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Abstract: The objective of this paper is to present a general definition of Neutrosophic Euclidean geometry. The mechanism of comparison between neutrosophic numbers is introduced, as well as the absolute value is extended to the neutrosophical case. In addition, the concept of neutrosophic plane with n neutrosophic dimensions is obtained. Also, Euclidean geometric concepts are extended neutrosophically such as neutrosophic distance, neutrosophic midpoint, neutrosophic vectors, neutrosophic circles, and lines. A connection between neutrosophic geometrical concepts and classical Euclidean geometry is described and established.

Keywords: Neutrosophic Plane, Neutrosophic Absolute Value, Neutrosophic Distance, Neutrosophic Midpoint, Neutrosophic Vectors, Neutrosophic Circles, Neutrosophic Lines.

1. Introduction

Neutrosophic logic is a generalization of intuitionistic fuzzy logic by adding an indeterminacy I with property $I=I^2$. Neutrosophic set concept has wide applications in different areas of science, such as decision making [7,20], health care [8,21], machine learning [9], artificial intelligence [10], soft computing [22], industry [23], and statistics [11].

On the other hand, neutrosophic sets played an interesting role in pure mathematics such as topology and analysis [12,13], spaces [1,2], and algebraic structures [3,4,5,6].

Neutrosophic spaces theory began with Agboola et.al [14], where they studied neutrosophic vector spaces and their properties. Recently, many studies have been carried out on these spaces, where AH-subspaces and homomorphisms were presented [15]. In [16,17,18,19], Hatip et. al studied neutrosophic modules (a generalized form of neutrosophic spaces) with its substructures such as homomorphisms and AH-submodules.

Euclidean geometry with two dimensions was built over points (a, b) taken from the space $R \times R$, Where constants are taken from the field of real numbers R. To extend the classical geometric system neutrosophicaly, we shall build an algebraic map which we call (AH-isometry) between the Cartesian product $R(I) \times R(I)$, and the classical space $R^2 \times R^2$.

In the classical geometry, multiplying by scalars is depending on taking these scalars from the field R , in neutrosophic geometry, multiplying is depending on neutrosophic real numbers from the neutrosophic field $R(I)$.

We show that the AH-isometry map will preserve addition and distances, in other words, it allows us to study neutrosophic vectors, points, shapes by going back to its corresponding classical concepts. This work is considered as a first step in the study of neutrosophic geometry, it has many benefits in the progression of neutrosophic studies and establishing neutrosophic functional analysis in the future, since we can define neutrosophic geometrical shapes and study their relation with classical geometrical shapes.

To reach our goals, we shall define an order relation between neutrosophic numbers as well as neutrosophic absolute values, thus we can build the concept of neutrosophic distances and norms in easy way.

This work clarifies a charming connection between neutrosophic algebra and geometry.

Motivation

We regard that there is not a geometrical system based on neutrosophic spaces, thus our motivation is to close this important research gap by defining the basic theoretical concepts of a new geometrical system based on neutrosophic numbers and spaces.

Preliminaries

Definition 2.1:[32] Let X be a non-empty fixed set. A neutrosophic set A is an object having the form $\{x, (\mu A(x), \delta A(x), \gamma A(x)): x \in X\}$, where $\mu A(x)$, $\delta A(x)\, and\, \gamma A(x)$ represent the degree of membership, the degree of indeterminacy, and the degree of non-membership respectively of each element $x \in X$ to the set A .

Definition 2.1: [31] Classical neutrosophic number has the form $a + bI$ where a, b are real or complex numbers and I is the indeterminacy such that $0 \cdot I = 0$ and $I^2 = I$ which results that $I^n = I$ for all positive integers n.

Definition 2.2 : [30] Let$(G,*)$be any group, the neutrosophic group is generated by I and G under * denoted by$N(G) = \{\langle G \cup I \rangle, *\}$.

Definition 2.3: [30] Let R be any ring. The neutrosophic ring $\langle R \cup I \rangle$ is also a ring generated by R and I under the operations of R.

Definition 2.4: [33] Let K be a field, the neutrosophic file generated by $\langle K \cup I \rangle$ which is denoted by $K(I) = \langle K \cup I \rangle$.

