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Neutrosophic Theory and Its Application in Various Queueing Models: Case Studies

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Abstract: Queueing theory is an important technique to study and evaluate the performance of system. Queueing theory is applied in many applications such as logistics, finance, emergency services and project management, etc. In this research we apply neutrosophic philosophy in queueing theory. We deal with several queue models such as M/M/1 queue, M/M/s queue and M/M/1/b queue. We illustrate, solve, and find the performance measures of M/M/1, M/M/s, and M/M/1/b crisp queue models via examples with exact arrival rate and service rate. Queueing models affect by many factors such as arrival rate, service rate, number of servers, etc. These factors are not constantly expressed by accurate times; hence we express the parameters of queueing system by the neutrosophic. We express arriving rates and serving rates by neutrosophic values. We also illustrate, solve, and find the performance measures of NM/NM/1, NM/NM/s, and NM/NM/1/b neutrosophic queue models via examples. We concluded that the performance measures of neutrosophic queue models is more accurate than crisp queue models.

Keywords: Neutrosophic Set, Queueing Theory, Poisson Process, Exponential Distribution.

1. Introduction

In 1909 Erlang developed queueing theory for modeling waiting lines and developing effectual systems that decrease waiting times of customers and makes it conceivable to serve more customers and growth profits of organizations.

In classical queueing theory the statement of well determined knowledge of queueing system's parameters such as arrival, service and departure rate are important and it is often imprecise in reality [1,2].

For dealing with problems of classical queueing theory, many researchers presented queueing theory in fuzzy environment to deal with uncertainty in parameters of queueing systems as in [3,4].

Since fuzzy and intuitionistic fuzzy theories does not represent reality efficiently and fails to simulate human thinking, Smarandache in 1995 presented neutrosophic logic. Neutrosophic logic is a generalization of fuzzy and intuitionistic fuzzy logic [5,6,7,8]. Neutrosophic logic able to deal with indeterminacy of data besides considering truth and falsity degrees. So, presenting queueing theory in neutrosophic environment makes decisions more competent [3,9,10,11,12,13,14,16].

In Neutrosophic set truth, indeterminacy, and falsity degrees are real values ranges from] $0^{-},\ 1^{+}$[with no restriction on the sum. For simplifying application of neutrosophic set in real cases, a single valued neutrosophic set is presented [15].

Now we can say that neutrosophic queueing theory has imprecise values of parameters. For example, let λ which is the arrival rate in the form $\lambda_N = \lambda + I$ and μ which is the service rate in the form $\mu_N = \mu + I$, where I determines the indeterminant part of the given values.

In this research we show the important role of neutrosophic theory [9,10] to deal with vague parameters of some queueing models that is: (NM/NM/1) :(FCFS/∞/∞) queue, (NM/NM/s) :(FCFS/∞/∞) queue and (NM/NM/1) :(FCFS/∞/b) queue, and we prove the applicability and superiority of neutrosophic performance via solving various examples.

The remaining parts of this research consist of the following: In Section 2, we briefly discussed the queueing theory preliminaries. Section 3 discusses the fundamental steps of the neutrosophic queueing theory. In section 4, real case studies are solved for showing important role of neutrosophic in queueing theory. Section 5 presents the conclusion, findings and offers future work suggestions.

2. Queueing Theory Preliminaries

In this section the major preliminaries and concepts of single server waiting line model and multi-server waiting line model are presented.

The structure of waiting line systems presented in Figure 1.

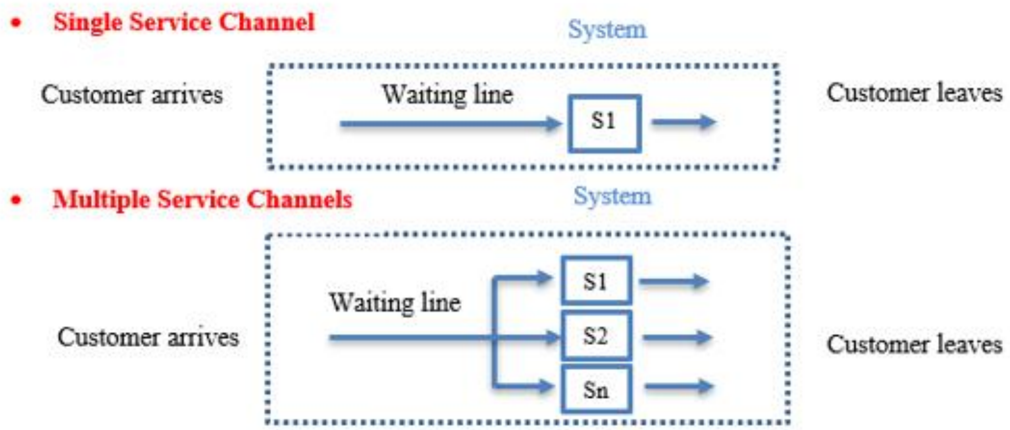

Fig.1. The structures **of** waiting line system

2.1 Single Server Waiting Line Model

2.1.1 (M/M/1) :(FCFS/∞/∞) [3,16]

