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A novel score and accuracy function for neutrosophic sets and their real-world applications to multi-criteria decision-making process

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Abstract: The purpose of this work is to understand the ranking order of the neutrosophic sets, where the uncertain or ambiguous information/data is stored in the terms of three independent variables i.e., degree of truthfulness, degree of indeterminacy, and degree of falseness. There exist many ranking tools in decision-making (DM) like score function (SF) and accuracy function (AF) that help to rank the single-valued neutrosophic set (SVNS) and the interval-valued neutrosophic set (IVNS) to make a better choice among all the available alternatives. An intensive study about all the existing score functions and an accuracy function reveals that the existing ranking method for SVNS and IVNS in DM problems holds for certain kinds of neutrosophic information and has its limitations. To validate these observations some well-defined examples are chosen, illustrating that the existing score functions and accuracy functions are like special cases for certain kinds of neutrosophic data. Since nothing in this world is an ultimate truth, hence this existing gap is a real motivation to come up with a more efficient SF and AF that would rank SVNS and IVNS in the real-life problems to make a better selection among all the available alternatives in DM problems in an efficient way. Hence, a new SF and AF have been proposed and multi-criteria decision-making (MCDM) method is developed based on these proposed SF and AF. Furthermore, a real-life problem from our immediate surroundings is taken and solved successfully, and also a comparative analysis of the solutions for the existing problems is made in detail.

Keywords: Accuracy function (AF), average operators, IVNS, multi-criteria decision-making (MCDM), score function (SF), SVNS.

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1. Introduction

Humans among all the living beings have evolved most intelligently since their existence and the reason behind it is a proper and timely DM in their environment. In this computer age, the scientific world is in continuous motion and whatever is new today is old in another hour, and the information or statistical data available is not always crisp, definite, constant, determinate, and consistent. Thus to deal with such kind of firsthand information, a new theory was evolved in 1965 by a great philosopher Zadeh, who had a farsighted vision and a penetrating understanding of the known and unknown data. He has come up with his intriguing theory of sets claiming that in real-life uncertainty is the only thing that is certain in life and named it as a fuzzy set (FS) [1] which deals with the concept of belongingness. This theory was reluctantly accepted in that period (i.e. in and around 1965) which tells that the available data is not always a real-value but it beholds the hand of uncertainty together and the study of this uncertainty or vagueness would be able to bring a huge revolution in the coming time with the real-life MCDM and MADM problems [2, 3]. After the acceptance of this theory of fuzzy sets, later with time, the scientific and intellectual world developed a keen interest in this concept of fuzziness, and then onwards various and wide extensions of fuzzy sets are propounded like- Atanassov proposed an intuitionistic fuzzy set (IFS) [4] who considered together both the concept of the degree of belongingness and the degree of non-belongingness. Since it is not always possible to evaluate any information in an exact value so, to define such data sometimes it is expressed in the interval, thus IFS was later expanded by Atanassov and Gargov to an interval-valued intuitionistic fuzzy set (IVIFS) [5]. Yager developed a Pythagorean fuzzy set (PFS) [6-8] which was extended by Zhang to an interval-valued Pythagorean fuzzy set (IVPFS) [9]. Smarandache introduced another extension of the FS as neutrosophic sets [10-12] which was more like a philosophical approach stating that together with membership and non-membership there is also an existence of one more component and he named it as indeterminacy such that, all these three values are independent of each other. IFS did not tell or explain indeterminate or inconsistent sets of information and hence neutrosophic set (NS) was able to handle such indeterminate data in a more efficient way. To apply this philosophy of NS into the real-world application Wang et al. [13, 14] proposed the concept of SVNSs and IVNSs along with their operators and properties respectively.

Since FS theory and its extensions lagged to deal with indeterminacy and inconsistent set of data therefore neutrosophic sets have successfully overcome these fuzzy drawbacks. A lot of exploration has been made till now in the area of SVNSs and IVNSs like neutrosophic sets are successfully applied in fuzzy linear optimization by using an important DM technique as linear programming by various researchers say Hezam et al. [15], Abdel-Basset et al. [16-17], Pramanik [18], Ye [19], Nafei et al. [20], Khatter [21], Bera et al. [22], Basumatary et al. [23], etc. Cubic fuzzy sets (CFSs) are introduced by YB Jun et al. [24] and, then YB Jun et al. [25] and M. Alia et al. [26] have extended CFSs to the neutrosophic environment and proposed neutrosophic cubic fuzzy sets (NCFSs) along with some of their basic operations. Recently, Ajay et al. [27] proposed aggregation operators on NCFSs. JC Kely [28] in 1963 introduced bitopological spaces which were extended in other fuzzy environments by many other researchers like Kandil et al. [29], Lee et al. [30] and Mwchahary et al. [31] recently proposed the concept and the propositions of neutrosophic bitopological spaces. Abdel-Basset et al. [32] proposed a method using quality function deployment (QFD) and plithogenic aggregation operations, and also, Abdel-Basset and Rehab [33] proposed a methodology based on plithogenic MCDM approach, utilizing both, techniques for order preference by similarity to ideal solution (TOPSIS) and criteria importance through inter-criteria correlation (CRITIC) techniques and applied in the study of telecommunications equipment categories. Lately, Nabeeh et al. [34] have contributed a lot in decision-making problems undertaken in the neutrosophic field like they have developed a neutrosophic MCDM framework to deal with inconsistent data related to environmental problems. Nabeeh et al. [35] have used integrated neutrosophic and TOSIS to deal with the personnel selection process. Nabeeh et al. [36] have applied the neutrosophic analytical hierarchy process (AHP) of the internet of things (IoT) in enterprises to estimate influential factors. Abdel-Basset et al. [37] proposed a hybrid combination of AHP and neutrosophic theory to deal with the uncertainty of IoT-based enterprises.

DM is a procedure that helps in selecting the best possible alternative among the set of feasible solutions. Since the world is in continuous motion, the societal structure is growing every second, and we need to make decisions under all these factors i.e., peer pressure, the vagueness of the imprecise data, limited funds, high-risk factor, environmental factors, biases, etc. which influences the DM of a decision-maker. There influencing factors are directly or indirectly associated with the unpredictability of the set of data which could be indeterminate, inconsistent, or uncertain, etc., occurring in different fields of life like economics, engineering, medical sciences, computer sciences, management sciences, psychology, meteorology, sociology, decision making. Since neutrosophic sets are quite efficient in dealing with indeterminate and inconsistent sets of data hence, many researchers in literature [15-42] have applied neutrosophic sets in real-life applications and can provide a more satisfactory solution to real-world applications like telecommunication, supply chain management, environment, personnel selection, enterprises, signal processing, pattern recognition, medical diagnosis, social sciences, engineering, management sciences, artificial intelligence, robotics, computer networks, DM, etc. MCDM helps the decision-maker to make his preferences by taking care of each criterion of the available alternatives, rank them by using some MCDM tools, and choose the best among the available alternatives.

This paper is the outcome of a deep study that has been made to understand the various existing ranking orders like SF and AF of the extensions of fuzzy sets like IFS [43-45], PFS [6-8], IVPFS [46-54], NS [17, 55-65], trapezoidal interval-valued neutrosophic numbers [66], etc., by the various researchers. After a rigorous analysis, it has been observed that the existing SF and AF [67, 68] for comparing single-valued neutrosophic sets (SVNSs) and the interval-valued neutrosophic sets (IVNSs) are more efficient for some special cases of SVNSs and IVNSs. Some well-defined counter-examples are chosen where the rating value of uncertainty is taken as SVNSs and IVNSs to claim that, the existing SF and AF rank these SVNSs and IVNSs correctly only to a certain limit. Hence to fulfill all the restrictions of the existing SF and AF, there is a need to propose a new SF and AF which would act as a helpful tool in the real-world DM problems. Taking this notion as an inspiration, an effort is made to suggest a new SF and AF for efficiently comparing SVNSs and IVNSs. Furthermore, based on these proposed SF and AF, an MCDM method is developed to solve the real-life applications and to validate these proposed SF and AF, the exact result of the real-life problem taken from our immediate surroundings is solved successfully in which the preference rating values are expressed by SVNSs and IVNSs and also, a detailed comparative analysis of the solutions with the existing approaches is presented respectively.

This paper is presented in the following manner: Section 2 - preliminaries; Section 3 - proposed SF and AF for SVNSs and IVNSs; Section 4 - MCDM method is proposed; Section 5 - real-life problem considered and solved; Section 6 - discussion and a comparative analysis of the obtained solutions; Section 7 – managerial insights; Section 8-conclusions.

2. Preliminaries - SVNSs, IVNSs and its SF and AF

This section states some requisites while dealing with SVNSs and IVNSs in the real-life application for the DM process.

Definition 2.1 [1] A set $\widetilde{A} = \{\langle x, \mu_{\widetilde{A}}(x)\rangle \mid x \in X,\ 0 \le \mu_{\widetilde{A}}(x) \le 1\}$, defined on the universal set X, is said to be an FS, where $\mu_{\widetilde{A}}(x)$ represents the degree of membership of the element x in $\widetilde{A}$.

Definition 2.2 [10] A set $\widetilde{A}^N = \{\langle x, T_{\widetilde{A}^N}(x),\ I_{\widetilde{A}^N}(x), F_{\widetilde{A}^N}(x)\rangle | x \in X,\ 0^- \le T_{\widetilde{A}^N}(x) \le 1^+,\ 0^- \le I_{\widetilde{A}^N}(x) \le 1^+,\ 0^- \le F_{\widetilde{A}^N} \le 1^+,\ T_{\widetilde{A}^N}(x) + I_{\widetilde{A}^N}(x) + F_{\widetilde{A}^N}(x) \le 3^+\}$, defined on the universal set X, is said to be an NS, where, $T_{\widetilde{A}^N}(x), I_{\widetilde{A}^N}(x)$, and $F_{\widetilde{A}^N}(x)$ represents the degree of truth-membership, the degree of indeterminacy-membership and degree of falsity-membership respectively of the element x in $\widetilde{A}^N$ as a real standard or real non-standard subsets of $]0^-, 1^+[$.

Definition 2.3 [13] A set $\widetilde{A}^N = \{\langle x, T_{\widetilde{A}^N}(x), I_{\widetilde{A}^N}(x), F_{\widetilde{A}^N}(x)\rangle | x \in X,\ 0 \le T_{\widetilde{A}^N}(x) \le 1,\ 0 \le I_{\widetilde{A}^N}(x) \le 1,\ 0 \le F_{\widetilde{A}^N} \le 1,\ T_{\widetilde{A}^N}(x) + I_{\widetilde{A}^N}(x) + F_{\widetilde{A}^N}(x) \le 3\}$, defined on the universal set X, is said to be an SVNS, where, $T_{\widetilde{A}^N}(x), I_{\widetilde{A}^N}(x)$, and $F_{\widetilde{A}^N}(x)$ represents the degree of truth-membership, the degree of indeterminacy-membership and degree of falsity-membership respectively of the element x in $\widetilde{A}^N$. For convenience, we may write the single-valued neutrosophic number (SVNN) as $\widetilde{A}^N = \langle \alpha, \beta, \gamma\rangle$.

Definition 2.4 [14] A set $\widetilde{A}^N = \{\langle x, [T^L_{\widetilde{A}^N}(x), T^U_{\widetilde{A}^N}(x)], [I^L_{\widetilde{A}^N}(x), I^U_{\widetilde{A}^N}(x)], [F^L_{\widetilde{A}^N}(x), F^U_{\widetilde{A}^N}(x)]\rangle | x \in X,\ 0 \le T^L_{\widetilde{A}^N}(x) \le T^U_{\widetilde{A}^N}(x) \le 1,\ 0 \le I^L_{\widetilde{A}^N}(x) \le I^U_{\widetilde{A}^N}(x) \le 1,\ 0 \le F^L_{\widetilde{A}^N}(x) \le F^U_{\widetilde{A}^N}(x) \le 1,\ T^U_{\widetilde{A}^N}(x) + I^U_{\widetilde{A}^N}(x) + F^U_{\widetilde{A}^N}(x) \le 3\}$, defined on the universal set X, is said to be an IVNS, where, $[T^L_{\widetilde{A}^N}(x), T^U_{\widetilde{A}^N}(x)]$, $[I^L_{\widetilde{A}^N}(x), I^U_{\widetilde{A}^N}(x)]$ and $[F^L_{\widetilde{A}^N}(x), F^U_{\widetilde{A}^N}(x)]$ represents the intervals of the degree of truth-membership, the degree of indeterminacy-membership and the degree of falsity-membership respectively of the element x in $\widetilde{A}^N$. For convenience, we may write interval-valued neutrosophic number (IVNN) as $\widetilde{A}^N = \langle [\alpha_1, \alpha_2)], [\beta_1, \beta_2], [\gamma_1, \gamma_2]\rangle$.

Definition 2.5 [67] Average operator for SVNSs:

Since $\widetilde{A}^N = \{\langle x, T_{\widetilde{A}^N}(x),\ I_{\widetilde{A}^N}(x), F_{\widetilde{A}^N}(x)\rangle | x \in X\}$ is an SVNS, then let $\widetilde{A}^N_k (k = 1,2, \dots, n)$ be n numbers of SVNSs.

(i) Weighted arithmetic average operator (WAM) for SVNSs is defined as

$$AO_{WA} = (\widetilde{A}^N_1, \widetilde{A}^N_2, \dots, \widetilde{A}^N_n) = \sum_{k=1}^{n} w_k \widetilde{A}^N_k$$
$$= \left(1 - \prod_{k=1}^{n}\left(1 - T_{\widetilde{A}^N_k}(x)\right)^{w_k}, \prod_{k=1}^{n}\left(I_{\widetilde{A}^N_k}(x)\right)^{w_k}, \prod_{k=1}^{n}\left(F_{\widetilde{A}^N_k}(x)\right)^{w_k}\right) \quad (1)$$

where w_k denotes the weight vector of SVNSs $\widetilde{A}^N_k (k = 1,2, \dots, n)$ and satisfies the conditions such that $w_k \in [0,1]$ and $\sum_{k=1}^{n} w_k = 1$.

(ii) Weighted geometric average operator (WGM) for SVNSs is defined as

$$AO_{WG} = (\widetilde{A}^N_1, \widetilde{A}^N_2, \dots, \widetilde{A}^N_n) = \prod_{k=1}^{n} w_k \widetilde{A}^N_k$$
$$= \left(\prod_{k=1}^{n}\left(T_{\widetilde{A}^N_k}(x)\right)^{w_k}, 1 - \prod_{k=1}^{n}\left(1 - I_{\widetilde{A}^N_k}(x)\right)^{w_k}, 1 - \prod_{k=1}^{n}\left(1 - F_{\widetilde{A}^N_k}(x)\right)^{w_k}\right) \quad (2)$$

where w_k denotes the weight vector of SVNSs $\widetilde{A}_k^N(k = 1,2,\dots,n)$ and satisfies the conditions such that $w_k \in [0,1]$ and $\sum_{k=1}^{n} w_k = 1$.