Definition 2.4: [29] Let $(M, +, .)$ be any R-module over a neutrosophic ring $R(I)$ The triple $(M(I), +, .)$ is called a strong neutrosophic R-module over a neutrosophic ring $R(I)$ generated by M and I.

3. Main concepts and discussion

In the beginning, we will define the basic concepts in neutrosophic Euclidean geometry and then we will study their relations with classical geometry.

Definition 3.1

Let $R(I) = \{a + bI;\ a, b \in R\}$ be the real neutrosophic field, we say that $a + bI \leq c + dI$ if and only if $a \leq c\ and\ a + b \leq c + d$.

Theorem 3.2

The relation defined in Definition 3.1 is a partial order relation.

Proof:

Let $x = a + bI, y = c + dI, z = m + nI \in R(I)$, we have

$x \leq x$ that is because $a \leq a \text{ and } a + b \leq a + b$.

Now, suppose that $x \leq y \text{ and } y \leq x$, then $a \leq c, a + b \leq c + d, c \leq a, c + d \leq a + b$, hence $a = c, a + b = c + d$, which means that $d = b$ and $x = y$.

Assume that $x \leq y \text{ and } y \leq z$, hence $a \leq c, a + b \leq c + d, c \leq m, c + d \leq m + n$, this implies that $a \leq m, a + b \leq m + n$, hence $x \leq z$. Thus $\leq$ is a partial order relation on R(I).

Remark 3.3

According to Theorem 3.2, we are able to define positive neutrosophic real numbers as follows:

$a + bI \geq 0 = 0 + 0.I$ implies that $a \geq 0, a + b \geq 0$.

Absolute value on R(I) can be defined as follows:

$|a + bI| = |a| + I[|a + b| - |a|]$, we can see that $|a + bI| \geq 0$.

We can compute the square root of a neutrosophic positive real number as follows:

$\sqrt{a + bI} = \sqrt{a} + I[\sqrt{a + b} - \sqrt{a}], \textit{it is clear that } (\sqrt{a} + I[\sqrt{a + b} - \sqrt{a}])^2 = a + bI \textit{ and } \sqrt{a + bI} \geq 0.$

Example 3.4

$x = 2 - I$ is a neutrosophic positive real number, since $2 \geq 0 \text{ and } (2 - 1) = 1 \geq 0$.

$2 + I \geq 2$, that is because $2 \geq 2 \text{ and } (2 + 1) = 3 \geq (2 + 0) = 2$.

$|1 + 3I| = |1| + I[|1 + 3| - |1|] = 1 + 3I$.

$\sqrt{9 + 4I} = \sqrt{9} + I[\sqrt{13} - \sqrt{9}] = 3 + (\sqrt{13} - 3)I$.

Definition 3.5

We define the neutrosophic plane with n neutrosophic dimensions (N-dimensions) as follows:

$R(I) \times R(I) \times R(I) \times \dots \times R(I)(n - times)$.

Example 3.6

$R(I) = \{a + bI; a, b \in R\}$ is a neutrosophic plane with one N-dimension.

$R(I)^2 = \{(a + bI, c + dI); a, b, c, d \in R\}$ is a neutrosophic plane with two S-dimensions.

- In the following, we will concentrate on the two N-dimensional neutrosophic plane.

Definition 3.7:

Let $A(a + bI, c + dI), B(x + yI, z + tI)$ be two neutrosophic points from $R(I)^2$, we define:

$\overrightarrow{AB} = ([x + yI] - [a + bI], [z + tI] - [c + dI])$, is called a neutrosophic vector with two N-dimensions.

Definition 3.8

Let $\vec{u} = (a + bI, c + dI)$

Be a neutrosophic vector, we define its norm as follows:

$\|\vec{u}\| = \sqrt{(a + bI)^2 + (c + dI)^2} = \sqrt{a^2 + c^2 + I[(a + b)^2 + (c + d)^2 - a^2 - c^2]}$.

It is easy to see that $\|\vec{u}\| \geq 0, \textit{according to Remark } 3.3$.