The waiting line model considers the elementary in server, queue, and stage. There assumptions on this model are as follows:

1. The customers do not leave the queue and their population is infinite.

2. The arrival of customer are specified by a Poisson distribution with a mean arrival rate λ, so the time between the arrival of consecutive customers is specified by an exponential distribution with an average of $1/\lambda$.

3. The service rate of customer is specified by a Poisson distribution with a mean service rate of μ , so the service time of customer is defined by an exponential distribution with an average of $1/\mu$.

4. The customers are served according to first-come, first-served.

We can calculate the operating features of a waiting line system using the following formulas:

$\lambda =$ mean arrival rate of customers

$\mu =$ mean service rate

$\rho = \frac{\lambda}{\mu}$ = average utilization of system (1)

$L_S = \frac{\lambda}{\mu - \lambda} =$ average number of customers in the system (2)

$L_Q = \rho\, L$= average number of customers waiting in line (3)

W_S= $\frac{1}{\mu - \lambda}$ = average time customers spent in the system (4)

W_Q= $\rho \;\; W_S$= average time customers spent waiting in line (5)

$P_n = (1 - P)P^n =$ the probability that n customers are in the service system at a given time (6)

2.1.2 (M/M/1) :(FCFS/∞/b) [3, 16]

The interarrival times and serving times are specified in this model according to exponential distribution, there is one server for customers. The customers are served according to FCFS policy, the calling source is infinite and system size is finite by b including the one being served.

The performance measures of the system are as follows:

$$\mathrm{P(k)} = \frac{\rho^{k}(1-\rho)}{(1-\rho^{b+1})} \tag{7}$$

$$L_Q = \frac{\rho^2[\,1 - b\rho^{b-1} + (b-1)\rho^{b}\,]}{(1-\rho)(1-\rho^{b+1})} \tag{8}$$

$$L_s = L_Q + Eff\rho \;\;;\;\; Eff\rho = \frac{Eff\,\lambda}{\mu} \quad , Eff\,\lambda = \lambda(1 - p(b)) \tag{9}$$

$$W_Q = \frac{L_Q}{Eff\,\lambda} \tag{10}$$

$$W_S = \frac{L_S}{Eff\,\lambda} \tag{11}$$

2.2 Multi-Server Waiting Line Model (M/M/s) :(FCFS/∞/∞) [3,16]

The assumptions on this model are described as follows:

1. The customers come from a single line.

2. The customers are served by the first server available.

3. There are s identical servers, the service time of each server is specified by exponential distribution and the mean service time is expressed by $1/\mu$. We can describe the operating features using the following formulas:

$s =$ the number of servers in the system

$$\rho = \frac{\lambda}{s\mu} = \text{the average utilization of system} \tag{12}$$

$$p_0 = \left[\sum_{n=0}^{s-1} \frac{\left(\lambda/\mu\right)^n}{n!} + \frac{\left(\lambda/\mu\right)^s}{s!}\left(\frac{1}{1-\rho}\right)\right]^{-1} \tag{13}$$

= the probability that no customers are in the system

$$L_Q = \frac{p_0\left(\lambda/\mu\right)^s \rho}{s!\,(1-\rho)^2} = \text{the average number of customers waiting in line} \tag{14}$$

$$W_Q = \frac{L_Q}{\lambda} = \text{the average time spent waiting in line} \tag{15}$$

$$W_S = W_Q + \frac{1}{\mu} = \text{the average time spent in the system, including service} \tag{16}$$

$$L_S = \lambda\ \mathrm{W} = \text{the average number of customers in the service system} \tag{17}$$

$$P_n = \begin{cases} \frac{\left(\lambda/\mu\right)^n}{n!} p_0 & for\ n \le s \\ \frac{\left(\lambda/\mu\right)^n}{s! s^{n-s}} p_0 & for\ n > s \end{cases} \tag{18}$$

= the probability that n customers are in the system at a given time

3. Neutrosophic Queueing Theory Preliminaries

In this section the major preliminaries and concepts of neutrosophic queues are presented.

