

3-10-2021

Compactness on Single-Valued Neutrosophic Ideal Topological Spaces

Fahad Alsharari

Florenentin Smarandache

Yaser Saber

Follow this and additional works at: https://digitalrepository.unm.edu/nss_journal

Recommended Citation

Alsharari, Fahad; Florenentin Smarandache; and Yaser Saber. "Compactness on Single-Valued Neutrosophic Ideal Topological Spaces." *Neutrosophic Sets and Systems* 41, 1 (2021).
https://digitalrepository.unm.edu/nss_journal/vol41/iss1/10

This Article is brought to you for free and open access by UNM Digital Repository. It has been accepted for inclusion in Neutrosophic Sets and Systems by an authorized editor of UNM Digital Repository. For more information, please contact disc@unm.edu.



Compactness on Single-Valued Neutrosophic Ideal Topological Spaces

Fahad Alsharari ^{1,*}, Florentin Smarandache ² and Yaser Saber ^{1,3}

¹ Department of Mathematics, College of Science and Human Studies, Hotat Sudair, Majmaah University, Majmaah 11952, Saudi Arabia; f.alsharari@mu.edu.sa

² Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA

³ Department of Mathematics, Faculty of Science Al-Azhar University, Assiut 71524, Egypt

* Correspondence: f.alsharari@mu.edu.sa

Abstract: In the current paper, particular achievements of single-valued neutrosophic continuity on a single-valued neutrosophic topological space $(\tilde{X}, \tilde{\tau}^{\tilde{\gamma}}, \tilde{\tau}^{\tilde{\eta}}, \tilde{\tau}^{\tilde{\mu}})$ are introduced. Some necessary implications between them are illustrated. The theories of r -single-valued neutrosophic compact, r -single-valued neutrosophic ideal compact, r -single-valued neutrosophic quasi H-closed and r -single-valued neutrosophic compact modulo an single-valued neutrosophic ideal $\tilde{J}$ are presented and investigated.

Keywords: single-valued neutrosophic (almost; weakly) continuous mapping; single-valued neutrosophic ideal (compact; quasi H-closed) and r -single-valued neutrosophic compact modulo.

1. Introduction

Using a fuzzy ideal $\tilde{J}$ defined on a fuzzy topological space (FTS) $(\tilde{X}, \tilde{\tau})$, a fuzzy ideal topological space (FITS) $(\tilde{X}, \tilde{\tau}, \tilde{J})$ is generated. It is a way of generalizing so many notions and results in $(\tilde{X}, \tilde{\tau})$. The main definition of fuzzy topology that is related to the results in this article was established by Šostak in [1]. The notion of fuzzy ideal was created in [2]. Tripathy et al. in [3 - 6] introduced different valuable research studies on (FITS) and gave several forms of fuzzy continuities. Saber and others [7 - 11] have considered several r -fuzzy compactnesses in (FITS) $(\tilde{X}, \tilde{\tau}, \tilde{J})$ and several types of fuzzy continuity.

Smarandache established the idea of the neutrosophic sets [12] in 1998. In terms of neutrosophic sets, there are a membership score ($\tilde{\gamma}$), an indeterminacy score ($\tilde{\eta}$) and a non-membership score ($\tilde{\mu}$) and a neutrosophic value is in the form $(\tilde{\gamma}, \tilde{\eta}, \tilde{\mu})$. In other meaning, in explaining an event or finding of a solution to a problem, a condition is handled according to its truth, not truth and resolution. Hence, the study of neutrosophic sets and neutrosophic logic are useful for decision-making applications in neutrosophic theories and led to too many researches and studies in the field as in [12-25]. It also gives the opportunity to others to establish some approaches in decision-making for neutrosophic theory as in [26-31]. Wang et al, [32] and Kim et al, [33] presented the theory of the neutrosophic equivalence relation single-valued. Single-valued neutrosophic

ideal ($\mathcal{SVNT}$) aspects in single-valued neutrosophic topological spaces ($\mathcal{SVNTS}$), have been introduced and considered by several authors from diverse viewpoints such as in [34-37].

In this research, we foreground the idea of r -single-valued neutrosophic (compact, ideal compact and quasi H-closed) in ($\mathcal{SVNTS}$) in the sense of Šostak. We are working on getting some of its important characteristics and results. Moreover, we investigate some properties of single-valued neutrosophic continuous mappings. Finally, some fascinating application of neutrosophic topology in reverse logistics arises could be found as in Abdel-Baset paper articles and others [38-41].

2. Preliminaries

Definition 2.1 [22] Suppose that $\tilde{\mathfrak{X}}$ is a non-empty set. We mean by a neutrosophic set (briefly, $\mathcal{NS}$) A the objects having the form

$$\mathcal{S} = \{(\omega, \tilde{\gamma}_S, \tilde{\eta}_S, \tilde{\mu}_S) : \omega \in \tilde{\mathfrak{X}}\}.$$

Anywhere $\tilde{\mu}_S$, $\tilde{\eta}_S$ and $\tilde{\gamma}_S$ indicate the degree of non-membership, the degree of indeterminacy, and the degree of membership, respectively of any element $\omega \in \tilde{\mathfrak{X}}$ to the set $\mathcal{S}$.

Definition 2.2 [32] Suppose that $\tilde{\mathfrak{X}}$ is a universal set. For $\forall \omega \in \tilde{\mathfrak{X}}$, $0 \leq \tilde{\gamma}_S(\omega) + \tilde{\eta}_S(\omega) + \tilde{\mu}_S(\omega) \leq 3$, by the meanings $\tilde{\gamma}_S: \mathcal{S} \rightarrow [0,1]$, $\tilde{\eta}_S: \mathcal{S} \rightarrow [0,1]$ and $\tilde{\mu}_S: \mathcal{S} \rightarrow [0,1]$, a single-valued neutrosophic set (briefly, $\mathcal{SVNS}$) on $\tilde{\mathfrak{X}}$ is defined by

$$\mathcal{S} = \{(\omega, \tilde{\gamma}_S, \tilde{\eta}_S, \tilde{\mu}_S) : \omega \in \tilde{\mathfrak{X}}\}.$$

Now, $\tilde{\mu}_S$, $\tilde{\eta}_S$ and $\tilde{\gamma}_S$ are the degrees of falsity, indeterminacy and trueness of $\omega \in \tilde{\mathfrak{X}}$, respectively. We will convey the set of all $\mathcal{SVNS}$ in $\mathcal{S}$ as $I^{\tilde{\mathfrak{X}}}$.

Definition 2.3 [32] The accompaniment of a $\mathcal{SVNS}$ $\mathcal{S}$ is indicated by $\mathcal{S}^c$ and is cleared by

$$\tilde{\gamma}_{\mathcal{S}^c}(\omega) = \tilde{\mu}_S(\omega), \quad \tilde{\eta}_{\mathcal{S}^c}(\omega) = 1 - \tilde{\eta}_S(\omega) \text{ and } \tilde{\mu}_{\mathcal{S}^c}(\omega) = \tilde{\gamma}_S(\omega).$$

for any $\omega \in \tilde{\mathfrak{X}}$,

Definition 2.4 [41] Let $\mathcal{S}, \mathcal{E} \in I^{\tilde{\mathfrak{X}}}$. Then,

1. $\mathcal{S} \subseteq \mathcal{E}$, if, for every $\omega \in \tilde{\mathfrak{X}}$,

$$\tilde{\gamma}_S(\omega) \leq \tilde{\gamma}_E(\omega), \quad \tilde{\eta}_S(\omega) \geq \tilde{\eta}_E(\omega), \quad \tilde{\mu}_S(\omega) \geq \tilde{\mu}_E(\omega)$$
2. $\mathcal{S} = \mathcal{E}$ if $\mathcal{S} \subseteq \mathcal{E}$ and $\mathcal{S} \supseteq \mathcal{E}$.
3. $\tilde{0} = \langle 0,1,1 \rangle$ and $\tilde{1} = \langle 1,0,0 \rangle$

Definition 2.5 [42] Let $\mathcal{S}, \mathcal{E} \in I^{\tilde{\mathfrak{X}}}$. Then,

1. $\mathcal{S} \cap \mathcal{E}$ is a $\mathcal{SVNS}$ in $\tilde{\mathfrak{X}}$ defined as:

$$\mathcal{S} \cap \mathcal{E} = (\tilde{\gamma}_S \cap \tilde{\gamma}_E, \tilde{\eta}_S \cup \tilde{\eta}_E, \tilde{\mu}_S \cup \tilde{\mu}_E).$$

Where, $(\tilde{\mu}_S \cup \tilde{\mu}_E)(\omega) = \tilde{\mu}_S(\omega) \cup \tilde{\mu}_E(\omega)$ and $(\tilde{\gamma}_S \cap \tilde{\gamma}_E)(\omega) = \tilde{\gamma}_S(\omega) \cap \tilde{\gamma}_E(\omega)$, for all $\omega \in \tilde{\mathfrak{X}}$,

1. $\mathcal{S} \cup \mathcal{E}$ is an $\mathcal{SVNS}$ on $\tilde{\mathfrak{X}}$ defined as:

$$\mathcal{S} \cup \mathcal{E} = (\tilde{\gamma}_{\mathcal{S}} \cup \tilde{\gamma}_{\mathcal{E}}, \tilde{\eta}_{\mathcal{S}} \cap \tilde{\eta}_{\mathcal{E}}, \tilde{\mu}_{\mathcal{S}} \cap \tilde{\mu}_{\mathcal{E}}).$$

Definition 2.6 [21] Suppose that $\tilde{\mathfrak{X}}$ is a nonempty set and $\mathcal{S} \in I^{\tilde{\mathfrak{X}}}$ is having the form $\mathcal{S} = \{(\omega, \tilde{\gamma}_{\mathcal{S}}, \tilde{\eta}_{\mathcal{S}}, \tilde{\mu}_{\mathcal{S}}): \omega \in \tilde{\mathfrak{X}}\}$ on $\tilde{\mathfrak{X}}$. Then,

1. $(\bigcap_{j \in \Delta} \mathcal{S}_j)(\omega) = \left(\bigcap_{j \in \Delta} \tilde{\gamma}_{\mathcal{S}_j}(\omega), \bigcup_{j \in \Delta} \tilde{\eta}_{\mathcal{S}_j}(\omega), \bigcup_{j \in \Delta} \tilde{\mu}_{\mathcal{S}_j}(\omega) \right),$
2. $(\bigcup_{j \in \Delta} \mathcal{S}_j)(\omega) = \left(\bigcup_{j \in \Delta} \tilde{\gamma}_{\mathcal{S}_j}(\omega), \bigcap_{j \in \Delta} \tilde{\eta}_{\mathcal{S}_j}(\omega), \bigcap_{j \in \Delta} \tilde{\mu}_{\mathcal{S}_j}(\omega) \right).$

Definition 2.7 [34] Let $s, t, k \in I_0$ and $s + t + k \leq 3$. A single-valued neutrosophic point ($\mathcal{SVNP}$) $x_{s,t,k}$ of $\tilde{\mathfrak{X}}$ is the $\mathcal{SVNS}$ in $I^{\tilde{\mathfrak{X}}}$ for every $\omega \in \mathcal{S}$, defined by

$$x_{s,t,k}(\omega) = \begin{cases} (s, t, k), & \text{if } x = \omega, \\ (0, 1, 1), & \text{if } x \neq \omega. \end{cases}$$

A $\mathcal{SVNP}$ $x_{s,t,k}$ is supposed to belong to a $\mathcal{SVNS}$ $\mathcal{S} = \{(\omega, \tilde{\gamma}_{\mathcal{S}}, \tilde{\eta}_{\mathcal{S}}, \tilde{\mu}_{\mathcal{S}}): \omega \in \tilde{\mathfrak{X}}\} \in I^{\tilde{\mathfrak{X}}}$, (notion: $x_{s,t,k} \in \mathcal{S}$ iff $s < \tilde{\gamma}_{\mathcal{S}}, t \geq \tilde{\eta}_{\mathcal{S}}$ and $k \geq \tilde{\mu}_{\mathcal{S}}$), and the set off all $\mathcal{SVNP}$ in $\tilde{\mathfrak{X}}$ indicated by $\mathcal{SVNP}(\tilde{\mathfrak{X}})$. $x_{s,t,k} \in \mathcal{SVNP}(\tilde{\mathfrak{X}})$ quasi-coincident with a $\mathcal{SVNS}$ $\mathcal{S} \in I^{\tilde{\mathfrak{X}}}$ denoted by $x_{s,t,k} q \mathcal{S}$, if

$$s + \tilde{\gamma}_{\mathcal{S}} > 1, t + \tilde{\eta}_{\mathcal{S}} \leq 1, k + \tilde{\mu}_{\mathcal{S}} \leq 1.$$

For every $\mathcal{S}, \mathcal{E} \in I^{\tilde{\mathfrak{X}}}$ $\mathcal{S}$ is quasi-coincident with $\mathcal{E}$ indicated by $\mathcal{S} q \mathcal{E}$, if there exists $x_{s,t,k} \in I^{\tilde{\mathfrak{X}}}$ s.t

$$\tilde{\gamma}_{\mathcal{E}} + \tilde{\gamma}_{\mathcal{S}} > 1, \tilde{\eta}_{\mathcal{E}} + \tilde{\eta}_{\mathcal{S}} \leq 1 \text{ and } \tilde{\mu}_{\mathcal{E}} + \tilde{\mu}_{\mathcal{S}} \leq 1.$$

Definition 2.8 [25] Let $\tilde{\tau}^{\tilde{\gamma}}, \tilde{\tau}^{\tilde{\eta}}, \tilde{\tau}^{\tilde{\mu}}: I^{\tilde{\mathfrak{X}}} \rightarrow I$ be mappings satisfying the following conditions:

1. $\tilde{\tau}^{\tilde{\gamma}}(\underline{0}) = \tilde{\tau}^{\tilde{\gamma}}(\underline{1}) = 1$ and $\tilde{\tau}^{\tilde{\eta}}(\underline{0}) = \tilde{\tau}^{\tilde{\eta}}(\underline{1}) = \tilde{\tau}^{\tilde{\mu}}(\underline{0}) = \tilde{\tau}^{\tilde{\mu}}(\underline{1}) = 0,$
2. $\tau^{\tilde{\gamma}}(\mathcal{S} \cap \mathcal{E}) \geq \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}) \cap \tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}), \tilde{\tau}^{\tilde{\eta}}(\mathcal{S} \cap \mathcal{E}) \leq \tau^{\tilde{\eta}}(\mathcal{S}) \cup \tilde{\tau}^{\tilde{\eta}}(\mathcal{E})$ and $\tilde{\tau}^{\tilde{\mu}}(\mathcal{S} \cap \mathcal{E}) \leq \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}) \cup \tilde{\tau}^{\tilde{\mu}}(\mathcal{E})$, for every $\mathcal{S}, \mathcal{E} \in I^{\tilde{\mathfrak{X}}}$,
3. $\tilde{\tau}^{\tilde{\gamma}}(\bigcup_{j \in \Gamma} \mathcal{S}_j) \geq \bigcap_{j \in \Gamma} \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j), \tilde{\tau}^{\tilde{\eta}}(\bigcup_{j \in \Gamma} \mathcal{S}_j) \leq \bigcup_{j \in \Gamma} \tau^{\tilde{\eta}}(\mathcal{S}_j)$ and $\tilde{\tau}^{\tilde{\mu}}(\bigcup_{j \in \Gamma} \mathcal{S}_j) \leq \bigcup_{j \in \Gamma} \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j)$, for every $\{\mathcal{S}_j, j \in \Gamma\} \in I^{\tilde{\mathfrak{X}}}$.

Then $(\tilde{\tau}^{\tilde{\gamma}}, \tilde{\tau}^{\tilde{\eta}}, \tilde{\tau}^{\tilde{\mu}})$ is called single valued neutrosophic topology $\mathcal{SVNT}$. Usually, we will write $\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}$ for $(\tilde{\tau}^{\tilde{\gamma}}, \tilde{\tau}^{\tilde{\eta}}, \tilde{\tau}^{\tilde{\mu}})$ and it will cause no indistinctness.

Definition 2.9 [34] Let $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be an $\mathcal{SVNTS}$. Then, for all $\mathcal{S} \in I^{\tilde{\mathfrak{X}}}$ and $r \in I_0$, the single valued neutrosophic (closure and interior) of $\mathcal{S}$ are define by:

$$C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r) = \bigcap \{ \mathcal{E} \in I^{\tilde{\mathfrak{X}}}: \mathcal{S} \leq \mathcal{E}, \quad \tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}^c) \geq r, \quad \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}^c) \leq 1 - r, \quad \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}^c) \leq 1 - r \}$$

$$int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r) = \bigcup \{ \mathcal{E} \in I^{\tilde{\mathfrak{X}}}: \mathcal{S} \geq \mathcal{E}, \quad \tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}) \geq r, \quad \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}) \leq 1 - r, \quad \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}) \leq 1 - r \}.$$

Definition 2.10 [34] A mapping $\tilde{\mathcal{J}}^{\tilde{\gamma}}, \tilde{\mathcal{J}}^{\tilde{\eta}}, \tilde{\mathcal{J}}^{\tilde{\mu}}: I^{\tilde{\mathfrak{X}}} \rightarrow I$ is said to be $\mathcal{SVNI}$ on $\tilde{\mathfrak{X}}$ if it satisfies the next three conditions for $\mathcal{S}, \mathcal{E} \in I^{\tilde{\mathfrak{X}}}$:

1. $\tilde{\mathcal{J}}^{\tilde{\eta}}(\underline{0}) = \tilde{\mathcal{J}}^{\tilde{\mu}}(\underline{0}) = 0, \tilde{\mathcal{J}}^{\tilde{\gamma}}(\underline{0}) = 1,$
2. If $\mathcal{S} \leq \mathcal{E}$ then $\tilde{\mathcal{J}}^{\tilde{\eta}}(\mathcal{E}) \geq \tilde{\mathcal{J}}^{\tilde{\eta}}(\mathcal{S}), \tilde{\mathcal{J}}^{\tilde{\mu}}(\mathcal{E}) \geq \tilde{\mathcal{J}}^{\tilde{\mu}}(\mathcal{S})$ and $\tilde{\mathcal{J}}^{\tilde{\gamma}}(\mathcal{E}) \leq \tilde{\mathcal{J}}^{\tilde{\gamma}}(\mathcal{S}).$
3. $\tilde{\mathcal{J}}^{\tilde{\eta}}(\mathcal{S} \cup \mathcal{E}) \leq \tilde{\mathcal{J}}^{\tilde{\eta}}(\mathcal{E}) \cup \tilde{\mathcal{J}}^{\tilde{\eta}}(\mathcal{S}), \tilde{\mathcal{J}}^{\tilde{\mu}}(\mathcal{S} \cup \mathcal{E}) \leq \tilde{\mathcal{J}}^{\tilde{\mu}}(\mathcal{S}) \cup \tilde{\mathcal{J}}^{\tilde{\mu}}(\mathcal{E})$ and $\tilde{\mathcal{J}}^{\tilde{\gamma}}(\mathcal{S} \cup \mathcal{E}) \geq \tilde{\mathcal{J}}^{\tilde{\gamma}}(\mathcal{S}) \cap \tilde{\mathcal{J}}^{\tilde{\gamma}}(\mathcal{E}).$

Then, $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathcal{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is said to be a single-valued neutrosophic ideal topological space ($\mathcal{SVNITS}$).

