Complex neutrosophic N-soft sets: A new model with applications

Muhammad Akram
Maria Shabir
Ather Ashraf
Complex neutrosophic $N$-soft sets: A new model with applications

Muhammad Akram$^1$, Maria Shabir$^1$ and Ather Ashraf$^2$
1. Department of Mathematics, University of the Punjab, Lahore, Pakistan.
E-mail: m.akram@pucit.edu.pk, mariashabir204@gmail.com
2. Punjab University College of Information Technology, University of the Punjab, Old Campus, Lahore-54000, Pakistan.
E-mail: atherashraf@gmail.com

Abstract: In this paper we establish the notion of complex single-valued neutrosophic $N$-soft set. It improves the traits of three general models, namely, single-valued neutrosophic sets, single-valued neutrosophic soft sets and single-valued neutrosophic $N$-soft sets, in such way that it makes two dimensional ambiguous information and parameterized grading evaluation compatible. We explain the modeling abilities of complex single-valued neutrosophic $N$-soft sets and investigate some of their fundamental properties. Moreover, the intended approach hinges on rational attributes to support the choice of the most suitable solution. The proposed method is explicated through an example from the islamic banking industry. We also perform a comparative analysis with respect to the neutrosophic TOPSIS method.

Keywords: Complex single-valued neutrosophic set, $N$-soft set, TOPSIS method, MAGDM.

1 Introduction

A fascinating research article by Smarandache [29] has attracted the attention of many researchers since 1998. Neutrosophic sets ($NS$) had been born that year. They are based on formal logic that contemplates the nature, origin, and scope of objectivities with their relations for numerous intellectual spectra. The neutrosophic theory comprises probability, set theory, logics, and statistics. As such it copes with real life events characterized by degree of satisfaction, dissatisfaction and indeterminacy. It is therefore acknowledged to provide a generalization of both classic set, fuzzy set, intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy sets and Pythagorean fuzzy sets [38, 17, 33]. Neutrosophic-inspired sets are classified into many subclasses like interval-valued neutrosophic sets, single-valued neutrosophic sets ($SVNS$), and the subclass known as simplified neutrosophic set. The $SVNS$ were introduced by Wang and Smarandache [31, 30]. They can be characterized by three real valued functions whose values are taken from the unit closed interval $[0, 1]$, therefore it is more convenient and applicable in many areas of science and engineering. After Wang and Smarandache, the single-valued neutrosophic environment has been scrutinized extensively. For example, Ye [34] provided a correlation coefficient between $SVNS$ which became a useful tool for decision making, and Akram and Luqman [6] illustrated the concept of $SVNS$ with the flavor of hypergraphs. Another breakthrough was Ramot et al. [26] who extended the 1-dimensional fuzzy perspective [38] to 2-dimensional phenomena. The resulting model was called complex fuzzy sets. This new perspective prompted many authors to adapt existing models to the complex spirit. Thus complex intuitionistic fuzzy sets [15] and complex Pythagorean fuzzy sets [37], which are precisely related to multi-attribute decision making (MAGDM) phenomena, were soon developed. The two aforementioned expressions of vagueness were made compatible by Ali and Smarandache [13]. These authors put forward the notion of complex neutrosophic set under the influence of both neutrosophic sets [29] and complex fuzzy sets [26]. In MAGDM problems, the opinions of people are not invariably expressed through binary evaluations. It is often easier to bring up decisions using non-binary evaluations, specifically in the case of qualitative information such as the perceived performance of banking industry, people’s morality, hospital assistance, etc. Hence, Fatimah et al. [21] firstly presented $NS$ sets and applied them on decision making methods based on non-binary evaluations. $N$-soft sets extended the scope of soft sets [25] whose foundation is that any alternative can be characterized by a selected list of attributes. Many real examples were given [11, 21]. Stimulated from the novel concept of $N$-soft set, Akram et al. [5] solved decision making problems using the hybrid combination of fuzzy set with $N$-soft set that improves the performance of fuzzy soft sets [10]. Further, Akram et al. [9] presented the novel idea of intuitionistic fuzzy $N$-soft sets ($IFNS$), Pythagorean fuzzy $N$-soft sets ($PFNS$) have been introduced by Zhang [39] in 2020, and recently the multi-fuzzy $N$-soft set model has been presented alongside its applications to decision-making [22]. This proves that $N$-soft sets are a trendy topic and that the model is amenable to hybridization from many standpoints including rough set theory [11] and hesitancy [4] in addition to the ideas discussed above. The theoretical models called neutrosophic soft sets ($NS$) and single-valued neutrosophic soft sets ($SVNS$) were put forward by Maji [40] and Jana et al. [23], respectively. The parametrized nature of the attributes that characterizes soft set theory is combined with neutrosophic...
information and the possibilities of these new models are discussed in detail. Ashraf and Butt [16] and Riaz et al. [27] first established a theoretical model for neutrosophic N-soft sets (NNS_{S_{f}}Ss). They made applications to business and the medical field supported by the TOPSIS method, respectively. Moreover, Sahin et al. [28] used the framework of (SVN_{S_{f}}Ss) for the development of a TOPSIS method which helped to find the most suitable supplier for a production industry. In 2015, Ye [36] introduced single-valued neutrosophic linguistic numbers (SVNLNs) as an extension of intuitionistic linguistic numbers and further set theoretical description for single-valued neutrosophic linguistic-TOPSIS method. More recently, Akram et al. [7, 8] have presented new decision making methods.

In this manuscript we present a quite general model known as complex single-valued neutrosophic -TOPSIS technique for solving MAGDM problems that require the use of CSVN_{S_{f}}S information. For this purpose, we define some basic notions and the CSVN_{S_{f}}Ss and CSVN_{S_{f}} averaging and geometric operators. These operators allow us to combine the decisions according to the performance of the alternatives and the weightage of the relevant attributes and experts. We also define score and accuracy function of CSVN_{S_{f}}Ns for the sake of CSVN_{S_{f}}-PIS and CSVN_{S_{f}}-NIS. Finally, we can sort out the alternatives using a revised closeness index whose values are totally based upon the normalized Euclidean distance.

The authenticity of the presented technique is verified by a numerical example that concerns the monitoring performance of the Islamic banking industry on the basis of the CAMELS rating system. Moreover, a comparison of the proposed model with the SVN-TOPSIS method substantiates the accuracy and reliability of the results and of our novel technique. For further useful notions related to N-soft sets not discussed in the paper, the readers are referred to [1, 2, 12].

The practical contribution of this article is the formalization of the CSVN_{S_{f}}S-TOPSIS technique for solving MAGDM problems that require the use of CSVN_{S_{f}}S information. For this purpose, we define some basic notions and the CSVN_{S_{f}}Ss and CSVN_{S_{f}} averaging and geometric operators. These operators allow us to combine the decisions according to the performance of the alternatives and the weightage of the relevant attributes and experts. We also define score and accuracy function of CSVN_{S_{f}}Ns for the sake of CSVN_{S_{f}}-PIS and CSVN_{S_{f}}-NIS. Finally, we can sort out the alternatives using a revised closeness index whose values are totally based upon the normalized Euclidean distance.

The authenticity of the presented technique is verified by a numerical example that concerns the monitoring performance of the Islamic banking industry on the basis of the CAMELS rating system. Moreover, a comparison of the proposed model with the SVN-TOPSIS method substantiates the accuracy and reliability of the results and of our novel technique. For further useful notions related to N-soft sets not discussed in the paper, the readers are referred to [1, 2, 12].

The arrangement of this paper is as follows. Section 2 contains some basic definitions related to the proposed model. In Section 3 we describe the main features of the presented theory with some operations and properties. Section 4 presents the score function, accuracy function and some aggregation operators related to CSVN_{S_{f}}Ns. Section 5, gives a brief description for the CSVN_{S_{f}}S-TOPSIS method with a specific algorithm. Section 6, models a MAGDM problem and applies the proposed technique to find a solution. Section 7 comprises the comparison analysis with the CSVN_{S}-TOPSIS method. In Section 8, we come to the conclusion with some ideas for future research works.

2 Preliminaries

Definition 1. [29] A neutrosophic set (NS) \( \Psi \) on a universe of discourse \( U \) has the form:
\[
\Psi = \{ (u, T_\Psi (u), I_\Psi (u), F_\Psi (u) : u \in U \},
\]
where, \( T_\Psi (u), I_\Psi (u) \) and \( F_\Psi (u) \) are degree of satisfaction, degree of indeterminacy and degree of dissatisfation, respectively, belongs to non-standard interval \([-0,1^+]\), for every \( u \in U \).

Definition 2. [31] A single-valued neutrosophic set (SVN) \( \Psi \) on a universe of discourse \( U \) has the form
\[
\Psi = \{ (u, T_\Psi (u), I_\Psi (u), F_\Psi (u) : u \in U \},
\]
where \( T_\Psi (u), I_\Psi (u), F_\Psi (u) \) : \( U \rightarrow [0,1] \) are the degree of truthness, degree of hesitancy and degree of falsity, respectively, without any condition on the sum of \( T_\Psi (u), I_\Psi (u) \) and \( F_\Psi (u) \) for all \( u \in U \). The triplet \( (T_\Psi , I_\Psi , F_\Psi ) \) is called single-valued neutrosophic number (SVN).

Definition 3. [13] A complex single-valued neutrosophic set (CSVN) \( \Psi \), on the universe \( U \) is defined as:
\[
\Psi = \{ (u, T_\Psi (u), I_\Psi (u), F_\Psi (u) : u \in U \},
\]
where \( T_\Psi (u) = p_\Psi (u)e^{i2\pi f_\Psi (u)}, I_\Psi (u) = q_\Psi (u)e^{i2\pi \omega_\Psi (u)} \) and \( F_\Psi (u) = r_\Psi (u)e^{i2\pi \omega_\Psi (u)} \), denote the degree of truthness, degree of hesitancy and degree of falsity, respectively; without any conditions on the sum of amplitude terms \( p_\Psi (u), q_\Psi (u), r_\Psi (u) : U \rightarrow [0,1] \) or the phase terms \( f_\Psi (u), \omega_\Psi (u), \omega_\Psi (u) : U \rightarrow [0,1] \) for all \( u \in U \). The triplet \( (p_\Psi (u)e^{i2\pi f_\Psi (u)}, q_\Psi (u)e^{i2\pi \omega_\Psi (u)}, r_\Psi (u)e^{i2\pi \omega_\Psi (u)} ) \) is called complex single-valued neutrosophic number (CSVN).

Definition 4. [25] Let \( U \) be a non-empty set and \( K \) be a set of parameters and \( Y \subseteq K \). A soft set \( S_{f}S \) over \( U \) is a pair \( (\Phi, Y) \), where \( \Phi : K \rightarrow P(U) \) is a set-valued function defined as:
\[
(\Phi, Y) = \{(y_{u}, \Phi(y_{u})) | y_{u} \in Y, \Phi(y_{u}) \in P(U)\}.
\]

M. Akram, M. Shabir, A. Ashraf, Complex neutrosophic N-soft sets: A new model with applications.
Definition 5. Let $\mathbb{U}$ be a non-empty set and $K$ be a set of parameters and $Y \subseteq K$. A complex single-valued neutrosophic soft set $CSV.NS_fS$ over $\mathbb{U}$ is a pair $(\Phi, Y)$, where $\Phi: K \rightarrow \mathcal{P}(CSV.NS)$ is a set-valued function defined as:

\[
(\Phi, Y) = \{(y_w, \Phi(y_w)) | y_w \in Y, \Phi(y_w) \in \mathcal{P}(CSV.NS)\} = \{(y_w, (u_s, (T_{us}, l_{us}, F_{us})))\} = \{(y_w, u_s, (p_{us} e^{\|e^{2\pi w_s} q_{us} e^{i2\pi w_s}, r_{us} e^{i2\pi f_{us}}}))\},
\]

where $\mathcal{P}(CSV.NS)$ is the collection of all subsets of $CSV.NS$s over the non-empty set $\mathbb{U}$ and $p_{us}, l_{us}, q_{us}, r_{us}, f_{us} \in [0, 1]$.

Definition 6. [21] Let $\mathbb{U}$ be a non-empty set and $K$ be a set of parameters and $Y \subseteq K$. Let $H = \{0, 1, 2, \ldots, N - 1\}$ be a set of ordered grades with $N \in \{2, 3, \ldots\}$. A triple $(\Phi, Y, N)$ is called $N$-soft set $(NS_fS)$ over $\mathbb{U}$ if $\Phi$ is a mapping define as $\Phi: Y \rightarrow 2^{\mathbb{U} \times H}$, that is there exist a unique pair $(u_s, h^*_w) \in \mathbb{U} \times H$ such that $(u_s, h^*_w) \in \Phi(y_w)$, where $u_s \in \mathbb{U}, h^*_w \in H$.

3 Complex single-valued neutrosophic $N$-soft sets

Definition 7. Let $\mathbb{U}$ be a non-empty set and $K$ be a set of parameters with $Y \subseteq K$. Let $H = \{0, 1, 2, \ldots, N - 1\}$ be a set of ordered grades with $N \in \{2, 3, \ldots\}$. A triple $(\Phi_y, Y, N)$ is called a complex single-valued neutrosophic $N$-soft set $(CSV.NS_fS)$ on $\mathbb{U}$, if $(\Phi, Y, N)$ is an $NS_fS$ on $\mathbb{U}$, and $\Phi_y: Y \rightarrow 2^{\mathbb{U} \times H}$ and $CSV.NS$ is a mapping, which is defined as:

\[
\Phi_y(y_w) = \{(\Phi(y_w), \Psi(y_w)) | y_w \in Y\} = \{((u_s, h^*_w), (T_{us}, l_{us}, F_{us}))\} = \{((u_s, h^*_w), (p_{us} e^{\|e^{2\pi w_s} q_{us} e^{i2\pi w_s}, r_{us} e^{i2\pi f_{us}}}))\},
\]

where $\Phi: Y \rightarrow 2^{\mathbb{U} \times H}$, $\Psi: Y \rightarrow CSV.NS$, and $CSV.NS$ denotes the collection of all complex single-valued neutrosophic numbers of $\mathbb{U}$, $h^*_w$ denotes the rank of parameter for the alternative $y_w$ and $p_{us}, l_{us}, q_{us}, r_{us}, f_{us} \in [0, 1]$, with no conditions on their sum.

