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Neutrosophic \mathcal{N} –structures on strong Sheffer stroke non-associative MV-algebras

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Abstract. The aim of the study is to examine a neutrosophic \mathcal{N} –subalgebra, a neutrosophic \mathcal{N} –filter, level sets of these neutrosophic \mathcal{N} –structures and their properties on a strong Sheffer stroke non-associative MV-algebra. We show that the level set of neutrosophic \mathcal{N} –subalgebras on this algebra is its strong Sheffer stroke non-associative MV-subalgebra and vice versa. Then it is proved that the family of all neutrosophic \mathcal{N} –subalgebras of a strong Sheffer stroke non-associative MV-algebra forms a complete distributive lattice. By defining a neutrosophic \mathcal{N} –filter of a strong Sheffer stroke non-associative MV-algebra, it is presented that every neutrosophic \mathcal{N} –filter of a strong Sheffer stroke non-associative MV-algebra is its neutrosophic \mathcal{N} –subalgebra but the inverse is generally not true, and some properties

Keywords: strong Sheffer stroke non-associative MV- algebra, filter, neutrosophic \mathcal{N} –subalgebra, neutrosophic \mathcal{N} –filter.

1. Introduction

The concept of fuzzy sets which has the truth (t) (membership) function was introduced by L. Zadeh [29]. Since a positive meaning of information is explained by means of fuzzy theory, researchers desire to deal with a negative meaning of information. Thus, Atanassov introduced intuitionistic fuzzy sets [2] which are fuzzy sets with the falsehood (f) (nonmembership) function. Then, Smarandache introduced neutrosophic sets which are intuitionistic fuzzy sets with the indeterminacy/neutrality (i) function [26,27]. Accordingly, neutrosophic sets are defined on three components: $(t, i, f) : (\text{truth}, \text{indeterminacy}, \text{falsehood})$ [32]. Specially, many scientists applied neutrosophic sets to the algebraic structures such as BCK/BCI-algebras, BE-algebras and semigroups [3, 4, 11–16, 24, 28, 30, 31].

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Sheffer stroke, which is also called the NAND operator in computer science, was firstly introduced by H. M. Sheffer [25]. Since any axioms and formulas in Boolean algebras can be written only by using this operation [17], Sheffer stroke can be applied to many logical algebras such as orthoimplication algebras [1], ortholattices [5], Hilbert algebras [18]- [19], BL-algebras [23], UP-algebras [20] and BG-algebras [21]. Therefore, it is easier to control a logical system consisting of Sheffer stroke itself. Moreover, C. C. Chang introduced MV-algebras which are algebraic counterparts of Lukasiewicz many-valued logic [9, 10]. Then Chajda et al. introduced and improved non-associative MV-algebras (briefly, NMV-algebras) which are generalizations of MV-algebras [7, 8]. Also, non-associative MV-algebras with Sheffer stroke [6] and their filters [22] are presented.

Basic definitions and notions about strong Sheffer stroke non-associative MV-algebras, \mathcal{N} -functions and neutrosophic \mathcal{N} -structures defined by the \mathcal{N} -functions on a nonempty universe X are presented. Then the concepts of a neutrosophic \mathcal{N} -subalgebra and a (a, b, c) -level set defined by \mathcal{N} -functions are given on strong Sheffer stroke non-associative MV-algebras. It is shown that the (a, b, c) -level set of a neutrosophic \mathcal{N} -subalgebra defined by \mathcal{N} -functions on strong Sheffer stroke non-associative MV-algebras is its strong Sheffer stroke non-associative MV-subalgebra and the inverse is true. In fact, we state that the family of all neutrosophic \mathcal{N} -subalgebras of this algebraic structure forms a complete distributive lattice. Some properties of neutrosophic \mathcal{N} -subalgebras of strong Sheffer stroke non-associative MV-algebras are analyzed. Also, it is investigated the images of the sequence under \mathcal{N} -functions on a strong Sheffer stroke non-associative MV-algebra. Besides, we examine that the case which \mathcal{N} -functions defining a neutrosophic \mathcal{N} -subalgebra of a strong Sheffer stroke non-associative MV-algebra are constant. After defining a neutrosophic \mathcal{N} -filter of a strong Sheffer stroke non-associative MV-algebra by \mathcal{N} -functions, some features of \mathcal{N} -functions defining the neutrosophic \mathcal{N} -filter are studied. We propound that (a, b, c) -level set of a neutrosophic \mathcal{N} -filter of a strong Sheffer stroke non-associative MV-algebra is its filter and that the subsets defined by \mathcal{N} -functions on a strong Sheffer stroke non-associative MV-algebra must be its filters so that a neutrosophic \mathcal{N} -structure on this algebra is a neutrosophic \mathcal{N} -filter. It is stated that every neutrosophic \mathcal{N} -filter of a strong Sheffer stroke non-associative MV-algebra is its neutrosophic \mathcal{N} -subalgebra while the inverse is usually not valid. In addition, new subsets of a strong Sheffer stroke non-associative MV-algebra are described by the \mathcal{N} -functions and certain elements in the algebra. We show that these subsets are filters of a strong Sheffer stroke non-associative MV-algebra for its neutrosophic \mathcal{N} -filter but the inverse does not mostly hold.

2. Preliminaries

In this section, we give basic definitions and notions about strong Sheffer stroke non-associative MV-algebras (briefly, strong Sheffer stroke NMV-algebras) and neutrosophic \mathcal{N} -structures.

Definition 2.1. [5] Let $\mathcal{A} = \langle A, | \rangle$ be a groupoid. The operation $|$ is said to be a *Sheffer stroke operation* if it satisfies the following conditions:

- (S1) $x|y = y|x$,
- (S2) $(x|x)|(x|y) = x$,
- (S3) $x|((y|z)|(y|z)) = ((x|y)|(x|y))|z$,
- (S4) $(x|((x|x)|(y|y))|(x|((x|x)|(y|y)))) = x$.

Definition 2.2. [6] A strong Sheffer stroke NMV-algebra is an algebra $(A, |, 1)$ of type $(2, 0)$ satisfying the identities for all $x, y, z \in A$:

- (n1) $x|y \approx y|x$,
- (n2) $x|0 \approx 1$,
- (n3) $(x|1)|1 \approx x$,
- (n4) $((x|1)|y)|y \approx ((y|1)|x)|x$,
- (n5) $(x|1)|((x|y)|1) \approx 1$,
- (n6) $x|((((x|y)|y)|z)|z)|1) \approx 1$,

where 0 denotes the algebraic constant $1|1$.

Proposition 2.3. [22] Let $(A, |, 1)$ be a strong Sheffer stroke NMV-algebra. Then the binary relation \leq defined by

$$x \leq y \text{ if and only if } x|(y|1) \approx 1$$

is a partial order on A . Hence, (A, \leq) is a poset with the least element 0 and the greatest element 1.