Definition 2.6 [67] Average operator for IVNSs:

Since $\widetilde{A}^N = \{\langle x, [T_{\widetilde{A}^N}^L(x), T_{\widetilde{A}^N}^U(x)], [I_{\widetilde{A}^N}^L(x), I_{\widetilde{A}^N}^U(x)], [F_{\widetilde{A}^N}^L(x), F_{\widetilde{A}^N}^U(x)]\rangle | x \in X\}$ is an IVNS, then let $\widetilde{A}_k^N(k = 1,2,\dots,n)$ be n numbers of IVNSs.

(i) Weighted arithmetic average operator (WAM) for IVNSs is defined as

$$AO_{WA} = (\widetilde{A}_1^N, \widetilde{A}_2^N, \dots, \widetilde{A}_n^N) = \sum_{k=1}^{n} w_k \widetilde{A}_k^N = \left(\left[1 - \prod_{k=1}^{n}\left(1 - T_{\widetilde{A}^N}^L(x)\right)^{w_k},\ 1 - \prod_{k=1}^{n}\left(1 - T_{\widetilde{A}^N}^U(x)\right)^{w_k}\right],\ \left[\prod_{k=1}^{n}\left(I_{\widetilde{A}^N}^L(x)\right)^{w_k},\ \prod_{k=1}^{n}\left(I_{\widetilde{A}^N}^U(x)\right)^{w_k}\right],\ \left[\prod_{k=1}^{n}\left(F_{\widetilde{A}^N}^L(x)\right)^{w_k}, \prod_{k=1}^{n}\left(F_{\widetilde{A}^N}^U(x)\right)^{w_k}\right]\right) \quad (3)$$

where w_k denotes the weight vector of IVNSs $\widetilde{A}_k^N(k = 1,2,\dots,n)$ and satisfies the conditions such that $w_k \in [0,1]$ and $\sum_{k=1}^{n} w_k = 1$.

(ii) Weighted geometric average operator (WGM) for IVNSs is defined as

$$AO_{WG} = (\widetilde{A}_1^N, \widetilde{A}_2^N, \dots, \widetilde{A}_n^N) = \prod_{k=1}^{n} w_k \widetilde{A}_k^N$$

$$= \left(\left[\prod_{k=1}^{n}\left(T_{\widetilde{A}^N}^L(x)\right)^{w_k}, \prod_{k=1}^{n}\left(T_{\widetilde{A}^N}^U(x)\right)^{w_k}\right],\ \left[1 - \prod_{k=1}^{n}\left(1 - I_{\widetilde{A}^N}^L(x)\right)^{w_k}, 1 - \prod_{k=1}^{n}\left(1 - I_{\widetilde{A}^N}^U(x)\right)^{w_k}\right], \left[1 - \prod_{k=1}^{n}\left(1 - F_{\widetilde{A}^N}^L(x)\right)^{w_k}, 1 - \prod_{k=1}^{n}\left(1 - F_{\widetilde{A}^N}^U(x)\right)^{w_k}\right]\right) \quad (4)$$

where w_k denotes the weight vector of IVNSs $\widetilde{A}_k^N(k = 1,2,\dots,n)$ and satisfies the conditions such that $w_k \in [0,1]$ and $\sum_{k=1}^{n} w_k = 1$.

SF and AF are defined as a metric method for ranking SVNSs and IVNSs which clearly and precisely order the available alternatives and helps in choosing the best alternative among all the present alternatives.

Definition 2.7 SF and AF for ranking SVNS

Let $\widetilde{A}^N = \{\langle x, T_{\widetilde{A}^N}(x), I_{\widetilde{A}^N}(x), F_{\widetilde{A}^N}(x)\rangle | x \in X\}$ be an SVNS, then SF and AF for SVNS is defined as,

(i) Existing SVNS SF [67] is defined as

$$\sigma_S(\widetilde{A}^N) = \frac{1+T_{\widetilde{A}^N}-2I_{\widetilde{A}^N}-F_{\widetilde{A}^N}}{2}, \qquad \text{where } \sigma_S(\widetilde{A}^N) \in [0,1]. \quad (5)$$

(ii) Existing SVNS AF [67] is defined as

$$\sigma_A(\widetilde{A}^N) = T_{\widetilde{A}^N} - I_{\widetilde{A}^N}(1 - T_{\widetilde{A}^N}) - F_{\widetilde{A}^N}(1 - I_{\widetilde{A}^N}), \qquad \text{where } \sigma_A(\widetilde{A}^N) \in [-1,1]. \quad (6)$$

(iii) Existing SVNS SF [68] is defined as

$$\tau_S(\widetilde{A}^N) = \frac{1+(T_{\widetilde{A}^N}-2I_{\widetilde{A}^N}-F_{\widetilde{A}^N})(2-T_{\widetilde{A}^N}-F_{\widetilde{A}^N})}{2}, \qquad \text{where } \tau_S(\widetilde{A}^N) \in [0,1]. \quad (7)$$

(iv) Existing SVNS AF [68] is defined as

$$\tau_A(\widetilde{A}^N) = T_{\widetilde{A}^N} - 2I_{\widetilde{A}^N} - F_{\widetilde{A}^N}, \qquad \text{where } \tau_A(\widetilde{A}^N) \in [-1,1]. \quad (8)$$

Definition 2.8 SF and AF for ranking IVNS

Let $\widetilde{A}^N = \{\langle x, [T_{\widetilde{A}^N}^L(x), T_{\widetilde{A}^N}^U(x)], [I_{\widetilde{A}^N}^L(x), I_{\widetilde{A}^N}^U(x)], [F_{\widetilde{A}^N}^L(x), F_{\widetilde{A}^N}^U(x)]\rangle | x \in X\}$ be an IVNS, then SF and AF for IVNS is defined as,

(i) Existing IVNS SF [67] is defined as

$$\chi_S(\widetilde{A}^N) = \frac{2+T^L_{\widetilde{A}^N}+T^U_{\widetilde{A}^N}-2I^L_{\widetilde{A}^N}-2I^U_{\widetilde{A}^N}-F^L_{\widetilde{A}^N}-F^U_{\widetilde{A}^N}}{4}, \quad \text{where } \chi_S(\widetilde{A}^N) \in [0,1]. \tag{9}$$

(ii) Existing IVNS AF [67] is defined as

$$\chi_A(\widetilde{A}^N) = \frac{1}{2}\left(T^L_{\widetilde{A}^N} + T^U_{\widetilde{A}^N} - I^U_{\widetilde{A}^N}(1-T^U_{\widetilde{A}^N}) - I^L_{\widetilde{A}^N}(1-T^L_{\widetilde{A}^N}) - F^U_{\widetilde{A}^N}(1-I^L_{\widetilde{A}^N}) - F^L_{\widetilde{A}^N}(1-I^U_{\widetilde{A}^N})\right),$$

where $\chi_A(\widetilde{A}^N) \in [-1,1]$. (10)

(iii) Existing IVNS SF [68] is defined as

$$\psi_S(\widetilde{A}^N) = \frac{4+\left(T^L_{\widetilde{A}^N}+T^U_{\widetilde{A}^N}-2I^L_{\widetilde{A}^N}-2I^U_{\widetilde{A}^N}-F^L_{\widetilde{A}^N}-F^U_{\widetilde{A}^N}\right)\left(4-T^L_{\widetilde{A}^N}-T^U_{\widetilde{A}^N}-F^L_{\widetilde{A}^N}-F^U_{\widetilde{A}^N}\right)}{8}, \quad \text{where } \psi_S(\widetilde{A}^N) \in [0,1]. \tag{11}$$

3. Proposed SF and AF for SVNSs and IVNSs

This section of the paper suggests a new SF and AF to obtain the correct ranking order of all the available alternatives of SVNSs and IVNSs and helps to choose the best alternative among all.

3.1 Proposed SF and AF for SVNSs

3.1.1 Proposed SF for SVNSs

Let $\tilde{A}^N = \{\langle x, T_{\tilde{A}^N}(x), I_{\tilde{A}^N}(x), F_{\tilde{A}^N}(x)\rangle | x \in X\}$ be an SVNS, then an SF in terms of the degree of truth-membership, the degree of indeterminacy-membership and the degree of falsity-membership respectively for SVNS, is defined by:

$$\varphi_S(\tilde{A}^N) = \frac{1+\left(T_{\tilde{A}^N}-2I_{\tilde{A}^N}-F_{\tilde{A}^N}\right)}{2\left(2-T_{\tilde{A}^N}-F_{\tilde{A}^N}\right)}, \quad where\ \varphi_S(\tilde{A}^N) \in [0,1]\ and\ T_{\tilde{A}^N}+F_{\tilde{A}^N} \neq 2. \tag{12}$$

Clearly, it is observed that if $T_{\tilde{A}^N}+F_{\tilde{A}^N}=1$, then $\varphi_S(\tilde{A}^N)=\sigma_S(\tilde{A}^N)$, therefore $T_{\tilde{A}^N}+F_{\tilde{A}^N}\neq 1$.

To validate the claim of the proposed SF $(\text{Eq.}\,(12))$, some well-defined SVNNs are chosen and evaluated. Let us consider the following examples.

Example 1. Let $\tilde{A}^N_1 = \langle T_{\tilde{A}^N_1}(x), I_{\tilde{A}^N_1}(x), F_{\tilde{A}^N_1}(x)\rangle$, and $\tilde{A}^N_2 = \langle T_{\tilde{A}^N_2}(x), I_{\tilde{A}^N_2}(x), F_{\tilde{A}^N_2}(x)\rangle$ be any two SVNNs, then the desirable alternative is selected according to the obtained value of SF using Eq. (12) among $\tilde{A}^N_1$ and $\tilde{A}^N_2$.

- **(i)** Let $\tilde{A}^N_1 = \langle 0.6, 0.3, 0.0\rangle$ and $\tilde{A}^N_2 = \langle 0.2, 0.1, 0.0\rangle$, then $\tilde{A}^N_1 > \tilde{A}^N_2$ in accordance with proposed SF $(\text{Eq.}\,(12))$.
- **(ii)** Let $\tilde{A}^N_1 = \langle 0.6,0.2,0.2\rangle$ and $\tilde{A}^N_2 = \langle 0.3,0.1,0.1\rangle$, then $\tilde{A}^N_1 > \tilde{A}^N_2$ in accordance with proposed SF $(\text{Eq.}\,(12))$.
- **(iii)** Let $\tilde{A}^N_1 = \langle 0.1,0.0,0.1\rangle$ and $\tilde{A}^N_2 = \langle 0.3,0.0,0.3\rangle$, then $\tilde{A}^N_2 > \tilde{A}^N_1$ in accordance with proposed SF $(\text{Eq.}\,(12))$.

For a deliberate comparison among various existing metric methods, for finding the correct ranking order of Example 1, a systematic tabular representation of the function values of various metric methods is presented in Table 1.

Table 1. SF ($\varphi_S(\tilde{A}^N)$) values in comparison with various existing metric methods

SVNNs	$\sigma_S(\tilde{A}^N)$	$\sigma_A(\tilde{A}^N)$	$\tau_S(\tilde{A}^N)$	$\tau_A(\tilde{A}^N)$	$\varphi_S(\tilde{A}^N)$
$\tilde{A}_1^N = \langle 0.6, 0.3, 0.0\rangle$	0.5	0.48	0.5	0.0	0.4545
$\tilde{A}_2^N = \langle 0.2, 0.1, 0.0\rangle$	0.5	0.12	0.5	0.0	0.2777
	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N > \tilde{A}_2^N$	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N > \tilde{A}_2^N$
$\tilde{A}_1^N = \langle 0.6, 0.2, 0.2\rangle$	0.5	0.36	0.5	0.0	0.4166
$\tilde{A}_2^N = \langle 0.3, 0.1, 0.1\rangle$	0.5	0.14	0.5	0.0	0.3125
	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N > \tilde{A}_2^N$	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N > \tilde{A}_2^N$
$\tilde{A}_1^N = \langle 0.1, 0.0, 0.1\rangle$	0.5	0.0	0.5	0.0	0.2777
$\tilde{A}_2^N = \langle 0.3, 0.0, 0.1\rangle$	0.5	0.0	0.5	0.0	0.3571
(Adopted from [61])	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_2^N > \tilde{A}_1^N$

From Table 1, it is expressed that there may exist several ranking methods for SVNNs which can rank the alternatives, $\tilde{A}_1^N$, $\tilde{A}_2^N$, and suggest which of the alternative is better among both. It has been observed that sometimes, the existing metric methods [67, 68] may or may not fail to rank, but the proposed SF $(\text{Eq.}\,(12))$ is providing desirable results respectively. Hence, it claims the validity of the proposed SF $(\text{Eq.}\,(12))$, stating that it is reasonable.

3.1.2 Proposed AF for SVNSs

It is observed that there may exist several SVNNs where, $T_{\tilde{A}^N} + F_{\tilde{A}^N} = 1$, then sometimes the proposed SF, Eq. (12) may or may not be able to rank the SVNNs desirably. Some of SVNNs $\tilde{A}_1^N$ and $\tilde{A}_2^N$, exhibiting such nature are considered as follows:

Example 2. Let $\tilde{A}_1^N = \langle T_{\tilde{A}_1^N}(x), I_{\tilde{A}_1^N}(x), F_{\tilde{A}_1^N}(x)\rangle$, and $\tilde{A}_2^N = \langle T_{\tilde{A}_2^N}(x), I_{\tilde{A}_2^N}(x), F_{\tilde{A}_2^N}(x)\rangle$ be any two SVNNs, then the desirable alternative is selected according to the obtained value of SF using Eq. (12) among $\tilde{A}_1^N$ and $\tilde{A}_2^N$.

(i) Let $\tilde{A}_1^N = \langle 0.6, 0.1, 0.4\rangle$ and $\tilde{A}_2^N = \langle 0.8, 0.3, 0.2\rangle$, then $\tilde{A}_1^N = \tilde{A}_2^N$ in accordance with proposed SF $(\text{Eq.}\,(12))$. While it is obvious that $\mathrm{A}_1 \neq \mathrm{A}_2$.

(ii) Let $\tilde{A}_1^N = \langle 0.9,0.4,0.1\rangle$ and $\tilde{A}_2^N = \langle 0.7,0.2,0.3\rangle$, then $\tilde{A}_1^N = \tilde{A}_2^N$ in accordance with proposed SF $(\text{Eq.}\,(12))$. While it is obvious that $A_1 \neq A_2$.

For a deliberate comparison among various existing metric methods, for finding the correct ranking order of Example 2, a systematic tabular representation of the function values of various metric methods is presented below in Table 2.

