K}}_n, \mathbf{Z}))^{r_{N_n}} =_{N_n} (\widetilde{\mathcal{H}}_n, \mathbf{Y})^{r_N} \circ (\widetilde{\mathcal{K}}_n, \mathbf{Z})^{r_{N_n}}$ *for each* $\diamond, \circ \in \{\Cap_{N_n}, \Cup_{N_n}\}$ *and* $\diamond \neq \circ$.

Proof. The proofs are straightforward. □

Proposition 3.16. *Let* $(\widetilde{\mathcal{H}}_n, \mathbf{Y}), (\widetilde{\mathcal{K}}_n, \mathbf{Z}), (\widetilde{\mathcal{L}}_n, \mathbf{T}) \in \mathfrak{C}_{N_n}\langle \mathfrak{A}, \mathbf{X} \rangle$. *Then, we have the following equalities.*

- **(i):** $(\widetilde{\mathcal{H}}_n, \mathbf{Y}) \diamond (\widetilde{\mathcal{K}}_n, \mathbf{Z}) =_{N_n} (\widetilde{\mathcal{K}}_n, \mathbf{Z}) \diamond (\widetilde{\mathcal{H}}, \mathbf{Y})$ *for each* $\diamond \in \{\sqcap_{N_n}, \sqcup_{N_n}\}$.
- **(ii):** $(\widetilde{\mathcal{H}}_n, \mathbf{Y}) \diamond ((\widetilde{\mathcal{K}}_n, \mathbf{Z}) \diamond (\widetilde{\mathcal{L}}_n, \mathbf{T})) =_{N_n} ((\widetilde{\mathcal{H}}_n, \mathbf{Y}) \diamond (\widetilde{\mathcal{K}}_n, \mathbf{Z})) \diamond (\widetilde{\mathcal{L}}_n, \mathbf{T})$ *for each* $\diamond \in \{\sqcap_{N_n}, \sqcup_{N_n}\}$.
- **(iii):** $(\widetilde{\mathcal{H}}_n, \mathbf{Y}) \diamond ((\widetilde{\mathcal{K}}_n, \mathbf{Z}) \circ (\widetilde{\mathcal{L}}_n, \mathbf{T})) =_{N_n} ((\widetilde{\mathcal{H}}_n, \mathbf{Y}) \diamond (\widetilde{\mathcal{K}}_n, \mathbf{Z})) \circ ((\widetilde{\mathcal{H}}_n, \mathbf{Y}) \diamond (\widetilde{\mathcal{L}}_n, \mathbf{T}))$ *for each* $\diamond, \circ \in \{\sqcap_{N_n}, \sqcup_{N_n}\}$.
- **(iv):** $((\widetilde{\mathcal{H}}_n, \mathbf{Y}) \diamond (\widetilde{\mathcal{K}}_n, \mathbf{Z}))^{r_{N_n}} =_{N_n} (\widetilde{\mathcal{H}}_n, \mathbf{Y})^{r_{N_n}} \circ (\widetilde{\mathcal{K}}_n, \mathbf{Z})^{r_{N_n}}$ *for each* $\diamond, \circ \in \{\sqcap_{N_n}, \sqcup_{N_n}\}$ *and* $\diamond \neq \circ$.

Proof. The proofs are straightforward. □

4. **Fuzzy Hypersoft Expert Sets**

In this section, we define the concept of fuzzy hypersoft expert set and give its basic operations with the properties.

Throughout this section, A is a universal set, $X_1, X_2, ..., X_m$ are the pairwise disjoint sets of parameters (i.e., $X_i \cap X_{i'} = \emptyset$ for each $i, i' \in I = \{1, 2, ..., m\}$ and $i \neq i'$), and $\mathbf{X} = \prod_{i \in I} X_i = X_1 \times X_2 \times ... \times X_m$. Also, $\mathcal{E}$ is a set of experts, $\mathcal{O}$ is a set of opinions, $\mathfrak{P} = \mathbf{X} \times \mathcal{E} \times \mathcal{O}$ and $\mathfrak{Q} \subseteq \mathfrak{P}$.

Definition 4.1. A pair $(\widetilde{\mathcal{H}}, \mathfrak{Q})$ is said to be a fuzzy hypersoft expert set over A, where $\widetilde{\mathcal{H}}$ is mapping given by

$$\widetilde{\mathcal{H}} : \mathfrak{Q} \to \mathfrak{F}(A). \tag{17}$$

Note 6. In this chapter, we suppose two-valued opinions only in the set $\mathcal{O}$, i.e., $\mathcal{O} = \{o_1 = agree(1),\ o_2 = disagree(0)\}$. However, the multi-valued opinions may be supposed as well.

Note 7. The set of all fuzzy hypersoft expert set over the universal set A for $\mathfrak{P}$ is denoted by $\mathfrak{C}_E\langle A, \mathfrak{P}\rangle$.

Example 4.2. Consider the problem in Example 2.14. Assume that he/se seeks the opinions of 3 experts with the intention of determining the optimal car(s) to buy. The set of experts is $\mathcal{E} = \{e_1, e_2, e_3\}$ and the set of opinions is $\mathcal{O} = \{o_1 = agree(1),\ o_2 = disagree(0)\}$. For

$$\mathfrak{Q} = \left\{ \begin{array}{l} (((x_1^1, x_1^2, x_1^3), e_1, 1), (((x_1^1, x_1^2, x_1^3), e_2, 1), (((x_1^1, x_1^2, x_1^3), e_3, 1), (((x_1^1, x_1^2, x_3^3), e_1, 1), \\ (((x_1^1, x_1^2, x_3^3), e_2, 1), (((x_1^1, x_1^2, x_3^3), e_3, 1), (((x_1^1, x_1^2, x_1^3), e_1, 0), (((x_1^1, x_1^2, x_1^3), e_2, 0), \\ (((x_1^1, x_1^2, x_1^3), e_3, 0), (((x_1^1, x_1^2, x_3^3), e_1, 0), (((x_1^1, x_1^2, x_3^3), e_2, 0), (((x_1^1, x_1^2, x_3^3), e_3, 0) \end{array} \right\} \subseteq \mathbf{X} \times \mathcal{E} \times \mathcal{O},$$

it is created the following fuzzy hypersoft expert set.

$$(\widetilde{\mathcal{H}}, \mathfrak{Q}) = \left\{ \begin{array}{l} (((x_1^1, x_1^2, x_1^3), e_1, 1), \{ {}^{(0.5)}a_1, {}^{(0.6)} a_2, {}^{(0.3)} a_3\}), \\ (((x_1^1, x_1^2, x_1^3), e_2, 1), \{ {}^{(0.2)}a_1, {}^{(0.4)} a_2, {}^{(0.7)} a_3\}), \\ (((x_1^1, x_1^2, x_1^3), e_3, 1), \{ {}^{(0.4)}a_1, {}^{(0.3)} a_2, {}^{(0.3)} a_3\}), \\ (((x_1^1, x_1^2, x_3^3), e_1, 1), \{ {}^{(0.7)}a_1, {}^{(0.2)} a_2, {}^{(0.4)} a_3\}), \\ (((x_1^1, x_1^2, x_3^3), e_2, 1), \{ {}^{(1)}a_1, {}^{(0.2)} a_2, {}^{(0.9)} a_3\}), \\ (((x_1^1, x_1^2, x_3^3), e_3, 1), \{ {}^{(0.3)}a_1, {}^{(0.3)} a_2, {}^{(0.6)} a_3\}), \\ (((x_1^1, x_1^2, x_1^3), e_1, 0), \{ {}^{(0.4)}a_1, {}^{(0.2)} a_2, {}^{(0.5)} a_3\}), \\ (((x_1^1, x_1^2, x_1^3), e_2, 0), \{ {}^{(0.5)}a_1, {}^{(0.6)} a_2, {}^{(0.3)} a_3\}), \\ (((x_1^1, x_1^2, x_1^3), e_3, 0), \{ {}^{(0.7)}a_1, {}^{(0.7)} a_2, {}^{(0.7)} a_3\}), \\ (((x_1^1, x_1^2, x_3^3), e_1, 0), \{ {}^{(0.1)}a_1, {}^{(0.6)} a_2, {}^{(0.5)} a_3\}), \\ (((x_1^1, x_1^2, x_3^3), e_2, 0), \{ {}^{(0.1)}a_1, {}^{(0.5)} a_2, {}^{(0.3)} a_3\}), \\ (((x_1^1, x_1^2, x_3^3), e_3, 0), \{ {}^{(0.7)}a_1, {}^{(0.6)} a_2, {}^{(0.5)} a_3\}) \end{array} \right\}.$$

Definition 4.3. Let $(\widetilde{\mathcal{H}}, \mathfrak{Q}) \in \mathfrak{C}_E\langle A, \mathfrak{P}\rangle$. Then,

- **(a):** $(\widetilde{\mathcal{H}}, \mathfrak{Q})^1 = \{(q, \widetilde{\mathcal{H}}(q)) : q \in \mathbf{X} \times \mathcal{E} \times \{1\} \subseteq \mathfrak{Q}\}$ is termed to be an agree-hypersoft expert set over A.
- **(b):** $(\widetilde{\mathcal{H}}, \mathfrak{Q})^0 = \{(q, \widetilde{\mathcal{H}}(q)) : q \in \mathbf{X} \times \mathcal{E} \times \{0\} \subseteq \mathfrak{Q}\}$ is termed to be a disagree-hypersoft expert set over A.

From the definition, it is obvious that $(\widetilde{\mathcal{H}}, \mathfrak{Q})^1 \cup (\widetilde{\mathcal{H}}, \mathfrak{Q})^0 = (\widetilde{\mathcal{H}}, \mathfrak{Q})$.

Example 4.4. We consider the fuzzy hypersoft expert set $(\widetilde{\mathcal{H}},\mathfrak{Q})$ given in Example 4.2. Then, the agree-fuzzy hypersoft expert set $(\widetilde{\mathcal{H}},\mathfrak{Q})^1$ and the disagree-fuzzy hypersoft expert set $(\widetilde{\mathcal{H}},\mathfrak{Q})^0$ are respectively

$$(\widetilde{\mathcal{H}},\mathfrak{Q})^1=\left\{\begin{array}{l}(((x_1^1,x_1^2,x_1^3),e_1,1),\{\,^{(0.5)}a_1,^{(0.6)}a_2,^{(0.3)}a_3\}),\\(((x_1^1,x_1^2,x_1^3),e_2,1),\{\,^{(0.2)}a_1,^{(0.4)}a_2,^{(0.7)}a_3\}),\\(((x_1^1,x_1^2,x_1^3),e_3,1),\{\,^{(0.4)}a_1,^{(0.3)}a_2,^{(0.3)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_1,1),\{\,^{(0.7)}a_1,^{(0.2)}a_2,^{(0.4)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_2,1),\{\,^{(1)}a_1,^{(0.2)}a_2,^{(0.9)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_3,1),\{\,^{(0.3)}a_1,^{(0.3)}a_2,^{(0.6)}a_3\})\end{array}\right\}.$$

and

$$(\widetilde{\mathcal{H}},\mathfrak{Q})^0=\left\{\begin{array}{l}(((x_1^1,x_1^2,x_1^3),e_1,0),\{\,^{(0.4)}a_1,^{(0.2)}a_2,^{(0.5)}a_3\}),\\(((x_1^1,x_1^2,x_1^3),e_2,0),\{\,^{(0.5)}a_1,^{(0.6)}a_2,^{(0.3)}a_3\}),\\(((x_1^1,x_1^2,x_1^3),e_3,0),\{\,^{(0.7)}a_1,^{(0.7)}a_2,^{(0.7)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_1,0),\{\,^{(0.1)}a_1,^{(0.6)}a_2,^{(0.5)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_2,0),\{\,^{(0.1)}a_1,^{(0.5)}a_2,^{(0.3)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_3,0),\{\,^{(0.7)}a_1,^{(0.6)}a_2,^{(0.5)}a_3\})\end{array}\right\}.$$

Definition 4.5. Let $(\widetilde{\mathcal{H}},\mathfrak{Q})\in\mathfrak{C}_E\langle A,\mathfrak{P}\rangle$.

- **(a):** If $\widetilde{\mathcal{H}}((\mathbf{x}^\mathbf{I},e,o))=\widehat{\emptyset}$ for each $(\mathbf{x}^\mathbf{I},e,o)\in\mathfrak{Q}$ then it is called a relative null fuzzy hypersoft expert set (with respect to $\mathfrak{Q}$), denoted by $\widehat{\emptyset}^E_\mathfrak{Q}$. If $\mathfrak{Q}=\mathfrak{P}$ then it is said to be a null fuzzy hypersoft expert set and denoted by $\widehat{\emptyset}^E_\mathfrak{P}$.
- **(b):** If $\widetilde{\mathcal{H}}^1((\mathbf{x}^\mathbf{I},e,1))=\widehat{\emptyset}$ for each $(\mathbf{x}^\mathbf{I},e,1)\in\mathfrak{Q}$ then it is termed to be a relative null agree-fuzzy hypersoft expert set (with respect to $\mathfrak{Q}$), denoted by $\widehat{\emptyset}^{E^1}_\mathfrak{Q}$. If $\mathfrak{Q}=\mathfrak{P}$ then it is named a null agree-fuzzy hypersoft expert set and denoted by $\widehat{\emptyset}^{E^1}_\mathfrak{P}$.
- **(c):** If $\widetilde{\mathcal{H}}^0((\mathbf{x}^\mathbf{I},e,0))=\widehat{\emptyset}$ for each $(\mathbf{x}^\mathbf{I},e,0)\in\mathfrak{Q}$ then it is termed to be a relative null disagree-fuzzy hypersoft expert set (with respect to $\mathfrak{Q}$), denoted by $\widehat{\emptyset}^{E^0}_\mathfrak{Q}$. If $\mathfrak{Q}=\mathfrak{P}$ then it is called a null disagree-fuzzy hypersoft expert set and denoted by $\widehat{\emptyset}^{E^0}_\mathfrak{P}$.
- **(d):** If $\widetilde{\mathcal{H}}((\mathbf{x}^\mathbf{I},e,o))=\widehat{A}$ for each $(\mathbf{x}^\mathbf{I},e,o)\in\mathfrak{Q}$ then it is said to be a relative whole fuzzy hypersoft expert set (with respect to $\mathfrak{Q}$), denoted by $\widehat{A}^E_\mathfrak{Q}$. If $\mathfrak{Q}=\mathfrak{P}$ then it is named an absolute fuzzy hypersoft expert set and denoted by $\widehat{A}^E_\mathfrak{P}$.
- **(e):** If $\widetilde{\mathcal{H}}^1((\mathbf{x}^\mathbf{I},e,1))=\widehat{A}$ for each $(\mathbf{x}^\mathbf{I},e,1)\in\mathfrak{Q}$ then it is said to be a relative whole agree-fuzzy hypersoft expert set (with respect to $\mathfrak{Q}$), denoted by $\widehat{A}^{E^1}_\mathfrak{Q}$. If $\mathfrak{Q}=\mathfrak{P}$ then it is named an absolute agree-fuzzy hypersoft expert set and denoted by $\widehat{A}^{E^1}_\mathfrak{P}$.
- **(f):** If $\widetilde{\mathcal{H}}^0((\mathbf{x}^\mathbf{I},e,0))=\widehat{A}$ for each $(\mathbf{x}^\mathbf{I},e,0)\in\mathfrak{Q}$ then it is called a relative whole disagree-fuzzy hypersoft expert set (with respect to $\mathfrak{Q}$), denoted by $\widehat{A}^{E^0}_\mathfrak{Q}$. If $\mathfrak{Q}=\mathfrak{P}$ then it is called an absolute disagree-fuzzy hypersoft expert set and denoted by $\widehat{A}^{E^0}_\mathfrak{P}$.

