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## Some Fundamental Operations on Interval Valued Neutrosophic Hypersoft Set with Their Properties

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# Some Fundamental Operations on Interval Valued Neutrosophic Hypersoft Set with Their Properties

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**Abstract:** Multi-criteria decision-making (MCDM) focuses on coordination, choice and planning issues, including multi-criteria. the neutrosophic soft set cannot handle environments involving multiple attributes. In order to overcome these obstacles, the neutrosophic hypersoft set (NHSS) and Interval Value neutrosophic hypersoft set (IVNHSS) are defined. In this paper, we extend the concept of IVNHSS with basic properties. We also developed some basic operations on IVNHSS such as union, intersection, addition, difference, Truth-favorite, and False-favorite, etc. with their desirable properties. Finally, the necessity and possibility operations on IVNHSS with properties are presented in the following research.

**Keywords:** Soft set; Neutrosophic Set; Interval-valued neutrosophic set; Hypersoft set; Interval-valued neutrosophic hypersoft set.

## 1. Introduction

Anxiety performs a dynamic part in lots of areas of life such as modeling, medicine, and engineering. However, people have raised a general question, that is, how can we verbalize anxiety in mathematical modeling. Several investigators all over the world have recommended and advised different methodologies to minimize uncertainty. First of all, Zadeh planned the idea of fuzzy sets [1] to resolve these complications which contain anxiety as well as ambiguity. It is seen that sometimes; fuzzy sets can't deal with scenarios. To overcome such scenarios, Turksen [2] suggested the concept of interval-valued fuzzy sets (IVFS). In some cases, we need to debate the suitable representation of the object under the circumstances of anxiety and uncertainty, and regard its unbiased membership value and non-membership value of the suitable representation of the object, that cannot be processed by these fuzzy sets or IVFS. To overcome such concerns, Atanassov projected the theory of IFS in [3]. The theory proposed by Atanassov only considers membership and non-membership values to deal with insufficient data, but the IFS theory cannot deal with incompatible and imprecise information. To deal with this incompatible and imprecise data, Smarandache proposed the idea of NS [4]. Molodtsov [5] proposed a general mathematical tool to deal with uncertain, ambiguous, and undefined substances, called soft sets (SS). Maji et al. [6] extended the work of SS and defined some operations and their attributes. In [7], they also use SS theory to make decisions. Ali et al. [8] Modified the Maji method of SS and developed some new operations with its properties. In [9], they proved De Morgan's SS theory and law by using different operators. Cagman and Enginoglu [10] proposed the concept of soft matrices with operations and discussed their properties. They also introduced a decision-making method to solve problems that contain uncertainty. In [11], they modified the

actions proposed by Molodtsov's SS. In [12], the author proposed some new operations for soft matrices, such as soft difference product, soft restricted difference product, soft extended difference product, and weak extended difference product.

Maji [13] put forward the idea of NSS with necessary operations and characteristics. The idea of Possibility NSS was proposed by Karaaslan [14] and introduced a neutrosophic soft decision method to solve those uncertain problems based on And-product. Broumi [15] developed a generalized NSS with certain operations and properties and used the proposed concept for decision-making. To solve the MCDM problem with single-valued neutrosophic numbers proposed by Deli and Subas in [16], they constructed the concept of the cut set of single-valued neutrosophic numbers. Based on the correlation of IFS, the term correlation coefficient of SVNNS is introduced [17]. In [18], the idea of simplifying NS introduced some algorithms and aggregation operators, such as weighted arithmetic operators and weighted geometric average operators. They constructed the MCDM method based on the proposed aggregation operator. Zulqarnain et al. [19] extended the fuzzy TOPSIS technique to the Neutrosophic TOPSIS technique and used the developed approach to solve the MCDM problem. Abdel-basset et al [20] presented the integration of TOPSIS methodology decision-making test as well as evaluation laboratory (DEMATEL) solution (TOPSIS) CIIC environment delivers a new method to pick out the proper project. Abdel-basset Mohamed [21] developed an MCDM model to discover along with display screen cancer addressing obscure, anxiety, the incompleteness of reported signs as well as handicapping apparently within cancer or replaceable ailments in the signs and symptoms. Abdel-Basset et al. [22] raised the issue of assessment of the smart emergency response techniques is interpreted as MCDM problem. they suggested a framework by combining three common MCDM strategies which are AHP, TOPSIS, and VIKOR.

All the above-mentioned studies cannot deal with the problems in which attributes of the alternates have their corresponding sub-attributes. To handle such compilations Smarandache [23] generalized the SS to HSS by converting the function to a multi-attribute function to deal with uncertainty. Saqlain et al. [24] developed the generalization of TOPSIS for the NHSS, by using accuracy function they transformed the fuzzy neutrosophic numbers to crisp form. Zulqarnain et al. [25] extended the notion of NHSSs and presented the generalized operations for NHSSs, they also developed the necessity and possibility operations and discussed their desirable features. In [26], the author's proposed the fuzzy Plithogenic hypersoft set in matrix form with some basic operations and properties. Saqlain et al. [27] proposed the aggregate operators on NHSS. In [28], the author extended the NHSS approach and introduced IVNHSS, m-polar, and m-polar IVNHSS. Zulqarnain et al. [29] presented the intuitionistic fuzzy hypersoft set, they developed the TOPSIS technique by developing a correlation coefficient to solve multi-attribute decision making problems. Many other novel researchers are done under neutrosophic environment and their applications in everyday life [30-34].

The following research is organized as follows: Some basic definitions recalled in section 2, which are used in the following research such as SS, NS, NSS, HSS, NHSS, and IVNHSS. We present different operators on IVNHSS such as union, intersection, addition, difference, extended union, extended intersection, truth-favorite, and false-favorite operations in section 3 with properties and prove the De Morgan laws by using union and intersection operators. We also proposed the necessity and possibility operators, OR, and operations with some properties in section 4.