Lemma 2.4. [22] In a strong Sheffer stroke NMV-algebra $(A, |, 1)$, the following properties hold for all $x, y, z \in A$:

- (i) $x|(x|1) \approx 1$,
- (ii) $x \leq y \Leftrightarrow y|1 \leq x|1$,
- (iii) $y \leq x|(y|1)$,
- (iv) $y|1 \leq x|y$,
- (v) $x \leq (x|y)|y$,
- (vi) $x \leq (((x|y)|y)|z)|z$,
- (vii) $((x|y)|y)|y \approx x|y$,

- (viii) $x|1 \approx x|x$,
 (ix) $x|(x|x) \approx 1$,
 (x) $1|(x|x) \approx x$,
 (xi) $x \leq y \Rightarrow y|z \leq x|z$,
 (xii) $x|(y|1) \leq (y|(z|1))|((x|(z|1))|1)$,
 (xiii) $x|(y|1) \leq (z|(x|1))|((z|(y|1))|1)$,
 (xiv) $x \leq y$ and $z \leq t$ imply $y|t \leq x|z$.

Definition 2.5. [22] A nonempty subset $F \subseteq A$ is called a filter of A if it satisfies the following properties:

- ($S_f - 1$) $1 \in F$,
 ($S_f - 2$) For all $x, y \in A$, $x|(y|1) \in F$ and $x \in F$ imply $y \in F$.

Lemma 2.6. [22] A nonempty subset $F \subseteq A$ is a filter of A if and only if $1 \in F$ and $x \leq y$ and $x \in F$ imply $y \in F$.

Definition 2.7. [11] $\mathcal{F}(X, [-1, 0])$ denotes the collection of functions from a set X to $[-1, 0]$ and a element of $\mathcal{F}(X, [-1, 0])$ is called a negative-valued function from X to $[-1, 0]$ (briefly, \mathcal{N} -function on X). An \mathcal{N} -structure refers to an ordered pair (X, f) of a set X and an \mathcal{N} -function f on X .

Definition 2.8. [16] A neutrosophic \mathcal{N} -structure over a nonempty universe X is defined by

$$X_N := \frac{X}{(T_N, I_N, F_N)} = \left\{ \frac{x}{(T_N(x), I_N(x), F_N(x))} : x \in X \right\},$$

where T_N, I_N and F_N are \mathcal{N} -function on X , called the negative truth membership function, the negative indeterminacy membership function and the negative falsity membership function, respectively.

Every neutrosophic \mathcal{N} -structure X_N over X satisfies the condition

$$(\forall x \in X)(-3 \leq T_N(x) + I_N(x) + F_N(x) \leq 0).$$

3. Neutrosophic \mathcal{N} -structures

In this section, we give neutrosophic \mathcal{N} -subalgebras and neutrosophic \mathcal{N} -filters on strong Sheffer stroke NMV-algebras. Unless indicated otherwise, A states a strong Sheffer stroke NMV-algebra.

Definition 3.1. A neutrosophic \mathcal{N} -subalgebra A_N on a strong Sheffer stroke NMV-algebra A is a neutrosophic \mathcal{N} -structure of A satisfying the conditions

$$\min\{T_N(x), T_N(y)\} \leq T_N(x|(y|1)),$$

$$\max\{I_N(x), I_N(y)\} \geq I_N(x|(y|1))$$

and

$$\max\{F_N(x), F_N(y)\} \geq F_N(x|(y|1)),$$

for all $x, y \in A$.

Example 3.2. Consider a strong Sheffer stroke NMV-algebra A in which the set $A = \{0, u, v, 1\}$ and the Sheffer operation $|$ on A has the following Cayley table:

TABLE 1

$ $	0	u	v	1
0	1	1	1	1
u	1	v	1	v
v	1	1	u	u
1	1	v	u	0

A neutrosophic \mathcal{N} -structure

$$A_N = \left\{ \frac{0}{(-0.79, -0.001, 0)}, \frac{u}{(-0.68, -0.72, -0.4)}, \frac{v}{(-0.68, -0.72, -0.4)}, \frac{1}{(0, -0.88, -1)} \right\}$$

on A is a neutrosophic \mathcal{N} -subalgebra of A .

Definition 3.3. Let A_N be a neutrosophic \mathcal{N} -structure on a strong Sheffer stroke NMV-algebra A and a, b, c be any elements of $[-1, 0]$ such that $-3 \leq a + b + c \leq 0$. For

$$T_N^a := \{x \in A : T_N(x) \geq a\},$$

$$I_N^b := \{x \in A : I_N(x) \leq b\}$$

and

$$F_N^c := \{x \in A : F_N(x) \leq c\},$$

the set

$$A_N(a, b, c) := \{x \in H : T_N(x) \geq a, I_N(x) \leq b \text{ and } F_N(x) \leq c\}$$

is called the (a, b, c) -level set of A_N . Moreover,

$$A_N(a, b, c) = T_N^a \cap I_N^b \cap F_N^c.$$

Definition 3.4. [22] A subset B of a strong Sheffer stroke NMV-algebra A is called a strong Sheffer stroke NMV-subalgebra of A if 1 of A is in B and $(B, |, 1)$ forms a strong Sheffer stroke NMV-algebra. Clearly, A itself and $\{1\}$ are strong Sheffer stroke NMV-subalgebras of A .

Lemma 3.5. *Let B be a nonempty subset of a strong Sheffer stroke NMV-algebra A . Then B is a strong Sheffer stroke NMV-subalgebra of A if and only if $x|(y|1) \in B$, for all $x, y \in B$.*

Proof. Let B be a nonempty subset of a strong Sheffer stroke NMV-algebra A such that $x|(y|1) \in B$, for all $x, y \in B$. Then $1 \approx x|(x|1) \in B$ from Lemma 2.4 (i). Since $B \subseteq A$, $(B, |, 1)$ satisfies (n1)-(n6), for all $x, y, z \in B$. Thus, $(B, |, 1)$ is a strong Sheffer stroke NMV-subalgebra A .

Conversely, let B be a strong Sheffer stroke NMV-subalgebra of A . Since B states a strong Sheffer stroke NMV-algebra, it must be closed under the Sheffer operation $|$, that is, $x|y \in B$, for all $x, y \in B$. Hence, $x|(y|1) \in B$, for all $x, y \in B$. \square

Theorem 3.6. *Let A_N be a neutrosophic \mathcal{N} -structure on a strong Sheffer stroke NMV-algebra A and a, b, c be any elements in $[-1, 0]$ such that $-3 \leq a + b + c \leq 0$. If A_N is a neutrosophic \mathcal{N} -subalgebra of A , then the nonempty level set $A_N(a, b, c)$ of A_N is a subalgebra of A .*

Proof. Let A_N be a neutrosophic \mathcal{N} -subalgebra of A and x, y be any elements in $A_N(a, b, c)$, for $a, b, c \in [-1, 0]$ with $-3 \leq a + b + c \leq 0$. Then $T_N(x) \geq a, I_N(x) \leq b, F_N(x) \leq c, T_N(y) \geq a, I_N(y) \leq b$ and $F_N(y) \leq c$. Since

$$T_N(x|(y|1)) \geq \min\{T_N(x), T_N(y)\} \geq a,$$

$$I_N(x|(y|1)) \leq \max\{I_N(x), I_N(y)\} \leq b$$

and

$$F_N(x|(y|1)) \leq \max\{F_N(x), F_N(y)\} \leq c,$$

for all $x, y \in A$, it follows that $x|(y|1) \in T_N^a, x|(y|1) \in I_N^b$ and $x|(y|1) \in F_N^c$, which implies that $x|(y|1) \in T_N^a \cap I_N^b \cap F_N^c = A_N(a, b, c)$. Thus, $A_N(a, b, c)$ is a subalgebra of A by Lemma 3.5. \square