Table 2. SF ($\varphi_S(\tilde{A}^N)$) value of some special SVNNs using various existing methods

SVNNs	$\sigma_S(\tilde{A}^N)$	$\tau_S(\tilde{A}^N)$	$\tau_A(\tilde{A}^N)$	$\varphi_S(\tilde{A}^N)$
$\tilde{A}_1^N = \langle 0.6, 0.1, 0.4\rangle$	0.5	0.5	0.0	0.5
$\tilde{A}_2^N = \langle 0.8, 0.3, 0.2\rangle$	0.5	0.5	0.0	0.5
	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N = \tilde{A}_2^N$
$\tilde{A}_1^N = \langle 0.9,0.4,0.1\rangle$	0.5	0.5	0.0	0.5
$\tilde{A}_2^N = \langle 0.7, 0.2, 0.3\rangle$	0.5	0.5	0.0	0.5
	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N = \tilde{A}_2^N$

From Table 2, it is observed that if there exist SVNNs where, $T_{\tilde{A}^N} + F_{\tilde{A}^N} = 1$, then sometimes may or may not the proposed SF, Eq. (12) lacks in providing a desirable solution.

Thus, to overcome this restriction, there is a need to find a new function that would be helpful to rank such alternatives $\tilde{A}_1^N$ and $\tilde{A}_2^N$ appropriately. Hence, a novel AF is proposed as follows:

$$\varphi_A(\tilde{A}^N) = 1 - I_{\tilde{A}^N} - 2F_{\tilde{A}^N}, \quad (13)$$

$where\ \varphi_A(\tilde{A}^N) \in [-1, 1]$, $T_{\tilde{A}^N} + F_{\tilde{A}^N} = 1$, and $I_{\tilde{A}^N} \neq 0$.

To validate the claim of the proposed AF $(\text{Eq.}(13))$, Example 2 is considered again and evaluated using the proposed AF $(\text{Eq.}(13))$ as follows:

(i) For Example 2 (i), where $\tilde{A}_1^N = \langle 0.6, 0.1, 0.4\rangle$ and $\tilde{A}_2^N = \langle 0.8, 0.3, 0.2\rangle$, then $\tilde{A}_2^N > \tilde{A}_1^N$ in accordance with the proposed AF $(\text{Eq.}(13))$.

(ii) For Example 2 (ii), where $\tilde{A}_1^N = \langle 0.9, 0.4, 0.1\rangle$ and $\tilde{A}_2^N = \langle 0.7, 0.2, 0.3\rangle$, then $\tilde{A}_1^N > \tilde{A}_2^N$ in accordance with proposed AF $(\text{Eq.}(13))$.

For a deliberate comparison among various existing metric methods for finding the correct ranking order of Example 2, a systematic tabular representation is presented in Table 3.

Table 3. AF ($\varphi_A(\tilde{A}^N)$) values in comparison with various metric methods

SVNNs	$\sigma_S(\tilde{A}^N)$	$\tau_S(\tilde{A}^N)$	$\tau_A(\tilde{A}^N)$	$\varphi_S(\tilde{A}^N)$	$\varphi_A(\tilde{A}^N)$	$\sigma_A(\tilde{A}^N)$
$\tilde{A}_1^N = \langle 0.6,0.1,0.4\rangle$	0.5	0.5	0.0	0.5	0.1	0.2
$\tilde{A}_2^N = \langle 0.8,0.3,0.2\rangle$	0.5	0.5	0.0	0.5	0.3	0.6
	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N = \tilde{A}_2^N$	$\boldsymbol{\tilde{A}_2^N > \tilde{A}_1^N}$	$\boldsymbol{\tilde{A}_2^N > \tilde{A}_1^N}$
$\tilde{A}_1^N = \langle 0.9,0.4,0.1\rangle$	0.5	0.5	0.0	0.5	0.4	0.8
$\tilde{A}_2^N = \langle 0.7,0.2,0.3\rangle$	0.5	0.5	0.0	0.5	0.2	0.4
	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N = \tilde{A}_2^N$	$\tilde{A}_1^N = \tilde{A}_2^N$	$\boldsymbol{\tilde{A}_1^N > \tilde{A}_2^N}$	$\boldsymbol{\tilde{A}_1^N > \tilde{A}_2^N}$

From Table 3, it is expressed that if there exist some SVNNs, exhibiting some peculiar behavior, then it can be ranked accordingly by using Eq. (13), and hence it produces desirable results and helps in selecting a better alternative among available SVNNs. Hence, it claims the validity of the proposed AF $(\text{Eq.}(13))$, stating that it is reasonable.

Furthermore, it is observed that, if any of the necessary condition of AF $(\text{Eq.}(13))$ is violated i.e.,

(i) $T_{\tilde{A}^N} + F_{\tilde{A}^N} = 1$, or

(ii) $I_{\tilde{A}^N} \neq 0$,

then the proposed AF $(\text{Eq.}(13))$, may or may not give a desirable result. Let us consider the following examples:

Example 3. Let $\tilde{A}_1^N = \langle T_{\tilde{A}_1^N}(x), I_{\tilde{A}_1^N}(x), F_{\tilde{A}_1^N}(x)\rangle$, and $\tilde{A}_2^N = \langle T_{\tilde{A}_2^N}(x), I_{\tilde{A}_2^N}(x), F_{\tilde{A}_2^N}(x)\rangle$ be any two SVNNs, then the desirable alternative is selected according to the obtained value of AF using Eq. (13) among these two SVNNs $\tilde{A}_1^N$ and $\tilde{A}_2^N$.

(i) Let $\tilde{A}_1^N = \langle 0.1,0.0,0.9\rangle$ and $\tilde{A}_2^N = \langle 0.5,0.0,0.9\rangle$, then $\tilde{A}_1^N = \tilde{A}_2^N = -0.8$ in accordance with proposed AF $(\text{Eq.}(13))$. While, it is obvious that $A_1 \neq A_2$, but we can see that the necessary conditions of AF, Eq. (13) are violated.

(ii) Let $\tilde{A}_1^N = \langle 0.1,0.0,0.4\rangle$ and $\tilde{A}_2^N = \langle 0.7,0.2,0.3\rangle$, then $\tilde{A}_1^N = \tilde{A}_2^N = 0.2$ in accordance with proposed AF $(\text{Eq.}(13))$. While, it is obvious that $A_1 \neq A_2$, but we can see that the necessary conditions of AF, Eq. (13) are violated.

Therefore, if we are using AF, Eq. (13), to obtain a reasonable solution then the must condition of AF, Eq. (13) should be necessarily followed. To validate this claim and to understand more precisely, let us consider some other example as follows:

Example 4. Let $\tilde{A}_1^N = \langle T_{\tilde{A}_1^N}(x), I_{\tilde{A}_1^N}(x), F_{\tilde{A}_1^N}(x)\rangle$, and $\tilde{A}_2^N = \langle T_{\tilde{A}_2^N}(x), I_{\tilde{A}_2^N}(x), F_{\tilde{A}_2^N}(x)\rangle$ be any two SVNNs, then the desirable alternative is selected according to the obtained value of AF using Eq. (13) among these two SVNNs $\tilde{A}_1^N$ and $\tilde{A}_2^N$.

(i) Let $\tilde{A}_1^N = \langle 0.7, 0.2, 0.3\rangle$ and $\tilde{A}_2^N = \langle 0.6, 0.0, 0.4\rangle$ be any two SVNNs, then on applying AF Eq. (13), the obtained values of $\tilde{A}_1^N$ and $\tilde{A}_2^N$ are $\tilde{A}_1^N = \tilde{A}_2^N = 0.2$ respectively.

(ii) Let $\tilde{A}_1^N = \langle 0.7, 0.2, 0.3\rangle$ and $\tilde{A}_2^N = \langle 0.6, 0.000001, 0.4\rangle$ be any two SVNNs, then on applying AF Eq. (13), the obtained values of $\tilde{A}_1^N = 0.2$ and $\tilde{A}_2^N = 0.1999999$ respectively.

Thus, we observe that in SVNN $\tilde{A}_2^N = \langle 0.6, 0.0, 0.4\rangle$, of Example 4 (i), $I_{\tilde{A}_2^N}(x) = 0.0$, the must condition of AF Eq. (13) i.e., $I_{\tilde{A}_2^N} \neq 0$, is violated, whereas in, SVNN $\tilde{A}_2^N = \langle 0.6, 0.000001, 0.4\rangle$ of Example 4 (ii), $I_{\tilde{A}_2^N}$ is nearly zero but is strictly not zero, i.e., $I_{\tilde{A}_2^N} = 0.0000001 \neq 0$, therefore, when AF Eq. (13) is applied on $\tilde{A}_2^N = \langle 0.6, 0.0, 0.4\rangle$ of Example 4 (i), the obtained value of $\tilde{A}_2^N$ is $\tilde{A}_2^N = 0.2$, and when AF Eq. (13) is applied on $\tilde{A}_2^N = \langle 0.6, 0.000001, 0.4\rangle$ of Example 4 (ii), the obtained value of $\tilde{A}_2^N$ is $\tilde{A}_2^N = 0.1999999$. Hence, we conclude that in Example 4 (i) alternatives $\tilde{A}_1^N$ and $\tilde{A}_2^N$ are equal i.e., $\tilde{A}_1^N = \tilde{A}_2^N = 0.2$ while in Example 4 (ii) alternatives $\tilde{A}_1^N$ is greater than $\tilde{A}_2^N$ i.e., $\tilde{A}_1^N > \tilde{A}_2^N$.

Therefore, we conclude from the above Example 3 and Example 4 that there may or may not exist several such SVNNs violating the must condition of the proposed AF, then Eq. (13) may or may not give an appropriate result. To handle such cases where the must condition for AF, Eq. (13) are not satisfied, then we can find the solution of such SVNNs from the AF, Eq. (6) of the literature [67] which can handle such special SVNNs in a better way.

Hence, it is claimed that the proposed AF Eq. (13) is simple but has restrictions in handling some special SVNNs, then such SVNNs can be ranked more appropriately by using the various other existing metric methods [67, 68], and the proposed SF Eq. (12) respectively. To validate the claim, Example 3 has been evaluated by using various other existing metric methods [67, 68], the proposed SF Eq. (12), and we conclude that in both the SVNSs, i.e., Example 3(i) and Example 3(ii), the desirable solution is $\tilde{A}_2^N$.

Thus, the detailed comparative analysis of Example 3 has been made using various other existing metric methods [67, 68] and the proposed SF Eq. (12), as shown below in Table 4.

Table 4. SF and AF values of various existing metric methods

SVNNs	$\sigma_S(\tilde{A}^N)$	$\sigma_A(\tilde{A}^N)$	$\tau_S(\tilde{A}^N)$	$\tau_A(\tilde{A}^N)$	$\varphi_S(\tilde{A}^N)$	$\varphi_A(\tilde{A}^N)$
$\tilde{A}_1^N = \langle 0.1,0.0,0.9\rangle$	0.1	−0.8	0.1	−0.8	0.1	−0.8
$\tilde{A}_2^N = \langle 0.5,0.0,0.9\rangle$	0.3	−0.4	0.38	−0.4	0.5	−0.8
	$\tilde{A}_2^N > \tilde{A}_1^N$	$\tilde{A}_2^N > \tilde{A}_1^N$	$\tilde{A}_2^N > \tilde{A}_1^N$	$\tilde{A}_2^N > \tilde{A}_1^N$	$\tilde{A}_2^N > \tilde{A}_1^N$	$\tilde{A}_1^N = \tilde{A}_2^N$

$\tilde{A}_1^N = \langle 0.1,0.0,0.4\rangle$ $\tilde{A}_2^N = \langle 0.7,0.2,0.3\rangle$	0.35 0.5 $\boldsymbol{\tilde{A}_2^N > \tilde{A}_1^N}$	-0.3 0.4 $\boldsymbol{\tilde{A}_2^N > \tilde{A}_1^N}$	0.2750 0.5 $\boldsymbol{\tilde{A}_2^N > \tilde{A}_1^N}$	-0.3 0.0 $\boldsymbol{\tilde{A}_2^N > \tilde{A}_1^N}$	0.2333 0.5 $\boldsymbol{\tilde{A}_2^N > \tilde{A}_1^N}$	0.2 0.2 $\tilde{A}_2^N = \tilde{A}_1^N$

Hence, based on the existing metric methods [67, 68] for comparing any two SVNSs $\tilde{A}_1^N = \langle T_{\tilde{A}_1^N}(x), I_{\tilde{A}_1^N}(x), F_{\tilde{A}_1^N}(x)\rangle$ and $\tilde{A}_2^N = \langle T_{\tilde{A}_2^N}(x), I_{\tilde{A}_2^N}(x), F_{\tilde{A}_2^N}(x)\rangle$ using SF $\varphi_S(\tilde{A}^N)$ and AF $\varphi_A(\tilde{A}^N)$, a comparison method can be defined as follows:

- If $\varphi_S(\tilde{A}_1^N) > \varphi_S(\tilde{A}_2^N)$ then $\tilde{A}_1^N > \tilde{A}_2^N$.
- If $\varphi_S(\tilde{A}_1^N) < \varphi_S(\tilde{A}_2^N)$ then $\tilde{A}_1^N < \tilde{A}_2^N$.
- If $\varphi_S(\tilde{A}_1^N) = \varphi_S(\tilde{A}_2^N)$ then check $\varphi_A(\tilde{A}^N)$ in the next step.
  - ✓If $\varphi_A(\tilde{A}_1^N) > \varphi_A(\tilde{A}_2^N)$ then $\tilde{A}_1^N > \tilde{A}_2^N$.
  - ✓If $\varphi_A(\tilde{A}_1^N) < \varphi_A(\tilde{A}_2^N)$ then $\tilde{A}_1^N < \tilde{A}_2^N$.
  - ✓If $\varphi_A(\tilde{A}_1^N) = \varphi_A(\tilde{A}_2^N)$ implies $\tilde{A}_1^N = \tilde{A}_2^N$ for special SVNNs, then check $\sigma_A(\tilde{A}^N)$ in the next step.
    - If $\sigma_A(\tilde{A}_1^N) > \sigma_A(\tilde{A}_2^N)$ then $\tilde{A}_1^N > \tilde{A}_2^N$.
    - If $\sigma_A(\tilde{A}_1^N) < \sigma_A(\tilde{A}_2^N)$ then $\tilde{A}_1^N < \tilde{A}_2^N$.
    - If $\sigma_A(\tilde{A}_1^N) = \sigma_A(\tilde{A}_2^N)$ implies $\tilde{A}_1^N = \tilde{A}_2^N$.

Thus, the proposed SF, Eq. (12) and the proposed AF, Eq. (13) can handle most of the SVNNs concerning its conditions and hence, are helpful in the DM process in a far better manner, and can give answers where the existing methods were having trouble in deriving the conclusions.

To validate the claim of the proposed SF, Eq. (12) and the proposed AF, Eq. (13), a detailed analysis of its properties are presented as follows:

Property 3.1. For SVNS $\tilde{A}^N = \langle \tilde{T}^N(x), \tilde{I}^N(x), \tilde{F}^N(x)\rangle$ the value of the proposed SF $\varphi_S(\tilde{A}^N)$, Eq. (12) lies between $[0,1]$ i.e., $\varphi_S(\tilde{A}^N) \in [0,1]$.