## 2. Preliminaries

In this section, we recollect some basic definitions such as SS, NSS, NHSS, and IVNHSS which use in the following sequel.

### Definition 2.1 [5]

The soft set is a pair  $(F, \Lambda)$  over  $\mathbb{U}$  if and only if  $F: \Lambda \rightarrow P(\mathbb{U})$  is a mapping. That is the parameterized family of subsets of  $\mathbb{U}$  known as a SS.

### Definition 2.2 [4]

Let  $\mathbb{U}$  be a universe and  $\Lambda$  be an NS on  $\mathbb{U}$  is defined as  $\Lambda = \{ \langle u, u_A(u), v_A(u), w_A(u) \rangle : u \in \mathbb{U} \}$ , where  $u, v, w: \mathbb{U} \rightarrow ]0^-, 1^+[$  and  $0^- \leq u_A(u) + v_A(u) + w_A(u) \leq 3^+$ .

**Definition 2.3** [13]

Let  $\mathbb{U}$  and  $\check{\mathbb{E}}$  are universal set and set of attributes respectively. Let  $P(\mathbb{U})$  be the set of Neutrosophic values of  $\mathbb{U}$  and  $\Lambda \subseteq \check{\mathbb{E}}$ . A pair  $(F, \Lambda)$  is called an NSS over  $\mathbb{U}$  and its mapping is given as

$$F: \Lambda \rightarrow (\mathbb{U})$$

**Definition 2.4** [35]

Let  $\mathbb{U}$  be a universal set, then interval valued neutrosophic set can be expressed by the set  $A = \{ \langle u, u_A(u), v_A(u), w_A(u) \rangle : u \in \mathbb{U} \}$ , where  $u_A, v_A$ , and  $w_A$  are truth, indeterminacy and falsity membership functions for  $A$  respectively,  $u_A, v_A$ , and  $w_A \subseteq [0, 1]$  for each  $u \in \mathbb{U}$ . Where

$$\begin{aligned} u_A(u) &= [u_A^L(u), u_A^U(u)] \\ v_A(u) &= [v_A^L(u), v_A^U(u)] \\ w_A(u) &= [w_A^L(u), w_A^U(u)] \end{aligned}$$

For each point  $u \in \mathbb{U}$ ,  $0 \leq u_A(u) + v_A(u) + w_A(u) \leq 3$  and  $IVN(\mathbb{U})$  represents the family of all interval valued neutrosophic sets.

**Definition 2.5** [23]

Let  $\mathbb{U}$  be a universal set and  $P(\mathbb{U})$  be a power set of  $\mathbb{U}$  and for  $n \geq 1$ , there are  $n$  distinct attributes such as  $k_1, k_2, k_3, \dots, k_n$  and  $K_1, K_2, K_3, \dots, K_n$  are sets for corresponding values attributes respectively with following conditions such as  $K_i \cap K_j = \emptyset (i \neq j)$  and  $i, j \in \{1,2,3 \dots n\}$ . Then the pair  $(F, K_1 \times K_2 \times K_3 \times \dots \times K_n)$  is said to be HSS over  $\mathbb{U}$  where  $F$  is a mapping from  $K_1 \times K_2 \times K_3 \times \dots \times K_n$  to  $P(\mathbb{U})$ .

**Definition 2.6** [23]

Let  $\mathbb{U}$  be a universal set and  $P(\mathbb{U})$  be a power set of  $\mathbb{U}$  and for  $n \geq 1$ , there are  $n$  distinct attributes such as  $k_1, k_2, k_3, \dots, k_n$  and  $K_1, K_2, K_3, \dots, K_n$  are sets for corresponding values attributes respectively with following conditions such as  $K_i \cap K_j = \emptyset (i \neq j)$  and  $i, j \in \{1,2,3 \dots n\}$ . Then the pair  $(F, \Lambda)$  is said to be NHSS over  $\mathbb{U}$  if there exists a relation  $K_1 \times K_2 \times K_3 \times \dots \times K_n = \Lambda$ .  $F$  is a mapping from  $K_1 \times K_2 \times K_3 \times \dots \times K_n$  to  $P(\mathbb{U})$  and  $F(K_1 \times K_2 \times K_3 \times \dots \times K_n) = \{ \langle u, u_A(u), v_A(u), w_A(u) \rangle : u \in \mathbb{U} \}$  where  $u, v, w$  are membership values for truthness, indeterminacy and falsity respectively such that  $u, v, w: \mathbb{U} \rightarrow ]0^-, 1^+[$  and  $0^- \leq u_A(u) + v_A(u) + w_A(u) \leq 3^+$ .

**Definition 2.7** [28]

Let  $\mathbb{U}$  be a universal set and  $P(\mathbb{U})$  be a power set of  $\mathbb{U}$  and for  $n \geq 1$ , there are  $n$  distinct attributes such as  $k_1, k_2, k_3, \dots, k_n$  and  $K_1, K_2, K_3, \dots, K_n$  are sets for corresponding values attributes respectively with following conditions such as  $K_i \cap K_j = \emptyset (i \neq j)$  and  $i, j \in \{1,2,3 \dots n\}$ . Then the pair  $(F, A)$  is said to be IVNHSS over  $\mathbb{U}$  if there exists a relation  $K_1 \times K_2 \times K_3 \times \dots \times K_n = A$ . Where

$$F: K_1 \times K_2 \times K_3 \times \dots \times K_n \rightarrow (\mathbb{U}) \text{ and}$$

$$F(K_1 \times K_2 \times K_3 \times \dots \times K_n) = \{ \langle u, [u_A^L(u), u_A^U(u)], [v_A^L(u), v_A^U(u)], [w_A^L(u), w_A^U(u)] \rangle : u \in \mathbb{U} \},$$

where  $u_A^L, v_A^L$ , and  $w_A^L$  are lower and  $u_A^U, v_A^U$ , and  $w_A^U$  are upper membership values for truthness, indeterminacy, and falsity respectively for  $A$  and  $[u_A^L(u), u_A^U(u)], [v_A^L(u), v_A^U(u)], [w_A^L(u), w_A^U(u)] \subseteq [0, 1]$  and  $0 \leq \sup u_A(u) + \sup v_A(u) + \sup w_A(u) \leq 3$  for each  $u \in \mathbb{U}$ .