3.1 Neutrosophic Queue

Neutrosophic queue is a queueing system in which queueing parameters such as average rate of customers entering the queueing system (λ), and average rate of customers being served (μ) are neutrosophic numbers [3,16].

In neutrosophic queueing λ is denoted by $\lambda_N = [\lambda^L, \lambda^U]$and μ is denoted by $\mu_N = [\mu^L, \mu^U]$. Then, the neutrosophic traffic intensity if we have s servers is denoted by

$$\rho_N = \frac{\lambda_N}{s\mu_N} = \frac{[\lambda^L, \lambda^U]}{s[\mu^L, \mu^U]}. \tag{19}$$

3.2 Arithmetic Operations of Interval Values

Let $[c_1, d_1], [c_2, d_2]$ be two Intervals where $c_1, c_2, d_1, d_2 \in \mathbb{R}$ and for practical cases set $c_1 > 0, c_2 > 0, d_1 > 0, d_2 > 0$ then:

$$[c_1, d_1] + [c_2, d_2] = [c_1 + c_2, d_1 + d_2] \tag{20}$$

$$[c_1, d_1] - [c_2, d_2] = [c_1 - d_2, d_1 - c_2] \tag{21}$$

$$[c_1, d_1] * [c_2, d_2] = [c_1 c_2, d_1 d_2] \tag{22}$$

$$[c_1, d_1]/[c_2, d_2] = [c_1, d_1] * \frac{1}{[c_2, d_2]} = \left[\frac{c_1}{d_2}, \frac{d_1}{c_2}\right] \tag{23}$$

3.3 (NM/NM/1) :(FCFS/∞/∞) Queue [16]

After replacing crisp parameters by neutrosophic parameters, the neutrosophic probability that arriving customer will find k customers in queue are as follows:

$NP(k) = (1 - \rho_N){\rho_N}^k; k = 0,1, \ldots$

$$NP(k) = \left(1 - \left[\frac{\lambda^L}{\mu^U}, \frac{\lambda^u}{\mu^L}\right]\right)\left[\frac{\lambda^L}{\mu^u}, \frac{\lambda^U}{\mu^L}\right]^k; k = 0,1, \ldots$$

$$NP(k) = \left[1 - \frac{\lambda^U}{\mu^L}, 1 - \frac{\lambda^L}{\mu^U}\right]\left[\left(\frac{\lambda^L}{\mu^U}\right)^k, \left(\frac{\lambda^U}{\mu^L}\right)^k\right]; k = 0,1, \ldots$$

$$NP(k) = \left[\left(1 - \frac{\lambda^U}{\mu^L}\right)\left(\frac{\lambda^L}{\mu^U}\right)^k, \left(1 - \frac{\lambda^L}{\mu^U}\right)\left(\frac{\lambda^U}{\mu^L}\right)^k\right]; k = 0,1, \ldots \tag{24}$$

The performance measures of the system are as follows:

- Neutrosophic expected number of customers in system:

  $NL_s = \rho_N/(1-\rho_N)$, then

$$NL_s = \left[\frac{\frac{\lambda^L}{\mu^U}}{1-\frac{\lambda^L}{\mu^U}}, \frac{\frac{\lambda^u}{\mu^L}}{1-\frac{\lambda^U}{\mu^L}}\right] \tag{25}$$

- Neutrosophic expected number of customers in queue:

  $NL_Q = {\rho_N}^2/(1-\rho_N)$, then

$$NL_Q = \left[\frac{\left(\frac{\lambda^L}{\mu^U}\right)^2}{1-\frac{\lambda^L}{\mu^U}}, \frac{\left(\frac{\lambda^U}{\mu^L}\right)^2}{1-\frac{\lambda^U}{\mu^L}}\right] \tag{26}$$

- Neutrosophic expected waiting time in system:

  $\mathrm{N}W_s = \frac{1}{\mu_N - \lambda_N}$, then

$$NW_s = \left[\frac{1}{\mu^U-\lambda^L}, \frac{1}{\mu^L-\lambda^U}\right] \tag{27}$$

- Neutrosophic expected waiting time in queue:

  $NW_Q = \rho_N/(\mu_N - \lambda_N)$, then

$$NW_Q = \left[\frac{\frac{\lambda^L}{\mu^U}}{\mu^U-\lambda^L}, \frac{\frac{\lambda^U}{\mu^L}}{\mu^L-\lambda^U}\right] \tag{28}$$

3.4 (NM/NM/s) :(FCFS/∞/∞) Queue [16]

NM/NM/s queue is the same as NM/NM/1 queue except that in NM/NM/s customers are being served by s parallel homogeneous servers according to FCFS policy.

