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R. Radha

A. Stanis Arul Mary

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Neutrosophic Pythagorean Soft Set With T and F as Dependent Neutrosophic Components

R. Radha [1] **and A. Stanis Arul Mary** [2]

[1] Research Scholar, Department of Mathematics, Nirmala College for Women, Coimbatore; radharmat2020@gmail.com
[2] Department of Mathematics, Nirmala College for Women, Coimbatore, stanisarulmary@gmail.com

[*] Correspondence: radharmat2020@gmail.com, kathirsubbu1987@gmail.com

Abstract: The aim of this paper is to introduce the new concept of Neutrosophic Pythagorean soft set with T and F as dependent components and have also discussed some of its properties.

Keywords: Neutrosophic set, Neutrosophic pythagorean set, Neutrosophic soft set, Neutrosophic pythagorean soft set.

1. Introduction

The fuzzy set was introduced by Zadeh [19] in 1965. In 1968, Chang [4] defined the concept of fuzzy topological space and generalized some basic notions of topology. Intuitionistic fuzzy set was introduced by Atanassov [2,3] in 1983.The concept of Neutrosophic set was introduced by F. Smarandache which is a mathematical tool for handling problems involving imprecise, indeterminancy and inconsistent data.

In 2018 Smarandache [16] generalized the gvlSoft Set to the Hyper Soft Set by transforming the classical uni-argument function F into a multi-argument function:

In 2016, F. Smarandache [13] introduced for the first time the degree of dependence between the components of fuzzy set and neutrosophic sets. The main idea of Neutrosophic sets is to characterize each value statement in a 3D – Neutrosophic space, where each dimension of the space represents respectively the truth membership, falsity membership and the indeterminacy, when two components T and F are dependent and I is independent then $T+I+F \leq 2$.

Pabitra kumar Maji had combined the Neutrosophic set with soft sets and introduced a new mathematical model Neutrosophic soft set. I.Arockiarani [2] introduced the new concept of fuzzy neutrosophic soft set. Yager introduced pythagorean fuzzy sets. R. Jhansi [6] introduced the concept of Pythagorean Neutrosophic set with T and F as dependent components.

In this we have to introduce the concept of neutrosophic pythagorean soft set with truth membership and false membership as dependent components and the indeterminacy as independent component and establish some of its properties.

2.Preliminaries

Definition:2.1[13]

Let U be a universe. A Neutrosophic set A on U can be defined as follows:

$A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in U\}$

Where $T_A, I_A, F_A: U \rightarrow [0,1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

Here, $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of indeterminancy and $F_A(x)$ is the degree of non-membership.

Definition:2.2[6]

Let U be a universe. A Pythagorean neutrosophic set with T and F are dependent neutrosophic components A on U is an object of the form

$A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in U\}$

Where $T_A, I_A, F_A: U \rightarrow [0,1]$ and $0 \leq (T_A(x))^2 + (I_A(x))^2 + (F_A(x))^2 \leq 2$

Here, $T_A(x)$ is the degree of membership, $I_A(x)$ is the degree of inderminancy and $F_A(x)$ is the degree of non-membership.

$Here$, $T_A(x)$ and $F_A(x)$are dependent components and $I_A(x)$ is an independent component.

Definition:2.3[2]

Let U be the initial universe set and E be set of parameters. Consider a non-empty set A on E, Let P(U) denote the set of all neutrosophic sets of U. The collection (F, A) is termed to be neutrosophic soft set over U, where F is a mapping given by F: A → P(U).

3.Neutrosophic Pythagorean Soft Set (NPSS or NPS Set)

Definition:3.1

Let U be the initial universe set and E be set of parameters. Consider a non-empty set A on E, Let P(U) denote the set of all neutrosophic pythagorean sets of U. The collection (F, A) is termed to be neutrosophic pythagorean soft set over U, where F is a mapping given by F: A → P(U).