Example 4.6. The following hypersoft expert sets are given as examples of relative null agree-fuzzy hypersoft expert set (with respect to $\mathfrak{Q}$) and relative whole disagree-fuzzy hypersoft expert set (with respect to $\mathfrak{Q}$) over A.

$$\widehat{\emptyset}_{\mathfrak{Q}}^{E^1} = \left\{ \begin{array}{l} (((x_1^1, x_1^2, x_1^3), e_1, 1), \{ {}^{(0)}a_1, {}^{(0)}a_2, {}^{(0)}a_3\}), \\ (((x_1^1, x_1^2, x_1^3), e_2, 1), \{ {}^{(0)}a_1, {}^{(0)}a_2, {}^{(0)}a_3\}), \\ (((x_1^1, x_1^2, x_1^3), e_3, 1), \{ {}^{(0)}a_1, {}^{(0)}a_2, {}^{(0)}a_3\}), \\ (((x_1^1, x_1^2, x_3^3), e_1, 1), \{ {}^{(0)}a_1, {}^{(0)}a_2, {}^{(0)}a_3\}), \\ (((x_1^1, x_1^2, x_3^3), e_2, 1), \{ {}^{(0)}a_1, {}^{(0)}a_2, {}^{(0)}a_3\}), \\ (((x_1^1, x_1^2, x_3^3), e_3, 1), \{ {}^{(0)}a_1, {}^{(0)}a_2, {}^{(0)}a_3\}) \end{array} \right\},$$

and

$$\widehat{A}_{\mathfrak{Q}}^{E^0} = \left\{ \begin{array}{l} (((x_1^1, x_1^2, x_1^3), e_1, 0), \{ {}^{(1)}a_1, {}^{(1)}a_2, {}^{(1)}a_3\}), \\ (((x_1^1, x_1^2, x_1^3), e_2, 0), \{ {}^{(1)}a_1, {}^{(1)}a_2, {}^{(1)}a_3\}), \\ (((x_1^1, x_1^2, x_1^3), e_3, 0), \{ {}^{(1)}a_1, {}^{(1)}a_2, {}^{(1)}a_3\}), \\ (((x_1^1, x_1^2, x_3^3), e_1, 0), \{ {}^{(1)}a_1, {}^{(1)}a_2, {}^{(1)}a_3\}), \\ (((x_1^1, x_1^2, x_3^3), e_2, 0), \{ {}^{(1)}a_1, {}^{(1)}a_2, {}^{(1)}a_3\}), \\ (((x_1^1, x_1^2, x_3^3), e_3, 0), \{ {}^{(1)}a_1, {}^{(1)}a_2, {}^{(1)}a_3\}) \end{array} \right\}.$$

Definition 4.7. Let $(\widetilde{\mathcal{H}}, \mathfrak{Q}), (\widetilde{\mathcal{K}}, \mathfrak{R}) \in \mathfrak{C}_E\langle A, \mathfrak{P}\rangle$.

- **(a):** $(\widetilde{\mathcal{H}}, \mathfrak{Q})$ is termed to be a fuzzy hypersoft expert subset of $(\widetilde{\mathcal{K}}, \mathfrak{R})$, denoted by $(\widetilde{\mathcal{H}}, \mathfrak{Q}) \sqsubseteq_E (\widetilde{\mathcal{K}}, \mathfrak{R})$, if $\mathfrak{Q} \subseteq \mathfrak{R}$ and $\widetilde{\mathcal{H}}((\mathbf{x}^{\mathbf{I}}, e, o)) \subseteq_f \widetilde{\mathcal{K}}((\mathbf{x}^{\mathbf{I}}, e, o))$ for each $(\mathbf{x}^{\mathbf{I}}, e, o) \in \mathfrak{Q}$.
- **(b):** The fuzzy hypersoft expert sets $(\widetilde{\mathcal{H}}, \mathfrak{Q})$ and $(\widetilde{\mathcal{K}}, \mathfrak{R})$ are named equal, denoted by $(\widetilde{\mathcal{H}}, \mathfrak{Q}) =_E (\widetilde{\mathcal{K}}, \mathfrak{R})$, if $(\widetilde{\mathcal{H}}, \mathfrak{Q}) \sqsubseteq_E (\widetilde{\mathcal{K}}, \mathfrak{R})$ and $(\widetilde{\mathcal{K}}, \mathfrak{R}) \sqsubseteq_E (\widetilde{\mathcal{H}}, \mathfrak{Q})$.

Example 4.8. Consider $(\widetilde{\mathcal{H}}, \mathfrak{Q})$ in Example 4.2 and the $(\widetilde{\mathcal{H}}, \mathfrak{Q})^1$ and $(\widetilde{\mathcal{H}}, \mathfrak{Q})^0$ Example 4.4. It is obvious that $(\widetilde{\mathcal{H}}, \mathfrak{Q})^1$ and $(\widetilde{\mathcal{H}}, \mathfrak{Q})^0$ are fuzzy hypersoft expert subsets of $(\mathcal{H}, \mathfrak{Q})$. Moreover, $\widehat{\emptyset}_{\mathfrak{Q}}^{E^1}$ is fuzzy hypersoft expert subset of $(\widetilde{\mathcal{H}}, \mathfrak{Q})$.

Definition 4.9. Let $(\widetilde{\mathcal{H}}, \mathfrak{Q}) \in \mathfrak{C}_E\langle A, \mathfrak{P}\rangle$. Then, the relative complement of fuzzy hypersoft expert set $(\widetilde{\mathcal{H}}, \mathfrak{Q})$, denoted by $(\widetilde{\mathcal{H}}, \mathfrak{Q})^{r_E}$, is defined as

$$(\widetilde{\mathcal{H}}, \mathfrak{Q})^{r_E} = (\widetilde{\mathcal{H}}^r, \mathfrak{Q}), \tag{18}$$

where $\widetilde{\mathcal{H}}^r((\mathbf{x}^{\mathbf{I}}, e, o))$ is the fuzzy complement of $\widetilde{\mathcal{H}}((\mathbf{x}^{\mathbf{I}}, e, o))$ for each $(\mathbf{x}^{\mathbf{I}}, e, o) \in \mathfrak{Q}$.

Example 4.10. We consider the fuzzy hypersoft expert set $(\widetilde{\mathcal{H}},\mathfrak{Q})$ in Example 4.2. Then, the relative complement of fuzzy hypersoft expert set $(\widetilde{\mathcal{H}},\mathfrak{Q})$ is

$$(\widetilde{\mathcal{H}},\mathfrak{Q})^{r_E}=\left\{\begin{array}{l}(((x_1^1,x_1^2,x_1^3),e_1,1),\{\,^{(0.5)}a_1,^{(0.4)}a_2,^{(0.7)}a_3\}),\\(((x_1^1,x_1^2,x_1^3),e_2,1),\{\,^{(0.8)}a_1,^{(0.6)}a_2,^{(0.3)}a_3\}),\\(((x_1^1,x_1^2,x_1^3),e_3,1),\{\,^{(0.6)}a_1,^{(0.7)}a_2,^{(0.7)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_1,1),\{\,^{(0.3)}a_1,^{(0.8)}a_2,^{(0.6)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_2,1),\{\,^{(0)}a_1,^{(0.8)}a_2,^{(0.1)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_3,1),\{\,^{(0.7)}a_1,^{(0.7)}a_2,^{(0.4)}a_3\}),\\(((x_1^1,x_1^2,x_1^3),e_1,0),\{\,^{(0.6)}a_1,^{(0.8)}a_2,^{(0.5)}a_3\}),\\(((x_1^1,x_1^2,x_1^3),e_2,0),\{\,^{(0.5)}a_1,^{(0.4)}a_2,^{(0.7)}a_3\}),\\(((x_1^1,x_1^2,x_1^3),e_3,0),\{\,^{(0.3)}a_1,^{(0.3)}a_2,^{(0.3)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_1,0),\{\,^{(0.9)}a_1,^{(0.4)}a_2,^{(0.5)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_2,0),\{\,^{(0.9)}a_1,^{(0.5)}a_2,^{(0.7)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_3,0),\{\,^{(0.3)}a_1,^{(0.4)}a_2,^{(0.5)}a_3\})\end{array}\right\}.$$

Proposition 4.11. *Let $(\widetilde{\mathcal{H}},\mathfrak{Q})\in\mathfrak{C}_E\langle A,\mathfrak{P}\rangle$. Then, we have the following.*

(i): $((\widetilde{\mathcal{H}},\mathfrak{Q})^{r_E})^{r_E}=_E(\widetilde{\mathcal{H}},\mathfrak{Q})$.
(ii): $(\widehat{A}^E_{\mathfrak{Q}})^{r_E}=_E(\widehat{\emptyset}^E_{\mathfrak{Q}})$.
(iii): $(\widehat{\emptyset}^E_{\mathfrak{Q}})^{r_E}=_E(\widehat{A}^E_{\mathfrak{Q}})$.

Proof. The proofs are straightforward. □

Definition 4.12. Let $(\widetilde{\mathcal{H}},\mathfrak{Q}),(\widetilde{\mathcal{K}},\mathfrak{R})\in\mathfrak{C}_E\langle A,\mathfrak{P}\rangle$. Then, the restricted intersection of fuzzy hypersoft expert sets $(\widetilde{\mathcal{H}},\mathfrak{Q})$ and $(\widetilde{\mathcal{K}},\mathfrak{R})$ is denoted and defined by $(\widetilde{\mathcal{L}},\mathfrak{S})=(\widetilde{\mathcal{H}},\mathfrak{Q})\Cap_E(\widetilde{\mathcal{K}},\mathfrak{R})$ where $\mathfrak{S}=\mathfrak{Q}\cap\mathfrak{R}$ and

$$\widetilde{\mathcal{L}}((\mathbf{x^I},e,o))=\widetilde{\mathcal{H}}((\mathbf{x^I},e,o))\cap_f\widetilde{\mathcal{K}}((\mathbf{x^I},e,o)) \tag{19}$$

for each $(\mathbf{x^I},e,o)\in\mathfrak{S}$.

Definition 4.13. Let $(\widetilde{\mathcal{H}},\mathfrak{Q}),(\widetilde{\mathcal{K}},\mathfrak{R})\in\mathfrak{C}_E\langle A,\mathfrak{P}\rangle$. Then, the extended intersection of fuzzy hypersoft expert sets $(\widetilde{\mathcal{H}},\mathfrak{Q})$ and $(\widetilde{\mathcal{K}},\mathfrak{R})$ is denoted and defined by $(\widetilde{\mathcal{L}},\mathfrak{S})=(\widetilde{\mathcal{H}},\mathfrak{Q})\sqcap_E(\widetilde{\mathcal{K}},\mathfrak{R})$ where $\mathfrak{S}=\mathfrak{Q}\cup\mathfrak{R}$ and

$$\widetilde{\mathcal{L}}((\mathbf{x^I},e,o))=\begin{cases}\widetilde{\mathcal{H}}((\mathbf{x^I},e,o)), & \text{if } (\mathbf{x^I},e,o)\in\mathfrak{Q},\\ \widetilde{\mathcal{K}}((\mathbf{x^I},e,o)), & \text{if } (\mathbf{x^I},e,o)\in\mathfrak{R},\\ \widetilde{\mathcal{H}}((\mathbf{x^I},e,o))\cap_f\widetilde{\mathcal{K}}((\mathbf{x^I},e,o)), & \text{if } (\mathbf{x^I},e,o)\in\mathfrak{Q}\cap\mathfrak{R},\end{cases} \tag{20}$$

for each $(\mathbf{x^I},e,o)\in\mathfrak{S}$.

Example 4.14. Consider the fuzzy hypersoft expert set $(\widetilde{\mathcal{H}},\mathfrak{Q})$ in Example 4.2. Also, we assume that

$$\mathfrak{R}=\left\{\begin{array}{l}(((x_1^1,x_1^2,x_2^3),e_1,1),(((x_1^1,x_1^2,x_2^3),e_3,1),(((x_1^1,x_1^2,x_3^3),e_1,1),(((x_1^1,x_1^2,x_3^3),e_2,1),\\(((x_1^1,x_1^2,x_2^3),e_1,0),(((x_1^1,x_1^2,x_2^3),e_3,0),(((x_1^1,x_1^2,x_3^3),e_1,0),(((x_1^1,x_1^2,x_3^3),e_2,0)\end{array}\right\}\subseteq\mathbf{X}\times\mathcal{E}\times\mathcal{O},$$

and

$$(\widetilde{\mathcal{K}},\mathfrak{R})=\left\{\begin{array}{l}(((x_1^1,x_1^2,x_2^3),e_1,1),\{\,^{(0.4)}a_1,^{(0)}a_2,^{(0.7)}a_3\}),\\(((x_1^1,x_1^2,x_2^3),e_3,1),\{\,^{(0.2)}a_1,^{(0.6)}a_2,^{(0.3)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_1,1),\{\,^{(0.7)}a_1,^{(0.4)}a_2,^{(0.4)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_2,1),\{\,^{(0.1)}a_1,^{(0.1)}a_2,^{(0.5)}a_3\}),\\(((x_1^1,x_1^2,x_2^3),e_1,0),\{\,^{(0.5)}a_1,^{(0.6)}a_2,^{(0.5)}a_3\}),\\(((x_1^1,x_1^2,x_2^3),e_3,0),\{\,^{(0.7)}a_1,^{(0.3)}a_2,^{(0.3)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_1,0),\{\,^{(0.3)}a_1,^{(0)}a_2,^{(0.4)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_2,0),\{\,^{(0.2)}a_1,^{(0.3)}a_2,^{(0.5)}a_3\})\end{array}\right\}.$$

Then, the restricted intersection and extended intersection of fuzzy hypersoft expert sets $(\widetilde{\mathcal{H}},\mathfrak{Q})$ and $(\widetilde{\mathcal{K}},\mathfrak{R})$ are respectively

$$(\widetilde{\mathcal{H}},\mathfrak{Q})\Cap_E(\widetilde{\mathcal{K}},\mathfrak{R})=\left\{\begin{array}{l}(((x_1^1,x_1^2,x_3^3),e_1,1),\{\,^{(0.7)}a_1,^{(0.2)}a_2,^{(0.4)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_2,1),\{\,^{(0.1)}a_1,^{(0.1)}a_2,^{(0.5)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_1,0),\{\,^{(0.1)}a_1,^{(0)}a_2,^{(0.4)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_2,0),\{\,^{(0.1)}a_1,^{(0.3)}a_2,^{(0.3)}a_3\})\end{array}\right\},$$

and

$$(\widetilde{\mathcal{H}},\mathfrak{Q})\sqcap_E(\widetilde{\mathcal{K}},\mathfrak{R})=\left\{\begin{array}{l}(((x_1^1,x_1^2,x_1^3),e_1,1),\{\,^{(0.5)}a_1,^{(0.6)}a_2,^{(0.3)}a_3\}),\\(((x_1^1,x_1^2,x_1^3),e_2,1),\{\,^{(0.2)}a_1,^{(0.4)}a_2,^{(0.7)}a_3\}),\\(((x_1^1,x_1^2,x_1^3),e_3,1),\{\,^{(0.4)}a_1,^{(0.3)}a_2,^{(0.3)}a_3\}),\\(((x_1^1,x_1^2,x_2^3),e_1,1),\{\,^{(0.4)}a_1,^{(0)}a_2,^{(0.7)}a_3\}),\\(((x_1^1,x_1^2,x_2^3),e_3,1),\{\,^{(0.2)}a_1,^{(0.6)}a_2,^{(0.3)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_1,1),\{\,^{(0.7)}a_1,^{(0.2)}a_2,^{(0.4)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_2,1),\{\,^{(0.1)}a_1,^{(0.1)}a_2,^{(0.5)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_3,1),\{\,^{(0.3)}a_1,^{(0.3)}a_2,^{(0.6)}a_3\}),\\(((x_1^1,x_1^2,x_1^3),e_1,0),\{\,^{(0.4)}a_1,^{(0.2)}a_2,^{(0.5)}a_3\}),\\(((x_1^1,x_1^2,x_1^3),e_2,0),\{\,^{(0.5)}a_1,^{(0.6)}a_2,^{(0.3)}a_3\}),\\(((x_1^1,x_1^2,x_1^3),e_3,0),\{\,^{(0.7)}a_1,^{(0.7)}a_2,^{(0.7)}a_3\}),\\(((x_1^1,x_1^2,x_2^3),e_1,0),\{\,^{(0.5)}a_1,^{(0.6)}a_2,^{(0.5)}a_3\}),\\(((x_1^1,x_1^2,x_2^3),e_3,0),\{\,^{(0.7)}a_1,^{(0.3)}a_2,^{(0.3)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_1,0),\{\,^{(0.1)}a_1,^{(0)}a_2,^{(0.4)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_2,0),\{\,^{(0.1)}a_1,^{(0.3)}a_2,^{(0.3)}a_3\}),\\(((x_1^1,x_1^2,x_3^3),e_3,0),\{\,^{(0.7)}a_1,^{(0.6)}a_2,^{(0.5)}a_3\})\end{array}\right\}.$$

Definition 4.15. Let $(\widetilde{\mathcal{H}},\mathfrak{Q}),(\widetilde{\mathcal{K}},\mathfrak{R}) \in \mathfrak{C}_E\langle A,\mathfrak{P}\rangle$. Then, the restricted union of fuzzy hypersoft expert sets $(\widetilde{\mathcal{H}},\mathfrak{Q})$ and $(\widetilde{\mathcal{K}},\mathfrak{R})$ is denoted and defined by $(\widetilde{\mathcal{L}},\mathfrak{S}) = (\widetilde{\mathcal{H}},\mathfrak{Q}) \Cup_E (\widetilde{\mathcal{K}},\mathfrak{R})$ where $\mathfrak{S} = \mathfrak{Q}\cap\mathfrak{R}$ and

$$\widetilde{\mathcal{L}}((\mathbf{x^I},e,o)) = \widetilde{\mathcal{H}}((\mathbf{x^I},e,o)) \cup_f \widetilde{\mathcal{K}}((\mathbf{x^I},e,o)) \tag{21}$$

for each $(\mathbf{x^I},e,o) \in \mathfrak{S}$.