Theorem 3.7. *Let A_N be a neutrosophic \mathcal{N} -structure on a strong Sheffer stroke NMV-algebra A and T_N^a, I_N^b and F_N^c be subalgebras of A , for all $a, b, c \in [-1, 0]$ with $-3 \leq a + b + c \leq 0$. Then A_N is a neutrosophic \mathcal{N} -subalgebra of A .*

Proof. Let T_N^a, I_N^b and F_N^c be subalgebras of A , for all $a, b, c \in [-1, 0]$ with $-3 \leq a + b + c \leq 0$. Assume that x and y are any elements in A such that $u_1 = T_N(x|(y|1)) < \min\{T_N(x), T_N(y)\} = v_1$. If $a_0 = \frac{1}{2}(u_1 + v_1) \in [-1, 0)$, then $u_1 < a_0 < v_1$. So, $x, y \in T_N^{a_0}$ while $x|(y|1) \notin T_N^{a_0}$, which is a contradiction. Thus, $\min\{T_N(x), T_N(y)\} \leq T_N(x|(y|1))$, for all $x, y \in A$.

Suppose that x and y are any elements in A such that $u_2 = \max\{I_N(x), I_N(y)\} < I_N(x|(y|1)) = v_2$. If $b_0 = \frac{1}{2}(u_2 + v_2) \in [-1, 0)$, then $u_2 < b_0 < v_2$, which implies that $x, y \in I_N^{b_0}$ but $x|(y|1) \notin I_N^{b_0}$. This is a contradiction. Thus, $I_N(x|(y|1)) \leq \max\{I_N(x), I_N(y)\}$, for all $x, y \in A$.

Assume that x and y are any elements in A such that $v_3 = F_N(x|(y|1)) > \max\{F_N(x), F_N(y)\} = u_3$. If $c_0 = \frac{1}{2}(u_3 + v_3) \in [-1, 0)$, then $u_3 < c_0 < v_3$. Thus, $x, y \in F_N^{c_0}$ but $x|(y|1) \notin F_N^{c_0}$, which is a contradiction. Thereby, $\max\{F_N(x), F_N(y)\} \geq F_N(x|(y|1))$, for all $x, y \in A$.

Therefore, A_N is a neutrosophic \mathcal{N} -subalgebra of A . \square

Theorem 3.8. *Let $\{A_{N_i} : i \in \mathbb{N}\}$ be a family of all neutrosophic \mathcal{N} -subalgebras of a strong Sheffer stroke NMV-algebra A . Then $\{A_{N_i} : i \in \mathbb{N}\}$ forms a complete distributive lattice.*

Proof. Let B be a nonempty subset of $\{A_{N_i} : i \in \mathbb{N}\}$. Since A_{N_i} is a neutrosophic \mathcal{N} -subalgebra of A , for all $A_{N_i} \in B$, it satisfies

$$\min\{T_N(x), T_N(y)\} \leq T_N(x|(y|1)),$$

$$I_N(x|(y|1)) \leq \max\{I_N(x), I_N(y)\}$$

and

$$F_N(x|(y|1)) \leq \max\{F_N(x), F_N(y)\},$$

for all $x, y \in A$. Then $\bigcap B$ satisfies these inequalities. Thus, $\bigcap B$ is a neutrosophic \mathcal{N} -subalgebra of A .

Let C be a family of all neutrosophic \mathcal{N} -subalgebras of A containing $\bigcup\{A_{N_i} : i \in \mathbb{N}\}$. Then $\bigcap C$ is also a neutrosophic \mathcal{N} -subalgebra of A .

If $\bigwedge_{i \in \mathbb{N}} A_{N_i} = \bigcap_{i \in \mathbb{N}} A_{N_i}$ and $\bigvee_{i \in \mathbb{N}} A_{N_i} = \bigcap C$, then $(\{A_{N_i} : i \in \mathbb{N}\}, \bigvee, \bigwedge)$ forms a complete lattice. Moreover, it is distributive by the definitions of \bigvee and \bigwedge . \square

Lemma 3.9. *If a neutrosophic \mathcal{N} -structure A_N on a strong Sheffer stroke NMV-algebra A is a neutrosophic \mathcal{N} -subalgebra of A , then $T_N(x) \leq T_N(1)$, $I_N(x) \geq I_N(1)$ and $F_N(x) \geq F_N(1)$, for all $x \in A$.*

Proof. Let a neutrosophic \mathcal{N} -structure A_N on a strong Sheffer stroke NMV-algebra A be a neutrosophic \mathcal{N} -subalgebra of A . By substituting $[y := x]$ in the inequalities in Definition 3.1, it is obtained from Lemma 2.4 (i) that

$$T_N(x) = \min\{T_N(x), T_N(x)\} \leq T_N(x|(x|1)) = T_N(1),$$

$$I_N(1) = I_N(x|(x|1)) \leq \max\{I_N(x), I_N(x)\} = I_N(x)$$

and

$$F_N(1) = F_N(x|(x|1)) \leq \max\{F_N(x), F_N(x)\} = F_N(x),$$

for all $x \in H$. \square

The inverse of Lemma 3.9 does not hold in general.

Example 3.10. Consider the strong Sheffer stroke NMV-algebra A in Example 3.2. Then a neutrosophic \mathcal{N} -structure

$$A_N = \left\{ \frac{0}{(-0.8, -0.7, -0.02)}, \frac{u}{(-0.5, -0.4, -0.3)}, \frac{v}{(-0.2, -0.1, -0.11)}, \frac{1}{(0, -1, -0.6)} \right\}$$

on A is not a neutrosophic \mathcal{N} -subalgebra of A since

$$\max\{I_N(u), I_N(0)\} = \max\{-0.4, -0.7\} = -0.4 < -0.1 = I_N(v) = I_N(u|(0|1)).$$