Definition 2.12 [36] A mapping $f: (\mathfrak{X}_1, \tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}) \rightarrow (\mathfrak{X}_2, \tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ from an $\mathcal{SVNTS}$ $(\mathfrak{X}_1, \tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ into another $\mathcal{SVNTS}$ $(\mathfrak{X}_2, \tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is said to be single-valued neutrosophic continuous (briefly, $\mathcal{SVN}$ -continuous) if and only if $\tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{S}) \leq \tilde{\tau}_1^{\tilde{\gamma}}(f^{-1}(\mathcal{S}))$, $\tilde{\tau}_2^{\tilde{\eta}}(\mathcal{S}) \geq \tilde{\tau}_1^{\tilde{\eta}}(f^{-1}(\mathcal{S}))$ and $\tilde{\tau}_2^{\tilde{\mu}}(\mathcal{S}) \geq \tilde{\tau}_1^{\tilde{\mu}}(f^{-1}(\mathcal{S}))$, for every $\mathcal{S} \in I^{\mathfrak{X}_2}$.

3. Single-Valued Neutrosophic (almost, weakly) Continuous Mappings

This section is dedicated to present the concepts of the single-valued neutrosophic (almost and weakly) mappings (briefly $\mathcal{SVN}$ – almost continuous, $\mathcal{SVN}$ – *weakly continuous*) mappings, respectively. It is also devoted to mark out the concepts of single-valued neutrosophic (preopen, regular-open) sets (briefly, r – $\mathcal{SVNPO}$, r – $\mathcal{SVNRO}$) sets, respectively.

Definition 3.1. Let $(\mathfrak{X}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be an $\mathcal{SVNTS}$ and $r \in I_0$. Then, $\mathcal{S} \in I^{\mathfrak{X}}$ is said to be:

1. r – $\mathcal{SVNPO}$ set iff $\mathcal{S} \leq \text{int}_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{C}_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r), r)$,
2. r – $\mathcal{SVNRO}$ set if $\mathcal{S} = \text{int}_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{C}_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r), r)$.

The complement of r – $\mathcal{SVNPO}$ (resp, r – $\mathcal{SVNRO}$) are said to be r – $\mathcal{SVNPC}$ (resp, r – $\mathcal{SVNRC}$), respectively.

Remark 3.2. Let $(\mathfrak{X}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be an $\mathcal{SVNTS}$ and $r \in I_0$, if $\mathcal{S}$ is an r – $\mathcal{SVNRO}$ set, then $\mathcal{S}$ is r – $\mathcal{SVNPO}$.

Example 3.3. Let $\mathfrak{X} = \{a, b\}$. Define $\mathcal{E}_1, \mathcal{E}_2 \in I^{\mathfrak{X}}$ as follows:

$$\mathcal{E}_1 = \langle (0 \cdot 5, 0.4, 0 \cdot 5), (0 \cdot 5, 0.4, 0 \cdot 5), (0 \cdot 5, 0.5, 0 \cdot 5) \rangle, \mathcal{E}_2 = \langle (0 \cdot 4, 0 \cdot 4, 0.4), (0 \cdot 5, 0 \cdot 4, 0.4), (0 \cdot 5, 0 \cdot 5, 0.4) \rangle.$$

Define $\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}} : I^{\mathfrak{X}} \rightarrow I$ as follows:

$$\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}) = \begin{cases} 1, & \text{if } \mathcal{S} = \tilde{0}, \\ 1, & \text{if } \mathcal{S} = \tilde{1}, \\ \frac{1}{2}, & \text{if } \mathcal{S} = \mathcal{E}_1, \\ 0, & \text{otherwise} \end{cases} \quad \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}) = \begin{cases} 0, & \text{if } \mathcal{S} = \tilde{0}, \\ 0, & \text{if } \mathcal{S} = \tilde{1}, \\ \frac{1}{2}, & \text{if } \mathcal{S} = \{\mathcal{E}_1, \mathcal{E}_2\}, \\ 1, & \text{otherwise} \end{cases}$$

$$\tilde{\tau}^{\tilde{\mu}}(\mathcal{S}) = \begin{cases} 0, & \text{if } \mathcal{S} = \tilde{0}, \\ 0, & \text{if } \mathcal{S} = \tilde{1}, \\ \frac{1}{2}, & \text{if } \mathcal{S} = \{\mathcal{E}_1, \mathcal{E}_2\}, \\ 1, & \text{otherwise} \end{cases}$$

Let, $\mathcal{E}_3 = \{ \langle \omega, (0 \cdot 5, 0.5, 0 \cdot 1), (0 \cdot 6, 0.3, 0 \cdot 1), (0 \cdot 6, 0.3, 0 \cdot 1) \rangle : \omega \in \mathfrak{X} \}$. Then, $\mathcal{E}_3$ is $\frac{1}{2}$ – $\mathcal{SVNPO}$ set but it is not $\frac{1}{2}$ – $\mathcal{SVNRO}$ set because, $\mathcal{E}_3 \neq \text{int}_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{C}_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_3, \frac{1}{2}), \frac{1}{2}) = \tilde{1}$.

Lemma 3.4. Let $\mathcal{S}$ be an $\mathcal{SVNS}$ in an $\mathcal{SVNTS}$ $(\mathfrak{X}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$. Then, for each $r \in I_0$.

1. If $\mathcal{S}$ is r – $\mathcal{SVNRO}$ set (resp, r – $\mathcal{SVNRC}$ set), then $[\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}) \leq 1 - r]$ (resp, $[\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1 - r]$),
2. $\mathcal{S}$ is r – $\mathcal{SVNRO}$ set if and only if $\mathcal{S}^c$ is r – $\mathcal{SVNRC}$ set.

Proof. Follows directly from Definition 3.1.

Lemma 3.5. Let $(\mathfrak{X}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be an $\mathcal{SVNTS}$. Then,

1. the union of two r – $\mathcal{SVNRC}$ sets is r – $\mathcal{SVNRC}$,
2. the intersection of two r – $\mathcal{SVNRO}$ sets, is r – $\mathcal{SVNRO}$.

Proof. (1) Let $\mathcal{S}, \mathcal{E}$ be any two r -SVNRC sets. By Lemma 3.4, $[\tilde{\tau}^{\tilde{Y}}(\mathcal{S}^c) \geq r, \tilde{\tau}^{\tilde{N}}(\mathcal{S}^c) \leq 1-r, \tilde{\tau}^{\tilde{M}}(\mathcal{S}^c) \leq 1-r]$ and $[\tilde{\tau}^{\tilde{Y}}(\mathcal{E}^c) \geq r, \tilde{\tau}^{\tilde{N}}(\mathcal{E}^c) \leq 1-r, \tilde{\tau}^{\tilde{M}}(\mathcal{E}^c) \leq 1-r]$. Then,

$$\tilde{\tau}^{*\tilde{Y}}(\mathcal{S} \cup \mathcal{E}) \geq \tilde{\tau}^{*\tilde{Y}}(\mathcal{S}) \cap \tilde{\tau}^{*\tilde{Y}}(\mathcal{E}), \tilde{\tau}^{*\tilde{N}}(\mathcal{S} \cup \mathcal{E}) \leq \tilde{\tau}^{*\tilde{N}}(\mathcal{S}) \cup \tilde{\tau}^{*\tilde{N}}(\mathcal{E}), \tilde{\tau}^{*\tilde{M}}(\mathcal{S} \cup \mathcal{E}) \leq \tilde{\tau}^{*\tilde{M}}(\mathcal{S}) \cup \tilde{\tau}^{*\tilde{M}}(\mathcal{E}),$$

but $\text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S} \cup \mathcal{E}, r) \leq \mathcal{S} \cup \mathcal{E}$, this suggests that

$$C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S} \cup \mathcal{E}, r), r) \leq C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S} \cup \mathcal{E}, r) = \mathcal{S} \cup \mathcal{E}.$$

Now,

$$\mathcal{S} = C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S}, r), r) \leq C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S} \cup \mathcal{E}, r), r),$$

and

$$\mathcal{E} = C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{E}, r), r) \leq C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S} \cup \mathcal{E}, r), r).$$

Thus, $\mathcal{S} \cup \mathcal{E} \leq C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S} \cup \mathcal{E}, r), r)$. So, $\mathcal{S} \cup \mathcal{E} = C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S} \cup \mathcal{E}, r), r)$. Hence, $\mathcal{S} \cup \mathcal{E}$ is r -SVNRC set.

(2) It can be ascertained by the same method.

Theorem 3.6. Let $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}})$ be an SVNTS. Then,

1. If $\mathcal{S} \in I^{\tilde{\mathfrak{X}}}$ s.t. $\tilde{\tau}^{\tilde{Y}}(\mathcal{S}^c) \geq r, \tilde{\tau}^{\tilde{N}}(\mathcal{S}^c) \leq 1-r, \tilde{\tau}^{\tilde{M}}(\mathcal{S}^c) \leq 1-r$, then, $\text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S}, r)$ is r -SVNRO set,
2. If $\mathcal{S} \in I^{\tilde{\mathfrak{X}}}$ s.t. $\tilde{\tau}^{\tilde{Y}}(\mathcal{S}) \geq r, \tilde{\tau}^{\tilde{N}}(\mathcal{S}) \leq 1-r$ and $\tilde{\tau}^{\tilde{M}}(\mathcal{S}) \leq 1-r$, then, $C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S}, r)$ is r -SVNRC set.

Proof. (1) Suppose that $\mathcal{S} \in I^{\tilde{\mathfrak{X}}}$ such that, $\tilde{\tau}^{\tilde{Y}}(\mathcal{S}^c) \geq r, \tilde{\tau}^{\tilde{N}}(\mathcal{S}^c) \leq 1-r, \tilde{\tau}^{\tilde{M}}(\mathcal{S}^c) \leq 1-r$. Clearly,

$$\text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S}, r) \leq \text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S}, r), r),$$

this denotes that, $\text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S}, r) \leq \text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S}, r), r), r)$. Now, since,

$$\tilde{\tau}^{\tilde{Y}}(\mathcal{S}^c) \geq r, \tilde{\tau}^{\tilde{N}}(\mathcal{S}^c) \leq 1-r, \tilde{\tau}^{\tilde{M}}(\mathcal{S}^c) \leq 1-r,$$

then $C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S}, r), r) \leq \mathcal{S}$; therefore,

$$\text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S}, r) \geq \text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S}, r), r), r).$$

Then, $\text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S}, r) = \text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S}, r), r), r)$. Hence, $\text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S}, r)$ is r -SVNRO set.

(2) Similar to the proof of (1).

Definition 3.7. A mapping $f: (\tilde{\mathfrak{X}}_1, \tilde{\tau}_1^{\tilde{Y}\tilde{N}\tilde{M}}) \rightarrow (\tilde{\mathfrak{X}}_2, \tilde{\tau}_2^{\tilde{Y}\tilde{N}\tilde{M}})$ from an SVNTS $(\tilde{\mathfrak{X}}_1, \tilde{\tau}_1^{\tilde{Y}\tilde{N}\tilde{M}})$ into another SVNTS $(\tilde{\mathfrak{X}}_2, \tilde{\tau}_2^{\tilde{Y}\tilde{N}\tilde{M}})$ is called:

1. SVN – almost continuous iff $\tilde{\tau}_1^{\tilde{Y}}(f^{-1}(\mathcal{S})) \geq r, \tilde{\tau}_1^{\tilde{N}}(f^{-1}(\mathcal{S})) \leq 1-r, \tilde{\tau}_1^{\tilde{M}}(f^{-1}(\mathcal{S})) \leq 1-r$, for each r -SVNRO set $\mathcal{S}$ of $\tilde{\mathfrak{X}}_2$,
2. SVN – weakly continuous iff $\tilde{\tau}_2^{\tilde{Y}}(\mathcal{S}) \geq r, \tilde{\tau}_2^{\tilde{N}}(\mathcal{S}) \leq 1-r$ and $\tilde{\tau}_2^{\tilde{M}}(\mathcal{S}) \leq 1-r$, implies $\tilde{\tau}_1^{\tilde{Y}}(f^{-1}(\mathcal{S})) \geq r, \tilde{\tau}_1^{\tilde{N}}(f^{-1}(\mathcal{S})) \leq 1-r, \tilde{\tau}_1^{\tilde{M}}(f^{-1}(\mathcal{S})) \leq 1-r$, for each $\mathcal{S} \in I^{\tilde{\mathfrak{X}}_2}$.

Remark 3.8. From Definition 3.7, it is clear that the next implications are correct for $r \in I_0$:

SVN – almost continuous mapping

$\Uparrow$

SVN – continuous mapping

$\Downarrow$

SVN – weakly continuous mapping

However, the one-sided suggestions are not correct in general, as presented by the next example.

Example 3.9. Suppose that $\tilde{\mathfrak{X}} = \{a, b, c\}$. Define $\mathcal{E}_1, \mathcal{E}_2 \in I^{\tilde{\mathfrak{X}}}$ as follows:

$$\mathcal{E}_1 = \langle (0 \cdot 5, 0.4, 0 \cdot 5), (0 \cdot 5, 0.4, 0 \cdot 5), (0 \cdot 5, 0.5, 0 \cdot 5) \rangle, \quad \mathcal{E}_2 = \langle (0 \cdot 5, 0 \cdot 4, 0.4), (0 \cdot 5, 0 \cdot 4, 0.4), (0 \cdot 5, 0 \cdot 5, .4) \rangle,$$

$$\mathcal{E}_3 = \langle (0 \cdot 3, 0.6, 0 \cdot 5), (0 \cdot 3, 0.6, 0 \cdot 5), 0 \cdot 3, 0.6, 0 \cdot 5 \rangle, \quad \mathcal{E}_4 = \langle (0 \cdot 4, 0 \cdot 4, 0.4), (0 \cdot 5, 0 \cdot 4, 0.4), (0 \cdot 5, 0 \cdot 5, .4) \rangle.$$

We define an $\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}} : I^{\tilde{\mathfrak{X}}} \rightarrow I$ as follows:

$$\tilde{\tau}_1^{\tilde{\gamma}}(\mathcal{S}) = \begin{cases} 1, & \text{if } \mathcal{S} = \tilde{0}, \\ 1, & \text{if } \mathcal{S} = \tilde{1}, \\ \frac{1}{2}, & \text{if } \mathcal{S} = \mathcal{E}_2, \\ 0, & \text{otherwise} \end{cases} \quad \tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{S}) = \begin{cases} 1, & \text{if } \mathcal{S} = \tilde{0}, \\ 1, & \text{if } \mathcal{S} = \tilde{1}, \\ \frac{1}{2}, & \text{if } \mathcal{S} = \{\mathcal{E}_2, \mathcal{E}_4\}, \\ 0, & \text{otherwise} \end{cases}$$

$$\tilde{\tau}_1^{\tilde{\eta}}(\mathcal{S}) = \begin{cases} 0, & \text{if } \mathcal{S} = \tilde{0}, \\ 0, & \text{if } \mathcal{S} = \tilde{1}, \\ \frac{1}{2}, & \text{if } \mathcal{S} = \{\mathcal{E}_1, \mathcal{E}_2\}, \\ 1, & \text{otherwise} \end{cases} \quad \tilde{\tau}_2^{\tilde{\eta}}(\mathcal{S}) = \begin{cases} 0, & \text{if } \mathcal{S} = \tilde{0}, \\ 0, & \text{if } \mathcal{S} = \tilde{1}, \\ \frac{1}{2}, & \text{if } \mathcal{S} = \{\mathcal{E}_2, \mathcal{E}_4\}, \\ 1, & \text{otherwise} \end{cases}$$

$$\tilde{\tau}_1^{\tilde{\mu}}(\mathcal{S}) = \begin{cases} 0, & \text{if } \mathcal{S} = \tilde{0}, \\ 0, & \text{if } \mathcal{S} = \tilde{1}, \\ \frac{1}{2}, & \text{if } \mathcal{S} = \{\mathcal{E}_2, \mathcal{E}_3\}, \\ 1, & \text{otherwise} \end{cases} \quad \tilde{\tau}_2^{\tilde{\mu}}(\mathcal{S}) = \begin{cases} 0, & \text{if } \mathcal{S} = \tilde{0}, \\ 0, & \text{if } \mathcal{S} = \tilde{1}, \\ \frac{1}{2}, & \text{if } \mathcal{S} = \{\mathcal{E}_2, \mathcal{E}_4\}, \\ 1, & \text{otherwise} \end{cases}$$

Then, the identity mapping, $f: (\tilde{\mathfrak{X}}_1, \tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}) \rightarrow (\tilde{\mathfrak{X}}_2, \tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is SVN – almost continuous, but it is not SVN – continuou. Since, $\tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{E}_4) = \frac{1}{2}$ and $\mathcal{E}_4$ is not $\frac{1}{2}$ – $SVNO$ set in $\tilde{\mathfrak{X}}_1$, because, $\tilde{\tau}_1^{\tilde{\gamma}}(f^{-1}(\mathcal{E}_4)) = 0 \not\geq \frac{1}{2}$, $\tilde{\tau}_1^{\tilde{\eta}}(f^{-1}(\mathcal{E}_4)) = 1 \not\leq \frac{1}{2}$ and $\tilde{\tau}_1^{\tilde{\mu}}(f^{-1}(\mathcal{E}_4)) = 1 \not\leq \frac{1}{2}$. Hence, $[\tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{E}_4) = \frac{1}{2} \not\leq 0 = \tilde{\tau}_1^{\tilde{\gamma}}(f^{-1}(\mathcal{E}_4))$, $\tilde{\tau}_2^{\tilde{\eta}}(\mathcal{E}_4) = \frac{1}{2} \not\geq 1 = \tilde{\tau}_1^{\tilde{\eta}}(f^{-1}(\mathcal{E}_4))$, $\tilde{\tau}_2^{\tilde{\mu}}(\mathcal{E}_4) = \frac{1}{2} \not\geq 1 = \tilde{\tau}_1^{\tilde{\mu}}(f^{-1}(\mathcal{E}_4))]$.

