Neutrosophic Sets and Systems

Volume 41 Article 5

3-6-2021

Pentapartitioned Neutrosophic Q-Ideals of Q-Algebra

Suman Das

Rakhal Das

Carlos Granados

Anjan Mukherjee

Neutrosophic Sets and Systems, Vol. 41, 2021
University of New Mexico

Pentapartitioned Neutrosophic *Q*-Ideals of *Q*-Algebra

Suman Das[1], Rakhal Das[2,*], Carlos Granados[3] and Anjan Mukherjee[4]

[1]Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India.

Email: [1]suman.mathematics@tripurauniv.in, sumandas18843@gmail.com

[2]Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India.

Email: [2]rakhaldas95@gmail.com, rakhal.mathematics@tripurauniv.in

[3]Universidad del Atlantico, Barranquilla, Colombia.

Email: [3]carlosgranadosortiz@outlook.es

[4]Department of Mathematics, Tripura University, Agartala, 799022, Tripura, India.

Email: [4]anjan2011_m@tripurauniv.in

*** Correspondence:** rakhaldas95@gmail.com

Abstract: In this paper, we procure the idea of pentapartitioned neutrosophic *Q*-ideal of *Q*-algebra. Then, we formulate some definitions and results on it. Further, we furnish some suitable examples.

Keywords: *Pentapartitioned Neutrosophic Set*; *PN-Q-Algebra*; *PN-Q-Ideal*; *PN-Q-Sub-Algebra*.

1. Introduction

Iseki and Tanaka [15] presented the concept of BCK-algebra in the year 1978. Later on, Negger and Kim [26] established the notion of *d*-algebra by extending the idea of BCK-algebra. In the year 1999, Negger et al. [25] defined the *d*-ideal in *d*-algebra. The notion of fuzzy set (FS) theory was established by Zadeh [28] in the year 1965. Thereafter, Atanassov [4] introduced the idea of intuitionistic fuzzy set (IFS) theory by generalizing the concept of FS. In the year 2013 F. Smarandache [27] extended the neutrosophic set to refined [n-valued] neutrosophic set, and to refined neutrosophic logic, and to refined neutrosophic probability. The notion of fuzzy *d*-ideals of *d*-algebras was studied by Jun et. al. [17] in the year 2000. The idea of intuitionistic fuzzy *d*-algebra was presented by Jun et al. [16]. In the year 2017, the concept of intuitionistic fuzzy *d*-ideal of *d*-algebra was introduced by Hasan [12]. Hasan [13] also studied the intuitionistic fuzzy *d*-filter of *d*-algebra. The notion of *Q*-algebra was grounded by Neggers et. al. [24] in the year 2001. Thereafter, Abdullah and Jawad [1] studied some new types of ideals in *Q*-algebra. Mostafa et. al. [22] introduced the notion of fuzzy *Q*-ideals in *Q*-algebras. Mostafa et. al. [23] also studied the intuitionistic fuzzy *Q*-ideals of *Q*-algebra. In the year 2005, Smarandache [27] grounded the idea of neutrosophic set by extending the IFS. Later on, the notion of neutrosophic *BCI*/*BCK*-algebras was presented by Agboola and Davvaz [2]. In the year 2016, Martina Jency and Arockiarani [21] established the notion of single valued neutrosophic ideals of *BCK*-algebras. In the year 2019, Mallick and Pramanik [20] presented the concept of pentapartitioned neutrosophic set and studied different operations on them. In this article, we procure the idea of pentapartitioned neutrosophic *Q*-ideals of *Q*-algebra.

The rest of the paper is designed as follows:

In section 2, we recall some preliminary definitions and results on *Q*-algebra, *Q*-ideal, fuzzy *Q*-algebra, fuzzy *Q*-ideal, intuitionistic fuzzy *Q*-algebra, intuitionistic fuzzy *Q*-ideal. In section-3, we introduce the notion of pentapartitioned neutrosophic *Q*-ideal of *Q*-algebra by generalizing the theory of intuitionistic fuzzy *Q*-ideal and neutrosophic *Q*-ideal. Further, we formulate some results on pentapartitioned neutrosophic *Q*-ideals of *Q*-algebra. In section 4, we conclude the work done in this article.

2. Relevant Definitions and Results:

Here we procure some basic definition and example which is needed for our work.

F. Smarandache[27] introduced the n-symbolic or numerical-Valued Refined Neutrosophic Logic

In general: T can be split into many types of truths: $T_1, T_2, ..., T_p$, and I into many types of indeterminacies: $I_1, I_2, ..., I_r$, and F into many types of falsities: $F_1, F_2, ..., Fs$,, where all p, r, s ≥ 1 are integers, and $p + r + s = n$.

All subcomponents T_j, I_k, F_l are symbolic or numerical for $j\in\{1,2,\ldots,p\}$, $k\in\{1,2,\ldots,r\}$, and $l\in\{1,2,\ldots,s\}$.

If at least one $I_k = T_j \wedge F_l$ = contradiction, we get again the Extenics.

We use five valued neutrosophic logic which is the particular case of the n-valued neutrosophic logic. The details for the *n*-valued neutrosophic logic one may refer to [27].

Definition 2.1.[20] Assume that W be a fixed set. A pentapartitioned neutrosophic set P over W is defined as follows:

$Y = \{(c, \hat{A}_Y(c), \hat{C}_Y(c), \hat{E}_Y(c), \check{D}_Y(c), \hat{U}_Y(c)) : c \in W\}$, where $\hat{A}_Y(c), \hat{C}_Y(c), \hat{E}_Y(c), \check{D}_Y(c), \hat{U}_Y(c)$ ($\in$]0,1[ ) are the truth, contradiction, ignorance, unknown and falsity membership values of each $c\in W$. So,

$$0 \leq \hat{A}_Y(c)+\hat{C}_Y(c)+\hat{E}_Y(c)+\check{D}_Y(c)+\hat{U}_Y(c) \leq 5.$$

Definition 2.3.[20] Suppose that $X = \{(c, \hat{A}_X(c), \hat{C}_X(c), \hat{E}_X(c), \check{D}_X(c), \hat{U}_X(c)) : c \in W\}$ and $Y = \{(c, \hat{A}_Y(c), \hat{C}_Y(c), \hat{E}_Y(c), \check{D}_Y(c), \hat{U}_Y(c)) : c \in W\}$ be two pentapartitioned neutrosophic sets over W. Then, $X\subseteq Y$ if and only if $\hat{A}_X(c)\leq \hat{A}_Y(c)$, $\hat{C}_X(c)\leq \hat{C}_Y(c)$, $\hat{E}_X(c)\geq \hat{E}_Y(c)$, $\check{D}_X(c)\geq \check{D}_Y(c)$, $\hat{U}_X(c)\geq \hat{U}_Y(c)$, for all $c\in W$.

Definition 2.4.[20] Suppose that $X = \{(c, \hat{A}_X(c), \hat{C}_X(c), \hat{E}_X(c), \check{D}_X(c), \hat{U}_X(c)) : c \in W\}$ and $Y = \{(c, \hat{A}_Y(c), \hat{C}_Y(c), \hat{E}_Y(c), \check{D}_Y(c), \hat{U}_Y(c)) : c \in W\}$ be two pentapartitioned neutrosophic sets over W. Then, $X \cap Y$ =

{(*c*, min {$\hat{A}_X(c)$, $\hat{A}_Y(c)$}, min {$\hat{C}_X(c)$, $\hat{C}_Y(c)$}, max {$\hat{E}_X(c)$, $\hat{E}_Y(c)$}, max {$\check{D}_X(c)$, $\check{D}_Y(c)$}, max {$\hat{U}_X(c)$, $\hat{U}_Y(c)$}): $c \in W$}.