Property 3.2. For SVNS $\tilde{A}^N = \langle \tilde{T}^N(x), \tilde{I}^N(x), \tilde{F}^N(x)\rangle$ or SVNN $\tilde{A}^N = \langle \alpha, \beta, \gamma\rangle$ (for convenience), if $\alpha + \gamma = 2$, then on using proposed SF, Eq. (12) no conclusion can be drawn.

Proof: Let us consider an example given below:

Let $\tilde{A}^N = \langle 1, 0.7, 1\rangle$ be any SVNN, where $\alpha + \gamma = 2$, then from the proposed SF, Eq. (12), we have

$$\varphi_S(\tilde{A}^N) = \frac{1+(\alpha-2(\beta)-\gamma)}{2\,(2-(\alpha+\gamma))} = \frac{1+(1-2(0.7)-1)}{2\,(2-1-1)} = \frac{1-1.4}{0} = \infty.$$

Since $\varphi_S(\tilde{A}^N) \in [0,1]$, hence, no conclusion can be drawn. Thus, for any SVNN $\tilde{A}^N = \langle \alpha, \beta, \gamma\rangle$, SF $\varphi_S(\tilde{A}^N)$ holds if $\alpha + \gamma \neq 2$

Property 3.3. For SVNS $\tilde{A}^N = \langle \tilde{T}^N(x), \tilde{I}^N(x), \tilde{F}^N(x)\rangle$ or SVNN $\tilde{A}^N = \langle \alpha, \beta, \gamma\rangle$ (for convenience), if $\alpha + \gamma = 1$, then proposed SF, Eq. (12) reduces to SF, Eq. (5) i.e., $\varphi_S(\tilde{A}^N) = \sigma_S(\tilde{A}^N)$.

Proof: Let $\alpha + \gamma = 1$, then from the proposed SF, Eq. (12), we have

$$\varphi_S(\tilde{A}^N) = \frac{1+(\alpha-2(\beta)-\gamma)}{2\,(2-(\alpha+\gamma))} = \frac{1+(\alpha-2(\beta)-\gamma)}{2\,(2-1)} = \frac{1+(\alpha-2(\beta)-\gamma)}{2} = \sigma_S(\tilde{A}^N).$$

Property 3.4. For SVNS $\tilde{A}^N = \langle \tilde{T}^N(x), \tilde{I}^N(x), \tilde{F}^N(x)\rangle$ or SVNN $\tilde{A}^N = \langle \alpha, \beta, \gamma\rangle$ (for convenience), the proposed SF, Eq. (12) is having a relationship with existing SF, $\sigma_S(\tilde{A}^N)$, and existing AF, $\tau_A(\tilde{A}^N)$ as follows:

(i) $\varphi_S(\tilde{A}^N) = \frac{1+(\alpha-2(\beta)-\gamma)}{2\,(2-\alpha-\gamma)} = \frac{\sigma_S(\tilde{A}^N)}{(2-\alpha-\gamma)}$

(ii) $\varphi_S(\tilde{A}^N) = \frac{1+(\alpha-2(\beta)-\gamma)}{2\,(2-\alpha-\gamma)} = \frac{1+\tau_A(\tilde{A}^N)}{2\,(2-\alpha-\gamma)}$.

Property 3.5. *One property*: If SVNN $\tilde{A}^N = \langle 1,0,0\rangle$, then $\varphi_S(\tilde{A}^N) = 1$, i.e., the maximum value of SVNN $\tilde{A}^N$ is 1.

Proof: Let $\tilde{A}^N = \langle 1,0,0\rangle$ be any SVNN, then from Eq. (12), we have

$$\varphi_S(\tilde{A}^N) = \frac{1+(1-2(0)-0)}{2\,(2-1-0)} = 1.$$

Property 3.6. *Zero property*: If SVNN $\tilde{A}^N = \langle 0,0,1\rangle$, then $\varphi_S(\tilde{A}^N) = 0$, i.e., the minimum value of SVNN $\tilde{A}^N$ is 0.

Proof: Let $\tilde{A}^N = \langle 0,0,1\rangle$ be any SVNN, then from Eq. (12), we have

$$\varphi_S(\tilde{A}^N) = \frac{1+(0-2(0)-1)}{2\,(2-0-0)} = 0.$$

Property 3.7. For any subset of SVNS $\tilde{A}^N = \langle \tilde{T}^N(x), \tilde{I}^N(x), \tilde{F}^N(x)\rangle$ or $\tilde{A}^N = \langle \alpha, \beta, \gamma\rangle$ (for convenience), the value of $\varphi_S(\tilde{A}^N) = \alpha - \beta$, if $\alpha + \gamma = 1$.

Proof: Let $\tilde{A}^N = \langle \alpha, \beta, \gamma\rangle$ be any subset of SVNS and $\alpha + \gamma = 1$.

(i) Let $\tilde{A}^N = \langle \alpha, \beta, 1-\alpha\rangle$, then from Eq. (12), we have

$$\varphi_S(\tilde{A}^N) = \frac{1+\left(\alpha-2(\beta)-(1-\alpha)\right)}{2\left(2-\alpha-(1-\alpha)\right)} = \frac{1+(2\alpha-2\beta-1)}{2} = \alpha - \beta.$$

(ii) Let $\tilde{A}^N = \langle 1-\gamma, \beta, \gamma\rangle$, then from Eq. (12), we have

$$\varphi_S(\tilde{A}^N) = \frac{1+\left((1-\gamma)-2\beta-\gamma\right)}{2\,(2-(1-\gamma)-\gamma)} = \frac{1+(1-2\beta-2\gamma)}{2} = \frac{2(1-\beta-\gamma)}{2} = \alpha - \beta.$$

Property 3.8. For SVNS $\tilde{A}^N = \langle \tilde{T}^N(x), \tilde{I}^N(x), \tilde{F}^N(x)\rangle$, the value of the proposed AF $\varphi_A(\tilde{A}^N)$, Eq. (13) lies between $[-1,1]$ i.e., $\varphi_A(\tilde{A}^N) \in [-1,1]$, provided $\alpha + \gamma = 1$, and $\beta \neq 0$.

Property 3.9. For SVNS $\tilde{A}^N = \langle \tilde{T}^N(x), \tilde{I}^N(x), \tilde{F}^N(x)\rangle$ or $\tilde{A}^N = \langle \alpha, \beta, \gamma\rangle$ (for convenience), the proposed AF, $\varphi_A(\tilde{A}^N) = 1 - \beta - 2\gamma$, is having a relation with SF Eq. (5) and AF, Eq. (6) as follows: i.e., $\varphi_A(\tilde{A}^N) = 1 - (\sigma_A - \sigma_S) - 3\gamma$ provided $\alpha + \gamma = 1$, and $\beta \neq 0$.

Proof: Let $\tilde{A}^N = \langle \alpha, \beta, \gamma\rangle$ be any SVNS, and $\alpha + \gamma = 1$, $\beta \neq 0$, then we have

$\varphi_A(\tilde{A}^N) = 1 - (\sigma_A - \sigma_S) - 3\gamma$

$= 1 - \left\{\left(\alpha - \beta(1-\alpha) - \gamma(1-\beta)\right) - \left(\frac{1+\alpha-2\beta-\gamma}{2}\right)\right\} - 3\gamma$

$= 1 - \left\{(\alpha - \beta + \alpha\beta - \gamma + \beta\gamma) - \left(\frac{1+1-\gamma-2\beta-\gamma}{2}\right)\right\} - 3\gamma$

$= 1 - \left\{(1 - \gamma - \beta + (1-\gamma)\beta - \gamma + \beta\gamma) - \left(\frac{2-2\gamma-2\beta}{2}\right)\right\} - 3\gamma$

$= 1 - \{(1 - \gamma - \beta + \beta - \beta\gamma - \gamma + \beta\gamma) - (1 - \gamma - \beta)\} - 3\gamma$

$= 1 - (-\gamma + \beta) - 3\gamma$

$= 1 - \beta - 2\gamma$.

3.2 Proposed SF for IVNSs

Let $\tilde{A}^N = \{\langle x, [T^L_{\tilde{A}^N}(x), T^U_{\tilde{A}^N}(x)], [I^L_{\tilde{A}^N}(x), I^U_{\tilde{A}^N}(x)], [F^L_{\tilde{A}^N}(x), F^U_{\tilde{A}^N}(x)]\rangle | x \in X\}$ be an IVNS, then a new SF in terms of the degree of truth-membership, the degree of indeterminacy-membership, and the degree of falsity-membership respectively for IVNS are defined by:

$$\omega_S(\tilde{A}^N) = \frac{2+\left(T^L_{\tilde{A}^N}+T^U_{\tilde{A}^N}-2I^L_{\tilde{A}^N}-2I^U_{\tilde{A}^N}-F^L_{\tilde{A}^N}-F^U_{\tilde{A}^N}\right)}{2\left(4-T^L_{\tilde{A}^N}-T^U_{\tilde{A}^N}-F^L_{\tilde{A}^N}-F^U_{\tilde{A}^N}\right)} \tag{14}$$

where $\omega_S(\tilde{A}^N) \in [0,1]$ *and* $T^L_{\tilde{A}^N} + T^U_{\tilde{A}^N} + F^L_{\tilde{A}^N} + F^U_{\tilde{A}^N} \neq 4$, as $0 \leq T^L_{\tilde{A}^N}(x) \leq T^U_{\tilde{A}^N}(x) \leq 1$, $0 \leq F^L_{\tilde{A}^N}(x) \leq F^U_{\tilde{A}^N}(x) \leq 1$.

Clearly, it is observed that if $T^L_{\tilde{A}^N} + T^U_{\tilde{A}^N} + F^L_{\tilde{A}^N} + F^U_{\tilde{A}^N} = 2$, then $\omega_S(\tilde{A}^N) = \chi_S(\tilde{A}^N)$.

To validate the claim of the proposed SF $(\text{Eq.}(14))$, some well-defined IVNNs are chosen and evaluated. Let us consider the following examples.

Example 5. Let $\tilde{A}^N_1 = \langle [T^L_{\tilde{A}^N_1}(x), T^U_{\tilde{A}^N_1}(x)], [I^L_{\tilde{A}^N_1}(x), I^U_{\tilde{A}^N_1}(x)], [F^L_{\tilde{A}^N_1}(x), F^U_{\tilde{A}^N_1}(x)]\rangle$ and $\tilde{A}^N_2 = \langle [T^L_{\tilde{A}^N_2}(x), T^U_{\tilde{A}^N_2}(x)], [I^L_{\tilde{A}^N_2}(x), I^U_{\tilde{A}^N_2}(x)], [F^L_{\tilde{A}^N_2}(x), F^U_{\tilde{A}^N_2}(x)]\rangle$ be any two IVNNs, then the desirable alternative is selected according to the obtained value of SF using Eq. (14) among $\tilde{A}^N_1$ and $\tilde{A}^N_2$.

(i) Let $\tilde{A}^N_1 = \langle [0.4,0.5], [0.1,0.2], [0.1,0.2]\rangle$ and $\tilde{A}^N_2 = \langle [0.48,0.52], [0.0,0.2], [0.2,0.4]\rangle$ then $\tilde{A}^N_2 > \tilde{A}^N_1$ in accordance with proposed SF $(\text{Eq.}(14))$.

(ii) Let $\tilde{A}^N_1 = \langle [0.4,0.6], [0.125,0.125], [0.1,0.4]\rangle$ and $\tilde{A}^N_2 = \langle [0.23,0.67], [0.1125,0.1125], [0.05,0.4]\rangle$ then $\tilde{A}^N_1 > \tilde{A}^N_2$ in accordance with proposed SF $(\text{Eq.}(14))$.

For a deliberate comparison among various existing metric methods, for finding the correct ranking order of Example 5, a systematic tabular representation of the function values of various metric methods is presented in Table 5.

Table 5. SF $\left(\omega_S(\tilde{A}^N)\right)$ values in comparison with various existing metric methods

IVNNs	$\chi_S(\tilde{A}^N)$	$\psi_S(\tilde{A}^N)$	$\omega_S(\tilde{A}^N)$	$\chi_A(\tilde{A}^N)$
$\tilde{A}^N_1 = \langle [0.4,0.5], [0.1,0.2], [0.1,0.2]\rangle$	0.5	0.5	0.3571	0.24
$\tilde{A}^N_2 = \langle [0.48,0.52], [0.0,0.2], [0.2,0.4]\rangle$	0.5	0.5	0.4167	0.27
	$\tilde{A}^N_1 = \tilde{A}^N_2$	$\tilde{A}^N_1 = \tilde{A}^N_2$	$\boldsymbol{\tilde{A}^N_2 > \tilde{A}^N_1}$	$\tilde{A}^N_2 > \tilde{A}^N_1$
$\tilde{A}^N_1 = \langle [0.4,0.6], [0.125,0.125], [0.1,0.4]\rangle$	0.5	0.5	0.4	0.2188
$\tilde{A}^N_2 = \langle [0.23,0.67], [0.1125,0.1125], [0.05,0.4]\rangle$	0.5	0.5	0.3774	0.1885
	$\tilde{A}^N_1 = \tilde{A}^N_2$	$\tilde{A}^N_1 = \tilde{A}^N_2$	$\boldsymbol{\tilde{A}^N_1 > \tilde{A}^N_2}$	$\tilde{A}^N_1 > \tilde{A}^N_2$

From Table 5, it is expressed that there may exist several ranking methods for IVNNs which can rank the alternatives, $\tilde{A}^N_1$, $\tilde{A}^N_2$, and suggest which of the alternative is better among both. It has been observed that sometimes, the existing metric methods [67, 68] may or may not fail to rank, but the proposed SF $(\text{Eq.}(14))$ is providing desirable results. Hence, it claims the validity of the proposed SF $(\text{Eq.}(14))$, stating that it is reasonable.

Also, it is observed that there may exist several IVNNs where, $T^L_{\tilde{A}^N} + T^U_{\tilde{A}^N} + F^L_{\tilde{A}^N} + F^U_{\tilde{A}^N} = 2$, then the proposed SF, Eq. (14) reduces to the existing SF (Eq. (9)) [67]. Some of IVNNs exhibiting such nature are considered as follows:

Example 6. Let $\tilde{A}^N = \langle [0.22, 0.78], [0.1, 0.3], [0.3, 0.7] \rangle$, then $\omega_S(\tilde{A}^N) = \chi_S(\tilde{A}^N) = 0.3000$ in accordance with obtained value of SF on using $(\text{Eq.}\,(14))$ and $(\text{Eq.}\,(9))$.

Example 7. Let $\tilde{A}^N = \langle [0.45, 0.55], [0.1, 0.2], [0.4, 0.6] \rangle$, then $\omega_S(\tilde{A}^N) = \chi_S(\tilde{A}^N) = 0.3500$ in accordance with obtained value of SF on using $(\text{Eq.}\,(14))$ and $(\text{Eq.}\,(9))$.

For a deliberate comparison among various existing metric methods, for finding the correct score value of Example 6 and Example 7, a systematic tabular representation of the function values of various metric methods is presented in Table 6.