Theorem 3.10. Let $f: (\tilde{\mathfrak{X}}_1, \tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}) \rightarrow (\tilde{\mathfrak{X}}_2, \tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be a mapping from an $SVNTS$ $(\tilde{\mathfrak{X}}_1, \tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ into another $SVNTS$ $(\tilde{\mathfrak{X}}_2, \tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$. Then the next statements are equivalent:

1. f is SVN – almost continuous,
2. $\tilde{\tau}_1^{\tilde{\gamma}}((f^{-1}(\mathcal{S}))^c) \geq r$, $\tilde{\tau}_1^{\tilde{\eta}}((f^{-1}(\mathcal{S}))^c) \leq 1 - r$, $\tilde{\tau}_1^{\tilde{\mu}}((f^{-1}(\mathcal{S}))^c) \leq 1 - r$, for any r – $SVNRC$ set $\mathcal{S}$ of $\tilde{\mathfrak{X}}_2$,
3. $f^{-1}(\mathcal{S}) \leq \text{int}_{\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(f^{-1}(\text{int}_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r), r)), r)$, for any $\mathcal{S}$ of $\tilde{\mathfrak{X}}_2$ such that $\tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{S}) \geq r$, $\tilde{\tau}_2^{\tilde{\eta}}(\mathcal{S}) \leq 1 - r$ and $\tilde{\tau}_2^{\tilde{\mu}}(\mathcal{S}) \leq 1 - r$,
4. $C_{\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(f^{-1}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\text{int}_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r), r)), r) \leq f^{-1}(\mathcal{S})$, for any $\mathcal{S}$ of $\tilde{\mathfrak{X}}_2$ such that $\tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{S}) \geq r$, $\tilde{\tau}_2^{\tilde{\eta}}(\mathcal{S}) \leq 1 - r$ and $\tilde{\tau}_2^{\tilde{\mu}}(\mathcal{S}) \leq 1 - r$.

Proof. (1) $\Rightarrow$ (2). Let $\mathcal{S}$ be an r – $SVNRC$ set of $\tilde{\mathfrak{X}}_2$. Then by Lemma 3.4, $\mathcal{S}^c$ is r – $SVNRO$ set in $\tilde{\mathfrak{X}}_2$. By (1), we obtain

$$\tilde{\tau}_1^{\tilde{\gamma}}(f^{-1}(\mathcal{S}^c)) = \tilde{\tau}_1^{\tilde{\gamma}}((f^{-1}(\mathcal{S}))^c) \geq r, \quad \tilde{\tau}_1^{\tilde{\eta}}(f^{-1}(\mathcal{S}^c)) = \tilde{\tau}_1^{\tilde{\eta}}((f^{-1}(\mathcal{S}))^c) \leq 1 - r,$$

$$\tilde{\tau}_1^{\tilde{\mu}}(f^{-1}(\mathcal{S}^c)) = \tilde{\tau}_1^{\tilde{\mu}}((f^{-1}(\mathcal{S}))^c) \leq 1 - r.$$

(2) $\Rightarrow$ (1). It is analogous to the proof of (1) $\Rightarrow$ (2).

(1) $\Rightarrow$ (3). Since, $[\tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{S}) \geq r$, $\tilde{\tau}_2^{\tilde{\eta}}(\mathcal{S}) \leq 1 - r$, $\tilde{\tau}_2^{\tilde{\mu}}(\mathcal{S}) \leq 1 - r]$, then, $\mathcal{S} = \text{int}_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r) \leq \text{int}_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r), r)$,

and hence, $f^{-1}(\mathcal{S}) = f^{-1}(\text{int}_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r), r))$, since

$$\tilde{\tau}_2^{\tilde{Y}}([C_{\tilde{\tau}_2^{\tilde{Y}}}(\mathcal{S}, r)]^c) \geq r, \quad \tilde{\tau}_2^{\tilde{\eta}}([C_{\tilde{\tau}_2^{\tilde{\eta}}}(\mathcal{S}, r)]^c) \leq 1 - r, \quad \tilde{\tau}_2^{\tilde{\mu}}([C_{\tilde{\tau}_2^{\tilde{\mu}}}(\mathcal{S}, r)]^c) \leq 1 - r,$$

then by Theorem 3.6 $\text{int}_{\tilde{\tau}_2^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r), r)$ is r -SVNRO set. So,

$$\tilde{\tau}_1^{\tilde{Y}}(f^{-1}(\text{int}_{\tilde{\tau}_2^{\tilde{Y}}}(C_{\tilde{\tau}_2^{\tilde{Y}}}(\mathcal{S}, r), r))) \geq r, \quad \tilde{\tau}_1^{\tilde{\eta}}(f^{-1}(\text{int}_{\tilde{\tau}_2^{\tilde{\eta}}}(C_{\tilde{\tau}_2^{\tilde{\eta}}}(\mathcal{S}, r), r))) \leq 1 - r, \quad \tilde{\tau}_1^{\tilde{\mu}}(f^{-1}(\text{int}_{\tilde{\tau}_2^{\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\mu}}}(\mathcal{S}, r), r))) \leq 1 - r.$$

Therefore, $f^{-1}(\mathcal{S}) \leq f^{-1}(\text{int}_{\tilde{\tau}_2^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r), r)) = \text{int}_{\tilde{\tau}_1^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(f^{-1}(\text{int}_{\tilde{\tau}_2^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r), r)))$.

(3) $\Rightarrow$ (1). Let $\mathcal{S}$ be an r -SVNRO set of $\tilde{\mathfrak{T}}_2$. Then, we get

$$f^{-1}(\mathcal{S}) \leq \text{int}_{\tilde{\tau}_1^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(f^{-1}(\text{int}_{\tilde{\tau}_2^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r), r)), r) = \text{int}_{\tilde{\tau}_1^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(f^{-1}(\mathcal{S}), r);$$

this suggests that, $f^{-1}(\mathcal{S}) = \text{int}_{\tilde{\tau}_1^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(f^{-1}(\mathcal{S}), r)$, then

$$\tilde{\tau}_1^{\tilde{Y}}(f^{-1}(\mathcal{S})) = \tilde{\tau}_1^{\tilde{Y}}(\text{int}_{\tilde{\tau}_1^{\tilde{Y}}}(f^{-1}(\mathcal{S}), r)) \geq r, \quad \tilde{\tau}_1^{\tilde{\eta}}(f^{-1}(\mathcal{S})) = \tilde{\tau}_1^{\tilde{\eta}}(\text{int}_{\tilde{\tau}_1^{\tilde{\eta}}}(f^{-1}(\mathcal{S}), r)) \leq 1 - r,$$

$$\tilde{\tau}_1^{\tilde{\mu}}(f^{-1}(\mathcal{S})) = \tilde{\tau}_1^{\tilde{\mu}}(\text{int}_{\tilde{\tau}_1^{\tilde{\mu}}}(f^{-1}(\mathcal{S}), r)) \leq 1 - r.$$

Therefore, f is $\mathcal{SVN}$ -almost continuous.

(2) $\Leftrightarrow$ (4). Can be proved similarly.

Theorem 3.11. Let $f: (\tilde{\mathfrak{T}}_1, \tilde{\tau}_1^{\tilde{Y}\tilde{\eta}\tilde{\mu}}) \rightarrow (\tilde{\mathfrak{T}}_2, \tilde{\tau}_2^{\tilde{Y}\tilde{\eta}\tilde{\mu}})$ be a map from an $\mathcal{SVNTS}$ $(\tilde{\mathfrak{T}}_1, \tilde{\tau}_1^{\tilde{Y}\tilde{\eta}\tilde{\mu}})$ into another $\mathcal{SVNTS}$ $(\tilde{\mathfrak{T}}_2, \tilde{\tau}_2^{\tilde{Y}\tilde{\eta}\tilde{\mu}})$. Then the following are equivalent:

1. f is $\mathcal{SVN}$ -weakly continuous,
2. $f(C_{\tilde{\tau}_1^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r)) \leq C_{\tilde{\tau}_2^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(f(\mathcal{S}), r)$ for each $\mathcal{S} \in I^{\tilde{\mathfrak{T}}_1}$

Proof. (1) $\Rightarrow$ (2). : Let $\mathcal{S} \in I^{\tilde{\mathfrak{T}}_1}$. Then,

$$\begin{aligned} f^{-1}(C_{\tilde{\tau}_2^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(f(\mathcal{S}), r)) &= f^{-1}\left[\bigcap\left\{\mathcal{E} \in I^{\tilde{\mathfrak{T}}_2}: \tilde{\tau}_2^{\tilde{Y}}(\mathcal{E}^c) \geq r, \tilde{\tau}_2^{\tilde{\eta}}(\mathcal{E}^c) \leq 1 - r, \tilde{\tau}_2^{\tilde{\mu}}(\mathcal{E}^c) \leq 1 - r, \quad \mathcal{E} \geq f(\mathcal{S})\right\}\right] \\ &\geq f^{-1}\left[\bigcap\left\{\mathcal{E} \in I^{\tilde{\mathfrak{T}}_2}: \tilde{\tau}_1^{\tilde{Y}}(f^{-1}(\mathcal{E}^c)) \geq r, \tilde{\tau}_1^{\tilde{\eta}}(f^{-1}(\mathcal{E}^c)) \leq 1 - r, \tilde{\tau}_1^{\tilde{\mu}}(f^{-1}(\mathcal{E}^c)) \leq 1 - r, \quad \mathcal{E} \geq f(\mathcal{S})\right\}\right] \\ &\geq f^{-1}\left[\bigcap\left\{\mathcal{E} \in I^{\tilde{\mathfrak{T}}_2}: \tilde{\tau}_1^{\tilde{Y}}((f^{-1}(\mathcal{E}))^c) \geq r, \tilde{\tau}_1^{\tilde{\eta}}((f^{-1}(\mathcal{E}))^c) \leq 1 - r, \tilde{\tau}_1^{\tilde{\mu}}((f^{-1}(\mathcal{E}))^c) \leq 1 - r, \quad \mathcal{E} \geq f(\mathcal{S})\right\}\right] \\ &\geq \bigcap\left\{f^{-1}(\mathcal{E}) \in I^{\tilde{\mathfrak{T}}_1}: \tilde{\tau}_1^{\tilde{Y}}((f^{-1}(\mathcal{E}))^c) \geq r, \tilde{\tau}_1^{\tilde{\eta}}((f^{-1}(\mathcal{E}))^c) \leq 1 - r, \tilde{\tau}_1^{\tilde{\mu}}((f^{-1}(\mathcal{E}))^c) \leq 1 - r, \quad f^{-1}(\mathcal{E}) \geq \mathcal{S}\right\} \\ &\geq \bigcap\left\{\mathcal{D} \in I^{\tilde{\mathfrak{T}}_1}: \tilde{\tau}_1^{\tilde{Y}}(\mathcal{D}^c) \geq r, \tilde{\tau}_1^{\tilde{\eta}}(\mathcal{D}^c) \leq 1 - r, \tilde{\tau}_1^{\tilde{\mu}}(\mathcal{D}^c) \leq 1 - r, \quad \mathcal{D} \geq \mathcal{S}\right\} = C_{\tilde{\tau}_1^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r). \end{aligned}$$

Hence, $f(C_{\tilde{\tau}_1^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r)) \leq f(f^{-1}(C_{\tilde{\tau}_2^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(f(\mathcal{S}), r))) \leq C_{\tilde{\tau}_2^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(f(\mathcal{S}), r)$.

(2) $\Rightarrow$ (1). It is similar to that of (1) $\Rightarrow$ (2).

Corollary 3.12. Let $f: \tilde{\mathfrak{T}}_1 \rightarrow \tilde{\mathfrak{T}}_2$ be an $\mathcal{SVN}$ -continuous mapping with respect to the $\mathcal{SVNTS}$ $\tilde{\tau}_1^{\tilde{Y}\tilde{\eta}\tilde{\mu}}$ and $\tilde{\tau}_2^{\tilde{Y}\tilde{\eta}\tilde{\mu}}$ respectively. Then, for each $\mathcal{S} \in I^{\tilde{\mathfrak{T}}_1}$, $f(C_{\tilde{\tau}_1^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r)) \leq C_{\tilde{\tau}_2^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(f(\mathcal{S}), r)$.

Theorem 3.13. Let $f: \tilde{\mathfrak{T}}_1 \rightarrow \tilde{\mathfrak{T}}_2$ be an $\mathcal{SVN}$ -continuous mapping with respect to the $\mathcal{SVNTS}$ $\tilde{\tau}_1^{\tilde{Y}\tilde{\eta}\tilde{\mu}}$ and $\tilde{\tau}_2^{\tilde{Y}\tilde{\eta}\tilde{\mu}}$, respectively. Then, for any $\mathcal{S} \in I^{\tilde{\mathfrak{T}}_2}$, $C_{\tilde{\tau}_1^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(f^{-1}(\mathcal{S}), r) \leq f^{-1}(C_{\tilde{\tau}_2^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r))$.

Proof. Let $\mathcal{S} \in I^{\tilde{\mathfrak{T}}_2}$. We get from Theorem 3.12, $C_{\tilde{\tau}_1^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(f^{-1}(\mathcal{S}), r) \leq f^{-1}(f(C_{\tilde{\tau}_1^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(f^{-1}(\mathcal{S}), r))) \leq f^{-1}(C_{\tilde{\tau}_2^{\tilde{Y}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r))$.

Hence, $C_{\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(f^{-1}(\mathcal{S}), r) \leq f^{-1}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}, r))$, for every $\mathcal{S} \in I^{\tilde{\mathfrak{X}}_2}$.

4. Compactness on Single-Valued Neutrosophic Ideal Topological Spaces

This section aims to establish new notions of r -single-valued neutrosophic aspects called (compact, ideal compact, ideal quasi H -closed, compact modulo an single-valued neutrosophic ideal) (briefly, $r - \mathcal{SVN} - compact$, $r - \mathcal{SVNI} - compact$, $r - \mathcal{SVNI} - quasi H - closed$, $r - \mathcal{SVNC}(\mathcal{I}) - compact$) in $\mathcal{SVNITS}$.

Definition 4.1. Let $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be an $\mathcal{SVNITS}$ and $r \in I_0$. Then $\tilde{\mathfrak{X}}$ is called $r - \mathcal{SVN} - compact$ iff for every family $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j) \leq 1 - r, j \in \Gamma\}$ such that $\bigcup_{j \in \Gamma} \mathcal{S}_j = \tilde{1}$, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ such that $\bigcup_{j \in \Gamma_0} \mathcal{S}_j = \tilde{1}$.

Definition 4.2. Let $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathfrak{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be an $\mathcal{SVNITS}$ and $r \in I_0$. Then,

- (1) $\tilde{\mathfrak{X}}$ is called $r - \mathcal{SVNI} - compact$ (resp., $r - \mathcal{SVNI} - quasi H - closed$) iff every family, $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j) \leq 1 - r, j \in \Gamma\}$ such that $\bigcup_{j \in \Gamma} \mathcal{S}_j = \tilde{1}$, there exists a finite subse $\Gamma_0 \subseteq \Gamma$ such that $\tilde{\mathfrak{J}}^{\tilde{\gamma}}(\bigcup_{j \in \Gamma_0} \mathcal{S}_j)^c \geq r, \tilde{\mathfrak{J}}^{\tilde{\eta}}(\bigcup_{j \in \Gamma_0} \mathcal{S}_j)^c \leq 1 - r, \tilde{\mathfrak{J}}^{\tilde{\mu}}(\bigcup_{j \in \Gamma_0} \mathcal{S}_j)^c \leq 1 - r$ (resp., $\tilde{\mathfrak{J}}^{\tilde{\gamma}}(\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{S}_j, r))^c \geq r, \tilde{\mathfrak{J}}^{\tilde{\eta}}(\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j, r))^c \leq 1 - r, \tilde{\mathfrak{J}}^{\tilde{\mu}}(\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j, r))^c \leq 1 - r$).
- (2) $\tilde{\mathfrak{X}}$ is called $r - \mathcal{SVNC}(\mathcal{I}) - compact$ if for any $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1 - r$ and every family $\{\mathcal{E}_j \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}_j) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}_j) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}_j) \leq 1 - r, j \in \Gamma\}$ such that $\mathcal{S} \leq \bigcup_{j \in \Gamma} \mathcal{E}_j$, there exists a finite subse $\Gamma_0 \subseteq \Gamma$ such that $\tilde{\mathfrak{J}}^{\tilde{\gamma}}(\mathcal{S} \cap [\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j, r)]^c) \geq r, \tilde{\mathfrak{J}}^{\tilde{\eta}}(\mathcal{S} \cap [\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j, r)]^c) \leq 1 - r, \tilde{\mathfrak{J}}^{\tilde{\mu}}(\mathcal{S} \cap [\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j, r)]^c) \leq 1 - r$.

Definition 4.3. Let $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathfrak{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be an $\mathcal{SVNITS}$ and $\mathcal{S} \in I^{\tilde{\mathfrak{X}}}$. Then $\mathcal{S}$ is called $r - \mathcal{SVNI} - compact$ iff every family $\{\mathcal{E}_j \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}_j) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}_j) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}_j) \leq 1 - r, j \in \Gamma\}$ such that $\mathcal{S} \leq \bigcup_{j \in \Gamma} \mathcal{E}_j$, there exists a finite subse $\Gamma_0 \subseteq \Gamma$ such that $\tilde{\mathfrak{J}}^{\tilde{\gamma}}(\mathcal{S} \cap [\bigcup_{j \in \Gamma_0} \mathcal{E}_j]^c) \geq r, \tilde{\mathfrak{J}}^{\tilde{\eta}}(\mathcal{S} \cap [\bigcup_{j \in \Gamma_0} \mathcal{E}_j]^c) \leq 1 - r, \tilde{\mathfrak{J}}^{\tilde{\mu}}(\mathcal{S} \cap [\bigcup_{j \in \Gamma_0} \mathcal{E}_j]^c) \leq 1 - r$.