Definition 4.16. Let $(\widetilde{\mathcal{H}},\mathfrak{Q}),(\widetilde{\mathcal{K}},\mathfrak{R}) \in \mathfrak{C}_E\langle A,\mathfrak{P}\rangle$. Then, the extended union of fuzzy hypersoft expert sets $(\widetilde{\mathcal{H}},\mathfrak{Q})$ and $(\widetilde{\mathcal{K}},\mathfrak{R})$ is denoted and defined by $(\widetilde{\mathcal{L}},\mathfrak{S}) = (\widetilde{\mathcal{H}},\mathfrak{Q}) \sqcup_E (\widetilde{\mathcal{K}},\mathfrak{R})$ where $\mathfrak{S} = \mathfrak{Q}\cup\mathfrak{R}$ and

$$\widetilde{\mathcal{L}}((\mathbf{x^I},e,o)) = \begin{cases} \widetilde{\mathcal{H}}((\mathbf{x^I},e,o)), & \text{if} \quad (\mathbf{x^I},e,o)\in\mathfrak{Q}, \\ \widetilde{\mathcal{K}}((\mathbf{x^I},e,o)), & \text{if} \quad (\mathbf{x^I},e,o)\in\mathfrak{R}, \\ \widetilde{\mathcal{H}}((\mathbf{x^I},e,o)) \cup_f \widetilde{\mathcal{K}}((\mathbf{x^I},e,o)), & \text{if} \quad (\mathbf{x^I},e,o)\in\mathfrak{Q}\cap\mathfrak{R}, \end{cases} \tag{22}$$

for each $(\mathbf{x^I},e,o) \in \mathfrak{S}$.

Example 4.17. We consider the fuzzy hypersoft expert sets $(\widetilde{\mathcal{H}},\mathfrak{Q})$ and $(\widetilde{\mathcal{K}},\mathfrak{R})$ in Examples 4.2 and 4.14, respectively. Then, the restricted union and extended union of fuzzy hypersoft expert sets $(\widetilde{\mathcal{H}},\mathfrak{Q})$ and $(\widetilde{\mathcal{K}},\mathfrak{R})$ are respectively

$$(\widetilde{\mathcal{H}},\mathfrak{Q}) \Cup_E (\widetilde{\mathcal{K}},\mathfrak{R}) = \left\{ \begin{array}{l} (((x_1^1,x_1^2,x_3^3),e_1,1),\{\,^{(0.7)}a_1,^{(0.4)}a_2,^{(0.4)}a_3\}), \\ (((x_1^1,x_1^2,x_3^3),e_2,1),\{\,^{(1)}a_1,^{(0.2)}a_2,^{(0.9)}a_3\}), \\ (((x_1^1,x_1^2,x_3^3),e_1,0),\{\,^{(0.3)}a_1,^{(0.6)}a_2,^{(0.5)}a_3\}), \\ (((x_1^1,x_1^2,x_3^3),e_2,0),\{\,^{(0.2)}a_1,^{(0.5)}a_2,^{(0.5)}a_3\}) \end{array} \right\},$$

and

$$(\widetilde{\mathcal{H}},\mathfrak{Q}) \sqcup_E (\widetilde{\mathcal{K}},\mathfrak{R}) = \left\{ \begin{array}{l} (((x_1^1,x_1^2,x_1^3),e_1,1),\{\,^{(0.5)}a_1,^{(0.6)}a_2,^{(0.3)}a_3\}), \\ (((x_1^1,x_1^2,x_1^3),e_2,1),\{\,^{(0.2)}a_1,^{(0.4)}a_2,^{(0.7)}a_3\}), \\ (((x_1^1,x_1^2,x_1^3),e_3,1),\{\,^{(0.4)}a_1,^{(0.3)}a_2,^{(0.3)}a_3\}), \\ (((x_1^1,x_1^2,x_2^3),e_1,1),\{\,^{(0.4)}a_1,^{(0)}a_2,^{(0.7)}a_3\}), \\ (((x_1^1,x_1^2,x_2^3),e_3,1),\{\,^{(0.2)}a_1,^{(0.6)}a_2,^{(0.3)}a_3\}), \\ (((x_1^1,x_1^2,x_3^3),e_1,1),\{\,^{(0.7)}a_1,^{(0.4)}a_2,^{(0.4)}a_3\}), \\ (((x_1^1,x_1^2,x_3^3),e_2,1),\{\,^{(1)}a_1,^{(0.2)}a_2,^{(0.9)}a_3\}), \\ (((x_1^1,x_1^2,x_3^3),e_3,1),\{\,^{(0.3)}a_1,^{(0.3)}a_2,^{(0.6)}a_3\}), \\ (((x_1^1,x_1^2,x_1^3),e_1,0),\{\,^{(0.4)}a_1,^{(0.2)}a_2,^{(0.5)}a_3\}), \\ (((x_1^1,x_1^2,x_1^3),e_2,0),\{\,^{(0.5)}a_1,^{(0.6)}a_2,^{(0.3)}a_3\}), \\ (((x_1^1,x_1^2,x_1^3),e_3,0),\{\,^{(0.7)}a_1,^{(0.7)}a_2,^{(0.7)}a_3\}), \\ (((x_1^1,x_1^2,x_2^3),e_1,0),\{\,^{(0.5)}a_1,^{(0.6)}a_2,^{(0.5)}a_3\}), \\ (((x_1^1,x_1^2,x_2^3),e_3,0),\{\,^{(0.7)}a_1,^{(0.3)}a_2,^{(0.3)}a_3\}), \\ (((x_1^1,x_1^2,x_3^3),e_1,0),\{\,^{(0.3)}a_1,^{(0.6)}a_2,^{(0.5)}a_3\}), \\ (((x_1^1,x_1^2,x_3^3),e_2,0),\{\,^{(0.2)}a_1,^{(0.5)}a_2,^{(0.5)}a_3\}), \\ (((x_1^1,x_1^2,x_3^3),e_3,0),\{\,^{(0.7)}a_1,^{(0.6)}a_2,^{(0.5)}a_3\}) \end{array} \right\}.$$

Proposition 4.18. *Let $(\widetilde{\mathcal{H}},\mathfrak{Q}),(\widetilde{\mathcal{K}},\mathfrak{R}),(\widetilde{\mathcal{L}},\mathfrak{S}) \in \mathfrak{C}_E\langle A,\mathfrak{P}\rangle$. Then, the following properties are acquired.*

(i): $(\widetilde{\mathcal{H}},\mathfrak{Q})\diamond(\widetilde{\mathcal{K}},\mathfrak{R}) =_E (\widetilde{\mathcal{K}},\mathfrak{R})\diamond(\widetilde{\mathcal{H}},\mathfrak{Q})$ *for each* $\diamond\in\{\Cap_E,\Cup_E\}$.

(ii): $(\widetilde{\mathcal{H}},\mathfrak{Q})\diamond((\widetilde{\mathcal{K}},\mathfrak{R})\diamond(\widetilde{\mathcal{L}},\mathfrak{S})) =_E ((\widetilde{\mathcal{H}},\mathfrak{Q})\diamond(\widetilde{\mathcal{K}},\mathfrak{R}))\diamond(\widetilde{\mathcal{L}},\mathfrak{S})$ *for each* $\diamond\in\{\Cap_E,\Cup_E\}$.

(iii): $(\widetilde{\mathcal{H}},\mathfrak{Q})\diamond((\widetilde{\mathcal{K}},\mathfrak{R})\circ(\widetilde{\mathcal{L}},\mathfrak{S})) =_E ((\widetilde{\mathcal{H}},\mathfrak{Q})\diamond(\widetilde{\mathcal{K}},\mathfrak{R}))\circ((\widetilde{\mathcal{H}},\mathfrak{Q})\diamond(\widetilde{\mathcal{L}},\mathfrak{S}))$ *for each* $\diamond,\circ\in\{\Cap_E,\Cup_E\}$.

(iv): $((\widetilde{\mathcal{H}},\mathfrak{Q})\diamond(\widetilde{\mathcal{K}},\mathfrak{R}))^{r_E} =_E (\widetilde{\mathcal{H}},\mathfrak{Q})^{r_E}\circ(\widetilde{\mathcal{K}},\mathfrak{R})^{r_E}$ *for each* $\diamond,\circ\in\{\Cap_E,\Cup_E\}$ *and* $\diamond\neq\circ$.

Proof. The proofs are straightforward. □

Proposition 4.19. *Let* $(\widetilde{\mathcal{H}},\mathfrak{Q}),(\widetilde{\mathcal{K}},\mathfrak{R}),(\widetilde{\mathcal{L}},\mathfrak{S})\in\mathfrak{C}_E\langle A,\mathfrak{P}\rangle$. *Then, the following properties are acquired.*

(i): $(\widetilde{\mathcal{H}},\mathfrak{Q})\diamond(\widetilde{\mathcal{K}},\mathfrak{R}) =_E (\widetilde{\mathcal{K}},\mathfrak{R})\diamond(\widetilde{\mathcal{H}},\mathfrak{Q})$ *for each* $\diamond\in\{\sqcap_E,\sqcup_E\}$.

(ii): $(\widetilde{\mathcal{H}},\mathfrak{Q})\diamond((\widetilde{\mathcal{K}},\mathfrak{R})\diamond(\widetilde{\mathcal{L}},\mathfrak{S})) =_E ((\widetilde{\mathcal{H}},\mathfrak{Q})\diamond(\widetilde{\mathcal{K}},\mathfrak{R}))\diamond(\widetilde{\mathcal{L}},\mathfrak{S})$ *for each* $\diamond\in\{\sqcap_E,\sqcup_E\}$.

(iii): $(\widetilde{\mathcal{H}},\mathfrak{Q})\diamond((\widetilde{\mathcal{K}},\mathfrak{R})\circ(\widetilde{\mathcal{L}},\mathfrak{S})) =_E ((\widetilde{\mathcal{H}},\mathfrak{Q})\diamond(\widetilde{\mathcal{K}},\mathfrak{R}))\circ((\widetilde{\mathcal{H}},\mathfrak{Q})\diamond(\widetilde{\mathcal{L}},\mathfrak{S}))$ *for each* $\diamond,\circ\in\{\sqcap_E,\sqcup_E\}$.

(iv): $((\widetilde{\mathcal{H}},\mathfrak{Q})\diamond(\widetilde{\mathcal{K}},\mathfrak{R}))^{r_E} =_E (\widetilde{\mathcal{H}},\mathfrak{Q})^{r_E}\circ(\widetilde{\mathcal{K}},\mathfrak{R})^{r_E}$ *for each* $\diamond,\circ\in\{\sqcap_E,\sqcup_E\}$ *and* $\diamond\neq\circ$.

Proof. The proofs are straightforward. □

5. n-ary Fuzzy Hypersoft Expert Sets

In this section, we initiate the theory of n-ary fuzzy hypersoft expert sets including both n-ary fuzzy hypersoft sets and fuzzy hypersoft expert sets.

Throughout this section, $\{A_j : j\in J=\{1,2,...,n\}\}$ is a collection of universal sets such that $A_j\cap A_{j'}=\emptyset$ for each $j,j'\in J=\{1,2,...,n\}$ and $j\neq j'$. $\mathfrak{F}(\mathfrak{A})=\prod_{j\in J}\mathfrak{F}(A_j)=\mathfrak{F}(A_1)\times\mathfrak{F}(A_2)\times...\times\mathfrak{F}(A_n)$ where $\mathfrak{F}(A_j)$ denotes the set of all fuzzy sets in A_j. Also, $X_1,X_2,...,X_m$ are the pairwise disjoint sets of parameters (i.e., $X_i\cap X_{i'}=\emptyset$ for each $i,i'\in I=\{1,2,...,m\}$ and $i\neq i'$), $\mathbf{X}=\prod_{i\in I}X_i=X_1\times X_2\times...\times X_m$, $\mathcal{E}$ is a set of experts, $\mathcal{O}$ is a set of opinions, $\mathfrak{P}=\mathbf{X}\times\mathcal{E}\times\mathcal{O}$ and $\mathfrak{Q}\subseteq\mathfrak{P}$.

Definition 5.1. A pair $(\widetilde{\mathcal{H}}_n,\mathfrak{Q})$ is called an n-ary fuzzy hypersoft expert set over $\mathfrak{A}=\{A_1,A_2,...,A_n\}$, where $\widetilde{\mathcal{H}}_n$ is mapping given by

$$\widetilde{\mathcal{H}}_n:\mathfrak{Q}\to\mathfrak{F}(\mathfrak{A}). \tag{23}$$

Simply, an n-ary fuzzy hypersoft expert set can be given as

$$\begin{aligned}(\widetilde{\mathcal{H}}_n,\mathfrak{Q}) &= \{((\mathbf{x^I},e,o),\widetilde{\mathcal{H}}_n((\mathbf{x^I},e,o))) : (\mathbf{x^I},e,o)\in\mathfrak{Q} \text{ and } \widetilde{\mathcal{H}}_n((\mathbf{x^I},e,o))\in\mathfrak{A}\}\\ &= \left\{\left((\mathbf{x^I},e,o),\begin{pmatrix}\{^{(\mu_{\widetilde{\mathcal{H}}_n((\mathbf{x^I},e,o))}(a^1))}a^1 : a^1\in A_1\},\\ \{^{(\mu_{\widetilde{\mathcal{H}}_n((\mathbf{x^I},e,o))}(a^2))}a^2 : a^2\in A_2\},\\ .\\ .\\ .\\ \{^{(\mu_{\widetilde{\mathcal{H}}_n((\mathbf{x^I},e,o))}(a^m))}a^m : a^m\in A_m\}\end{pmatrix}\right) : (\mathbf{x^I},e,o)\in\mathfrak{Q}\right\},\end{aligned}$$

where $\widetilde{\mathcal{H}}_{n(j)}((\mathbf{x^I}, e, o)) = \{^{(\mu_{\widetilde{\mathcal{H}}_n((\mathbf{x^I},e,o))}(a^j))}a^j : a^j \in A_j\}$ for $j = 1, 2, ..., n$ and it is termed to be an A_j-part of $\widetilde{\mathcal{H}}_n((\mathbf{x^I}, e, o))$.

Especially, if $n = 2, 3, 4$ and 5 then it is called a binary fuzzy hypersoft expert set, ternary fuzzy hypersoft expert set, quaternary fuzzy hypersoft expert set, quinary fuzzy hypersoft expert set, respectively.

Note 8. The set of all n-ary fuzzy hypersoft expert sets over $\mathfrak{A} = \{A_1, A_2, ..., A_n\}$ for $\mathfrak{P}$ is denoted by $\mathfrak{C}_{EN_n}\langle\mathfrak{A}, \mathfrak{P}\rangle$.