Definition 2.5.[20] Suppose that X = {(c, $\hat{A}_X(c)$, $\hat{C}_X(c)$, $\hat{E}_X(c)$, $\check{D}_X(c)$, $\hat{U}_X(c)$) : c ∈W} and Y = {(c, $\hat{A}_Y(c)$, $\hat{C}_Y(c)$, $\hat{E}_Y(c)$, $\check{D}_Y(c)$, $\hat{U}_Y(c)$) : c ∈W} be two pentapartitioned neutrosophic sets over *W*. Then, $X \cup Y$ = {(*c*, max {$\hat{A}_X(c)$, $\hat{A}_Y(c)$}, max {$\hat{C}_X(c)$, $\hat{C}_Y(c)$}, min {$\hat{E}_X(c)$, $\hat{E}_Y(c)$}, min {$\check{D}_X(c)$, $\check{D}_Y(c)$}, min {$\hat{U}_X(c)$, $\hat{U}_Y(c)$}): $c \in W$}.

Definition 2.6.[20] Suppose that X = {(c, $\hat{A}_X(c)$, $\hat{C}_X(c)$, $\hat{E}_X(c)$, $\check{D}_X(c)$, $\hat{U}_X(c)$) : c ∈W} be a pentapartitioned neutrosophic set over *W*. Then, X^c = {(*c*, $\hat{U}_X(c)$, $\check{D}_X(c)$, 1-$\hat{E}_X(c)$, $\hat{C}_X(c)$, $\hat{A}_X(c)$): $q \in W$}.

Definition 2.1.[15] Suppose that W be a fixed set. Let 0 be a constant in W and ∗ be a binary operation defined on W. Then (W, ∗, 0) is called a BCK-algebra if the following holds:

(i) $((x * y) * (x * z)) * (z * y) = 0$,

(ii) $(x * (x * y)) * y = 0$,

(iii) $x * x = 0$,

(iv) $x * y = y * x = 0 \Rightarrow x = y$,

(v) $0 * x = 0$, for all x, y, z ∈ X .

Definition 2.2.[15] Let W be a BCK-algebra with binary operator ∗ and a constant 0. Then $I \subseteq W$ is called a BCK-ideal of W if the following holds:

(i) $0 \in I$;

(ii) $h * d \in I$ and $d \in I \Rightarrow h \in I$, ∀ h, d∈ W.

Definition 2.3.[22] Suppose that W be a fixed set. Let 0 be a constant in W and ∗ be a binary operation defined on W. Then (W, ∗, 0) is called a *Q*-algebra if the followings hold:

(i) $h * h = 0$, ∀ h ∈ W

(ii) $0 * h = h = h * 0$, ∀ h ∈ W

(iii) $(h * d) * e = (h * e) * d$, ∀ h, d, e ∈ W.

Sometime, one can refer to $h \leq d$ if and only if $h * d = 0$.

Definition 2.4.[24] Let (W, ∗, 0) be a *Q*-algebra. Then, (W, ∗, 0) is said to be commutative *Q*-algebra if $c * (c * d) = d * (d * c)$, ∀c, d ∈W, and $d * (d * c)$ is denoted by $(c \wedge d)$.

Definition 2.5.[24] A *Q*-algebra W is called bounded if there exist g ∈ W such that $h \leq g$ for all h ∈ W, i.e. $h * g = 0$, ∀ h ∈ W.

Definition 2.6.[24] Let W be a *Q*-algebra with binary operator ∗ and H ($\neq 0_N$) ⊆ W. Then, H is called a *Q*-sub-algebra of W, if h, d ∈A implies $h * d \in H$.

Definition 2.7.[1] Let W be a *Q*-algebra with binary operator ∗ and a constant 0. Then, $I \subseteq W$ is called a *Q*-ideal of W if the following holds:

(i) $0 \in I$;

(ii) $(h * d) * e \in I$ and $d \in I \Rightarrow h * e \in I$, for all h, d, e∈ W.

Proposition 2.1.[1] Let (W, ∗, 0) be called a *Q*-algebra. Let I be a *Q*-ideal of W. Then, I be a BCK-ideal of W.

Definition 2.8.[3] A fuzzy set Y={(c,$T_Y(c)$): c∈W} over a BCK-algebra W is called the fuzzy BCK-ideal if the following two conditions holds:

(i) $T_Y(0) \geq T_Y(c)$, for all c∈ W;

(ii) $T_Y(c) \geq \min\{T_Y((c * d), T_Y(d)\}$ for all c, d∈W.

Definition 2.9.[22] A fuzzy set Y={(c,T_Y(c)): c∈W} over a *Q*-algebra W is called the fuzzy *Q*-ideal if the following holds:

(i) $T_Y(0) \geq T_Y(c)$, for all c∈ W;

(ii) $T_Y(c * d) \geq \min\{T_Y((c * h) * d), T_Y(h)\}$ for all c, h, d∈W.

Lemma 2.1.[22] Let (W, ∗, 0) be a *Q*-algebra. If Y = {(c,T_Y(c)): c∈W} is a fuzzy *Q*-ideal of W, then it is also a fuzzy BCK-ideal of W.

Definition 2.10.[23] An intuitionistic fuzzy set Y={(c,T_Y(c),F_Y(c)): c∈W} over a *Q*-algebra W is called the intuitionistic fuzzy *Q*-ideal if it satisfies the following inequalities:

(i) $T_Y(0) \geq T_Y(c)$, $F_Y(0) \leq F_Y(c)$, for all c∈ W;

(ii) $T_Y(c * d) \geq \min\{T_Y((c * h) * d), T_Y(h)\}$;

(iii) $F_Y(c * d) \leq \max\{F_Y((c * h) * d), F_Y(h)\}$.

Lemma 2.2.[23] Let Y={(c,T_Y(c),F_Y(c)): c∈W} be an intuitionistic fuzzy *Q*-ideal over a *Q*-algebra W. If $c * d \leq h$, for all c, d, h ∈W, then $T_Y(c) \geq \min\{T_Y(d), T_Y(h)\}$ and $F_Y(c) \geq \max\{F_Y(d), F_Y(h)\}$, for all c∈W.

Lemma 2.3.[23] Let Y={(c,T_Y(c),F_Y(c)): c∈W} be an intuitionistic fuzzy *Q*-ideal over a *Q*-algebra W. If $c \leq d$, , for all c, d∈ W, then $T_Y(c) \geq T_Y(d)$ and $F_Y(c) \leq F_Y(d)$.

Lemma 2.4.[23] If Y={(c,T_Y(c),F_Y(c)): c∈W} be an intuitionistic fuzzy *Q*-ideal over a *Q*-algebra W, then the sets $\alpha\text{-}T_Y = \{c: c\in W, T_Y(c) \geq \alpha\}$ and $\alpha\text{-}F_Y = \{c: c\in W, F_Y(c) \leq \alpha\}$ are *Q*-ideal of *Q*-algebra W.

3. Pentapartitioned Neutrosophic *Q*-Ideals of *Q*-Algebra:

In this section, we procure the notion of pentapartitioned neutrosophic *Q*-ideal (PN-*Q*-Ideal) of pentapartitioned neutrosophic *Q*-algebra (PN-*Q*-Algebra). Then, we formulate some definitions and results on PN-*Q*-Ideal and PN-*Q*-Algebra.

Definition 3.1. Suppose that *W* be a *Q*-algebra and Y = {(c, $\hat{A}_Y$(c), $\hat{C}_Y$(c), $\hat{E}_Y$(c), $\check{D}_Y$(c), $\hat{U}_Y$(c)) : c ∈W} be a pentapartitioned neutrosophic set over W. Then, Y is said to be a pentapartitioned neutrosophic *Q*-algebra (PN-*Q*-algebra) if and only if the following holds:

(i) $\hat{A}_Y(c * d) \geq \min\{\hat{A}_Y((c * h) * d), \hat{A}_Y(h)\}$;

(ii) $\hat{C}_Y(c * d) \geq \min\{\hat{C}_Y((c * h) * d), \hat{C}_Y(h)\}$;

(iii) $\hat{E}_Y(c * d) \leq \max\{\hat{E}_Y((c * h) * d), \hat{E}_Y(h)$;

(iv) $\check{D}_Y(c * d) \leq \max\{\check{D}_Y((c * h) * d), \check{D}_Y(h)$;

(v) $\hat{U}_Y(c * d) \leq \max\{\hat{U}_Y((c * h) * d), \hat{U}_Y(h)\}$, where c, d∈ W.