Example 1. Let $\mathbb{U} = \{U_1 = Emirates, U_2 = Eithad Airways, U_3 = Turkish airlines, U_4 = Flynas\}$ be the set of airlines from Pakistan to Turkey and $Y = \{Y_1 = Price, Y_2 = Entertainment, Y_3 = luxuries, Y_4 = Safety\}$ be the characteristics which are experienced by the passengers and then passengers assigned ratings to these airlines. These ratings are aggregated by the experts and form a 6-soft set given Table 1, where

0 means ‘very Bad’
1 means ‘Bad’
2 means ‘Ok’
3 means ‘Good’
4 means ‘Great’
5 means ‘Excellent’

<table>
<thead>
<tr>
<th>$Y/U$</th>
<th>$U_1$</th>
<th>$U_2$</th>
<th>$U_3$</th>
<th>$U_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_1$</td>
<td>3</td>
<td>5</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$Y_2$</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>$Y_3$</td>
<td>2</td>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>$Y_4$</td>
<td>5</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

For handling the alternatives with fuzziness property related to parameters, we need $CSV.NS_fS$s. Therefore, authorities defined grading criteria, given in Table 2, for the evaluation of airlines under the environment of $CSV.NS_fS$s, where Table 2 is evaluated from the following criteria:

\[
\begin{align*}
\text{when } h^*_w &= 0, \quad -4.00 \leq S(\Psi) < -3.30, \\
\text{when } h^*_w &= 1, \quad -3.30 \leq S(\Psi) < -2.20, \\
\text{when } h^*_w &= 2, \quad -2.20 \leq S(\Psi) < -1.00, \\
\text{when } h^*_w &= 3, \quad -1.00 \leq S(\Psi) < 0.20, \\
\text{when } h^*_w &= 4, \quad 0.20 \leq S(\Psi) < 1.20, \\
\text{when } h^*_w &= 5, \quad 1.20 \leq S(\Psi) \leq 2.00. \\
\end{align*}
\]

M. Akram, M. Shabir, A. Ashraf, Complex neutrosophic $N$-soft sets: A new model with applications.
Table 2: Grading criteria for CSVN6SS

<table>
<thead>
<tr>
<th>$h^w_u / J$</th>
<th>degree of truthness</th>
<th>degree of indeterminacy</th>
<th>degree of falsity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p_w$</td>
<td>$q_w$</td>
<td>$r_w$</td>
</tr>
<tr>
<td>$h^w_u = 0$</td>
<td>$0.00, 0.15$</td>
<td>$(0.0, 0.3\pi)$</td>
<td>$(1.8\pi, 2.0\pi)$</td>
</tr>
<tr>
<td>$h^w_u = 1$</td>
<td>$(0.15, 0.30)$</td>
<td>$(0.3\pi, 0.6\pi)$</td>
<td>$(1.4\pi, 1.8\pi)$</td>
</tr>
<tr>
<td>$h^w_u = 2$</td>
<td>$(0.30, 0.50)$</td>
<td>$(0.6\pi, 1.0\pi)$</td>
<td>$(1.0\pi, 1.4\pi)$</td>
</tr>
<tr>
<td>$h^w_u = 3$</td>
<td>$(0.50, 0.70)$</td>
<td>$(1.0\pi, 1.4\pi)$</td>
<td>$(0.6\pi, 1.0\pi)$</td>
</tr>
<tr>
<td>$h^w_u = 4$</td>
<td>$(0.70, 0.90)$</td>
<td>$(1.4\pi, 1.8\pi)$</td>
<td>$(0.3\pi, 0.6\pi)$</td>
</tr>
<tr>
<td>$h^w_u = 5$</td>
<td>$(0.90, 1.00)$</td>
<td>$(1.8\pi, 2.0\pi)$</td>
<td>$(0.0, 0.3\pi)$</td>
</tr>
</tbody>
</table>

Using the prescribed information, the CSVN6SS, shown in 3, is defined as:

\[
\Phi(Y_1) = \{((U_1), 3) \cdot (0.6e^{1.26\pi}, 0.35e^{0.68\pi}, 0.4e^{0.84\pi})\}, ((U_2), 5) \cdot (0.95e^{1.92\pi}, 0.05e^{0.12\pi}, 0.12e^{0.26\pi}))
\]

\[
\Phi(Y_2) = \{((U_1), 1) \cdot (0.17e^{0.40\pi}, 0.75e^{1.48\pi}, 0.81e^{1.66\pi})\}, ((U_2), 4) \cdot (0.81e^{1.66\pi}, 0.22e^{0.42\pi}, 0.25e^{0.48\pi}))
\]

\[
\Phi(Y_3) = \{((U_1), 2) \cdot (0.36e^{0.74\pi}, 0.58e^{1.18\pi}, 0.54e^{1.10\pi})\}, ((U_2), 1) \cdot (0.24e^{0.50\pi}, 0.86e^{1.70\pi}, 0.87e^{1.72\pi}))
\]

\[
\Phi(Y_4) = \{((U_1), 3) \cdot (0.17e^{1.06\pi}, 0.52e^{1.12\pi}, 0.52e^{1.06\pi})\}, ((U_2), 3) \cdot (0.75e^{1.52\pi}, 0.17e^{0.36\pi}, 0.20e^{0.38\pi}))
\]

\[
\Phi(Y_5) = \{((U_1), 5) \cdot (0.98e^{1.94\pi}, 0.01e^{0.04\pi}, 0.1e^{0.24\pi})\}, ((U_2), 0) \cdot (0.03e^{0.10\pi}, 0.91e^{1.84\pi}, 0.93e^{1.88\pi}))
\]

Table 3: The CSVN6SS ($\Phi, Y, 6$)

<table>
<thead>
<tr>
<th>($\Phi, Y, 6$)</th>
<th>$U_1$</th>
<th>$U_2$</th>
<th>$U_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi(Y_1)$</td>
<td>$(0.6e^{1.26\pi}, 0.35e^{0.68\pi}, 0.4e^{0.84\pi})$</td>
<td>$(0.95e^{1.92\pi}, 0.05e^{0.12\pi}, 0.12e^{0.26\pi})$</td>
<td>$(0.17e^{0.40\pi}, 0.75e^{1.48\pi}, 0.81e^{1.66\pi})$</td>
</tr>
<tr>
<td>$\Phi(Y_2)$</td>
<td>$(0.81e^{1.66\pi}, 0.22e^{0.42\pi}, 0.25e^{0.48\pi})$</td>
<td>$(0.36e^{0.74\pi}, 0.58e^{1.18\pi}, 0.54e^{1.10\pi})$</td>
<td>$(0.24e^{0.50\pi}, 0.86e^{1.70\pi}, 0.87e^{1.72\pi})$</td>
</tr>
<tr>
<td>$\Phi(Y_3)$</td>
<td>$(0.75e^{1.52\pi}, 0.17e^{0.36\pi}, 0.20e^{0.38\pi})$</td>
<td>$(0.98e^{1.94\pi}, 0.01e^{0.04\pi}, 0.1e^{0.24\pi})$</td>
<td>$(0.03e^{0.10\pi}, 0.91e^{1.84\pi}, 0.93e^{1.88\pi})$</td>
</tr>
<tr>
<td>$\Phi(Y_5)$</td>
<td>$(0.98e^{1.94\pi}, 0.01e^{0.04\pi}, 0.1e^{0.24\pi})$</td>
<td>$(0.03e^{0.10\pi}, 0.91e^{1.84\pi}, 0.93e^{1.88\pi})$</td>
<td>$(0.24e^{0.50\pi}, 0.86e^{1.70\pi}, 0.87e^{1.72\pi})$</td>
</tr>
</tbody>
</table>

Definition 8. A CSVN6S($\Phi, Y, N$) over a non-empty set $U$ is said to be efficient where $(\Phi, Y, N)$ is an $NS_{fS}$ if $\Phi(y_w) = (u_s, N - 1), 1, 0, 0$ for some $y_w \in Y, u_s \in U$.

Example 2. Let $(\Phi, Y, 6)$ be CSVN6S, as in Example 1. From Table 3, it is clear that Example 1 is not efficient.

Definition 9. Let $(\Phi, Y, N_1)$ and $(\chi_A, C, N_2)$ be two CSVN6Ss on a universe of discourse $U$. Then, they are said to be equal if and only if $\Phi = \chi$, $\Psi = A$, $Y = C$ and $N_1 = N_2$.

Definition 10. Let $(\Phi, Y, N)$ be a CSVN6S on $U$. The weak complement of CSVN6S is defined as the weak complement of the N-soft set $(\Phi, Y, N)$, that is, any $N$-soft set such that $\Phi^w(y_w) \cap \Phi(y_w) = \emptyset$ for all $y_w \in Y$. The weak complement of CSVN6S of $(\Phi, Y, N)$ is represented as $(\Phi^w, Y, N)$.

Example 3. Let $(\Phi, Y, 6)$ be CSVN6S, as in Example 1. The weak complement $(\Phi^w, Y, N)$ is given in Table 4.

Table 4: A weak complement of the CSVN6S ($\Phi^w, Y, 6$)

<table>
<thead>
<tr>
<th>($\Phi^w, Y, 6$)</th>
<th>$U_1$</th>
<th>$U_2$</th>
<th>$U_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\Phi^w, Y_1)$</td>
<td>$(0.98e^{1.94\pi}, 0.01e^{0.04\pi}, 0.1e^{0.24\pi})$</td>
<td>$(0.03e^{0.10\pi}, 0.91e^{1.84\pi}, 0.93e^{1.88\pi})$</td>
<td>$(0.24e^{0.50\pi}, 0.86e^{1.70\pi}, 0.87e^{1.72\pi})$</td>
</tr>
<tr>
<td>$(\Phi^w, Y_2)$</td>
<td>$(0.75e^{1.52\pi}, 0.17e^{0.36\pi}, 0.20e^{0.38\pi})$</td>
<td>$(0.98e^{1.94\pi}, 0.01e^{0.04\pi}, 0.1e^{0.24\pi})$</td>
<td>$(0.03e^{0.10\pi}, 0.91e^{1.84\pi}, 0.93e^{1.88\pi})$</td>
</tr>
<tr>
<td>$(\Phi^w, Y_3)$</td>
<td>$(0.03e^{0.10\pi}, 0.91e^{1.84\pi}, 0.93e^{1.88\pi})$</td>
<td>$(0.24e^{0.50\pi}, 0.86e^{1.70\pi}, 0.87e^{1.72\pi})$</td>
<td>$(0.03e^{0.10\pi}, 0.91e^{1.84\pi}, 0.93e^{1.88\pi})$</td>
</tr>
</tbody>
</table>

M. Akram, M. Shabir, A. Ashraf, Complex neutrosophic N-soft sets: A new model with applications.
Definition 11. Let \((\Phi, Y, N)\) be a CSV\(\tilde{N}\)S\(\tilde{J}\)S\(\tilde{S}\) on \(U\). The strong complement of CSV\(\tilde{N}\)S\(\tilde{J}\)S\(\tilde{S}\), denoted as \((\Phi', Y, N)\), is defined as:
\[
\Phi'(y_u) = \begin{cases} 
  h^c_w - 1, & \text{if } h^c_w = (N - 1) - h_w, \\
  (N - 1) - h_w, & \text{otherwise,}
\end{cases}
\]
for all \(y_u \in Y\) and \(u_s \in U\), satisfying the condition \((\Phi, Y, N) \cap (\Phi', Y, N) = \emptyset\).

Example 4. Let \((\Phi, Y, 6)\) be CSV\(\tilde{N}\)S\(\tilde{J}\)S\(\tilde{S}\), then the strong complement \((\Phi', Y, 6)\) of Example 1 is given in Table 5 such that \((\Phi, Y, 6) \cap (\Phi', Y, 6) = \emptyset\).

Table 5: Strong complement of \((\Phi, Y, 6)\)
\[
\begin{array}{cccc}
\Phi(Y, 6) & \Phi'(Y, 6) & U_1 & U_2 \\
(1) & (2) & (3) & (4) \\
\end{array}
\]

Proposition 12. A strong complement of CSV\(\tilde{N}\)S\(\tilde{J}\)S\(\tilde{S}\) is also a weak complement but weak complement may or may not be strong complement.

Proof. The proof is straightforward from the definitions of strong complement and weak complement.

Definition 13. Let \((\Phi, Y, N)\) be a CSV\(\tilde{N}\)S\(\tilde{J}\)S\(\tilde{S}\) on \(U\). The complex single-valued neutrosophic complement of CSV\(\tilde{N}\)S\(\tilde{J}\)S\(\tilde{S}\) is denoted as \((\Phi\tilde{c}, Y, N)\) and is defined as
\[
\Psi(Y, u_s) = \left\{ \left( u_s, h^c_w, (F_{u_s}, 1 - \Pi_{u_s}, T_{u_s}) \right) = \left( u_s, h^c_w, (r_{u_s}e^{2\pi f_{u_s}}, (1 - q_{u_s})e^{2\pi(1-\omega_{u_s})}, p_{u_s}e^{2\pi \tau_{u_s}}) \right) \right\}.
\]

Example 5. Let \((\Phi, Y, 6)\) be CSV\(\tilde{N}\)S\(\tilde{J}\)S\(\tilde{S}\), as in Example 1. The complex single-valued neutrosophic complement \((\Phi\tilde{c}, Y, N)\), is given in Table 6.

Table 6: The complex single-valued neutrosophic complement \((\Phi\tilde{c}, Y, N)\) of the CSV\(\tilde{N}\)S\(\tilde{J}\)S\(\tilde{S}\)
\[
\begin{array}{cccc}
\Phi(Y, 6) & \Phi\tilde{c}(Y, 6) & U_1 & U_2 \\
(1) & (2) & (3) & (4) \\
\end{array}
\]

Definition 14. Let \((\Phi, Y, N)\) be a CSV\(\tilde{N}\)S\(\tilde{J}\)S\(\tilde{S}\) on \(U\). \(F_{\tilde{c}}(Y, Z, N)\) is referred to as a weak complex single-valued neutrosophic complement of \((\Phi\tilde{c}, Y, N)\) if and only if \((\Phi, Y, N)\) is a weak complement and \((\Phi\tilde{c}, Y, N)\) is a complex single-valued neutrosophic complement of \((\Phi, Y, N)\).