Lemma 3.11. Let A_N be a neutrosophic \mathcal{N} -subalgebra of a strong Sheffer stroke NMV-algebra A . If there exists a sequence $\{a_n\}$ on A such that

$$\lim_{n \rightarrow \infty} T_N(a_n) = 0, \lim_{n \rightarrow \infty} I_N(a_n) = -1 \text{ and } \lim_{n \rightarrow \infty} F_N(a_n) = -1,$$

then

$$T_N(1) = 0, I_N(1) = -1 \text{ and } F_N(1) = -1.$$

Proof. Let A_N be a neutrosophic \mathcal{N} -subalgebra of a strong Sheffer stroke NMV-algebra A . Suppose that there exists a sequence $\{a_n\}$ on A such that $\lim_{n \rightarrow \infty} T_N(a_n) = 0$ and $\lim_{n \rightarrow \infty} I_N(a_n) = -1 = \lim_{n \rightarrow \infty} F_N(a_n)$. Since $T_N(a_n) \leq T_N(1)$, $I_N(a_n) \geq I_N(1)$ and $F_N(a_n) \geq F_N(1)$, for every $n \in \mathbb{N}$ from Lemma 3.9, it is obtained that

$$\begin{aligned} 0 &= \lim_{n \rightarrow \infty} T_N(a_n) \leq \lim_{n \rightarrow \infty} T_N(1) = T_N(1) \leq 0, \\ -1 \leq I_N(1) &= \lim_{n \rightarrow \infty} I_N(1) \leq \lim_{n \rightarrow \infty} I_N(a_n) = -1 \end{aligned}$$

and

$$-1 \leq F_N(1) = \lim_{n \rightarrow \infty} F_N(1) \leq \lim_{n \rightarrow \infty} F_N(a_n) = -1.$$

Thus, $T_N(1) = 0$ and $I_N(1) = F_N(1) = -1$. \square

Lemma 3.12. A neutrosophic \mathcal{N} -subalgebra A_N of a strong Sheffer stroke NMV-algebra A satisfies $T_N(x) \leq T_N(x|(y|1))$, $I_N(x) \geq I_N(x|(y|1))$ and $F_N(x) \geq F_N(x|(y|1))$, for all $x, y \in A$ if and only if T_N, I_N and F_N are constant.

Proof. Let A_N be a neutrosophic \mathcal{N} -subalgebra of a strong Sheffer stroke NMV-algebra A satisfying $T_N(x) \leq T_N(x|(y|1))$, $I_N(x) \geq I_N(x|(y|1))$ and $F_N(x) \geq F_N(x|(y|1))$, for all $x, y \in A$. Since $T_N(1) \leq T_N(1|(x|1)) = T_N((x|1)|1) = T_N(x)$, $I_N(1) \geq I_N(1|(x|1)) = I_N((x|1)|1) = I_N(x)$ and $F_N(1) \geq F_N(1|(x|1)) = F_N((x|1)|1) = F_N(x)$ from (n1) and (n3), it follows from Lemma 3.9 that $T_N(x) = T_N(1)$, $I_N(x) = I_N(1)$ and $F_N(x) = F_N(1)$, for all $x \in A$.

Conversely, every neutrosophic \mathcal{N} -subalgebra A_N of a strong Sheffer stroke NMV-algebra A satisfies $T_N(x) \leq T_N(x|(y|1))$, $I_N(x) \geq I_N(x|(y|1))$ and $F_N(x) \geq F_N(x|(y|1))$, for all $x, y \in A$ because T_N, I_N and F_N are constant. \square

Definition 3.13. A neutrosophic \mathcal{N} -structure A_N on a strong Sheffer stroke NMV-algebra A is called a neutrosophic \mathcal{N} -filter of A if

$$\min\{T_N(x|(y|1)), T_N(x)\} \leq T_N(y) \leq T_N(1),$$

$$I_N(1) \leq I_N(y) \leq \max\{I_N(x|(y|1)), I_N(x)\}$$

and

$$F_N(1) \leq F_N(y) \leq \max\{F_N(x|(y|1)), F_N(x)\},$$

for all $x, y \in A$.

Example 3.14. Consider the strong Sheffer stroke NMV-algebra A in Example 3.2. Then a neutrosophic \mathcal{N} -structure

$$A_N = \left\{ \frac{0}{(-0.23, -0.3, -0.01)}, \frac{u}{(-0.02, -0.98, -0.11)}, \frac{v}{(-0.23, -0.3, -0.01)}, \frac{1}{(-0.02, -0.98, -0.11)} \right\}$$

on A is a neutrosophic \mathcal{N} -filter of A .

Lemma 3.15. Every a neutrosophic \mathcal{N} -filter A_N of a strong Sheffer stroke NMV-algebra A satisfies that $x \leq y$ implies $T_N(x) \leq T_N(y)$, $I_N(x) \geq I_N(y)$ and $F_N(x) \geq F_N(y)$, for all $x, y \in A$.

Proof. Let A_N be a neutrosophic \mathcal{N} -filter of a strong Sheffer stroke NMV-algebra A and $x \leq y$. Then $x|(y|1) \approx 1$ from Proposition 2.3. Thus,

$$T_N(x) = \min\{T_N(1), T_N(x)\} = \min\{T_N(x|(y|1)), T_N(x)\} \leq T_N(y),$$

$$I_N(x) = \max\{I_N(1), I_N(x)\} = \max\{I_N(x|(y|1)), I_N(x)\} \geq I_N(y)$$

and

$$F_N(x) = \max\{F_N(1), F_N(x)\} = \max\{F_N(x|(y|1)), F_N(x)\} \geq F_N(y),$$

for any $x, y \in A$. \square

The inverse of Lemma 3.15 is generally not true.

Example 3.16. Consider the neutrosophic \mathcal{N} -filter of A in Example 3.14. Then $v \not\leq u$ when $-0.98 = I_N(u) \leq I_N(v) = -0.3$.

Lemma 3.17. *Let A_N be a neutrosophic \mathcal{N} -filter of a strong Sheffer stroke NMV-algebra A . Then*

$$\begin{aligned}
 T_N((x|(y|1)|(z|1)) &\leq T_N((x|(z|1)|((y|(z|1))|1)), \\
 I_N((x|(y|1)|(z|1)) &\geq I_N((x|(z|1)|((y|(z|1))|1)), \\
 &\text{and} \\
 F_N((x|(y|1)|(z|1)) &\geq F_N((x|(z|1)|((y|(z|1))|1)),
 \end{aligned}
 \tag{1}$$

for all $x, y, z \in A$.

Proof. Let A_N be a neutrosophic \mathcal{N} -filter of a strong Sheffer stroke NMV-algebra A . Since $(x|(y|1)|(z|1) \leq y|(z|1) \leq (x|(z|1)|((y|(z|1))|1)$ from Lemma 2.4 (iii) and (xi), it follows from Lemma 3.15 that

$$\begin{aligned}
 T_N((x|(y|1)|(z|1)) &\leq T_N((x|(z|1)|((y|(z|1))|1)), \\
 I_N((x|(y|1)|(z|1)) &\geq I_N((x|(z|1)|((y|(z|1))|1))
 \end{aligned}$$

and

$$F_N((x|(y|1)|(z|1)) \geq F_N((x|(z|1)|((y|(z|1))|1)),$$

for all $x, y, z \in A$. \square

The inverse of Lemma 3.17 does not usually hold.