By the structure [(W, Y), ∗, 0], we denotes the PN-*Q*-algebra Y over W.

Theorem 3.1. If {Y_i : i∈Δ} be the collection of PN-*Q*-algebra's of W, then, $\bigcap_{i\in\Delta} Y_i$ is also a PN-*Q*-algebra of W.

Proof. Assume that {Y_i : i∈Δ} be a family of PN-*Q*-algebras of W. It is clear that, $\bigcap_{i\in\Delta} Y_i$ = {(c, $\wedge\hat{A}_{Y_i}$(c), $\wedge\hat{C}_{Y_i}$(c), $\vee\hat{E}_{Y_i}$(c), $\vee\check{D}_{Y_i}$(c), $\vee\hat{U}_{Y_i}$(c)) : c∈W}.

Now,

$$\begin{aligned}\wedge\hat{A}_{Y_i}(c * d) &= \wedge\{\hat{A}_{Y_i}(c * d): i\in\Delta\}\\ &\geq \wedge\{\min\{\hat{A}_{Y_i}((c * h) * d), \hat{A}_{Y_i}(h)\}\}\\ &= \min\{\wedge\hat{A}_{Y_i}((c * h) * d), \wedge\hat{A}_{Y_i}(h)\}\end{aligned}$$

$$\Rightarrow \wedge\hat{A}_{Y_i}(c * d) \geq \min\{\wedge\hat{A}_{Y_i}((c * h) * d), \wedge\hat{A}_{Y_i}(h)\}\}.$$

Now,

$\wedge\hat{C}_{Y_i}(c*d) = \wedge\{\hat{C}_{Y_i}(c*d): i\in\Delta\}$

$\geq \wedge\{\min\{\hat{C}_{Y_i}((c*h)*d),\ \hat{C}_{Y_i}(h)\}\}$

$= \min\{\wedge\hat{C}_{Y_i}((c*h)*d), \wedge\hat{C}_{Y_i}(h)\}\}$

$\Rightarrow \wedge\hat{C}_{Y_i}(c*d) \geq \min\{\wedge\hat{C}_{Y_i}((c*h)*d), \wedge\hat{C}_{Y_i}(h)\}\}.$

Now,

$\vee\hat{E}_{Y_i}(c*d) = \vee\{\hat{E}_{Y_i}(c*d): i\in\Delta\}$

$\geq \vee\{\min\{\hat{E}_{Y_i}((c*h)*d),\ \hat{E}_{Y_i}(h)\}\}$

$= \min\{\vee\hat{E}_{Y_i}((c*h)*d), \vee\hat{E}_{Y_i}(h)\}\}$

$\Rightarrow \vee\hat{E}_{Y_i}(c*d) \geq \min\{\vee\hat{E}_{Y_i}((c*h)*d), \vee\hat{E}_{Y_i}(h)\}\}.$

Now,

$\vee\check{D}_{Y_i}(c*d) = \vee\{\check{D}_{Y_i}(c*d): i\in\Delta\}$

$\geq \vee\{\min\{\check{D}_{Y_i}((c*h)*d),\ \check{D}_{Y_i}(h)\}\}$

$= \min\{\vee\check{D}_{Y_i}((c*h)*d), \vee\check{D}_{Y_i}(h)\}\}$

$\Rightarrow \vee\check{D}_{Y_i}(c*d) \geq \min\{\vee\check{D}_{Y_i}((c*h)*d), \vee\check{D}_{Y_i}(h)\}\}.$

Now,

$\vee\hat{U}_{Y_i}(c*d) = \vee\{\hat{U}_{Y_i}(c*d): i\in\Delta\}$

$\geq \vee\{\min\{\hat{U}_{Y_i}((c*h)*d),\ \hat{U}_{Y_i}(h)\}\}$

$= \min\{\vee\hat{U}_{Y_i}((c*h)*d), \vee\hat{U}_{Y_i}(h)\}\}$

$\Rightarrow \vee\hat{U}_{Y_i}(c*d) \geq \min\{\vee\hat{U}_{Y_i}((c*h)*d), \vee\hat{U}_{Y_i}(h)\}\}.$

Therefore, $\bigcap_{i\in\Delta} Y_i$ is also a PN-*Q*-algebra of W.

Definition 3.2. Suppose that Y = {(c, $\hat{A}_Y$(c), $\hat{C}_Y$(c), $\hat{E}_Y$(c), $\check{D}_Y$(c), $\hat{U}_Y$(c)) : c ∈W} be a pentapartitioned neutrosophic set over W. Then, Y is said to be a pentapartitioned neutrosophic *Q*-sub-algebra (PN-*Q*-sub-algebra) if and only if the following holds:

(i) $\hat{A}_Y(c*d) \geq \min\{\hat{A}_Y(c), \hat{A}_Y(d)\}$;

(ii) $\hat{C}_Y(c*d) \geq \min\{\hat{C}_Y(c), \hat{C}_Y(d)\}$;

(iii) $\hat{E}_Y(c*d) \leq \max\{\hat{E}_Y(c), \hat{E}_Y(d)\}$;

(iv) $\check{D}_Y(c*d) \leq \max\{\check{D}_Y(c), \check{D}_Y(d)\}$;

(v) $\hat{U}_Y(c*d) \leq \max\{\hat{U}_Y(c), \hat{U}_Y(d)\}$; where c, d∈ W.

Theorem 3.2. Let Y = {(c, $\hat{A}_Y$(c), $\hat{C}_Y$(c), $\hat{E}_Y$(c), $\check{D}_Y$(c), $\hat{U}_Y$(c)) : c ∈W} be a PN-*Q*-sub-algebra of a *Q*-algebra W. Then, the following holds:

(i) $\hat{A}_Y(0)\geq \hat{A}_Y(c)$, for all c ∈W;

(ii) $\hat{C}_Y(0)\geq \hat{C}_Y(c)$, for all c ∈W;

(iii) $\hat{E}_Y(0)\leq \hat{E}_Y(c)$, for all c ∈W;

(iv) $\check{D}_Y(0)\leq \check{D}_Y(c)$, foĈr all c ∈W;

(v) $\hat{U}_Y(0)\leq \hat{U}_Y(c)$, for all c ∈W.

Proof. Assume that Y = {(c, $\hat{A}_Y$(c), $\hat{C}_Y$(c), $\hat{E}_Y$(c), $\check{D}_Y$(c), $\hat{U}_Y$(c)) : c ∈W} be a PN-*Q*-sub-algebra of a *Q*-algebra W. Hence, $\hat{A}_Y(c*d) \geq \min\{\hat{A}_Y(c), \hat{A}_Y(d)\}$, $\hat{C}_Y(c*d) \geq \min\{\hat{C}_Y(c), \hat{C}_Y(d)\}$, $\hat{E}_Y(c*d) \leq \max\{\hat{E}_Y(c), \hat{E}_Y(d)\}$, $\check{D}_Y(c*d) \leq \max\{\check{D}_Y(c), \check{D}_Y(d)\}$, $\hat{U}_Y(c*d) \leq \max\{\hat{U}_Y(c), \hat{U}_Y(d)\}$, for all c, d∈ W.

Now we have,

$\hat{A}_Y(0) = \hat{A}_Y(c*c)$

$\geq \min\{\hat{A}_Y(c), \hat{A}_Y(c)\}$

$= \hat{A}_Y(c)$

$\Rightarrow \hat{A}_Y(0) \geq \hat{A}_Y(c)$, for all $c \in W$.

$\hat{C}_Y(0) = \hat{C}_Y(c * c)$

$\geq \min\{\hat{C}_Y(c), \hat{C}_Y(c)\}$

$= \hat{C}_Y(c)$

$\Rightarrow \hat{C}_Y(0) \geq \hat{C}_Y(c)$, for all $c \in W$.