**Example 1** Assume  $\mathbb{U} = \{u_1, u_2\}$  be a universe of discourse and  $E = \{x_1, x_2, x_3, x_4\}$  be a set of attributes. Consider  $F_A$  be an IVNHSS over  $\mathbb{U}$  can be expressed as follows

$$F_A = \{(x_1, \{\langle u_1, [ . 6, . 8], [ . 5, 0.9], [ . 1, . 4] \rangle, \langle u_2, [ . 4, . 7], [ . 3, . 9], [ . 2, . 6] \rangle\}),$$

$$(x_2, \{\langle u_1, [ . 4, . 7], [ . 3, . 9], [ . 3, . 5] \rangle, \langle u_2, [ 0, . 3], [ . 6, . 8], [ . 3, . 7] \rangle\}),$$

$$(x_3, \{\langle u_1, [ . 2, . 9], [ . 1, . 5], [ . 7, . 8] \rangle, \langle u_2, [ . 4, . 9], [ . 1, . 6], [ . 5, . 7] \rangle\}),$$

$$(x_4, \{\langle u_1, [ . 6, . 9], [ . 6, . 9], [ 1, 1] \rangle, \langle u_2, [ . 5, . 9], [ . 6, . 8], [ . 1, . 8] \rangle\}).$$

Tablur representation of IVNHSS  $F_A$  over  $\mathbb{U}$  given as follows

**Table 1: Tablur representation of IVNHSS  $F_A$**

$\mathbb{U}$	$u_1$	$u_1$
$x_1$	$\langle [ . 6, . 8], [ . 5, . 9], [ . 1, . 4] \rangle$	$\langle [ . 4, . 7], [ . 3, . 9], [ . 2, . 6] \rangle$
$x_2$	$\langle [ . 4, . 7], [ . 3, . 9], [ . 3, . 5] \rangle$	$\langle [ 0, . 3], [ . 6, . 8], [ . 3, . 7] \rangle$
$x_3$	$\langle [ . 2, . 9], [ . 1, . 5], [ . 7, . 8] \rangle$	$\langle [ . 4, . 9], [ . 1, . 6], [ . 5, . 7] \rangle$
$x_4$	$\langle [ . 6, . 9], [ . 6, . 9], [ 1, 1] \rangle$	$\langle [ . 5, . 9], [ . 6, . 8], [ . 1, . 8] \rangle$

### 3. Operations on Interval Valued Neutrosophic Hypersoft Set with Properties

In this section, we extend the concept of IVNHSS and introduce some fundamental operations on IVNHSS with their properties.

#### Definition 3.1

Let  $F_A$  and  $G_B \in$  IVNHSS over  $\mathbb{U}$ , then  $F_A \subseteq G_B$  if

$$\inf u_A(u) \leq \inf u_B(u), \sup u_A(u) \leq \sup u_B(u)$$

$$\inf v_A(u) \geq \inf v_B(u), \sup v_A(u) \geq \sup v_B(u)$$

$$\inf w_A(u) \geq \inf w_B(u), \sup w_A(u) \geq \sup w_B(u)$$

**Example 2** Assume  $\mathbb{U} = \{u_1, u_2\}$  be a universe of discourse and  $E = \{x_1, x_2, x_3, x_4\}$  be a set of attributes. Consider  $G_B$  be an IVNHSS over  $\mathbb{U}$  can be expressed as follows and  $F_A$  given in example 1

$$G_B = \{(x_1, \{\langle u_1, [ . 6, . 9], [ . 3, . 7], [ . 1, . 3] \rangle, \langle u_2, [ . 6, . 9], [ . 3, . 5], [ . 1, . 4] \rangle\}),$$

$$(x_2, \{\langle u_1, [ . 6, . 8], [ . 2, . 5], [ . 2, . 3] \rangle, \langle u_2, [ . 3, . 5], [ . 4, . 7], [ . 1, . 4] \rangle\}),$$

$$(x_3, \{\langle u_1, [ . 4, . 9], [ . 1, . 3], [ . 4, . 6] \rangle, \langle u_2, [ . 6, 1], [ . 1, . 4], [ . 3, . 4] \rangle\}),$$

$$(x_4, \{\langle u_1, [ . 7, . 9], [ . 4, . 6], [ . 6, 1] \rangle, \langle u_2, [ . 5, . 7], [ . 4, . 7], [ . 1, . 4] \rangle\}).$$

Thus

$$F_A \subseteq G_B.$$

#### Definition 3.2

Let  $F_A \in$  IVNHSS over  $\mathbb{U}$ , then

- i. Empty IVNHSS can be represented as  $F_{\emptyset}$ , and defined as follows  $F_{\emptyset} = \{ \langle u, [0, 0], [1, 1], [1, 1] \rangle : u \in \mathbb{U} \}$ .
- ii. Universal IVNHSS can be represented as  $F_E$ , and defined as follows  $F_E = \{ \langle u, [0, 0], [1, 1], [1, 1] \rangle : u \in \mathbb{U} \}$ .

- iii. The complement of IVNHSS can be defined as follows  $F_A^c = \{ \langle u, [w_A^L(u), w_A^U(u)], [1 - v_A^U(u), 1 - v_A^L(u)], [u_A^L(u), u_A^U(u)] \rangle : u \in \mathbb{U} \}$ .

**Example 3** Assume  $\mathbb{U} = \{u_1, u_2\}$  be a universe of discourse and  $E = \{x_1, x_2, x_3, x_4\}$  be a set of attributes. The tabular representation of  $F_{\bar{0}}$  and  $F_{\bar{E}}$  given as follows in table 2 and table 3 respectively.