Example 3.18. Consider the strong Sheffer stroke NMV-algebra A in Example 3.2. Then a neutrosophic \mathcal{N} -structure

$$A_N = \left\{ \frac{0}{(-0.69, -0.12, 0)}, \frac{u}{(-0.58, -0.87, -0.22)}, \frac{v}{(-0.58, -0.87, -0.22)}, \frac{1}{(-0.14, -0.93, 0.96)} \right\}$$

on A satisfies the condition (1) in Lemma 3.17 but it is not a neutrosophic \mathcal{N} -filter of A since

$$\min\{T_N(u|(0|1)), T_N(u)\} = \min\{T_N(v), T_N(u)\} = -0.58 > -0.69 = T_N(0).$$

Lemma 3.19. *Let A_N be a neutrosophic \mathcal{N} -structure on a strong Sheffer stroke NMV-algebra A and a, b, c be any elements of $[-1, 0]$ with $-3 \leq a + b + c \leq 0$. If A_N is a neutrosophic \mathcal{N} -filter of A , then the nonempty subset $A_N(a, b, c)$ is a filter of A .*

Proof. Let A_N be a neutrosophic \mathcal{N} -filter of a strong Sheffer stroke NMV-algebra A and $A_N(a, b, c) \neq \emptyset$ for $a, b, c \in [-1, 0]$ with $-3 \leq a + b + c \leq 0$. Since $a \leq T_N(x) \leq T_N(1), b \geq I_N(x) \geq I_N(1)$ and $c \geq F_N(x) \geq F_N(1)$, for all $x \in A_N(a, b, c)$, we have $1 \in A_N(a, b, c)$. Let $x|(y|1), x \in A_N(a, b, c)$. Then $a \leq T_N(x), I_N(x) \leq b, F_N(x) \leq c, a \leq T_N(x|(y|1)), I_N(x|(y|1)) \leq b$ and $F_N(x|(y|1)) \leq c$. Since

$$a \leq \min\{T_N(x|(y|1)), T_N(x)\} \leq T_N(y),$$

$$I_N(y) \leq \max\{I_N(x|(y|1)), I_N(x)\} \leq b$$

and

$$F_N(y) \leq \max\{F_N(x|(y|1)), F_N(x)\} \leq c,$$

for all $x, y \in A$, it is obtained $y \in A_N(a, b, c)$. Hence, $A_N(a, b, c)$ is a filter of A . \square

Theorem 3.20. *Let A_N be a neutrosophic \mathcal{N} -structure on a strong Sheffer stroke NMV-algebra A and T_N^a, I_N^b, F_N^c be filters of A , for all $a, b, c \in [-1, 0]$ with $-3 \leq a + b + c \leq 0$. Then A_N is a neutrosophic \mathcal{N} -filter of A .*

Proof. Let A_N be a neutrosophic \mathcal{N} -structure on a strong Sheffer stroke NMV-algebra A and T_N^a, I_N^b, F_N^c be filters of A , for all $a, b, c \in [-1, 0]$ with $-3 \leq a + b + c \leq 0$. Assume that $T_N(1) < T_N(x_0)$, $I_N(y_0) < I_N(1)$ and $F_N(z_0) < F_N(1)$. If $a_0 = \frac{1}{2}(T_N(1) + T_N(x_0))$, $b_0 = \frac{1}{2}(I_N(1) + I_N(y_0))$ and $c_0 = \frac{1}{2}(F_N(1) + F_N(z_0))$ in $[-1, 0)$, then $T_N(1) < a_0 < T_N(x_0)$, $I_N(1) > b_0 > I_N(y_0)$ and $F_N(1) > c_0 > F_N(z_0)$. Thus, $1 \notin T_N^{a_0}, 1 \notin I_N^{b_0}$ and $1 \notin F_N^{c_0}$, which contradict with $(S_f - 1)$. Hence, $T_N(x) \leq T_N(1)$, $I_N(x) \geq I_N(1)$ and $F_N(x) \geq F_N(1)$, for all $x \in A$. Suppose that x_1, x_2, x_3, y_1, y_2 and y_3 are any elements of A such that

$$v_1 = T_N(y_1) < \min\{T_N(x_1|(y_1|1)), T_N(x_1)\} = u_1,$$

$$u_2 = \max\{I_N(x_2|(y_2|1)), I_N(x_2)\} < I_N(y_2) = v_2,$$

and

$$u_3 = \max\{F_N(x_3|(y_3|1)), F_N(x_3)\} < F_N(y_3) = v_3.$$

If $a' = \frac{1}{2}(u_1 + v_1)$, $b' = \frac{1}{2}(u_2 + v_2)$ and $c' = \frac{1}{2}(u_3 + v_3)$ in $[-1, 0)$, then $v_1 < a' < u_1$, $u_2 < b' < v_2$ and $u_3 < c' < v_3$. So, $y_1 \notin T_N^{a'}, y_2 \notin I_N^{b'}$ and $y_3 \notin F_N^{c'}$ when $x_1|(y_1|1), x_1 \in T_N^{a'}$, $x_2|(y_2|1), x_2 \in I_N^{b'}$ and $x_3|(y_3|1), x_3 \in F_N^{c'}$. This is a contradiction. Thereby,

$$\min\{T_N(x|(y|1)), T_N(x)\} \leq T_N(y),$$

$$I_N(y) \leq \max\{I_N(x|(y|1)), I_N(x)\}$$

and

$$F_N(y) \leq \max\{F_N(x|(y|1)), F_N(x)\},$$

for all $x, y \in A$. Therefore, A_N is a neutrosophic \mathcal{N} -filter of A . \square

Lemma 3.21. *Let A_N be a neutrosophic \mathcal{N} -structure on a strong Sheffer stroke NMV-algebra A . Then A_N is a neutrosophic \mathcal{N} -filter of A if and only if $z \leq y|(x|1)$ implies*

$$\min\{T_N(y), T_N(z)\} \leq T_N(x),$$

$$I_N(x) \leq \max\{I_N(y), I_N(z)\}$$

and

$$F_N(x) \leq \max\{F_N(y), F_N(z)\},$$

for all $x, y, z \in A$.

Proof. Let A_N be a neutrosophic \mathcal{N} -filter of A and x, y and z be any elements of A such that $z \leq y|(x|1)$. Since $T_N(z) \leq T_N(y|(x|1))$, $I_N(z) \geq I_N(y|(x|1))$ and $F_N(z) \geq F_N(y|(x|1))$ from Lemma 3.15, it follows that

$$\min\{T_N(y), T_N(z)\} \leq \min\{T_N(y|(x|1)), T_N(y)\} \leq T_N(x),$$

$$I_N(x) \leq \max\{I_N(y|(x|1)), I_N(y)\} \leq \max\{I_N(y), I_N(z)\}$$

and

$$F_N(x) \leq \max\{F_N(y|(x|1)), F_N(y)\} \leq \max\{F_N(y), F_N(z)\},$$

for all $x, y, z \in A$.