Example 5.2. Consider the problem in Examples 3.2 and 4.2. Assume that he/se seeks the opinions of 3 experts with the intention of determining the optimal car(s) for each segment (i.e., A_1, A_2 and A_3), simultaneously. The set of experts is $\mathcal{E} = \{e_1, e_2, e_3\}$ and the set of opinions is $\mathcal{O} = \{o_1 = agree(1),\ o_2 = disagree(0)\}$. For

$$\mathfrak{Q} = \left\{ \begin{array}{l} (((x_1^1, x_1^2, x_1^3), e_1, 1), (((x_1^1, x_1^2, x_1^3), e_2, 1), (((x_1^1, x_1^2, x_1^3), e_3, 1), (((x_1^1, x_1^2, x_3^3), e_1, 1), \\ (((x_1^1, x_1^2, x_3^3), e_2, 1), (((x_1^1, x_1^2, x_3^3), e_3, 1), (((x_1^1, x_1^2, x_1^3), e_1, 0), (((x_1^1, x_1^2, x_1^3), e_2, 0), \\ (((x_1^1, x_1^2, x_1^3), e_3, 0), (((x_1^1, x_1^2, x_3^3), e_1, 0), (((x_1^1, x_1^2, x_3^3), e_2, 0), (((x_1^1, x_1^2, x_3^3), e_3, 0) \end{array} \right\} \subseteq \mathbf{X} \times \mathcal{E} \times \mathcal{O},$$

it is created the following ternary fuzzy hypersoft expert set.

$$(\widetilde{\mathcal{H}}_3, \mathfrak{Q}) = \left\{ \begin{array}{l} (((x_1^1, x_1^2, x_1^3), e_1, 1), (\{^{(0.5)}a_1^1, ^{(0.6)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.5)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.6)}a_2^3, ^{(0.2)}a_3^3\})), \\ (((x_1^1, x_1^2, x_1^3), e_2, 1), (\{^{(0.2)}a_1^1, ^{(0.4)}a_2^1, ^{(0.7)}a_3^1\}, \{^{(0.3)}a_1^2, ^{(0.6)}a_2^2\}, \{^{(0.4)}a_1^3, ^{(0.4)}a_2^3, ^{(0.1)}a_3^3\})), \\ (((x_1^1, x_1^2, x_1^3), e_3, 1), (\{^{(0.4)}a_1^1, ^{(0.3)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.7)}a_1^2, ^{(0.2)}a_2^2\}, \{^{(0.4)}a_1^3, ^{(0.3)}a_2^3, ^{(0.5)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_1, 1), (\{^{(0.7)}a_1^1, ^{(0.2)}a_2^1, ^{(0.4)}a_3^1\}, \{^{(0.3)}a_1^2, ^{(0.6)}a_2^2\}, \{^{(0.3)}a_1^3, ^{(0.4)}a_2^3, ^{(1)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_2, 1), (\{^{(1)}a_1^1, ^{(0.2)}a_2^1, ^{(0.9)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.4)}a_2^2\}, \{^{(0.5)}a_1^3, ^{(0.2)}a_2^3, ^{(0.3)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_3, 1), (\{^{(0.3)}a_1^1, ^{(0.3)}a_2^1, ^{(0.6)}a_3^1\}, \{^{(0.5)}a_1^2, ^{(0.3)}a_2^2\}, \{^{(0.4)}a_1^3, ^{(0.2)}a_2^3, ^{(0.6)}a_3^3\})), \\ (((x_1^1, x_1^2, x_1^3), e_1, 0), (\{^{(0.4)}a_1^1, ^{(0.2)}a_2^1, ^{(0.5)}a_3^1\}, \{^{(0.6)}a_1^2, ^{(0.2)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.2)}a_2^3, ^{(0.3)}a_3^3\})), \\ (((x_1^1, x_1^2, x_1^3), e_2, 0), (\{^{(0.5)}a_1^1, ^{(0.6)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.3)}a_1^2, ^{(0.3)}a_2^2\}, \{^{(0.4)}a_1^3, ^{(0.5)}a_2^3, ^{(0.1)}a_3^3\})), \\ (((x_1^1, x_1^2, x_1^3), e_3, 0), (\{^{(0.7)}a_1^1, ^{(0.7)}a_2^1, ^{(0.7)}a_3^1\}, \{^{(0.5)}a_1^2, ^{(0.2)}a_2^2\}, \{^{(0)}a_1^3, ^{(0.4)}a_2^3, ^{(0.3)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_1, 0), (\{^{(0.1)}a_1^1, ^{(0.6)}a_2^1, ^{(0.5)}a_3^1\}, \{^{(0.7)}a_1^2, ^{(0.4)}a_2^2\}, \{^{(0.2)}a_1^3, ^{(0.5)}a_2^3, ^{(0.1)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_2, 0), (\{^{(0.1)}a_1^1, ^{(0.5)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.2)}a_1^2, ^{(0)}a_2^2\}, \{^{(1)}a_1^3, ^{(0.5)}a_2^3, ^{(0.3)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_3, 0), (\{^{(0.7)}a_1^1, ^{(0.6)}a_2^1, ^{(0.5)}a_3^1\}, \{^{(0.7)}a_1^2, ^{(0.7)}a_2^2\}, \{^{(0.7)}a_1^3, ^{(0.2)}a_2^3, ^{(0.7)}a_3^3\})) \end{array} \right\}.$$

Definition 5.3. Let $(\widetilde{\mathcal{H}}_n, \mathfrak{Q}) \in \mathfrak{C}_{EN_n}\langle\mathfrak{A}, \mathfrak{P}\rangle$. Then,

- **(a):** $(\widetilde{\mathcal{H}}_n, \mathfrak{Q})^1 = \{(q, \widetilde{\mathcal{H}}_n(q)) : q \in \mathbf{X} \times \mathcal{E} \times \{1\} \subseteq \mathfrak{Q}\}$ is named to be an agree n-ary fuzzy hypersoft expert set over $\mathfrak{A}$.
- **(b):** $(\widetilde{\mathcal{H}}_n, \mathfrak{Q})^0 = \{(q, \widetilde{\mathcal{H}}_n(q)) : q \in \mathbf{X} \times \mathcal{E} \times \{0\} \subseteq \mathfrak{Q}\}$ is named to be a disagree n-ary fuzzy hypersoft expert set over $\mathfrak{A}$.

From the definition, it is clear that $(\widetilde{\mathcal{H}}_n, \mathfrak{Q})^1 \cup (\widetilde{\mathcal{H}}_n, \mathfrak{Q})^0 = (\widetilde{\mathcal{H}}_n, \mathfrak{Q})$.

Example 5.4. We consider the ternary fuzzy hypersoft expert set $(\widetilde{\mathcal{H}}_3, \mathfrak{Q}) \in \mathfrak{C}_{EN_3}\langle\mathfrak{A}, \mathfrak{P}\rangle$ given in Example 5.2. Then, the agree ternary fuzzy hypersoft expert set $(\widetilde{\mathcal{H}}_3, \mathfrak{Q})^1$ and the disagree ternary fuzzy hypersoft expert set $(\widetilde{\mathcal{H}}_3, \mathfrak{Q})^0$ are respectively

$$(\widetilde{\mathcal{H}}_3, \mathfrak{Q})^1 = \left\{ \begin{array}{l} (((x_1^1, x_1^2, x_1^3), e_1, 1), (\{^{(0.5)}a_1^1, ^{(0.6)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.5)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.6)}a_2^3, ^{(0.2)}a_3^3\})), \\ (((x_1^1, x_1^2, x_1^3), e_2, 1), (\{^{(0.2)}a_1^1, ^{(0.4)}a_2^1, ^{(0.7)}a_3^1\}, \{^{(0.3)}a_1^2, ^{(0.6)}a_2^2\}, \{^{(0.4)}a_1^3, ^{(0.4)}a_2^3, ^{(0.1)}a_3^3\})), \\ (((x_1^1, x_1^2, x_1^3), e_3, 1), (\{^{(0.4)}a_1^1, ^{(0.3)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.7)}a_1^2, ^{(0.2)}a_2^2\}, \{^{(0.4)}a_1^3, ^{(0.3)}a_2^3, ^{(0.5)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_1, 1), (\{^{(0.7)}a_1^1, ^{(0.2)}a_2^1, ^{(0.4)}a_3^1\}, \{^{(0.3)}a_1^2, ^{(0.6)}a_2^2\}, \{^{(0.3)}a_1^3, ^{(0.4)}a_2^3, ^{(1)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_2, 1), (\{^{(1)}a_1^1, ^{(0.2)}a_2^1, ^{(0.9)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.4)}a_2^2\}, \{^{(0.5)}a_1^3, ^{(0.2)}a_2^3, ^{(0.3)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_3, 1), (\{^{(0.3)}a_1^1, ^{(0.3)}a_2^1, ^{(0.6)}a_3^1\}, \{^{(0.5)}a_1^2, ^{(0.3)}a_2^2\}, \{^{(0.4)}a_1^3, ^{(0.2)}a_2^3, ^{(0.6)}a_3^3\})) \end{array} \right\},$$

and

$$(\widetilde{\mathcal{H}}_3, \mathfrak{Q})^0 = \left\{ \begin{array}{l} (((x_1^1, x_1^2, x_1^3), e_1, 0), (\{^{(0.4)}a_1^1, ^{(0.2)}a_2^1, ^{(0.5)}a_3^1\}, \{^{(0.6)}a_1^2, ^{(0.2)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.2)}a_2^3, ^{(0.3)}a_3^3\})), \\ (((x_1^1, x_1^2, x_1^3), e_2, 0), (\{^{(0.5)}a_1^1, ^{(0.6)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.3)}a_1^2, ^{(0.3)}a_2^2\}, \{^{(0.4)}a_1^3, ^{(0.5)}a_2^3, ^{(0.1)}a_3^3\})), \\ (((x_1^1, x_1^2, x_1^3), e_3, 0), (\{^{(0.7)}a_1^1, ^{(0.7)}a_2^1, ^{(0.7)}a_3^1\}, \{^{(0.5)}a_1^2, ^{(0.2)}a_2^2\}, \{^{(0)}a_1^3, ^{(0.4)}a_2^3, ^{(0.3)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_1, 0), (\{^{(0.1)}a_1^1, ^{(0.6)}a_2^1, ^{(0.5)}a_3^1\}, \{^{(0.7)}a_1^2, ^{(0.4)}a_2^2\}, \{^{(0.2)}a_1^3, ^{(0.5)}a_2^3, ^{(0.1)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_2, 0), (\{^{(0.1)}a_1^1, ^{(0.5)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.2)}a_1^2, ^{(0)}a_2^2\}, \{^{(1)}a_1^3, ^{(0.5)}a_2^3, ^{(0.3)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_3, 0), (\{^{(0.7)}a_1^1, ^{(0.6)}a_2^1, ^{(0.5)}a_3^1\}, \{^{(0.7)}a_1^2, ^{(0.7)}a_2^2\}, \{^{(0.7)}a_1^3, ^{(0.2)}a_2^3, ^{(0.7)}a_3^3\})) \end{array} \right\}.$$

Definition 5.5. Let $(\widetilde{\mathcal{H}}_n, \mathfrak{Q}) \in \mathfrak{C}_{EN_n}\langle \mathfrak{A}, \mathfrak{P} \rangle$.

- **(a):** If $\widetilde{\mathcal{H}}_n((\mathbf{x^I}, e, o)) = (\widehat{\emptyset}, \widehat{\emptyset}, ..., \widehat{\emptyset})$ (i.e., $\widetilde{\mathcal{H}}_{n(j)}((\mathbf{x^I}, e, o)) = \widehat{\emptyset} \quad \forall j \in J$) for each $(\mathbf{x^I}, e, o) \in \mathfrak{Q}$ then it is termed a relative null n-ary fuzzy hypersoft expert set (with respect to $\mathfrak{Q}$), denoted by $\widehat{\emptyset}_{\mathfrak{Q}}^{EN_n}$. If $\mathfrak{Q} = \mathfrak{P}$ then it is called a null n-ary fuzzy hypersoft expert set and denoted by $\widehat{\emptyset}_{\mathfrak{P}}^{EN_n}$.
- **(b):** If $\widetilde{\mathcal{H}}_n^1((\mathbf{x^I}, e, 1)) = (\widehat{\emptyset}, \widehat{\emptyset}, ..., \widehat{\emptyset})$ (i.e., $\widetilde{\mathcal{H}}_{n(j)}^1((\mathbf{x^I}, e, 1)) = \widehat{\emptyset} \quad \forall j \in J$) for each $(\mathbf{x^I}, e, 1) \in \mathfrak{Q}$ then it is termed to be a relative null agree n-ary fuzzy hypersoft expert set (with respect to $\mathfrak{Q}$), denoted by $\widehat{\emptyset}_{\mathfrak{Q}}^{EN_n^1}$. If $\mathfrak{Q} = \mathfrak{P}$ then it is called a null agree n-ary fuzzy hypersoft expert set and denoted by $\widehat{\emptyset}_{\mathfrak{P}}^{E_n^1}$.
- **(c):** If $\widetilde{\mathcal{H}}_n^0((\mathbf{x^I}, e, 0)) = (\widehat{\emptyset}, \widehat{\emptyset}, ..., \widehat{\emptyset})$ (i.e., $\widetilde{\mathcal{H}}_{n(j)}^1((\mathbf{x^I}, e, 0)) = \widehat{\emptyset} \quad \forall j \in J$) for each $(\mathbf{x^I}, e, 0) \in \mathfrak{Q}$ then it is named to be a relative null disagree n-ary fuzzy hypersoft expert set (with respect to $\mathfrak{Q}$), denoted by $\widehat{\emptyset}_{\mathfrak{Q}}^{EN_n^0}$. If $\mathfrak{Q} = \mathfrak{P}$ then it is termed to be a null disagree n-ary fuzzy hypersoft expert set and denoted by $\widehat{\emptyset}_{\mathfrak{P}}^{EN_n^0}$.
- **(d):** If $\widetilde{\mathcal{H}}_n((\mathbf{x^I}, e, o)) = (\widehat{A}_1, \widehat{A}_2, ..., \widehat{A}_n)$ (i.e., $\widetilde{\mathcal{H}}_{n(j)}((\mathbf{x^I}, e, o)) = \widehat{A}_j \quad \forall j \in J$) for each $(\mathbf{x^I}, e, o) \in \mathfrak{Q}$ then it is termed to be a relative whole n-ary fuzzy hypersoft expert set (with respect to $\mathfrak{Q}$), denoted by $\widehat{A}_{\mathfrak{Q}}^{EN_n}$. If $\mathfrak{Q} = \mathfrak{P}$ then it is called an absolute n-ary fuzzy hypersoft expert set and denoted by $\widehat{A}_{\mathfrak{P}}^{EN_n}$.
- **(e):** If $\widetilde{\mathcal{H}}_n^1((\mathbf{x^I}, e, 1)) = (\widehat{A}_1, \widehat{A}_2, ..., \widehat{A}_n)$ (i.e., $\widetilde{\mathcal{H}}_{n(j)}^1((\mathbf{x^I}, e, 1)) = \widehat{A}_j \quad \forall j \in J$) for each $(\mathbf{x^I}, e, 1) \in \mathfrak{Q}$ then it is called a relative whole agree n-ary fuzzy hypersoft expert set (with respect to $\mathfrak{Q}$), denoted by $\widehat{A}_{\mathfrak{Q}}^{EN_n^1}$. If $\mathfrak{Q} = \mathfrak{P}$ then it is called an absolute agree n-ary fuzzy hypersoft expert set and denoted by $\widehat{A}_{\mathfrak{P}}^{EN_n^1}$.
- **(f):** If $\widetilde{\mathcal{H}}_n^0((\mathbf{x^I}, e, 0)) = (\widehat{A}_1, \widehat{A}_2, ..., \widehat{A}_n)$ (i.e., $\widetilde{\mathcal{H}}_{n(j)}^0((\mathbf{x^I}, e, 0)) = \widehat{A}_j \quad \forall j \in J$) for each $(\mathbf{x^I}, e, 0) \in \mathfrak{Q}$ then it is named a relative whole disagree n-ary fuzzy hypersoft expert set (with respect to $\mathfrak{Q}$),

denoted by $\widehat{A}_{\mathfrak{Q}}^{EN_n^0}$. If $\mathfrak{Q} = \mathfrak{P}$ then it is called an absolute disagree n-ary fuzzy hypersoft expert set and denoted by $\widehat{A}_{\mathfrak{P}}^{EN_n^0}$.