*Table 2: Tabular representation of IVNHSS  $F_{\bar{0}}$*

$\mathbb{U}$	$u_1$	$u_1$
$x_1$	$\langle [0, 0], [1, 1], [1, 1] \rangle$	$\langle [0, 0], [1, 1], [1, 1] \rangle$
$x_2$	$\langle [0, 0], [1, 1], [1, 1] \rangle$	$\langle [0, 0], [1, 1], [1, 1] \rangle$
$x_3$	$\langle [0, 0], [1, 1], [1, 1] \rangle$	$\langle [0, 0], [1, 1], [1, 1] \rangle$
$x_4$	$\langle [0, 0], [1, 1], [1, 1] \rangle$	$\langle [0, 0], [1, 1], [1, 1] \rangle$

*Table 3: Tabular representation of IVNHSS  $F_{\bar{E}}$*

$\mathbb{U}$	$u_1$	$u_1$
$x_1$	$\langle [1, 1], [0, 0], [0, 0] \rangle$	$\langle [1, 1], [0, 0], [0, 0] \rangle$
$x_2$	$\langle [1, 1], [0, 0], [0, 0] \rangle$	$\langle [1, 1], [0, 0], [0, 0] \rangle$
$x_3$	$\langle [1, 1], [0, 0], [0, 0] \rangle$	$\langle [1, 1], [0, 0], [0, 0] \rangle$
$x_4$	$\langle [1, 1], [0, 0], [0, 0] \rangle$	$\langle [1, 1], [0, 0], [0, 0] \rangle$

**Proposition 3.3**

If  $F_A \in$  IVNHSS, then

1.  $(F_A^c)^c = F_A$
2.  $(F_{\bar{0}})^c = F_{\bar{E}}$
3.  $(F_{\bar{E}})^c = F_{\bar{0}}$

**Proof 1** Let  $F_A = \{ \langle u, [u_A^L(u), u_A^U(u)], [v_A^L(u), v_A^U(u)], [w_A^L(u), w_A^U(u)] \rangle : u \in \mathbb{U} \}$  be an IVNHSS. Then by using definition 3.3(iii), we have

$$F_A^c = \{ \langle u, [w_A^L(u), w_A^U(u)], [1 - v_A^U(u), 1 - v_A^L(u)], [u_A^L(u), u_A^U(u)] \rangle : u \in \mathbb{U} \}$$

Thus

$$(F_A^c)^c = \{ \langle u, [u_A^L(u), u_A^U(u)], [1 - (1 - v_A^L(u)), 1 - (1 - v_A^U(u))], [w_A^L(u), w_A^U(u)] \rangle : u \in \mathbb{U} \}$$

$$(F_A^c)^c = \{ \langle u, [u_A^L(u), u_A^U(u)], [v_A^L(u), v_A^U(u)], [w_A^L(u), w_A^U(u)] \rangle : u \in \mathbb{U} \}$$

$$(F_A^c)^c = F_A$$

**Proof 2**

As we know that  $F_{\bar{0}} = \{ \langle u, [0, 0], [1, 1], [1, 1] \rangle : u \in \mathbb{U} \}$

By using definition 3.3(iii), we get

$$(F_{\bar{0}})^c = \{ \langle u, [1, 1], [0, 0], [0, 0] \rangle : u \in \mathbb{U} \} = F_{\bar{E}}$$

Similarly, we can prove 3.

**Definition 3.4**

Let  $F_A$  and  $G_B \in$  IVNHSS over  $\mathbb{U}$ , then

$$F_A \cup G_B = \left\{ \begin{array}{l} (< u, [\max\{\inf u_A(u), \inf u_B(u)\}, \max\{\sup u_A(u), \sup u_B(u)\}], \\ \quad [\min\{\inf v_A(u), \inf v_B(u)\}, \min\{\sup v_A(u), \sup v_B(u)\}], \\ [\min\{\inf w_A(u), \inf w_B(u)\}, \min\{\sup w_A(u), \sup w_B(u)\}] >/ u \in \mathbb{U} \end{array} \right\}. \tag{1}$$

**Example 4** Assume  $\mathbb{U} = \{u_1, u_2\}$  be a universe of discourse and  $E = \{x_1, x_2, x_3, x_4\}$  be a set of attributes. Consider  $F_A$  and  $G_B$  are IVNHSS over  $\mathbb{U}$  can be given as follows

$$\begin{aligned} F_A &= \{(x_1, \{\langle u_1, [ .6, .8], [ .5, .9], [ .1, .4] \rangle, \langle u_2, [ .4, .7], [ .3, .9], [ .2, .6] \rangle\}), \\ &\quad (x_2, \{\langle u_1, [ .4, .7], [ .3, .9], [ .3, .5] \rangle, \langle u_2, [ .2, .8], [ .6, .8], [ .3, .7] \rangle\}), \\ &\quad (x_3, \{\langle u_1, [ .2, .9], [ .1, .5], [ .4, .7] \rangle, \langle u_2, [ .4, .9], [ .1, .6], [ .5, .7] \rangle\}), \\ &\quad (x_4, \{\langle u_1, [ .6, .9], [ .6, .9], [ .1, .1] \rangle, \langle u_2, [ .5, .9], [ .6, .8], [ .1, .8] \rangle\}) \\ G_B &= \{(x_1, \{\langle u_1, [ .5, .7], [ .5, .7], [ .4, .6] \rangle, \langle u_2, [ .3, .9], [ .3, .6], [ .4, .7] \rangle\}), \\ &\quad (x_2, \{\langle u_1, [ .3, .8], [ .4, .5], [ .4, .9] \rangle, \langle u_2, [ .4, .7], [ .5, .9], [ .4, .6] \rangle\}), \\ &\quad (x_3, \{\langle u_1, [ .3, .5], [ .2, .6], [ .3, .8] \rangle, \langle u_2, [ .3, .1], [ .2, .7], [ .3, .8] \rangle\}), \\ &\quad (x_4, \{\langle u_1, [ .4, .6], [ .7, .8], [ .4, .1] \rangle, \langle u_2, [ .4, .8], [ .3, .6], [ .2, .6] \rangle\}) \end{aligned}$$