Definition:3.2

A neutrosophic pythagorean soft set A is contained in another neutrosophic pythagorean soft set B (i.e) A$\subseteq B$ if $T_A(x) \leq T_B(x), I_A(x) \leq I_B(x)$ and $F_A(x) \geq F_B(x)$

Definition:3.3

The complement of a neutrosophic pythagorean soft set (F, A) Denoted by $(F,A)^c$ and is defined as

F^c(x)= $\{< x, F_A(x), 1 - I_A(x), T_A(x) >: x \in U\}$

Definition:3.4

Let U be a non-empty set, A = $< x, T_A(x), I_A(x), F_A(x) >$ and

B = $< x, T_B(x), I_B(x), F_B(x) >$ are neutrosophic pythagorean soft (NPS) sets. Then

A ∪ B = $< x, max(T_A(x), T_B(x)), max(I_A(x), I_B(x)), min(F_A(x), F_B(x)) >$

A ∩ B = $< x, min(T_A(x), T_B(x)), min(I_A(x), I_B(x)), max(F_A(x), F_B(x)) >$

Definition:3.5

A neutrosophic pythagorean soft set (F, A) over the universe U is said to be empty neutrosophic pythagorean soft set with respect to the parameter A if $T_{F(e)} = 0, I_{F(e)} = 0, F_{F(e)}$=1,$\forall x \in U, \forall e \in A$. It is denoted by 0_N

Definition:3.6

A neutrosophic pythagorean soft set (F, A) over the universe U is said to be universe neutrosophic pythagorean soft set with respect to the parameter A if $T_{F(e)} = 1, I_{F(e)} = 1, F_{F(e)}$=0,$\forall x \in U, \forall e \in A$. It is denoted by 1_N

Remark: $0_N^c = 1_N$ and $1_N^c = 0_N$

Definition:3.7

Let A and B be two neutrosophic pythagorean soft sets then A\B may be defined as

A\B = $< x, min(T_A(x), F_B(x)), min(I_A(x), 1 - I_B(x)), max(F_A(x), T_B(x)) >$

Definition:3.8

FE is called neutrosophic pythagorean soft set over U if F(e) = 1_N for any $e \in E$. We denote it by U_E

F_E is called relative null neutrosophic pythagorean soft set over U if F(e) = 0_N for any $e \in E$. We denote it by $\emptyset_E$

Obviously $\emptyset_E = U_E^c$ and $U_E = \emptyset_E^c$

Definition:3.9

The complement of a neutrosophic pythagorean soft set (F, A) can also be defined as

$(F, A)^c = U_E \backslash F(e)$ for all $e \in A$.

Note: We denote U_E by U in the proofs of proposition.

Definition:3.10

If (F, A) and (G, B) be two neutrosophic pythagorean soft set then "(F, A) AND (G, B)" is a denoted by (F, A) ∧(G, B) and is defined by (F, A)∧ (G, B) = (H, A ×B)

where H (a, b) = F(a) ∩ G(b) $\forall a \in A$ and $\forall b \in B$, where ∩ is the operation intersection of NPS set.

Definition:3.11

If (F, A) and (G, B) be two neutrosophic pythagorean soft set then "(F, A) OR (G, B)" is a denoted by (F, A) ∨(G, B) and is defined by (F, A)∨ (G, B) = (K, A ×B)

where K (a, b) = F(a) ∪ G(b) $\forall a \in A$ and $\forall b \in B$, where ∪ is the operation union of NPS set.

Theorem :3.12

Let (F, A) and (G, B) be NPS set in $NPSS(U)_A$ .Then the following are true.

(i) (F, A)⊆ (G, A) iff (F, A) ∩ (G, A) = (F, A)

(ii) (F, A)⊆ (G, A) iff (F, A) ∪ (G, A) = (F, A)

Proof:

(i)Suppose that (F, A)⊆ (G, A), then F(e) ⊆G(e) for all e ∈ A. Let (F, A) ∩ (G, A) = (H, A).

Since H(e) = F(e) ∩ G(e) =F(e) for all e ∈ A, by definition (H, A) = (F, A).

Consider (F, A) $\cap$ (G, A) = (F, A). Let (F, A) $\cap$ (G, A) = (H, A). Since H(e) = F(e) $\cap$ G(e) =F(e) for all e $\in$ A, we know that F(e) $\subseteq$ G(e) for all e $\in$ A. Hence (F, A)$\subseteq$ (G, A).