Table 6. Function values of some special IVNNs using various existing methods

IVNNs	$\chi_S(\tilde{A}^N)$	$\psi_S(\tilde{A}^N)$	$\omega_S(\tilde{A}^N)$	$\chi_A(\tilde{A}^N)$
$\tilde{A}^N = \langle [0.22, 0.78], [0.1, 0.3], [0.3, 0.7] \rangle$	0.3000	0.3000	0.3000	0.0080
$\tilde{A}^N = \langle [0.45, 0.55], [0.1, 0.2], [0.4, 0.6] \rangle$	0.3500	0.3500	0.3500	−0.0025

Also, it is observed that there may exist several IVNNs where, $T^L_{\tilde{A}^N} + T^U_{\tilde{A}^N} + F^L_{\tilde{A}^N} + F^U_{\tilde{A}^N} = 4$, then the proposed score functions Eq. (14), have its limitation. Let us consider an example of IVNNs exhibiting such nature as follows:

Example 8. Let $\tilde{A}^N_1 = \langle [1, 1], [0.2, 0.7], [1, 1] \rangle$ and $\tilde{A}^N_2 = \langle [1, 1], [0.5, 0.9], [1, 1] \rangle$ be any two IVNNs, then $\tilde{A}^N_1 = \tilde{A}^N_2 = \infty$ on using the proposed SF, Eq. (14), since it is violating the must condition for the proposed SF Eq. (14) hence, no conclusion can be drawn.

Furthermore, on analysis, it is observed that the existing AF $\chi_A(\tilde{A}^N)$, $(\text{Eq.}\,(10))$ [67] is successful in giving a desirable solution for such IVNNs, where sometimes all the existing [67, 68] and the proposed SF i.e., $\text{Eq.}\,(9)$, $\text{Eq.}\,(11)$ and $\text{Eq.}\,(14)$ may or may not be able to give an appropriate solution. To validate the claim, above stated Example 8 is evaluated using AF $\chi_A(\tilde{A}^N)$, $(\text{Eq.}\,(10))$ [67] which states that, $\tilde{A}^N_2 > \tilde{A}^N_1$ i.e., $\tilde{A}^N_2$ is the best alternative among $\tilde{A}^N_2$ and $\tilde{A}^N_1$ as shown in Table 7.

For a deliberate comparison among various existing metric methods, for finding the correct ranking order of some special SVNN concerning an existing AF $\chi_A(\tilde{A}^N)$, $(\text{Eq.}\,(10))$ [67], is presented in Table 7 as follows:

Table 7. Function values of some special IVNNs using various existing metric methods

IVNNs	$\chi_S(\tilde{A}^N)$	$\psi_S(\tilde{A}^N)$	$\omega_S(\tilde{A}^N)$	$\chi_A(\tilde{A}^N)$
$\tilde{A}^N_1 = \langle [1, 1], [0.2, 0.7], [1, 1] \rangle$	0.05	0.5	∞	−0.05
$\tilde{A}^N_2 = \langle [1, 1], [0.5, 0.9], [1, 1] \rangle$	−0.2 $(\alpha_S(\tilde{A}^N) \notin [0, 1])$	0.5	∞	0.7

Thus, from Table 7, it is concluded that there may or may not exists several such SVNNs for which sometimes all the existing [67, 68] and the proposed SF i.e., Eq. (9), Eq. (11) and Eq. (14) may not suggest an appropriate solution among $\tilde{A}_1^N$ and $\tilde{A}_2^N$ but the existing AF $\chi_A(\tilde{A}^N)$, (Eq. (10)) [67] is successful in providing a satisfactory solution so far.

Hence, based on the existing metric methods [67, 68] for comparing any two IVNSs $\tilde{A}_1^N = \langle[T^L_{\tilde{A}_1^N}(x), T^U_{\tilde{A}_1^N}(x)], [I^L_{\tilde{A}_1^N}(x), I^U_{\tilde{A}_1^N}(x)], [F^L_{\tilde{A}_1^N}(x), F^U_{\tilde{A}_1^N}(x)]\rangle$ and $\tilde{A}_2^N = \langle[T^L_{\tilde{A}_2^N}(x), T^U_{\tilde{A}_2^N}(x)], [I^L_{\tilde{A}_2^N}(x), I^U_{\tilde{A}_2^N}(x)], [F^L_{\tilde{A}_2^N}(x), F^U_{\tilde{A}_2^N}(x)]\rangle$ using SF and AF, a comparison method can be defined as follows:

- If $\omega_S(\tilde{A}_1^N) > \omega_S(\tilde{A}_2^N)$ then $\tilde{A}_1^N > \tilde{A}_2^N$.
- If $\omega_S(\tilde{A}_1^N) < \omega_S(\tilde{A}_2^N)$ then $\tilde{A}_1^N < \tilde{A}_2^N$.
- If $\omega_S(\tilde{A}_1^N) = \omega_S(\tilde{A}_2^N)$ or no conclusion can be drawn, then check $\chi_A(\tilde{A}^N)$ in the next step.
  - If $\chi_A(\tilde{A}_1^N) > \chi_A(\tilde{A}_2^N)$ then $\tilde{A}_1^N > \tilde{A}_2^N$.
  - If $\chi_A(\tilde{A}_1^N) < \chi_A(\tilde{A}_2^N)$ then $\tilde{A}_1^N < \tilde{A}_2^N$.
  - If $\chi_A(\tilde{A}_1^N) = \chi_A(\tilde{A}_2^N)$ then $\tilde{A}_1^N = \tilde{A}_2^N$.

Thus, the proposed SF, Eq. (14) can handle most of the IVNNs along with its conditions and hence, is helpful in the DM process in a much better way, also can give answers where the existing methods were not leading the solution to anywhere.

To validate the claim of the proposed SF, Eq. (14), a detailed analysis of its properties are presented as follows:

Property 3.10. For IVNS $\tilde{A}^N = \langle[T^L_{\tilde{A}^N}(x), T^U_{\tilde{A}^N}(x)], [I^L_{\tilde{A}^N}(x), I^U_{\tilde{A}^N}(x)], [F^L_{\tilde{A}^N}(x), F^U_{\tilde{A}^N}(x)]\rangle$ the value of the proposed SF $\omega_S(\tilde{A}^N)$, Eq. (14) lies between [0,1] i.e., $\omega_S(\tilde{A}^N) \in [0,1]$.

Property 3.11. For IVNS $\tilde{A}^N = \langle[T^L_{\tilde{A}^N}(x), T^U_{\tilde{A}^N}(x)], [I^L_{\tilde{A}^N}(x), I^U_{\tilde{A}^N}(x)], [F^L_{\tilde{A}^N}(x), F^U_{\tilde{A}^N}(x)]\rangle$ or $\tilde{A}^N = \langle[\alpha_1, \alpha_2)], [\beta_1, \beta_2], [\gamma_1, \gamma_2]\rangle$ (for convenience), if $\alpha_1 + \alpha_2 + \gamma_1 + \gamma_2 = 4$, then on using proposed SF, Eq. (14) no conclusion can be drawn.

Proof: Let us consider an example given below:

Let $\tilde{A}^N = \langle\langle[1,1], [0.25, 0.571], [1,1]\rangle\rangle$ be any IVNN, where $\alpha_1 + \alpha_2 + \gamma_1 + \gamma_2 = 4$, then from the proposed SF, Eq. (14), we have

$$\omega_S(\tilde{A}^N) = \frac{2+(1+1-2(0.25)-2(0.571)-1-1)}{2\,(4-1-1-1-1)} = \frac{2+(-1.6420)}{0} = \frac{0.3580}{0} = \infty.$$

Since $\omega_S(\tilde{A}^N) \in [0,1]$, hence, no conclusion can be drawn. Thus, for any IVNN $\tilde{A}^N = \langle[\alpha_1, \alpha_2)], [\beta_1, \beta_2], [\gamma_1, \gamma_2]\rangle$, SF $\omega_S(\tilde{A}^N)$ holds if $\alpha_1 + \alpha_2 + \gamma_1 + \gamma_2 \neq 4$.

Property 3.12. For IVNS $\tilde{A}^N = \langle[T^L_{\tilde{A}^N}(x), T^U_{\tilde{A}^N}(x)], [I^L_{\tilde{A}^N}(x), I^U_{\tilde{A}^N}(x)], [F^L_{\tilde{A}^N}(x), F^U_{\tilde{A}^N}(x)]\rangle$ or $\tilde{A}^N = \langle[\alpha_1, \alpha_2)], [\beta_1, \beta_2], [\gamma_1, \gamma_2]\rangle$ (for convenience), if $\alpha_1 + \alpha_2 + \gamma_1 + \gamma_2 = 2$, , then proposed SF, Eq. (14) reduces to SF, Eq. (9) i.e., $\omega_S(\tilde{A}^N) = \chi_S(\tilde{A}^N)$.

Proof: Let $\alpha_1 + \alpha_2 + \gamma_1 + \gamma_2 = 2$, then from the proposed SF, Eq. (14), we have

$$\omega_S(\tilde{A}^N) = \frac{2+(\alpha_1+\alpha_2-2\beta_1-\beta_2-\gamma_1-\gamma_2)}{2\,(4-\alpha_1-\alpha_2-\gamma_1-\gamma_2)} = \frac{2+(\alpha_1+\alpha_2-2\beta_1-\beta_2-\gamma_1-\gamma_2)}{2\,(4-(\alpha_1+\alpha_2+\gamma_1+\gamma_2))} = \frac{2+(\alpha_1+\alpha_2-2\beta_1-\beta_2-\gamma_1-\gamma_2)}{2\,(4-2)}$$

$$= \frac{2+(\alpha_1+\alpha_2-2\beta_1-\beta_2-\gamma_1-\gamma_2)}{4}$$

$= \chi_S(\tilde{A}^N)$.

Property 3.13. For IVNS $\tilde{A}^N = \langle [T^L_{\tilde{A}^N}(x), T^U_{\tilde{A}^N}(x)], [I^L_{\tilde{A}^N}(x), I^U_{\tilde{A}^N}(x)], [F^L_{\tilde{A}^N}(x), F^U_{\tilde{A}^N}(x)] \rangle$ or $\tilde{A}^N = \langle [\alpha_1, \alpha_2)], [\beta_1, \beta_2], [\gamma_1, \gamma_2] \rangle$ (for convenience), the proposed SF $\omega_S(\tilde{A}^N)$, Eq. (14) is having a relation with SF $\chi_S(\tilde{A}^N)$, Eq. (9) as follows: i.e., $\omega_S(\tilde{A}^N) = \frac{\chi_S}{(4-\alpha_1-\alpha_2-\gamma_1-\gamma_2)}$.

Property 3.14. *One property*: If IVNN $\tilde{A}^N = \langle [1,1], [0,0], [0,0] \rangle$, then $\omega_S(\tilde{A}^N) = 1$, i.e., the maximum value of IVNN $\tilde{A}^N$ is 1.

Proof: Let $\tilde{A}^N = \langle [1,1], [0,0], [0,0] \rangle$ be any IVNN, then from Eq. (14), we have

$$\omega_S(\tilde{A}^N) = \frac{2+(1+1-0-0-0-0)}{2\,(4-1-1-0-0)} = \frac{4}{4} = 1.$$

Property 3.15. *Zero property*: If IVNN $\tilde{A}^N = \langle [0,0], [0,0], [1,1] \rangle$, then $\omega_S(\tilde{A}^N) = 0$, i.e., the minimum value of IVNN $\tilde{A}^N$ is 0.

Proof: Let $\tilde{A}^N = \langle [0,0], [0,0], [1,1] \rangle$ be any IVNN, then from Eq. (14), we have

$$\omega_S(\tilde{A}^N) = \frac{2+(0+0-0-0-1-1)}{2\,(4-0-0-1-1)} = \frac{0}{4} = 0.$$

4. MCDM method based on proposed SF and AF under neutrosophic environment

In this section MCDM method is proposed for both SVNSs and IVNSs using proposed SF and proposed AF, which is pictorially presented in Figure 1.

4.1. MCDM method based on proposed SF and AF under SVNSs

Let us consider an MCDM problem having m number of alternatives i.e., $\tilde{A}^N = \{\tilde{A}^N_1, \tilde{A}^N_2, \dots, \tilde{A}^N_m\}$ which are evaluated on n number of criteria i.e., $\tilde{G}^N = \{\tilde{G}^N_1, \tilde{G}^N_2, \dots, \tilde{G}^N_n\}$. Suppose that the weight allotted to each criterion by the decision-maker is $w_j \in [0,1]$ and $\sum_{j=1}^n w_j = 1$. Also, the characteristics of an alternatives $\tilde{A}^N_i (i = 1,2,\dots,m)$ per criterion $\tilde{G}^N_j (j = 1,2,\dots,n)$ can be represented by an SVNS i.e., $\tilde{A}^N_i = \left\{ \langle \tilde{G}^N_j, T_{\tilde{A}^N_i}(\tilde{G}^N_j), I_{\tilde{A}^N_i}(\tilde{G}^N_j), F_{\tilde{A}^N_i}(\tilde{G}^N_j) \rangle \mid \tilde{G}^N_j \in \tilde{G}^N \right\}$, where $T_{\tilde{A}^N_i}(\tilde{G}^N_j) + I_{\tilde{A}^N_i}(\tilde{G}^N_j) + F_{\tilde{A}^N_i}(\tilde{G}^N_j) \le 3$ and $T_{\tilde{A}^N_i}(\tilde{G}^N_j) \ge 0$, $I_{\tilde{A}^N_i}(\tilde{G}^N_j) \ge 0$, $F_{\tilde{A}^N_i}(\tilde{G}^N_j) \ge 0$, for all $i = 1\ to\ m$ and $j = 1\ to\ n$, for convenience it is denoted as $\Psi_{ij} = \langle \alpha_{ij}, \beta_{ij}, \gamma_{ij} \rangle$. The single-valued neutrosophic decision matrix (SVNDM) derived from the collected single-valued neutrosophic data available for m number of alternatives with respect to n number of the criterion is represented as

$D = (\Psi_{ij})_{m\times n} = (\langle \alpha_{ij}, \beta_{ij}, \gamma_{ij} \rangle)_{m\times n}$ i.e.,

$$(\Psi_{ij})_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccccc} G_1 & & G_2 & & G_n \end{array} \\ \begin{bmatrix} \langle \alpha_{11}, \beta_{11}, \gamma_{11} \rangle & \langle \alpha_{12}, \beta_{12}, \gamma_{12} \rangle & \cdots & \langle \alpha_{1n}, \beta_{1n}, \gamma_{1n} \rangle \\ \langle \alpha_{21}, \beta_{21}, \gamma_{21} \rangle & \langle \alpha_{22}, \beta_{22}, \gamma_{22} \rangle & \cdots & \langle \alpha_{2n}, \beta_{2n}, \gamma_{2n} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \alpha_{m1}, \beta_{m1}, \gamma_{m1} \rangle & \langle \alpha_{m2}, \beta_{m2}, \gamma_{m2} \rangle & \cdots & \langle \alpha_{mn}, \beta_{mn}, \gamma_{mn} \rangle \end{bmatrix} \end{array}.$$

For evaluating the MCDM problem for SVNSs we need a step-wise procedure which is summarized as follows:

Step 1: Check that all the criteria of the SVNDM, D are of the same type or not.