Conversely, suppose that A_N is a neutrosophic \mathcal{N} -structure on A such that $z \leq y|(x|1)$ implies

$$\min\{T_N(y), T_N(z)\} \leq T_N(x),$$

$$I_N(x) \leq \max\{I_N(y), I_N(z)\}$$

and

$$F_N(x) \leq \max\{F_N(y), F_N(z)\},$$

for all $x, y, z \in A$. Since $x \leq 1 \approx x|0 \approx x|(1|1)$ from (n2), it is obtained that $T_N(x) \leq T_N(1)$, $I_N(1) \leq I_N(x)$ and $F_N(1) \leq F_N(x)$, for all $x \in A$. Since $x \leq (x|(y|1))|(y|1)$ from Lemma 2.4 (v), we have

$$\min\{T_N(x|(y|1)), T_N(x)\} \leq T_N(y),$$

$$I_N(y) \leq \max\{I_N(x|(y|1)), I_N(x)\}$$

and

$$F_N(y) \leq \max\{F_N(x|(y|1)), F_N(x)\},$$

for all $x, y \in A$. Hence, A_N is a neutrosophic \mathcal{N} -filter of A . \square

Theorem 3.22. *Every neutrosophic \mathcal{N} -filter of a strong Sheffer stroke NMV-algebra A is a neutrosophic \mathcal{N} -subalgebra of A .*

Proof. Let A_N be a neutrosophic \mathcal{N} -filter of A . Since

$$\begin{aligned} \min\{T_N(x), T_N(y)\} &\leq \min\{T_N(1), T_N(y)\} \\ &= \min\{T_N(((y|1)|1)|((y|1)|x)|1)), T_N(y)\} \\ &= \min\{T_N(y|((x|(y|1))|1)), T_N(y)\} \\ &\leq T_N(x|(y|1)), \end{aligned}$$

and similarly,

$$I_N(x|(y|1)) \leq \max\{I_N(x), I_N(y)\}$$

and

$$F_N(x|(y|1)) \leq \max\{F_N(x), F_N(y)\},$$

from (n1), (n3) and (n5), it follows that A_N is a neutrosophic \mathcal{N} -subalgebra of A . \square

The inverse of Theorem 3.22 does not usually hold.

Example 3.23. The neutrosophic \mathcal{N} -subalgebra A_N of A in Example 3.2. Then it is not a neutrosophic \mathcal{N} -filter of A since $\min\{T_N(u|(0|1)), T_N(u)\} = \min\{T_N(v), T_N(u)\} = -0.68 > -0.79 = T_N(0)$.

Definition 3.24. Let A be a strong Sheffer stroke NMV-algebra. Define

$$\begin{aligned} A_N^{x_t} &:= \{x \in A : T_N(x_t) \leq T_N(x)\}, \\ A_N^{x_i} &:= \{x \in A : I_N(x) \leq I_N(x_i)\} \end{aligned}$$

and

$$A_N^{x_f} := \{x \in A : F_N(x) \leq F_N(x_f)\},$$

for all $x_t, x_i, x_f \in A$. Obviously, $x_t \in A_N^{x_t}, x_i \in A_N^{x_i}$ and $x_f \in A_N^{x_f}$.

Example 3.25. Consider the strong Sheffer stroke NMV-algebra A in Example 3.2. Let $T_N(0) = -0.113, T_N(u) = -0.12, T_N(v) = -0.13, T_N(1) = 0, I_N(0) = -0.21, I_N(u) = -0.22, I_N(v) = -0.23, I_N(1) = -1, F_N(0) = -0.31, F_N(u) = -0.32, F_N(v) = -0.33, F_N(1) = -0.34, x_t = u, x_i = v$ and $x_f = 0$. Then

$$\begin{aligned} A_N^{x_t} &= \{x \in A : T_N(u) \leq T_N(x)\} = \{0, u, 1\}, \\ A_N^{x_i} &= \{x \in A : I_N(x) \leq I_N(v)\} = \{v, 1\} \end{aligned}$$

and

$$A_N^{x_f} = \{x \in A : F_N(x) \leq F_N(0)\} = A.$$

Theorem 3.26. Let x_t, x_i and x_f be any elements of a strong Sheffer stroke NMV-algebra A . If A_N is a neutrosophic \mathcal{N} -filter of A , then $A_N^{x_t}, A_N^{x_i}$ and $A_N^{x_f}$ are filters of A .

Proof. Let A_N be a neutrosophic \mathcal{N} -filter of a strong Sheffer stroke NMV-algebra A . Since $T_N(x_t) \leq T_N(1)$, $I_N(1) \leq I_N(x_i)$ and $F_N(1) \leq F_N(x_f)$, for any $x_t, x_i, x_f \in A$, we have $1 \in A_N^{x_t}$, $1 \in A_N^{x_i}$ and $1 \in A_N^{x_f}$. Let $x_1|(y_1|1), x_1 \in A_N^{x_t}$, $x_2|(y_2|1), x_2 \in A_N^{x_i}$ and $x_3|(y_3|1), x_3 \in A_N^{x_f}$. Then $T_N(x_t) \leq T_N(x_1|(y_1|1))$, $T_N(x_t) \leq T_N(x_1)$, $I_N(x_2|(y_2|1)) \leq I_N(x_i)$, $I_N(x_2) \leq I_N(x_i)$ and $F_N(x_3|(y_3|1)) \leq F_N(x_f)$, $F_N(x_3) \leq F_N(x_f)$. Since

$$T_N(x_t) \leq \min\{T_N(x_1|(y_1|1)), T_N(x_1)\} \leq T_N(y_1),$$

$$I_N(y_2) \leq \max\{I_N(x_2|(y_2|1)), I_N(x_2)\} \leq I_N(x_i)$$

and

$$F_N(y_3) \leq \max\{F_N(x_3|(y_3|1)), F_N(x_3)\} \leq F_N(x_f),$$

we get $y_1 \in A_N^{x_t}$, $y_2 \in A_N^{x_i}$ and $y_3 \in A_N^{x_f}$. Thus, $A_N^{x_t}$, $A_N^{x_i}$ and $A_N^{x_f}$ are filters of A . \square

Example 3.27. Consider the strong Sheffer stroke NMV-algebra A in Example 3.2. For a neutrosophic \mathcal{N} -filter

$$A_N = \left\{ \frac{0}{(-0.32, -0.29, -0.07)}, \frac{u}{(-0.32, -0.29, -0.07)}, \frac{v}{(-0.1, -0.78, -0.17)}, \frac{1}{(-0.1, -0.78, -0.17)} \right\}$$

of A , $x_t = u$, $x_i = v$ and $x_f = 1 \in A$, the subsets

$$A_N^{x_t} = \{x \in A : T_N(u) \leq T_N(x)\} = A,$$

$$A_N^{x_i} = \{x \in A : I_N(x) \leq I_N(v)\} = \{v, 1\}$$

and

$$A_N^{x_f} = \{x \in A : F_N(x) \leq F_N(1)\} = \{v, 1\}$$

of A are filters of A .

Theorem 3.28. Let x_t, x_i and x_f be any elements of a strong Sheffer stroke NMV-algebra A and A_N be a neutrosophic \mathcal{N} -structure on A .

(a) If $A_N^{x_t}, A_N^{x_i}$ and $A_N^{x_f}$ are filters of A , then

$$T_N(x) \leq \min\{T_N(y|(z|1)), T_N(y)\} \Rightarrow T_N(x) \leq T_N(z),$$

$$I_N(x) \geq \max\{I_N(y|(z|1)), I_N(y)\} \Rightarrow I_N(x) \geq I_N(z) \tag{2}$$

and

$$F_N(x) \geq \max\{F_N(y|(z|1)), F_N(y)\} \Rightarrow F_N(x) \geq F_N(z),$$

for all $x, y, z \in A$.