Then

$$\begin{aligned} F_A \cup G_B &= \{(x_1, \{\langle u_1, [ .6, .8], [ .5, .7], [ .1, .4] \rangle, \langle u_2, [ .4, .9], [ .3, .6], [ .2, .6] \rangle\}), \\ &\quad (x_2, \{\langle u_1, [ .4, .8], [ .3, .5], [ .3, .5] \rangle, \langle u_2, [ .4, .8], [ .5, .8], [ .3, .6] \rangle\}), \\ &\quad (x_3, \{\langle u_1, [ .3, .9], [ .1, .5], [ .3, .7] \rangle, \langle u_2, [ .4, .1], [ .1, .6], [ .3, .7] \rangle\}), \\ &\quad (x_4, \{\langle u_1, [ .6, .9], [ .6, .8], [ .4, .1] \rangle, \langle u_2, [ .5, .9], [ .3, .6], [ .1, .6] \rangle\}) \end{aligned}$$

**Proposition 3.5**

Let  $\mathcal{F}_{\tilde{A}}, \mathcal{G}_{\tilde{B}}, \mathcal{H}_{\tilde{C}} \in$  IVNHSS over  $\mathbb{U}$ . Then

1.  $\mathcal{F}_{\tilde{A}} \cup \mathcal{F}_{\tilde{A}} = \mathcal{F}_{\tilde{A}}$
2.  $\mathcal{F}_{\tilde{A}} \cup \mathcal{F}_{\tilde{0}} = \mathcal{F}_{\tilde{0}}$
3.  $\mathcal{F}_{\tilde{A}} \cup \mathcal{F}_{\tilde{E}} = \mathcal{F}_{\tilde{A}}$
4.  $\mathcal{F}_{\tilde{A}} \cup \mathcal{G}_{\tilde{B}} = \mathcal{G}_{\tilde{B}} \cup \mathcal{F}_{\tilde{A}}$
5.  $(\mathcal{F}_{\tilde{A}} \cup \mathcal{G}_{\tilde{B}}) \cup \mathcal{H}_{\tilde{C}} = \mathcal{F}_{\tilde{A}} \cup (\mathcal{G}_{\tilde{B}} \cup \mathcal{H}_{\tilde{C}})$

**Proof** By using definition 3.4 we can prove easily.

**Definition 3.6**

Let  $F_A$  and  $G_B \in$  IVNHSS over  $\mathbb{U}$ , then

$$F_A \cap G_B = \left\{ \begin{array}{l} (< u, [\min\{\inf u_A(u), \inf u_B(u)\}, \min\{\sup u_A(u), \sup u_B(u)\}], \\ \quad [\max\{\inf v_A(u), \inf v_B(u)\}, \max\{\sup v_A(u), \sup v_B(u)\}], \\ [\max\{\inf w_A(u), \inf w_B(u)\}, \max\{\sup w_A(u), \sup w_B(u)\}] >/ u \in \mathbb{U} \end{array} \right\}. \tag{2}$$

**Example 5** Reconsider example 4

$$\begin{aligned} F_A &= \{(x_1, \{\langle u_1, [ .6, .8], [ .5, .9], [ .1, .4] \rangle, \langle u_2, [ .4, .7], [ .3, .9], [ .2, .6] \rangle\}), \\ &\quad (x_2, \{\langle u_1, [ .4, .7], [ .3, .9], [ .3, .5] \rangle, \langle u_2, [ .2, .8], [ .6, .8], [ .3, .7] \rangle\}), \end{aligned}$$



$$\begin{aligned}
 & (x_3, \{\langle u_1, [2, 9], [1, 5], [4, 7] \rangle, \langle u_2, [4, 9], [1, 6], [5, 7] \rangle\}), \\
 & (x_4, \{\langle u_1, [6, 9], [6, 9], [1, 1] \rangle, \langle u_2, [5, 9], [6, 8], [1, 8] \rangle\}) \\
 G_B = & \{(x_1, \{\langle u_1, [5, 7], [5, 7], [4, 6] \rangle, \langle u_2, [3, 9], [3, 6], [4, 7] \rangle\}), \\
 & (x_2, \{\langle u_1, [3, 8], [4, 5], [4, 9] \rangle, \langle u_2, [4, 7], [5, 9], [4, 6] \rangle\}), \\
 & (x_3, \{\langle u_1, [3, 5], [2, 6], [3, 8] \rangle, \langle u_2, [3, 1], [2, 7], [3, 8] \rangle\}), \\
 & (x_4, \{\langle u_1, [4, 6], [7, 8], [4, 1] \rangle, \langle u_2, [4, 8], [3, 6], [2, 6] \rangle\})
 \end{aligned}$$

Then

$$\begin{aligned}
 F_A \cap G_B = & \{(x_1, \{\langle u_1, [5, 7], [5, 9], [4, 6] \rangle, \langle u_2, [3, 7], [3, 9], [4, 7] \rangle\}), \\
 & (x_2, \{\langle u_1, [3, 7], [4, 9], [4, 9] \rangle, \langle u_2, [2, 7], [6, 9], [4, 7] \rangle\}), \\
 & (x_3, \{\langle u_1, [2, 5], [2, 6], [4, 8] \rangle, \langle u_2, [3, 9], [2, 7], [5, 8] \rangle\}), \\
 & (x_4, \{\langle u_1, [4, 6], [7, 9], [1, 1] \rangle, \langle u_2, [4, 8], [6, 8], [2, 8] \rangle\})
 \end{aligned}$$