Case (i) If all the criteria are of the same type then go to Step 2.

Case (ii) If some criteria are of benefit type and others are of cost types then normalize the SVNDM by transforming the cost criterion into benefit criterion, in the following manner: If the p^{th} criterion is cost criterion then replace all the elements $\langle \alpha_{ip}, \beta_{ip}, \gamma_{ip} \rangle$ of the p^{th} column of the decision matrix, D with $\langle \gamma_{ip}, 1-\beta_{ip}, \alpha_{ip} \rangle$.

Step 2: Evaluate the SVNSs Ψ_{ij} for each $\tilde{A}_i^N$ into an SVNN Ψ_i using WAM, Eq. (1), or the WGM, Eq. (2).

Step 3: After aggregating (by applying either of the approaches i.e., WAM or WGM) according to Step 2, now obtain the crisp value of Ψ_i $(i = 1,2,\ldots,m)$ by using SF $\varphi_S(\tilde{A}^N)$, Eq. (12) or AF $\varphi_A(\tilde{A}^N)$, Eq. (13).

Step 4: After Step 3, rank all the alternatives as per the obtained value of $\varphi_S(\tilde{A}^N)$ or $\varphi_A(\tilde{A}^N)$ and choose the best alternative.

4.2. MCDM method based on proposed SF and AF under IVNSs

Let us consider an MCDM problem having m number of alternatives i.e., $\tilde{A}^N = \{\tilde{A}_1^N, \tilde{A}_2^N, \ldots, \tilde{A}_m^N\}$ which are evaluated on n number of criteria i.e., $\tilde{G}^N = \{\tilde{G}_1^N, \tilde{G}_2^N, \ldots, \tilde{G}_n^N\}$. Suppose that the weight allotted to each criterion by the decision-maker is $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. Also, the characteristics of an alternatives $\tilde{A}_i^N (i = 1,2,\ldots,m)$ as per criterion $\tilde{G}_j^N (j = 1,2,\ldots,n)$ can be represented by an IVNS i.e., $\tilde{A}_i^N = \left\{ \langle \tilde{G}_j^N, \left[T^L_{\tilde{A}_i^N}(\tilde{G}_j^N), T^U_{\tilde{A}_i^N}(\tilde{G}_j^N)\right], \left[I^L_{\tilde{A}_i^N}(\tilde{G}_j^N), I^U_{\tilde{A}_i^N}(\tilde{G}_j^N)\right], \left[F^L_{\tilde{A}_i^N}(\tilde{G}_j^N), F^U_{\tilde{A}_i^N}(\tilde{G}_j^N)\right] \rangle \mid \tilde{G}_j^N \in \tilde{G}^N \right\}$, where $T^U_{\tilde{A}_i^N}(\tilde{G}_j^N) + I^U_{\tilde{A}_i^N}(\tilde{G}_j^N) + F^U_{\tilde{A}_i^N}(\tilde{G}_j^N) \leq 3$ and $0 \leq T^L_{\tilde{A}_i^N}(\tilde{G}_j^N) \leq T^U_{\tilde{A}_i^N}(\tilde{G}_j^N) \leq 1$, $0 \leq I^L_{\tilde{A}_i^N}(\tilde{G}_j^N) \leq I^U_{\tilde{A}_i^N}(\tilde{G}_j^N) \leq 1$, $0 \leq F^L_{\tilde{A}_i^N}(\tilde{G}_j^N) \leq F^U_{\tilde{A}_i^N}(\tilde{G}_j^N) \leq 1$, for all $i = 1\ to\ m$ and $j = 1\ to\ n$, for convenience it is denoted as $\Psi_{ij} = \langle \alpha_{ij}, \beta_{ij}, \gamma_{ij} \rangle = \langle [\alpha_{ij}^L, \alpha_{ij}^U], [\beta_{ij}^L, \beta_{ij}^U], [\gamma_{ij}^L, \gamma_{ij}^U] \rangle$. The interval-valued neutrosophic decision-matrix (IVNDM) derived from the collected interval-valued neutrosophic data available for m number of alternatives for n number of the criterion is represented as

$$D = (\Psi_{ij})_{m\times n} = (\langle \alpha_{ij}, \beta_{ij}, \gamma_{ij} \rangle)_{m\times n} = (\langle [\alpha_{ij}^L, \alpha_{ij}^U], [\beta_{ij}^L, \beta_{ij}^U], [\gamma_{ij}^L, \gamma_{ij}^U] \rangle)_{m\times n} \text{ i.e.,}$$

$$(\Psi_{ij})_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} G_1 & G_2 & & G_n \end{array} \\ \begin{bmatrix} \langle \alpha_{11}, \beta_{11}, \gamma_{11} \rangle & \langle \alpha_{12}, \beta_{12}, \gamma_{12} \rangle & \cdots & \langle \alpha_{1n}, \beta_{1n}, \gamma_{1n} \rangle \\ \langle \alpha_{21}, \beta_{21}, \gamma_{21} \rangle & \langle \alpha_{22}, \beta_{22}, \gamma_{22} \rangle & \cdots & \langle \alpha_{2n}, \beta_{2n}, \gamma_{2n} \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle \alpha_{m1}, \beta_{m1}, \gamma_{m1} \rangle & \langle \alpha_{m2}, \beta_{m2}, \gamma_{m2} \rangle & \cdots & \langle \alpha_{mn}, \beta_{mn}, \gamma_{mn} \rangle \end{bmatrix} \end{array}.$$

For evaluating the MCDM problem for IVNSs we need a step-wise procedure which is summarized as follows:

Step 1: Check that all the criteria of the IVNDM, D are of the same type or not.

Case (i) If all the criteria are of the same type then go to Step 2.

Case (ii) If some criteria are of benefit types and others are of cost types then normalize the IVNDM by transforming the cost criterion into benefit criterion, in the following manner: If the

p^{th} criterion is of cost type then replace all the elements $\langle \alpha_{ip}, \beta_{ip}, \gamma_{ip} \rangle$ of the p^{th} column of the decision matrix, D with $\langle \gamma_{ip}, 1-\beta_{ip}, \alpha_{ip} \rangle$.

Step 2: Evaluate the IVNSs Ψ_{ij} for each $\tilde{A}_i^N$ into an IVNN Ψ_i using WAM, Eq. (3) or the WGM, Eq. (4).

Step 3: After aggregating (by applying either of the approaches i.e., WAM or WGM) according to Step 2, now obtain the crisp value of Ψ_i $(i = 1,2,\dots,m)$ by using SF $\omega_S(\tilde{A}^N)$, Eq. (14) or AF $\chi_A(\tilde{A}^N)$, Eq. (10).

Step 4: After Step 3, rank all the alternatives as per the obtained value of $\omega_S(\tilde{A}^N)$ or $\chi_A(\tilde{A}^N)$ and choose the best alternative.

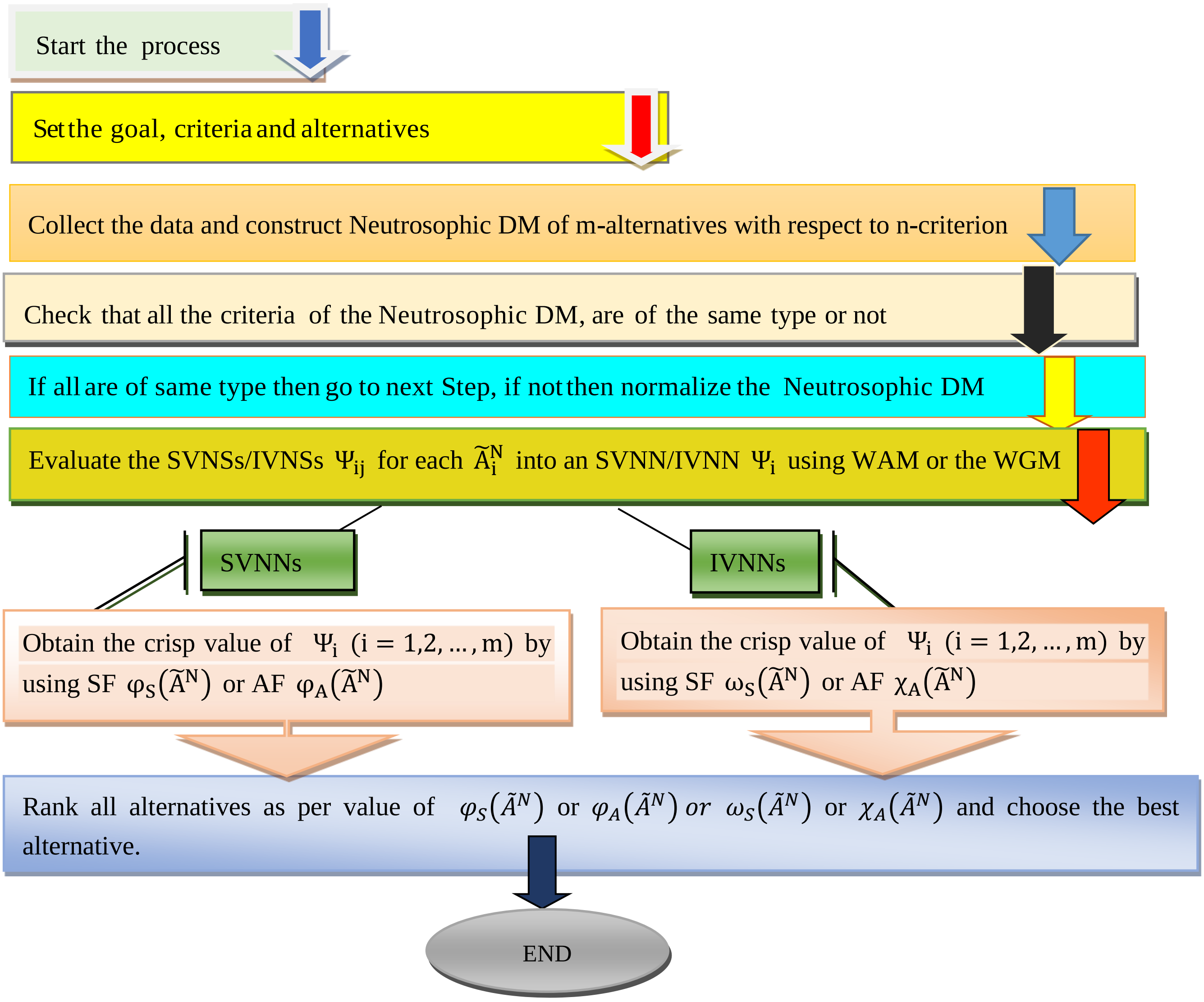

Figure 1. The flowchart of the proposed MCDM method

5. Real-world problem on SVNSs and IVNSs

In this section, a very common example is taken from a real-life which helps in validating the proposed approach.

Example 5.1. Consider an MCDM problem of selecting a pre-school for the first time, by the parents of a kindergarten child. To make the best selection, parents have collected the data in terms of the neutrosophic set (SVNSs or IVNSs) of 05 possible pre-schools, as per their liking, which are their prospective alternatives $\tilde{A}^N = \{\tilde{A}_1^N, \tilde{A}_2^N, \tilde{A}_3^N, \tilde{A}_4^N, \tilde{A}_5^N\}$ respectively. The data of these 05 possible alternatives are based on 03 different criteria $\tilde{G}^N = \{\tilde{G}_1^N, \tilde{G}_2^N, \tilde{G}_3^N\}$ where $\tilde{G}_1^N$ represents "near to the house, and safety of the child", $\tilde{G}_2^N$ represents "fee, infrastructure, and rapport" and $\tilde{G}_3^N$ represents "teaching methods in terms of effective learning concerning, cognitive, conative, affective, and physical activity" and the weight vectors are chosen for each criterion is $w_j = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)^T$. Thus, when these five schools w.r.t the above-stated criteria are assessed by the parents (decision-maker), using the above-mentioned procedure stated in Section 4.1 and Section 4.2 as represented pictorially in Figure 2, the best alternative is obtained.

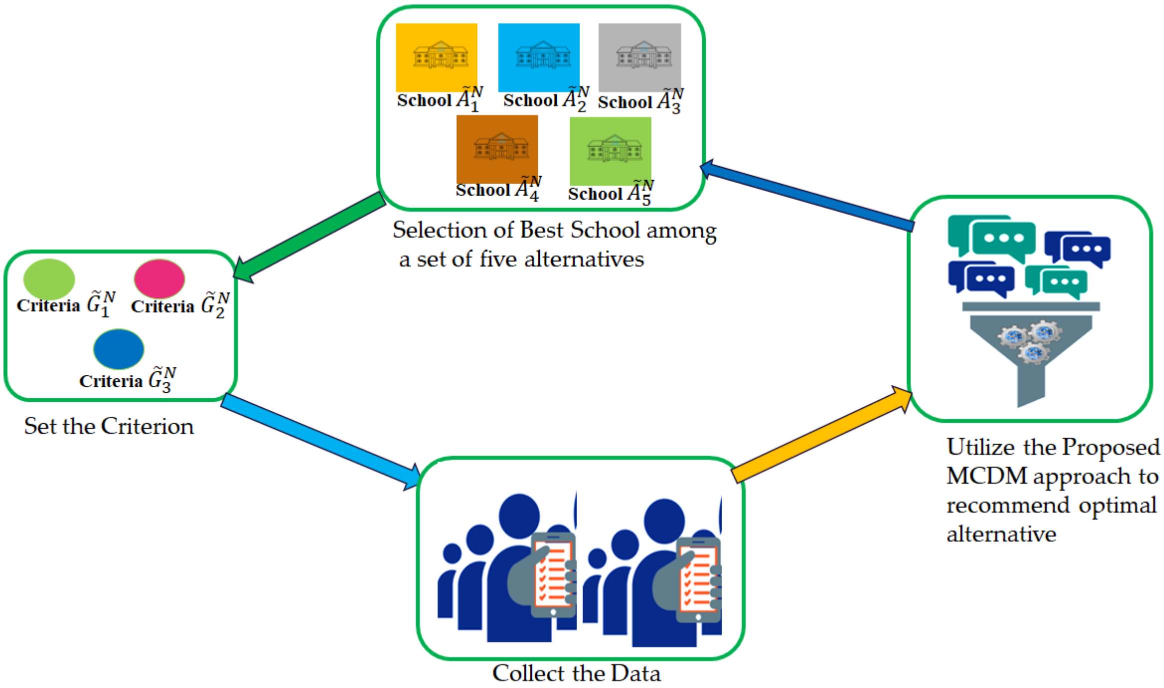

Figure 2. A Framework of Proposed MCDM approach for a real-life problem

5.1 Real-world problem on SVNSs

On applying the procedure mentioned in Section 4.1 on Example 5.1, where the collected data by the decision-maker is in terms of SVNSs, the best solution is derived as follows:

Step 1: Using the Step 1 of Section 4.1, the obtained SVNDM, D as per the collected SVNS information is presented in Table 8.