Example 5.6. The following ternary fuzzy hypersoft expert sets are given as examples of relative null disagree ternary fuzzy hypersoft expert set (with respect to $\mathfrak{Q}$) and relative whole agree ternary fuzzy hypersoft expert set (with respect to $\mathfrak{Q}$) over $\mathfrak{A}$.

$$\widehat{\emptyset}_{\mathfrak{Q}}^{EN_3^0} = \left\{ \begin{array}{l} (((x_1^1, x_1^2, x_1^3), e_1, 0), (\{^{(0)}a_1^1, ^{(0)}a_2^1, ^{(0)}a_3^1\}, \{^{(0)}a_1^2, ^{(0)}a_2^2\}, \{^{(0)}a_1^3, ^{(0)}a_2^3, ^{(0)}a_3^3\})), \\ (((x_1^1, x_1^2, x_1^3), e_2, 0), (\{^{(0)}a_1^1, ^{(0)}a_2^1, ^{(0)}a_3^1\}, \{^{(0)}a_1^2, ^{(0)}a_2^2\}, \{^{(0)}a_1^3, ^{(0)}a_2^3, ^{(0)}a_3^3\})), \\ (((x_1^1, x_1^2, x_1^3), e_3, 0), (\{^{(0)}a_1^1, ^{(0)}a_2^1, ^{(0)}a_3^1\}, \{^{(0)}a_1^2, ^{(0)}a_2^2\}, \{^{(0)}a_1^3, ^{(0)}a_2^3, ^{(0)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_1, 0), (\{^{(0)}a_1^1, ^{(0)}a_2^1, ^{(0)}a_3^1\}, \{^{(0)}a_1^2, ^{(0)}a_2^2\}, \{^{(0)}a_1^3, ^{(0)}a_2^3, ^{(0)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_2, 0), (\{^{(0)}a_1^1, ^{(0)}a_2^1, ^{(0)}a_3^1\}, \{^{(0)}a_1^2, ^{(0)}a_2^2\}, \{^{(0)}a_1^3, ^{(0)}a_2^3, ^{(0)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_3, 0), (\{^{(0)}a_1^1, ^{(0)}a_2^1, ^{(0)}a_3^1\}, \{^{(0)}a_1^2, ^{(0)}a_2^2\}, \{^{(0)}a_1^3, ^{(0)}a_2^3, ^{(0)}a_3^3\})) \end{array} \right\}.$$

and

$$\widehat{A}_{\mathfrak{Q}}^{EN_3^1} = \left\{ \begin{array}{l} (((x_1^1, x_1^2, x_1^3), e_1, 1), (\{^{(1)}a_1^1, ^{(1)}a_2^1, ^{(1)}a_3^1\}, \{^{(1)}a_1^2, ^{(1)}a_2^2\}, \{^{(1)}a_1^3, ^{(1)}a_2^3, ^{(1)}a_3^3\})), \\ (((x_1^1, x_1^2, x_1^3), e_2, 1), (\{^{(1)}a_1^1, ^{(1)}a_2^1, ^{(1)}a_3^1\}, \{^{(1)}a_1^2, ^{(1)}a_2^2\}, \{^{(1)}a_1^3, ^{(1)}a_2^3, ^{(1)}a_3^3\})), \\ (((x_1^1, x_1^2, x_1^3), e_3, 1), (\{^{(1)}a_1^1, ^{(1)}a_2^1, ^{(1)}a_3^1\}, \{^{(1)}a_1^2, ^{(1)}a_2^2\}, \{^{(1)}a_1^3, ^{(1)}a_2^3, ^{(1)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_1, 1), (\{^{(1)}a_1^1, ^{(1)}a_2^1, ^{(1)}a_3^1\}, \{^{(1)}a_1^2, ^{(1)}a_2^2\}, \{^{(1)}a_1^3, ^{(1)}a_2^3, ^{(1)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_2, 1), (\{^{(1)}a_1^1, ^{(1)}a_2^1, ^{(1)}a_3^1\}, \{^{(1)}a_1^2, ^{(1)}a_2^2\}, \{^{(1)}a_1^3, ^{(1)}a_2^3, ^{(1)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_3, 1), (\{^{(1)}a_1^1, ^{(1)}a_2^1, ^{(1)}a_3^1\}, \{^{(1)}a_1^2, ^{(1)}a_2^2\}, \{^{(1)}a_1^3, ^{(1)}a_2^3, ^{(1)}a_3^3\})) \end{array} \right\}.$$

Definition 5.7. Let $(\widetilde{\mathcal{H}}_n, \mathfrak{Q}), (\widetilde{\mathcal{K}}_n, \mathfrak{R}) \in \mathfrak{C}_{EN_n}\langle \mathfrak{A}, \mathfrak{P} \rangle$.

(a): $(\widetilde{\mathcal{H}}_n, \mathfrak{Q})$ is called an n-ary fuzzy hypersoft expert subset of $(\widetilde{\mathcal{K}}_n, \mathfrak{R})$, denoted by $(\widetilde{\mathcal{H}}_n, \mathfrak{Q}) \sqsubseteq_{EN_n} (\widetilde{\mathcal{K}}_n, \mathfrak{R})$, if $\mathfrak{Q} \subseteq \mathfrak{R}$ and

$$\widetilde{\mathcal{H}}_{n(j)}((\mathbf{x}^{\mathbf{I}}, e, o)) \subseteq_f \widetilde{\mathcal{K}}_{n(j)}((\mathbf{x}^{\mathbf{I}}, e, o)) \quad \forall j \in J \tag{24}$$

for each $(\mathbf{x}^{\mathbf{I}}, e, o) \in \mathfrak{Q}$.

(b): The n-ary fuzzy hypersoft expert sets $(\widetilde{\mathcal{H}}_n, \mathfrak{Q})$ and $(\widetilde{\mathcal{K}}_n, \mathfrak{R})$ are called equal, denoted by $(\widetilde{\mathcal{H}}_n, \mathfrak{Q}) =_{EN_n} (\widetilde{\mathcal{K}}_n, \mathfrak{R})$, if $(\widetilde{\mathcal{H}}_n, \mathfrak{Q}) \sqsubseteq_{EN_n} (\widetilde{\mathcal{K}}_n, \mathfrak{R})$ and $(\widetilde{\mathcal{K}}_n, \mathfrak{R}) \sqsubseteq_{EN_n} (\widetilde{\mathcal{H}}_n, \mathfrak{Q})$.

Example 5.8. Consider $(\widetilde{\mathcal{H}}_3, \mathfrak{Q})$ in Example 5.2 and the $(\widetilde{\mathcal{H}}_3, \mathfrak{Q})^1$ and $(\widetilde{\mathcal{H}}_3, \mathfrak{Q})^0$ Example 5.4. It is clear that $(\widetilde{\mathcal{H}}_3, \mathfrak{Q})^1$ and $(\widetilde{\mathcal{H}}_3, \mathfrak{Q})^0$ are ternary fuzzy hypersoft expert subsets of $(\widetilde{\mathcal{H}}_3, \mathfrak{Q})$. Furthermore, $\widehat{\emptyset}_{\mathfrak{Q}}^{EN_3^0}$ is ternary fuzzy hypersoft expert subset of $(\widetilde{\mathcal{H}}_3, \mathfrak{Q})$.

Definition 5.9. Let $(\widetilde{\mathcal{H}}_n, \mathfrak{Q}) \in \mathfrak{C}_{EN_n}\langle \mathfrak{A}, \mathfrak{P} \rangle$. Then, the relative complement of n-ary fuzzy hypersoft expert set $(\widetilde{\mathcal{H}}_n, \mathfrak{Q})$, denoted by $(\widetilde{\mathcal{H}}_n, \mathfrak{Q})^{r_{EN_n}}$, is defined as

$$(\widetilde{\mathcal{H}}_n, \mathfrak{Q})^{r_{EN_n}} = (\widetilde{\mathcal{H}}_n^r, \mathfrak{Q}), \tag{25}$$

where $\widetilde{\mathcal{H}}_{n(j)}^r((\mathbf{x}^{\mathbf{I}}, e, o))$ is the fuzzy complement of $\widetilde{\mathcal{H}}_{n(j)}((\mathbf{x}^{\mathbf{I}}, e, o))$ $(\forall j \in J)$ for each $(\mathbf{x}^{\mathbf{I}}, e, o) \in \mathfrak{Q}$.

Example 5.10. Consider the ternary fuzzy hypersoft expert set $(\widetilde{\mathcal{H}}_3, \mathfrak{Q})$ in Example 5.2. Then, the relative complement of ternary fuzzy hypersoft expert set $(\widetilde{\mathcal{H}}_3, \mathfrak{Q})$ is

$$(\widetilde{\mathcal{H}}_3, \mathfrak{Q})^{r_{EN_3}} = \left\{ \begin{array}{l} (((x_1^1, x_1^2, x_1^3), e_1, 1), (\{^{(0.5)}a_1^1, ^{(0.4)}a_2^1, ^{(0.7)}a_3^1\}, \{^{(0.6)}a_1^2, ^{(0.5)}a_2^2\}, \{^{(0.9)}a_1^3, ^{(0.4)}a_2^3, ^{(0.8)}a_3^3\})), \\ (((x_1^1, x_1^2, x_1^3), e_2, 1), (\{^{(0.8)}a_1^1, ^{(0.6)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.7)}a_1^2, ^{(0.4)}a_2^2\}, \{^{(0.6)}a_1^3, ^{(0.6)}a_2^3, ^{(0.9)}a_3^3\})), \\ (((x_1^1, x_1^2, x_1^3), e_3, 1), (\{^{(0.6)}a_1^1, ^{(0.7)}a_2^1, ^{(0.7)}a_3^1\}, \{^{(0.3)}a_1^2, ^{(0.8)}a_2^2\}, \{^{(0.6)}a_1^3, ^{(0.7)}a_2^3, ^{(0.5)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_1, 1), (\{^{(0.3)}a_1^1, ^{(0.8)}a_2^1, ^{(0.6)}a_3^1\}, \{^{(0.7)}a_1^2, ^{(0.4)}a_2^2\}, \{^{(0.7)}a_1^3, ^{(0.6)}a_2^3, ^{(0)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_2, 1), (\{^{(1)}a_1^1, ^{(0.8)}a_2^1, ^{(0.1)}a_3^1\}, \{^{(0.6)}a_1^2, ^{(0.6)}a_2^2\}, \{^{(0.5)}a_1^3, ^{(0.8)}a_2^3, ^{(0.7)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_3, 1), (\{^{(0.7)}a_1^1, ^{(0.7)}a_2^1, ^{(0.4)}a_3^1\}, \{^{(0.5)}a_1^2, ^{(0.7)}a_2^2\}, \{^{(0.6)}a_1^3, ^{(0.8)}a_2^3, ^{(0.4)}a_3^3\})), \\ (((x_1^1, x_1^2, x_1^3), e_1, 0), (\{^{(0.6)}a_1^1, ^{(0.8)}a_2^1, ^{(0.5)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.8)}a_2^2\}, \{^{(0.9)}a_1^3, ^{(0.8)}a_2^3, ^{(0.7)}a_3^3\})), \\ (((x_1^1, x_1^2, x_1^3), e_2, 0), (\{^{(0.5)}a_1^1, ^{(0.4)}a_2^1, ^{(0.7)}a_3^1\}, \{^{(0.7)}a_1^2, ^{(0.7)}a_2^2\}, \{^{(0.6)}a_1^3, ^{(0.5)}a_2^3, ^{(0.9)}a_3^3\})), \\ (((x_1^1, x_1^2, x_1^3), e_3, 0), (\{^{(0.3)}a_1^1, ^{(0.3)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.5)}a_1^2, ^{(0.8)}a_2^2\}, \{^{(1)}a_1^3, ^{(0.5)}a_2^3, ^{(0.7)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_1, 0), (\{^{(0.9)}a_1^1, ^{(0.4)}a_2^1, ^{(0.5)}a_3^1\}, \{^{(0.3)}a_1^2, ^{(0.6)}a_2^2\}, \{^{(0.8)}a_1^3, ^{(0.5)}a_2^3, ^{(0.9)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_2, 0), (\{^{(0.9)}a_1^1, ^{(0.5)}a_2^1, ^{(0.7)}a_3^1\}, \{^{(0.8)}a_1^2, ^{(1)}a_2^2\}, \{^{(0)}a_1^3, ^{(0.5)}a_2^3, ^{(0.3)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_3, 0), (\{^{(0.3)}a_1^1, ^{(0.4)}a_2^1, ^{(0.5)}a_3^1\}, \{^{(0.3)}a_1^2, ^{(0.3)}a_2^2\}, \{^{(0.3)}a_1^3, ^{(0.8)}a_2^3, ^{(0.3)}a_3^3\})) \end{array} \right\}.$$

Proposition 5.11. *Let $(\widetilde{\mathcal{H}}_n, \mathfrak{Q}) \in \mathfrak{C}_{EN_n}\langle \mathfrak{A}, \mathfrak{P} \rangle$. Then, we have the following.*

- **(i):** $((\widetilde{\mathcal{H}}_n, \mathfrak{Q})^{r_{EN_n}})^{r_{EN_n}} =_{EN_n} (\widetilde{\mathcal{H}}_n, \mathfrak{Q})$.
- **(ii):** $(\widehat{A}_{\mathfrak{Q}}^{EN_n})^{r_{EN}} =_{EN_n} (\widehat{\emptyset}_{\mathfrak{Q}}^{EN_n})$.
- **(iii):** $(\widehat{\emptyset}_{\mathfrak{Q}}^{EN_n})^{r_{EN_n}} =_{EN_n} (\widehat{A}_{\mathfrak{Q}}^{EN_n})$.

Proof. The proofs are straightforward. □

Definition 5.12. Let $(\widetilde{\mathcal{H}}_n, \mathfrak{Q}), (\widetilde{\mathcal{K}}_n, \mathfrak{R}) \in \mathfrak{C}_{EN_n}\langle \mathfrak{A}, \mathfrak{P} \rangle$. Then, the restricted intersection of n-ary fuzzy hypersoft expert sets $(\widetilde{\mathcal{H}}_n, \mathfrak{Q})$ and $(\widetilde{\mathcal{K}}_n, \mathfrak{R})$ is denoted and defined by $(\widetilde{\mathcal{L}}_n, \mathfrak{S}) = (\widetilde{\mathcal{H}}_n, \mathfrak{Q}) \Cap_{EN_n} (\widetilde{\mathcal{K}}_n, \mathfrak{R})$ where $\mathfrak{S} = \mathfrak{Q} \cap \mathfrak{R}$ and

$$\widetilde{\mathcal{L}}_{n(j)}((\mathbf{x}^{\mathbf{I}}, e, o)) = \widetilde{\mathcal{H}}_{n(j)}((\mathbf{x}^{\mathbf{I}}, e, o)) \cap_f \widetilde{\mathcal{K}}_{n(j)}((\mathbf{x}^{\mathbf{I}}, e, o)) \quad \forall j \in J \tag{26}$$

for each $(\mathbf{x}^{\mathbf{I}}, e, o) \in \mathfrak{S}$.

Definition 5.13. Let $(\widetilde{\mathcal{H}}_n, \mathfrak{Q}), (\widetilde{\mathcal{K}}_n, \mathfrak{R}) \in \mathfrak{C}_{EN_n}\langle \mathfrak{A}, \mathfrak{P} \rangle$. Then, the extended intersection of n-ary fuzzy hypersoft expert sets $(\widetilde{\mathcal{H}}_n, \mathfrak{Q})$ and $(\widetilde{\mathcal{K}}_n, \mathfrak{R})$ is denoted and defined by $(\widetilde{\mathcal{L}}_n, \mathfrak{S}) = (\widetilde{\mathcal{H}}_n, \mathfrak{Q}) \sqcap_{EN_n} (\widetilde{\mathcal{K}}_n, \mathfrak{R})$ where $\mathfrak{S} = \mathfrak{Q} \cup \mathfrak{R}$ and

$$\widetilde{\mathcal{L}}_n((\mathbf{x}^{\mathbf{I}}, e, o)) = \begin{cases} \widetilde{\mathcal{H}}_n((\mathbf{x}^{\mathbf{I}}, e, o)), & \text{if } (\mathbf{x}^{\mathbf{I}}, e, o) \in \mathfrak{Q}, \\ \widetilde{\mathcal{K}}_n((\mathbf{x}^{\mathbf{I}}, e, o)), & \text{if } (\mathbf{x}^{\mathbf{I}}, e, o) \in \mathfrak{R}, \\ \widetilde{\mathcal{H}}_n((\mathbf{x}^{\mathbf{I}}, e, o)) \cap_f \widetilde{\mathcal{K}}_n((\mathbf{x}^{\mathbf{I}}, e, o)), & \text{if } (\mathbf{x}^{\mathbf{I}}, e, o) \in \mathfrak{Q} \cap \mathfrak{R}, \end{cases} \tag{27}$$

for each $(\mathbf{x}^{\mathbf{I}}, e, o) \in \mathfrak{S}$, where Eq. (26) is applied to obtain $\widetilde{\mathcal{H}}_n((\mathbf{x}^{\mathbf{I}}, e, o)) \cap_f \widetilde{\mathcal{K}}_n(\mathbf{x}^{\mathbf{I}})$.