(ii)The proof is similar to (i).

Theorem :3.13

Let (F, A), (G, A), (H, A), and (S, A) be NPS set in $NPSS(U)_A$ .Then the following are true.

(i) If (F, A) $\cap$ (G, A) =$\emptyset_A$, then (F, A)$\subseteq$ (G, A)^c

(ii) If (F, A)$\subseteq$ (G, A) and (G, A)$\subseteq$ (H, A) then (F, A)$\subseteq$ (H, A)

(iii) If (F, A)$\subseteq$ (G, A) and (H, A)$\subseteq$ (S, A) then (F, A) $\cap$ (H, A) $\subseteq$ (G, A) $\cap$ (S, A)

(iv) (F, A) $\subseteq$ (G, A) iff (G, A)^c $\subseteq$(F, A)^c

Proof:

(i)Suppose that (F, A) $\cap$ (G, A) =$\emptyset_A$.Then F(e)$\cap$ G(e) = $\emptyset$. So, F(e) $\subseteq$ U\G(e)= $G^c(e)$ for all e $\in$ A. therefore we have (F, A)$\subseteq$ (G, A)^c

Proof of (ii) and (iii) are obvious.

(iv) (F, A)$\subseteq$ (G, A)$\Leftrightarrow$ F(e) $\subseteq$ G(e) for all e $\in$ A.

$\Leftrightarrow$ (G(e))^c $\subseteq$ (F(e))^c for all e $\in$ A.

$\Leftrightarrow$(G, A)^c $\subseteq$(F, A)^c

Definition:3.14

Let I be an arbitrary index $\{(F_i, A)\}_{i\in I}$ be a subfamily of NPSS(U)$_A$.

(i)The union of these NPSS is the NPSS (H, A) where H(e) = $\cup_{i\in I} F_i\ (e)$ for each e $\in A$.

We write $\cup_{i\in I}(F_i\,, A)$= (H, A)

(ii)The intersection of these NPSS is the NPSS (M, A) where M(e) = $\cap_{i\in I}\ F_i(e)$ for each e $\in A$.

We write $\cap_{i\in I}\,(F_i, A)$ = (M, A)

Theorem:3.15

Let I be an arbitrary index set and $\{(F_i, A)\}_{i\in I}$ be a subfamily of NPSS(U)$_A$. Then

(i) $(\bigcup_{i\in I}(F_i, A))^C = \bigcap_{i\in I}(F_i, A)^C$

(ii) $(\bigcap_{i\in I}(F_i, A))^C = \bigcup_{i\in I}(F_i, A)^C$

Proof:

(i) $(\bigcup_{i\in I}(F_i, A))^C$ = (H, A)^C, By definition $H^C(e) = U_E \setminus H(e) = U_E \setminus \bigcup_{i\in I} F_i(e) = \bigcap_{i\in I}(U_E \setminus F_i(e))$

for all e $\in A$. On the other hand, $(\bigcap_{i\in I}(F_i, A))^C$(K, A).

By definition, K(e)= $\bigcap_{i\in I} F_i^C(e) = \bigcap_{i\in I}(U - F_i(e))$ for all e $\in A$.

(ii) It is obvious.

Note: We denote $\emptyset_E$ *by* $\emptyset$ and U_E *by* U.

Theorem:3.16

(i) $(\emptyset, A)^c$ = (U, A)

(ii) $(U, A)^c = (\emptyset, A)$

Proof:

Let ($\emptyset$, A) = (F, A)

Then $\forall e \in A$,

F(e) = $\{< x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x) >: x \in U\}$

= $\{(x,0,0,1): x \in U\}$

Now, $(\emptyset, A)^c = (F, A)^c$

Then $\forall e \in A$,

$(F(e))^c = \{< x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x) >: x \in U\}^c$

$= \{< x, F_{F(e)}(x), 1 - I_{F(e)}(x), T_{F(e)}(x) >: x \in U\}$

$=\{(x1,1,0): x \in U\}$ = U

Thus $(\emptyset, A)^c$ = (U, A)