Definition 3.9

Let $A(a + bI, c + dI), B(x + yI, z + tI)$ be two neutrosophic points from $R(I)^2$, we define:

(a) The midpoint of [AB] is $C\left(\frac{a+bI+x+yI}{2}, \frac{c+dI+z+tI}{2}\right)$.

(b) The neutrosophic distance between A and B is equal to $\|\overrightarrow{AB}\|$.

Example 3.10

Consider the following neutrosophic points $A(1+I, 2-3I), B(-I, -1+2I)$,

The neutrosophic vector $\overrightarrow{AB} = (-1-2I, -3+5I)$, the square of neutrosophic distance between A and B is $\|\overrightarrow{AB}\|^2 = 1+9+I[9+4-1-9] = 10+3I$.

Hence the neutrosophic distance is equal to $\sqrt{10+3I} = \sqrt{10} + I[\sqrt{13} - \sqrt{10}]$. We can find easily that $(\sqrt{10+3I})^2 = (\sqrt{10} + I[\sqrt{13} - \sqrt{10}])^2 = 10+3I$.

Let C be the neutrosophic midpoint of [AB], then $C\left(\frac{1}{2}, \frac{1}{2} - \frac{1}{2}I\right)$.

Now, we list some geometrical and algebraic properties of the classical space $R^2 \times R^2$. We will need them in forthcoming sections.

Remark 3.11

Let $V = R^2 \times R^2$ be the Cartesian product of the classical Euclidean plane with itself, we have

(a) V has a module structure over the ring $R \times R$, with respect to the following operations:

Addition: $\big((a,b),(c,d)\big) + \big((x,y),(z,t)\big) = \big((a+x, b+y),(c+z, d+t)\big)$,

Multiplication by a duplet scalar from $R \times R$: $(m,n).\big((a,b),(c,d)\big) = \big((m.a, n.b),(m.c, n.d)\big)$.

(b) The norm of any vector in V can be defined as a duplet number from $R \times R$, as follows:

$$\|((a,b),(c,d))\| = \left(\sqrt{a^2+c^2}, \sqrt{b^2+d^2}\right).$$

Example 3.12

Consider the following two points from the space V , $A((1,2),(2,5)), B((-1,4),(3,-2))$, we have:(a) $\overrightarrow{AB} = \big((-2,2),(1,-7)\big)$.

(b) $\|\overrightarrow{AB}\| = \left(\sqrt{(-2)^2+(1)^2}, \sqrt{(2)^2+(-7)^2}\right) = (\sqrt{5}, \sqrt{53})$..

(c) Let $r = (5,8) \in R \times R$ be a duplet scalar, we have:

$r.\overrightarrow{AB} = \big((-10,16),(5,-56)\big)$,it is clear that $\|r.\overrightarrow{AB}\| = r.\|\overrightarrow{AB}\|$.

4. The connection between neutrosophic and classical geometry

This section is devoted to clarify the relationships between neutrosophic coordinates defined above, and between classical geometrical coordinates.

Many important questions arise according to section 3. The first one is about famous relations in classical geometry for example is the midpoint of $[AB]$ has the same neutrosophic distance from A and B?. If the answer is no, then our geometrical system is weak and has no importance because it contradicts with logical statements.

The second, do the neutrosophic points have relationships with classical points? This question is the most important one, that is because if it has a positive answer, then we are able to study geometrical shapes in neutrosophic plane.

The third is about how can we define neutrosophic lines, circles, elliptic curves,… etc.

We try to answer these important questions by using algebra, since the neutrosophic plane with two N-dimensions is a module over the ring $R(I)$.

Definition 4.1: (a) **(Two-dimensional AH-isometry)** Let $M = R(I)^2 = R(I) \times R(I), V = R^2 \times R^2$ Be the neutrosophic plane with two N-dimensions and the Cartesian product of the classical Euclidean space R^2 with itself, we define the AH-isometry map as follows:

$f: M \to V; f(a + bI, c + dI) = ((a, a + b), (c, c + d))$.

(b) (**One dimensional AH-isometry**) We can define the one-dimensional isometry between R(I) and the space R×R as follows:

$g: R(I) \to R \times R; g(a + bI) = (a, a + b)$.