The neutrosophic probability that K customers in the queue will be:

$$NP(k)=\begin{cases}\frac{(s\rho_N)^k}{k!}NP(0); k<s\\ \frac{{\rho_N}^k s^s}{s!}NP(0); k\geq s\end{cases}$$

$$NP(k)=\begin{cases}\frac{\left(s\left[\frac{\lambda^L}{\mu^U},\frac{\lambda^U}{\mu^L}\right]\right)^k}{k!}NP(0); k<s\\ \frac{\left[\frac{\lambda^L}{\mu^l},\frac{\lambda^U}{\mu^L}\right]^k s^s}{s!}NP(0); k\geq s\end{cases} \tag{29}$$

Where: $NP(0)=\left(\sum_{n=0}^{s-1}\frac{(s\rho_N)^n}{n!}+\frac{(s\rho_N)^s}{s!}\cdot\frac{1}{1-\rho_N}\right)^{-1}$

$$NP(0)=\left(\sum_{n=0}^{s-1}\frac{\left(s\left[\frac{\lambda^L}{\mu},\frac{\lambda^u}{\mu^L}\right]\right)^n}{n!}+\frac{\left(s\left[\frac{\lambda^L}{\mu^u},\frac{\lambda^u}{\mu^L}\right]\right)^s}{s!}\cdot\frac{1}{1-\left[\frac{\lambda^L}{\mu^U},\frac{\lambda^U}{\mu^L}\right]}\right)^{-1} \quad \text{OR}$$

$$NP(0)=\left[\sum_{n=0}^{s-1}\frac{\left({}^{\lambda_N}/_{\mu_N}\right)^n}{n!}+\frac{\left({}^{\lambda_N}/_{\mu_N}\right)^s}{s!}\left(\frac{1}{1-\rho_N}\right)\right]^{-1} \tag{30}$$

The neutrosophic performance measures will be as follows:

- The average number of customers waiting in line:

$$NL_Q=\frac{NP(0)\left(\frac{\lambda_N}{\mu_N}\right)^S\rho_N}{S!(1-\rho_N)^2} \tag{31}$$

- The average waiting time of customer in line:

$$NW_Q=\frac{NL_Q}{\lambda_N} \tag{32}$$

- The average waiting time of customer in the system:

$$NW_S=NW_Q+\frac{1}{\mu_N} \tag{33}$$

- The average number of customers in the system:

$$NL_s=\ \lambda_N NW_S \tag{34}$$

3.5 (NM/NM/1) :(FCFS/∞/b) Queue [16]

In this model the interarrival times and serving times are specified according to neutrosophic exponential distribution, there is one server for customers. The customers are served according to FCFS policy, the calling source is infinite and system size is finite by b including the one being served. The neutrosophic probability that K customer in the queue will be:

$$NP(k) = \frac{{\rho_N}^{k}(1-\rho_N)}{(1-{\rho_N}^{b+1})}; k = 0..b$$

$$NP(k) = \frac{\left[\frac{\lambda^L}{\mu^U}, \frac{\lambda^U}{\mu^L}\right]^k \left(1-\left[\frac{\lambda^L}{\mu^L}, \frac{\lambda^U}{\mu^L}\right]\right)}{\left(1-\left[\frac{\lambda^L}{\mu^U}, \frac{\lambda^U}{\mu^L}\right]^{b+1}\right)}; k = 0..b \tag{35}$$

The performance measures will be then as follows:

- The average number of customers waiting in line:

$$NL_Q = \frac{{\rho_N}^{2}[1-b{\rho_N}^{b-1}+(b-1){\rho_N}^{b}]}{(1-\rho_N)(1-{\rho_N}^{b+1})}$$, then

$$NL_Q = \frac{\left[\frac{\lambda^L}{\mu^U}, \frac{\lambda^U}{\mu^L}\right]^2 \left[1-b\left[\frac{\lambda^L}{\mu^U}, \frac{\lambda^U}{\mu^L}\right]^{b-1} + (b-1)\left[\frac{\lambda^L}{\mu^U}, \frac{\lambda^v}{\mu^L}\right]^b\right]}{\left[1-\frac{\lambda^U}{\mu^L}, 1-\frac{\lambda^L}{\mu^U}\right]\left(1-\left[\frac{\lambda^L}{\mu^U}, \frac{\lambda^U}{\mu^L}\right]^{b+1}\right)} \tag{36}$$