Example 6. Let \((\Phi, Y, 6)\) be CSV\(\tilde{N}\)S\(\tilde{J}\)S\(\tilde{S}\), as in Example 1. The weak complex single-valued neutrosophic complement \((\Phi\tilde{c}, Y, N)\), is given in Table 7.

Table 7: The weak complex-valued neutrosophic complement \((\Phi\tilde{c}, Y, 6)\) of the CSV\(\tilde{N}\)S\(\tilde{J}\)S\(\tilde{S}\)
\[
\begin{array}{cccc}
\Phi(Y, 6) & \Phi\tilde{c}(Y, 6) & U_1 & U_2 \\
(1) & (2) & (3) & (4) \\
\end{array}
\]

Definition 15. Let \((\Phi, Y, N)\) be a CSV\(\tilde{N}\)S\(\tilde{J}\)S\(\tilde{S}\) on \(U\), then the strong complex single-valued neutrosophic complement \((\Phi\tilde{c}, Y, N)\) is defined as a strong complement \((\Phi, Y, N)\) and a complex single-valued neutrosophic complement \((\Phi\tilde{c}, Y, N)\) of \((\Phi, Y, N)\), defined as:
\[
\Phi\tilde{c}(y_u) = \begin{cases} 
  \left( h^c_w - 1, (r_{u_s}e^{2\pi f_{u_s}}, (1 - q_{u_s})e^{2\pi(1-\omega_{u_s})}, p_{u_s}e^{2\pi \tau_{u_s}}) \right) & \text{if } h^c_w = (N - 1) - h_w, \\
  \left( (N - 1) - h^c_w, (r_{u_s}e^{2\pi f_{u_s}}, (1 - q_{u_s})e^{2\pi(1-\omega_{u_s})}, p_{u_s}e^{2\pi \tau_{u_s}}) \right) & \text{otherwise,}
\end{cases}
\]
for all \(y_u \in Y\) and \(u_s \in U\).
Example 7. Let \((\Phi, Y, 6)\) be CSVN\(S_3S_5S_7 S\) on \(U\), then the strong single-valued neutrosophic complement \((\Phi^c, Y, N)\) of \((\Phi, Y, 6)\) arranged in Table 3, is calculated in Table 8.

<table>
<thead>
<tr>
<th>((\Phi, Y, 6))</th>
<th>(U_1)</th>
<th>(U_2)</th>
<th>(U_3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Y_1)</td>
<td>((2, 0.42^0.12\pi, 0.16^0.12\pi, 0.71^0.12\pi), 0.69^1\pi)</td>
<td>((3, 0.21^0.12\pi, 0.16^0.12\pi, 0.71^0.12\pi), 0.69^1\pi)</td>
<td>((2, 0.42^0.12\pi, 0.16^0.12\pi, 0.71^0.12\pi), 0.69^1\pi)</td>
</tr>
<tr>
<td>(Y_2)</td>
<td>((3, 0.21^0.12\pi, 0.16^0.12\pi, 0.71^0.12\pi), 0.69^1\pi)</td>
<td>((3, 0.21^0.12\pi, 0.16^0.12\pi, 0.71^0.12\pi), 0.69^1\pi)</td>
<td>((3, 0.21^0.12\pi, 0.16^0.12\pi, 0.71^0.12\pi), 0.69^1\pi)</td>
</tr>
<tr>
<td>(Y_3)</td>
<td>((3, 0.21^0.12\pi, 0.16^0.12\pi, 0.71^0.12\pi), 0.69^1\pi)</td>
<td>((3, 0.21^0.12\pi, 0.16^0.12\pi, 0.71^0.12\pi), 0.69^1\pi)</td>
<td>((3, 0.21^0.12\pi, 0.16^0.12\pi, 0.71^0.12\pi), 0.69^1\pi)</td>
</tr>
</tbody>
</table>

Proposition 16. Let \((\Phi, Y, N)\) be weak and strong complex single-valued neutrosophic complement of CSVN\(S_3S_5S_7 S\) \((\Phi, Y, N)\), then

1. \(((\Phi^c)^c, Y, N) \neq (\Phi, Y, N)\)
2. \(((\Phi^c)^c, Y, N) \neq (\Phi, Y, N)\)

3. \((\Phi, Y, N) \neq (\Phi, Y, N)\)
4. \((\Phi, Y, N) \neq (\Phi, Y, N)\)

Proof. The proof is straightforward from the definitions.

Definition 17. Let \(U\) be a non-empty set and \((\Phi, Y, N)\) and \((X, C, N)\) be CSVN\(S_3S_5S_7 S\) and CSVN\(S_3S_5S_7 S\) on \(U\), respectively, their restricted intersection is defined as \((L_M, G, O) = (\Phi, Y, N) \cap (X, C, N)\), with \(L_M = \Phi \cap X, G = Y \cap C, O = \min(N, N)\), etc., \(\forall x \in G, u_x \in U\) we have

\[
L_M(x_u) = \langle (h^1_u, h^2_u, \tau^1_u, \tau^2_u, F_u) \rangle,
\]

\[
\langle \min(h^1_u, h^2_u), \min(\tau^1_u, \tau^2_u) \rangle e^{2\pi \max(\omega^1_x, \omega^2_x)}, \max(q^1_u, q^2_u) e^{2\pi \max(\omega^1_x, \omega^2_x)}, \max(r^1_u, r^2_u) e^{2\pi \max(\omega^1_x, \omega^2_x)} \rangle, \quad \text{where} \quad \langle h^1_u, \tau^1_u \rangle = (h^1_u, \tau^1_u), \quad \langle h^2_u, \tau^2_u \rangle = (h^2_u, \tau^2_u)
\]

Example 8. The restricted intersection \((L_M, G, O)\) of \((\Phi, Y, 6)\) and \((X, C, 5)\), given in Table 3 and Table 9, arranged in Table 10.

<table>
<thead>
<tr>
<th>((L_M, G, 56))</th>
<th>(U_1)</th>
<th>(U_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Y_1)</td>
<td>((2, 0.42^0.12\pi, 0.16^0.12\pi, 0.71^0.12\pi), 0.69^1\pi)</td>
<td>((3, 0.21^0.12\pi, 0.16^0.12\pi, 0.71^0.12\pi), 0.69^1\pi)</td>
</tr>
<tr>
<td>(Y_2)</td>
<td>((3, 0.81^0.14\pi, 0.17^0.14\pi, 0.90^0.14\pi))</td>
<td>((3, 0.81^0.14\pi, 0.17^0.14\pi, 0.90^0.14\pi))</td>
</tr>
<tr>
<td>(Y_3)</td>
<td>((3, 0.81^0.14\pi, 0.17^0.14\pi, 0.90^0.14\pi))</td>
<td>((3, 0.81^0.14\pi, 0.17^0.14\pi, 0.90^0.14\pi))</td>
</tr>
<tr>
<td>(Y_4)</td>
<td>((3, 0.81^0.14\pi, 0.17^0.14\pi, 0.90^0.14\pi))</td>
<td>((3, 0.81^0.14\pi, 0.17^0.14\pi, 0.90^0.14\pi))</td>
</tr>
</tbody>
</table>

Definition 18. Let \((\Phi, Y, N_1)\) and \((X, C, N_2)\) be CSVN\(S_3S_5S_7 S\) and CSVN\(S_3S_5S_7 S\) on \(U\), respectively, their extended intersection is defined as \((D_Q, T, S) = (\Phi, Y, N) \cap (X, C, N)\), with \(D_Q = \Phi \cap X, T = Y \cup C, S = \max(N_1, N_2)\), that is, \(\forall x \in T \in U\) and \(u_x \in U\) we have

\[
D_Q(x_u) = \begin{cases} 
(h^1_u, \tau^1_u), & \text{if } x_u \in Y - C, \\
(h^2_u, \tau^2_u), & \text{if } x_u \in C - Y, \\
\left(\min(h^1_u, h^2_u), \min(\tau^1_u, \tau^2_u)\right) e^{2\pi \max(\omega^1_x, \omega^2_x)}, \max(q^1_u, q^2_u) e^{2\pi \max(\omega^1_x, \omega^2_x)}, \max(r^1_u, r^2_u) e^{2\pi \max(\omega^1_x, \omega^2_x)} & \text{if } x_u \in C \cap Y
\end{cases}
\]
where \((h_{1w}^{1s}, (T_{ws}^{1w} I_{ws}^{1w}, F_{ws}^{1w})) = (h_{1w}^{1s}, (p_{ws} e^{(2\pi r_{ws}^{1w})}, q_{ws} e^{(2\pi r_{ws}^{1w})}, r_{ws} e^{(2\pi r_{ws}^{1w})})) \in \Phi_{w} \) and \((h_{2w}^{2s}, (T_{ws}^{2w} I_{ws}^{2w}, F_{ws}^{2w})) = (h_{w}^{2s}, (p_{ws}^{2} e^{(2\pi r_{ws}^{2w})}, q_{ws}^{2} e^{(2\pi r_{ws}^{2w})}, r_{ws}^{2} e^{(2\pi r_{ws}^{2w})})) \in \chi_{A} \).

**Example 9.** The extended intersection \((D_{Q}, T, 6)\) of \((\Phi_{w}, Y, 6)\) and \((\chi_{A}, C, 5)\), given in Table 3 and Table 9, arranged in 11.

**Table 11: The extended intersection \((D_{Q}, T, \mathcal{G})\)**

<table>
<thead>
<tr>
<th>((D_{Q}, T, \mathcal{G}))</th>
<th>(U_{1})</th>
<th>(U_{2})</th>
<th>(U_{3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Y_{1})</td>
<td>((0.60 e^{(11.26\pi)}, 0.35 e^{(0.68\pi)}, 0.40 e^{(0.84\pi)}))</td>
<td>((0.95 e^{(11.26\pi)}, 0.05 e^{(0.12\pi)}, 0.12 e^{(0.26\pi)}))</td>
<td>((0.60 e^{(11.26\pi)}, 0.35 e^{(0.68\pi)}, 0.40 e^{(0.84\pi)}))</td>
</tr>
<tr>
<td>(Y_{2})</td>
<td>((2.0 e^{(0.82\pi)}, 0.51 e^{(1.04\pi)}, 0.56 e^{(1.10\pi)}))</td>
<td>((4.0 e^{(1.78\pi)}, 0.99 e^{(0.16\pi)}, 0.06 e^{(0.10\pi)}))</td>
<td>((0.60 e^{(11.26\pi)}, 0.35 e^{(0.68\pi)}, 0.40 e^{(0.84\pi)}))</td>
</tr>
<tr>
<td>(Y_{3})</td>
<td>((2.0 e^{(0.82\pi)}, 0.51 e^{(1.04\pi)}, 0.56 e^{(1.10\pi)}))</td>
<td>((4.0 e^{(1.78\pi)}, 0.99 e^{(0.16\pi)}, 0.06 e^{(0.10\pi)}))</td>
<td>((0.60 e^{(11.26\pi)}, 0.35 e^{(0.68\pi)}, 0.40 e^{(0.84\pi)}))</td>
</tr>
</tbody>
</table>

**Example 10.** The restricted union \((L_{Q}, G, O)\) of \((\Phi_{w}, Y, 6)\) and \((\chi_{A}, C, 5)\), given in Table 3 and Table 9, arranged in 12.

**Table 12: Restricted union \((L_{Q}, G, O)\)**

<table>
<thead>
<tr>
<th>((L_{Q}, G, O))</th>
<th>(U_{1})</th>
<th>(U_{2})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Y_{1})</td>
<td>((3.0 e^{(11.26\pi)}, 0.35 e^{(0.68\pi)}, 0.40 e^{(0.84\pi)}))</td>
<td>((5.0 e^{(11.26\pi)}, 0.05 e^{(0.12\pi)}, 0.12 e^{(0.26\pi)}))</td>
</tr>
<tr>
<td>(Y_{2})</td>
<td>((2.0 e^{(0.82\pi)}, 0.51 e^{(1.04\pi)}, 0.56 e^{(1.10\pi)}))</td>
<td>((4.0 e^{(1.78\pi)}, 0.99 e^{(0.16\pi)}, 0.06 e^{(0.10\pi)}))</td>
</tr>
<tr>
<td>(Y_{3})</td>
<td>((2.0 e^{(0.82\pi)}, 0.51 e^{(1.04\pi)}, 0.56 e^{(1.10\pi)}))</td>
<td>((4.0 e^{(1.78\pi)}, 0.99 e^{(0.16\pi)}, 0.06 e^{(0.10\pi)}))</td>
</tr>
</tbody>
</table>

**Definition 19.** Let \(U\) be a non-empty set and \((\Phi_{w}, Y, N_{1})\) and \((\chi_{A}, C, N_{2})\) be CSVN1,S,S and CSVN2,S,S on \(U\), respectively, their restricted union is defined as \((L_{M}, \mathcal{G}, D) = (\Phi_{w}, Y, N_{1}) \cup (\chi_{A}, C, N_{2})\), with \(L_{M} = \Phi_{w} \cup \chi_{A}, \mathcal{G} = Y \cap C, D = \max(N_{1}, N_{2})\), i.e., \(\forall x_{w} \in \mathcal{G}, u_{s} \in U\) we have

\[
L_{M}(x_{w}) = \left\{ \begin{array}{l}
(h_{1w}^{1s} (T_{ws}^{1w} I_{ws}^{1w} F_{ws}^{1w})), \\
(\min(h_{1w}^{1s}, h_{2w}^{2s}), \min(T_{ws}^{1w}, T_{ws}^{2w}), \max(I_{ws}^{1w}, I_{ws}^{2w}), \max(F_{ws}^{1w}, F_{ws}^{2w})), \\
(\max(h_{1w}^{1s}, h_{2w}^{2s}), \max(p_{ws}^{1s}, p_{ws}^{2s}) e^{2\pi \min(t_{ws}^{1w}, t_{ws}^{2w})}, \min(q_{ws}^{1s}, q_{ws}^{2s}) e^{2\pi \min(w_{ws}^{1w}, w_{ws}^{2w})}, \min(r_{ws}^{1s}, r_{ws}^{2s}) e^{2\pi \min(f_{ws}^{1w}, f_{ws}^{2w})})
\end{array} \right.
\]

**Example 11.** The extended union \((L_{M}, G, O)\) of \((\Phi_{w}, Y, 6)\) and \((\chi_{A}, C, 5)\), given in Table 3 and Table 9, arranged in 13.