(i) Proof is similar to (i)

Theorem:3.17

(i) (F, A)∪ (∅, A) = (F, A)

(ii) (F, A)∪ (U, A) = (U, A)

Proof:

(i) (F, A) = {e, $(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)): x \in U\}$ ∀e ∈ A

(∅, A) = {e, (x,0,0,1): x ∈ U} ∀e ∈ A

(F, A)∪ (∅, A) = {e, $(x, max(T_{F(e)}(x),0), max(I_{F(e)}(x),0), min(F_{F(e)}(x),1)): x \in U\}$ ∀e ∈ A

= {e, $(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)): x \in U\}$ ∀e ∈ A

= (F, A)

(ii) Proof is similar to (i).

Theorem:3.18

(i) (F, A)∩ (∅, A) = (∅, A)

(ii) (F, A)∩ (U, A) = (F, A)

Proof:

(i) (F, A) = {e, $(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)): x \in U\}$ ∀e ∈ A

(∅, A) = {e, (x,0,0,1): x ∈ U} ∀e ∈ A

(F, A)∩ (∅, A) = {e, $(x, min(T_{F(e)}(x),0), min(I_{F(e)}(x),0), max(F_{F(e)}(x),1)): x \in U\}$ ∀e ∈ A

= {e, , $(x, 0,0,1): x \in U\}$ ∀e ∈ A

= (∅, A)

(ii) Proof is similar to (i).

Theorem:3.19

(i) (F, A)∪ (∅, B) = (F, A) iff B⊆A

(ii) (F, A)∪ (U, B) = (U, A) iff A⊆ B

Proof:

(i) We have for (F, A)

F(e) = $\{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\}$ ∀e ∈ A

Also let (∅, B) = (G, B) then

G(e) = {(x, 0,0,1): x ∈ U} ∀e ∈ B

Let (F, A)∪ (∅, B) = (F, A)∪ (G, B) = (H, C) where C = A ∪B and for all $e \in C$

H(e) may be defined as

$$\begin{cases} \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\}\ if\ e \in A - B \\ \{(x, T_{G(e)}(x), I_{G(e)}(x), F_{G(e)}(x)) : x \in U\}\ if\ e \in B - A \\ \{(x, max(T_{F(e)}(x), T_{G(e)}(x)), max(I_{F(e)}(x), I_{G(e)}(x)), min(F_{F(e)}(x), F_{G(e)}(x)) : x \in U\}\ if\ e \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\}\ if\ e \in A - B \\ \{(x, 0,0,1) : x \in U\}\ if\ e \in B - A \\ \{(x, max(T_{F(e)}(x), 0), max(I_{F(e)}(x), 0), min(F_{F(e)}(x), 1) : x \in U\}\ if\ e \in A \cap B \end{cases}$$

$$= \begin{cases} \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\}\ if\ e \in A - B \\ \{(x, 0,0,1) : x \in U\}\ if\ e \in B - A \\ \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\}\ if\ e \in A \cap B \end{cases}$$

Let B⊆A

Then H (e) = $\begin{cases} \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\}\ if\ e \in A - B \\ \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)) : x \in U\}\ if\ e \in A \cap B \end{cases}$

= F(e) $\forall e \in A$

Conversely Let $(F, A) \cup (\emptyset, B) = (F, A)$

Then A = A ∪ B⟹ $B \subseteq A$

(ii) Proof is similar to (i)

Theorem:3.20

(i) (F, A)∩ (∅, B) = (∅, A∩B)

(ii) (F, A)∩ (𝑈, B) = (F, A∩B)

Proof:

(i) We have for (F, A)

F(e) = $\{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)): x \in U\}$ ∀e ∈ A

Also let (∅, B) = (G, B) then

G(e) = {(x, 0,0,1): x ∈ U} ∀e ∈ B

Let (F, A) ∩ (∅,B) = (F, A) ∩(G, B) = (H, C) where C = A ∩B and ∀e ∈ C

H(e) = $\{(x, min(T_{F(e)}(x), T_{G(e)}(x)), min(I_{F(e)}(x), I_{G(e)}(x)), max(F_{F(e)}(x), F_{G(e)}(x)): x \in U\}$

= $\{ (x, min(T_{F(e)}(x), 0), min(I_{F(e)}(x), 0), max(F_{F(e)}(x), 1)): x \in U\}$

= $\{(x, 0,0,1): x \in U\}$

= (G, B) = (∅, B)

Thus (F, A)∩ (∅, B) = (∅, B) = (∅, A∩B)

(ii) Proof is similar to (i).