**Proposition 3.7**

Let  $\mathcal{F}_{\tilde{A}}, \mathcal{G}_{\tilde{B}}, \mathcal{H}_{\tilde{C}} \in \text{IVNHSS}$  over  $\mathbb{U}$ . Then

1.  $\mathcal{F}_{\tilde{A}} \cap \mathcal{F}_{\tilde{A}} = \mathcal{F}_{\tilde{A}}$
2.  $\mathcal{F}_{\tilde{A}} \cap \mathcal{F}_{\tilde{0}} = \mathcal{F}_{\tilde{A}}$
3.  $\mathcal{F}_{\tilde{A}} \cap \mathcal{F}_{\tilde{E}} = \mathcal{F}_{\tilde{E}}$
4.  $\mathcal{F}_{\tilde{A}} \cap \mathcal{G}_{\tilde{B}} = \mathcal{G}_{\tilde{B}} \cap \mathcal{F}_{\tilde{A}}$
5.  $(\mathcal{F}_{\tilde{A}} \cap \mathcal{G}_{\tilde{B}}) \cap \mathcal{H}_{\tilde{C}} = \mathcal{F}_{\tilde{A}} \cap (\mathcal{G}_{\tilde{B}} \cap \mathcal{H}_{\tilde{C}})$

**Proof** By using definition 3.6 we can prove easily.

**Proposition 3.8**

Let  $F_A$  and  $G_B \in \text{IVNHSS}$  over  $\mathbb{U}$ , then

1.  $(F_A \cup G_B)^c = F_A^c \cap G_B^c$
2.  $(F_A \cap G_B)^c = F_A^c \cup G_B^c$

**Proof 1** As we know that

$$\begin{aligned}
 F_A = & \{ \langle u, u_A(u), v_A(u), w_A(u) \rangle : u \in \mathbb{U} \} \text{ and } G_B = \{ \langle u, u_B(u), v_B(u), w_B(u) \rangle : u \in \mathbb{U} \}. \text{ Where} \\
 u_A(u) = & [\inf u_A(u), \sup u_A(u)] \text{ or } [u_A^L(u), u_A^U(u)], u_A^L(u) = \inf u_A(u) \text{ and } u_A^U(u) = \sup u_A(u) \\
 v_A(u) = & [\inf v_A(u), \sup v_A(u)] \text{ or } [v_A^L(u), v_A^U(u)], v_A^L(u) = \inf v_A(u) \text{ and } v_A^U(u) = \sup v_A(u) \\
 w_A(u) = & [\inf w_A(u), \sup w_A(u)] \text{ or } [w_A^L(u), w_A^U(u)], w_A^L(u) = \inf w_A(u) \text{ and } w_A^U(u) = \sup w_A(u) \\
 u_B(u) = & [\inf u_B(u), \sup u_B(u)] \text{ or } [u_B^L(u), u_B^U(u)], u_B^L(u) = \inf u_B(u) \text{ and } u_B^U(u) = \sup u_B(u) \\
 v_B(u) = & [\inf v_B(u), \sup v_B(u)] \text{ or } [v_B^L(u), v_B^U(u)], v_B^L(u) = \inf v_B(u) \text{ and } v_B^U(u) = \sup v_B(u) \\
 w_B(u) = & [\inf w_B(u), \sup w_B(u)] \text{ or } [w_B^L(u), w_B^U(u)], w_B^L(u) = \inf w_B(u) \text{ and } w_B^U(u) = \sup w_B(u)
 \end{aligned}$$

Then by using Equation 1

$$F_A \cup G_B = \left\{ \begin{aligned} & \langle u, [\max\{\inf u_A(u), \inf u_B(u)\}, \max\{\sup u_A(u), \sup u_B(u)\}], \\ & [\min\{\inf v_A(u), \inf v_B(u)\}, \min\{\sup v_A(u), \sup v_B(u)\}], \\ & [\min\{\inf w_A(u), \inf w_B(u)\}, \min\{\sup w_A(u), \sup w_B(u)\}] \rangle / u \in \mathbb{U} \end{aligned} \right\}$$

By using definition 3.3(iii), we get

$$(F_A \cup G_B)^C = \left\{ \begin{array}{l} (< u, [\min\{inf w_A(u), inf w_B(u)\}, \min\{sup w_A(u), sup w_B(u)\}], \\ [1 - \min\{sup v_A(u), sup v_B(u)\}, 1 - \min\{inf v_A(u), inf v_B(u)\}], \\ [\max\{inf u_A(u), inf u_B(u)\}, \max\{sup u_A(u), sup u_B(u)\}] >/ u \in \mathbb{U} \end{array} \right\}$$

Now

$$F_A^C = \{< u, [inf w_A(u), sup w_A(u)], [1 - sup v_A(u), 1 - inf v_A(u)], [inf u_A(u), sup u_A(u)] > : u \in \mathbb{U}\}$$

$$G_B^C = \{< u, [inf w_B(u), sup w_B(u)], [1 - sup v_B(u), 1 - inf v_B(u)], [inf u_B(u), sup u_B(u)] > : u \in \mathbb{U}\}$$

$$F_A^C \cap G_B^C = \left\{ \begin{array}{l} (< u, [\min\{inf w_A(u), inf w_B(u)\}, \min\{sup w_A(u), sup w_B(u)\}], \\ [1 - \min\{sup v_A(u), sup v_B(u)\}, 1 - \min\{inf v_A(u), inf v_B(u)\}], \\ [\max\{inf u_A(u), inf u_B(u)\}, \max\{sup u_A(u), sup u_B(u)\}] >/ u \in \mathbb{U} \end{array} \right\}$$

$$F_A^C \cap G_B^C = \left\{ \begin{array}{l} (< u, [\min\{inf w_A(u), inf w_B(u)\}, \min\{sup w_A(u), sup w_B(u)\}], \\ [1 - \min\{sup v_A(u), sup v_B(u)\}, 1 - \min\{inf v_A(u), inf v_B(u)\}], \\ [\max\{inf u_A(u), inf u_B(u)\}, \max\{sup u_A(u), sup u_B(u)\}] >/ u \in \mathbb{U} \end{array} \right\}$$

Hence

$$(F_A \cup G_B)^C = F_A^C \cap G_B^C$$

**Proof 2**

Similar to assertion 1.