$\hat{E}_Y(0) = \hat{E}_Y(c * c)$

$\leq \max\{\hat{E}_Y(c), \hat{E}_Y(d)\}$

$= \hat{E}_Y(c)$

$\Rightarrow \hat{E}_Y(0) \leq \hat{E}_Y(c)$, for all $c \in W$.

$\check{D}_Y(0) = \check{D}_Y(c * c)$

$\leq \max\{\check{D}_Y(c), \check{D}_Y(d)\}$

$= \check{D}_Y(c)$

$\Rightarrow \check{D}_Y(0) \leq \check{D}_Y(c)$, for all $c \in W$.

$\hat{U}_Y(0) = \hat{U}_Y(c * c)$

$\leq \max\{\hat{U}_Y(c), \hat{U}_Y(d)\}$

$= \hat{U}_Y(c)$

$\Rightarrow \hat{U}_Y(0) \leq \hat{U}_Y(c)$, for all $c \in W$.

Definition 3.3. A pentapartitioned neutrosophic set $Y = \{(c, \hat{A}_Y(c), \hat{C}_Y(c), \hat{E}_Y(c), \check{D}_Y(c), \hat{U}_Y(c)) : c \in W\}$ over a *Q*-algebra W is said to be a pentapartitioned neutrosophic *Q*-ideal (PN-*Q*-ideal) if and only if the following inequalities holds:

(i) $\hat{A}_Y(0) \geq \hat{A}_Y(c)$ & $\hat{A}_Y(c * d) \geq \min\{\hat{A}_Y((c * h) * d), \hat{A}_Y(h)\}$, for all $c, d, h \in W$;

(ii) $\hat{C}_Y(0) \geq \hat{C}_Y(c)$ & $\hat{C}_Y(c * d) \geq \min\{\hat{C}_Y((c * h) * d), \hat{C}_Y(h)\}$, for all $c, d, h \in W$;

(iii) $\hat{E}_Y(0) \leq \hat{E}_Y(c)$ & $\hat{E}_Y(c * d) \leq \max\{\hat{E}_Y((c * h) * d), \hat{E}_Y(h)\}$, for all $c, d, h \in W$;

(iv) $\check{D}_Y(0) \leq \check{D}_Y(c)$ & $\check{D}_Y(c * d) \leq \max\{\check{D}_Y((c * h) * d), \check{D}_Y(h)\}$, for all $c, d, h \in W$;

(v) $\hat{U}_Y(0) \leq \hat{U}_Y(c)$ & $\hat{U}_Y(c * d) \leq \max\{\hat{U}_Y((c * h) * d), \hat{U}_Y(h)\}$, for all $c, d, h \in W$.

Remark 3.1. Every PN-*Q*-ideal of a *Q*-algebra W is also a PN-*Q*-sub-algebra.

Theorem 3.3. Suppose that $\{D_i : i \in \Delta\}$ be a family of PN-*Q*-ideals of *Q*-algebra W. Then, $\bigcap_{i\in\Delta} D_i$ is also a PN-*Q*-ideal of *Q*-algebra W.

Proof. Let $\{D_i : i \in \Delta\}$ be a family of PN-*Q*-ideals of *Q*-algebra W. Therefore,

(i) $\hat{A}_{D_i}(0) \geq \hat{A}_{D_i}(c)$ & $\hat{A}_{D_i}(c * d) \geq \min\{\hat{A}_{D_i}((c * h) * d), \hat{A}_{D_i}(h)\}$, for all $c, d, h \in W$ and $i \in \Delta$;

(ii) $\hat{C}_{D_i}(0) \geq \hat{C}_{D_i}(c)$ & $\hat{C}_{D_i}(c * d) \geq \min\{\hat{C}_{D_i}((c * h) * d), \hat{C}_{D_i}(h)\}$, for all $c, d, h \in W$ and $i \in \Delta$;

(iii) $\hat{E}_{D_i}(0) \leq \hat{E}_{D_i}(c)$ & $\hat{E}_{D_i}(c * d) \leq \max\{\hat{E}_{D_i}((c * h) * d), \hat{E}_{D_i}(h)\}$, for all $c, d, h \in W$ and $i \in \Delta$;

(iv) $\check{D}_{D_i}(0) \leq \check{D}_{D_i}(c)$ & $\check{D}_{D_i}(c * d) \leq \max\{\check{D}_{D_i}((c * h) * d), \check{D}_{D_i}(h)\}$, for all $c, d, h \in W$ and $i \in \Delta$;

(v) $\hat{U}_{D_i}(0) \leq \hat{U}_{D_i}(c)$ & $\hat{U}_{D_i}(c * d) \leq \max\{\hat{U}_{D_i}((c * h) * d), \hat{U}_{D_i}(h)\}$, for all $c, d, h \in W$ and $i \in \Delta$.

Clearly, $\bigcap_{i\in\Delta} D_i = \{< c, \wedge\hat{A}_{D_i}(c), \wedge\hat{C}_{D_i}(c), \vee\hat{E}_{D_i}(c), \vee\check{D}_{D_i}(c), \vee\hat{U}_{D_i}(c) > : c \in W\}$.

Now, we have

$\hat{A}_{D_i}(0) \geq \hat{A}_{D_i}(c)$, for all $c \in W$ and $i \in \Delta$

$\Rightarrow \wedge\hat{A}_{D_i}(0) \geq \wedge\hat{A}_{D_i}(c)$.

$\hat{C}_{D_i}(0) \geq \hat{C}_{D_i}(c)$, for all c ∈W and i∈Δ

$\Rightarrow \wedge\hat{C}_{D_i}(0) \geq \wedge\hat{C}_{D_i}(c)$.

$\hat{E}_{D_i}(0) \leq \hat{E}_{D_i}(c)$, for all c ∈W and i∈Δ

$\Rightarrow \vee\hat{E}_{D_i}(0) \leq \vee\hat{E}_{D_i}(c)$.

$\check{D}_{D_i}(0) \leq \check{D}_{D_i}(c)$, for all c ∈W and i∈Δ

$\Rightarrow \vee\check{D}_{D_i}(0) \leq \vee\check{D}_{D_i}(c)$.

$\hat{U}_{D_i}(0) \leq \hat{U}_{D_i}(c)$, for all c ∈W and i∈Δ

$\Rightarrow \vee\hat{U}_{D_i}(0) \leq \vee\hat{U}_{D_i}(c)$.

Further, we have

$\hat{A}_{D_i}(c * d) \geq \min\{\hat{A}_{D_i}((c * h) * d), \hat{A}_{D_i}(h)\}$, for all c, d, h∈W and i∈Δ.

$\Rightarrow \wedge\hat{A}_{D_i}(c * d) \geq \wedge\min\{\hat{A}_{D_i}((c * h) * d), \hat{A}_{D_i}(h)\}$

$= \min\{\wedge\hat{A}_{D_i}((c * h) * d), \wedge\hat{A}_{D_i}(h)\}$

$\Rightarrow \wedge\hat{A}_{D_i}(c * d) \geq \min\{\wedge\hat{A}_{D_i}((c * h) * d), \wedge\hat{A}_{D_i}(h)\}$.

$\hat{C}_{D_i}(c * d) \geq \min\{\hat{C}_{D_i}((c * h) * d), \hat{C}_{D_i}(h)\}$, for all c, d, h∈W and i∈Δ.

$\Rightarrow \wedge\hat{C}_{D_i}(c * d) \geq \wedge\min\{\hat{C}_{D_i}((c * h) * d), \hat{C}_{D_i}(h)\}$

$= \min\{\wedge\hat{C}_{D_i}((c * h) * d), \wedge\hat{C}_{D_i}(h)\}$

$\Rightarrow \wedge\hat{C}_{D_i}(c * d) \geq \min\{\wedge\hat{C}_{D_i}((c * h) * d), \wedge\hat{C}_{D_i}(h)\}$.