Remark: The one-dimensional isometry is an isometry, i.e., an algebraic isomorphism between R(I) and $R \times R$. Also, it preserves distances on $R(I)$.

Proof: Let $a + bI, c + dI$ be two neutrosophic real numbers, then

$g(a + bI + c + dI) = g([a + c] + [b + d]I) = (a + c, a + c + b + d) = (a, a + b) + (c, c + d) = g(a + bI) + g(c + dI)$.

$g([a + bI].[c + dI]) = g(ac + I[ad + bc + bd]) = (ac, ac + ad + bc + bd) = (a, a + b).(c, c + d) = g(a + bI).g(c + dI)$.

g is a correspondence one-to-one, that is because $Ker(g) = \{0\}$, and for every pair $(a, b) \in R \times R$, there exists $a + (b - a)I \in R(I) such\ that\ g(a + [b - a]I) = (a, b)$. Thus g is an isomorphism.

The distance on $R(I)$ can be defined as follows:

Let $A = a + bI, B = c + dI$ be two neutrosophic real numbers, then $L = \|\overrightarrow{AB}\| = d[(a + bI, c + dI)] = |a + bI - (c + dI)| = |(a - c) + I(b - d)| = |a - c| + I[|a + b - c - d| - |a - c|]$.

(According to the definition of the absolute value in Remark 3.3).

On the other hand, we have:

$g(\|\overrightarrow{AB}\|) = (|a - c|, |(a + b) - (c + d)|) = (d(a, c), d(a + b, c + d)) = d[(a, a + b), (c, c + d)]$=$d(T(a + bI), T(c + dI))$

=$\|g(\overrightarrow{AB})\|$.

This implies that the distance is preserved up to isometry. i.e. $\|g(AB)\| = g(\|AB\|)$

Example 4.2

Consider the following neutrosophic point$A(1 + I, 3 - 6I)$, its isometric image is $((1,2), (3, -3))$.

Consider the following neutrosophic vector $\vec{u} = (2 - I, 4 + I), its\ isometric\ vector\ is\ \vec{v} = ((2,1), (4,5))$.

The idea behind the AH-isometry is to deal with neutrosophic points as classical points, and to explore their properties using classical Euclidean geometry.

The following theorem is considered as the fundamental theorem in neutrosophic Euclidean geometry, since it describes the relation between neutrosophic space with two N-dimensions and the classical module generated by the Cartesian product of the classical Euclidean space by itself.

Theorem 4.3: (Fundamental Theorem In neutrosophic Euclidean Geometry)

Let $f: M \to V; f(a + bI, c + dI) = ((a, a + b), (c, c + d))$ be the AH-isometry defined above, we have:

(a) f preserves addition operation between vectors.

(b) f preserves distances between points.

(c) f is a bijection one-to-one between M and V.

(d) Multiplying a neutrosophic vector by a neutrosophic real number is preserved up to isometry, i.e. The direct image of a neutrosophic vector multiplied by a neutrosophic real number is exactly equal to its AH-isometric image multiplied by the one-dimensional isometric image of the corresponding neutrosophic real number.

Proof:

(a) Let $\vec{u} = (a + bI, c + dI), \vec{v} = (x + yI, z + tI)$ be two neutrosophic vectors, we have

$f(\vec{u} + \vec{v}) = f(a + x + I[b + y], c + z + I[d + t]) = ((a + x, a + x + b + y), (c + z, c + z + d + t))$
$= \big((a, a + b), (c, c + d)\big) + ((x, x + y), (z, z + t))$=$f(\vec{u}) + f(\vec{v})$

(b) We must prove that the norm of the classical vector $\overrightarrow{f(u)}$,
is exactly equal to the one-dimensional isometric image of the norm of neutrosophic vector $\vec{u}$.
$\|f(\vec{u})\|^2 = (a^2 + c^2, (a + b)^2 + (c + d)^2)$, on the other hand, we have
$g(\|\vec{u}\|^2) = g(a^2 + c^2 + I[(a + b)^2 + (c + d)^2 - a^2 - c^2] = (a^2 + c^2, (a + b)^2 + (c + d)^2) = \|f(\vec{u})\|^2$.