Theorem:3.21

(i) ((F, A)∪ (𝐺, B))^C ⊆ (F, A)^C ∪ (G, Bj)^C

(ii) (F, A)^C∩ (G, B)^C ⊆((F, A)∩ (𝐺, B))^C

Proof:

Let (F, A)∪(G, B) = (H, C) Where C = A∪B and∀ e∈ C

H(e) may be defined as

$$\begin{cases} \{(x, T_{F(e)}(x), I_{F(e)}(x), F_{F(e)}(x)): x \in U\} \text{ if } e \in A - B \\ \{(x, T_{G(e)}(x), I_{G(e)}(x), F_{G(e)}(x)): x \in U\} \text{ if } e \in B - A \\ \{(x, max(T_{F(e)}(x), T_{G(e)}(x)), max(I_{F(e)}(x), I_{G(e)}(x)), min(F_{F(e)}(x), F_{G(e)}(x)): x \in U\} \text{ if } e \in A \cap B \end{cases}$$

Thus (F,A)∪(G,B))^C = (H,C)^C Where C = A∪B and∀ e∈ C

$$(H(e))^C = \begin{cases} (F(e))^C \text{ if } e \in A - B \\ (G(e))^C \text{ if } e \in B - A \\ (F(e) \cup G(e))^C \text{ if } e \in A \cap B \end{cases}$$

=

$$\begin{cases} \{(x, F_{F(e)}(x), 1 - I_{F(e)}(x), T_{F(e)}(x)) : x \in U\} \text{ if } e \in A - B \\ \{(x, F_{G(e)}(x), 1 - I_{G(e)}(x), T_{G(e)}(x)) : x \in U\} \text{ if } e \in B - A \\ \{(x, min(F_{F(e)}(x), F_{G(e)}(x)), 1 - max(I_{F(e)}(x), I_{G(e)}(x)), max(T_{F(e)}(x), T_{G(e)}(x)) : x \in U\} \text{ if } e \in A \cap B \end{cases}$$

Again (F, A) ^{C} ∪ (G, B) ^{C} = (I, J) say J = A∪B and ∀ e∈ J

$$I(e) = \begin{cases} (F(e))^C \text{ if } e \in A - B \\ (G(e))^C \text{ if } e \in B - A \\ (F(e) \cup G(e))^C \text{ if } e \in A \cap B \end{cases}$$

=

$$\begin{cases} \{(x, F_{F(e)}(x), 1 - I_{F(e)}(x), T_{F(e)}(x)) : x \in U\} \text{ if } e \in A - B \\ \{(x, F_{G(e)}(x), 1 - I_{G(e)}(x), T_{G(e)}(x)) : x \in U\} \text{ if } e \in B - A \\ \{(x, min(F_{F(e)}(x), F_{G(e)}(x)), 1 - max(I_{F(e)}(x), I_{G(e)}(x)), max(T_{F(e)}(x), T_{G(e)}(x)) : x \in U\} \text{ if } e \in A \cap B \end{cases}$$

So, C⊆ J ∀ e∈ J, $(H(e))^C$ ⊆I(e)

Thus (F, A)∪(G, B)) ^{C} ⊆ (F, A) ^{C} ∪ (G, B) ^{C}

(ii) Let (F, A)∩(G, B) = (H, C) Where C = A∩B and∀ e∈ C

H(e) = F(e)∩G(e)

$$=\{(x, min(T_{F(e)}(x), T_{G(e)}(x)), min(I_{F(e)}(x), I_{G(e)}(x)), max(F_{F(e)}(x), F_{G(e)}(x))\}$$