Theorem 4.4. Let $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\mathfrak{J}}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ be an $\mathcal{SVNITS}$ and $r \in I_0$. Then,

- (1) $r - \mathcal{SVN} - compact \Rightarrow r - \mathcal{SVNI} - compact$,
- (2) $r - \mathcal{SVNI} - compact \Rightarrow r - \mathcal{SVNC}(\mathcal{I}) - compact$,
- (3) $r - \mathcal{SVNI} - compact \Rightarrow r - \mathcal{SVNI} - quasi H - closed$.

Proof. (1) For every family $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}_j) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j) \leq 1 - r, j \in \Gamma\}$ such that $\bigcup_{j \in \Gamma} \mathcal{S}_j = \tilde{1}$. By $r - \mathcal{SVN} - compactness$ of $\tilde{\mathfrak{X}}$, there exists a finite subse $\Gamma_0 \subseteq \Gamma$ such that $\bigcup_{j \in \Gamma_0} \mathcal{S}_j = \tilde{1}$. Now, since $[\bigcup_{j \in \Gamma_0} \mathcal{S}_j]^c = \tilde{0}$, we have $\tilde{\mathfrak{J}}^{\tilde{\gamma}}([\bigcup_{j \in \Gamma_0} \mathcal{S}_j]^c) \geq r, \tilde{\mathfrak{J}}^{\tilde{\eta}}([\bigcup_{j \in \Gamma_0} \mathcal{S}_j]^c) \leq 1 - r, \tilde{\mathfrak{J}}^{\tilde{\mu}}([\bigcup_{j \in \Gamma_0} \mathcal{S}_j]^c) \leq 1 - r$.

(2) For every $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1 - r$ and evrey family $\{\mathcal{E}_j \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}_j) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}_j) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}_j) \leq 1 - r, j \in \Gamma\}$ such that $\mathcal{S} \leq \bigcup_{j \in \Gamma} \mathcal{E}_j$. By $r - \mathcal{SVNI} - compactness$ of $\mathcal{S}$, there exists a finite subse $\Gamma_0 \subseteq \Gamma$ such that $\tilde{\mathfrak{J}}^{\tilde{\gamma}}(\mathcal{S} \cap [\bigcup_{j \in \Gamma_0} \mathcal{E}_j]^c) \geq r, \tilde{\mathfrak{J}}^{\tilde{\eta}}(\mathcal{S} \cap [\bigcup_{j \in \Gamma_0} \mathcal{E}_j]^c) \leq 1 - r, \tilde{\mathfrak{J}}^{\tilde{\mu}}(\mathcal{S} \cap [\bigcup_{j \in \Gamma_0} \mathcal{E}_j]^c) \leq 1 - r$. Since, $\mathcal{S} \cap [\bigcup_{j \in \Gamma_0} \mathcal{E}_j]^c \geq \mathcal{S} \cap [\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r)]^c$, we have

$$\tilde{\mathfrak{J}}^{\tilde{\gamma}}\left(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j, r)\right]^c\right) \geq r, \quad \tilde{\mathfrak{J}}^{\tilde{\eta}}\left(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j, r)\right]^c\right) \leq 1 - r, \quad \tilde{\mathfrak{J}}^{\tilde{\mu}}\left(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j, r)\right]^c\right) \leq 1 - r$$

Hence, $\tilde{\mathfrak{X}}$ is $r - \mathcal{SVNC}(\mathcal{I}) - compact$.

(3) Let $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\mathfrak{V}}}(\mathcal{S}_j) \geq r, \tilde{\tau}^{\tilde{\mathfrak{N}}}(\mathcal{S}_j) \leq 1 - r, \tilde{\tau}^{\tilde{\mathfrak{M}}}(\mathcal{S}_j) \leq 1 - r: j \in \Gamma\}$ be a family such that $\bigcup_{j \in \Gamma} \mathcal{S}_j = \tilde{\mathbf{I}}$. By $r - \mathcal{SVNJ} - compactness$ of $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}}, \tilde{\mathfrak{J}}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}})$, there exists a finite subfamily $\Gamma_0 \subseteq \Gamma$ such that $\tilde{\mathfrak{J}}^{\tilde{\mathfrak{V}}}([\bigcup_{j \in \Gamma_0} \mathcal{S}_j]^c) \geq r$, $\tilde{\mathfrak{J}}^{\tilde{\mathfrak{N}}}([\bigcup_{j \in \Gamma_0} \mathcal{S}_j]^c) \leq 1 - r$, $\tilde{\mathfrak{J}}^{\tilde{\mathfrak{M}}}([\bigcup_{j \in \Gamma_0} \mathcal{S}_j]^c) \leq 1 - r$. Since, $[\bigcup_{j \in \Gamma_0} \mathcal{S}_j]^c \supseteq [\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}}(\mathcal{S}_j, r)}]^c$, we have

$$\tilde{\mathfrak{J}}^{\tilde{\mathfrak{V}}}\left(\left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}}(\mathcal{S}_j, r)}\right]^c\right) \geq r, \quad \tilde{\mathfrak{J}}^{\tilde{\mathfrak{N}}}\left(\left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}}(\mathcal{S}_j, r)}\right]^c\right) \leq 1 - r, \quad \tilde{\mathfrak{J}}^{\tilde{\mathfrak{M}}}\left(\left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}}(\mathcal{S}_j, r)}\right]^c\right) \leq 1 - r$$

Hence, $\tilde{\mathfrak{X}}$ is $r - \mathcal{SVNI} - quasi H - closed$.

Theorem 4.5. The next statements are equivalent in an $\mathcal{SVNJTS}(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}}, \tilde{\mathfrak{J}}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}})$:

- (1) $\tilde{\mathfrak{X}}$ is $r - \mathcal{SVNJ} - compact$,
- (2) For any family $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\mathfrak{V}}}(\mathcal{S}_j^c) \geq r, \tilde{\tau}^{\tilde{\mathfrak{N}}}(\mathcal{S}_j^c) \leq 1 - r, \tilde{\tau}^{\tilde{\mathfrak{M}}}(\mathcal{S}_j^c) \leq 1 - r, j \in \Gamma\}$ with $\bigcap_{j \in \Gamma} \mathcal{S}_j = \tilde{\mathbf{0}}$, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ with $\tilde{\mathfrak{J}}^{\tilde{\mathfrak{V}}}(\bigcap_{j \in \Gamma_0} \mathcal{S}_j) \geq r$, $\tilde{\mathfrak{J}}^{\tilde{\mathfrak{N}}}(\bigcap_{j \in \Gamma_0} \mathcal{S}_j) \leq 1 - r$, $\tilde{\mathfrak{J}}^{\tilde{\mathfrak{M}}}(\bigcap_{j \in \Gamma_0} \mathcal{S}_j) \leq 1 - r$.

Proof. (1) $\Rightarrow$ (2). For each family $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\mathfrak{V}}}(\mathcal{S}_j^c) \geq r, \tilde{\tau}^{\tilde{\mathfrak{N}}}(\mathcal{S}_j^c) \leq 1 - r, \tilde{\tau}^{\tilde{\mathfrak{M}}}(\mathcal{S}_j^c) \leq 1 - r, j \in \Gamma\}$ with $\bigcap_{j \in \Gamma} \mathcal{S}_j = \tilde{\mathbf{0}}$. Then, $\bigcup_{j \in \Gamma} \mathcal{S}_j^c = \tilde{\mathbf{I}}$. By $r - \mathcal{SVNJ} - compactness$ of $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}}, \tilde{\mathfrak{J}}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}})$, there exists a finite subse $\Gamma_0 \subseteq \Gamma$ such that $\tilde{\mathfrak{J}}^{\tilde{\mathfrak{V}}}([\bigcup_{j \in \Gamma_0} \mathcal{S}_j^c]^c) \geq r$, $\tilde{\mathfrak{J}}^{\tilde{\mathfrak{N}}}([\bigcup_{j \in \Gamma_0} \mathcal{S}_j^c]^c) \leq 1 - r$, $\tilde{\mathfrak{J}}^{\tilde{\mathfrak{M}}}([\bigcup_{j \in \Gamma_0} \mathcal{S}_j^c]^c) \leq 1 - r$, this implies that,

$$\tilde{\mathfrak{J}}^{\tilde{\mathfrak{V}}}\left(\bigcap_{j \in \Gamma_0} \mathcal{S}_j\right) \geq r, \quad \tilde{\mathfrak{J}}^{\tilde{\mathfrak{N}}}\left(\bigcap_{j \in \Gamma_0} \mathcal{S}_j\right) \leq 1 - r, \quad \tilde{\mathfrak{J}}^{\tilde{\mathfrak{M}}}\left(\bigcap_{j \in \Gamma_0} \mathcal{S}_j\right) \leq 1 - r.$$

(2) $\Rightarrow$ (1). Let $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\mathfrak{V}}}(\mathcal{S}_j) \geq r, \tilde{\tau}^{\tilde{\mathfrak{N}}}(\mathcal{S}_j) \leq 1 - r, \tilde{\tau}^{\tilde{\mathfrak{M}}}(\mathcal{S}_j) \leq 1 - r, j \in \Gamma\}$ be a family such that $\bigcup_{j \in \Gamma} \mathcal{S}_j = \tilde{\mathbf{I}}$. Then, $\bigcap_{j \in \Gamma} \mathcal{S}_j^c = \tilde{\mathbf{0}}$, by (2), there exists a finite subse $\Gamma_0 \subseteq \Gamma$ such that $\tilde{\mathfrak{J}}^{\tilde{\mathfrak{V}}}(\bigcap_{j \in \Gamma_0} \mathcal{S}_j^c) \geq r$, $\tilde{\mathfrak{J}}^{\tilde{\mathfrak{N}}}(\bigcap_{j \in \Gamma_0} \mathcal{S}_j^c) \leq 1 - r$, $\tilde{\mathfrak{J}}^{\tilde{\mathfrak{M}}}(\bigcap_{j \in \Gamma_0} \mathcal{S}_j^c) \leq 1 - r$ this implies that $\tilde{\mathfrak{J}}^{\tilde{\mathfrak{V}}}([\bigcup_{j \in \Gamma_0} \mathcal{S}_j]^c) \geq r$, $\tilde{\mathfrak{J}}^{\tilde{\mathfrak{N}}}([\bigcup_{j \in \Gamma_0} \mathcal{S}_j]^c) \leq 1 - r$, $\tilde{\mathfrak{J}}^{\tilde{\mathfrak{M}}}([\bigcup_{j \in \Gamma_0} \mathcal{S}_j]^c) \leq 1 - r$. Therefore $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}}, \tilde{\mathfrak{J}}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}})$ is $r - \mathcal{SVNJ} - compact$.

Remark 4.6. Let $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}}, \tilde{\mathfrak{J}}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}})$ be an $\mathcal{SVNJTS}$. The simplest $\mathcal{SVNJ}$ on $\tilde{\mathfrak{X}}$ is $\tilde{\mathfrak{J}}_0^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}}: I^{\tilde{\mathfrak{X}}} \rightarrow I$, where

$$\tilde{\mathfrak{J}}_0^{\tilde{\mathfrak{V}}}(\mathcal{S}) = \begin{cases} 1, & \text{if } \mathcal{S} = \tilde{\mathbf{0}} \\ 0, & \text{otherwise,} \end{cases} \quad \tilde{\mathfrak{J}}_0^{\tilde{\mathfrak{N}}}(\mathcal{S}) = \begin{cases} 0, & \text{if } \mathcal{S} = \tilde{\mathbf{0}} \\ 1, & \text{otherwise,} \end{cases} \quad \tilde{\mathfrak{J}}_0^{\tilde{\mathfrak{M}}}(\mathcal{S}) = \begin{cases} 0, & \text{if } \mathcal{S} = \tilde{\mathbf{0}} \\ 1, & \text{otherwise,} \end{cases}$$

If $\tilde{\mathfrak{J}}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}} = \tilde{\mathfrak{J}}_0^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}}$ then $r - \mathcal{SVN} - compact$ and $r - \mathcal{SVNJ} - compact$ are equivalent

Definition 4.7. An $\mathcal{SVNJTS}(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}})$ is said to be r -single-valued neutrosophic regular ($r - \mathcal{SVN} - regular$) iff for every $\tilde{\tau}^{\tilde{\mathfrak{V}}}(\mathcal{S}) \geq r$, $\tilde{\tau}^{\tilde{\mathfrak{N}}}(\mathcal{S}) \leq 1 - r$, $\tilde{\tau}^{\tilde{\mathfrak{M}}}(\mathcal{S}) \leq 1 - r$ and $r \in I_0$,

$$\mathcal{S} = \bigcup \{\mathcal{E} \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\mathfrak{V}}}(\mathcal{E}) \geq r, \quad \tilde{\tau}^{\tilde{\mathfrak{N}}}(\mathcal{E}) \leq 1 - r, \quad \tilde{\tau}^{\tilde{\mathfrak{M}}}(\mathcal{E}) \leq 1 - r, \quad C_{\tilde{\tau}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}}(\mathcal{E}, r)} = \mathcal{S}\}.$$

Theorem 4.8. Let $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}}, \tilde{\mathfrak{J}}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}})$ be an $r - \mathcal{SVNJ} - quasi H - closed$ and $r - \mathcal{SVN} - regular$. Then $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}}, \tilde{\mathfrak{J}}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}})$ is $r - \mathcal{SVNJ} - compact$.

Proof. For every family $\{\mathcal{S} \in I^{\tilde{\mathfrak{X}}}: \tilde{\tau}^{\tilde{\mathfrak{V}}}(\mathcal{S}_j) \geq r, \tilde{\tau}^{\tilde{\mathfrak{N}}}(\mathcal{S}_j) \leq 1 - r, \tilde{\tau}^{\tilde{\mathfrak{M}}}(\mathcal{S}_j) \leq 1 - r, j \in \Gamma\}$ such that $\bigcup_{j \in \Gamma} \mathcal{S}_j = \tilde{\mathbf{I}}$. By $r - \mathcal{SVN} - regularity$ of $(\tilde{\mathfrak{X}}, \tilde{\tau}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}}, \tilde{\mathfrak{J}}^{\tilde{\mathfrak{V}}\tilde{\mathfrak{N}}\tilde{\mathfrak{M}}})$, for any $\tilde{\tau}^{\tilde{\mathfrak{V}}}(\mathcal{S}_j) \geq r$, $\tilde{\tau}^{\tilde{\mathfrak{N}}}(\mathcal{S}_j) \leq 1 - r$, $\tilde{\tau}^{\tilde{\mathfrak{M}}}(\mathcal{S}_j) \leq 1 - r$, we have

$$\mathcal{S}_j = \bigcup_{j_\Delta \in \Delta_j} \{ \mathcal{S}_{j_\Delta} : \tilde{\tau}^{\tilde{Y}}(\mathcal{S}_{j_\Delta}) \geq r, \quad \tilde{\tau}^{\tilde{N}}(\mathcal{S}_{j_\Delta}) \leq 1-r, \quad \tilde{\tau}^{\tilde{M}}(\mathcal{S}_{j_\Delta}) \leq 1-r, \quad C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S}_{j_\Delta}, r) \leq \mathcal{S}_j \}.$$

Thus, $\bigcup_{j \in \Gamma} (\bigcup_{j_\Delta \in \Delta_j} \mathcal{S}_{j_\Delta}) = \tilde{I}$. Since $(\tilde{\mathfrak{I}}, \tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}, \tilde{J}^{\tilde{Y}\tilde{N}\tilde{M}})$ is $r - \mathcal{SVNJ}$ -quasi H-closed, there exists a finite subset $K \times \Delta_K$ such that

$$\tilde{J}^{\tilde{Y}} \left(\left[\bigcup_{k \in K} \left(\bigcup_{k_\Delta \in \Delta_k} C_{\tilde{\tau}^{\tilde{Y}}}(\mathcal{S}_{k_\Delta}, r) \right) \right]^c \right) \geq r, \quad \tilde{J}^{\tilde{N}} \left(\left[\bigcup_{k \in K} \left(\bigcup_{k_\Delta \in \Delta_k} C_{\tilde{\tau}^{\tilde{N}}}(\mathcal{S}_{k_\Delta}, r) \right) \right]^c \right) \leq 1-r, \quad \tilde{J}^{\tilde{M}} \left(\left[\bigcup_{k \in K} \left(\bigcup_{k_\Delta \in \Delta_k} C_{\tilde{\tau}^{\tilde{M}}}(\mathcal{S}_{k_\Delta}, r) \right) \right]^c \right) \leq 1-r.$$

For each $k \in K$, since $\bigcup_{k_\Delta \in \Delta_k} C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S}_{k_\Delta}, r) \leq \mathcal{S}_k$. It implies that $\left[\bigcup_{k \in K} (\bigcup_{k_\Delta \in \Delta_k} C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{S}_{k_\Delta}, r)) \right]^c \geq \left[\bigcup_{k \in K} \mathcal{S}_k \right]^c$. Thus,

$$\tilde{J}^{\tilde{Y}} \left(\left[\bigcup_{k \in K} \mathcal{S}_k \right]^c \right) \geq \tilde{J}^{\tilde{Y}} \left(\left[\bigcup_{k \in K} \left(\bigcup_{k_\Delta \in \Delta_k} C_{\tilde{\tau}^{\tilde{Y}}}(\mathcal{S}_{k_\Delta}, r) \right) \right]^c \right) \geq r, \quad \tilde{J}^{\tilde{N}} \left(\left[\bigcup_{k \in K} \mathcal{S}_k \right]^c \right) \leq \tilde{J}^{\tilde{N}} \left(\left[\bigcup_{k \in K} \left(\bigcup_{k_\Delta \in \Delta_k} C_{\tilde{\tau}^{\tilde{N}}}(\mathcal{S}_{k_\Delta}, r) \right) \right]^c \right) \leq 1-r$$

$$\tilde{J}^{\tilde{M}} \left(\left[\bigcup_{k \in K} \mathcal{S}_k \right]^c \right) \leq \tilde{J}^{\tilde{M}} \left(\left[\bigcup_{k \in K} \left(\bigcup_{k_\Delta \in \Delta_k} C_{\tilde{\tau}^{\tilde{M}}}(\mathcal{S}_{k_\Delta}, r) \right) \right]^c \right) \leq 1-r.$$

Hence, $(\tilde{\mathfrak{I}}, \tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}, \tilde{J}^{\tilde{Y}\tilde{N}\tilde{M}})$ is $r - \mathcal{SVNJ}$ -compact.