(c) Suppose that $f(a + bI, c + dI) = f(x + yI, z + tI)$, hence$\big((a, a + b), (c, c + d)\big) = \big((x, x + y), (z, z + t)\big)$, thus $x = a, b = y, z = c, d = t$, so that f *is* injective.

It is clear that f is surjective, thus it is a bijection.

(d) Consider the following neutrosophic vector $\vec{u} = (a + bI, c + dI)$

With the following neutrosophic real number $m + nI$, *we have*

$(m + nI).\vec{u} = ((m + nI)(a + bI), (m + nI)(c + dI) = ((ma + I[mb + na + nb]), (mc + I[md + nc + nd])$, on the other hand, we have

$$f\big((m + nI).\vec{u}\big) = \Big(\big(ma, (ma + mb + na + nb)\big), (mc, mc + md + nc + nd)\Big)$$

$$= (m, m + n).\big((a, a + b), (c, c + d)\big) = g(m + nI).f(a + bI, c + dI).$$

Example 4.4

Consider the following two neutrosophic points $A(1 + 2I, I), B(3I, -2 + I)$, we have:

(a) The isometric points of A, B are $A' = \big((1,3), (0,1)\big), B' = \big((0,3), (-2, -1)\big)$.

(b) $\overrightarrow{AB} = (-1 + I, -2)$, *the corresponding isometric vector is* $\overrightarrow{A'B'} = \big((-1,0), (-2, -2)\big) = f(\overrightarrow{AB})$.

(c) The neutrosophic distance $[AB] = \sqrt{1 + 4 + I[0 + 4 - 1 - 4]}$
$= \sqrt{5 - I} = \sqrt{5} + I[4 - \sqrt{5}]$. The classical distance between isometric images is

$$[A'B'] = \left(\sqrt{(-1)^2 + (-2)^2}, \sqrt{(0)^2 + (-2)^2}\right) = (\sqrt{5}, 4) = g([AB]).$$

Theorem 4.3 introduces an algorithm to transform any neutrosophic point to a classical Cartesian product of two classical points. The following theorem describes the inverse relation between classical coordinates and neutrosophic coordinates, i.e. It clarifies how to go back from classical coordinates to neutrosophic coordinates.

Theorem 4.5

Let $A((a, b), (c, d))$ be a Cartesian product of two classical points, then the inverse isometric image (the corresponding neutrosophic point) is

$B(a + (b - a)I, c + (d - c)I)$.

Proof:

It holds directly by taking the image of B with respect to AH-isometry, the point A is obtained.

Example 4.6:

Consider the following classical point $A((1,2),(-1,4))$, its corresponding neutrosophic point is $B(1+I,-1+5I)$.

As a result of Section 4, we can find that all geometrical famous properties is still true in neutrosophic Euclidean geometry, that is because we can transform any neutrosophic point to a corresponding classical point with preserving addition, distances, and multiplication by scalars.

5. Some neutrosophic geometrical shapes with two N-dimensions

Definition 5.1: (Neutrosophic circle)

Let $M(a+bI,c+dI)$ be a fixed neutrosophic point, we define the neutrosophic circle with centre M and radius $R=r_1+r_2I\geq 0$ to be the set of all two N-dimensional points $N(X,Y)=N(x_0+x_1I,y_0+y_1I); dist(M,N)=R=const.$

Theorem 5.2:

Let $M(a+bI,c+dI)$ be a fixed neutrosophic point, $R=r_1+r_2I$ be a neutrosophic real positive number, we have:

(a) The equation of the circle with center M and radius R is $([(x_0+x_1I]-[a+bI])^2+([y_0+Iy_1]-[c+dI])^2=R^2$.

(b) The previous neutrosophic circle is equivalent to the following direct product of two classical circles

$C_1:(x_0-a)^2+(y_o-c)^2={r_1}^2, C_2:([x_0+x_1]-[a+b])^2+([y_0+y_1]-[c+d])^2=(r_1+r_2)^2.$

Proof:

(a) By using the neutrosophic distance form defined in Definition 3.8 and Definition 3.9, we get is $([(x_0+x_1I]-[a+bI])^2+([y_0+Iy_1]-[c+dI])^2=R^2$.