- The average number of customers in the system:

$$NL_S = NL_Q + Eff\rho_N, \tag{37}$$

$$Eff\ \ \rho_N = \frac{Eff\lambda_N}{\mu_N}, Eff\lambda_N = \lambda_N(1-NP(b)) \tag{38}$$

$$Eff\ \ \lambda_N = [\lambda^{\mathrm{L}}, \lambda^U](1-NP(b)) \tag{39}$$

- The average waiting time of customer in line:

$$NW_Q = \frac{1}{Eff\lambda_N} NL_Q \tag{40}$$

- The average waiting time of customer in the system:

$$NW_S = \frac{1}{Eff\lambda_N} NL_S \tag{41}$$

The methodology for solving neutrosophic queueing models presented in Figure 2.

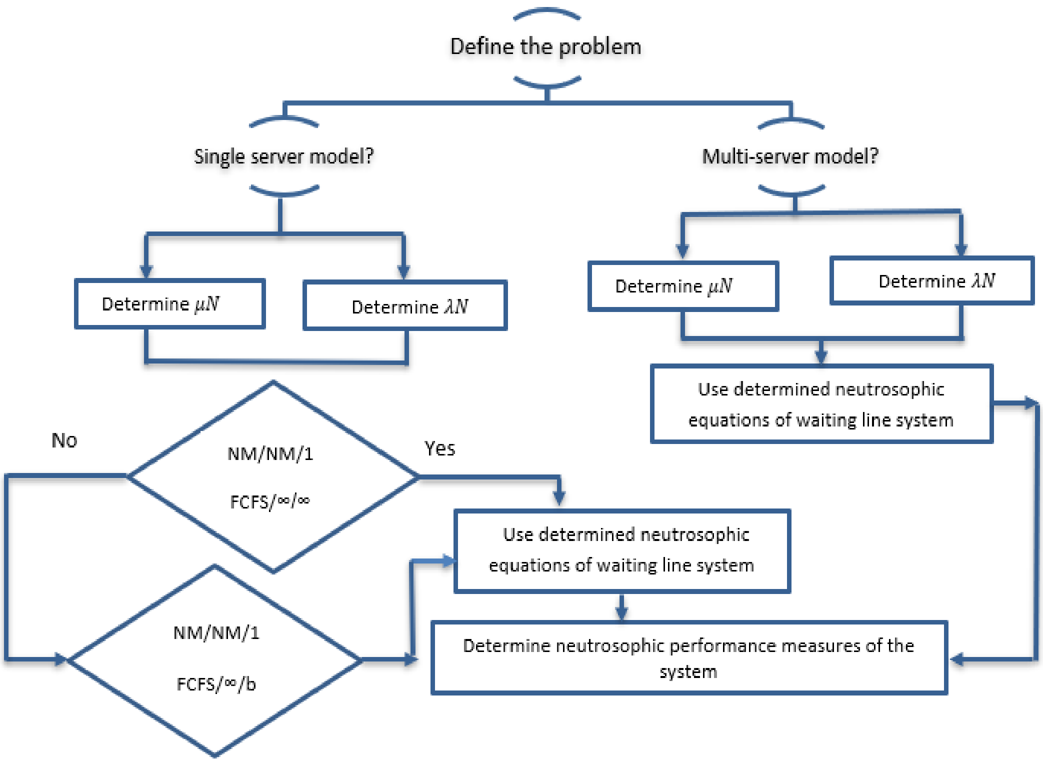

Fig.2. The methodology for solving neutrosophic queueing models

4. Case Studies

In this section various case studies on crisp and neutrosophic queues are presented and solved.

4.1 Example on ((M/M/1) :(FCFS/∞/∞) Crisp Queue Model

The computer lab at State University helps the students by help desk. The students stand in front of the desk to wait for help. Students are served according to priority rule first-come, first-served. Students arrive according to Poisson process with a mean arrival rate 15 students per hour. Students are served by service rate exponentially distributed with an average 20 students per hour. Find the performance measures of the system.

(a) The average utilization of the system

(b) The average number of students in the system

(c) The average number of students waiting in queue

(d) The average waiting time in the system

(e) The average waiting time in queue

Crisp solution

(a) By using Eq. (1), the average utilization is as follows: $\rho = \frac{15}{20}$ = 0.75, or 75%.