Table 8. SVNDM

	$\tilde{G}_1^N$	$\tilde{G}_2^N$	$\tilde{G}_3^N$
$\tilde{A}_1^N$	$\langle 0.6,0.3,0.0\rangle$	$\langle 0.6,0.1,0.4\rangle$	$\langle 0.4,0.3,0.8\rangle$
$\tilde{A}_2^N$	$\langle 0.2,0.1,0.0\rangle$	$\langle 0.8,0.3,0.2\rangle$	$\langle 0.9,0.2,0.6\rangle$
$\tilde{A}_3^N$	$\langle 0.4,0.2,0.3\rangle$	$\langle 0.4,0.2,0.3\rangle$	$\langle 0.2,0.2,0.5\rangle$
$\tilde{A}_4^N$	$\langle 0.3,0.2,0.3\rangle$	$\langle 0.5,0.2,0.3\rangle$	$\langle 0.5,0.3,0.2\rangle$
$\tilde{A}_5^N$	$\langle 0.7,0.0,0.1\rangle$	$\langle 0.6,0.1,0.2\rangle$	$\langle 0.4,0.3,0.2\rangle$

Step 2: Using the Step 2 of Section 4.1, the obtained aggregated SVNN Ψ_i, for SVNSs Ψ_{ij}, for each $\tilde{A}_i^N$ using WAM, Eq. (1) and the WGM, Eq. (2) are shown in Table 9.

Table 9. SVNN using AO_{WA} and AO_{WG}

	AO_{WA}	AO_{WG}
Ψ_1	$\langle 0.5421, 0.2080, 0.0\rangle$	$\langle 0.5241, 0.2388, 0.5068\rangle$
Ψ_2	$\langle 0.7480, 0.1817, 0.0\rangle$	$\langle 0.5241, 0.2042, 0.3160\rangle$
Ψ_3	$\langle 0.3396, 0.2, 0.3557\rangle$	$\langle 0.3175, 0.2, 0.3743\rangle$
Ψ_4	$\langle 0.4407, 0.2289, 0.2621\rangle$	$\langle 0.4217, 0.2348, 0.2681\rangle$
Ψ_5	$\langle 0.5840, 0.0, 0.1587\rangle$	$\langle 0.5518, 0.1427, 0.1680\rangle$

Step 3: After Step 2, using the Step 3 of Section 4.1, the score value, φ_S for each Ψ_i $(i = 1,2,\dots,m)$ are obtained by using Eq. (12) as follows:

Approach 1 (Using WAM). Aggregated SVNN Ψ_i $(i = 1,2,\dots,m)$ on using, WAM Eq. (1), the obtained score values $\varphi_S(\tilde{A}_i^N)$ are as follows:

$\varphi_S(\Psi_1) = 0.3862$, $\varphi_S(\Psi_2) = 0.5530$, $\varphi_S(\Psi_3) = 0.2238$, $\varphi_S(\Psi_4) = 0.2778$, $\varphi_S(\Psi_5) = 0.5668$.

Approach 2 (Using WGM). Aggregated SVNN Ψ_i $(i = 1,2,\dots,m)$ on using, WGM Eq. (2), the obtained score values $\varphi_S(\tilde{A}_i^N)$ are as follows:

$\varphi_S(\Psi_1) = 0.2785$, $\varphi_S(\Psi_2) = 0.3448$, $\varphi_S(\Psi_3) = 0.2076$, $\varphi_S(\Psi_4) = 0.2610$, $\varphi_S(\Psi_5) = 0.4290$.

Step 4: According to the obtained values of SF in Step 3, the following results are deduced, i.e.,

(i) For approach 1, the obtained ranking order of the alternatives is $\tilde{A}_5^N > \tilde{A}_2^N > \tilde{A}_1^N > \tilde{A}_4^N > \tilde{A}_3^N$, hence, $\tilde{A}_5^N$ is the best alternative according to the obtained score value $\varphi_S(\tilde{A}^N)$ for each Ψ_i $(i = 1,2,\dots,m)$.

(ii) For approach 2, the obtained ranking order of the alternatives is $\tilde{A}_5^N > \tilde{A}_2^N > \tilde{A}_1^N > \tilde{A}_4^N > \tilde{A}_3^N$, hence, $\tilde{A}_5^N$ is the best alternative according to the obtained score value $\varphi_S(\tilde{A}^N)$ for each Ψ_i $(i = 1,2,\dots,m)$.

Furthermore, to validate the above results obtained from the proposed method $\varphi_S(\tilde{A}^N)$, a detailed comparative analysis of given data in Table 9 is done with the existing methods $\sigma_S(\tilde{A}^N)$ [67], and $\tau_S(\tilde{A}^N)$ [68], and the obtained values of their respective score functions are represented in Table 10 and Table 11, on applying both the approaches of aggregation i.e., WAM and WGM respectively.

According to the obtained values of the SF on using the existing methods $\sigma_S(\tilde{A}^N)$ [67], $\tau_S(\tilde{A}^N)$ [68], and the proposed method $\varphi_S(\tilde{A}^N)$ for WAM, the obtained ranking order of all the alternatives is the same, i.e., $\tilde{A}_5^N > \tilde{A}_2^N > \tilde{A}_1^N > \tilde{A}_4^N > \tilde{A}_3^N$, as shown in Table 10, hence we conclude that $\tilde{A}_5^N$ is the best alternative.

Table 10. Comparative analysis of SF of various methods for WAM

	SVNNs	$\boldsymbol{\sigma_S(\tilde{A}^N)}$	$\boldsymbol{\tau_S(\tilde{A}^N)}$	$\boldsymbol{\varphi_S(\tilde{A}^N)}$
$\varphi_S(\Psi_1)$	$\langle 0.5421, 0.2080, 0.0\rangle$	0.5631	0.5919	0.3862
$\varphi_S(\Psi_2)$	$\langle 0.7480, 0.1817, 0.0\rangle$	0.6923	0.7408	0.5530
$\varphi_S(\Psi_3)$	$\langle 0.3396, 0.2, 0.3557\rangle$	0.2919	0.2286	0.2238
$\varphi_S(\Psi_4)$	$\langle 0.4407, 0.2289, 0.2621\rangle$	0.3604	0.3189	0.2778
$\varphi_S(\Psi_5)$	$\langle 0.5840, 0.0, 0.1587\rangle$	**0.7127**	**0.7674**	**0.5668**
Ranking order		$\tilde{A}_5^N > \tilde{A}_2^N > \tilde{A}_1^N > \tilde{A}_4^N > \tilde{A}_3^N$	$\tilde{A}_5^N > \tilde{A}_2^N > \tilde{A}_1^N > \tilde{A}_4^N > \tilde{A}_3^N$	$\tilde{A}_5^N > \tilde{A}_2^N > \tilde{A}_1^N > \tilde{A}_4^N > \tilde{A}_3^N$

Similarly, according to the obtained values of the SF on using the existing methods $\sigma_S(\tilde{A}^N)$ [67], $\tau_S(\tilde{A}^N)$ [68] and the proposed method $\varphi_S(\tilde{A}^N)$ for WGM, the obtained ranking order of the best and the second-best alternatives is the same as shown in Table 11, thus we conclude that $\tilde{A}_5^N$ is the best alternative.

Table 11. Comparative analysis of SF of various methods for WGM

	SVNNs	$\boldsymbol{\sigma_S(\tilde{A}^N)}$	$\boldsymbol{\tau_S(\tilde{A}^N)}$	$\boldsymbol{\varphi_S(\tilde{A}^N)}$
$\varphi_S(\Psi_1)$	$\langle 0.5241, 0.2388, 0.5068\rangle$	0.2699	0.2770	0.2785
$\varphi_S(\Psi_2)$	$\langle 0.5241, 0.2042, 0.3160\rangle$	0.3999	0.3838	0.3448
$\varphi_S(\Psi_3)$	$\langle 0.3175, 0.2, 0.3743\rangle$	0.2716	0.2012	0.2076
$\varphi_S(\Psi_4)$	$\langle 0.4217, 0.2348, 0.2681\rangle$	0.3420	0.2930	0.2610
$\varphi_S(\Psi_5)$	$\langle 0.5518, 0.1427, 0.1680\rangle$	**0.5492**	**0.5630**	**0.4290**
Ranking order		$\tilde{A}_5^N > \tilde{A}_2^N > \tilde{A}_4^N > \tilde{A}_3^N > \tilde{A}_1^N$	$\tilde{A}_5^N > \tilde{A}_2^N > \tilde{A}_4^N > \tilde{A}_1^N > \tilde{A}_3^N$	$\tilde{A}_5^N > \tilde{A}_2^N > \tilde{A}_1^N > \tilde{A}_4^N > \tilde{A}_3^N$

Hence, we can conclude that the proposed score function $\varphi_S(\tilde{A}^N)$ is justified and is giving reasonable results on applying in real-world applications.

5.2 Real-world problem on IVNSs

On applying the procedure mentioned in Section 4.2 on Example 5.1, where the rating value of the collected data by the decision-maker is in terms of IVNSs, the best solution is derived as follows:

Step 1: Using the Step 1 of Section 4.2, the obtained IVNDM, D as per the collected IVNS information is presented in Table 12.

Table 12. IVNDM

	$\tilde{G}_1^N$	$\tilde{G}_2^N$	$\tilde{G}_3^N$
$\tilde{A}_1^N$	$\langle[0.1, 0.5], [0.1, 0.2], [0, 0]\rangle$	$\langle[0.1, 0.5], [0, 0.1], [0.2, 0.2]\rangle$	$\langle[0.1, 0.3], [0.1, 0.2], [0.3, 0.5]\rangle$
$\tilde{A}_2^N$	$\langle[0.1, 0.1], [0.05, 0.95], [0,0]\rangle$	$\langle[0.1, 0.7], [0.1, 0.2], [0.5, 0.15]\rangle$	$\langle[0.1, 0.8], [0.1, 0.1], [0.3, 0.3]\rangle$
$\tilde{A}_3^N$	$\langle[0.1, 0.3], [0.1, 0.1], [0.15,0.15]\rangle$	$\langle[0.1, 0.3], [0.1, 0.1], [0, 0.3]\rangle$	$\langle[0.1, 0.1], [0.1, 0.1], [0.2, 0.3]\rangle$
$\tilde{A}_4^N$	$\langle[0.11, 0.19], [0.05, 0.15], [0, 0.27]$	$\langle[0.1, 0.4], [0.1, 0.1], [0.15, 0.15$	$\langle[0.1, 0.4], [0.1, 0.2], [0.05, 0.15]\rangle$
$\tilde{A}_5^N$	$\langle[0.1, 0.6], [0, 0], [0.02, .08]\rangle$	$\langle[0.1, 0.5], [0, 0.1], [0.1, 0.1]\rangle$	$\langle[0.1, 0.3], [0.1, 0.2], [0.1, 0.1]\rangle$

Step 2: Using the Step 2 of Section 4.2, the obtained aggregated IVNN Ψ_i, for IVNSs Ψ_{ij}, for each $\tilde{A}_i^N$ using WAM, Eq. (3) and the WGM, Eq. (4) are shown in Table 13.

Table 13. IVNN using AO_{WA} and AO_{WG}

	AO_{WA}	AO_{WG}
Ψ_1	$\langle[0.1, 0.4407], [0, 0.1587], [0,0]\rangle$	$\langle[0.1, 0.4217], [0.0678, 0.1680], [0.1757, 0.2632]\rangle$
Ψ_2	$\langle[0.1, 0.6220], [0.0794, 0.2668], [0,0]\rangle$	$\langle[0.1, 0.3826], [0.0836, 0.6698], [0.1271, 0.1589]\rangle$
Ψ_3	$\langle[0.1, 0.2388], [0.1, 0.1], [0, 0.2381]\rangle$	$\langle[0.1, 0.2080], [0.1, 0.1], [0.1206, 0.2532]\rangle$
Ψ_4	$\langle[0.1033, 0.3369], [0.0794, 0.1442], [0, 0.1825]\rangle$	$\langle[0.1032, 0.3121], [0.0836, 0.1510], [0.0688, 0.1920]\rangle$
Ψ_5	$\langle[0.1, 0.4808], [0, 0], [0.0585, 0.0928]\rangle$	$\langle[0.1, 0.4481], [0.0345, 0.1037], [0.0741, 0.0934]\rangle$

Step 3: After Step 2, using the Step 3 of Section 4.2, the score value, $\omega_S(\tilde{A}^N)$ for each Ψ_i $(i = 1,2,\dots,m)$ are obtained by using Eq. (14) as follows:

Approach 1 (Using WAM). Aggregated IVNN Ψ_i $(i = 1,2,\dots,m)$ on using, WAM Eq. (3), the obtained score values $\omega_S(\tilde{A}_i^N)$ are as follows:

$\omega_S(\Psi_1) = 0.3214$, $\omega_S(\Psi_2) = 0.3096$, $\omega_S(\Psi_3) = 0.2484$, $\omega_S(\Psi_4) = 0.2680$, $\omega_S(\Psi_5) = 0.3717$.

Approach 2 (Using WGM). Aggregated IVNN Ψ_i $(i = 1,2,\dots,m)$ on using, WGM Eq. (4), the obtained score values $\omega_S(\tilde{A}_i^N)$ are as follows:

$\omega_S(\Psi_1) = 0.2651$, $\omega_S(\Psi_2) = 0.1067$, $\omega_S(\Psi_3) = 0.2312$, $\omega_S(\Psi_4) = 0.2535$, $\omega_S(\Psi_5) = 0.3203$.

Step 4: According to the obtained values of SF in Step 3, the following results are deduced, i.e.,

(i) For approach 1, the obtained ranking order of the alternatives is $\tilde{A}_5^N > \tilde{A}_1^N > \tilde{A}_2^N > \tilde{A}_4^N > \tilde{A}_3^N$, hence, $\tilde{A}_5^N$ is the best alternative according to the obtained score value $\omega_S(\tilde{A}^N)$ for each Ψ_i $(i = 1,2,\dots,m)$.

(ii) For approach 2, the obtained ranking order of the alternatives is $\tilde{A}_5^N > \tilde{A}_1^N > \tilde{A}_4^N > \tilde{A}_3^N > \tilde{A}_2^N$ hence $\tilde{A}_5^N$ is the best alternative according to the obtained score value $\omega_S(\tilde{A}^N)$ for each Ψ_i $(i = 1,2,\dots,m)$.

Furthermore, to validate the above-obtained results from the proposed method $\omega_S(\tilde{A}^N)$, a detailed comparative analysis of given data in Table 13 is done with the existing methods $\chi_S(\tilde{A}^N)$ [67], $\psi_S(\tilde{A}^N)$ [68], and the obtained values of their respective score functions are represented in Table 14 and Table 15, on applying both the approaches of aggregation i.e., WAM and WGM respectively.