(b) If A_N satisfies the condition (3.2) and

$$T_N(x) \leq T_N(1), \quad I_N(1) \leq I_N(x) \quad \text{and} \quad F_N(1) \leq F_N(x), \quad \text{for all } x \in A, \quad (3)$$

then $A_N^{x_t}, A_N^{x_i}$ and $A_N^{x_f}$ are filters of A , for all $x_t \in T_N^{-1}$, $x_i \in I_N^{-1}$ and $x_f \in F_N^{-1}$.

Proof. Let A_N be a neutrosophic \mathcal{N} -structure on a strong Sheffer stroke NMV-algebra A .

(a) Let $A_N^{x_t}, A_N^{x_i}$ and $A_N^{x_f}$ be filters of A , for any $x_t, x_i, x_f \in A$, and x, y, z be any elements of A such that $T_N(x) \leq \min\{T_N(y|(z|1)), T_N(y)\}$, $I_N(x) \geq \max\{I_N(y|(z|1)), I_N(y)\}$ and $F_N(x) \geq \max\{F_N(y|(z|1)), F_N(y)\}$. Since $y|(z|1), y \in A_N^{x_t}, y|(z|1), y \in A_N^{x_i}$ and $y|(z|1), y \in A_N^{x_f}$, where $x_t = x_i = x_f = x$, it follows from $(S_f - 2)$ that $z \in A_N^{x_t}, z \in A_N^{x_i}$ and $z \in A_N^{x_f}$, where $x_t = x_i = x_f = x$. Thus, $T_N(x) \leq T_N(z), I_N(z) \leq I_N(x)$ and $F_N(z) \leq F_N(x)$, for all $x, y, z \in A$.

(b) Let A_N be a neutrosophic \mathcal{N} -structure on A satisfying the conditions (2) and (3), for $x_t \in T_N^{-1}$, $x_i \in I_N^{-1}$ and $x_f \in F_N^{-1}$. Then $1 \in A_N^{x_t}, 1 \in A_N^{x_i}$ and $1 \in A_N^{x_f}$ from the condition (3). Let $x_1|(y_1|1), x_1 \in X_N^{x_t}, x_2|(y_2|1), x_2 \in A_N^{x_i}$ and $x_3|(y_3|1), x_3 \in A_N^{x_f}$. Thus, $T_N(x_t) \leq T_N(x_1|(y_1|1)), T_N(x_t) \leq T_N(x_1), I_N(x_2|(y_2|1)) \leq I_N(x_i), I_N(x_2) \geq I_N(x_i)$ and $F_N(x_3|(y_3|1)) \leq F_N(x_f), F_N(x_3) \leq F_N(x_f)$. Since

$$T_N(x_t) \leq \min\{T_N(x_1|(y_1|1)), T_N(x_1)\},$$

$$\max\{I_N(x_2|(y_2|1)), I_N(x_2)\} \leq I_N(x_i)$$

and

$$\max\{F_N(x_3|(y_3|1)), F_N(x_3)\} \leq F_N(x_f),$$

it follows from the condition (2) that $T_N(x_t) \leq T_N(y_1), I_N(y_2) \leq I_N(x_i)$ and $F_N(y_3) \leq F_N(x_f)$. Hence, $y_1 \in A_N^{x_t}, y_2 \in A_N^{x_i}$ and $y_3 \in A_N^{x_f}$. Therefore, $A_N^{x_t}, A_N^{x_i}$ and $A_N^{x_f}$ are filters of A . \square

Example 3.29. Consider the strong Sheffer stroke NMV-algebra A in Example 3.2. Let $T_N(0) = T_N(v) = -1, T_N(u) = T_N(1) = 0, I_N(0) = I_N(v) = 0, I_N(u) = I_N(1) = -1, F_N(0) = F_N(v) = -0.71, F_N(u) = F_N(1) = -0.5$. Then the filters

$$A_N^{x_t} = A, A_N^{x_i} = \{u.1\} \quad \text{and} \quad A_N^{x_f} = A$$

of A satisfy the condition (2) in Theorem 3.28, for $x_t = v, x_i = u$ and $x_f = 1 \in A$.

Moreover, let

$$A_N = \left\{ \frac{0}{(-0.99, 0, -0.01)}, \frac{u}{(-0.99, 0, -0.01)}, \frac{v}{(-0.99, 0, -0.01)}, \frac{1}{(0, -1, -1)} \right\}$$

be a neutrosophic \mathcal{N} -structure on A satisfying the conditions (2) and (3) in Theorem 3.28.

Then the subsets

$$A_N^{x_t} = \{x \in A : T_N(1) \leq T_N(x)\} = \{1\},$$

$$A_N^{x_i} = \{x \in A : I_N(x) \leq I_N(0)\} = A$$

and

$$A_N^{x_f} = \{x \in A : F_N(x) \leq F_N(u)\} = A$$

of A are filters of A , where $x_t = 1, x_i = 0$ and $x_f = u \in A$.

4. Conclusion

In this study, neutrosophic \mathcal{N} -structures defined by \mathcal{N} -functions on strong Sheffer stroke NMV-algebras have been investigated. Basic definitions and notions about strong Sheffer stroke NMV-algebras and neutrosophic \mathcal{N} -structures defined by \mathcal{N} -functions on a nonempty universe X are presented and then a neutrosophic \mathcal{N} -subalgebra and a (a, b, c) -level set of a neutrosophic \mathcal{N} -structure are defined by the help of \mathcal{N} -functions on strong Sheffer stroke NMV-algebras. It is shown that the (a, b, c) -level set of a neutrosophic \mathcal{N} -subalgebra of a strong Sheffer stroke NMV-algebra is its strong Sheffer stroke NMV-subalgebra and vice versa. Also, it is proved that the family of all neutrosophic \mathcal{N} -subalgebras of this algebraic structure forms a complete distributive lattice. It is illustrated that every neutrosophic \mathcal{N} -subalgebra of a strong Sheffer stroke NMV-algebra satisfies $T_N(x) \leq T_N(1), I_N(1) \leq I_N(x)$ and $F_N(1) \leq F_N(x)$, for all elements x in this algebra but a neutrosophic \mathcal{N} -structure on a strong Sheffer stroke NMV-algebra satisfying this property is generally not its neutrosophic \mathcal{N} -subalgebra. Besides, it is interpreted the images of the sequence under \mathcal{N} -functions on a strong Sheffer stroke NMV-algebra. Moreover, it is stated the case which \mathcal{N} -functions determining a neutrosophic \mathcal{N} -subalgebra of a strong Sheffer stroke NMV-algebra are constant. Then a neutrosophic \mathcal{N} -filter of a strong Sheffer stroke NMV-algebra is defined via \mathcal{N} -functions and shown that the functions T_N, I_N and F_N defining the neutrosophic \mathcal{N} -filter satisfies $T_N(x) \leq T_N(y), I_N(x) \geq I_N(y)$ and $F_N(x) \geq F_N(y)$ when $x \leq y$, but the inverse does not usually hold. It is demonstrated that (a, b, c) -level set of a neutrosophic \mathcal{N} -filter of a strong Sheffer stroke NMV-algebra is its filter. Indeed, it is given that the subsets defined by \mathcal{N} -functions on a strong Sheffer stroke NMV-algebra must be its filters so that a neutrosophic \mathcal{N} -structure on this algebra is a neutrosophic \mathcal{N} -filter. It is proved that every neutrosophic \mathcal{N} -filter of a strong Sheffer stroke NMV-algebra is its neutrosophic \mathcal{N} -subalgebra whereas the inverse is not true in general. Additionally, new three subsets $A_N^{x_t}, A_N^{x_i}$ and $A_N^{x_f}$ of a strong Sheffer stroke NMV-algebra are defined by \mathcal{N} -functions and any elements x_t, x_i and x_f of the algebra. We show that these subsets are filters of a strong Sheffer stroke NMV-algebra for its neutrosophic \mathcal{N} -filter but the inverse holds under special conditions.