Definition 4.9. A family $\{\mathcal{S}_j\}_{j \in \Gamma}$ in $\tilde{\mathfrak{I}}$ has the finite intersection property (**I-FIP**) iff the intersection of no finite sub-family $\Gamma_0 \subseteq \Gamma$ s.t. $\tilde{J}^{\tilde{Y}}(\bigcap_{j \in \Gamma_0} \mathcal{S}_j) \geq r$, $\tilde{J}^{\tilde{N}}(\bigcap_{j \in \Gamma_0} \mathcal{S}_j) \leq 1-r$, $\tilde{J}^{\tilde{M}}(\bigcap_{j \in \Gamma_0} \mathcal{S}_j) \leq 1-r$.

Theorem 4.10. An $\mathcal{SVNJTS}$ $(\tilde{\mathfrak{I}}, \tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}, \tilde{J}^{\tilde{Y}\tilde{N}\tilde{M}})$ is $r - \mathcal{SVNJ}$ -compact, iff every family $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{I}}} : \tilde{\tau}^{\tilde{Y}}(\mathcal{S}_j^c) \geq r, \tilde{\tau}^{\tilde{N}}(\mathcal{S}_j^c) \leq 1-r, \tilde{\tau}^{\tilde{M}}(\mathcal{S}_j^c) \leq 1-r, j \in \Gamma\}$ having the finite intersection property (**I-FIP**) has a non-empty intersection.

Proof. Obvious.

Theorem 4.11. Suppose that $(\tilde{\mathfrak{I}}, \tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}, \tilde{J}^{\tilde{Y}\tilde{N}\tilde{M}})$ is an $\mathcal{SVNJTS}$, $\mathcal{S}$ is $r - \mathcal{SVNJ}$ -compact. Then for every collection $\{\mathcal{E}_j \in I^{\tilde{\mathfrak{I}}} : \mathcal{E}_j \leq \text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{E}_j, r), r), j \in \Gamma\}$ with $\mathcal{S} \leq \bigcup_{j \in \Gamma} \mathcal{E}_j$, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ s.t,

$$\tilde{J}^{\tilde{Y}} \left(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} \text{int}_{\tilde{\tau}^{\tilde{Y}}}(C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{E}_j, r), r) \right]^c \right) \geq r, \quad \tilde{J}^{\tilde{N}} \left(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} \text{int}_{\tilde{\tau}^{\tilde{N}\tilde{M}}}(C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{E}_j, r), r) \right]^c \right) \leq 1-r$$

$$\tilde{J}^{\tilde{M}} \left(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} \text{int}_{\tilde{\tau}^{\tilde{M}}}(C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{E}_j, r), r) \right]^c \right) \leq 1-r.$$

Proof. Let $\{\mathcal{E}_j \in I^{\tilde{\mathfrak{I}}} : \mathcal{E}_j \leq \text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{E}_j, r), r), j \in \Gamma\}$ with $\mathcal{S} \leq \bigcup_{j \in \Gamma} \mathcal{E}_j$. Then, $\mathcal{S} \leq \bigcup_{j \in \Gamma} \text{int}_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{E}_j, r), r)$, $[\tilde{\tau}^{\tilde{Y}}(\text{int}_{\tilde{\tau}^{\tilde{Y}}}(C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{E}_j, r), r)) \geq r, \tilde{\tau}^{\tilde{N}}(\text{int}_{\tilde{\tau}^{\tilde{N}\tilde{M}}}(C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{E}_j, r), r)) \leq 1-r, \tilde{\tau}^{\tilde{M}}(\text{int}_{\tilde{\tau}^{\tilde{M}}}(C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{E}_j, r), r)) \leq 1-r]$. By $r - \mathcal{SVNJ}$ -compactness of $\mathcal{S}$, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ s.t,

$$\tilde{J}^{\tilde{Y}} \left(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} \text{int}_{\tilde{\tau}^{\tilde{Y}}}(C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{E}_j, r), r) \right]^c \right) \geq r, \quad \tilde{J}^{\tilde{N}} \left(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} \text{int}_{\tilde{\tau}^{\tilde{N}\tilde{M}}}(C_{\tilde{\tau}^{\tilde{Y}\tilde{N}\tilde{M}}}(\mathcal{E}_j, r), r) \right]^c \right) \leq 1-r$$

$$\tilde{J}^{\tilde{\mu}}\left(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} \text{int}_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{C}_{\tilde{\tau}^{\tilde{\mu}}\tilde{\eta}^{\tilde{\mu}}}(\mathcal{E}_j, r), r)\right]^c\right) \leq 1 - r.$$

Definition 4.12. Let $(\tilde{\mathcal{X}}, \tilde{\tau}^{\tilde{\nu}}\tilde{\eta}^{\tilde{\mu}})$ be an $\mathcal{SVN}\mathcal{T}\mathcal{S}$ and $\mathcal{S} \in I^{\tilde{\mathcal{X}}}$. Then $\mathcal{S}$ is called r -single-valued neutrosophic locally closed iff $\mathcal{S} = \mathcal{E} \cap \mathcal{D}$ where $[\tilde{\tau}^{\tilde{\nu}}(\mathcal{E}) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}) \leq 1 - r], [\tilde{\tau}^{\tilde{\nu}}(\mathcal{D}^c) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{D}^c) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{D}^c) \leq 1 - r]$.

Lemma 4.13. Let $(\tilde{\mathcal{X}}, \tilde{\tau}^{\tilde{\nu}}\tilde{\eta}^{\tilde{\mu}})$ be an $\mathcal{SVN}\mathcal{T}\mathcal{S}$ and $\mathcal{S} \in I^{\tilde{\mathcal{X}}}$. Then $\tilde{\tau}^{\tilde{\nu}}(\mathcal{S}) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}) \leq 1 - r$ iff $\mathcal{S}$ both r -single-valued neutrosophic locally closed and r -SVNPO set.

Proof. It is trivial.

Lemma 4.14. If $\mathcal{S}$ is r -SVN $\mathcal{J}$ -compact, then for every collection $\{\mathcal{E}_j \in I^{\tilde{\mathcal{X}}}: \mathcal{E}_j \text{ is both } r\text{-SVNPO and } r\text{-single-valued neutrosophic locally closed sets, } j \in \Gamma\}$ with $\mathcal{S} \leq \bigcup_{j \in \Gamma} \mathcal{E}_j$, there exists a finite subfamily $\Gamma_0 \subseteq \Gamma$ such that $\tilde{J}^{\tilde{\nu}}(\mathcal{S} \cap [\bigcup_{j \in \Gamma_0} \mathcal{E}_j]^c) \geq r, \tilde{J}^{\tilde{\eta}}(\mathcal{S} \cap [\bigcup_{j \in \Gamma_0} \mathcal{E}_j]^c) \leq 1 - r, \tilde{J}^{\tilde{\mu}}(\mathcal{S} \cap [\bigcup_{j \in \Gamma_0} \mathcal{E}_j]^c) \leq 1 - r$.

Proof. Follows from Lemma 4.13.

Theorem 4.15. Let $(\tilde{\mathcal{X}}, \tilde{\tau}^{\tilde{\nu}}\tilde{\eta}^{\tilde{\mu}}, \tilde{J}^{\tilde{\nu}}\tilde{\eta}^{\tilde{\mu}})$ be an $\mathcal{SVN}\mathcal{J}\mathcal{T}\mathcal{S}$, $\mathcal{S}_1$ and $\mathcal{S}_2$ are r -SVN $\mathcal{J}$ -compact. Then, $\mathcal{S} \cup \mathcal{E}$ is r -SVN $\mathcal{J}$ -compact subset relative to $\tilde{\mathcal{X}}$.

Proof. Let $\{\mathcal{E}_j \in I^{\tilde{\mathcal{X}}}: \tilde{\tau}^{\tilde{\nu}}(\mathcal{E}_j) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}_j) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}_j) \leq 1 - r, j \in \Gamma\}$ be a family such that $\mathcal{S}_1 \cup \mathcal{S}_2 \leq \bigcup_{j \in \Gamma} \mathcal{E}_j$. Then $\mathcal{S}_1 \leq \bigcup_{j \in \Gamma} \mathcal{E}_j$ and $\mathcal{S}_2 \leq \bigcup_{j \in \Gamma} \mathcal{E}_j$. Since $\mathcal{S}_1$ and $\mathcal{S}_2$ are r -SVN $\mathcal{J}$ -compact, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ such that

$$\tilde{J}^{\tilde{\nu}}\left(\mathcal{S}_k \cap \left[\bigcup_{j \in \Gamma_0} \mathcal{E}_j\right]^c\right) \geq r, \quad \tilde{J}^{\tilde{\eta}}\left(\mathcal{S}_k \cap \left[\bigcup_{j \in \Gamma_0} \mathcal{E}_j\right]^c\right) \leq 1 - r, \quad \tilde{J}^{\tilde{\mu}}\left(\mathcal{S}_k \cap \left[\bigcup_{j \in \Gamma_0} \mathcal{E}_j\right]^c\right) \leq 1 - r,$$

for $k = 1, 2$, since $(\mathcal{S}_1 \cap [\bigcup_{j \in \Gamma_0} \mathcal{E}_j]^c) \cup (\mathcal{S}_2 \cap [\bigcup_{j \in \Gamma_0} \mathcal{E}_j]^c) = (\mathcal{S}_1 \cup \mathcal{S}_2) \cap [\bigcup_{j \in \Gamma_0} \mathcal{E}_j]^c$. Then,

$$\tilde{J}^{\tilde{\nu}}\left((\mathcal{S}_1 \cup \mathcal{S}_2) \cap \left[\bigcup_{j \in \Gamma_0} \mathcal{E}_j\right]^c\right) \geq r, \quad \tilde{J}^{\tilde{\eta}}\left((\mathcal{S}_1 \cup \mathcal{S}_2) \cap \left[\bigcup_{j \in \Gamma_0} \mathcal{E}_j\right]^c\right) \leq 1 - r, \quad \tilde{J}^{\tilde{\mu}}\left((\mathcal{S}_1 \cup \mathcal{S}_2) \cap \left[\bigcup_{j \in \Gamma_0} \mathcal{E}_j\right]^c\right) \leq 1 - r.$$

This shown that $(\mathcal{S}_1 \cup \mathcal{S}_2)$ is r -SVN $\mathcal{J}$ -compact.

Theorem 4.16. Suppose $(\tilde{\mathcal{X}}, \tilde{\tau}^{\tilde{\nu}}\tilde{\eta}^{\tilde{\mu}}, \tilde{J}^{\tilde{\nu}}\tilde{\eta}^{\tilde{\mu}})$ be an $\mathcal{SVN}\mathcal{J}\mathcal{T}\mathcal{S}$, $r \in I_0$. Then the next statements are equivalent:

- (1) $(\tilde{\mathcal{X}}, \tilde{\tau}^{\tilde{\nu}}\tilde{\eta}^{\tilde{\mu}}, \tilde{J}^{\tilde{\nu}}\tilde{\eta}^{\tilde{\mu}})$ is r -SVN $\mathcal{J}$ -quasi H -closed,
- (2) For every collection $\{\mathcal{S}_j \in I^{\tilde{\mathcal{X}}}: \tilde{\tau}^{\tilde{\nu}}(\mathcal{S}_j^c) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j^c) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j^c) \leq 1 - r, j \in \Gamma\}$ with $\bigcap_{j \in \Gamma} \mathcal{S}_j = \tilde{0}$, there exists $\Gamma_0 \subseteq \Gamma$ such that $\tilde{J}^{\tilde{\nu}}(\bigcap_{j \in \Gamma_0} \text{int}_{\tilde{\tau}^{\tilde{\nu}}}(\mathcal{S}_j, r)) \geq r, \tilde{J}^{\tilde{\eta}}(\bigcap_{j \in \Gamma_0} \text{int}_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j, r)) \leq 1 - r, \tilde{J}^{\tilde{\mu}}(\bigcap_{j \in \Gamma_0} \text{int}_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j, r)) \leq 1 - r,$
- (3) $\bigcap_{j \in \Gamma} \mathcal{S}_j \neq \tilde{0}$, holds for any collection $\{\mathcal{S}_j \in I^{\tilde{\mathcal{X}}}: \tilde{\tau}^{\tilde{\nu}}(\mathcal{S}_j^c) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j^c) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j^c) \leq 1 - r, j \in \Gamma\}$ such that $\{\text{int}_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}^{\tilde{\mu}}}(\mathcal{S}_j, r): \tilde{\tau}^{\tilde{\nu}}(\mathcal{S}_j^c) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j^c) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j^c) \leq 1 - r, j \in \Gamma\}$ has the **I-FIP**,
- (4) For any collection $\{\mathcal{S}_j \in I^{\tilde{\mathcal{X}}}: \mathcal{S}_j \text{ is } r\text{-SVNRO sets, } j \in \Gamma\}$ such taht $\bigcup_{j \in \Gamma} \mathcal{S}_j = \tilde{1}$, there exists $\Gamma_0 \subseteq \Gamma$ such that $\tilde{J}^{\tilde{\nu}}([\bigcup_{j \in \Gamma_0} \mathcal{C}_{\tilde{\tau}^{\tilde{\nu}}}(\mathcal{S}_j, r)]^c) \geq r, \tilde{J}^{\tilde{\eta}}([\bigcup_{j \in \Gamma_0} \mathcal{C}_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j, r)]^c) \leq 1 - r, \tilde{J}^{\tilde{\mu}}([\bigcup_{j \in \Gamma_0} \mathcal{C}_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j, r)]^c) \leq 1 - r,$

- (5) For every collection $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{T}}}: \mathcal{S}_j \text{ is } r\text{-SVNRC set, } j \in \Gamma\}$ such that $\bigcap_{j \in \Gamma} \mathcal{S}_j = \tilde{0}$, there exists $\Gamma_0 \subseteq \Gamma$ such that $\tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\bigcap_{j \in \Gamma_0} \text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r)) \geq r$, $\tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\bigcap_{j \in \Gamma_0} \text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r)) \leq 1 - r$, $\tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\bigcap_{j \in \Gamma_0} \text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r)) \leq 1 - r$,
- (6) $\bigcap_{j \in \Gamma} \mathcal{S}_j \neq \tilde{0}$, holds for every collection $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{T}}}: \mathcal{S}_j \text{ is } r\text{-SVNRC set, } j \in \Gamma\}$ such that $\{\text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r): \mathcal{S}_j \text{ is } r\text{-SVNRC set, } j \in \Gamma\}$ has the **I-FIP**.

Proof. (1) $\Rightarrow$ (2). Let $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{T}}}: \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\mathcal{S}_j^c) \geq r, \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\mathcal{S}_j^c) \leq 1 - r, \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\mathcal{S}_j^c) \leq 1 - r, j \in \Gamma\}$ be a family with $\bigcap_{j \in \Gamma} \mathcal{S}_j = \tilde{0}$. Then, $\bigcup_{j \in \Gamma} \mathcal{S}_j^c = \tilde{1}$. Since, $(\tilde{\mathfrak{T}}, \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}, \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}})$ is $r\text{-SVNJ-quasi } H\text{-closed}$, there exists $\Gamma_0 \subseteq \Gamma$ such that $\tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\bigcup_{j \in \Gamma_0} C_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j^c, r))^c \geq r$, $\tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\bigcup_{j \in \Gamma_0} C_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j^c, r))^c \leq 1 - r$, $\tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\bigcup_{j \in \Gamma_0} C_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j^c, r))^c \leq 1 - r$. Since, $[\bigcup_{j \in \Gamma_0} C_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j^c, r)]^c = \bigcap_{j \in \Gamma_0} \text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r)$, we have

$$\tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}} \left(\bigcap_{j \in \Gamma_0} \text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r) \right) \geq r, \quad \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}} \left(\bigcap_{j \in \Gamma_0} \text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r) \right) \leq 1 - r, \quad \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}} \left(\bigcap_{j \in \Gamma_0} \text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r) \right) \leq 1 - r.$$

(2) $\Rightarrow$ (1). Let $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{T}}}: \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\mathcal{S}_j) \geq r, \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\mathcal{S}_j) \leq 1 - r, \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\mathcal{S}_j) \leq 1 - r, j \in \Gamma\}$ be a family s.t $\bigcup_{j \in \Gamma} \mathcal{S}_j = \tilde{1}$. Then, $\bigcap_{j \in \Gamma} \mathcal{S}_j^c = \tilde{0}$ and by hypothesis, there exists $\Gamma_0 \subseteq \Gamma$ s.t, $\tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\bigcap_{j \in \Gamma_0} \text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j^c, r)) \geq r$, $\tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\bigcap_{j \in \Gamma_0} \text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j^c, r)) \leq 1 - r$, $\tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\bigcap_{j \in \Gamma_0} \text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j^c, r)) \leq 1 - r$. Since, $\bigcap_{j \in \Gamma_0} \text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j^c, r) = [\bigcup_{j \in \Gamma_0} C_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r)]^c$,

$$\tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}} \left(\left[\bigcup_{j \in \Gamma_0} C_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r) \right]^c \right) \geq r, \quad \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}} \left(\left[\bigcup_{j \in \Gamma_0} C_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r) \right]^c \right) \leq 1 - r, \quad \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}} \left(\left[\bigcup_{j \in \Gamma_0} C_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r) \right]^c \right) \leq 1 - r.$$

Thus, $(\tilde{\mathfrak{T}}, \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}, \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}})$ is $r\text{-SVNJ-quasi } H\text{-closed}$,

(1) $\Rightarrow$ (3). For any family $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{T}}}: \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\mathcal{S}_j^c) \geq r, \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\mathcal{S}_j^c) \leq 1 - r, \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\mathcal{S}_j^c) \leq 1 - r, j \in \Gamma\}$ such that $\{\text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r): \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\mathcal{S}_j^c) \geq r, \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\mathcal{S}_j^c) \leq 1 - r, \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\mathcal{S}_j^c) \leq 1 - r, j \in \Gamma\}$ has the **I-FIP**. If $\bigcap_{j \in \Gamma} \mathcal{S}_j = \tilde{0}$, then $\bigcup_{j \in \Gamma} \mathcal{S}_j^c = \tilde{1}$. Since $(\tilde{\mathfrak{T}}, \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}, \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}})$ is $r\text{-SVNJ-quasi } H\text{-closed}$, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ such that

$$\tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}} \left(\left[\bigcup_{j \in \Gamma_0} C_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j^c, r) \right]^c \right) \geq r, \quad \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}} \left(\left[\bigcup_{j \in \Gamma_0} C_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j^c, r) \right]^c \right) \leq 1 - r, \quad \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}} \left(\left[\bigcup_{j \in \Gamma_0} C_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j^c, r) \right]^c \right) \leq 1 - r.$$

Since, $[\bigcup_{j \in \Gamma_0} C_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j^c, r)]^c = \bigcap_{j \in \Gamma_0} \text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r)$, we have

$$\tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}} \left(\bigcap_{j \in \Gamma_0} \text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r) \right) \geq r, \quad \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}} \left(\bigcap_{j \in \Gamma_0} \text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r) \right) \leq 1 - r, \quad \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}} \left(\bigcap_{j \in \Gamma_0} \text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r) \right) \leq 1 - r.$$

Which is a contradiction.