**Proposition 3.9**

Let  $\mathcal{F}_{\tilde{A}}, \mathcal{G}_{\tilde{B}}, \mathcal{H}_{\tilde{C}} \in \text{IVNHSS}$  over  $\mathbb{U}$ . Then

1.  $\mathcal{F}_{\tilde{A}} \cup (\mathcal{G}_{\tilde{B}} \cap \mathcal{H}_{\tilde{C}}) = (\mathcal{F}_{\tilde{A}} \cup \mathcal{G}_{\tilde{B}}) \cap (\mathcal{F}_{\tilde{A}} \cup \mathcal{H}_{\tilde{C}})$
2.  $\mathcal{F}_{\tilde{A}} \cap (\mathcal{G}_{\tilde{B}} \cup \mathcal{H}_{\tilde{C}}) = (\mathcal{F}_{\tilde{A}} \cap \mathcal{G}_{\tilde{B}}) \cup (\mathcal{F}_{\tilde{A}} \cap \mathcal{H}_{\tilde{C}})$
3.  $\mathcal{F}_{\tilde{A}} \cup (\mathcal{F}_{\tilde{A}} \cap \mathcal{G}_{\tilde{B}}) = \mathcal{F}_{\tilde{A}}$
4.  $\mathcal{F}_{\tilde{A}} \cap (\mathcal{F}_{\tilde{A}} \cup \mathcal{G}_{\tilde{B}}) = \mathcal{F}_{\tilde{A}}$

**Proof 1** From Equation 2, we have

$$\mathcal{G}_{\tilde{B}} \cap \mathcal{H}_{\tilde{C}} = \left\{ \begin{array}{l} (< u, [\min\{inf u_B(u), inf u_C(u)\}, \min\{sup u_B(u), sup u_C(u)\}], \\ [\max\{inf v_B(u), inf v_C(u)\}, \max\{sup v_B(u), sup v_C(u)\}], \\ [\max\{inf w_B(u), inf w_C(u)\}, \max\{sup w_B(u), sup w_C(u)\}] >/ u \in \mathbb{U} \end{array} \right\}$$

$$\mathcal{F}_{\tilde{A}} \cup (\mathcal{G}_{\tilde{B}} \cap \mathcal{H}_{\tilde{C}}) =$$

$$\left\{ \begin{array}{l} (< u, [\max\{inf u_A(u), \min\{inf u_B(u), inf u_C(u)\}\}, \max\{sup u_A(u), \min\{sup u_B(u), sup u_C(u)\}\}], \\ [\min\{inf v_A(u), \max\{inf v_B(u), inf v_C(u)\}\}, \min\{sup v_A(u), \max\{sup v_B(u), sup v_C(u)\}\}], \\ [\min\{inf w_A(u), \max\{inf w_B(u), inf w_C(u)\}\}, \min\{sup w_A(u), \max\{sup w_B(u), sup w_C(u)\}\}] >/ u \in \mathbb{U} \end{array} \right\}$$

$$\mathcal{F}_{\tilde{A}} \cup \mathcal{G}_{\tilde{B}} = \left\{ \begin{array}{l} (< u, [\max\{inf u_A(u), inf u_B(u)\}, \max\{sup u_A(u), sup u_B(u)\}], \\ [\min\{inf v_A(u), inf v_B(u)\}, \min\{sup v_A(u), sup v_B(u)\}], \\ [\min\{inf w_A(u), inf w_B(u)\}, \min\{sup w_A(u), sup w_B(u)\}] >/ u \in \mathbb{U} \end{array} \right\}$$

$$\mathcal{F}_{\tilde{A}} \cup \mathcal{H}_{\tilde{C}} = \left\{ \begin{array}{l} (< u, [\max\{inf u_A(u), inf u_C(u)\}, \max\{sup u_A(u), sup u_C(u)\}], \\ [\min\{inf v_A(u), inf v_C(u)\}, \min\{sup v_A(u), sup v_C(u)\}], \\ [\min\{inf w_A(u), inf w_C(u)\}, \min\{sup w_A(u), sup w_C(u)\}] >/ u \in \mathbb{U} \end{array} \right\}$$

$$(\mathcal{F}_{\tilde{A}} \cup \mathcal{G}_{\tilde{B}}) \cap (\mathcal{F}_{\tilde{A}} \cup \mathcal{H}_{\tilde{C}}) =$$

$$\left( \begin{array}{l} < u, [\min\{\max\{\inf u_A(u), \inf u_B(u)\}, \max\{\inf u_A(u), \inf u_C(u)\}, \min\{\max\{\sup u_A(u), \sup u_B(u)\}, \max\{\sup u_A(u), \sup u_C(u)\}\}, \\ \quad [\max\{\min\{\inf v_A(u), \inf v_B(u)\}, \min\{\inf v_A(u), \inf v_C(u)\}, \max\{\min\{\sup v_A(u), \sup v_B(u)\}, \min\{\sup v_A(u), \sup v_C(u)\}\}, \\ \quad [\max\{\min\{\inf w_A(u), \inf w_B(u)\}, \min\{\inf w_A(u), \inf w_C(u)\}, \max\{\min\{\sup w_A(u), \sup w_B(u)\}, \sup w_A(u), \sup w_C(u)\}] >/ u \in \mathbb{U} \end{array} \right)$$

$$(\mathcal{F}_{\tilde{A}} \cup \mathcal{G}_{\tilde{B}}) \cap (\mathcal{F}_{\tilde{A}} \cup \mathcal{H}_{\tilde{C}}) =$$

$$\left( \begin{array}{l} < u, [\max\{\inf u_A(u), \min\{\inf u_B(u), \inf u_C(u)\}\}, \max\{\sup u_A(u), \min\{\sup u_B(u), \sup u_C(u)\}\}, \\ \quad [\min\{\inf v_A(u), \max\{\inf v_B(u), \inf v_C(u)\}\}, \min\{\sup v_A(u), \max\{\sup v_B(u), \sup v_C(u)\}\}, \\ \quad [\min\{\inf w_A(u), \max\{\inf w_B(u), \inf w_C(u)\}\}, \min\{\sup w_A(u), \max\{\sup w_B(u), \sup w_C(u)\}] >/ u \in \mathbb{U} \end{array} \right)$$