In our future works, we wish to introduce new Sheffer stroke algebraic structures and investigate their neutrosophic \mathcal{N} -structures.

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References

1. Abbott, J. C. Implicational algebras. *Bulletin Mathématique de la Société des Sciences Mathématiques de la République Socialiste de Roumanie* 1967, 11(1), pp. 3-23.
2. Atanassov, K. T. Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* 1986, 20, pp. 87-96.
3. Borumand Saeid, A.; Jun, Y. B. Neutrosophic subalgebras of BCK/BCI-algebras based on neutrosophic points. *Ann. Fuzzy Math. Inform.* 2017, 14, pp. 87-97.
4. Borzooei, R. A.; Zhang, X.; Smarandache, F.; Jun, Y. B. Commutative generalized neutrosophic ideals in BCK-algebras. *Symmetry* 2018, 10(350).
5. Chajda, I. Sheffer operation in ortholattices. *Acta Universitatis Palackianae Olomucensis, Facultas Rerum Naturalium. Mathematica* 2005, 44(1), pp. 19-23.
6. Chajda, I.; Halaš, R.; Länger, H. Operations and structures derived from non-associative MV-algebras. *Soft Computing* 2019, 23(12), pp. 3935-3944.
7. Chajda, I.; Kühr, J. A non-associative generalization of MV-algebras. *Math. Slov.* 2007, 57, pp. 301-312.
8. Chajda, I.; Länger, H. Properties of non-associative MV-algebras. *Math. Slov.* 2017, 67, pp. 1095-1104.
9. Chang, C. C. Algebraic analysis of many-valued logics, *Transactions of the American Mathematical Society* 1958, 88, pp. 467-490.
10. Chang, C. C. A new proof of the completeness of the Łukasiewicz axioms. *Transactions of the American Mathematical Society* 1959, 93, pp. 74-80.
11. Jun, Y. B.; Lee, K. J.; Song, S. Z. \mathcal{N} -ideals of BCK/BCI-algebras. *J. Chungcheong Math. Soc.* 2009, 22, pp. 417-437.
12. Jun, Y. B.; Smarandache, F.; Bordbar, H. Neutrosophic \mathcal{N} -structures applied to BCK/BCI-algebras. *Information* 2017, 8(128), pp. 1-12.
13. Jun, Y. B. Neutrosophic subalgebras of several types in BCK/BCI-algebras. *Ann. Fuzzy Math. Inform.* 2017, 14, pp. 75-86.
14. Jun, Y. B.; Kim, S. J.; Smarandache, F. Interval neutrosophic sets with applications in BCK/BCI-algebras. *Axioms* 2018, 7(23), pp. 1-13.
15. Jun, Y. B.; Smarandache, F.; Song, S. Z.; Khan, M., Neutrosophic positive implicative \mathcal{N} -ideals in BCK-algebras. *Axioms* 2018, 7(3), pp. 1-13.
16. Khan, M.; Anis, S.; Smarandache, F.; Jun, Y. B. Neutrosophic \mathcal{N} -structures and their applications in semigroups. *Infinite Study* 2017.
17. McCune, W.; Veroff, R.; Fitelson, B.; Harris, K.; Feist, A.; Wos, L. Short single axioms for Boolean algebra. *Journal of Automated Reasoning* 2002, 29(1), pp. 1-16.
18. Oner, T.; Katican, T.; Borumand Saeid, A. Relation between Sheffer Stroke and Hilbert Algebras. *Categories and General Algebraic Structures with Applications* 2021, (in press).
19. Oner, T.; Katican, T.; Borumand Saeid, A. Fuzzy filters of Sheffer stroke Hilbert algebras. *Journal of Intelligent and Fuzzy Systems* 2021, 40 (1), pp. 759-772.
20. Oner, T.; Katican, T.; Borumand Saeid, A. On Sheffer stroke UP-algebras. *Discussiones Mathematicae – General Algebra and Applications* 2021, (in press)
21. Oner, T.; Kalkan, T.; Kircali Gursoy, N. Sheffer stroke BG-algebras. *International Journal of Maps in Mathematics* 2021, 4(1), pp. 27-3.
22. Oner, T.; Katican, T.; Borumand Saeid, A.; Terziler, M. Filters of strong Sheffer stroke non-associative MV-algebras. *Analele Stiintifice ale Universitatii Ovidius Constanta* 2021, (in press).

23. Oner, T.; Katican T.; Borumand Saeid, A. (Fuzzy) filters of Sheffer stroke BL-algebras. *Kragujevac Journal of Mathematics* 2023, 47(1), pp. 39-55.
24. Rezaei, A.; Borumand Saeid, A.; Smarandache, F. Neutrosophic filters in BE-algebras. *Ratio Mathematica* 2015, 99, pp. 65-79.
25. Sheffer, H. M. A set of five independent postulates for Boolean algebras, with application to logical constants. *Transactions of the American Mathematical Society* 1913, 14(4), pp. 481-488.
26. Smarandache, F. A unifying field in logic. *Neutrosophy: neutrosophic probability, set and logic*. American Research Press: Rehoboth, NM, USA, 1999.
27. Smarandache, F. Neutrosophic set-a generalization of the intuitionistic fuzzy set. *Int. J. Pure Appl. Math.* 2005, 24, pp. 287-297.
28. Song, S. Z.; Smarandache, F.; Jun, Y. B. Neutrosophic commutative \mathcal{N} -ideals in BCK-algebras. *Information* 2017, 8(130).
29. Zadeh L. A. Fuzzy sets. *Inf. Control* 1965, 8, pp. 338-353.
30. Zhang, X.; Yu, P.; Smarandach, F.; Park, C. Redefined neutrosophic filters in BE-algebras. *Italian Journal of Pure and Applied Mathematics* 2020, 43, pp. 158-176.
31. Öztürk, M. A.; Jun, Y. B. Neutrosophic ideals in BCK/BCI-algebras based on neutrosophic points. *J. Int. Math. Virtual Inst.* 2018, 8, pp. 1-17.
32. <http://fs.gallup.unm.edu/FlorentinSmarandache.htm>

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