Example 5.14. Consider the ternary fuzzy hypersoft expert set $(\widetilde{\mathcal{H}}_3, \mathfrak{Q})$ in Example 5.2. Also, we suppose that

$$\mathfrak{R} = \left\{ \begin{array}{l} (((x_1^1, x_1^2, x_2^3), e_1, 1), (((x_1^1, x_1^2, x_2^3), e_3, 1), (((x_1^1, x_1^2, x_3^3), e_1, 1), (((x_1^1, x_1^2, x_3^3), e_2, 1), \\ (((x_1^1, x_1^2, x_2^3), e_1, 0), (((x_1^1, x_1^2, x_2^3), e_3, 0), (((x_1^1, x_1^2, x_3^3), e_1, 0), (((x_1^1, x_1^2, x_3^3), e_2, 0) \end{array} \right\} \subseteq \mathbf{X} \times \mathcal{E} \times \mathcal{O},$$

and

$$(\widetilde{\mathcal{K}}_3,\mathfrak{R})=\left\{\begin{array}{l}(((x_1^1,x_1^2,x_2^3),e_1,1),(\{{}^{(0.4)}a_1^1,{}^{(0)}a_2^1,{}^{(0.7)}a_3^1\},\{{}^{(0.3)}a_1^2,{}^{(0.1)}a_2^2\},\{{}^{(0.6)}a_1^3,{}^{(0.4)}a_2^3,{}^{(0.3)}a_3^3\})),\\(((x_1^1,x_1^2,x_2^3),e_3,1),(\{{}^{(0.2)}a_1^1,{}^{(0.6)}a_2^1,{}^{(0.3)}a_3^1\},\{{}^{(0.4)}a_1^2,{}^{(0.2)}a_2^2\},\{{}^{(0)}a_1^3,{}^{(0.4)}a_2^3,{}^{(0.7)}a_3^3\})),\\(((x_1^1,x_1^2,x_3^3),e_1,1),(\{{}^{(0.7)}a_1^1,{}^{(0.4)}a_2^1,{}^{(0.4)}a_3^1\},\{{}^{(0.5)}a_1^2,{}^{(0.3)}a_2^2\},\{{}^{(0.3)}a_1^3,{}^{(0.2)}a_2^3,{}^{(0.2)}a_3^3\})),\\(((x_1^1,x_1^2,x_3^3),e_2,1),(\{{}^{(0.1)}a_1^1,{}^{(0.1)}a_2^1,{}^{(0.5)}a_3^1\},\{{}^{(0)}a_1^2,{}^{(0.4)}a_2^2\},\{{}^{(0.5)}a_1^3,{}^{(0.2)}a_2^3,{}^{(0.2)}a_3^3\})),\\(((x_1^1,x_1^2,x_2^3),e_1,0),(\{{}^{(0.5)}a_1^1,{}^{(0.6)}a_2^1,{}^{(0.5)}a_3^1\},\{{}^{(0.7)}a_1^2,{}^{(0.4)}a_2^2\},\{{}^{(0.3)}a_1^3,{}^{(0.1)}a_2^3,{}^{(0.1)}a_3^3\})),\\(((x_1^1,x_1^2,x_2^3),e_3,0),(\{{}^{(0.7)}a_1^1,{}^{(0.3)}a_2^1,{}^{(0.3)}a_3^1\},\{{}^{(1)}a_1^2,{}^{(0.3)}a_2^2\},\{{}^{(0.4)}a_1^3,{}^{(0.1)}a_2^3,{}^{(0)}a_3^3\})),\\(((x_1^1,x_1^2,x_3^3),e_1,0),(\{{}^{(0.3)}a_1^1,{}^{(0)}a_2^1,{}^{(0.4)}a_3^1\},\{{}^{(0.4)}a_1^2,{}^{(0.4)}a_2^2\},\{{}^{(0.2)}a_1^3,{}^{(0.5)}a_2^3,{}^{(0.6)}a_3^3\})),\\(((x_1^1,x_1^2,x_3^3),e_2,0),(\{{}^{(0.2)}a_1^1,{}^{(0.3)}a_2^1,{}^{(0.5)}a_3^1\},\{{}^{(0.5)}a_1^2,{}^{(0.6)}a_2^2\},\{{}^{(0.2)}a_1^3,{}^{(0.4)}a_2^3,{}^{(0.5)}a_3^3\}))\end{array}\right\}.$$

Then, the restricted intersection and extended intersection of ternary fuzzy hypersoft expert sets $(\widetilde{\mathcal{H}}_3,\mathfrak{Q})$ and $(\widetilde{\mathcal{K}}_3,\mathfrak{R})$ are respectively

$$(\widetilde{\mathcal{H}}_3,\mathfrak{Q})\Cap_{EN_3}(\widetilde{\mathcal{K}}_3,\mathfrak{R})$$
$$=\left\{\begin{array}{l}(((x_1^1,x_1^2,x_3^3),e_1,1),(\{{}^{(0.7)}a_1^1,{}^{(0.2)}a_2^1,{}^{(0.4)}a_3^1\},\{{}^{(0.3)}a_1^2,{}^{(0.3)}a_2^2\},\{{}^{(0.3)}a_1^3,{}^{(0.2)}a_2^3,{}^{(0.2)}a_3^3\})),\\(((x_1^1,x_1^2,x_3^3),e_2,1),(\{{}^{(0.1)}a_1^1,{}^{(0.1)}a_2^1,{}^{(0.5)}a_3^1\},\{{}^{(0)}a_1^2,{}^{(0.4)}a_2^2\},\{{}^{(0.5)}a_1^3,{}^{(0.2)}a_2^3,{}^{(0.2)}a_3^3\})),\\(((x_1^1,x_1^2,x_3^3),e_1,0),(\{{}^{(0.1)}a_1^1,{}^{(0)}a_2^1,{}^{(0.4)}a_3^1\},\{{}^{(0.4)}a_1^2,{}^{(0.4)}a_2^2\},\{{}^{(0.2)}a_1^3,{}^{(0.5)}a_2^3,{}^{(0.1)}a_3^3\})),\\(((x_1^1,x_1^2,x_3^3),e_2,0),(\{{}^{(0.1)}a_1^1,{}^{(0.3)}a_2^1,{}^{(0.3)}a_3^1\},\{{}^{(0.2)}a_1^2,{}^{(0)}a_2^2\},\{{}^{(0.2)}a_1^3,{}^{(0.4)}a_2^3,{}^{(0.3)}a_3^3\}))\end{array}\right\},$$

and

$$(\widetilde{\mathcal{H}}_3,\mathfrak{Q})\sqcap_{EN_3}(\widetilde{\widetilde{\mathcal{H}}}_3,\mathfrak{R})$$
$$=\left\{\begin{array}{l}(((x_1^1,x_1^2,x_1^3),e_1,1),(\{{}^{(0.5)}a_1^1,{}^{(0.6)}a_2^1,{}^{(0.3)}a_3^1\},\{{}^{(0.4)}a_1^2,{}^{(0.5)}a_2^2\},\{{}^{(0.1)}a_1^3,{}^{(0.6)}a_2^3,{}^{(0.2)}a_3^3\})),\\(((x_1^1,x_1^2,x_1^3),e_2,1),(\{{}^{(0.2)}a_1^1,{}^{(0.4)}a_2^1,{}^{(0.7)}a_3^1\},\{{}^{(0.3)}a_1^2,{}^{(0.6)}a_2^2\},\{{}^{(0.4)}a_1^3,{}^{(0.4)}a_2^3,{}^{(0.1)}a_3^3\})),\\(((x_1^1,x_1^2,x_1^3),e_3,1),(\{{}^{(0.4)}a_1^1,{}^{(0.3)}a_2^1,{}^{(0.3)}a_3^1\},\{{}^{(0.7)}a_1^2,{}^{(0.2)}a_2^2\},\{{}^{(0.4)}a_1^3,{}^{(0.3)}a_2^3,{}^{(0.5)}a_3^3\})),\\(((x_1^1,x_1^2,x_2^3),e_1,1),(\{{}^{(0.4)}a_1^1,{}^{(0)}a_2^1,{}^{(0.7)}a_3^1\},\{{}^{(0.3)}a_1^2,{}^{(0.1)}a_2^2\},\{{}^{(0.6)}a_1^3,{}^{(0.4)}a_2^3,{}^{(0.3)}a_3^3\})),\\(((x_1^1,x_1^2,x_2^3),e_3,1),(\{{}^{(0.2)}a_1^1,{}^{(0.6)}a_2^1,{}^{(0.3)}a_3^1\},\{{}^{(0.4)}a_1^2,{}^{(0.2)}a_2^2\},\{{}^{(0)}a_1^3,{}^{(0.4)}a_2^3,{}^{(0.7)}a_3^3\})),\\(((x_1^1,x_1^2,x_3^3),e_1,1),(\{{}^{(0.7)}a_1^1,{}^{(0.2)}a_2^1,{}^{(0.4)}a_3^1\},\{{}^{(0.3)}a_1^2,{}^{(0.3)}a_2^2\},\{{}^{(0.3)}a_1^3,{}^{(0.2)}a_2^3,{}^{(0.2)}a_3^3\})),\\(((x_1^1,x_1^2,x_3^3),e_2,1),(\{{}^{(0.1)}a_1^1,{}^{(0.1)}a_2^1,{}^{(0.5)}a_3^1\},\{{}^{(0)}a_1^2,{}^{(0.4)}a_2^2\},\{{}^{(0.5)}a_1^3,{}^{(0.2)}a_2^3,{}^{(0.2)}a_3^3\})),\\(((x_1^1,x_1^2,x_3^3),e_3,1),(\{{}^{(0.3)}a_1^1,{}^{(0.3)}a_2^1,{}^{(0.6)}a_3^1\},\{{}^{(0.5)}a_1^2,{}^{(0.3)}a_2^2\},\{{}^{(0.4)}a_1^3,{}^{(0.2)}a_2^3,{}^{(0.6)}a_3^3\})),\\(((x_1^1,x_1^2,x_1^3),e_1,0),(\{{}^{(0.4)}a_1^1,{}^{(0.2)}a_2^1,{}^{(0.5)}a_3^1\},\{{}^{(0.6)}a_1^2,{}^{(0.2)}a_2^2\},\{{}^{(0.1)}a_1^3,{}^{(0.2)}a_2^3,{}^{(0.3)}a_3^3\})),\\(((x_1^1,x_1^2,x_1^3),e_2,0),(\{{}^{(0.5)}a_1^1,{}^{(0.6)}a_2^1,{}^{(0.3)}a_3^1\},\{{}^{(0.3)}a_1^2,{}^{(0.3)}a_2^2\},\{{}^{(0.4)}a_1^3,{}^{(0.5)}a_2^3,{}^{(0.1)}a_3^3\})),\\(((x_1^1,x_1^2,x_1^3),e_3,0),(\{{}^{(0.7)}a_1^1,{}^{(0.7)}a_2^1,{}^{(0.7)}a_3^1\},\{{}^{(0.5)}a_1^2,{}^{(0.2)}a_2^2\},\{{}^{(0)}a_1^3,{}^{(0.4)}a_2^3,{}^{(0.3)}a_3^3\})),\\(((x_1^1,x_1^2,x_2^3),e_1,0),(\{{}^{(0.5)}a_1^1,{}^{(0.6)}a_2^1,{}^{(0.5)}a_3^1\},\{{}^{(0.7)}a_1^2,{}^{(0.4)}a_2^2\},\{{}^{(0.3)}a_1^3,{}^{(0.1)}a_2^3,{}^{(0.1)}a_3^3\})),\\(((x_1^1,x_1^2,x_2^3),e_3,0),(\{{}^{(0.7)}a_1^1,{}^{(0.3)}a_2^1,{}^{(0.3)}a_3^1\},\{{}^{(1)}a_1^2,{}^{(0.3)}a_2^2\},\{{}^{(0.4)}a_1^3,{}^{(0.1)}a_2^3,{}^{(0)}a_3^3\})),\\(((x_1^1,x_1^2,x_3^3),e_1,0),(\{{}^{(0.1)}a_1^1,{}^{(0)}a_2^1,{}^{(0.4)}a_3^1\},\{{}^{(0.4)}a_1^2,{}^{(0.4)}a_2^2\},\{{}^{(0.2)}a_1^3,{}^{(0.5)}a_2^3,{}^{(0.1)}a_3^3\})),\\(((x_1^1,x_1^2,x_3^3),e_2,0),(\{{}^{(0.1)}a_1^1,{}^{(0.3)}a_2^1,{}^{(0.3)}a_3^1\},\{{}^{(0.2)}a_1^2,{}^{(0)}a_2^2\},\{{}^{(0.2)}a_1^3,{}^{(0.4)}a_2^3,{}^{(0.3)}a_3^3\})),\\(((x_1^1,x_1^2,x_3^3),e_3,0),(\{{}^{(0.7)}a_1^1,{}^{(0.6)}a_2^1,{}^{(0.5)}a_3^1\},\{{}^{(0.7)}a_1^2,{}^{(0.7)}a_2^2\},\{{}^{(0.7)}a_1^3,{}^{(0.2)}a_2^3,{}^{(0.7)}a_3^3\}))\end{array}\right\}.$$

Definition 5.15. Let $(\widetilde{\mathcal{H}}_n, \mathfrak{Q}), (\widetilde{\mathcal{K}}_n, \mathfrak{R}) \in \mathfrak{C}_{EN_n}\langle \mathfrak{A}, \mathfrak{P} \rangle$. Then, the restricted union of n-ary fuzzy hypersoft expert sets $(\widetilde{\mathcal{H}}_n, \mathfrak{Q})$ and $(\widetilde{\mathcal{K}}_n, \mathfrak{R})$ is denoted and defined by $(\widetilde{\mathcal{L}}_n, \mathfrak{S}) = (\widetilde{\mathcal{H}}_n, \mathfrak{Q}) \Cup_{EN_n} (\widetilde{\mathcal{K}}_n, \mathfrak{R})$ where $\mathfrak{S} = \mathfrak{Q} \cap \mathfrak{R}$ and

$$\widetilde{\mathcal{L}}_{n(j)}((\mathbf{x}^{\mathbf{I}}, e, o)) = \widetilde{\mathcal{H}}_{n(j)}((\mathbf{x}^{\mathbf{I}}, e, o)) \cup_f \widetilde{\mathcal{K}}_{n(j)}((\mathbf{x}^{\mathbf{I}}, e, o)) \quad \forall j \in J \tag{28}$$

for each $(\mathbf{x}^{\mathbf{I}}, e, o) \in \mathfrak{S}$.

Definition 5.16. Let $(\widetilde{\mathcal{H}}_n, \mathfrak{Q}), (\widetilde{\mathcal{K}}_n, \mathfrak{R}) \in \mathfrak{C}_{EN_n}\langle \mathfrak{A}, \mathfrak{P} \rangle$. Then, the extended union of n-ary fuzzy hypersoft expert sets $(\widetilde{\mathcal{H}}_n, \mathfrak{Q})$ and $(\widetilde{\mathcal{K}}_n, \mathfrak{R})$ is denoted and defined by $(\widetilde{\mathcal{L}}_n, \mathfrak{S}) = (\widetilde{\mathcal{H}}_n, \mathfrak{Q}) \sqcup_{EN_n} (\widetilde{\mathcal{K}}_n, \mathfrak{R})$ where $\mathfrak{S} = \mathfrak{Q} \cup \mathfrak{R}$ and

$$\widetilde{\mathcal{L}}_n((\mathbf{x}^{\mathbf{I}}, e, o)) = \begin{cases} \widetilde{\mathcal{H}}_n((\mathbf{x}^{\mathbf{I}}, e, o)), & \text{if } \ (\mathbf{x}^{\mathbf{I}}, e, o) \in \mathfrak{Q}, \\ \widetilde{\mathcal{K}}_n((\mathbf{x}^{\mathbf{I}}, e, o)), & \text{if } \ (\mathbf{x}^{\mathbf{I}}, e, o) \in \mathfrak{R}, \\ \widetilde{\mathcal{H}}_n((\mathbf{x}^{\mathbf{I}}, e, o)) \cup_f \widetilde{\mathcal{K}}_n((\mathbf{x}^{\mathbf{I}}, e, o)), & \text{if } \ (\mathbf{x}^{\mathbf{I}}, e, o) \in \mathfrak{Q} \cap \mathfrak{R}, \end{cases} \tag{29}$$

for each $(\mathbf{x}^{\mathbf{I}}, e, o) \in \mathfrak{S}$, where Eq. (28) is applied to obtain $\widetilde{\mathcal{H}}_n((\mathbf{x}^{\mathbf{I}}, e, o)) \cup_f \widetilde{\mathcal{K}}_n(\mathbf{x}^{\mathbf{I}})$.