(3) $\Rightarrow$ (1). For any family $\{\mathcal{S}_j \in I^{\tilde{\mathfrak{T}}}: \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\mathcal{S}_j) \geq r, \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\mathcal{S}_j) \leq 1 - r, \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\mathcal{S}_j) \leq 1 - r, j \in \Gamma\}$ such that $\bigcup_{j \in \Gamma} \mathcal{S}_j = \tilde{1}$, with the property that for no finite $\Gamma_0 \subseteq \Gamma$ such that $\tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\bigcup_{j \in \Gamma_0} C_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r))^c \geq r, \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\bigcup_{j \in \Gamma_0} C_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r))^c \leq 1 - r, \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\bigcup_{j \in \Gamma_0} C_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r))^c \leq 1 - r$. Since,

$$\left[\bigcup_{j \in \Gamma_0} C_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r) \right]^c = \bigcap_{j \in \Gamma_0} \text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j^c, r).$$

The family $\{\text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j^c, r): \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\mathcal{S}_j) \geq r, \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\mathcal{S}_j) \leq 1 - r, \tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\mathcal{S}_j) \leq 1 - r, j \in \Gamma\}$ has the **I-FIP**. By (3). $\bigcap_{j \in \Gamma} \mathcal{S}_j^c \neq \tilde{0}$. Then, $\bigcup_{j \in \Gamma} \mathcal{S}_j \neq \tilde{1}$. It is a contradiction.

(1) $\Rightarrow$ (4). Let $\{\mathcal{S}_j\}_{j \in \Gamma}$ be a family of $r\text{-SVNRO}$ set such that $\bigcup_{j \in \Gamma} \mathcal{S}_j = \tilde{1}$. Then, $\bigcup_{j \in \Gamma} \text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(C_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r), r) = \tilde{1}$, since, $\tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(C_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r), r)) \geq r$, $\tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(C_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r), r)) \leq 1 - r$, $\tilde{\mathcal{J}}^{\tilde{\mathfrak{T}}}(\text{int}_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(C_{\tilde{\mathfrak{T}}^{\tilde{\mathfrak{T}}}}(\mathcal{S}_j, r), r)) \leq 1 - r$ and $\tilde{\mathfrak{T}}$ is $r\text{-SVNJ-quasi } H\text{-closed}$, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ such that

$$\begin{aligned} \tilde{J}^{\tilde{\nu}} \left(\left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\nu}}} (int_{\tilde{\tau}^{\tilde{\nu}}} (C_{\tilde{\tau}^{\tilde{\nu}}} (\mathcal{S}_j, r), r), r) \right]^c \right) &\geq r, & \tilde{J}^{\tilde{\eta}} \left(\left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}} (int_{\tilde{\tau}^{\tilde{\eta}}} (C_{\tilde{\tau}^{\tilde{\eta}}} (\mathcal{S}_j, r), r), r) \right]^c \right) &\leq 1 - r, \\ \tilde{J}^{\tilde{\mu}} \left(\left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}} (int_{\tilde{\tau}^{\tilde{\mu}}} (C_{\tilde{\tau}^{\tilde{\mu}}} (\mathcal{S}_j, r), r), r) \right]^c \right) &\leq 1 - r. \end{aligned}$$

Since, for $\tilde{\tau}^{\tilde{\nu}}(\mathcal{S}_j) \geq r$, $\tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j) \leq 1 - r$, $\tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j) \leq 1 - r$ we have $C_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}}(int_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}}(C_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_j, r), r), r) = C_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_j, r)$. Hence, $\tilde{J}^{\tilde{\nu}}([\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\nu}}}(\mathcal{S}_j, r)]^c) \geq r$, $\tilde{J}^{\tilde{\eta}}([\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j, r)]^c) \leq 1 - r$, $\tilde{J}^{\tilde{\mu}}([\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j, r)]^c) \leq 1 - r$.

(4) $\Rightarrow$ (5). Let $\{\mathcal{S}_j \in I^{\tilde{\tau}}: j \in \Gamma\}$ be a family of r -SVNRC sets such that $\bigcap_{j \in \Gamma} \mathcal{S}_j = \tilde{0}$. Then, $\bigcup_{j \in \Gamma} \mathcal{S}_j^c = \tilde{1}$, and $\{\mathcal{S}_j^c \in I^{\tilde{\tau}}: j \in \Gamma\}$ is a family of r -SVNRO sets. By (4), there will be a finite subset $\Gamma_0 \subseteq \Gamma$ such that $\tilde{J}^{\tilde{\nu}}([\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\nu}}}(\mathcal{S}_j^c, r)]^c) \geq r$, $\tilde{J}^{\tilde{\eta}}([\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j^c, r)]^c) \leq 1 - r$, $\tilde{J}^{\tilde{\mu}}([\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j^c, r)]^c) \leq 1 - r$. Thus,

$$\tilde{J}^{\tilde{\nu}} \left(\bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\nu}}} (\mathcal{S}_j, r) \right) \geq r, \quad \tilde{J}^{\tilde{\eta}} \left(\bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\eta}}} (\mathcal{S}_j, r) \right) \leq 1 - r, \quad \tilde{J}^{\tilde{\mu}} \left(\bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}} (\mathcal{S}_j, r) \right) \leq 1 - r.$$

(5) $\Rightarrow$ (1). Let $\{\mathcal{S}_j \in I^{\tilde{\tau}}: \tilde{\tau}^{\tilde{\nu}}(\mathcal{S}_j) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j) \leq 1 - r, j \in \Gamma\}$ be a family such that $\bigcup_{j \in \Gamma} \mathcal{S}_j = \tilde{1}$. Then, $\bigcup_{j \in \Gamma} int_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}}(C_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_j, r), r) = \tilde{1}$. Thus, $\bigcap_{j \in \Gamma} C_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}}(int_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_j^c, r), r) = \tilde{0}$ and $C_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}}(int_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_j^c, r), r)$ is r -SVNRC. For the hypothesis, there exists $\Gamma_0 \subseteq \Gamma$ such that

$$\begin{aligned} \tilde{J}^{\tilde{\nu}} \left(\bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\nu}}} (C_{\tilde{\tau}^{\tilde{\nu}}} (int_{\tilde{\tau}^{\tilde{\nu}}} (\mathcal{S}_j^c, r), r), r) \right) &\geq r, & \tilde{J}^{\tilde{\eta}} \left(\bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\eta}}} (C_{\tilde{\tau}^{\tilde{\eta}}} (int_{\tilde{\tau}^{\tilde{\eta}}} (\mathcal{S}_j^c, r), r), r) \right) &\leq 1 - r, \\ \tilde{J}^{\tilde{\mu}} \left(\bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}} (C_{\tilde{\tau}^{\tilde{\mu}}} (int_{\tilde{\tau}^{\tilde{\mu}}} (\mathcal{S}_j^c, r), r), r) \right) &\leq 1 - r \end{aligned}$$

Since, for $\tilde{\tau}^{\tilde{\nu}}(\mathcal{S}_j) \geq r$, $\tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j) \leq 1 - r$, $\tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j) \leq 1 - r$ we have $C_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}}(int_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}}(C_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_j, r), r), r) = C_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_j, r)$, and hence, $\bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}}(C_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}}(int_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_j^c, r), r), r) = [\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_j, r)]^c$. Therefore, $\tilde{J}^{\tilde{\nu}}([\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\nu}}}(\mathcal{S}_j, r)]^c) \geq r$, $\tilde{J}^{\tilde{\eta}}([\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j, r)]^c) \leq 1 - r$, $\tilde{J}^{\tilde{\mu}}([\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j, r)]^c) \leq 1 - r$. Hence, $(\tilde{\tau}, \tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}, \tilde{J}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu})$ is r -SVN $\mathcal{J}$ -quasi H -closed,

(6) $\Leftrightarrow$ (4) is proved similarly like (3) $\Leftrightarrow$ (1).

Theorem 4.17. Let $(\tilde{\tau}, \tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}, \tilde{J}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu})$ be an SVN $\mathcal{JTS}$ and $r \in I_0$. Then the next statements are equivalent:

- (1) $(\tilde{\tau}, \tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}, \tilde{J}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu})$ is r -SVN $\mathcal{J}$ -quasi H -closed,
- (2) For any family $\{\mathcal{S}_j \in I^{\tilde{\tau}}: \mathcal{S}_j \leq int_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}}(C_{\tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}}(\mathcal{S}_j, r), r)\}$ with $\bigcup_{j \in \Gamma} \mathcal{S}_j = \tilde{1}$, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ such that $\tilde{J}^{\tilde{\nu}}([\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\nu}}}(\mathcal{S}_j, r)]^c) \geq r$, $\tilde{J}^{\tilde{\eta}}([\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j, r)]^c) \leq 1 - r$, $\tilde{J}^{\tilde{\mu}}([\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j, r)]^c) \leq 1 - r$,
- (3) For any family $\{\mathcal{S}_j \in I^{\tilde{\tau}}: \tilde{\tau}^{\tilde{\nu}}(\mathcal{S}_j^c) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}_j^c) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}_j^c) \leq 1 - r, j \in \Gamma\}$ such that $\bigcap_{j \in \Gamma} \mathcal{S}_j = \tilde{0}$, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ such that $\tilde{J}^{\tilde{\nu}}(\bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\nu}}}(\mathcal{S}_j, r)) \geq r$, $\tilde{J}^{\tilde{\eta}}(\bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{S}_j, r)) \leq 1 - r$, $\tilde{J}^{\tilde{\mu}}(\bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{S}_j, r)) \leq 1 - r$.

Proof. Obvious.

Theorem 4.18. Let $(\tilde{\tau}, \tilde{\tau}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu}, \tilde{J}^{\tilde{\nu}}\tilde{\eta}\tilde{\mu})$ be an SVN $\mathcal{JTS}$ and $r \in I_0$. Then the next statements are equivalent:

- (1) $(\mathfrak{T}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\gamma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is $r - SVN C(\mathcal{I}) - compact$,
- (2) For each family $\{\mathcal{E}_j \in I^{\mathfrak{T}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}_j^c) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}_j^c) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}_j^c) \leq 1 - r, j \in \Gamma\}$ and every $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1 - r$ with $\bigcap_{j \in \Gamma} \mathcal{E}_j \bar{q} \mathcal{S}$, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ such that $\tilde{\gamma}^{\tilde{\gamma}}(\mathcal{S} \cap \bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j, r)) \geq r, \tilde{\gamma}^{\tilde{\eta}}(\mathcal{S} \cap \bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j, r)) \leq 1 - r, \tilde{\gamma}^{\tilde{\mu}}(\mathcal{S} \cap \bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j, r)) \leq 1 - r$.
- (3) $\bigcap_{j \in \Gamma} \mathcal{E}_j q \mathcal{S}$ holds for each family $\{\mathcal{E}_j \in I^{\mathfrak{T}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}_j^c) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}_j^c) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}_j^c) \leq 1 - r, j \in \Gamma\}$ and any $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1 - r$ with $\{int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r) q \mathcal{S}, j \in \Gamma\}$ has the **I - FIP**,
- (4) For each family $\{\mathcal{E}_j \in I^{\mathfrak{T}}: \mathcal{E}_j \text{ is } r - SVNRO, j \in \Gamma\}$ and any $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1 - r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1 - r$ with $\mathcal{S} \leq \bigcup_{j \in \Gamma} \mathcal{E}_j$, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ such that,

$$\tilde{\gamma}^{\tilde{\gamma}}\left(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j, r)\right]^c\right) \geq r, \tilde{\gamma}^{\tilde{\eta}}\left(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j, r)\right]^c\right) \leq 1 - r, \tilde{\gamma}^{\tilde{\mu}}\left(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j, r)\right]^c\right) \leq 1 - r.$$

- (5) For each family $\{\mathcal{E}_j \in I^{\mathfrak{T}}: \mathcal{E}_j \text{ is } r - SVNRC, j \in \Gamma\}$ and any $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1 - r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1 - r$, with $\bigcap_{j \in \Gamma} \mathcal{E}_j \bar{q} \mathcal{S}$, there exists $\Gamma_0 \subseteq \Gamma$ such that,

$$\tilde{\gamma}^{\tilde{\gamma}}\left(\bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j, r) \cap \mathcal{S}\right) \geq r, \tilde{\gamma}^{\tilde{\eta}}\left(\bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j, r) \cap \mathcal{S}\right) \leq 1 - r, \tilde{\gamma}^{\tilde{\mu}}\left(\bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j, r) \cap \mathcal{S}\right) \leq 1 - r,$$

- (6) $\bigcap_{j \in \Gamma} \mathcal{E}_j q \mathcal{S}$ holds for each family $\{\mathcal{E}_j \in I^{\mathfrak{T}}: \mathcal{E}_j \text{ is } r - SVNRC, j \in \Gamma\}$ and any $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1 - r$ such that $\{int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r) \cap \mathcal{S}: j \in \Gamma\}$ has the **I - FIP**.

Proof. (1) $\Rightarrow$ (2). Let $\{\mathcal{E}_j \in I^{\mathfrak{T}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}_j^c) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}_j^c) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}_j^c) \leq 1 - r, j \in \Gamma\}$ and $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1 - r$ with $\bigcap_{j \in \Gamma} \mathcal{E}_j \bar{q} \mathcal{S}$. Then, $\tilde{\gamma}_{\bigcap_{j \in \Gamma} \mathcal{E}_j} + \tilde{\gamma}_{\mathcal{S}} \leq 1, \tilde{\eta}_{\bigcap_{j \in \Gamma} \mathcal{E}_j} + \tilde{\eta}_{\mathcal{S}} \geq 1, \tilde{\mu}_{\bigcap_{j \in \Gamma} \mathcal{E}_j} + \tilde{\mu}_{\mathcal{S}} \geq 1$. It implies that $\mathcal{S} \leq \bigcup_{j \in \Gamma} \mathcal{E}_j^c$. By $r - SVN C(\mathcal{I}) - compactness$ of $(\mathfrak{T}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\gamma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ such that,

$$\tilde{\gamma}^{\tilde{\gamma}}\left(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j^c, r)\right]^c\right) \geq r, \tilde{\gamma}^{\tilde{\eta}}\left(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j^c, r)\right]^c\right) \leq 1 - r, \tilde{\gamma}^{\tilde{\mu}}\left(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j^c, r)\right]^c\right) \leq 1 - r.$$

Since, $\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j^c, r)\right]^c = \mathcal{S} \cap \bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r)$. Then

$$\tilde{\gamma}^{\tilde{\gamma}}\left(\mathcal{S} \cap \bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j, r)\right) \geq r, \tilde{\gamma}^{\tilde{\eta}}\left(\mathcal{S} \cap \bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j, r)\right) \leq 1 - r, \tilde{\gamma}^{\tilde{\mu}}\left(\mathcal{S} \cap \bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j, r)\right) \leq 1 - r.$$

(2) $\Rightarrow$ (3). It is trivial.

(3) $\Rightarrow$ (1). Let $\{\mathcal{E}_j \in I^{\mathfrak{T}}: \tilde{\tau}^{\tilde{\gamma}}(\mathcal{E}_j) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{E}_j) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{E}_j) \leq 1 - r, j \in \Gamma\}$ be a family and $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1 - r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1 - r$ such that $\mathcal{S} \leq \bigcup_{j \in \Gamma} \mathcal{E}_j$ with property that for no finite subfamily Γ_0 of Γ one has, $\tilde{\gamma}^{\tilde{\gamma}}(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j, r)\right]^c) \geq r, \tilde{\gamma}^{\tilde{\eta}}(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j, r)\right]^c) \leq 1 - r, \tilde{\gamma}^{\tilde{\mu}}(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j, r)\right]^c) \leq 1 - r$. Since, $\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j, r)\right]^c = \bigcap_{j \in \Gamma_0} \{int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j^c, r) \cap \mathcal{S}\}$, the family $\{\bigcap_{j \in \Gamma_0} \{int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j^c, r) \cap \mathcal{S}, j \in \Gamma\}$ has the **I - FIP**. By (3), $\bigcap_{j \in \Gamma} \mathcal{E}_j^c q \mathcal{S}$ implies that $\bigcup_{j \in \Gamma} \mathcal{E}_j \leq \mathcal{S}$. It is a contradiction.