Thus ((F, A)∩(G, B)) ^{C} = (H, C^C Where C = A∩B and∀ e∈ C

$$(H(e))^C = \{(x, min(T_{F(e)}(x), T_{G(e)}(x)), min(I_{F(e)}(x), I_{G(e)}(x)), max(F_{F(e)}(x), F_{G(e)}(x))\}^C$$

$$=\{(x, max(F_{F(e)}(x), F_{G(e)}(x)), 1 - min(I_{F(e)}(x), I_{G(e)}(x)), min(T_{F(e)}(x), T_{G(e)}(x))\}$$

Again (F, A) ^{C}∩ (G, B) ^{C} = (I, J) say where J= A∩ B and ∀ e∈ J

I(e) = (F(e)) ^{C}∩ (G(e)) ^{C}

$$=\{(x, min(F_{F(e)}(x), F_{G(e)}(x)), min(1 - I_{F(e)}(x), 1 - I_{G(e)}(x)), max(T_{F(e)}(x), T_{G(e)}(x))\}$$

We see that C = J and ∀ e∈ J, I(e) ⊆(H(e)) ^{C}

Thus (F, A) ^{C} ∩ (G, B) ^{C} ⊆((F, A)∩ (G, B)) ^{C}

Theorem :3.22

Let (F, A) and (G, A) are two neutrosophic pythagorean soft sets over the same universe U. We have the following

(i) ((F, A)∪ (G, A)) ^{C} = (F, A) ^{C} ∩ (G, A) ^{C}

(ii) ((F, A)∩ (G, A)) ^{C} = (F, A) ^{C}∪ (G, A) ^{C}

Proof:

(i) Let (F, A) ∪ (G, A) = (H, A) ∀ e∈ A

H(e) = F(e)∪G(e)

$$=\{(x, max(T_{F(e)}(x), T_{G(e)}(x)), max(I_{F(e)}(x), I_{G(e)}(x)), min(F_{F(e)}(x), F_{G(e)}(x))\}$$

Thus (F, A) ∪ (G, A))^C = (H, A)^C ∀ e∈ A

(H(e)) ^{C} = (F(e)∪G(e)) ^{C}

$$=\{(x, max(T_{F(e)}(x), T_{G(e)}(x)), max(I_{F(e)}(x), I_{G(e)}(x)), min(F_{F(e)}(x), F_{G(e)}(x))\}^C$$

$$=\{(x, min(F_{F(e)}(x), F_{G(e)}(x)), 1 - max(I_{F(e)}(x), I_{G(e)}(x)), min(T_{F(e)}(x), T_{G(e)}(x))$$

Again (F, A) ^{C}∩ (G, A) ^{C} = (I, A) where∀ $e \in A$

I(e) = (F(e)) ^{C}∩ (G(e)) ^{C}

$$=\{(x, min(F_{F(e)}(x), F_{G(e)}(x)), min(1 - I_{F(e)}(x), 1 - I_{G(e)}(x)), max(T_{F(e)}(x), T_{G(e)}(x))\}$$

$$=\{(x, min(F_{F(e)}(x), F_{G(e)}(x)), 1 - max(I_{F(e)}(x), I_{G(e)}(x)), max(T_{F(e)}(x), T_{G(e)}(x))$$

Thus ((F, A)∪ (G, A)) ^{C} = (F, A) ^{C} ∩ (G, A) ^{C}

(ii) Let (F, A) ∩ (G, A) = (H, A) ∀ e∈ A

H(e) = F(e)∩G(e)

$$=\{(x, min(T_{F(e)}(x), T_{G(e)}(x)), min(I_{F(e)}(x), I_{G(e)}(x)), max(F_{F(e)}(x), F_{G(e)}(x))\} \ \forall \ e \in A$$

Thus (F, A) ∩ (G, A)) ^{C} = (H, A) ^{C}

(H(e)) ^{C} = (F(e)∩G(e)) ^{C}

$= \{(x, min(T_{F(e)}(x), T_{G(e)}(x)), min(I_{F(e)}(x), I_{G(e)}(x)), max(F_{F(e)}(x), F_{G(e)}(x))\ \}^C$ =