Example 5.17. We consider the ternary fuzzy hypersoft expert sets $(\widetilde{\mathcal{H}}_3, \mathfrak{Q})$ and $(\widetilde{\mathcal{K}}_3, \mathfrak{R})$ in Examples 5.2 and 5.14, respectively. Then, the restricted union and extended union of ternary fuzzy hypersoft expert sets $(\widetilde{\mathcal{H}}_3, \mathfrak{Q})$ and $(\widetilde{\mathcal{K}}_3, \mathfrak{R})$ are respectively

$$(\widetilde{\mathcal{H}}_3, \mathfrak{Q}) \Cup_{EN_3} (\widetilde{\mathcal{K}}_3, \mathfrak{R})$$
$$= \left\{ \begin{array}{l} (((x_1^1, x_1^2, x_3^3), e_1, 1), (\{^{(0.7)}a_1^1, ^{(0.4)}a_2^1, ^{(0.4)}a_3^1\}, \{^{(0.5)}a_1^2, ^{(0.6)}a_2^2\}, \{^{(0.3)}a_1^3, ^{(0.4)}a_2^3, ^{(1)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_2, 1), (\{^{(1)}a_1^1, ^{(0.2)}a_2^1, ^{(0.9)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.4)}a_2^2\}, \{^{(0.5)}a_1^3, ^{(0.2)}a_2^3, ^{(0.3)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_1, 0), (\{^{(0.3)}a_1^1, ^{(0.6)}a_2^1, ^{(0.5)}a_3^1\}, \{^{(0.7)}a_1^2, ^{(0.4)}a_2^2\}, \{^{(0.2)}a_1^3, ^{(0.5)}a_2^3, ^{(0.6)}a_3^3\})), \\ (((x_1^1, x_1^2, x_3^3), e_2, 0), (\{^{(0.2)}a_1^1, ^{(0.5)}a_2^1, ^{(0.5)}a_3^1\}, \{^{(0.5)}a_1^2, ^{(0.6)}a_2^2\}, \{^{(1)}a_1^3, ^{(0.5)}a_2^3, ^{(0.5)}a_3^3\})) \end{array} \right\},$$

and

$$(\widetilde{\mathcal{H}}_3,\mathfrak{Q})\sqcup_{EN_3}(\widetilde{\mathcal{K}}_3,\mathfrak{R})$$
$$=\left\{\begin{array}{l}
(((x_1^1,x_1^2,x_1^3),e_1,1),(\{^{(0.5)}a_1^1,^{(0.6)}a_2^1,^{(0.3)}a_3^1\},\{^{(0.4)}a_1^2,^{(0.5)}a_2^2\},\{^{(0.1)}a_1^3,^{(0.6)}a_2^3,^{(0.2)}a_3^3\})),\\
(((x_1^1,x_1^2,x_1^3),e_2,1),(\{^{(0.2)}a_1^1,^{(0.4)}a_2^1,^{(0.7)}a_3^1\},\{^{(0.3)}a_1^2,^{(0.6)}a_2^2\},\{^{(0.4)}a_1^3,^{(0.4)}a_2^3,^{(0.1)}a_3^3\})),\\
(((x_1^1,x_1^2,x_1^3),e_3,1),(\{^{(0.4)}a_1^1,^{(0.3)}a_2^1,^{(0.3)}a_3^1\},\{^{(0.7)}a_1^2,^{(0.2)}a_2^2\},\{^{(0.4)}a_1^3,^{(0.3)}a_2^3,^{(0.5)}a_3^3\})),\\
(((x_1^1,x_1^2,x_2^3),e_1,1),(\{^{(0.4)}a_1^1,^{(0)}a_2^1,^{(0.7)}a_3^1\},\{^{(0.3)}a_1^2,^{(0.1)}a_2^2\},\{^{(0.6)}a_1^3,^{(0.4)}a_2^3,^{(0.3)}a_3^3\})),\\
(((x_1^1,x_1^2,x_2^3),e_3,1),(\{^{(0.2)}a_1^1,^{(0.6)}a_2^1,^{(0.3)}a_3^1\},\{^{(0.4)}a_1^2,^{(0.2)}a_2^2\},\{^{(0)}a_1^3,^{(0.4)}a_2^3,^{(0.7)}a_3^3\})),\\
(((x_1^1,x_1^2,x_3^3),e_1,1),(\{^{(0.7)}a_1^1,^{(0.4)}a_2^1,^{(0.4)}a_3^1\},\{^{(0.5)}a_1^2,^{(0.6)}a_2^2\},\{^{(0.3)}a_1^3,^{(0.4)}a_2^3,^{(1)}a_3^3\})),\\
(((x_1^1,x_1^2,x_3^3),e_2,1),(\{^{(1)}a_1^1,^{(0.2)}a_2^1,^{(0.9)}a_3^1\},\{^{(0.4)}a_1^2,^{(0.4)}a_2^2\},\{^{(0.5)}a_1^3,^{(0.2)}a_2^3,^{(0.3)}a_3^3\})),\\
(((x_1^1,x_1^2,x_3^3),e_3,1),(\{^{(0.3)}a_1^1,^{(0.3)}a_2^1,^{(0.6)}a_3^1\},\{^{(0.5)}a_1^2,^{(0.3)}a_2^2\},\{^{(0.4)}a_1^3,^{(0.2)}a_2^3,^{(0.6)}a_3^3\})),\\
(((x_1^1,x_1^2,x_1^3),e_1,0),(\{^{(0.4)}a_1^1,^{(0.2)}a_2^1,^{(0.5)}a_3^1\},\{^{(0.6)}a_1^2,^{(0.2)}a_2^2\},\{^{(0.1)}a_1^3,^{(0.2)}a_2^3,^{(0.3)}a_3^3\})),\\
(((x_1^1,x_1^2,x_1^3),e_2,0),(\{^{(0.5)}a_1^1,^{(0.6)}a_2^1,^{(0.3)}a_3^1\},\{^{(0.3)}a_1^2,^{(0.3)}a_2^2\},\{^{(0.4)}a_1^3,^{(0.5)}a_2^3,^{(0.1)}a_3^3\})),\\
(((x_1^1,x_1^2,x_1^3),e_3,0),(\{^{(0.7)}a_1^1,^{(0.7)}a_2^1,^{(0.7)}a_3^1\},\{^{(0.5)}a_1^2,^{(0.2)}a_2^2\},\{^{(0)}a_1^3,^{(0.4)}a_2^3,^{(0.3)}a_3^3\})),\\
(((x_1^1,x_1^2,x_2^3),e_1,0),(\{^{(0.5)}a_1^1,^{(0.6)}a_2^1,^{(0.5)}a_3^1\},\{^{(0.7)}a_1^2,^{(0.4)}a_2^2\},\{^{(0.3)}a_1^3,^{(0.1)}a_2^3,^{(0.1)}a_3^3\})),\\
(((x_1^1,x_1^2,x_2^3),e_3,0),(\{^{(0.7)}a_1^1,^{(0.3)}a_2^1,^{(0.3)}a_3^1\},\{^{(1)}a_1^2,^{(0.3)}a_2^2\},\{^{(0.4)}a_1^3,^{(0.1)}a_2^3,^{(0)}a_3^3\})),\\
(((x_1^1,x_1^2,x_3^3),e_1,0),(\{^{(0.3)}a_1^1,^{(0.6)}a_2^1,^{(0.5)}a_3^1\},\{^{(0.7)}a_1^2,^{(0.4)}a_2^2\},\{^{(0.2)}a_1^3,^{(0.5)}a_2^3,^{(0.6)}a_3^3\})),\\
(((x_1^1,x_1^2,x_3^3),e_2,0),(\{^{(0.2)}a_1^1,^{(0.5)}a_2^1,^{(0.5)}a_3^1\},\{^{(0.5)}a_1^2,^{(0.6)}a_2^2\},\{^{(1)}a_1^3,^{(0.5)}a_2^3,^{(0.5)}a_3^3\})),\\
(((x_1^1,x_1^2,x_3^3),e_3,0),(\{^{(0.7)}a_1^1,^{(0.6)}a_2^1,^{(0.5)}a_3^1\},\{^{(0.7)}a_1^2,^{(0.7)}a_2^2\},\{^{(0.7)}a_1^3,^{(0.2)}a_2^3,^{(0.7)}a_3^3\}))
\end{array}\right\}.$$

Proposition 5.18. *Let* $(\widetilde{\mathcal{H}}_n,\mathfrak{Q}),(\widetilde{\mathcal{K}}_n,\mathfrak{R}),(\widetilde{\mathcal{L}}_n,\mathfrak{S})\in\mathfrak{C}_{EN_n}\langle\mathfrak{A},\mathfrak{P}\rangle$*. Then, the following properties are satisfied.*

- **(i):** $(\widetilde{\mathcal{H}}_n,\mathfrak{Q})\diamond(\widetilde{\mathcal{K}}_n,\mathfrak{R})=_{EN_n}(\widetilde{\mathcal{K}}_n,\mathfrak{R})\diamond(\widetilde{\mathcal{H}}_n,\mathfrak{Q})$ *for each* $\diamond\in\{\Cap_{EN_n},\Cup_{EN_n}\}$.
- **(ii):** $(\widetilde{\mathcal{H}}_n,\mathfrak{Q})\diamond((\widetilde{\mathcal{K}}_n,\mathfrak{R})\diamond(\widetilde{\mathcal{L}}_n,\mathfrak{S}))=_{EN_n}((\widetilde{\mathcal{H}}_n,\mathfrak{Q})\diamond(\widetilde{\mathcal{K}}_n,\mathfrak{R}))\diamond(\widetilde{\mathcal{L}}_n,\mathfrak{S})$ *for each* $\diamond\in\{\Cap_{EN_n},\Cup_{EN_n}\}$.
- **(iii):** $(\widetilde{\mathcal{H}}_n,\mathfrak{Q})\diamond((\widetilde{\mathcal{H}}_n,\mathfrak{R})\circ(\widetilde{\mathcal{L}}_n,\mathfrak{S}))=_{EN_n}((\widetilde{\mathcal{H}}_n,\mathfrak{Q})\diamond(\widetilde{\mathcal{K}}_n,\mathfrak{R}))\circ((\widetilde{\mathcal{H}}_n,\mathfrak{Q})\diamond(\widetilde{\mathcal{L}}_n,\mathfrak{S}))$ *for each* $\diamond,\circ\in\{\Cap_{EN_n},\Cup_{EN_n}\}$.
- **(iv):** $((\widetilde{\mathcal{H}}_n,\mathfrak{Q})\diamond(\widetilde{\mathcal{K}}_n,\mathfrak{R}))^{r_{EN_n}}=_{EN_n}(\widetilde{\mathcal{H}}_n,\mathfrak{Q})^{r_{EN_n}}\circ(\widetilde{\mathcal{K}}_n,\mathfrak{R})^{r_{EN_n}}$ *for each* $\diamond,\circ\in\{\Cap_{EN_n},\Cup_{EN_n}\}$ *and* $\diamond\neq\circ$.

Proof. The proofs are straightforward. □

Proposition 5.19. *Let* $(\widetilde{\mathcal{H}}_n,\mathfrak{Q}),(\widetilde{\mathcal{K}}_n,\mathfrak{R}),(\widetilde{\mathcal{L}}_n,\mathfrak{S})\in\mathfrak{C}_{EN_n}\langle\mathfrak{A},\mathfrak{P}\rangle$*. Then, the following properties are satisfied.*

- **(i):** $(\widetilde{\mathcal{H}}_n,\mathfrak{Q})\diamond(\widetilde{\mathcal{K}}_n,\mathfrak{R})=_{EN_n}(\widetilde{\mathcal{K}}_n,\mathfrak{R})\diamond(\widetilde{\mathcal{H}}_n,\mathfrak{Q})$ *for each* $\diamond\in\{\sqcap_{EN_n},\sqcup_{EN_n}\}$.
- **(ii):** $(\widetilde{\mathcal{H}}_n,\mathfrak{Q})\diamond((\widetilde{\mathcal{K}}_n,\mathfrak{R})\diamond(\widetilde{\mathcal{L}}_n,\mathfrak{S}))=_{EN_n}((\widetilde{\mathcal{H}}_n,\mathfrak{Q})\diamond(\widetilde{\mathcal{K}}_n,\mathfrak{R}))\diamond(\widetilde{\mathcal{L}}_n,\mathfrak{S})$ *for each* $\diamond\in\{\sqcap_{EN_n},\sqcup_{EN_n}\}$.
- **(iii):** $(\widetilde{\mathcal{H}}_n,\mathfrak{Q})\diamond((\widetilde{\mathcal{H}}_n,\mathfrak{R})\circ(\widetilde{\mathcal{L}}_n,\mathfrak{S}))=_{EN_n}((\widetilde{\mathcal{H}}_n,\mathfrak{Q})\diamond(\widetilde{\mathcal{K}}_n,\mathfrak{R}))\circ((\widetilde{\mathcal{H}}_n,\mathfrak{Q})\diamond(\widetilde{\mathcal{L}}_n,\mathfrak{S}))$ *for each* $\diamond,\circ\in\{\sqcap_{EN_n},\sqcup_{EN_n}\}$.
- **(iv):** $((\widetilde{\mathcal{H}}_n,\mathfrak{Q})\diamond(\widetilde{\mathcal{K}}_n,\mathfrak{R}))^{r_{EN_n}}=_{EN_n}(\widetilde{\mathcal{H}}_n,\mathfrak{Q})^{r_{EN_n}}\circ(\widetilde{\mathcal{K}}_n,\mathfrak{R})^{r_{EN_n}}$ *for each* $\diamond,\circ\in\{\sqcap_{EN_n},\sqcup_{EN_n}\}$ *and* $\diamond\neq\circ$.

Proof. The proofs are straightforward. □

6. An Application of n-ary Fuzzy Hypersoft Expert Sets

In this section, we present a possible application of n-ary fuzzy hypersoft expert set theory in real-life problem.

We construct the following algorithm to determine the optimal choice in each universal set in the n-ary fuzzy hypersoft expert set setting.

Algorithm 1.

(1) Input the n-ary fuzzy hypersoft expert set.
(2) Construct (in the tabular form) the agree n-ary fuzzy hypersoft expert set and the disagree n-ary fuzzy hypersoft expert set.
(3) For the agree n-ary fuzzy hypersoft expert set, calculate $c_k^j = \sum_l (a_k^j)_l$ for each $j \in J$.
(4) For the disagree n-ary fuzzy hypersoft expert set, calculate $d_k^j = \sum_l (a_k^j)_l$ for each $j \in J$.
(5) Compute score value $s_k^j = c_k^j - d_k^j$ for each $j \in J$.
(6) Find γ_j for each $j \in J$ which $s_{\gamma_j} = \max s_k^j$ for each $j \in J$, and then determine the optimal choice a_γ^j in each universal set A_j.

In the tables of agree n-ary fuzzy hypersoft expert set and disagree n-ary fuzzy hypersoft expert set, $(a_k^j)_l$ corresponds the fuzzy value in the l-th row for a_k^j.

Now, we ready to give an application of n-ary fuzzy hypersoft expert set theory in handling real-life problem.