(b) By using Eq.(2), the average number of students in the system is as follows: L_s = $\frac{15}{20-15} = 3$ students

(c) By using Eq.(3), the average number of students waiting in queue: $L_Q = 0.75 \times 3$ =2.25 students

(d) By using Eq.(4), the average waiting time in the system: W_s = $\frac{1}{20-15}$= 0.2 hours, or 12 minutes

(e) By using Eq.(5), the average waiting time in queue: W_Q = 0.75 × 0.2 = 0.15 hours, or 9 minutes

4.2 Example on (NM/NM/1) :(FCFS/∞/∞) Neutrosophic Queue Model

The computer lab at State University helps the students by help desk. The students stand in front of the desk to wait for help. Students are served according to priority rule first-come, first-served. Students arrive according to Poisson process with a mean arrival rate between 14 and 16 students per hour. Students are served by service rate exponentially distributed with an average 19 and 21 students per hour. Find the performance measures of the system.

(a) The average utilization of the system

(b) The average number of students in the system

(c) The average number of students waiting in queue

(d) The average waiting time in the system

(e) The average waiting time in queue

Neutrosophic solution

$\lambda_N = [\,14{,}16]$ students per hour.
$\mu_N = [19{,}21]$ students per hour.

a) Average utilization: $\rho_N = \frac{\lambda_N}{\mu_N} = \frac{[\,14,16]}{[19,21]}$ = [0.66, 0.84]. We can say the efficiency of the system ranges between 0.66 and 0.84 and 0.75 (crisp value) ∈ [0.66,0.84].

b) By using Eq.(25), the average number of students in the system: $NL_s = \frac{[0.66\,,0.84]}{(1-[0.66\,,0.84])} = \frac{[0.66\,,0.84]}{[0.16\,,0.34]} = [1.94\,,5.25]$. Which means that expected number of students in system ranges between 1.94 and 5.25 and 3 (crisp value) ∈ [1.94 , 5.25].

c) By using Eq. (26), the average number of students in queue: $NL_Q = \frac{[0.66\,,0.84]^2}{(1-[0.66\,,0.84])} =$

$\frac{[0.4356\,,0.7056]}{[0.16\,,0.34]} =$ [1.28, 4.41]. Which means that expected number of students in queue ranges between 1.28 and 4.41 and 2.25 (crisp value) $\in$ [1.28 , 4.41].

d) By using Eq. (27), the average waiting time in the system: $NW_s = \frac{1}{[3\,,7]}$ = [0.14,0.33].

Which means that mean waiting time in system ranges between 8.4 mins and 19.8 mins and 12 mins (crisp value) $\in$ [8.4 , 19.8].

e) By using Eq. (28), the average waiting time a student in queue: $NW_Q = \frac{[0.66\,,0.84]}{[3,7]} =$

[0.09, 0.28]. Which means that mean waiting time in queue ranges between 5.4 mins and 16.8 mins and 9 mins (crisp value) $\in$ [5.4 , 16.8].

4.3 Example on ((M/M/s) :(FCFS/∞/∞) Crisp Queue Model

State University has intended to maximize the number of assignments. Instead of a single person working at the help desk, the university planned to have three servers. The students will arrive at a rate of 45 per hour, according to a poison distribution. The service rate for each of the three servers is 18 students per hour with exponential service times. Find the following performance measures of the system.

(a) The average utilization of the help desk

(b) The average number of students in the queue

(c) The average waiting time in the queue

(d) The average waiting time in the system

(e) The average number of students in the system

Crisp solution

(a) Average utilization: $\rho = \frac{\lambda}{s\mu} = \frac{45}{3\times 18} =$ = 0.833, or 83.3%

(b) The average number of students in the queue:
Firstly, we find the probability that there are no students in the system using Eq. (13) as follows:

$$p_0 = \left[\frac{(45/18)^0}{0!} + \frac{(45/18)^1}{1!} + \frac{(45/18)^2}{2!} + \left(\frac{(45/18)^3}{3!}\left(\frac{1}{1-0.833}\right)\right)\right]^{-1}$$

$= \frac{1}{22.215}$ = 0.045, or 4.5% of having no students in the system

By using Eq. (14), the average number of students in the queue is as follows:

$$L_Q = \frac{0.045(45/18)^3 \times 0.833}{3! \times (1-0.833)^2} = \frac{0.5857}{0.1673} = 3.5 \text{ students}$$

(c) By using Eq. (15), the average waiting time in the queue: $W_q = \frac{3.5}{45} =$ 0.078 hour, or 4.68 minutes

(d) By using Eq. (16), the average waiting time in the system: W_s =0.078 + $\frac{1}{18}$ = 0.134 hour, or 8.04 minutes

(e) By using Eq. (17), the average number of students in the system: $L_s = 45(0.134) =$ 6.03 students

4.4 Example on (NM/NM/s) :(FCFS/∞/∞) Neutrosophic Queue Model

State University has intended to maximize the number of assignments. Instead of a single person working at the help desk, the university planned to have three servers. The students will arrive at a rate of [44, 46] students per hour, according to Poisson distribution. The service rate for each of the three servers is [17,19] students per hour with exponential service times. Find the following performance measures of the system.