**M. Akram, M. Shabir, A. Ashraf, Complex neutrosophic N-soft sets: A new model with applications.**
Table 13: Extended union \((P_Q, T, \mathfrak{B})\)

<table>
<thead>
<tr>
<th>((P_Q, T, \mathfrak{B}))</th>
<th>(U_1)</th>
<th>(U_2)</th>
<th>(U_3)</th>
<th>(U_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Y_1)</td>
<td>([0.668^{0.041}, 0.332^{0.467}, 0.467^{0.563}])</td>
<td>([0.956^{0.127}, 0.056^{0.332}, 0.127^{0.842}])</td>
<td>([0.668^{0.041}, 0.332^{0.467}, 0.467^{0.563}])</td>
<td>([0.243^{0.467}, 0.861^{0.127}, 0.842^{0.041}])</td>
</tr>
<tr>
<td>(Y_2)</td>
<td>([0.420^{0.082}, 0.510^{0.144}, 0.560^{0.101}])</td>
<td>([0.886^{0.078}, 0.090^{0.162}, 0.060^{0.101}])</td>
<td>([0.360^{0.078}, 0.580^{0.144}, 0.540^{0.101}])</td>
<td>([0.086^{0.101}, 0.960^{0.078}, 0.940^{0.144}])</td>
</tr>
<tr>
<td>(Y_3)</td>
<td>([0.810^{0.144}, 0.310^{0.368}, 0.190^{0.407}])</td>
<td>([0.856^{0.144}, 0.270^{0.368}, 0.306^{0.387}])</td>
<td>([0.756^{0.127}, 0.170^{0.387}, 0.200^{0.307}])</td>
<td>([0.69^{0.127}, 0.410^{0.387}, 0.480^{0.307}])</td>
</tr>
<tr>
<td>(Y_4)</td>
<td>([0.980^{0.127}, 0.020^{0.368}, 0.100^{0.407}])</td>
<td>([0.260^{0.127}, 0.560^{0.368}, 0.110^{0.407}])</td>
<td>([0.101^{0.260}, 0.80^{0.144}, 0.530^{0.101}])</td>
<td>([0.301^{0.260}, 0.50^{0.127}, 0.550^{0.144}])</td>
</tr>
</tbody>
</table>

Now we discuss some properties and their proofs.

**Theorem 21.** Let \((\Phi_\Psi, Y, N_1)\) be a CSV\(\{N_1\}\)S over a non-empty set \(U\). Then,

1. \((\Phi_\Psi, Y, N_1) \cap (\Phi_\Psi, Y, N_1) = (\Phi_\Psi, Y, N_1)\)
2. \((\Phi_\Psi, Y, N_1) \cap (\Phi_\Psi, Y, N_1) = (\Phi_\Psi, Y, N_1)\)
3. \((\Phi_\Psi, Y, N_1) \cup (\Phi_\Psi, Y, N_1) = (\Phi_\Psi, Y, N_1)\)
4. \((\Phi_\Psi, Y, N_1) \cap (\Phi_\Psi, Y, N_1) = (\Phi_\Psi, Y, N_1)\)

**Proof.**

1. 

\[ R.H.S = (\Phi_\Psi, Y, N_1) \cap (\Phi_\Psi, Y, N_1) \]

where the extended intersection of two CSV\(\{N_1\}\)Ss is calculated as:

\[ (D_Q, T, \mathfrak{S}) = (\Phi_\Psi, Y, N_1) \cap (\Phi_\Psi, Y, N_1) \]

with \(T = Y \cup Y, \mathfrak{S} = \max(N_1, N_1)\) and

\[ D_Q(x_w) = \begin{cases} 
(\min h_w^1, \min(T^1_w, 1_{w,2}, F^1_w)), & \text{if } x_w \in Y - Y, \\
(\min h_w^1, \min(T^1_w, 1_{w,2}, F^1_w)), & \text{if } x_w \in Y - Y, \\
(\min h_w^1, \min(T^1_w, 1_{w,2}, F^1_w)), & \text{if } x_w \in Y - Y, \\
(\min h_w^1, \min(T^1_w, 1_{w,2}, F^1_w)), & \text{if } x_w \in Y - Y. 
\end{cases} \]

Case 1: If \(x_w \in Y - Y = \emptyset\),

\[ D_Q(x_w) = \Phi_\Psi(x_w) \]

Case 2: If \(x_w \in Y - Y = \emptyset\),

\[ D_Q(x_w) = \Phi_\Psi(x_w) \]

Case 3: If \(x_w \in Y - Y = Y, \)

\[ D_Q(x_w) = \begin{cases} 
(\min h_w^1, \min(T^1_w, 1_{w,2}, F^1_w)), & \text{if } x_w \in Y - Y, \\
(\min h_w^1, \min(T^1_w, F^1_w)), & \text{if } x_w \in Y - Y, \\
(\min h_w^1, \min(T^1_w, 1_{w,2}, F^1_w)), & \text{if } x_w \in Y - Y. 
\end{cases} \]

From Equations 2, 3 and 4, \((D_Q, T, \mathfrak{S}) = (\Phi_\Psi, Y, N_1)\) and further Eq.1 implies \((\Phi_\Psi, Y, N_1) \cap (\Phi_\Psi, Y, N_1) = (\Phi_\Psi, Y, N_1)\).

2. 

\[ R.H.S = (\Phi_\Psi, Y, N_1) \cap (\Phi_\Psi, Y, N_1) \]

where the restricted intersection of two CSV\(\{N_1\}\)Ss is calculated as:

\[ (L_M, G, O) = (\Phi_\Psi, Y, N_1) \cap (\Phi_\Psi, Y, N_1) \]

with \(G = Y \cap Y = Y, O = \min(N_1, N_1) = N_1\) and

\[ L_M(x_w) = \begin{cases} 
(\min h_w^1, h_w^1), & \text{if } x_w \in Y - Y, \\
(\min(T^1_w, 1_{w,2}, F^1_w)), & \text{if } x_w \in Y - Y, \\
(\min h_w^1, \min(T^1_w, 1_{w,2}, F^1_w)), & \text{if } x_w \in Y - Y. 
\end{cases} \]

clearly, from Equations 6, 7 and 8, we get the required result.

3. 

\[ R.H.S = (\Phi_\Psi, Y, N_1) \cap (\Phi_\Psi, Y, N_1) \]
where the extended union of two CSVNNSs is calculated as:

\[(P_Q, \mathcal{T}, \mathcal{B}) = (\Phi_\psi, Y, N_1) \cup (\Phi_\psi, Y, N_1), \quad (10)\]

with \( \mathcal{T} = Y \cup Y, \mathcal{B} = \max(N_1, N_1) \) and

\[
P_Q(x_w) = \begin{cases} 
(h_{w}^{1_1}, (T_{w,x}, I_{w,x}, F_{w,x})), & \text{if } x_w \in Y \cap Y, \\
(h_{w}^{1_2}, (T_{w,x}, I_{w,x}, F_{w,x})), & \text{if } x_w \in Y \cap \mathcal{T}, \\
(\max(h_{w}^{1_1}, h_{w}^{1_2}), (\max(T_{w,x}, T_{w,x}), \min(I_{w,x}, I_{w,x}), \min(F_{w,x}, F_{w,x}))), & \text{if } x_w \in Y \cap Y.
\end{cases}
\]

Case 1: If \( x_w \in Y = Y = 0 \),

\[
P_Q(x_w) = \Phi_\psi(x_w).
\]

Case 2: If \( x_w \in Y - Y = 0 \),

\[
P_Q(x_w) = \Phi_\psi(x_w).
\]

Case 3: If \( x_w \in Y \cap Y = Y \),

\[
P_Q(x_w) = (\max(h_{w}^{1_1}, h_{w}^{1_2}), (\max(T_{w,x}, T_{w,x}), \min(I_{w,x}, I_{w,x}), \min(F_{w,x}, F_{w,x}))),
\]

\[
= (h_{w}^{1_2}, (T_{w,x}, I_{w,x}, F_{w,x})),
\]

\[
= \Phi_\psi(x_w).
\]

From Equations 9, 10, 11, 12 and 13, we get

\[(\Phi_\psi, Y, N_1) \cup (\Phi_\psi, Y, N_1) = (\Phi_\psi, Y, N_1).
\]

4. \( R.H.S = (\Phi_\psi, Y, N_1) \cup (\Phi_\psi, Y, N_1) \),

where the restricted union of two CSVNNSs is calculated as:

\[(L_M, \mathcal{B}, \mathcal{O}) = (\Phi_\psi, Y, N_1) \cup (\Phi_\psi, Y, N_1), \quad (15)\]

with \( \mathcal{B} = Y \cap Y = Y, \mathcal{O} = \max(N_1, N_1) = N_1 \) and

\[
L_M(x_w) = (\max(h_{w}^{1_1}, h_{w}^{1_2}), (\max(T_{w,x}, T_{w,x}), \min(I_{w,x}, I_{w,x}), \min(F_{w,x}, F_{w,x}))),
\]

\[
= (h_{w}^{1_1}, (T_{w,x}, I_{w,x}, F_{w,x})),
\]

\[
= \Phi_\psi(x_w).
\]

Clearly, from Equations 14, 15 and 16, we get the required result.

\[\Box\]

**Theorem 22.** Let \((\Phi_\psi, Y, N_1)\) and \((\chi_A, C, N_2)\) be CSVNNSs and CSVNNSs, respectively, over the same universe \(U\), then the absorption properties hold:

1. \((\Phi_\psi, Y, N_1) \cup (\chi_A, C, N_2) \cap (\Phi_\psi, E, N_1) = (\Phi_\psi, Y, N_1)\)
2. \((\Phi_\psi, Y, N_1) \cup ((\chi_A, C, N_2) \cap (\Phi_\psi, E, N_1)) = (\Phi_\psi, Y, N_1)\)
3. \((\Phi_\psi, Y, N_1) \cap ((\chi_A, C, N_2) \cup (\Phi_\psi, E, N_1)) = (\Phi_\psi, Y, N_1)\)
4. \((\Phi_\psi, Y, N_1) \cap ((\chi_A, C, N_2) \cap (\Phi_\psi, E, N_1)) = (\Phi_\psi, Y, N_1)\)

**Proof.**

1. Let the extended union of CSVNNSs \((\Phi_\psi, Y, N_1)\) and CSVNNSs \((\chi_A, C, N_2)\), be

\[
(P_Q, \mathcal{T}, \mathcal{B}) = (\Phi_\psi, Y, N_1) \cup (\chi_A, C, N_2),
\]

with \( \mathcal{T} = Y \cup C, \mathcal{B} = \max(N_1, N_2) \) and

\[
P_Q(x_w) = (h_{w}^{1_1}, (T_{w,x}, I_{w,x}, F_{w,x})),
\]

\[
= \begin{cases} 
(h_{w}^{1_1}, (T_{w,x}, I_{w,x}, F_{w,x})), & \text{if } x_w \in Y - C, \\
(h_{w}^{1_2}, (T_{w,x}, I_{w,x}, F_{w,x})), & \text{if } x_w \in C - Y, \\
(\max(h_{w}^{1_1}, h_{w}^{1_2}), (\max(T_{w,x}, T_{w,x}), \min(I_{w,x}, I_{w,x}), \min(F_{w,x}, F_{w,x}))), & \text{if } x_w \in Y \cap C.
\end{cases}
\]

Now, consider the restricted intersection of \((P_Q, \mathcal{T}, \mathcal{B})\) and \((\Phi_\psi, Y, N_1)\), that is defined as

\[
(L_M, G, O) = (P_Q, \mathcal{T}, \mathcal{B}) \cap (\Phi_\psi, Y, N_1),
\]

with \( G = \mathcal{T} \cap Y, O = \min(\mathcal{B}, N_1) = N_1 \) and

\[
L_M(x_w) = (\min(h_{w}^{1_1}, h_{w}^{1_2}), (\min(T_{w,x}, T_{w,x}), \max(I_{w,x}, I_{w,x}), \max(F_{w,x}, F_{w,x}))).
\]

\[\Box\]
for all \( x_w \in G = Y \cap C \), so that \( x_w \in W, x_w \in C \). If \( x_w \in W \), then there are three cases.

**Case 1:** if \( x_w \in Y - C \), using Equations 17 and 18, we get,

\[
L_M(x_w) = (\min(h_w^{1x}, h_w^{2x}), (\min(T_{w,x}^{1}, T_{w,x}^{2}), \max(T_{w,x}^{1}, T_{w,x}^{2}), \max(F_{w,x}^{1}, F_{w,x}^{2})))
\]

\[
= (h_w^{1x}, T_{w,x}^{1} - F_{w,x}^{1})
\]

\[
\Phi(x_w)
\]

(19)

**Case 2:** if \( x_w \in C - Y \), since \( x_w \in G = Y \cap C \) implies \( x_w \in Y \), therefore, this case is omitted.

**Case 3:** if \( x_w \in C \cap Y \), using Equations 17 and 18, we get,

\[
L_M(x_w) = (\min(\max(h_w^{1x}, h_w^{2x}), h_w^{1x}), (\min(\max(T_{w,x}^{1}, T_{w,x}^{2}), T_{w,x}^{1}), \max(\min(T_{w,x}^{1}, T_{w,x}^{2}), T_{w,x}^{1}), \max(\min(F_{w,x}^{1}, F_{w,x}^{2}), F_{w,x}^{1})))
\]

\[
= (h_w^{1x}, T_{w,x}^{1} - F_{w,x}^{1})
\]

\[
\Phi(x_w)
\]

(20)

Thus from Equations 19 and 20, we get \( ((\Phi_{\Psi}, E, N_1) \cup (\chi_A, C, N_2)) \cap (\Phi_{\Psi}, E, N_1) = (\Phi_{\Psi}, E, N_1) \).