(b) To obtain the classical equivalent geometrical system of the neutrosophic circle, it is sufficient to take its isometric image as follows:

$f(([(x_0+x_1I]-[a+bI])^2)+f(([y_0+Iy_1]-[c+dI])^2)=f(R^2)$,hence

$((x_0-a)^2,(x_0+x_1-[a+b])^2)+((y_0-c)^2,(y_0+y_1-[c+d])^2)=({r_1}^2,(r_1+r_2)^2)$,thus

$$\big(((x_0-a)^2+(y_0-c)^2),((x_0+x_1-[a+b])^2+(y_0+y_1-[c+d])^2)\big)=({r_1}^2,(r_1+r_2)^2),$$

Thus, we get $(x_0-a)^2+(y_o-c)^2={r_1}^2$ and $([x_0+x_1]-[a+b])^2+([y_0+y_1]-[c+d])^2=(r_1+r_2)^2$.

Example 5.3:

Consider the following neutrosophic circle: $C:(X-I)^2+(Y-(2-3I))^2=(2+I)^2$

It is equivalent to the direct product of the following two classical circles:

$C_1:(x_0-0)^2+(y_0-2)^2=2^2, C_2:([x_0+x_1]-[-1])^2+([y_0+y_1]-[-1])^2=(2+1)^2.$

Definition 5.4: (Neutrosophic line)

We define the neutrosophic line by the set of all two N-dimensional points (X,Y) with the property

$$AX+BY+C=0; X=x_0+x_1I, Y=y_0+y_1I, A=a_0+a_1I, B=b_0+b_1I, C=c_0+c_1I.$$

Theorem 5.5:

Let $AX+BY+C=0$ be an equation of a neutrosophic line d, this line is equivalent to the direct product of the following two classical lines:

$$d_1: a_0x_0+b_0y_0+c_0=0, d_2:(a_0+a_1)(x_0+x_1)+(b_0+b_1)(y_0+y_1)+c_0+c_1=0.$$

Proof:

By taking the isometric image to the equation $AX + BY + C = 0$, we get the proof.

Example 5.6:

Consider the following neutrosophic line $(1 + I)X + (2 - 4I)Y + 1 - 3I = 0$, it is equivalent to the following two classical lines

$$d_1: x_0 + 2y_0 + 1 = 0, d_2: 2\,(x_0 + x_1) - 2(y_0 + y_1) - 2 = 0.$$

Remark 5.7:

(a) If we have two classical circles $C_1: (x_0 - a)^2 + (y_0 - c)^2 = (r_1)^2, C_2: (x_1 - b)^2 + (y_1 - d)^2 = (r_2)^2$, then we can transform the set of their direct product $C_1 \times C_2$, into one neutrosophic circle by using the inverse image of the AH-isometry as follows:

$C: (X - M)^2 + (Y - N)^2 = r^2; X = x_0 + (x_1 - x_0)I, Y = y_0 + (y_1 - y_0)I, M = a + (b - a)I, N = c + (d - c)I, r = r_1 + (r_2 - r_1)I.$

The proof holds easily by taking the inverse image with respect to AH-isometry.

(b) By the same argument, if we have two classical lines:

$a_0x_0 + b_0y_0 + c_0 = 0, a_1x_1 + b_1y_1 + c_1 = 0$.We can transform the set of their direct product into one neutrosophic line as follows:

$$AX + BY + C = 0; A = a_0 + (a_1 - a_0)I, B = b_0 + (b_1 - b_0)I, X = x_0 + (x_1 - x_0)I, Y = y_0 + (y_1 - y_0)I, C = c_0 + (c_1 - c_0)I.$$

4. Conclusions

In this article, we presented a general definition of neutrosophic Euclidean geometry. We studied the mechanism of comparison between neutrosophic numbers. In the luminosity of our findings, we make a connection between neutrosophic geometrical concepts and classical Euclidean geometry. Further the concept of neutrosophic plane with n neutrosophic dimensions is obtained. Also, Euclidean geometric concepts are extended neutrosophically such as neutrosophic distance, neutrosophic midpoint, neutrosophic vectors, neutrosophic circles, and lines.

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