(a) The average utilization of the help desk

(b) The average number of students in the queue

(c) The average waiting time in the queue

(d) The average waiting time in the system

(e) The average number of students in the system

Neutrosophic solution

$\lambda_N = [\,44{,}46]$ students per hour.

$\mu_N = [17{,}19]$ students per hour.

a) Average utilization: $\rho_N = \frac{\lambda_N}{s\mu_N} = \frac{[\,44,46]}{3[17,19]} = \frac{[\,44,46]}{[51,57]}$ = [0.77,0.90].We can say that the efficiency of the system ranges between .0.77 and 0.9 and 0.83 (crisp value) ∈ [0.77 , 0.90].

b) The average number of students in the queue:
Firstly, we find the probability that there are no students in the system using Eq. (30) as follows:

$$NP(0) = \left[\frac{([2.3,2.7])^0}{0!} + \frac{([2.3,2.7])^1}{1!} + \frac{([2.3,2.7])^2}{2!} + \left(\frac{([2.3,2.7])^3}{3!}\left(\frac{1}{1-[0.77,0.9]}\right)\right)\right]^{-1}$$

$=\left[[5.9,7.3] + \left(\frac{[12.16,19.8]}{6} \left(\frac{1}{[0.1,0.23]}\right)\right)\right]^{-1}$

$=[[5.9\ ,7.3] + ([2.02\ ,3.3][4.3\ ,10])]^{-1}$

= [0.024, 0.068] and we can say that the probability that we will find no student in the system ranges between .0.0.024 and 0.068 and 0.045 (crisp value) ∈ [0.024, 0.68].

By using Eq. (31), the average number of students waiting in line is as follows:

$NL_Q = \frac{[0.024,0.068]\ ([2.3,2.7])^3 [0.77,0.9]}{3!\ ([0.1,0.23])^2}$ = [0.69, 20].

Which means that expected number of students in queue ranges between 0.69 and 20 and 3.5 (crisp value) ∈ [0.069, 20].

(c) By using Eq. (32), the average waiting time in the queue is as follows:

$NW_Q = \frac{[0.69,\ 20]}{[44\ ,46]} = [0.015\ ,0.45]$ hour = $[0.9\ ,27]$ minutes which means that mean waiting time in queue ranges between 0.9 mins and 27 mins and 4.68 (crisp value) ∈ $[0.9\ ,27]$minutes.

(d) By using Eq. (33), the average waiting time in the system is as follows:

NW_S =[0.015,0.45] + $\frac{1}{[17,19]}$ = [0.06,0.5] hour = [3.6,30] minutes which means that mean waiting time in system ranges between 3.6 mins and 30 mins and 8.04 (crisp value) ∈ [3.6,30] minutes.

(e) By using Eq. (34), the average number of students in the system is as follows:

NL_s= [44,46] [0.06,0.5] = [2.64,23] students, which means that expected number of students in system ranges between 2.64 and 23 and 6.03 (crisp value) ∈ [2.64,23].

4.5 Example on (M/M/1) :(FCFS/∞/b) Crisp Queue Model

The packets of wireless access gateway arrive at a mean rate of 125 packets per second, they are buffered until they can be transmitted. The gateway takes 500 seconds to transmit a packet. The gateway currently has 13 places (including the packet being transmitted) and packets that arrive when the buffer is full are lost. Find the probability that a new packet is going to be lost, then find the performance measures of the system.

Crisp solution

By using Eq. (1), $\rho = \frac{125}{500} = 0.25$

Then, by using Eq. (7) the probability that a new packet is going to be lost is as follows:

$$\mathrm{P(k)} = \frac{(0.25)^{13}(0.75)}{1-(0.25)^{14}} = 1.12 \times 10^{-8}$$

The performance measures of the system are as follows:

(a) By using Eq. (8), the average number of packets waiting in the queue is as follows:

$$L_Q = \frac{(0.25)^2[\,1-13(0.25)^{12}+12\,(0.25)^{13}]}{0.75\,[1-(0.25)^{14}]} = 0.0834$$