2. proofs of 2, 3 and 4 are same as above.

\[\square\]

**Theorem 23.** Let \( (\Phi_{\Psi}, Y, N_1), (\chi_A, C, N_2) \) and \( (\chi_{\kappa}, \varrho, N_3) \) be any three CSVN\'s \( S_f S \), CSVN\'s \( S_f S \), and CSVN\'s \( S_f S \), and over the same universe \( U \), then the following properties hold:

1. \( (\Phi_{\Psi}, Y, N_1) \cup (\chi_A, C, N_2) = (\chi_A, C, N_2) \cup (\Phi_{\Psi}, Y, N_1) \),
2. \( (\Phi_{\Psi}, Y, N_1) \cap (\chi_A, C, N_2) = (\chi_A, C, N_2) \cap (\Phi_{\Psi}, Y, N_1) \),
3. \( (\Phi_{\Psi}, Y, N_1) \cap (\chi_{\kappa}, \varrho, N_3) = (\phi_{\Psi}, Y, N_1) \cap (\chi_{\kappa}, \varrho, N_3) \),
4. \( (\Phi_{\Psi}, Y, N_1) \cap (\chi_A, C, N_2) = (\chi_A, C, N_2) \cap (\Phi_{\Psi}, Y, N_1) \),
5. \( ((\Phi_{\Psi}, Y, N_1) \cup (\chi_A, C, N_2) ) \cup (\chi_{\kappa}, \varrho, N_3) = (\Phi_{\Psi}, Y, N_1) \cup ((\chi_A, C, N_2) \cup (\chi_{\kappa}, \varrho, N_3)) \),
6. \( ((\Phi_{\Psi}, Y, N_1) \cup (\chi_A, C, N_2) ) \cup (\chi_{\kappa}, \varrho, N_3) = (\Phi_{\Psi}, Y, N_1) \cup ((\chi_A, C, N_2) \cup (\chi_{\kappa}, \varrho, N_3)) \),
7. \( ((\Phi_{\Psi}, Y, N_1) \cap (\chi_A, C, N_2) ) \cap (\chi_{\kappa}, \varrho, N_3) = (\Phi_{\Psi}, Y, N_1) \cap ((\chi_A, C, N_2) \cap (\chi_{\kappa}, \varrho, N_3)) \),
8. \( ((\Phi_{\Psi}, Y, N_1) \cap (\chi_A, C, N_2) ) \cap (\chi_{\kappa}, \varrho, N_3) = (\Phi_{\Psi}, Y, N_1) \cap ((\chi_A, C, N_2) \cap (\chi_{\kappa}, \varrho, N_3)) \),
9. \( ((\Phi_{\Psi}, Y, N_1) \cap (\chi_A, C, N_2) ) \cap (\chi_{\kappa}, \varrho, N_3) = (\Phi_{\Psi}, Y, N_1) \cap ((\chi_A, C, N_2) \cap (\chi_{\kappa}, \varrho, N_3)) \),
10. \( ((\Phi_{\Psi}, Y, N_1) \cap (\chi_A, C, N_2) ) \cap (\chi_{\kappa}, \varrho, N_3) = (\Phi_{\Psi}, Y, N_1) \cap ((\chi_A, C, N_2) \cap (\chi_{\kappa}, \varrho, N_3)) \),
11. \( ((\Phi_{\Psi}, Y, N_1) \cup (\chi_A, C, N_2) ) \cup (\chi_{\kappa}, \varrho, N_3) = (\Phi_{\Psi}, Y, N_1) \cup ((\chi_A, C, N_2) \cup (\chi_{\kappa}, \varrho, N_3)) \),
12. \( ((\Phi_{\Psi}, Y, N_1) \cup (\chi_A, C, N_2) ) \cup (\chi_{\kappa}, \varrho, N_3) = (\Phi_{\Psi}, Y, N_1) \cup ((\chi_A, C, N_2) \cup (\chi_{\kappa}, \varrho, N_3)) \).

4. Complex single-valued neutrosophic \( N \)-soft number

**Definition 24.** Let \( \Phi_{\Psi}(y_w) = ((u_w, h_w\delta, p_wse^{\delta tw_{w}}q_wse^{\delta tw_{w}}, r_wse^{\delta tw_{w}})) \) be a CSVN\'s \( S_f S \). Then the complex single-valued neutrosophic \( N \)-soft number \( (CSVN\'s \ S \ N) \) is defined as:

\[
\alpha_{w_s} = (h_w\delta, p_wse^{\delta tw_{w}}q_wse^{\delta tw_{w}}, r_wse^{\delta tw_{w}}),
\]

**Definition 25.** Consider a CCSS\( \ S \ N \) \( \alpha_{w_s} = (h_w\delta, p_wse^{\delta tw_{w}}q_wse^{\delta tw_{w}}, r_wse^{\delta tw_{w}}) \). The score function \( S(\alpha_{w_s}) \) is:

\[
S(\alpha_{w_s}) = \frac{h_w\delta}{N-1} + (p_wse^{\delta tw_{w}}q_wse^{\delta tw_{w}}, r_wse^{\delta tw_{w}})
\]

where \( S(\alpha_{w_s}) \in [-4, 3] \). The accuracy function \( A(\alpha_{w_s}) \) is:

\[
A(\alpha_{w_s}) = \frac{h_w\delta}{N-1} + (p_wse^{\delta tw_{w}}q_wse^{\delta tw_{w}}, r_wse^{\delta tw_{w}})
\]

where \( A(\alpha_{w_s}) \in [0, 7] \), respectively.

**Definition 26.** Let \( \alpha_{w_s} = (h_w\delta, p_wse^{\delta tw_{w}}q_wse^{\delta tw_{w}}, r_wse^{\delta tw_{w}}) \) and \( \alpha_{w_s} = (h_w\delta, p_wse^{\delta tw_{w}}q_wse^{\delta tw_{w}}, r_wse^{\delta tw_{w}}) \) be two CSVN\'s \( \ S \ N \).
1. If $S_{\alpha ws} < S_{\alpha ls}$, then $\alpha ws < \alpha ls$ (\(\alpha ws\) is inferior to \(\alpha ls\)),
2. If $S_{\alpha ws} = S_{\alpha ls}$, then
   i. $A_{\alpha ws} < A_{\alpha ls}$, then $\alpha ws < \alpha ls$ (\(\alpha ws\) is inferior to \(\alpha ls\)),
   ii. $A_{\alpha ws} = A_{\alpha ls}$, then $\alpha ws \sim \alpha ls$ (\(\alpha ws\) is equivalent to \(\alpha ls\)).

### Definition 27.

Let $\alpha ws = (h^w_s p_{ws} e^{2\pi i w_s}, q_{ws} e^{2\pi i w_s}, r_{ws} e^{2\pi i f_{ws}})$ and $\alpha ls = (h^l_s p_{ls} e^{2\pi i t_{ls}}, q_{ls} e^{2\pi i t_{ls}}, r_{ls} e^{2\pi i f_{ls}})$ be two CSV NNS\(_f\)Ns and $\beta > 0$. Some operation for CSV NNS\(_f\)Ns are:

\[
\beta \alpha ws = \left( h^w_s, \left[ 1 - (1 - p_{ws})^\beta \right] e^{2\pi [1 - (1 - t_{ws})^\beta]}, q_{ws} e^{2\pi i t_{ws}}, r_{ws} e^{2\pi f_{ws}} \right),
\]

\[
\alpha ws \oplus \alpha ls = \max(h^w_s, h^l_s), (p_{ws} + p_{ls} - p_{ws} p_{ls}) e^{2\pi i (t_{ws} + t_{ls} - t_{ws} t_{ls})}, (q_{ws} q_{ls}) e^{2\pi (\omega_{ws} + \omega_{ls})}, (r_{ws} r_{ls}) e^{2\pi (f_{ws} + f_{ls})},
\]

\[
\alpha ws \odot \alpha ls = \left( \min(h^w_s, h^l_s), (p_{ws} p_{ls}) e^{2\pi i (t_{ws} t_{ls})}, (q_{ws} + q_{ls} - q_{ws} q_{ls}) e^{2\pi (\omega_{ws} + \omega_{ls} - \omega_{ws} \omega_{ls})}, (r_{ws} + r_{ls} - r_{ws} r_{ls}) e^{2\pi (f_{ws} + f_{ls} - f_{ws} f_{ls})} \right).
\]

### Definition 28.

Let $\alpha ws = (h^w_s p_{ws} e^{2\pi i w_s}, q_{ws} e^{2\pi i w_s}, r_{ws} e^{2\pi i f_{ws}})$ and $\alpha ls = (h^l_s p_{ls} e^{2\pi i t_{ls}}, q_{ls} e^{2\pi i t_{ls}}, r_{ls} e^{2\pi i f_{ls}})$ be two CSV NNS\(_f\)Ns and $\beta > 0$, then the following properties hold:

1. $\alpha ws \oplus \alpha ls = \alpha ls \oplus \alpha ws$,
2. $\alpha ws \odot \alpha ls = \alpha ls \odot \alpha ws$,
3. $\beta \alpha ws \oplus \beta \alpha ls = \beta (\alpha ws \oplus \alpha ls)$, $\beta > 0$,
4. $\beta_1 \alpha ws \oplus \beta_1 \alpha ls = (\beta_1 + \beta_2) \alpha ws$, $\beta_1, \beta_2 > 0$,
5. $\alpha ws \oplus \alpha ls = (\alpha ls \oplus \alpha ws)^\beta$, $\beta > 0$,
6. $\alpha ws \odot \alpha ls = (\alpha ls \odot \alpha ws)^{\beta + \beta_2}$, $\beta_1, \beta_2 > 0$.

**Proof.**

1. $\alpha ws \oplus \alpha ls = \left( \max(h^w_s, h^l_s), (p_{ws} + p_{ls} - p_{ws} p_{ls}) e^{2\pi i (t_{ws} + t_{ls} - t_{ws} t_{ls})}, (q_{ws} q_{ls}) e^{2\pi (\omega_{ws} + \omega_{ls})}, (r_{ws} r_{ls}) e^{2\pi (f_{ws} + f_{ls})} \right),$

\[
= \alpha ls \oplus \alpha ws,
\]

2. $\alpha ws \odot \alpha ls = \left( \min(h^w_s, h^l_s), (p_{ws} p_{ls}) e^{2\pi i (t_{ws} t_{ls})}, (q_{ws} + q_{ls} - q_{ws} q_{ls}) e^{2\pi (\omega_{ws} + \omega_{ls} - \omega_{ws} \omega_{ls})}, (r_{ws} + r_{ls} - r_{ws} r_{ls}) e^{2\pi (f_{ws} + f_{ls} - f_{ws} f_{ls})} \right),$

\[
= \alpha ls \odot \alpha ws,
\]

3. $\beta \alpha ws \oplus \beta \alpha ls = \left( h^w_s, \left[ 1 - (1 - p_{ws})^\beta \right] e^{2\pi [1 - (1 - t_{ws})^\beta]}, q_{ws} e^{2\pi i t_{ws}}, r_{ws} e^{2\pi f_{ws}} \right) \oplus \left( h^l_s, \left[ 1 - (1 - p_{ls})^\beta \right] e^{2\pi [1 - (1 - t_{ls})^\beta]}, q_{ls} e^{2\pi i t_{ls}}, r_{ls} e^{2\pi f_{ls}} \right),$
Definition 29. Let \( \alpha_{uws} = (h^s_w, p_{uws}e^{2\pi i w s}, q_{uws}e^{2\pi i w s}, r_{uws}e^{2\pi i f_{uws}}) \) be a collection of CSV\(\alpha\)NS\(\nu\)s and \( \nu_w \) be the weight vectors of \( \alpha_{uws} \) with \( \nu_w > 0 \) and \( \sum_{w=1}^{k} \nu_w = 1 \). The complex single-valued neutrosophic \( N\)-soft weighted average operator (CSV\(\alpha\)NS\(\nu\)WA) is a mapping CSV\(\alpha\)NS\(\nu\)WA : \( \mathcal{J}^k \rightarrow \mathcal{J} \), where \( \mathcal{J} \) is the set of CSV\(\alpha\)NS\(\nu\)s, defined as follows:

\[
\text{CSV}^{\text{\alpha} \text{NS}^{\text{\nu}} \text{WA}}(\alpha_{1s}, \alpha_{2s}, \ldots, \alpha_{ks}) = (\nu_1 \alpha_{q(1s)} \oplus \nu_2 \alpha_{q(2s)} \oplus \cdots \oplus \nu_k \alpha_{q(k s)})
\]

\[
= \left( \max_{w=1}^{k}(h^s_w), [1 - \Pi_{w=1}^{k}(1 - p_{uws}^{\nu_w})]e^{2\pi i (1 - f_{uws}^{\nu_w})}, [\Pi_{w=1}^{k}(q_{uws}^{\nu_w})]e^{2\pi i (1 - \omega_{uws}^{\nu_w})}, [\Pi_{w=1}^{k}(r_{uws}^{\nu_w})]e^{2\pi i (1 - f_{uws}^{\nu_w})} \right).
\]

Definition 30. Let \( \alpha_{uws} = (h^s_w, p_{uws}e^{2\pi i w s}, q_{uws}e^{2\pi i w s}, r_{uws}e^{2\pi i f_{uws}}) \) be a collection of CSV\(\alpha\)NS\(\nu\)s and \( \nu_w \) be the weight vectors of \( \alpha_{uws} \) with \( \nu_w > 0 \) and \( \sum_{w=1}^{k} \nu_w = 1 \). The complex single-valued neutrosophic \( N\)-soft weighted geometric operator (CSV\(\alpha\)NS\(\nu\)WG) is a mapping CSV\(\alpha\)NS\(\nu\)WG : \( \mathcal{J}^k \rightarrow \mathcal{J} \), where \( \mathcal{J} \) is the set of CSV\(\alpha\)NS\(\nu\)s, defined as follows:

\[
\text{CSV}^{\text{\alpha} \text{NS}^{\text{\nu}} \text{WG}}(\alpha_{1s}, \alpha_{2s}, \ldots, \alpha_{ks}) = (\alpha_{q(1s)}^s \oplus \alpha_{q(2s)}^s \oplus \cdots \oplus \alpha_{q(k s)}^s)
\]

\[
= \left( \min_{w=1}^{k}(h^s_w), [\Pi_{w=1}^{k}(p_{uws})]e^{2\pi i (1 - f_{uws})}, [\Pi_{w=1}^{k}(q_{uws})]e^{2\pi i (1 - \omega_{uws})}, [\Pi_{w=1}^{k}(r_{uws})]e^{2\pi i (1 - f_{uws})} \right).
\]