$\hat{E}_{D_i}(c * d) \leq \max\{\hat{E}_{D_i}((c * h) * d), \hat{E}_{D_i}(h)\}$, for all c, d, h∈W and i∈Δ.

$\Rightarrow \vee\hat{E}_{D_i}(c * d) \leq \vee\max\{\hat{E}_{D_i}((c * h) * d), \hat{E}_{D_i}(h)\}$

$= \max\{\vee\hat{E}_{D_i}((c * h) * d), \vee\hat{E}_{D_i}(h)\}$

$\Rightarrow \vee\hat{E}_{D_i}(c * d) \leq \max\{\vee\hat{E}_{D_i}((c * h) * d), \vee\hat{E}_{D_i}(h)\}$.

$\check{D}_{D_i}(c * d) \leq \max\{\check{D}_{D_i}((c * h) * d), \check{D}_{D_i}(h)\}$, for all c, d, h∈W and i∈Δ.

$\Rightarrow \vee\check{D}_{D_i}(c * d) \leq \vee\max\{\check{D}_{D_i}((c * h) * d), \check{D}_{D_i}(h)\}$

$= \max\{\vee\check{D}_{D_i}((c * h) * d), \vee\check{D}_{D_i}(h)\}$

$\Rightarrow \vee\check{D}_{D_i}(c * d) \leq \max\{\vee\check{D}_{D_i}((c * h) * d), \vee\check{D}_{D_i}(h)\}$.

$\hat{U}_{D_i}(c * d) \leq \max\{\hat{U}_{D_i}((c * h) * d), \hat{U}_{D_i}(h)\}$, for all c, d, h∈W and i∈Δ.

$\Rightarrow \vee\hat{U}_{D_i}(c * d) \leq \vee\max\{\hat{U}_{D_i}((c * h) * d), \hat{U}_{D_i}(h)\}$

$= \max\{\vee\hat{U}_{D_i}((c * h) * d), \vee\hat{U}_{D_i}(h)\}$

$\Rightarrow \vee\hat{U}_{D_i}(c * d) \leq \max\{\vee\hat{U}_{D_i}((c * h) * d), \vee\hat{U}_{D_i}(h)\}$.

Therefore, $\bigcap_{i\in\Delta} D_i$ is a PN-*Q*-ideal of *Q*-algebra W.

Corollary 3.1. Assume that Y = {(c, $\hat{A}_Y(c)$, $\hat{C}_Y(c)$, $\hat{E}_Y(c)$, $\check{D}_Y(c)$, $\hat{U}_Y(c)$) : c ∈W} be a PN-*Q*-ideal of a *Q*-algebra W. Then, Y is a neutrosophic BCK-ideal of the BCK-algebra W.

Theorem 3.4. Assume that Y = {(c, $\hat{A}_Y(c)$, $\hat{C}_Y(c)$, $\hat{E}_Y(c)$, $\check{D}_Y(c)$, $\hat{U}_Y(c)$) : c ∈W} be a PN-*Q*-ideal over a *Q*-algebra W. If c, h, d ∈ W such that $c * h \leq d$, then $\hat{A}_Y(c) \geq \min\{\hat{A}_Y(h), \hat{A}_Y(d)\}$, $\hat{C}_Y(c) \geq \min\{\hat{C}_Y(h), \hat{C}_Y(d)\}$, $\hat{E}_Y(c) \leq \max\{\hat{E}_Y(h), \hat{E}_Y(d)\}$, $\check{D}_Y(c) \leq \max\{\check{D}_Y(h), \check{D}_Y(d)\}$ and $\hat{U}_Y(c) \leq \max\{\hat{U}_Y(h), \hat{U}_Y(d)\}$.

Proof. Let Y = {(c, $\hat{A}_Y(c)$, $\hat{C}_Y(c)$, $\hat{E}_Y(c)$, $\check{D}_Y(c)$, $\hat{U}_Y(c)$) : c ∈W} be a PN-*Q*-ideal over a *Q*-algebra W. Suppose that c, h, d ∈W such that c * h ≤ d. Therefore, $(c * h) * d = 0$.

Now, we have

$\hat{A}_Y(c) = \hat{A}_Y(c * 0) \geq \min\{\hat{A}_Y((c * h) * 0), \hat{A}_Y(h)\}$

$= \min\{\hat{A}_Y((c * h)), \hat{A}_Y(h)\}$

$\geq \min\{\min\{\hat{A}_Y((c * d) * h), \hat{A}_Y(d)\}, \hat{A}_Y(h)\}$

$= \min \{\hat{A}_Y((c * h) * d), \hat{A}_Y(d), \hat{A}_Y(h)\}$

$= \min \{\hat{A}_Y(0), \hat{A}_Y(d), \hat{A}_Y(h)\}$

$= \min \{\hat{A}_Y(h), \hat{A}_Y(d)\}$

$\Rightarrow \hat{A}_Y(c) \geq \min \{ \hat{A}_Y(h), \hat{A}_Y(d)\}$.

$\hat{C}_Y(c) = \hat{C}_Y(c * 0) \geq \min \{\hat{C}_Y((c * h) * 0), \hat{C}_Y(h)\}$

$= \min \{\hat{C}_Y((c * h)), \hat{C}_Y(h)\}$

$\geq \min \{\min \{\hat{C}_Y((c * d) * h), \hat{C}_Y(d)\}, \hat{C}_Y(h)\}$

$= \min \{\hat{C}_Y((c * h) * d), \hat{C}_Y(d), \hat{C}_Y(h)\}$

$= \min \{\hat{C}_Y(0), \hat{C}_Y(d), \hat{C}_Y(h)\}$

$= \min \{\hat{C}_Y(h), \hat{C}_Y(d)\}$

$\Rightarrow \hat{C}_Y(c) \geq \min \{ \hat{C}_Y(h), \hat{C}_Y(d)\}$.

$\hat{E}_Y(c) = \hat{E}_Y(c * 0) \leq \max \{ \hat{E}_Y((c * h) * 0), \hat{E}_Y(h)\}$

$= \max \{ \hat{E}_Y((c * h)), \hat{E}_Y(h)\}$

$\leq \max \{ \max\{ \hat{E}_Y((c * d) * h), \hat{E}_Y(d)\}, \hat{E}_Y(h)\}$

$= \max\{ \hat{E}_Y((c * h) * d), \hat{E}_Y(d), \hat{E}_Y(h)\}$

$= \max\{ \hat{E}_Y(0), \hat{E}_Y(d), \hat{E}_Y(h)\}$

$= \max\{ \hat{E}_Y(h), \hat{E}_Y(d)\}$

$\Rightarrow \hat{E}_Y(c) \leq \max\{ \hat{E}_Y(h), \hat{E}_Y(d)\}$.

$\check{D}_Y(c) = \check{D}_Y(c * 0) \leq \max \{ \check{D}_Y((c * h) * 0), \check{D}_Y(h)\}$

$= \max \{ \check{D}_Y((c * h)), \check{D}_Y(h)\}$

$\leq \max \{ \max\{ \check{D}_Y((c * d) * h), \check{D}_Y(d)\}, \check{D}_Y(h)\}$

$= \max\{ \check{D}_Y((c * h) * d), \check{D}_Y(d), \check{D}_Y(h)\}$

$= \max\{ \check{D}_Y(0), \check{D}_Y(d), \check{D}_Y(h)\}$

$= \max\{ \check{D}_Y(h), \check{D}_Y(d)\}$

$\Rightarrow \check{D}_Y(c) \leq \max\{ \check{D}_Y(h), \check{D}_Y(d)\}$.

Further, we have

$\hat{U}_Y(c) = \hat{U}_Y(c * 0) \leq \max \{ \hat{U}_Y((c * h) * 0), \hat{U}_Y(h)\}$

$= \max \{ \hat{U}_Y((c * h)), \hat{U}_Y(h)\}$

$\leq \max \{ \max\{ \hat{U}_Y((c * d) * h), \hat{U}_Y(d)\}, \hat{U}_Y(h)\}$

$= \max \{ \hat{U}_Y((c * h) * d), \hat{U}_Y(d), \hat{U}_Y(h)\}$

$= \max \{ \hat{U}_Y(0), \hat{U}_Y(d), \hat{U}_Y(h)\}$

$= \max \{ \hat{U}_Y(h), \hat{U}_Y(d)\}$

$\Rightarrow \hat{U}_Y(c) \leq \max \{\hat{U}_Y(h), \hat{U}_Y(d)\}$.