According to the obtained values of the SF on using the existing methods $\chi_S(\tilde{A}^N)$ [67], $\psi_S(\tilde{A}^N)$ [68] and the proposed method $\omega_S(\tilde{A}^N)$ for WAM, the obtained ranking order of the first three alternatives is the same, i.e., $\tilde{A}_5^N > \tilde{A}_1^N > \tilde{A}_2^N$, as shown in Table 14, we conclude that $\tilde{A}_5^N$ is the best alternative.

Table 14. Comparative analysis of SF of various methods for WAM

	SVNNs	$\chi_S(\tilde{A}^N)$	$\psi_S(\tilde{A}^N)$	$\omega_S(\tilde{A}^N)$
$\omega_S(\Psi_1)$	$\langle[0.1, 0.4407], [0, 0.1587], [0,0]\rangle$	0.5558	0.5966	0.3214
$\omega_S(\Psi_2)$	$\langle[0.1, 0.6220], [0.0794, 0.2668], [0,0]\rangle$	0.5074	0.5121	0.3096
$\omega_S(\Psi_3)$	$\langle[0.1, 0.2388], [0.1, 0.1], [0, 0.2381]\rangle$	0.4252	0.3719	0.2484
$\omega_S(\Psi_4)$	$\langle[0.1033, 0.3369], [0.0794, 0.1442], [0, 0.1825]\rangle$	0.4526	0.4200	0.2680
$\omega_S(\Psi_5)$	$\langle[0.1, 0.4808], [0, 0], [0.0585, 0.0928]\rangle$	**0.6074**	**0.6754**	**0.3717**
	Ranking order	$\tilde{A}_5^N > \tilde{A}_1^N > \tilde{A}_2^N > \tilde{A}_4^N > \tilde{A}_3^N$	$\tilde{A}_5^N > \tilde{A}_1^N > \tilde{A}_2^N > \tilde{A}_4^N > \tilde{A}_3^N$	$\tilde{A}_5^N > \tilde{A}_1^N > \tilde{A}_2^N > \tilde{A}_4^N > \tilde{A}_3^N$

Similarly, according to the obtained values of the SF on using the existing methods $\chi_S(\tilde{A}^N)$ [67], $\psi_S(\tilde{A}^N)$ [68] and the proposed method $\omega_S(\tilde{A}^N)$ for WGM, the obtained ranking order suggests that $\tilde{A}_5^N$ is the best alternative by all the existing and the proposed method, as shown below in Table 15, hence, we conclude that $\tilde{A}_5^N$ is the best alternative.

Table 15. Comparative analysis of SF of various methods for WGM

	SVNNs	$\chi_S(\tilde{A}^N)$	$\psi_S(\tilde{A}^N)$	$\omega_S(\tilde{A}^N)$
$\omega_S(\Psi_1)$	$\langle[0.1, 0.4217], [0.0678, 0.1680], [0.1757, 0.2632]\rangle$	0.4028	0.3523	0.2651
$\omega_S(\Psi_2)$	$\langle[0.1, 0.3826], [0.0836, 0.6698], [0.1271, 0.1589]\rangle$	0.1724	-0.0292	0.1067
$\omega_S(\Psi_3)$	$\langle[0.1, 0.2080], [0.1, 0.1], [0.1206, 0.2532]\rangle$	0.3836	0.3068	0.2312

$\omega_S(\Psi_4)$	$\langle [0.1032, 0.3121], [0.0836, 0.1510], [0.0688, 0.1920] \rangle$	0.4213	0.3692	0.2535
$\omega_S(\Psi_5)$	$\langle [0.1, 0.4481], [0.0345, 0.1037], [0.0741, 0.0934] \rangle$	**0.5261**	**0.5428**	**0.3203**
	Ranking order	$\tilde{A}_5^N > \tilde{A}_4^N > \tilde{A}_1^N > \tilde{A}_3^N > \tilde{A}_2^N$	$\tilde{A}_5^N > \tilde{A}_4^N > \tilde{A}_1^N > \tilde{A}_3^N > \tilde{A}_2^N$	$\tilde{A}_5^N > \tilde{A}_1^N > \tilde{A}_4^N > \tilde{A}_3^N > \tilde{A}_2^N$

Hence, we can conclude that the proposed score function $\omega_S(\tilde{A}^N)$ is justified and is giving reasonable results on applying in real-world applications.

6. Discussion and Comparative Analysis

In this section, the SVNSs and IVNSs from the existing literature [67-69] are considered and solved by the existing and the proposed method, and the obtained solutions are presented in Table 16 given below. The obtained Table 16 argues well that the proposed methods are giving the same or the better results for all the considered problems reasonably and also it highlights, that the existing methods [67, 68] are behaving well for some particular SVNSs or IVNSs but fails under certain restrictions, then to deal with such SVNSs or IVNSs the proposed approaches works well and a desirable conclusion can be drawn respectively. Hence, it is claimed that the proposed SF and AF are better to evaluate MCDM problems and can be easily applied in solving real-life problems.

Table 16. A comparative analysis of SVNSs and IVNSs with various existing metric methods

SVNNs						
	$\sigma_S(\tilde{A}^N)$	$\sigma_A(\tilde{A}^N)$	$\tau_S(\tilde{A}^N)$	$\tau_A(\tilde{A}^N)$	$\varphi_S(\tilde{A}^N)$	$\varphi_A(\tilde{A}^N)$
$\tilde{A}_1^N = \langle 0.5,0.2,0.6 \rangle$ $\tilde{A}_2^N = \langle 0.6,0.4,0.2 \rangle$ (Adopted from [67])	0.25 0.3 $\tilde{A}_2^N > \tilde{A}_1^N$	-0.08 0.32 $\tilde{A}_2^N > \tilde{A}_1^N$	0.2750 0.2600 $\tilde{A}_1^N > \tilde{A}_2^N$	-0.5 -0.4 $\tilde{A}_2^N > \tilde{A}_1^N$	0.2778 0.25 $\tilde{A}_1^N > \tilde{A}_2^N$	-0.4 0.2 $\tilde{A}_2^N > \tilde{A}_1^N$
$\tilde{A}_1^N = \langle 0.5,0.2,0.6 \rangle$ $\tilde{A}_2^N = \langle 0.2,0.2,0.3 \rangle$ (Adopted from [68])	0.25 0.25 $\tilde{A}_1^N = \tilde{A}_2^N$	-0.08 -0.2 $\tilde{A}_1^N > \tilde{A}_2^N$	0.2750 0.1250 $\tilde{A}_1^N > \tilde{A}_2^N$	-0.5 -0.5 $\tilde{A}_1^N = \tilde{A}_2^N$	0.2778 0.1667 $\tilde{A}_1^N > \tilde{A}_2^N$	-0.4 0.2 $\tilde{A}_2^N > \tilde{A}_1^N$
$\tilde{A}_1^N = \langle 0.5,0.0,0.2 \rangle$ $\tilde{A}_2^N = \langle 0.4,0.0,0.1 \rangle$ (Adopted from [68])	0.65 0.65 $\tilde{A}_1^N = \tilde{A}_2^N$	0.3 0.3 $\tilde{A}_1^N = \tilde{A}_2^N$	0.6950 0.7250 $\tilde{A}_2^N > \tilde{A}_1^N$	0.3 0.3 $\tilde{A}_1^N = \tilde{A}_2^N$	0.5 0.433 $\tilde{A}_1^N > \tilde{A}_2^N$	0.6 0.8 $\tilde{A}_2^N > \tilde{A}_1^N$
$\tilde{A}_1^N = \langle 0.8,0.1,0.6 \rangle$ $\tilde{A}_2^N = \langle 0.8,0.2,0.4 \rangle$	0.5	0.24	0.5	0.0	0.8333	-0.3

(Adopted from [69])	0.5 $\widetilde{A}_1^N = \widetilde{A}_2^N$	0.44 $\widetilde{A}_2^N > \widetilde{A}_1^N$	0.5 $\widetilde{A}_1^N = \widetilde{A}_2^N$	0.0 $\widetilde{A}_1^N = \widetilde{A}_2^N$	0.625 $\widetilde{A}_1^N > \widetilde{A}_2^N$	0.0 $\widetilde{A}_2^N > \widetilde{A}_1^N$
$\widetilde{A}_1^N = \langle 0.1,0.0,0.1 \rangle$ $\widetilde{A}_2^N = \langle 0.3,0.0,0.3 \rangle$ (Adopted from [69])	0.5 0.5 $\widetilde{A}_1^N = \widetilde{A}_2^N$	0.0 0.0 $\widetilde{A}_1^N = \widetilde{A}_2^N$	0.5 0.5 $\widetilde{A}_1^N = \widetilde{A}_2^N$	0.0 0.0 $\widetilde{A}_1^N = \widetilde{A}_2^N$	0.2778 0.3571 $\widetilde{A}_2^N > \widetilde{A}_1^N$	0.8 0.4 $\widetilde{A}_1^N > \widetilde{A}_2^N$
IVNNs						
			$\chi_S(\widetilde{A}^N)$	$\psi_S(\widetilde{A}^N)$	$\omega_S(\widetilde{A}^N)$	$\chi_A(\widetilde{A}^N)$
$\widetilde{A}_1^N = \langle [0.6,0.4], [0.3, 0.1], [0.1, 0.3] \rangle$ $\widetilde{A}_2^N = \langle [0.1, 0.6], [0.2, 0.3], [0.1, 0.4] \rangle$ (Adopted from [67])			0.45 0.3 $\widetilde{A}_1^N > \widetilde{A}_2^N$	0.4375 0.22 $\widetilde{A}_1^N > \widetilde{A}_2^N$	0.3462 0.2143 $\widetilde{A}_1^N > \widetilde{A}_2^N$	0.26 0.005 $\widetilde{A}_1^N > \widetilde{A}_2^N$
$\widetilde{A}_1^N = \langle [0.4,0.6], [0.2, 0.3], [0.5, 0.7] \rangle$ $\widetilde{A}_2^N = \langle [0.2, 0.7], [0.1, 0.2], [0.1, 0.3] \rangle$ (Adopted from [68])			0.2 0.4750 $\widetilde{A}_2^N > \widetilde{A}_1^N$	0.23 0.4663 $\widetilde{A}_2^N > \widetilde{A}_1^N$	0.2222 0.3519 $\widetilde{A}_2^N > \widetilde{A}_1^N$	−0.0750 0.2050 $\widetilde{A}_2^N > \widetilde{A}_1^N$
$\widetilde{A}_1^N = \langle [0.1,0.7], [0.05, 0.15], [0.1, 0.3] \rangle$ $\widetilde{A}_2^N = \langle [0.2, 0.8], [0.05, 0.15], [0.2, 0.4] \rangle$ (Adopted from [69])			0.5 0.5 $\widetilde{A}_1^N = \widetilde{A}_2^N$	0.5 0.5 $\widetilde{A}_1^N = \widetilde{A}_2^N$	0.2857 0.4167 $\widetilde{A}_2^N > \widetilde{A}_1^N$	0.17 0.19 $\widetilde{A}_2^N > \widetilde{A}_1^N$
$\widetilde{A}_1^N = \langle [0.1,0.7], [0.1, 0.1], [0.1, 0.3] \rangle$ $\widetilde{A}_2^N = \langle [0.2, 0.8], [0.1, 0.1], [0.2, 0.4] \rangle$ (Adopted from [69])			0.5 0.5 $\widetilde{A}_1^N = \widetilde{A}_2^N$	0.5 0.5 $\widetilde{A}_1^N = \widetilde{A}_2^N$	0.3571 0.4167 $\widetilde{A}_2^N > \widetilde{A}_1^N$	0.16 0.18 $\widetilde{A}_2^N > \widetilde{A}_1^N$
$\widetilde{A}_1^N = \langle [0.1,0.7], [0.0, 0.2], [0.1, 0.3] \rangle$ $\widetilde{A}_2^N = \langle [0.2, 0.8], [0.0, 0.2], [0.2, 0.4] \rangle$ (Adopted from [69])			0.5 0.5 $\widetilde{A}_1^N = \widetilde{A}_2^N$	0.5 0.5 $\widetilde{A}_1^N = \widetilde{A}_2^N$	0.3571 0.4167 $\widetilde{A}_2^N > \widetilde{A}_1^N$	0.18 0.2 $\widetilde{A}_2^N > \widetilde{A}_1^N$
$\widetilde{A}_1^N = \langle [0.1,0.2], [0.0, 0.0], [0.1, 0.2] \rangle$ $\widetilde{A}_2^N = \langle [0.4, 0.5], [0.0, 0.0], [0.4, 0.5] \rangle$ (Adopted from [69])			0.5 0.5 $\widetilde{A}_1^N = \widetilde{A}_2^N$	0.5 0.5 $\widetilde{A}_1^N = \widetilde{A}_2^N$	0.2941 0.4545 $\widetilde{A}_2^N > \widetilde{A}_1^N$	0.0 0.0 $\widetilde{A}_1^N = \widetilde{A}_2^N$

7. Managerial insights

The study adopted DM when multiple criteria are involved and the decision-maker is supposed to find the best alternative among all the present alternatives on the basis of their corresponding criteria. The data involved indeterminacy and inconsistent sets of information, since neutrosophic sets deal with such data in the best possible manner, so the neutrosophic environment has been chosen to deal with the real-life problem. On using the proposed MCDM methodology all the available alternatives are evaluated under the neutrosophic environment, by the proposed SF and AF which lead to the best option available in the alternative based on their criteria. The application of the proposed MCDM methodology based on SF and AF provides a judicious solution to the decision-maker by considering all the available information in real-world applications in comparison to all the existing metric methods. Hence, the proposed MCDM methodology is more reliable in terms of its derived solutions.

8. Conclusions

In this paper, a new SF and AF for SVNSs and IVNSs are proposed, also, an MCDM method is developed based on the new ranking tools for SVNSs and IVNSs. In the proposed MCDM method, the score value of the aggregated SVNSs or IVNSs is obtained by applying the proposed SF or AF. According to the obtained results on applying the proposed SF or AF, the alternatives are ordered and the most desirable alternative i.e., the alternative with the highest value of SF or AF is chosen in the DM problem. To illustrate the efficiency and the validity of the proposed SF and AF for SVNSs and IVNSs a real-life application is solved successfully and the obtained results sync completely with the existing methods [67, 68]. Since neutrosophic sets are efficient enough to consider indeterminate and inconsistent information/data, hence, our proposed method would play an effective role in dealing with MCDM problems in several real-life applications like personnel selection, enterprises, signal processing, pattern recognition, medical diagnosis, engineering, management, DM, etc. having indeterminacy and inconsistent set of data. The only limitations of the proposed method would be that the data must be analyzed properly and all the restrictions should be followed in order to derive an accurate solution to the problem on applying the proposed SF and AF. Moreover, neutrosophic sets are still being in their prime have a lot to reveal concerning real-life applications. In the future, these proposed SF and AF for SVNSs and IVNSs will be expanded and will be generalized to the other domains of the neutrosophic sets like-refined neutrosophic sets, neutrosophic soft sets, neutrosophic cubic fuzzy sets, and their applications.

Compliance with Ethical Standards

Conflict of Interest

The authors declare that they do not have any financial or associative interest indicating a conflict of interest in about submitted work.

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