Definition 31. Let \( \alpha_{uws} = (h^s_w, p_{uws}e^{2\pi i w s}, q_{uws}e^{2\pi i w s}, r_{uws}e^{2\pi i f_{uws}}) \) be a collection of CSV\(\alpha\)NS\(\nu\)s and \( \nu_w \) be the weight vectors of \( \alpha_{uws} \) with \( \nu_w > 0 \) and \( \sum_{w=1}^{k} \nu_w = 1 \). The complex single-valued neutrosophic \( N\)-soft weighted ordered geometric operator (CSV\(\alpha\)NS\(\nu\)OWG) is a mapping CSV\(\alpha\)NS\(\nu\)OWG : \( \mathcal{J}^k \rightarrow \mathcal{J} \), where \( \mathcal{J} \) is the set of CSV\(\alpha\)NS\(\nu\)s, defined as follows:

\[
\text{CSV}^{\text{\alpha} \text{NS}^{\text{\nu}} \text{OWG}}(\alpha_{1s}, \alpha_{2s}, \ldots, \alpha_{ks}) = (\alpha_{q(1s)}^s \oplus \alpha_{q(2s)}^s \oplus \cdots \oplus \alpha_{q(k s)}^s)
\]

\[
= \left( \min_{w=1}^{k}(h^s_w), [\Pi_{w=1}^{k}(p_{uws})]e^{2\pi i (1 - f_{uws})}, [\Pi_{w=1}^{k}(q_{uws})]e^{2\pi i (1 - \omega_{uws})}, [\Pi_{w=1}^{k}(r_{uws})]e^{2\pi i (1 - f_{uws})} \right).
\]

5 Complex single-valued neutrosophic \( N\)-soft TOPSIS method

In this section, we developed methodology for TOPSIS method under the framework of CSV\(\alpha\)NS\(\nu\)s for solving multi-attribute group decision making (MAGDM) problem. For the optimal solution of the MADM problem, TOPSIS method specifically used ideal solutions of that problem. Consider a MAGDM problem with \( U = \{ U_1, U_2, U_3, \ldots, U_n \} \) and \( Y = \{ Y_1, Y_2, Y_3, \ldots, Y_l \} \) be the set of alternative and attributes decided by the experts \( Z_1, Z_2, Z_3, \ldots, Z_f \), where the experts weight vector for this MAGDM problem is \( \nu = (\nu_1, \nu_2, \nu_3, \ldots, \nu_l)^T \). The procedure for CSV\(\alpha\)NS\(\nu\)TOPSIS method is as follows:

5.1 Organizing the complex single-valued neutrosophic \( N\)-soft decision matrix

After studied the MADM problem properly, decision makers use rating system for assigning rank to each alternative, parallel to each semantic term, relative to the attributes that indeed form a NS\(\nu\)S. Further, decision making panel associate CSV\(\alpha\)NS\(\nu\)N corresponding to each rank (ordered
grade) by defining grading criteria related to the aptitude of the MADM problem. Therefore, a complex single-valued neutrosophic N-soft decision matrix (CSVNNSjDM) \( \mathbf{H} = (\mathbf{H}_{ws}^{(j)})_{(s \times t)} \) is organized as follows:

\[
\mathbf{H}^{(j)} = \left( \begin{array}{cccc}
(h_1^{(j)}(\pi_1, \pi_2, \pi_3), p_1^{(j)}, q_1^{(j)}, r_1^{(j)}) & (h_2^{(j)}(\pi_1, \pi_2, \pi_3), p_2^{(j)}, q_2^{(j)}, r_2^{(j)}) & \cdots & (h_t^{(j)}(\pi_1, \pi_2, \pi_3), p_t^{(j)}, q_t^{(j)}, r_t^{(j)}) \\
(h_1^{(j)}(\pi_1, \pi_2, \pi_3), p_1^{(j)}, q_1^{(j)}, r_1^{(j)}) & (h_2^{(j)}(\pi_1, \pi_2, \pi_3), p_2^{(j)}, q_2^{(j)}, r_2^{(j)}) & \cdots & (h_t^{(j)}(\pi_1, \pi_2, \pi_3), p_t^{(j)}, q_t^{(j)}, r_t^{(j)}) \\
\vdots & \vdots & \ddots & \vdots \\
(h_1^{(j)}(\pi_1, \pi_2, \pi_3), p_1^{(j)}, q_1^{(j)}, r_1^{(j)}) & (h_2^{(j)}(\pi_1, \pi_2, \pi_3), p_2^{(j)}, q_2^{(j)}, r_2^{(j)}) & \cdots & (h_t^{(j)}(\pi_1, \pi_2, \pi_3), p_t^{(j)}, q_t^{(j)}, r_t^{(j)})
\end{array} \right)
\]

where, \( \mathbf{H}_{ws}^{(j)} = ((h_s^{(j)}, \pi_1, \pi_2, \pi_3), p_s^{(j)}, q_s^{(j)}, r_s^{(j)}) = (h_{ws}, p_{ws}e^{2\pi i w_s}, q_{ws}e^{2\pi i w_s}, r_{ws}e^{2\pi i f_{ws}}), s = \{1, 2, 3, \ldots, t\}, j = \{1, 2, 3, \ldots, f\}, \) and \( w = \{1, 2, 3, \ldots, k\} \).

### 5.2 Aggregated complex single-valued neutrosophic N-soft decision matrix

As the decision makers (experts) are not equally weighted in MAGDM problems, therefore by utilizing the weightage of each expert decided by the panel we cumulate the decision of all experts and get aggregated complex single-valued neutrosophic N-soft decision matrix (ACSVNNSjDM). The CSVNNSjWA operator or CSVNNSjWG operator are precisely used to cumulate the CSVNNSjDM (\( \mathbf{H} \)) as follows:

\[
\mathcal{H}_{ws} = \text{CSVNNSjWA}(\mathbf{H}_{ws}^{(1)}, \mathbf{H}_{ws}^{(2)}, \ldots, \mathbf{H}_{ws}^{(f)});
\]

\( (OR) = \text{CSVNNSjWG}(\mathbf{H}_{ws}^{(1)}, \mathbf{H}_{ws}^{(2)}, \ldots, \mathbf{H}_{ws}^{(f)}); \)

where, \( \mathcal{H}_{ws} = (h_1, T_{ws}, I_{ws}, F_{ws}) = (h_{ws}, p_{ws}e^{2\pi i w_s}, q_{ws}e^{2\pi i w_s}, r_{ws}e^{2\pi i f_{ws}}). \)

The ACSVNNSjSDM denoted as:

\[
\mathcal{H} = \left( \begin{array}{cccc}
(h_1^{(1)}, T_{11}, I_{11}, F_{11}) & (h_1^{(2)}, T_{12}, I_{12}, F_{12}) & \cdots & (h_1^{(f)}, T_{1f}, I_{1f}, F_{1f}) \\
(h_2^{(1)}, T_{21}, I_{21}, F_{21}) & (h_2^{(2)}, T_{22}, I_{22}, F_{22}) & \cdots & (h_2^{(f)}, T_{2f}, I_{2f}, F_{2f}) \\
\vdots & \vdots & \ddots & \vdots \\
(h_t^{(1)}, T_{t1}, I_{t1}, F_{t1}) & (h_t^{(2)}, T_{t2}, I_{t2}, F_{t2}) & \cdots & (h_t^{(f)}, T_{tf}, I_{tf}, F_{tf})
\end{array} \right)
\]

### 5.3 Weights for parameters

To highlight the influence of the parameters in the MAGDM problem, experts judged each parameter and assign grades as the weight of the parameter. Further, CSVNNSjNs are associated to each grade using the grading criteria finalized by the panel. Let \( \theta_{ws}^{(j)} = (h_{ws}, T_{ws}, I_{ws}, F_{ws}) \) be the weight of wth parameter given by the jth expert in the MAGDM problem. Let \( \theta = (\theta_1, \theta_2, \ldots, \theta_k)^T = (h_{ws}, T_{ws}, I_{ws}, F_{ws}) \) be the weight vector of attributes that is summarized, by CSVNNSjWA operator or CSVNNSjWG operator, as follows:

\[
\theta_{ws} = \text{CSVNNSjWA}(\theta_1^{(1)}, \theta_2^{(1)}, \ldots, \theta_k^{(1)});
\]

\( (OR) = \text{CSVNNSjWG}(\theta_1^{(1)}, \theta_2^{(1)}, \ldots, \theta_k^{(1)}). \)

where, \( \theta_{ws} = (h_1, T_{ws}, I_{ws}, F_{ws}) = (h_{ws}, p_{ws}e^{2\pi i w_s}, q_{ws}e^{2\pi i w_s}, r_{ws}e^{2\pi i f_{ws}}). \)

### 5.4 Aggregated weighted complex single-valued neutrosophic N-soft decision matrix

The ACSVNNSjSDM \( \mathcal{H} \) is used within the weight vector \( (\theta_1, \theta_2, \ldots, \theta_k)^T \) of parameter for the formulation of aggregated weighted single-valued neutrosophic N-soft decision matrix (AWCSVNNSjDM). The calculations for are performed as follows:

\[
\tilde{H}_{ws} = \mathcal{H}_{ws} \otimes \theta_{ws}
\]

\[
= \left( \begin{array}{c}
\min((h_{ws}, h_{ws}), (T_{ws}T_{ws}), (I_{ws} + I_{ws} - I_{ws}I_{ws}), (F_{ws} + F_{ws} - F_{ws}F_{ws}))
\end{array} \right)
\]

\[
= \left( \begin{array}{c}
\min(h_{ws}, h_{ws}), p_{ws}p_{ws}e^{2\pi i w_s f_{ws}}, (q_{ws} + q_{ws} - q_{ws}q_{ws})e^{2\pi i [w_{ws} + w_{ws} - w_{ws}w_{ws}]}, (r_{ws} + r_{ws} - r_{ws}r_{ws})e^{2\pi i [f_{ws} + f_{ws} - f_{ws}f_{ws}]}
\end{array} \right)
\]

\[
= \left( \begin{array}{c}
(h_{ws}, T_{ws}, I_{ws}, F_{ws})
\end{array} \right)
\]

\[
= \left( \begin{array}{c}
\tilde{h}_{ws}, \tilde{F}_{ws}e^{2\pi i f_{ws}}, \tilde{q}_{ws}e^{2\pi i w_{ws}}, \tilde{r}_{ws}e^{2\pi i f_{ws}}
\end{array} \right).
\]

---

M. Akram, M. Shabir, A. Ashraf, Complex neutrosophic N-soft sets: A new model with applications.
The AWCSV\textit{NS}_{f}\textit{DM} is:

\[
\hat{H}_{wss} = \left( \begin{array}{cccc}
(h_1^{w}, \tilde{T}_{11}, \tilde{1}_{11}, \tilde{1}_{12}) & (h_1^{w}, \tilde{T}_{12}, \tilde{1}_{12}, \tilde{1}_{13}) & \cdots & (h_1^{w}, \tilde{T}_{1k}, \tilde{1}_{1k}, \tilde{1}_{1k}) \\
(h_2^{w}, \tilde{T}_{11}, \tilde{1}_{11}, \tilde{1}_{12}) & (h_2^{w}, \tilde{T}_{12}, \tilde{1}_{12}, \tilde{1}_{13}) & \cdots & (h_2^{w}, \tilde{T}_{1k}, \tilde{1}_{1k}, \tilde{1}_{1k}) \\
\vdots & \vdots & \ddots & \vdots \\
(h_{N}^{w}, \tilde{T}_{11}, \tilde{1}_{11}, \tilde{1}_{12}) & (h_{N}^{w}, \tilde{T}_{12}, \tilde{1}_{12}, \tilde{1}_{13}) & \cdots & (h_{N}^{w}, \tilde{T}_{1k}, \tilde{1}_{1k}, \tilde{1}_{1k}) \\
\end{array} \right) .
\]

### 5.5 Complex single-valued neutrosophic N-soft ideal solutions

Let BT be the collection of benefit-type criteria and CT be the collection of cost-type criteria opted from the number of parameters, keeping in view the expertise of the given problem. Using these collection we are able to evaluate the complex single-valued neutrosophic positive ideal solution CSV\textit{NS}_{f}\textit{S-PIS} and complex single-valued neutrosophic N-soft negative ideal solution CSV\textit{NS}_{f}\textit{S-NIS} of the MAGDM problem. The CSV\textit{NS}_{f}\textit{S-PIS}, related to the parameter $Y_w$, is defined as:

\[
\hat{H}^{PIS}_w = \begin{cases} 
\max_{j=1}^{s} \hat{H}_{wss}, & \text{if } Y_w \in \text{BT,} \\
\min_{j=1}^{s} \hat{H}_{wss}, & \text{if } Y_w \in \text{CT,} 
\end{cases}
\]

and the CSV\textit{NS}_{f}\textit{S-NIS} is defined as:

\[
\hat{H}^{NIS}_w = \begin{cases} 
\max_{j=1}^{s} \hat{H}_{wss}, & \text{if } Y_w \in \text{CT,} \\
\min_{j=1}^{s} \hat{H}_{wss}, & \text{if } Y_w \in \text{BT.} 
\end{cases}
\]

The CSV\textit{NS}_{f}\textit{S-PIS} and CSV\textit{NS}_{f}\textit{S-NIS} are denoted as: $\hat{H}^{PIS}_w = (\hat{h}_w, \hat{\tilde{T}}_w e^{i2\pi f_w}, \hat{q}_w e^{i2\pi \omega_w}, \hat{r}_w e^{i2\pi f_w})$, and $\hat{H}^{NIS}_w = (\tilde{h}_w, \tilde{\tilde{T}}_w e^{i2\pi f_w}, \tilde{q}_w e^{i2\pi \omega_w}, \tilde{r}_w e^{i2\pi f_w})$, respectively.