Theorem 3.5. Let $Y = \{(c, \hat{A}_Y(c), \hat{C}_Y(c), \hat{E}_Y(c), \check{D}_Y(c), \hat{U}_Y(c)) : c \in W\}$ be a PN-*Q*-ideal over a *Q*-algebra W. If c, h $\in$W such that $c \leq h$, then $\hat{A}_Y(c) \geq \hat{A}_Y(h)$, $\hat{C}_Y(c) \geq \hat{C}_Y(h)$, $\hat{E}_Y(c) \leq \hat{E}_Y(h)$, $\check{D}_Y(c) \leq \check{D}_Y(h)$ and $\hat{U}_Y(c) \leq \hat{U}_Y(h)$.

Proof. Assume that $Y = \{(c, \hat{A}_Y(c), \hat{C}_Y(c), \hat{E}_Y(c), \check{D}_Y(c), \hat{U}_Y(c)) : c \in W\}$ be a PN-*Q*-ideal over a *Q*-algebra W. Suppose c, h be two elements of W such that $c \leq h$. Therefore, $c * h=0$.

Now, we have

$\hat{A}_Y(c) = \hat{A}_Y(c * 0) \geq \min \{\hat{A}_Y((c * h) * 0), \hat{A}_Y(h)\}$

$= \min \{\hat{A}_Y((c * h)), \hat{A}_Y(h)\}$

$= \min\{\hat{A}_Y(0), \hat{A}_Y(h)\}$
$= \hat{A}_Y(h)$
$\Rightarrow \hat{A}_Y(c) \geq \hat{A}_Y(h)$.
$\hat{C}_Y(c) = \hat{C}_Y(c * 0) \geq \min\{\hat{C}_Y((c * h) * 0), \hat{C}_Y(h)\}$
$= \min\{\hat{C}_Y((c * h)), \hat{C}_Y(h)\}$
$= \min\{\hat{C}_Y(0), \hat{C}_Y(h)\}$
$= \hat{C}_Y(h)$
$\Rightarrow \hat{C}_Y(c) \geq \hat{C}_Y(h)$.
$\hat{E}_Y(c) = \hat{E}_Y(c * 0) \leq \max\{\hat{E}_Y((c * h) * 0), \hat{E}_Y(h)\}$
$= \max\{\hat{E}_Y((c * h)), \hat{E}_Y(h)\}$
$= \max\{\hat{E}_Y(0), \hat{E}_Y(h)\}$
$= \hat{E}_Y(h)$
$\Rightarrow \hat{E}_Y(c) \leq \hat{E}_Y(h)$.
$\check{D}_Y(c) = \check{D}_Y(c * 0) \leq \max\{\check{D}_Y((c * h) * 0), \check{D}_Y(h)\}$
$= \max\{\check{D}_Y((c * h)), \check{D}_Y(h)\}$
$= \max\{\check{D}_Y(0), \check{D}_Y(h)\}$
$= \check{D}_Y(h)$
$\Rightarrow \check{D}_Y(c) \leq \check{D}_Y(h)$.
$\hat{U}_Y(c) = \hat{U}_Y(c * 0) \leq \max\{\hat{U}_Y((c * h) * 0), \hat{U}_Y(h)\}$
$= \max\{\hat{U}_Y((c * h)), \hat{U}_Y(h)\}$
$= \max\{\hat{U}_Y(0), \hat{U}_Y(h)\}$
$= \hat{U}_Y(h)$
$\Rightarrow \hat{U}_Y(c) \leq \hat{U}_Y(h)$.

Theorem 3.6. If $Y = \{(c, \hat{A}_Y(c), \hat{C}_Y(c), \hat{E}_Y(c), \check{D}_Y(c), \hat{U}_Y(c)) : c \in W\}$ is a PN-*Q*-sub-algebra over a *Q*-algebra W, then the sets $\alpha\text{-}\hat{A}_Y = \{c: c \in W, \hat{A}_Y(c) \geq \alpha\}$, $\alpha\text{-}\hat{C}_Y = \{c: c \in W, \hat{C}_Y(c) \geq \alpha\}$, $\alpha\text{-}\hat{E}_Y = \{c: c \in W, \hat{E}_Y(c) \leq \alpha\}$, $\alpha\text{-}\check{D}_Y = \{c: c \in W, \check{D}_Y(c) \leq \alpha\}$ and $\alpha\text{-}\hat{U}_Y = \{c: c \in W, \hat{U}_Y(c) \leq \alpha\}$ are the *Q*-sub-algebra of W.

Proof. Suppose that $Y = \{(c, \hat{A}_Y(c), \hat{C}_Y(c), \hat{E}_Y(c), \check{D}_Y(c), \hat{U}_Y(c)) : c \in W\}$ be a PN-*Q*-sub-algebra over a *Q*-algebra W. Therefore,

(i) $\hat{A}_Y(c * d) \geq \min\{\hat{A}_Y(c), \hat{A}_Y(d)\}$;
(ii) $\hat{C}_Y(c * d) \geq \min\{\hat{C}_Y(c), \hat{C}_Y(d)\}$;
(iii) $\hat{E}_Y(c * d) \leq \max\{\hat{E}_Y(c), \hat{E}_Y(d)\}$;
(iv) $\check{D}_Y(c * d) \leq \max\{\check{D}_Y(c), \check{D}_Y(d)\}$;
(v) $\hat{U}_Y(c * d) \leq \max\{\hat{U}_Y(c), \hat{U}_Y(d)\}$; where $c, d \in W$.

Let $c, d \in \alpha\text{-}\hat{A}_Y$. This implies, $\hat{A}_Y(c) \geq \alpha$, $\hat{A}_Y(d) \geq \alpha$.
Therefore, $\hat{A}_Y(c * d) \geq \min\{\hat{A}_Y(c), \hat{A}_Y(d)\} \geq \min\{\alpha, \alpha\} \geq \alpha$.
Hence, $\alpha\text{-}\hat{A}_Y = \{c: c \in W, \hat{A}_Y(c) \geq \alpha\}$ is a *Q*-sub-algebra of W.
Let $c, d \in \alpha\text{-}\hat{C}_Y$. This implies, $\hat{C}_Y(c) \geq \alpha$, $\hat{C}_Y(d) \geq \alpha$.
Therefore, $\hat{C}_Y(c * d) \geq \min\{\hat{C}_Y(c), \hat{C}_Y(d)\} \geq \min\{\alpha, \alpha\} \geq \alpha$.
Hence, $\alpha\text{-}\hat{C}_Y = \{c: c \in W, \hat{C}_Y(c) \geq \alpha\}$ is a *Q*-sub-algebra of W.
Let $c, d \in \alpha\text{-}\hat{E}_Y$. This implies, $\hat{E}_Y(c) \leq \alpha$, $\hat{E}_Y(d) \leq \alpha$.
Therefore, $\hat{E}_Y(c * d) \leq \max\{\hat{E}_Y(c), \hat{E}_Y(d)\} \leq \max\{\alpha, \alpha\} \leq \alpha$.