(1) $\Rightarrow$ (4). Let $\{\mathcal{E}_j \in I^{\mathfrak{T}}: j \in \Gamma\}$ be a family of $r - SVNRO$ sets and $\tilde{\tau}^{\tilde{\gamma}}(\mathcal{S}^c) \geq r, \tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1 - r, \tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1 - r$ with $\mathcal{S} \leq \bigcup_{j \in \Gamma} \mathcal{E}_j$. Then, $\mathcal{S} \leq \bigcup_{j \in \Gamma} int_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r), r)$. By $r - SVN C(\mathcal{I}) - compactness$ of $(\mathfrak{T}, \tilde{\tau}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\gamma}^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$, there exists a finite subset $\Gamma_0 \subseteq \Gamma$ such that,

$$\tilde{\gamma}^{\tilde{\gamma}}\left(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\gamma}}}(int_{\tilde{\tau}^{\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j, r), r), r)\right]^c\right) \geq r, \tilde{\gamma}^{\tilde{\eta}}\left(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(int_{\tilde{\tau}^{\tilde{\mu}}}(C_{\tilde{\tau}^{\tilde{\gamma}}}(\mathcal{E}_j, r), r), r)\right]^c\right) \leq 1 - r,$$

$$\tilde{J}^{\tilde{\mu}}\left(\mathcal{S} \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j, r), r)\right]^c\right) \leq 1 - r$$

Since, for $\tilde{\tau}^{\tilde{\nu}}(\mathcal{E}_j) \geq r$, $\tilde{\tau}^{\tilde{\eta}}(\mathcal{E}_j) \leq 1 - r$, $\tilde{\tau}^{\tilde{\mu}}(\mathcal{E}_j) \leq 1 - r$, $C_{\tilde{\tau}^{\tilde{\eta}\tilde{\mu}}}(int_{\tilde{\tau}^{\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r), r) = C_{\tilde{\tau}^{\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r)$. Therefore, $\tilde{J}^{\tilde{\nu}}(\mathcal{S} \cap [\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\nu}}}(\mathcal{E}_j, r)]^c) \geq r$, $\tilde{J}^{\tilde{\eta}}(\mathcal{S} \cap [\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j, r)]^c) \leq 1 - r$, $\tilde{J}^{\tilde{\mu}}(\mathcal{S} \cap [\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j, r)]^c) \leq 1 - r$.

(4) $\Rightarrow$ (1). It is trivial.

(4) $\Rightarrow$ (5). Let $\{\mathcal{E}_j\}_{j \in \Gamma}$ be a family of r -SVNRC sets and every $\tilde{\tau}^{\tilde{\nu}}(\mathcal{S}^c) \geq r$, $\tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1 - r$, $\tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1 - r$ such that $\bigcap_{j \in \Gamma} \mathcal{E}_j \bar{q} \mathcal{S}$. Then, $\mathcal{S} \leq \bigcup_{j \in \Gamma} \mathcal{E}_j^c$ and $\{\mathcal{E}_j^c \in I^{\tilde{\tau}}: j \in \Gamma\}$ be a family of r -SVNRO sets. By (4), there exists a finite subset $\Gamma_0 \subseteq \Gamma$ such that $\tilde{J}^{\tilde{\nu}}(\mathcal{S} \cap [\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\nu}}}(\mathcal{E}_j^c, r)]^c) \geq r$, $\tilde{J}^{\tilde{\eta}}(\mathcal{S} \cap [\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j^c, r)]^c) \leq 1 - r$, $\tilde{J}^{\tilde{\mu}}(\mathcal{S} \cap [\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j^c, r)]^c) \leq 1 - r$ implies that

$$\tilde{J}^{\tilde{\nu}}\left(\mathcal{S} \cap \bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\nu}}}(\mathcal{E}_j, r)\right) \geq r, \quad \tilde{J}^{\tilde{\eta}}\left(\mathcal{S} \cap \bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j, r)\right) \leq 1 - r, \quad \tilde{J}^{\tilde{\mu}}\left(\mathcal{S} \cap \bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j, r)\right) \leq 1 - r.$$

(5) $\Rightarrow$ (6). Let $\{\mathcal{E}_j\}_{j \in \Gamma}$ be a family of r -SVNRC sets and every $\tilde{\tau}^{\tilde{\nu}}(\mathcal{S}^c) \geq r$, $\tilde{\tau}^{\tilde{\mu}}(\mathcal{S}^c) \leq 1 - r$, $\tilde{\tau}^{\tilde{\eta}}(\mathcal{S}^c) \leq 1 - r$ such that $\{int_{\tilde{\tau}^{\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r) \cap \mathcal{S}: j \in \Gamma\}$ has the **I-FIP**. If $\bigcap_{j \in \Gamma} \mathcal{E}_j \bar{q} \mathcal{S}$. By (5), there exists a finite subset $\Gamma_0 \subseteq \Gamma$ such that $\tilde{J}^{\tilde{\nu}}(\bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\nu}}}(\mathcal{E}_j, r) \cap \mathcal{S}) \geq r$, $\tilde{J}^{\tilde{\eta}}(\bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\eta}}}(\mathcal{E}_j, r) \cap \mathcal{S}) \leq 1 - r$, $\tilde{J}^{\tilde{\mu}}(\bigcap_{j \in \Gamma_0} int_{\tilde{\tau}^{\tilde{\mu}}}(\mathcal{E}_j, r) \cap \mathcal{S}) \leq 1 - r$. It is a contradiction.

(6) $\Rightarrow$ (4). It is trivial.

Theorem 4.19. Let $(\tilde{\mathcal{X}}_1, \tilde{\tau}_1^{\tilde{\nu}\tilde{\eta}\tilde{\mu}}, \tilde{J}_1^{\tilde{\nu}\tilde{\eta}\tilde{\mu}})$, $(\tilde{\mathcal{X}}_2, \tilde{\tau}_2^{\tilde{\nu}\tilde{\eta}\tilde{\mu}}, \tilde{J}_2^{\tilde{\nu}\tilde{\eta}\tilde{\mu}})$ be two SVNJS's and $f: \tilde{\mathcal{X}}_1 \rightarrow \tilde{\mathcal{X}}_2$ a surjective SVN-continuous. If $(\tilde{\mathcal{X}}_1, \tilde{\tau}_1^{\tilde{\nu}\tilde{\eta}\tilde{\mu}}, \tilde{J}_1^{\tilde{\nu}\tilde{\eta}\tilde{\mu}})$ is r -SVNJ₁-compact and $\tilde{J}_1^{\tilde{\nu}}(\mathcal{S}) \leq \tilde{J}_2^{\tilde{\nu}}(f(\mathcal{S}))$, $\tilde{J}_1^{\tilde{\eta}}(\mathcal{S}) \geq \tilde{J}_2^{\tilde{\eta}}(f(\mathcal{S}))$, $\tilde{J}_1^{\tilde{\mu}}(\mathcal{S}) \geq \tilde{J}_2^{\tilde{\mu}}(f(\mathcal{S}))$. Then, $(\tilde{\mathcal{X}}_2, \tilde{\tau}_2^{\tilde{\nu}\tilde{\eta}\tilde{\mu}}, \tilde{J}_2^{\tilde{\nu}\tilde{\eta}\tilde{\mu}})$ is r -SVNJ₂-compact.

Proof. Let $\{\mathcal{E}_j \in I^{\tilde{\tau}}: \tilde{\tau}_2^{\tilde{\nu}}(\mathcal{E}_j) \geq r, \tilde{\tau}_2^{\tilde{\eta}}(\mathcal{E}_j) \leq 1 - r, \tilde{\tau}_2^{\tilde{\mu}}(\mathcal{E}_j) \leq 1 - r, j \in \Gamma\}$ be a family such that $\bigcup_{j \in \Gamma} \mathcal{E}_j = \tilde{\mathcal{I}}$. Then, $\bigcup_{j \in \Gamma} f^{-1}(\mathcal{E}_j) = \tilde{\mathcal{I}}$. Since, f is SVN-continuous, for each $j \in \Gamma$, $\tilde{\tau}_1^{\tilde{\nu}}(f^{-1}(\mathcal{E}_j)) \geq r$, $\tilde{\tau}_1^{\tilde{\eta}}(f^{-1}(\mathcal{E}_j)) \leq 1 - r$, $\tilde{\tau}_1^{\tilde{\mu}}(f^{-1}(\mathcal{E}_j)) \leq 1 - r$. By r -SVNJ₁-compactness of $(\tilde{\mathcal{X}}_1, \tilde{\tau}_1^{\tilde{\nu}\tilde{\eta}\tilde{\mu}}, \tilde{J}_1^{\tilde{\nu}\tilde{\eta}\tilde{\mu}})$, there exists a finite $\Gamma_0 \subseteq \Gamma$ such that $\tilde{J}_1^{\tilde{\nu}}([\bigcup_{j \in \Gamma_0} f^{-1}(\mathcal{E}_j)]^c) \geq r$, $\tilde{J}_1^{\tilde{\eta}}([\bigcup_{j \in \Gamma_0} f^{-1}(\mathcal{E}_j)]^c) \leq 1 - r$, $\tilde{J}_1^{\tilde{\mu}}([\bigcup_{j \in \Gamma_0} f^{-1}(\mathcal{E}_j)]^c) \leq 1 - r$. Since $\tilde{J}_1^{\tilde{\nu}}(\mathcal{S}) \leq \tilde{J}_2^{\tilde{\nu}}(f(\mathcal{S}))$, $\tilde{J}_1^{\tilde{\eta}}(\mathcal{S}) \geq \tilde{J}_2^{\tilde{\eta}}(f(\mathcal{S}))$, $\tilde{J}_1^{\tilde{\mu}}(\mathcal{S}) \geq \tilde{J}_2^{\tilde{\mu}}(f(\mathcal{S}))$, for $j \in \Gamma_0$, $\tilde{J}_2^{\tilde{\nu}}(f([\bigcup_{j \in \Gamma_0} f^{-1}(\mathcal{E}_j)]^c)) \geq r$, $\tilde{J}_2^{\tilde{\eta}}(f([\bigcup_{j \in \Gamma_0} f^{-1}(\mathcal{E}_j)]^c)) \leq 1 - r$, $\tilde{J}_2^{\tilde{\mu}}(f([\bigcup_{j \in \Gamma_0} f^{-1}(\mathcal{E}_j)]^c)) \leq 1 - r$. From the surjectivity of f we obtain $f([\bigcup_{j \in \Gamma_0} f^{-1}(\mathcal{E}_j)]^c) = [\bigcup_{j \in \Gamma_0} \mathcal{E}_j]^c$. Hence, $\tilde{J}_2^{\tilde{\nu}}([\bigcup_{j \in \Gamma_0} \mathcal{E}_j]^c) \geq r$, $\tilde{J}_2^{\tilde{\eta}}([\bigcup_{j \in \Gamma_0} \mathcal{E}_j]^c) \leq 1 - r$, $\tilde{J}_2^{\tilde{\mu}}([\bigcup_{j \in \Gamma_0} \mathcal{E}_j]^c) \leq 1 - r$. Thus, $(\tilde{\mathcal{X}}_2, \tilde{\tau}_2^{\tilde{\nu}\tilde{\eta}\tilde{\mu}}, \tilde{J}_2^{\tilde{\nu}\tilde{\eta}\tilde{\mu}})$ is r -SVNJ₂-compact.

Theorem 4.20. Let $(\tilde{\mathcal{X}}_1, \tilde{\tau}_1^{\tilde{\nu}\tilde{\eta}\tilde{\mu}}, \tilde{J}_1^{\tilde{\nu}\tilde{\eta}\tilde{\mu}})$, $(\tilde{\mathcal{X}}_2, \tilde{\tau}_2^{\tilde{\nu}\tilde{\eta}\tilde{\mu}}, \tilde{J}_2^{\tilde{\nu}\tilde{\eta}\tilde{\mu}})$ be two SVNJS's and $f: \tilde{\mathcal{X}}_1 \rightarrow \tilde{\mathcal{X}}_2$ a surjective SVN-continuous. If $(\tilde{\mathcal{X}}_1, \tilde{\tau}_1^{\tilde{\nu}\tilde{\eta}\tilde{\mu}}, \tilde{J}_1^{\tilde{\nu}\tilde{\eta}\tilde{\mu}})$ is r -SVNC(J)₁-compact and $\tilde{J}_1^{\tilde{\nu}}(\mathcal{S}) \leq \tilde{J}_2^{\tilde{\nu}}(f(\mathcal{S}))$, $\tilde{J}_1^{\tilde{\eta}}(\mathcal{S}) \geq \tilde{J}_2^{\tilde{\eta}}(f(\mathcal{S}))$, $\tilde{J}_1^{\tilde{\mu}}(\mathcal{S}) \geq \tilde{J}_2^{\tilde{\mu}}(f(\mathcal{S}))$. Then, $(\tilde{\mathcal{X}}_2, \tilde{\tau}_2^{\tilde{\nu}\tilde{\eta}\tilde{\mu}}, \tilde{J}_2^{\tilde{\nu}\tilde{\eta}\tilde{\mu}})$ is r -SVNC(J)₂-compact.

Proof. Let $\tilde{\tau}_2^{\tilde{\nu}}(\mathcal{S}) \geq r$, $\tilde{\tau}_2^{\tilde{\eta}}(\mathcal{S}) \leq 1 - r$, $\tilde{\tau}_2^{\tilde{\mu}}(\mathcal{S}) \leq 1 - r$ and every family $\{\mathcal{E}_j \in I^{\tilde{\tau}}: \tilde{\tau}_2^{\tilde{\nu}}(\mathcal{E}_j) \geq r, \tilde{\tau}_2^{\tilde{\eta}}(\mathcal{E}_j) \leq 1 - r\}$ with $\mathcal{S} \leq \bigcup_{j \in \Gamma} \mathcal{E}_j$. Then, $f^{-1}(\mathcal{S}) \leq \bigcup_{j \in \Gamma} f^{-1}(\mathcal{E}_j)$. Since, f is SVN-continuous for each $j \in \Gamma$, $\tilde{\tau}_1^{\tilde{\nu}}(f^{-1}(\mathcal{E}_j)) \geq r$, $\tilde{\tau}_1^{\tilde{\eta}}(f^{-1}(\mathcal{E}_j)) \leq 1 - r$, $\tilde{\tau}_1^{\tilde{\mu}}(f^{-1}(\mathcal{E}_j)) \leq 1 - r$. By r -SVNC(J)₁-compactness of $(\tilde{\mathcal{X}}_1, \tilde{\tau}_1^{\tilde{\nu}\tilde{\eta}\tilde{\mu}}, \tilde{J}_1^{\tilde{\nu}\tilde{\eta}\tilde{\mu}})$, there exists a finite $\Gamma_0 \subseteq \Gamma$ such that

$$\begin{aligned} \tilde{J}_1^{\tilde{\gamma}} \left(f^{-1}(\mathcal{S}) \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}_1^{\tilde{\gamma}}}(f^{-1}(\mathcal{E}_j), r) \right]^c \right) &\geq r, & \tilde{J}_1^{\tilde{\eta}} \left(f^{-1}(\mathcal{S}) \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}_1^{\tilde{\eta}}}(f^{-1}(\mathcal{E}_j), r) \right]^c \right) &\leq 1 - r, \\ \tilde{J}_1^{\tilde{\mu}} \left(f^{-1}(\mathcal{S}) \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}_1^{\tilde{\mu}}}(f^{-1}(\mathcal{E}_j), r) \right]^c \right) &\leq 1 - r. \end{aligned}$$

Since, f is $\mathcal{SVN}$ -continuous mapping, $C_{\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(f^{-1}(\mathcal{S}_j), r) \leq f^{-1}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{S}_j), r)$ for every $\mathcal{S} \in I^{\tilde{\mathcal{X}}_2}$. Therefore, $f^{-1}(\mathcal{S}) \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(f^{-1}(\mathcal{E}_j), r) \right]^c = f^{-1}(\mathcal{S}) \cap \left[\bigcup_{j \in \Gamma_0} f^{-1}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j), r) \right]^c$. Hence,

$$\begin{aligned} \tilde{J}_1^{\tilde{\gamma}} \left(f^{-1}(\mathcal{S}_j) \cap \left[\bigcup_{j \in \Gamma_0} f^{-1}(C_{\tilde{\tau}_2^{\tilde{\gamma}}}(\mathcal{S}, r)) \right]^c \right) &\geq r, & \tilde{J}_1^{\tilde{\eta}} \left(f^{-1}(\mathcal{S}_j) \cap \left[\bigcup_{j \in \Gamma_0} f^{-1}(C_{\tilde{\tau}_2^{\tilde{\eta}}}(\mathcal{S}, r)) \right]^c \right) &\leq 1 - r, \\ \tilde{J}_1^{\tilde{\mu}} \left(f^{-1}(\mathcal{S}_j) \cap \left[\bigcup_{j \in \Gamma_0} f^{-1}(C_{\tilde{\tau}_2^{\tilde{\mu}}}(\mathcal{S}, r)) \right]^c \right) &\leq 1 - r. \end{aligned}$$

Since, $\tilde{J}_1^{\tilde{\gamma}}(\mathcal{S}) \leq \tilde{J}_2^{\tilde{\gamma}}(f(\mathcal{S}))$, $\tilde{J}_1^{\tilde{\eta}}(\mathcal{S}) \geq \tilde{J}_2^{\tilde{\eta}}(f(\mathcal{S}))$, $\tilde{J}_1^{\tilde{\mu}}(\mathcal{S}) \geq \tilde{J}_2^{\tilde{\mu}}(f(\mathcal{S}))$, for each $j \in \Gamma_0$ we have,

$$\begin{aligned} \tilde{J}_2^{\tilde{\gamma}} \left(f[f^{-1}(\mathcal{S}_j) \cap \left[\bigcup_{j \in \Gamma_0} f^{-1}(C_{\tilde{\tau}_2^{\tilde{\gamma}}}(\mathcal{S}, r)) \right]^c] \right) &\geq r, & \tilde{J}_2^{\tilde{\eta}} \left(f[f^{-1}(\mathcal{S}_j) \cap \left[\bigcup_{j \in \Gamma_0} f^{-1}(C_{\tilde{\tau}_2^{\tilde{\eta}}}(\mathcal{S}, r)) \right]^c] \right) &\leq 1 - r, \\ \tilde{J}_2^{\tilde{\mu}} \left(f[f^{-1}(\mathcal{S}_j) \cap \left[\bigcup_{j \in \Gamma_0} f^{-1}(C_{\tilde{\tau}_2^{\tilde{\mu}}}(\mathcal{S}, r)) \right]^c] \right) &\leq 1 - r. \end{aligned}$$

Since, f is surjective,

$$\tilde{J}_2^{\tilde{\gamma}} \left(\mathcal{S}_j \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}_2^{\tilde{\gamma}}}(\mathcal{S}, r) \right]^c \right) \geq r, \quad \tilde{J}_2^{\tilde{\eta}} \left(\mathcal{S}_j \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}_2^{\tilde{\eta}}}(\mathcal{S}, r) \right]^c \right) \leq 1 - r, \quad \tilde{J}_2^{\tilde{\mu}} \left(\mathcal{S}_j \cap \left[\bigcup_{j \in \Gamma_0} C_{\tilde{\tau}_2^{\tilde{\mu}}}(\mathcal{S}, r) \right]^c \right) \leq 1 - r.$$

Thus, $(\tilde{\mathcal{X}}_2, \tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{J}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ is $r - \mathcal{SVN}(\mathcal{I})_2 - compact$.