Example 6.1. Suppose that an association wants to determine the best films of the year in the fields of drama, comedy and documentary and to award these films at an award ceremony to be organized by the association. The sets of nominated films of drama, comedy and documentary are $A_1 = \{a_1^1, a_2^1, a_3^1\}$, $A_2 = \{a_1^2, a_2^2\}$ and $A_3 = \{a_1^3, a_2^3, a_3^3\}$, respectively. Also, the association hires the jury members (experts) to determine the best of the films in each category. Assume that $\mathcal{E} = \{e_1, e_2\}$ is a set of jury. The jury should analyze the characteristics or attributes of these films. Therefore, they consider the disjoint parameter sets X_1, X_2 and X_3 based on the story, message and narration of film, respectively. These sets are $X_1 = \{x_1^1 = originality,\ x_2^1 = fiction\}$, $X_2 = \{x_1^2 = ease\ of\ perception,\ x_2^2 = essence\}$, and $X_3 = \{x_1^3 = narrative\ style,\ x_2^3 = audiovisual\ quality\}$, and thereby $\mathbf{X} = X_1 \times X_2 \times X_3$. Following the serious discussion, the jury constructs the following ternary fuzzy hypersoft expert set for $\mathfrak{P} = \mathbf{X} \times \mathcal{E} \times \mathcal{O}$.

$$
(\tilde{\tilde{\mathcal{H}}}, \mathfrak{P}) = \left\{ \begin{array}{l}
(((x_1^1, x_1^2, x_1^3), e_1, 1), (\{^{(0.4)}a_1^1, ^{(0.3)}a_2^1, ^{(0.2)}a_3^1\}, \{^{(0.1)}a_1^2, ^{(0.7)}a_2^2\}, \{^{(0.3)}a_1^3, ^{(0.4)}a_2^3, ^{(0.1)}a_3^3\})), \\
(((x_1^1, x_1^2, x_1^3), e_2, 1), (\{^{(0.4)}a_1^1, ^{(0.6)}a_2^1, ^{(0.4)}a_3^1\}, \{^{(0.3)}a_1^2, ^{(0.2)}a_2^2\}, \{^{(0.6)}a_1^3, ^{(0.8)}a_2^3, ^{(0.1)}a_3^3\})), \\
(((x_1^1, x_1^2, x_2^3), e_1, 1), (\{^{(0.7)}a_1^1, ^{(0.5)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.2)}a_2^2\}, \{^{(0.2)}a_1^3, ^{(0.1)}a_2^3, ^{(0.4)}a_3^3\})), \\
(((x_1^1, x_1^2, x_2^3), e_2, 1), (\{^{(0.4)}a_1^1, ^{(0.4)}a_2^1, ^{(0.2)}a_3^1\}, \{^{(0.2)}a_1^2, ^{(0.5)}a_2^2\}, \{^{(0.3)}a_1^3, ^{(0.1)}a_2^3, ^{(0.2)}a_3^3\})), \\
(((x_1^1, x_2^2, x_1^3), e_1, 1), (\{^{(0.4)}a_1^1, ^{(0.2)}a_2^1, ^{(0.2)}a_3^1\}, \{^{(0.6)}a_1^2, ^{(0.4)}a_2^2\}, \{^{(0.3)}a_1^3, ^{(0.2)}a_2^3, ^{(0.1)}a_3^3\})), \\
(((x_1^1, x_2^2, x_1^3), e_2, 1), (\{^{(0.4)}a_1^1, ^{(0.2)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.2)}a_1^2, ^{(0.5)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.4)}a_2^3, ^{(0.1)}a_3^3\})), \\
(((x_1^1, x_2^2, x_2^3), e_1, 1), (\{^{(0.1)}a_1^1, ^{(0.6)}a_2^1, ^{(0.5)}a_3^1\}, \{^{(0.5)}a_1^2, ^{(0)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.3)}a_2^3, ^{(0.5)}a_3^3\})), \\
(((x_1^1, x_2^2, x_2^3), e_2, 1), (\{^{(0.4)}a_1^1, ^{(0.7)}a_2^1, ^{(0.6)}a_3^1\}, \{^{(0.8)}a_1^2, ^{(0.7)}a_2^2\}, \{^{(0)}a_1^3, ^{(0.6)}a_2^3, ^{(1)}a_3^3\})), \\
(((x_2^1, x_1^2, x_1^3), e_1, 1), (\{^{(0.4)}a_1^1, ^{(0.4)}a_2^1, ^{(0.2)}a_3^1\}, \{^{(0.3)}a_1^2, ^{(0.6)}a_2^2\}, \{^{(0.8)}a_1^3, ^{(0.4)}a_2^3, ^{(0.4)}a_3^3\})), \\
(((x_2^1, x_1^2, x_1^3), e_2, 1), (\{^{(0.2)}a_1^1, ^{(0.5)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.2)}a_1^2, ^{(0.1)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.5)}a_2^3, ^{(0.3)}a_3^3\})), \\
(((x_2^1, x_1^2, x_2^3), e_1, 1), (\{^{(0.5)}a_1^1, ^{(0.6)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.5)}a_2^2\}, \{^{(0.4)}a_1^3, ^{(0.6)}a_2^3, ^{(0.2)}a_3^3\})), \\
(((x_2^1, x_1^2, x_2^3), e_2, 1), (\{^{(0.3)}a_1^1, ^{(0.6)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.3)}a_1^2, ^{(0.5)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.6)}a_2^3, ^{(0.2)}a_3^3\})), \\
(((x_2^1, x_2^2, x_1^3), e_1, 1), (\{^{(0.1)}a_1^1, ^{(0.6)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.4)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.6)}a_2^3, ^{(0.2)}a_3^3\})), \\
(((x_2^1, x_2^2, x_1^3), e_2, 1), (\{^{(0.7)}a_1^1, ^{(0.7)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.2)}a_1^2, ^{(0.5)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.6)}a_2^3, ^{(0.2)}a_3^3\})), \\
(((x_2^1, x_2^2, x_2^3), e_1, 1), (\{^{(0.7)}a_1^1, ^{(0.3)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.3)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.8)}a_2^3, ^{(0.7)}a_3^3\})), \\
(((x_2^1, x_2^2, x_2^3), e_2, 1), (\{^{(0.7)}a_1^1, ^{(0.1)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.5)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.6)}a_2^3, ^{(0.2)}a_3^3\})), \\
(((x_1^1, x_1^2, x_1^3), e_1, 0), (\{^{(0.5)}a_1^1, ^{(0.5)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(1)}a_2^2\}, \{^{(0.2)}a_1^3, ^{(0.6)}a_2^3, ^{(0.5)}a_3^3\})), \\
(((x_1^1, x_1^2, x_1^3), e_2, 0), (\{^{(0.4)}a_1^1, ^{(0.6)}a_2^1, ^{(0.4)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.4)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.4)}a_2^3, ^{(0.4)}a_3^3\})), \\
(((x_1^1, x_1^2, x_2^3), e_1, 0), (\{^{(0.6)}a_1^1, ^{(0.8)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.5)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.6)}a_2^3, ^{(0.2)}a_3^3\})), \\
(((x_1^1, x_1^2, x_2^3), e_2, 0), (\{^{(0.9)}a_1^1, ^{(0.5)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.5)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.6)}a_2^3, ^{(0.2)}a_3^3\})), \\
(((x_1^1, x_2^2, x_1^3), e_1, 0), (\{^{(0.3)}a_1^1, ^{(0.3)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.5)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.6)}a_2^3, ^{(0.4)}a_3^3\})), \\
(((x_1^1, x_2^2, x_1^3), e_2, 0), (\{^{(0.3)}a_1^1, ^{(0.4)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.6)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.6)}a_2^3, ^{(0.4)}a_3^3\})), \\
(((x_1^1, x_2^2, x_2^3), e_1, 0), (\{^{(0.2)}a_1^1, ^{(0.2)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.2)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.6)}a_2^3, ^{(0.2)}a_3^3\})), \\
(((x_1^1, x_2^2, x_2^3), e_2, 0), (\{^{(0.2)}a_1^1, ^{(0.2)}a_2^1, ^{(0.4)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.5)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.6)}a_2^3, ^{(0.3)}a_3^3\})), \\
(((x_2^1, x_1^2, x_1^3), e_1, 0), (\{^{(0.6)}a_1^1, ^{(0.6)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.5)}a_1^2, ^{(0.5)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.6)}a_2^3, ^{(0.4)}a_3^3\})), \\
(((x_2^1, x_1^2, x_1^3), e_2, 0), (\{^{(0.8)}a_1^1, ^{(0.6)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.2)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.6)}a_2^3, ^{(0.2)}a_3^3\})), \\
(((x_2^1, x_1^2, x_2^3), e_1, 0), (\{^{(0.4)}a_1^1, ^{(0.5)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(1)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.6)}a_2^3, ^{(0.1)}a_3^3\})), \\
(((x_2^1, x_1^2, x_2^3), e_2, 0), (\{^{(0.3)}a_1^1, ^{(0.4)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.5)}a_1^2, ^{(0.5)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.4)}a_2^3, ^{(0.2)}a_3^3\})), \\
(((x_2^1, x_2^2, x_1^3), e_1, 0), (\{^{(0.5)}a_1^1, ^{(0.5)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.6)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.6)}a_2^3, ^{(0.3)}a_3^3\})), \\
(((x_2^1, x_2^2, x_1^3), e_2, 0), (\{^{(0.4)}a_1^1, ^{(0.5)}a_2^1, ^{(0.2)}a_3^1\}, \{^{(0.4)}a_1^2, ^{(0.4)}a_2^2\}, \{^{(0.5)}a_1^3, ^{(0.6)}a_2^3, ^{(0.4)}a_3^3\})), \\
(((x_2^1, x_2^2, x_2^3), e_1, 0), (\{^{(0.2)}a_1^1, ^{(0.5)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.8)}a_1^2, ^{(0.3)}a_2^2\}, \{^{(0.1)}a_1^3, ^{(0.7)}a_2^3, ^{(0.3)}a_3^3\})), \\
(((x_2^1, x_2^2, x_2^3), e_2, 0), (\{^{(0.3)}a_1^1, ^{(0.7)}a_2^1, ^{(0.3)}a_3^1\}, \{^{(0.2)}a_1^2, ^{(0.6)}a_2^2\}, \{^{(0.3)}a_1^3, ^{(0.7)}a_2^3, ^{(0.1)}a_3^3\}))
\end{array} \right\}.
$$

The steps of Algorithm 1 may be followed by the association to determine the best of the films in each category.

In Tables 1 and 2, we give the agree ternary fuzzy hypersoft expert set and the disagree ternary fuzzy hypersoft expert set, respectively.

TABLE 1. Agree ternary fuzzy hypersoft expert set

	a_1^1	a_2^1	a_3^1	a_1^2	a_2^2	a_1^3	a_2^3	a_3^3
$((x_1^1,x_1^2,x_1^3),e_1,1)$	0.4	0.3	0.2	0.1	0.7	0.3	0.4	0.1
$((x_1^1,x_1^2,x_1^3),e_2,1)$	0.4	0.6	0.4	0.3	0.2	0.6	0.8	0.1
$((x_1^1,x_1^2,x_2^3),e_1,1)$	0.7	0.5	0.3	0.4	0.2	0.2	0.1	0.4
$((x_1^1,x_1^2,x_2^3),e_2,1)$	0.4	0.4	0.2	0.2	0.5	0.3	0.1	0.2
$((x_1^1,x_2^2,x_1^3),e_1,1)$	0.4	0.2	0.2	0.6	0.4	0.3	0.2	0.1
$((x_1^1,x_2^2,x_1^3),e_2,1)$	0.4	0.2	0.3	0.2	0.5	0.1	0.4	0.1
$((x_1^1,x_2^2,x_2^3),e_1,1)$	0.1	0.6	0.5	0.5	0	0.1	0.3	0.5
$((x_1^1,x_2^2,x_2^3),e_2,1)$	0.4	0.7	0.6	0.8	0.7	0	0.6	1
$((x_2^1,x_1^2,x_1^3),e_1,1)$	0.4	0.4	0.2	0.3	0.6	0.8	0.4	0.4
$((x_2^1,x_1^2,x_1^3),e_2,1)$	0.2	0.5	0.3	0.2	0.1	0.1	0.5	0.3
$((x_2^1,x_1^2,x_2^3),e_1,1)$	0.5	0.6	0.3	0.4	0.5	0.4	0.6	0.2
$((x_2^1,x_1^2,x_2^3),e_2,1)$	0.3	0.6	0.3	0.3	0.5	0.1	0.6	0.2
$((x_2^1,x_2^2,x_1^3),e_1,1)$	0.1	0.6	0.3	0.4	0.4	0.1	0.6	0.2
$((x_2^1,x_2^2,x_1^3),e_2,1)$	0.7	0.7	0.3	0.2	0.5	0.1	0.6	0.2
$((x_2^1,x_2^2,x_2^3),e_1,1)$	0.7	0.3	0.3	0.4	0.3	0.1	0.8	0.7
$((x_2^1,x_2^2,x_2^3),e_2,1)$	0.7	0.1	0.3	0.4	0.5	0.1	0.6	0.2

TABLE 2. Disagree ternary fuzzy hypersoft expert set

	a_1^1	a_2^1	a_3^1	a_1^2	a_2^2	a_1^3	a_2^3	a_3^3
$((x_1^1,x_1^2,x_1^3),e_1,0)$	0.5	0.5	0.3	0.4	1	0.2	0.6	0.5
$((x_1^1,x_1^2,x_1^3),e_2,0)$	0.4	0.6	0.4	0.4	0.4	0.1	0.4	0.4
$((x_1^1,x_1^2,x_2^3),e_1,0)$	0.6	0.8	0.3	0.4	0.5	0.1	0.6	0.2
$((x_1^1,x_1^2,x_2^3),e_2,0)$	0.9	0.5	0.3	0.4	0.5	0.1	0.6	0.2
$((x_1^1,x_2^2,x_1^3),e_1,0)$	0.3	0.3	0.3	0.4	0.5	0.1	0.6	0.4
$((x_1^1,x_2^2,x_1^3),e_2,0)$	0.3	0.4	0.3	0.4	0.6	0.1	0.6	0.4
$((x_1^1,x_2^2,x_2^3),e_1,0)$	0.2	0.2	0.3	0.4	0.2	0.1	0.6	0.2
$((x_1^1,x_2^2,x_2^3),e_2,0)$	0.2	0.2	0.4	0.4	0.5	0.1	0.6	0.3
$((x_2^1,x_1^2,x_1^3),e_1,0)$	0.6	0.6	0.3	0.5	0.5	0.1	0.6	0.4
$((x_2^1,x_1^2,x_1^3),e_2,0)$	0.8	0.6	0.3	0.4	0.2	0.1	0.6	0.2
$((x_2^1,x_1^2,x_2^3),e_1,0)$	0.4	0.5	0.3	0.4	1	0.1	0.6	0.1
$((x_2^1,x_1^2,x_2^3),e_2,0)$	0.3	0.4	0.3	0.5	0.5	0.1	0.4	0.2
$((x_2^1,x_2^2,x_1^3),e_1,0)$	0.5	0.5	0.3	0.4	0.6	0.1	0.6	0.3
$((x_2^1,x_2^2,x_1^3),e_2,0)$	0.4	0.5	0.2	0.4	0.4	0.5	0.6	0.4
$((x_2^1,x_2^2,x_2^3),e_1,0)$	0.2	0.5	0.3	0.8	0.3	0.1	0.7	0.3
$((x_2^1,x_2^2,x_2^3),e_2,0)$	0.3	0.7	0.3	0.2	0.6	0.3	0.7	0.1

From Tables 1 and Table 2, we have the score values in Table 3.

TABLE 3. Score values s_k^j

	a_1^1	a_2^1	a_3^1	a_1^2	a_2^2	a_1^3	a_2^3	a_3^3
$c_k^j = \sum_l (a_k^j)_l$	$c_1^1 = 6.8$	$c_2^1 = 7.3$	$c_3^1 = 5$	$c_1^2 = 5.7$	$c_2^2 = 6.6$	$c_1^3 = 3.7$	$c_2^3 = 7.6$	$c_3^3 = 5.1$
$d_k^j = \sum_l (a_k^j)_l$	$d_1^1 = 6.9$	$d_2^1 = 7.8$	$d_3^1 = 4.9$	$d_1^2 = 6.8$	$d_2^2 = 8.3$	$c_1^3 = 2.3$	$d_2^3 = 9.4$	$d_3^3 = 4.6$
$s_k^j = c_k^j - d_k^j$	$s_1^1 = -0.1$	$s_2^1 = -0.5$	$s_3^1 = 0.1$	$s_1^2 = -1.1$	$s_2^2 = -1.7$	$s_1^3 = 1.4$	$s_2^3 = -1.8$	$s_3^3 = 0.5$

Since $\max s_{\gamma_1}^1 = s_3^1$ and $\max s_{\gamma_2}^2 = s_1^2$ and $\max s_{\gamma_3}^3 = s_1^3$ from Table 3, the best films of the year in the fields of drama, comedy and documentary are determined as s_3^1, s_1^2 and s_1^3, respectively.

7. Conclusions

In this chapter, the concepts of n-ary fuzzy hypersoft set, fuzzy hypersoft expert set and n-ary fuzzy hypersoft expert set, which are effective mathematical models for dealing with many kinds of uncertainties in the real world, were introduced. Also, the basic operations such as complement, intersection and union of these emerged types of fuzzy hypersoft sets were defined and some of their remarkable properties were discussed. An application was given to illustrate how the n-ary fuzzy hypersoft expert set sets can be useful in solving a problem in real-life. The topics of future research may be developing the n-ary fuzzy hypersoft sets, fuzzy hypersoft expert sets and n-ary fuzzy hypersoft expert sets in theoretical aspects such as describing new operations and characteristic properties in more general frameworks, and also investigating their practical applications in decision making, medical diagnosis and game theory.

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