Hence, α-$\hat{E}_Y$ = {c: c∈ W, $\hat{E}_Y(c) \leq \alpha$} is a *Q*-sub-algebra of W.
Let c, d ∈α-$\check{D}_Y$. This implies, $\check{D}_Y(c) \leq \alpha$, $\check{D}_Y(d) \leq \alpha$.
Therefore, $\check{D}_Y(c * d) \leq \max\{\check{D}_Y(c), \check{D}_Y(d)\} \leq \max\{\alpha, \alpha\} \leq \alpha$.
Hence, α-$\check{D}_Y$ = {c: c∈ W, $\check{D}_Y(c) \leq \alpha$} is a *Q*-sub-algebra of W.
Let c, d ∈α-$\hat{U}_Y$. This implies, $\hat{U}_Y(c) \leq \alpha$, $\hat{U}_Y(d) \leq \alpha$.
Therefore, $\hat{U}_Y(c * d) \leq \max\{\hat{U}_Y(c), \hat{U}_Y(d)\} \leq \max\{\alpha, \alpha\} \leq \alpha$.
Hence, α-$\hat{U}_Y$ = {c: c∈ W, $\hat{U}_Y(c) \leq \alpha$} is a *Q*-sub-algebra of W.

Theorem 3.7. Suppose that Y = {(c, $\hat{A}_Y(c)$, $\hat{C}_Y(c)$, $\hat{E}_Y(c)$, $\check{D}_Y(c)$, $\hat{U}_Y(c)$) : c ∈W} be a PN-*Q*-ideal of W, then the sets W($\hat{A}$)={c∈W: $\hat{A}_Y(c)=\hat{A}_Y(0)$}, W($\hat{C}$)={c∈W: $\hat{C}_Y(c)=\hat{C}_Y(0)$}, W($\hat{E}$)={c∈W: $\hat{E}_Y(c)= \hat{E}_Y(0)$}, W($\check{D}$)={c∈W: $\check{D}_Y(c)= \check{D}_Y(0)$}, and W($\hat{U}$)={c∈W: $\hat{U}_Y(c)= \hat{U}_Y(0)$} are *Q*-ideals of W.

Proof. Suppose that Y = {(c, $\hat{A}_Y(c)$, $\hat{C}_Y(c)$, $\hat{E}_Y(c)$, $\check{D}_Y(c)$, $\hat{U}_Y(c)$) : c ∈W} be a PN-*Q*-ideal of W. Therefore,

(i) $\hat{A}_Y(0)\geq \hat{A}_Y(c)$ & $\hat{A}_Y(c * d) \geq \min\{\hat{A}_Y((c * h) * d), \hat{A}_Y(h)\}$, for all c, d, h∈W;
(ii) $\hat{C}_Y(0)\geq \hat{C}_Y(c)$ & $\hat{C}_Y(c * d) \geq \min\{\hat{C}_Y((c * h) * d), \hat{C}_Y(h)\}$, for all c, d, h∈W;
(iii) $\hat{E}_Y(0)\leq \hat{E}_Y(c)$ & $\hat{E}_Y(c * d) \leq \max\{\hat{E}_Y((c * h) * d), \hat{E}_Y(h)\}$, for all c, d, h∈W;
(iv) $\check{D}_Y(0)\leq \check{D}_Y(c)$ & $\check{D}_Y(c * d) \leq \max\{\check{D}_Y((c * h) * d), \check{D}_Y(h)\}$, for all c, d, h∈W;
(v) $\hat{U}_Y(0)\leq \hat{U}_Y(c)$ & $\hat{U}_Y(c * d) \leq \max\{\hat{U}_Y((c * h) * d), \hat{U}_Y(h)\}$, for all c, d, h∈W.

Since, $\hat{A}_Y(0)=\hat{A}_Y(0)$, so 0 ∈ W($\hat{A}$).
Since, $\hat{C}_Y(0)= \hat{C}_Y(0)$, so 0 ∈ W($\hat{C}$).
Since, $\hat{E}_Y(0)= \hat{E}_Y(0)$, so 0 ∈ W($\hat{E}$).
Since, $\check{D}_Y(0)= \check{D}_Y(0)$, so 0 ∈ W($\check{D}$).
Since, $\hat{U}_Y(0)= \hat{U}_Y(0)$, so 0 ∈ W($\hat{U}$).
Let (h ∗ d) ∗ e∈W($\hat{A}$) and d∈W($\hat{A}$). Therefore, $\hat{A}_Y((h * d) * e)=\hat{A}_Y(0)$ and $\hat{A}_Y(d)=\hat{A}_Y(0)$.
It is clear that $\hat{A}_Y(0)\geq \hat{A}_Y(h * e)$ (1)
Now, we have
$\hat{A}_Y(h * e) \geq \min\{\hat{A}_Y((h * d) * e), \hat{A}_Y(d)\}= \min\{\hat{A}_Y(0), \hat{A}_Y(0)\}= \hat{A}_Y(0)$
$\Rightarrow \hat{A}_Y(h * e) \geq \hat{A}_Y(0)$ (2)
From (1) and (2), we get
$\hat{A}_Y(h * e) = \hat{A}_Y(0)$.
This implies, h ∗ e∈ W($\hat{A}$). Therefore, the set W($\hat{A}$)={c∈W: $\hat{A}_Y(c)=\hat{A}_Y(0)$} is a *Q*-ideal of W.
Similarly, it can be shown that, the sets W($\hat{C}$)={c∈W: $\hat{C}_Y(c)= \hat{C}_Y(0)$}, W($\hat{E}$)={c∈W: $\hat{E}_Y(c)= \hat{E}_Y(0)$}, W($\check{D}$)={c∈W: $\check{D}_Y(c)= \check{D}_Y(0)$} and W($\hat{U}$)={c∈W: $\hat{U}_Y(c)= \hat{U}_Y(0)$} are *Q*-ideals of W.

Theorem 3.8. Assume that Y = {(c, $\hat{A}_Y(c)$, $\hat{C}_Y(c)$, $\hat{E}_Y(c)$, $\check{D}_Y(c)$, $\hat{U}_Y(c)$) : c ∈W} be a PN-*Q*-ideal of *Q*-algebra W. Then, the fuzzy sets {(c, $\hat{A}_Y(c)$): c∈W}, {(c, $\hat{C}_Y(c)$): c∈W}, {(c,1- $\hat{E}_Y(c)$): c∈W}, {(c,1- $\check{D}_Y(c)$): c∈W}, {(c,1- $\hat{U}_Y(c)$): c∈W} are fuzzy *Q*-ideals of W.

Proof. Let Y = {(c, $\hat{A}_Y(c)$, $\hat{C}_Y(c)$, $\hat{E}_Y(c)$, $\check{D}_Y(c)$, $\hat{U}_Y(c)$) : c ∈W} be a PN-*Q*-ideal of a *Q*-algebra W. Therefore,

(i) $\hat{A}_Y(0) \geq \hat{A}_Y(c)$ & $\hat{A}_Y(c * d) \geq \min\{\hat{A}_Y((c * h) * d), \hat{A}_Y(h)\}$, for all c, d, h∈W;
(ii) $\hat{C}_Y(0) \geq \hat{C}_Y(c)$ & $\hat{C}_Y(c * d) \geq \min\{\hat{C}_Y((c * h) * d), \hat{C}_Y(h)\}$, for all c, d, h∈W;
(iii) $\hat{E}_Y(0) \leq \hat{E}_Y(c)$ & $\hat{E}_Y(c * d) \leq \max\{\hat{E}_Y((c * h) * d), \hat{E}_Y(h)\}$, for all c, d, h∈W;
(iv) $\check{D}_Y(0) \leq \check{D}_Y(c)$ & $\check{D}_Y(c * d) \leq \max\{\check{D}_Y((c * h) * d), \check{D}_Y(h)\}$, for all c, d, h∈W;
(v) $\hat{U}_Y(0) \leq \hat{U}_Y(c)$ & $\hat{U}_Y(c * d) \leq \max\{\hat{U}_Y((c * h) * d), \hat{U}_Y(h)\}$, for all c, d, h∈W.