Theorem 4.21. The image of an $r - \mathcal{SVN}\mathcal{I}_1 - compact$ under a surjective $\mathcal{SVN} - almost continuous$ mapping and $\tilde{J}_1^{\tilde{\gamma}}(\mathcal{S}) \leq \tilde{J}_2^{\tilde{\gamma}}(f(\mathcal{S}))$, $\tilde{J}_1^{\tilde{\eta}}(\mathcal{S}) \geq \tilde{J}_2^{\tilde{\eta}}(f(\mathcal{S}))$, $\tilde{J}_1^{\tilde{\mu}}(\mathcal{S}) \geq \tilde{J}_2^{\tilde{\mu}}(f(\mathcal{S}))$ is $r - \mathcal{SVN}\mathcal{C}(\mathcal{I})_2 - compact$.

Proof. Let $\mathcal{S} \in I^{\tilde{\mathcal{X}}_1}$ be an $r - \mathcal{SVN}\mathcal{I}_1 - compact$ in $(\tilde{\mathcal{X}}_1, \tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{J}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ and $f: (\tilde{\mathcal{X}}_1, \tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{J}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}) \rightarrow (\tilde{\mathcal{X}}_2, \tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{J}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$ a surjective $\mathcal{SVN} - almost continuous$. If $\tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{S}^c) \geq r$, $\tilde{\tau}_2^{\tilde{\eta}}(\mathcal{S}^c) \leq 1 - r$, $\tilde{\tau}_2^{\tilde{\mu}}(\mathcal{S}^c) \leq 1 - r$ and each family $\{\mathcal{E}_j \in I^{\tilde{\mathcal{X}}_2} : \tilde{\tau}_2^{\tilde{\gamma}}(\mathcal{E}_j) \geq r, \tilde{\tau}_2^{\tilde{\eta}}(\mathcal{E}_j) \leq 1 - r, \tilde{\tau}_2^{\tilde{\mu}}(\mathcal{E}_j) \leq 1 - r\}$ with $f(\mathcal{S}) \leq \bigcup_{j \in \Gamma} \mathcal{E}_j$, then $f(\mathcal{S}) \leq \bigcup_{j \in \Gamma} int_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r), r)$ and

since for $j \in \Gamma$,

$$int_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(int_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r), r), r) = int_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r), r).$$

By $\mathcal{SVN} - almost continuous$ of f we have $\mathcal{S} \leq \bigcup_{j \in \Gamma} f^{-1}(int_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r), r))$ and

$$\tilde{\tau}_1^{\tilde{\gamma}}(f^{-1}(int_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r), r))) \geq r, \quad \tilde{\tau}_1^{\tilde{\eta}}(f^{-1}(int_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tilde{\tau}_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r), r))) \leq 1 - r,$$

$$\tau_1^{\tilde{\mu}} \left(f^{-1}(\text{int}_{\tau_2^{\tilde{\mu}}}(C_{\tau_2^{\tilde{\mu}}}(\mathcal{E}_j, r), r)) \right) \leq 1 - r.$$

By $r - \mathcal{SVNJ}_1$ - compactness of $\mathcal{S}$ in $(\tilde{\tau}_1, \tau_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}, \tilde{\tau}_1^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}})$, there exists a finite $\Gamma_0 \subseteq \Gamma$ such that

$$\tilde{\tau}_1^{\tilde{\gamma}} \left(\mathcal{S}_j \cap \left[\bigcup_{j \in \Gamma_0} f^{-1}(\text{int}_{\tau_2^{\tilde{\gamma}}}(C_{\tau_2^{\tilde{\gamma}}}(\mathcal{E}_j, r), r)) \right]^c \right) \geq r, \quad \tilde{\tau}_1^{\tilde{\eta}} \left(\mathcal{S}_j \cap \left[\bigcup_{j \in \Gamma_0} f^{-1}(\text{int}_{\tau_2^{\tilde{\eta}}}(C_{\tau_2^{\tilde{\eta}}}(\mathcal{E}_j, r), r)) \right]^c \right) \leq 1 - r,$$

$$\tilde{\tau}_1^{\tilde{\mu}} \left(\mathcal{S}_j \cap \left[\bigcup_{j \in \Gamma_0} f^{-1}(\text{int}_{\tau_2^{\tilde{\mu}}}(C_{\tau_2^{\tilde{\mu}}}(\mathcal{E}_j, r), r)) \right]^c \right) \leq 1 - r.$$

Since $\tilde{\tau}_1^{\tilde{\gamma}}(\mathcal{S}) \leq \tilde{\tau}_2^{\tilde{\gamma}}(f(\mathcal{S}))$, $\tilde{\tau}_1^{\tilde{\eta}}(\mathcal{S}) \geq \tilde{\tau}_2^{\tilde{\eta}}(f(\mathcal{S}))$, $\tilde{\tau}_1^{\tilde{\mu}}(\mathcal{S}) \geq \tilde{\tau}_2^{\tilde{\mu}}(f(\mathcal{S}))$, we have

$$\tilde{\tau}_2^{\tilde{\gamma}} \left(f(\mathcal{S}_j \cap \left[\bigcup_{j \in \Gamma_0} f^{-1}(\text{int}_{\tau_2^{\tilde{\gamma}}}(C_{\tau_2^{\tilde{\gamma}}}(\mathcal{E}_j, r), r)) \right]^c) \right) \geq r, \quad \tilde{\tau}_2^{\tilde{\eta}} \left(f(\mathcal{S}_j \cap \left[\bigcup_{j \in \Gamma_0} f^{-1}(\text{int}_{\tau_2^{\tilde{\eta}}}(C_{\tau_2^{\tilde{\eta}}}(\mathcal{E}_j, r), r)) \right]^c) \right) \leq 1 - r,$$

$$\tilde{\tau}_2^{\tilde{\mu}} \left(f(\mathcal{S}_j \cap \left[\bigcup_{j \in \Gamma_0} f^{-1}(\text{int}_{\tau_2^{\tilde{\mu}}}(C_{\tau_2^{\tilde{\mu}}}(\mathcal{E}_j, r), r)) \right]^c) \right) \leq 1 - r.$$

By surjectivity of f , $f(\mathcal{S}_j \cap \left[\bigcup_{j \in \Gamma_0} f^{-1}(\text{int}_{\tau_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(C_{\tau_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r), r)) \right]^c) = f(\mathcal{S}_j) \cap \left[\bigcup_{j \in \Gamma_0} (C_{\tau_2^{\tilde{\gamma}\tilde{\eta}\tilde{\mu}}}(\mathcal{E}_j, r)) \right]^c$. Thus,

$$\tilde{\tau}_2^{\tilde{\gamma}} \left(f(\mathcal{S}_j) \cap \left[\bigcup_{j \in \Gamma_0} (C_{\tau_2^{\tilde{\gamma}}}(\mathcal{E}_j, r)) \right]^c \right) \geq r, \quad \tilde{\tau}_2^{\tilde{\eta}} \left(f(\mathcal{S}_j) \cap \left[\bigcup_{j \in \Gamma_0} (C_{\tau_2^{\tilde{\eta}}}(\mathcal{E}_j, r)) \right]^c \right) \leq 1 - r,$$

$$\tilde{\tau}_2^{\tilde{\mu}} \left(f(\mathcal{S}_j) \cap \left[\bigcup_{j \in \Gamma_0} (C_{\tau_2^{\tilde{\mu}}}(\mathcal{E}_j, r)) \right]^c \right) \leq 1 - r.$$

and hence, $f(\mathcal{S})$ is $r - \mathcal{SVNC}(\mathcal{I})_2$ - compact.

Theorem 4.22. The image of an $r - \mathcal{SVNJ}_1$ - compact under a surjective $\mathcal{SVN}$ - weakly continuous mapping and $\tilde{\tau}_1^{\tilde{\gamma}}(\mathcal{S}) \leq \tilde{\tau}_2^{\tilde{\gamma}}(f(\mathcal{S}))$, $\tilde{\tau}_1^{\tilde{\eta}}(\mathcal{S}) \geq \tilde{\tau}_2^{\tilde{\eta}}(f(\mathcal{S}))$, $\tilde{\tau}_1^{\tilde{\mu}}(\mathcal{S}) \geq \tilde{\tau}_2^{\tilde{\mu}}(f(\mathcal{S}))$, is $r - \mathcal{SVNJ}_2$ -quasi H-closed.

Proof. Similar to proof of Theorem 4.21.

5. Conclusions

In the current research paper, we found some results of single-valued neutrosophic continuous mappings called almost continuous and weakly continuous. These instances are kinds of some generalizations of fuzzy continuity in view of the definition of Šostak. We brought counterexamples whenever such properties fail to be preserved. We also introduced and studied several kinds of r -single-valued neutrosophic compactness defined on the single-valued neutrosophic ideal topological spaces.

References

1. Šostak, A. P. On a fuzzy topological structure. Suppl. Rend. Circ. Matm. Palerms Ser.II. 1985, vol. 11, pp. 89-103

2. Saber, Y. M.; Abdel-Sattar, M. A. Ideals on fuzzy topological spaces. *Applied Mathematical Sciences*, 2014, vol. 8, pp. 1667-1691.
3. Tripathy, B. C.; Ray, G. C. On Mixed fuzzy topological spaces and countability. *Soft Computing*, 2012, vol. 16, no. 10. pp. 1691–1695.
4. Tripathy, B. C.; Ray, G. C. Mixed fuzzy ideal topological spaces. *Applied Mathematics and Computations*. 2013, vol. 220, pp. 602–607.
5. Tripathy, B. C.; Ray, G. C. On δ -continuity in mixed fuzzy topological spaces, *Boletim da Sociedade Paranaense de Matematica*. 2014, vol. 32, no. 2, pp. 175–187.
6. Tripathy, B. C.; Ray, G. C. Weakly continuous functions on mixed fuzzy topological spaces. *Acta Scientiarum. Technology*. 2014, vol. 36, no. 2. pp. 331–335.
7. Saber, Y. M.; Abdel-Sattar, M. A. Ideals on fuzzy topological spaces. *Applied Mathematical Sciences*, 2014, vol. 8, pp. 1667-1691.
8. Zahran, A. M.; Abd El-baki S. A.; Saber Y. M. Decomposition of fuzzy ideal continuity via fuzzy idealization. *International Journal of Fuzzy Logic and Intelligent Systems*. 2009, vol. 9, no. 2. pp. 83-93.
9. Abd El-baki S. A.; Saber Y. M. Fuzzy extremally disconnected ideal topological spaces. *International Journal of Fuzzy Logic and Intelligent Systems*. 2010, vol. 10, no. 1, pp. 1-6.
10. Saber, Y. M.; Alsharari F. Generalized fuzzy ideal closed sets on fuzzy topological spaces in Sostak sense. *International Journal of Fuzzy Logic and Intelligent Systems*. 2018, vol. 18, pp. 161-166.
11. Alsharari, F.; Saber, Y. M. $\mathcal{G}\theta_{\tau_i}^{*rj}$ -Fuzzy closure operator. *New Mathematics and Natural Computation*. 2020, vol. 16, pp. 123-141.
12. Smarandache, F. *Neutrosophy, Neutrisophic Property, Sets, and Logic*. American Research Press: Rehoboth, DE, US. 1998.
13. Fatimah M.; Sarah W. Generalized b Closed Sets and Generalized b Open Sets in Fuzzy Neutrosophic bi-Topological Spaces, *Neutrosophic Sets and Systems*. 2020, vol. 35, pp. 188-197.
14. Riaz, M.; Smarandache, F.; Karaaslan, F.; Hashmi, M.; Nawaz, I. Neutrosophic Soft Rough Topology and its Applications to Multi-Criteria Decision-Making. *Neutrosophic Sets and Systems*. 2020, vol. 35, pp.198-219.
15. Porselvi, K.; Elavarasan, B.; Smarandache F.; Jun, Y. B. Neutrosophic N-bi-ideals in semigroups. *Neutrosophic Sets and Systems*,. 2020, vol. 35, pp. 422-434.
16. Singh, N.; Chakraborty, A.; Biswas, S. B.; Majumdar, M. Impact of Social Media in Banking Sector under Triangular Neutrosophic Arena Using MCGDM Technique. *Neutrosophic Sets and Systems*. 2020, vol. 35, pp.153-176.
17. Salama, A. A.; Broumi, S.; Smarandache, F. Some types of neutrosophic crisp sets and neutrosophic crisp relations. *I. J. Inf. Eng. Electron. Bus.* 2014,
Available online: <http://fs.unm.edu/Neutro-SomeTypeNeutrosophicCrisp.pdf> (accessed on 10 February 2019).
18. Salama, A. A.; Smarandache, F. *Neutrosophic Crisp Set Theory*. The Educational Publisher Columbus: Columbus, OH, USA, 2015.
19. Hur, K.; Lim, P.K.; Lee, J.G.; Kim, J. The category of neutrosophic crisp sets. *Ann. Fuzzy Math. Inform.* 2017, vol. 14, pp. 43-54.
20. Hur, K.; Lim, P.K.; Lee, J.G.; Kim, J. The category of neutrosophic sets. *Neutrosophic Sets and Syst.* 2016, vol. 14, 12-20.
21. Salama, A.A.; Alblowi, S.A. Neutrosophic set and neutrosophic topological spaces. *IOSR J. Math.* 2012, vol. 3, pp. 31-35.
22. Smarandache, F. *A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability and Statistics*. 6th ed.; Info Learn Quest: Ann Arbor, MI, USA, 2007.
23. Ye, J. A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. *J. Intell. Fuzzy Syst.* 2014, vol. 26, pp. 2450-2466
24. Yang, H.L.; Guo, Z.L.; Liao, X. On single valued neutrosophic relations. *J. Intell. Fuzzy Syst.* 2016, vol. 30, pp. 1045-1056.
25. El-Gayyar, M. Smooth Neutrosophic Topological Spaces. *Neutrosophic Sets Syst.* 2016, vol. 65, pp. 65-72.

26. Broumi, S., Topal, S., Bakali, A., Talea, M., & Smarandache, F. A Novel Python Toolbox for Single and Interval-Valued Neutrosophic Matrixs. In *Neutrosophic Sets in Decision Analysis and Operations Research*. (2020) (pp. 281-330). IGI Global,
27. Ye J. Similarity measures between interval neutrosophic sets and their applications in multicriteria decision – making. *J. Intell. Fuzzy Syst.*, (2012), 26 (1) 165 – 172
28. Bakbak D, Uluçay V. Chapter Eight Multiple Criteria Decision Making in Architecture Based on QNeutrosophic Soft Expert Multiset. In *Neutrosophic Triplet Structures*, Pons Editions Brussels, Belgium, EU, 2019 vol. 9, 108 - 124
29. Şahin M., Ecemiş O., Uluçay V. and Kargin A. Some new generalized aggregation operators based on centroid single valued triangular neutrosophic numbers and their applications in multi-attribute decision making, *Asian Journal of Mathematics and Computer Research*. (2017) , 16(2): 63-84
30. Uluçay, V., Kiliç, A., Yildiz, I., & Sahin, M. A new approach for multi-attribute decision-making problems in bipolar neutrosophic sets. *Neutrosophic Sets and Systems*. 2018, 23(1), 12.
31. Bakbak, D., Uluçay, V., & Şahin, M. Neutrosophic soft expert multi set and their application to multiple criteria decision making. *Mathematics*. (2019), 7(1), 50.
32. Wang, H.; Smarandache, F.; Zhang, Y.Q.; Sunderraman, R. Single valued neutrosophic sets. *Multispace Multistruct*. 2010, vol. 4, pp. 410-413.
33. Kim, J.; Lim, P.K.; Lee, J.G.; Hur, K. Single valued neutrosophic relations. *Ann. Fuzzy Math. Inform.* 2018, vol. 16, pp. 201–221.
34. Saber, Y. M.; Alsharari, F.; Smarandache, F. On Single-valued neutrosophic ideals in Šostak sense. *Symmetry*. 2020, vol. 12, 194.
35. Saber Y. M.; Alsharari, F.; Smarandache, F.; Abdel-Sattar M. Connectedness and stratification of single-valued neutrosophic topological spaces. *Symmetry*. 2020, vol. 12, 1464.
36. Alsharari, F. $\mathcal{L}$ -Single valued extremally disconnected ideal neutrosophic topological spaces. *Symmetry*. 2021, 13(1), 53
37. Alsharari, F. Decomposition of single-valued neutrosophic ideal continuity via fuzzy idealization. *Neutrosophic Sets Syst*. 2020, vol. 38, pp. 145-163
38. Abdel-Basst, M., Mohamed, R., & Elhoseny, M. (2020). A model for the effective COVID-19 identification in uncertainty environment using primary symptoms and CT scans. *Health Informatics Journal*, 1460458220952918.
39. Abdel-Basset, M.; Mohamed, R.; Abd El-Nasser, H.; Gamal, A.; Smarandache, F. Solving the supply chain problem using the best-worst method based on a novel Plithogenic model. *Optimization Theory Based on Neutrosophic and Plithogenic Sets*. 2020, (pp. 1-19). Academic Press.
40. Abdel-Basst, M., Mohamed, R., & Elhoseny, M. A novel framework to evaluate innovation value proposition for smart product-service systems. *Environmental Technology & Innovation*. 2020, vol. 20, 101036.
41. Abdel-Basset, M., Gamal, A., Chakraborty, R. K., & Ryan, M. A new hybrid multi-criteria decision-making approach for location selection of sustainable offshore wind energy stations: A case study. *Journal of Cleaner Production*, 280, 124462.
42. Ye, J. A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. *J. Intell. Fuzzy Syst.* 2014, vol. 26, pp. 2450-2466.
43. Yang, H.L.; Guo, Z.L.; Liao, X. On single valued neutrosophic relations. *J. Intell. Fuzzy Syst.* 2016, vol. 30, pp. 1045-1056.

Received: Jan 13, 2021. Accepted: March 10, 2021.