$\{(x, max(F_{F(e)}(x), F_{G(e)}(x)), 1 - min(I_{F(e)}; (x), I_{G(e)}(x)), min(T_{F(e)}(x), T_{G(e)}(x))\} \forall\ e \in\ A$

Again (F, A) ^{C}∪ (G, A) ^{C} = (I, A) where∀ $e \in\ A$

I(e) = (F(e)) ^{C}∪ (G(e)) ^{C}

$= \{(x, max(F_{F(e)}(x), F_{G(e)}(x)), max(1 - \ I_{F(e)}(x), 1 - \ I_{G(e)}(x)), min(T_{F(e)}(x), T_{G(e)}(x))\}$

$= \{(x, max(F_{F(e)}(x), F_{G(e)}(x)), 1 - min(I_{F(e)}(x), I_{G(e)}(x)), min(T_{F(e)}(x), T_{G(e)}(x))$

Thus ((F, A)∩ (G, A)) ^{C} = (F, A) ^{C} ∪ (G, A) ^{C}

Theorem:3.23

Let (F, A) and (G, A) are two neutrosophic pythagorean soft sets over the same universe U. We have the following

(i) ((F, A)∧ (G, A)) ^{C} = (F, A) ^{C} ∨ (G, A) ^{C}

(ii) ((F, A)∨ (G, A)) ^{C} = (F, A) ^{C}∧ (G, A) ^{C}

Proof:

Let (F, A) ∧ (G, B) = (H, A ×B) where H (a, b) =F(a) ∩G(b) ∀ $a \in\ A$ and ∀ $b \in\ B$ where ∩ is the operation intersection of NPSS.

Thus H (a, b) = F(a) ∩G(b)

$= \{(x, min(T_{F(a)}(x), T_{G(b)}(x)), min(I_{F(a)}(x), I_{G(b)}(x)), max(F_{F(a)}(x), F_{G(b)}(x))\}$

((F, A) ∧ (G, B)) ^{C} = (H, A ×B) ^{C} $\forall(\ a, b) \in\ A \times B$

Thus (H (a, b)) ^{C}=$\{(x, min(T_{F(a)}(x), T_{G(b)}(x)), min(I_{F(a)}(x), I_{G(b)}(x)), max(F_{F(a)}(x), F_{G(b)}(x))\}^C$

=

$\{(x, max(F_{F(a)}(x), F_{G(b)}(x)), 1 - \ min(I_{F(a)}(x), I_{G(b)}(x)), min(T_{F(a)}(x), T_{G(b)}(x))\}$

Let (F, A) ^{C} ∨ (G, A) ^{C} = (R, A ×B) where R (a, b) =(F(a)) ^{C} ∪ (G(b)) ^{C} ∀ $a \in\ A$ and ∀ $b \in\ B$ where ∪ is the operation union of NPSS.

R (a, b) = $\{(x, max(F_{F(a)}(x), F_{G(b)}(x)), max(1 - I_{F(a)}(x), 1 - I_{F(b)}(x)), min(T_{F(a)}(x), T_{G(b)}(x))\}$

$=\{(x, max(F_{F(a)}(x), F_{G(b)}(x)), 1 - min(I_{F(a)}(x), I_{G(b)}(x)), min(T_{F(a)}(x), T_{G(b)}(x))\}$

Hence ((F, A)$\wedge$ (G, A)) ^{C} = (F, A) ^{C} $\vee$ (G, A) ^{C}

Similarly, we can prove (ii)

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Conclusion

In this paper, I have defined the concept of neutrosophic pythagorean soft sets with dependent components by combining the concept of neutrosophic pythagorean set and neutrosophic set. Then we have discussed the properties of union, intersection and complement of neutrosophic pythagorean soft set. This may helpful in future study of generalized neutrosophic pythagorean soft set in neutrosophic pythagorean soft topological spaces. This may lead to the new properties of separation axioms in neutrosophic pythagorean soft topological space.

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