It is clear that, $\hat{A}_Y(0) \geq \hat{A}_Y(c)$ & $\hat{A}_Y(c * d) \geq \min\{\hat{A}_Y((c * h) * d), \hat{A}_Y(h)\}$, for all c, d, h∈W. Therefore, the fuzzy set $\{(c, \hat{A}_Y(c)): c \in W\}$ is a fuzzy *Q*-ideal of W.
It is clear that, $\hat{C}_Y(0) \geq \hat{C}_Y(c)$ & $\hat{C}_Y(c * d) \geq \min\{\hat{C}_Y((c * h) * d), \hat{C}_Y(h)\}$, for all c, d, h∈W. Therefore, the fuzzy set $\{(c, \hat{C}_Y(c)): c \in W\}$ is a fuzzy *Q*-ideal of W.
Now, for all c, d, h∈W,
$\hat{E}_Y(c * d) \leq \max\{\hat{E}_Y((c * h) * d), \hat{E}_Y(h)\} \Rightarrow 1- \hat{E}_Y(c) \geq \min\{1- \hat{E}_Y((c * h) * d), 1-\hat{E}_Y(h)\}$
and $\hat{E}_Y(0) \leq \hat{E}_Y(c) \Rightarrow 1- \hat{E}_Y(0) \geq 1- \hat{E}_Y(c)$.
Therefore, the fuzzy set $\{(c,1-\hat{E}_Y(c)): c \in W\}$ is a fuzzy *Q*-ideal of W.
Now, for all c, d, h∈W,
$\check{D}_Y(c * d) \leq \max\{\check{D}_Y((c * h) * d), \check{D}_Y(h)\} \Rightarrow 1- \check{D}_Y(c) \geq \min\{1- \check{D}_Y((c * h) * d), 1-\check{D}_Y(h)\}$
and $\check{D}_Y(0) \leq \check{D}_Y(c) \Rightarrow 1- \check{D}_Y(0) \geq 1- \check{D}_Y(c)$.
Therefore, the fuzzy set $\{(c,1-\check{D}_Y(c)): c \in W\}$ is a fuzzy *Q*-ideal of W.
Further, for all c, d, h∈W,
$\hat{U}_Y(c * d) \leq \max\{ \hat{U}_Y((c * h) * d), \hat{U}_Y(h)\} \Rightarrow 1-\hat{U}_Y(c) \geq \min\{1- \hat{U}_Y((c * h) * d), 1-\hat{U}_Y(h)\}$
and $\hat{U}_Y(0) \leq \hat{U}_Y(c) \Rightarrow 1- \hat{U}_Y(0) \geq 1- \hat{U}_Y(c)$.
Therefore, the fuzzy set $\{(c, 1- \hat{U}_Y(c)): c \in W\}$ is a fuzzy *Q*-ideal of W.

4. Conclusions:

In this paper, we have established the notion of PN-*Q*-ideals of PN-*Q*-algebra. By defining PN-*Q*-ideals, we have formulated some results on PN-*Q*-algebra from the point of view of neutrosophic set. It is just the beginning of the concept of PN-*Q*-algebra. In the future, we hope that based on the notions of PN-*Q*-ideals many new investigations can be done.

References:

1. Abdullah, H.K., & Jawad, H.K. (2018). New types of ideals in *Q*-algebra. *Journal University of Kerbala*, 16 (4), scientific.
2. Agboola, A.A.A., & Davvaz, B. (2015). Introduction to neutrosophic *BCI*/*BCK*-algebras. *International Journal of Mathematics and Mathematical Sciences*, Vol. 2015, Article ID 370267, 1-6.
3. Alcheikh, M., & Sabouh, A. (2019). A study of fuzzy ideals in BCK algebra. *Journal of Mathematics Research*, 11 (5), 11-15.
4. Atanassov, K.T. (1986). Intuitionistic fuzzy sets. *Fuzzy sets and Systems*, 35, 87–96.
5. Das, S., Das, R., & Tripathy, B.C. (2020). Multi-criteria group decision making model using single-valued neutrosophic set. *LogForum*, 16 (3), 421-429.
6. Das, S., Granados, C. Neutrosophic *Q*-Ideals of Neutrosophic *Q*-Algebra. *Iraqi Journal of Science*. (Under Communication).
7. Das, S., & Pramanik, S. (2020). Generalized neutrosophic *b*-open sets in neutrosophic topological space. *Neutrosophic Sets and Systems*, 35, 522-530.
8. Das, S., & Pramanik, S. (2020). Neutrosophic Φ-open sets and neutrosophic Φ-continuous functions. *Neutrosophic Sets and Systems*. 38, 355-367.

9. Das, S., & Pramanik, S. (2020). Neutrosophic simply soft open set in neutrosophic soft topological space. *Neutrosophic Sets and Systems*, 38, 235-243.
10. Das, S., & Tripathy, B.C. (2020). Pairwise neutrosophic-*b*-open set in neutrosophic bitopological spaces. *Neutrosophic Sets and Systems*, 38, 135-144.
11. Das, S., & Tripathy, B.C. (2021). Neutrosophic simply *b*-open set in neutrosophic topological spaces. *Iraqi Journal of Science*. (In Press)
12. Hasan, A.K. (2017). On intuitionistic fuzzy *d*-ideal of *d*-algebra. *Journal University of Kerbala*, 15 (1), 161-169.
13. Hasan, A.K. (2020). Intuitionistic fuzzy *d*-filter of *d*-algebra. *Journal of mechanics of continua and mathematical sciences*, 15 (6), 360-370.
14. Iami, Y., & Iseki, K. (1966). On axiom system of propositional calculi XIV. *Proceedings of the Japan Academy*, 42, 19-20.
15. Iseki, K., & Tanaka, S. (1978). An introduction to the theory of BCK-algebra. *Mathematica Japonicae*, 23 (1), 1-26.
16. Jun, Y.B., Kim, H.S, & Yoo, D.S. (e-2006). Intuitionistic fuzzy *d*-algebra. *Scientiae Mathematicae Japonicae Online*, e-2006, 1289–1297.
17. Jun, Y.B., Neggers, J., & Kim, H.S. (2000). Fuzzy *d*-ideals of *d*-algebras. *Journal of Fuzzy Mathematics*, 8 (1), 123-130.
18. Kadhum, H.J., & Kareem, A.H. (2018). Some types of fuzzy pseudo ideals in pseudo *Q*-algebra. *Thesis, Department of mathematics, University of Kufa*, pp. 1-109.
19. Kareem, A.H., & Ahmed, H.Z. (2016). Complete *BCK*-ideal. *European Journal of Scientific Research*, 137 (3), 302-314.
20. Mallick, R., & Pramanik, S. (2020). Pentapartitioned neutrosophic set and its properties. *Neutrosophic Sets and Systems*, 36, 184-192.
21. Martina Jency, J., & Arockiarani. I. (2016). Single valued neutrosophic ideals of *BCK*-algebras. *International Journal of Mathematics Trends and Technology*, 39 (1), 8-17.
22. Mostafa, S.M., Abdel Naby, M.A., & Elgendy, O.R. (2012). Fuzzy *Q*-ideals in *Q*-algebras. *World Applied Programming*, 2 (2), 69-80.
23. Mostafa, S.M., Abdel Naby, M.A., & Elgendy, O.R. (2012). Intuitionistic fuzzy *Q*-ideals in *Q*-algebra. *Global Journal of Pure and Applied Mathematics*, 8 (2), 135-144.
24. Neggers, J., Ahn, S.S., & Kim, H.S. (2001). On *Q*-algebra. *International Journal of Mathematics and Mathematical Sciences*, 27 (12), 749-757.
25. Neggers, J., Jun, Y.B., & Kim, H.S. (1999). On *d*-ideals in *d*-algebras. *Mathematica Slovaca*, 49 (3), 243-251.
26. Neggers, J., & Kim, H.S. (1999). On *d*-algebra. *Mathematica Slovaca*, 49 (1), 19-26.
27. Smarandache, F. (2005). Neutrosophic set: a generalization of the intuitionistic fuzzy sets. *International Journal of Pure and Applied Mathematics*, 24, 287-297.
28. Smarandache, F. (2013). n-Valued Refined Neutrosophic Logic and Its Applications in Physics, *Progress in Physics*, 4, 143-146.
29. Zadeh, L.A. (1965). Fuzzy set. *Information and Control*, 8, 338-353.

Received: Jan 5, 2021. Accepted: March 6, 2021.