(b) By using Eq. (9), the average number of packets in the system is as follows:

$$Eff\,\lambda = 125(1-1.12\times 10^{-8}) = 124.9$$

$$Eff\rho = \frac{124.9}{500} = 0.249$$

Hence $L_s = 0.0834 + 0.249 = 0.3324$

(a) By using Eq. (10), the average waiting time in the queue is as follows:

$W_Q = \frac{0.0834}{124.9}$ =0.00066 seconds

(b) By using Eq. (11), the average waiting time in the system is as follows:

$W_S = \frac{0.3324}{124.9} = 0.00266$ seconds

4.6 Example on (NM/NM/1) :(FCFS/∞/b) Neutrosophic Queue Model

The packets of wireless access gateway arrive at a mean rate of [124,126] packets per second, and they are buffered until they can be transmitted. The gateway takes [499,501] seconds to transmit a packet. The gateway currently has 13 places (including the packet being transmitted) and packets that arrive when the buffer is full are lost. Calculate the probability that a new packet is going to be lost. Find the performance measures of the system.

Neutrosophic solution

$\lambda_N = [\ 124{,}126]$ packets per second.

$\mu_N = [499{,}501]$ packets per second.

$\rho_N = \frac{\lambda_N}{\mu_N} = \frac{[\ 124,126]}{[499,501]} = [0.247{,}0.252]$ and 0.25 (crisp value) $\in [0.247\ , 0.252]$.

By using Eq. (35), the probability that a new packet is going to be lost is as follows:

$$\mathrm{NP(k)} = \frac{[0.247\ ,0.252]^{13}(1-[0.247\ ,0.252])}{(1-[0.247\ ,0.252]^{14})} = [95 \times 10^{-10}, 124 \times 10^{-10}]$$

and 1.12×10^{-8} (crisp value) $\in [95 \times 10^{-10}, 124 \times 10^{-10}]$

The performance measures of the system are as follows:

(a) By using Eq. (36), the average number of packets waiting in line is as follows:

$$NL_Q = \frac{[0.247\ ,0.252]^2[\ 1-13[0.247\ ,0.252]^{12}+\ 12\ [0.247\ ,0.252]^{13}]}{(1-[0.247\ ,0.252])(1-[0.247\ ,0.252]^{14})} = \frac{[0.060{,}0.0629]}{[0.747{,}0.752]} = [0.079{,}0.084]$$

Means that average number of waiting packets will be between 0.079 and 0.084 and 0.0834 (crisp value) $\in [0.079{,}0.084]$.

(b) By using Eq. (37), Eq. (38), and Eq. (39), the average number of packets in the system is as follows:

$$Eff\ \lambda_N = [124{,}126][1-(95 \times 10^{-10}, 124 \times 10^{-10})] =$$

$$[124{,}126][0.99999998{,}0.99999999] = [123.9{,}125.9]$$

$$Eff\rho = \frac{[123.9,125.9]}{[499,501]} = [0.2473{,}0.2523]$$

Hence, $NL_s = [0.079{,}0.084] + [0.2473{,}0.2523] = [0.3263{,}0.3363]$

Means that average number of packets in the system will be between 0.3263 and 0.3363 and 0.3324 (crisp value) $\in [0.3263{,}0.3363]$.

(c) By using Eq. (40), the average waiting time in the queue is as follows:

$NW_Q = \frac{[0.079,0.084]}{[123.9,125.9]} = [0.00062{,}0.00067]$ seconds and 0.00066 (crisp value) $\in [0.00062{,}0.00067]$.

(d) By using Eq. (41), the average waiting time in the system is as follows:

$$NW_S = \frac{[0.3263,0.3363]}{[123.9,125.9]} = [0.0025,0.0027] \text{ seconds and } 0.0026 \text{ (crisp value) } \in [0.0025,0.0027].$$

5. Conclusions and Future Directions

We concluded that the neutrosophic queueing theory is better than the crisp queueing theory when we deal with imprecise data. We have presented three types of queues in neutrosophic environment: (NM/NM/1) :(FCFS/∞/∞) queue, (NM/NM/s) :(FCFS/∞/∞) queue and (NM/NM/1) :(FCFS/∞/b) queue. We evaluate the neutrosophic performance measures for three queueing models according to crisp and neutrosophic queueing models. Neutrosophic queueing models gives better results than crisp queueing models.

In the future we can study other types of queueing systems in neutrosophic environment. We can also use triangular and trapezoidal neutrosophic numbers in various queueing theory models. Also, various types of neutrosophic sets such as single, interval and bipolar neutrosophic sets will apply in our future research in queueing theory.

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