### 5.6 Formulation of normalized Euclidean distance

For evaluating the alternatives distance from the ideal solution, we can used similarity measures or distance measure. Moreover, from distance measures we used the normalized Euclidean distance. The normalized Euclidean distance of any of the alternative $U_s$ from the CSV\textit{NS}_{f}\textit{S-PIS} is defined as:

\[
d(\hat{H}^{PIS}_w, U_s) = \left( \frac{1}{l^{w}} \sum_{w=1}^{k} \left( \frac{\hat{h}_w}{N-1} - \frac{\tilde{h}_w}{N-1} \right)^2 + (\hat{\tilde{T}}_w - \tilde{\tilde{T}}_w)^2 + (\hat{q}_w - \tilde{q}_w)^2 + (\hat{r}_w - \tilde{r}_w)^2 + (\hat{1}_w - \tilde{1}_w)^2 + 
(\hat{\omega}_w - \tilde{\omega}_w)^2 + (\hat{f}_w - \tilde{f}_w)^2 \right) \right) 
\]

The normalized Euclidean distance between the CSV\textit{NS}_{f}\textit{S-NIS} and any of the alternative $U_s$, can be evaluated as follows:

\[
d(\hat{H}^{NIS}_w, U_s) = \left( \frac{1}{l^{w}} \sum_{w=1}^{k} \left( \frac{\hat{h}_w}{N-1} - \frac{\tilde{h}_w}{N-1} \right)^2 + (\hat{\tilde{T}}_w - \tilde{\tilde{T}}_w)^2 + (\hat{q}_w - \tilde{q}_w)^2 + (\hat{r}_w - \tilde{r}_w)^2 + (\hat{1}_w - \tilde{1}_w)^2 + 
(\hat{\omega}_w - \tilde{\omega}_w)^2 + (\hat{f}_w - \tilde{f}_w)^2 \right) \right) 
\]

### 5.7 Revised closeness index

In TOPSIS method, at last we left with two values related to the alternative that prescribed the distance of that particular alternative from CSV\textit{NS}_{f}\textit{S-PIS} and CSV\textit{NS}_{f}\textit{S-NIS}. Therefore, revised closeness index is utilized for the choice of right solution. The revised closeness index $\Lambda(U_s)$ is calculated as:

\[
\Lambda(U_s) = \frac{d(\hat{H}^{PIS}_w, U_s)}{\min_s d(\hat{H}^{PIS}_w, U_s)} - \frac{d(\hat{H}^{NIS}_w, U_s)}{\max_s d(\hat{H}^{NIS}_w, U_s)},
\]

where, $s = 1, 2, \ldots, t$. 

---

M. Akram, M. Shabir, A. Ashraf, Complex neutrosophic N-soft sets: A new model with applications.
5.8 Identify dominant alternative

For the evaluation of dominant alternative with respect to their performance in MAGDM problem, revised closeness index related to each alternative arranged in ascending order. So that the alternative with least revised closeness index will be the required one.

For solving a MAGDM problem, the Algorithm 1 is given as:

Algorithm 1: Steps to deal MAGDM problem by CSVNN_Nf-TOPSIS method

1. **Input**:
   - \( U \): Set of alternatives,
   - \( Y \): Set of attributes,
   - \( \nu \): Weight vector for experts \( \bar{Z}_i \),
   - \( NS_f : (\Phi, Y, N) \) with \( H = \{0, 1, 2, 3, \ldots, N - 1\} \), \( N \in \{1, 2, 3, \ldots\} \).

2. Construct the CSVNN_NfDM \( \mathcal{H}^{(j)} \), using the input data.

3. Evaluate the ACSVNN_NfDM as follows:

   \[
   \mathcal{H}_{ws} = \left( \max_{j=1}^{f}(h_{ws}^{(j)}), [1 - \Pi_{f=1}^{f}(1 - p_{uj}^{(j)\nu_{ws}})]e^{12\pi[1 - \Pi_{f=1}^{f}(1 - t_{uj}^{(j)\nu_{ws}})]}, [\Pi_{f=1}^{f}(q_{uj}^{(j)\nu_{ws}})]e^{12\pi[\Pi_{f=1}^{f}(\omega_{uj}^{(j)})\nu_{ws}]} \right)
   \]

4. Calculating the weight vector \( \theta = (\theta_1, \theta_2, \ldots, \theta_k)^T \) for parameters as:

   \[
   \theta_{ws} = \left( \max_{j=1}^{f}(h_{ws}^{(j)}), [1 - \Pi_{f=1}^{f}(1 - p_{uj}^{(j)\nu_{ws}})]e^{12\pi[1 - \Pi_{f=1}^{f}(1 - t_{uj}^{(j)\nu_{ws}})]}, [\Pi_{f=1}^{f}(q_{uj}^{(j)\nu_{ws}})]e^{12\pi[\Pi_{f=1}^{f}(\omega_{uj}^{(j)})\nu_{ws}]} \right)
   \]

5. Compute the AWCSVNN_NfDM using ACSVNN_NfDM and the weight vector of attributes \( \theta_{ws} \), as follows:

   \[
   \tilde{H}_{ws} = \left( \min(h_{ws}, h_w), p_{ws}p_we^{12\pi t_{ws}t_w}, (q_{ws} + q_w - q_{ws}q_w)e^{12\pi[\omega_{ws}+\omega_w-\omega_{ws}\omega_w]}, (r_{ws} + r_w - r_{ws}r_w)e^{12\pi[f_{ws}+f_w-f_{ws}f_w]} \right)
   \]

6. Evaluate the CSVNN_Nf PIS and CSVNN_Nf NIS.

7. Evaluate the normalized Euclidean distance \( d(\tilde{H}_{ws}^{PIS}, U_s) \) and \( d(\tilde{H}_{ws}^{NIS}, U_s) \).

8. Evaluate the revised closeness index \( \Lambda(U_s) \).

9. Arranged revised closeness index in ascending order.

**Output**: Choose the alternative with minimum revised closeness index.

6 Application

In this section, we solve a MAGDM problem using CSVNN_Nf-TOPSIS method for analyzing the performance of Islamic banks in Pakistan with CAMELS rating system.

6.1 Monitoring performance of Islamic banking industry on the basis of CAMELS rating system.

The banks are more closely monitored other than any field of economy because of their constitution and important role in the economy of the country. Analyzing the banking system create more assurance and reliability in making both short and long term decisions, that in return give on to healthier business in the country. In banking industry, one of the flourishing institute is Islamic banking that follow the rules of Islamic Shariah and promote the Islamic principles to the transaction of financial banking. The evaluation of financial performance of Islamic banking in Pakistan using the CAMELS model and TOPSIS method is necessary for higher level of efficiency that further help to set a benchmark for the country. In this MAGDM problem, following Islamic banks are considered as alternatives:

- \( U_1 \): Bank Albarka(BA)
- \( U_2 \): Bank Islamic (BIL)
- \( U_3 \): Dubai Islamic Bank (DIB)

\[M. \text{ Akram, M. Shabir, A. Ashraf, Complex neutrosophic N-soft sets: A new model with applications.}\]
U₄ : Muslim Commercial Bank (MCB)
U₅ : Meezan Bank (MBL)

For this MAGDM problem, decision making panel consists of three experts ˜Z₁, ˜Z₂, ˜Z₃ that collected data from the official websites of the banks according to the CAMELS model. CAMELS model is generally apply to analyze the performance of the banks on the basis of five different attributes described as follow:

Y₁ : Capital adequacy: Experts rank the capital adequacy by checking the factors of growth plan and capacity to control financial risk and loan.
Y₂ : Asset quality: In this attribute the banking stability is measure whenever the bank faced loss of values of the assets.
Y₃ : Management: Experts rate this attribute by measuring the efficiency of banks while dealing with daily activities.
Y₄ : Earning capacity: This attribute includes the existing assets, earnings and growth of the banks, as well as to remain competitive in economy.
Y₅ : Liquidity: This attribute examine on the basis of the availability of adequate funds by converting assets into the cash.

We solve this MAGDM problem by following the CSVN⁴−TOPSIS method.

**Step 1:** According to these attributes each expert model 5-soft set in Table14 where

0 means ‘Bad’
1 means ‘Ok’
2 means ‘Good’
3 means ‘Great’
4 means ‘Excellent’

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Alternatives</th>
<th>˜Z₁</th>
<th>˜Z₂</th>
<th>˜Z₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y₁</td>
<td>U₁</td>
<td>****=4</td>
<td>***=3</td>
<td>**=2</td>
</tr>
<tr>
<td></td>
<td>U₂</td>
<td>****=4</td>
<td>* =1</td>
<td>***=3</td>
</tr>
<tr>
<td></td>
<td>U₃</td>
<td>****=3</td>
<td>**=2</td>
<td>* =1</td>
</tr>
<tr>
<td></td>
<td>U₄</td>
<td>****=3</td>
<td>* =1</td>
<td>***=3</td>
</tr>
<tr>
<td></td>
<td>U₅</td>
<td>****=4</td>
<td>* =0</td>
<td>* =1</td>
</tr>
<tr>
<td>Y₂</td>
<td>U₁</td>
<td>****=3</td>
<td>**=2</td>
<td>***=3</td>
</tr>
<tr>
<td></td>
<td>U₂</td>
<td>****=3</td>
<td>****=4</td>
<td>****=4</td>
</tr>
<tr>
<td></td>
<td>U₃</td>
<td>****=4</td>
<td>* =0</td>
<td>**=2</td>
</tr>
<tr>
<td></td>
<td>U₄</td>
<td>****=4</td>
<td>**=3</td>
<td>****=4</td>
</tr>
<tr>
<td></td>
<td>U₅</td>
<td>****=4</td>
<td>* =1</td>
<td>** =2</td>
</tr>
<tr>
<td>Y₃</td>
<td>U₁</td>
<td>* =0</td>
<td>* =1</td>
<td>* =1</td>
</tr>
<tr>
<td></td>
<td>U₂</td>
<td>****=4</td>
<td>****=4</td>
<td>****=3</td>
</tr>
<tr>
<td></td>
<td>U₃</td>
<td>* =0</td>
<td>**=2</td>
<td>* =1</td>
</tr>
<tr>
<td></td>
<td>U₄</td>
<td>* =0</td>
<td>**=3</td>
<td>****=4</td>
</tr>
<tr>
<td></td>
<td>U₅</td>
<td>* =0</td>
<td>* =0</td>
<td>* =1</td>
</tr>
<tr>
<td>Y₄</td>
<td>U₁</td>
<td>* =0</td>
<td>* =1</td>
<td>* =0</td>
</tr>
<tr>
<td></td>
<td>U₂</td>
<td>* =0</td>
<td>****=3</td>
<td>** =2</td>
</tr>
<tr>
<td></td>
<td>U₃</td>
<td>* =1</td>
<td>**=2</td>
<td>****=3</td>
</tr>
<tr>
<td></td>
<td>U₄</td>
<td>****=4</td>
<td>****=4</td>
<td>****=3</td>
</tr>
<tr>
<td></td>
<td>U₅</td>
<td>* =0</td>
<td>* =0</td>
<td>* =1</td>
</tr>
<tr>
<td>Y₅</td>
<td>U₁</td>
<td>**=2</td>
<td>* =0</td>
<td>* =1</td>
</tr>
<tr>
<td></td>
<td>U₂</td>
<td>****=3</td>
<td>**=2</td>
<td>* =1</td>
</tr>
<tr>
<td></td>
<td>U₃</td>
<td>****=3</td>
<td>****=4</td>
<td>**=2</td>
</tr>
<tr>
<td></td>
<td>U₄</td>
<td>****=4</td>
<td>****=3</td>
<td>**=2</td>
</tr>
<tr>
<td></td>
<td>U₅</td>
<td>* =0</td>
<td>* =1</td>
<td>* =0</td>
</tr>
</tbody>
</table>

To assign CSVN⁴NSf to each rank in Table14, experts defined grading criteria given in Table 15 and Tables 16, 17, 18 representing the decision of the experts ˜Z₁, ˜Z₂, ˜Z₃, respectively.
Table 15: Grading criteria for CSVN5SS

<table>
<thead>
<tr>
<th>$h^w / J$</th>
<th>degree of truthness</th>
<th>degree of indeterminacy</th>
<th>degree of falsity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h^w$ = 0</td>
<td>0.00, 0.15</td>
<td>0.0, 0.3π</td>
<td>0.85, 1.00</td>
</tr>
<tr>
<td>$h^w$ = 1</td>
<td>0.15, 0.35</td>
<td>0.3π, 0.7π</td>
<td>0.65, 0.85</td>
</tr>
<tr>
<td>$h^w$ = 2</td>
<td>0.35, 0.65</td>
<td>0.7π, 1.3π</td>
<td>0.35, 0.65</td>
</tr>
<tr>
<td>$h^w$ = 3</td>
<td>0.65, 0.85</td>
<td>1.3π, 1.7π</td>
<td>0.15, 0.35</td>
</tr>
<tr>
<td>$h^w$ = 4</td>
<td>0.85, 1.00</td>
<td>1.7π, 2.0π</td>
<td>0.00, 0.15</td>
</tr>
</tbody>
</table>

Table 16: CSVN5DM related to expert $\tilde{Z}_1$.

$\nu = (0.33, 0.40, 0.27)^T$ be the weight vector for the experts so that we get $ACSVN5FDM$ summarized in Table 19.

Table 17: CSVN5DM related to expert $\tilde{Z}_2$.

Table 18: CSVN5DM related to expert $\tilde{Z}_3$.

Step 2: The decision of all experts cumulated using the $CSVNNSW$ operator with $\nu = (0.33, 0.40, 0.27)^T$ be the weight vector for the experts so that we get $ACSVN5FDM$ summarized in Table 19.
Step 3: In CAMELS model each attribute has its own weight and value that continuously change as the time passing out, therefore experts rank them and then assigned CSVNNSfP as accordingly. We summarized the weights of the experts related to the attributes, are arranged in Table 20, using the CSVNNSfWA operator and get the weight vector $\theta$, given as:

$$
\chi = \begin{pmatrix}
(2, 0.0079e^{0.0168\pi}, 0.9893e^{-1.9794\pi}, 0.9902e^{1.9814\pi}) \\
(4, 0.0387e^{0.1720\pi}, 0.9388e^{1.8814\pi}, 0.9425e^{1.8841\pi}) \\
(3, 0.0408e^{0.0804\pi}, 0.9458e^{1.8948\pi}, 0.9489e^{1.9086\pi}) \\
(3, 0.0180e^{0.0372\pi}, 0.9642e^{1.9304\pi}, 0.9842e^{1.9672\pi})
\end{pmatrix}
$$

Step 4: The weight vector $\theta$ and ACSVNNSfDM are encapsulated using the CSVNNSfWG operator into AWCSVNNSfDM, compile in Table 21.

Step 5 The groundwork of the TOPSIS method that differentiate it from others is to evaluate the PIS and NIS that help to find out optimal solution using the tool of distance measure. The criteria evaluated for this MAGDM problem based on CAMELS model and all are related to benefit-type criteria. Therefore, the CSVNNSf-PIS and CSVNNSf-NIS, taking into account the nature of the attributes, are arranged in Table 22.
Step 6 For distance measure, normalized Euclidean distance is used that precisely evaluate the distance between the alternatives and the ideal solutions, simultaneously. Table 23 describe the distance of each alternative from CSV-NS\textsubscript{1}-PIS and CSV-NS\textsubscript{1}-NIS, respectively.

<table>
<thead>
<tr>
<th>( U_s )</th>
<th>( d(H_{wPIS}^{NS}, U_s) )</th>
<th>( d(H_{wNIS}^{NS}, U_s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_1 )</td>
<td>0.133746</td>
<td>0.059764</td>
</tr>
<tr>
<td>( U_2 )</td>
<td>0.005061</td>
<td>0.179298</td>
</tr>
<tr>
<td>( U_3 )</td>
<td>0.084647</td>
<td>0.13363</td>
</tr>
<tr>
<td>( U_4 )</td>
<td>0.003998</td>
<td>0.1793085</td>
</tr>
<tr>
<td>( U_5 )</td>
<td>0.174320</td>
<td>0.042260</td>
</tr>
</tbody>
</table>

Step 7: Revised closeness index is used for ranking the alternatives having the properties of closeness and far-away from the ideal solution at a time. The numeric values of revised closeness index calculated in Table ??

<table>
<thead>
<tr>
<th>( U_s )</th>
<th>( \Lambda(U_s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_1 )</td>
<td>33.1199</td>
</tr>
<tr>
<td>( U_2 )</td>
<td>0.26594</td>
</tr>
<tr>
<td>( U_3 )</td>
<td>20.4343</td>
</tr>
<tr>
<td>( U_4 )</td>
<td>0.00000</td>
</tr>
<tr>
<td>( U_5 )</td>
<td>43.3661</td>
</tr>
</tbody>
</table>

Step 8: Clearly, from the values of revised closeness index we can easily highlight the bank with best performance that is actually the \( U_4 = MCB \) opting as best performer in Pakistan, where, the ascending order of the values of revised closeness index describe the ranks of the banks on the basis of the CAMELS model and TOPSIS method, shown in Table 25

<table>
<thead>
<tr>
<th>Alternative</th>
<th>( U_1 )</th>
<th>( U_2 )</th>
<th>( U_3 )</th>
<th>( U_4 )</th>
<th>( U_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ranking</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

7 Comparison

To prove the versatility of the CSV-NS\textsubscript{1}-TOPSIS method we compare the proposed method with SVN\textsubscript{N}-TOPSIS method [28] by solving the describe MAGDM problem of “Monitoring performance of Islamic banking industry on the basis of CAMELS rating syste” by SVN-TOPSIS method[28]. The evaluation of the problem by SVN-TOPSIS method [28] is as follows:

Step 1 For the implication of SVN-TOPSIS method on the proposed MAGDM problem we have to exclude the grading part as well as reduce the periodic terms to zero in the CSV\textsubscript{N}-NS\textsubscript{1}, so that experts \( \bar{Z}_1, \bar{Z}_2, \bar{Z}_3 \) assigned SVNs to each rank given in Tables 26, 27 and 28, respectively.

<table>
<thead>
<tr>
<th>( Y_1 )</th>
<th>( Y_2 )</th>
<th>( Y_3 )</th>
<th>( Y_4 )</th>
<th>( Y_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_1 )</td>
<td>(0.86, 0.08, 0.07)</td>
<td>(0.71, 0.31, 0.29)</td>
<td>(0.11, 0.91, 0.93)</td>
<td>(0.12, 0.87, 0.86)</td>
</tr>
<tr>
<td>( U_2 )</td>
<td>(0.87, 0.09, 0.08)</td>
<td>(0.66, 0.27, 0.31)</td>
<td>(0.89, 0.04, 0.11)</td>
<td>(0.13, 0.87, 0.86)</td>
</tr>
<tr>
<td>( U_3 )</td>
<td>(0.69, 0.19, 0.22)</td>
<td>(0.88, 0.06, 0.10)</td>
<td>(0.14, 0.88, 0.89)</td>
<td>(0.34, 0.66, 0.67)</td>
</tr>
<tr>
<td>( U_4 )</td>
<td>(0.82, 0.18, 0.21)</td>
<td>(0.91, 0.02, 0.03)</td>
<td>(0.13, 0.88, 0.86)</td>
<td>(0.93, 0.04, 0.01)</td>
</tr>
<tr>
<td>( U_5 )</td>
<td>(0.87, 0.13, 0.12)</td>
<td>(0.90, 0.07, 0.10)</td>
<td>(0.02, 0.95, 0.97)</td>
<td>(0.03, 0.96, 0.98)</td>
</tr>
</tbody>
</table>
Step 2 The weights of experts $\nu = (0.33, 0.40, 0.27)^T$ and averaging operator [28], we can cumulate the aggregated single-valued neutrosophic decision matrix ($ASVN\!DM$), as follows:

$$H_{w\nu} = 
\left(1 - \prod_{j=1}^{r} (1 - p_{w(j)}^{(j)})^{\nu_{w}}, \prod_{j=1}^{r} (q_{w(j)}^{(j)})^{\nu_{w}}, \prod_{j=1}^{r} (r_{w(j)}^{(j)})^{\nu_{w}}\right).$$

The $ASVN\!DM$ is arranged in Table 29.

Step 3 The weights for attributes are calculated, by summarizing the experts opinion about the nature of attributes given in Table 30, as follows:

$$\theta_{w} = 
\left(1 - \prod_{j=1}^{r} (1 - p_{w(j)}^{(j)})^{\nu_{w}}, \prod_{j=1}^{r} (q_{w(j)}^{(j)})^{\nu_{w}}, \prod_{j=1}^{r} (r_{w(j)}^{(j)})^{\nu_{w}}\right).$$

Thus we have,

$$\theta = 
\begin{pmatrix}
(0.0079, 0.9893, 0.9902) \\
(0.0387, 0.9388, 0.9425) \\
(0.0820, 0.9298, 0.9243) \\
(0.0408, 0.9458, 0.9489) \\
(0.0180, 0.9642, 0.9842)
\end{pmatrix}.\]
To measure distance of alternatives from PIS and NIS, Euclidean distance used. The calculated values are given in Table 33.

**Step 4** The aggregated weighted single-valued neutrosophic decision matrix $(AWSVNDM)$, shown in Table 31, calculated as:

$$R_{ws} = \left( p_{ws}p_w, (q_{ws} + q_w - q_{ws}q_w), (r_{ws} + r_w - r_{ws}r_w) \right).$$

**Step 5** Keeping in view the nature of data, Equation 26 and 27 is used for the evaluation of the single-valued neutrosophic positive ideal solution and negative ideal solution arranged in Table 32.

$$H_{w}^{PIS} = \left\{ \begin{array}{ll} \max_s \tilde{T}_{w,s}, & \text{if } Y_w \in BT, \\
\min_s \tilde{T}_{w,s}, & \text{if } Y_w \in CT, \\
\end{array} \right. \quad (26)$$

and

$$H_{w}^{NIS} = \left\{ \begin{array}{ll} \min_s \tilde{T}_{w,s}, & \text{if } Y_w \in BT, \\
\max_s \tilde{T}_{w,s}, & \text{if } Y_w \in CT, \\
\end{array} \right. \quad (27)$$

**Step 6** To measure distance of alternatives from PIS and NIS, Euclidean distance used. The calculated values are given in Table 33.

<table>
<thead>
<tr>
<th>Table 30: Weights for attributes from experts</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_1$</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>$Y_1$</td>
</tr>
<tr>
<td>$Y_2$</td>
</tr>
<tr>
<td>$Y_3$</td>
</tr>
<tr>
<td>$Y_4$</td>
</tr>
<tr>
<td>$Y_5$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 31: $AWSVNDM$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_1$</td>
</tr>
<tr>
<td>$U_1$</td>
</tr>
<tr>
<td>$U_2$</td>
</tr>
<tr>
<td>$U_3$</td>
</tr>
<tr>
<td>$U_4$</td>
</tr>
<tr>
<td>$U_5$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 32: $SVN$-PIS and $SVN$-NIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_1$</td>
</tr>
<tr>
<td>$U_2$</td>
</tr>
<tr>
<td>$U_3$</td>
</tr>
<tr>
<td>$U_4$</td>
</tr>
<tr>
<td>$U_5$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 33: Distance measures of alternatives from ideal solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_s$</td>
</tr>
<tr>
<td>$U_1$</td>
</tr>
<tr>
<td>$U_2$</td>
</tr>
<tr>
<td>$U_3$</td>
</tr>
<tr>
<td>$U_4$</td>
</tr>
<tr>
<td>$U_5$</td>
</tr>
</tbody>
</table>
Step 7: The revised closeness index calculated using Equation 28, is tabulated in Table 34 and the ranks evaluated through the index values are arranged in Table 35 in descending order, according to which $U_4$ is the best performer.

$$\Lambda(U_s) = \frac{d(H_w^{NIS}, U_s)}{d(H_w^{PIS}, U_s) + d(H_w^{NIS}, U_s)},$$

where, $s = 1, 2, \ldots, k$.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>$\Lambda(U_s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_1$</td>
<td>0.0769</td>
</tr>
<tr>
<td>$U_2$</td>
<td>0.4641</td>
</tr>
<tr>
<td>$U_3$</td>
<td>0.2429</td>
</tr>
<tr>
<td>$U_4$</td>
<td>0.6041</td>
</tr>
<tr>
<td>$U_5$</td>
<td>0.1900</td>
</tr>
</tbody>
</table>

Table 35: Ranking in single-valued neutrosophic environment

<table>
<thead>
<tr>
<th>Alternative</th>
<th>$U_1$</th>
<th>$U_2$</th>
<th>$U_3$</th>
<th>$U_4$</th>
<th>$U_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ranking</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

7.1 Discussion

1. The comparison of the $CSVN^{N}NS_f$-TOPSIS method with the existing SVN-TOPSIS method have same findings for the Islamic bank as best performer in Pakistan but the consequences relevant to the ranks of other banks have no analogy given in Table 36.

<table>
<thead>
<tr>
<th>Model</th>
<th>Ranks</th>
<th>Best Performer</th>
</tr>
</thead>
<tbody>
<tr>
<td>SVN-TOPSIS [28]</td>
<td>$U_4 &gt; U_2 &gt; U_3 &gt; U_5 &gt; U_1$</td>
<td>$U_4$</td>
</tr>
<tr>
<td>$CSVN^{N}NS_f$-TOPSIS(Proposed)</td>
<td>$U_4 &gt; U_2 &gt; U_3 &gt; U_1 &gt; U_5$</td>
<td>$U_4$</td>
</tr>
</tbody>
</table>

2. The expertise of the presented methodology $CSVN^{N}NS_f$-TOPSIS method to manipulate the indeterminacy degree and two dimensional information in the MAGDM problems by using the frame of $CSVN^{N}NS_f$.

3. The presented methodology of $CSVN^{N}NS_f$-TOPSIS method has potential to operate the problems of $IFNS_f$S, being the generalization of the $IFS_f$.

4. The presented model has proficiency to overcome the latest problems characterized by parameterized ordered evaluation system but the existing methods have no grip on such problems.

5. By employing $N = 2$ and periodic terms equal to zero, we switch from $CSVN^{N}NS_f$ environment to single-valued environment so that the $CSVN^{N}NS_f$-TOPSIS method could sensibly handled the daily life problems under single-valued environment.

8 Conclusion

In this paper we have merged the idea of single-valued neutrosophic set with $N$-soft sets, and in doing so, we have initiated the idea of $CSVN^{N}NS_f$S. These sets combine the 2-dimensional single-valued neutrosophic nature of the attributes with parameterized ordered grades which demonstrates their superiority over $FNS_f$S, $IFNS_f$S and $N^{N}S_f$S. A MAGDM model of TOPSIS method is extended to handle the real life problems under the frame of $CSVN^{N}NS_f$S in which the ordered grades are assigned to each alternative as initial evaluation that are further characterized by $CSVN^{N}NS_f$S. The PIS and NIS in $CSVN^{N}NS_f$-TOPSIS method have been determined by the score function which has been further employed to quantify the distance measures and the closeness index that sort the alternatives from highest to lowest rank. An example from the banking industry and the comparison with single-valued neutrosophic TOPSIS method have clarified the accuracy and superiority of the presented technique. The new model and method pioneer a promising avenue for research in the decision making arena that we have only hinted at in this paper. Moreover, the proposed $CSVN^{N}NS_f$-TOPSIS method does not evaluate the relative importance of the normalized Euclidean distances. Therefore we will work for the extension of the VIKOR method under a $CSVN^{N}NS_f$ environment, which might be more credible and trustworthy.

M. Akram, M. Shabir, A. Ashraf, Complex neutrosophic $N$-soft sets: A new model with applications.
Data availability: No data were used to support this study.

Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

References


Received: Jan. 7, 2021. Accepted: April 6, 2021.