Hence

$$\mathcal{F}_{\tilde{A}} \cup (\mathcal{G}_{\tilde{B}} \cap \mathcal{H}_{\tilde{C}}) = (\mathcal{F}_{\tilde{A}} \cup \mathcal{G}_{\tilde{B}}) \cap (\mathcal{F}_{\tilde{A}} \cup \mathcal{H}_{\tilde{C}}).$$

Similarly, we can prove other results.

**Definition 3.10**

Let  $F_A, G_B \in \text{IVNHSS}$ , then their extended union is

$$u (F_A \cup G_B) = \begin{cases} [\inf u_A(u), \sup u_A(u)] & \text{if } u \in A - B \\ [\inf u_B(u), \sup u_B(u)] & \text{if } u \in B - A \\ [\max\{\inf u_A(u), \inf u_B(u)\}, \max\{\sup u_A(u), \sup u_B(u)\}] & \text{if } u \in A \cap B \end{cases}$$

$$v (F_A \cup G_B) = \begin{cases} [\inf v_A(u), \sup v_A(u)] & \text{if } u \in A - B \\ [\inf v_B(u), \sup v_B(u)] & \text{if } u \in B - A \\ [\min\{\inf v_A(u), \inf v_B(u)\}, \min\{\sup v_A(u), \sup v_B(u)\}] & \text{if } u \in A \cap B \end{cases}$$

$$w (F_A \cup G_B) = \begin{cases} [\inf w_A(u), \sup w_A(u)] & \text{if } u \in A - B \\ [\inf w_B(u), \sup w_B(u)] & \text{if } u \in B - A \\ [\min\{\inf w_A(u), \inf w_B(u)\}, \min\{\sup w_A(u), \sup w_B(u)\}] & \text{if } u \in A \cap B \end{cases}$$

**Definition 3.11**

Let  $F_A, G_B \in \text{IVNHSS}$ , then their extended intersection is

$$u (F_A \cap G_B) = \begin{cases} [\inf u_A(u), \sup u_A(u)] & \text{if } u \in A - B \\ [\inf u_B(u), \sup u_B(u)] & \text{if } u \in B - A \\ [\min\{\inf u_A(u), \inf u_B(u)\}, \min\{\sup u_A(u), \sup u_B(u)\}] & \text{if } u \in A \cap B \end{cases}$$

$$v (F_A \cap G_B) = \begin{cases} [\inf v_A(u), \sup v_A(u)] & \text{if } u \in A - B \\ [\inf v_B(u), \sup v_B(u)] & \text{if } u \in B - A \\ [\max\{\inf v_A(u), \inf v_B(u)\}, \max\{\sup v_A(u), \sup v_B(u)\}] & \text{if } u \in A \cap B \end{cases}$$

$$w (F_A \cap G_B) = \begin{cases} [\inf w_A(u), \sup w_A(u)] & \text{if } u \in A - B \\ [\inf w_B(u), \sup w_B(u)] & \text{if } u \in B - A \\ [\max\{\inf w_A(u), \inf w_B(u)\}, \max\{\sup w_A(u), \sup w_B(u)\}] & \text{if } u \in A \cap B \end{cases}$$

**Definition 3.12**

Let  $F_A$  and  $G_B \in \text{IVNHSS}$  over  $\mathbb{U}$ , then their difference defined as follows

$$F_A \setminus G_B = \left\{ \begin{array}{l} < u, [\min\{\inf u_A(u), \inf u_B(u)\}, \min\{\sup u_A(u), \sup u_B(u)\}], \\ \quad [\max\{\inf v_A(u), 1 - \sup v_B(u)\}, \max\{\sup v_A(u), 1 - \inf v_B(u)\}], \\ \quad [\max\{\inf w_A(u), \inf w_B(u)\}, \max\{\sup w_A(u), \sup w_B(u)\}] >/ u \in \mathbb{U} \end{array} \right\}. \tag{3}$$

**Example 6** Reconsider example 4

$$F_A \setminus G_B = \{ \langle x_1, \{ \langle u_1, [ .5, .7 ], [ .5, .9 ], [ .4, .6 ] \rangle, \langle u_2, [ .3, .7 ], [ .4, .9 ], [ .4, .7 ] \rangle \} \rangle, \\ \langle x_2, \{ \langle u_1, [ .3, .7 ], [ .5, .9 ], [ .4, .9 ] \rangle, \langle u_2, [ .2, .7 ], [ .6, .8 ], [ .4, .7 ] \rangle \} \rangle \}$$

$$\begin{aligned}
 & (x_3, \{\langle u_1, [2, 5], [4, 8], [4, 8] \rangle, \langle u_2, [3, 9], [3, 8], [5, 8] \rangle\}), \\
 & (x_4, \{\langle u_1, [4, 6], [6, 9], [1, 1] \rangle, \langle u_2, [4, 8], [6, 8], [2, 8] \rangle\})
 \end{aligned}$$

**Definition 3.13**

Let  $F_A$  and  $G_B \in$  IVNHSS over  $\mathbb{U}$ , then their addition defined as follows

$$F_A + G_B = \left\{ \begin{aligned} & (< u, [\min\{\inf u_A(u) + \inf u_B(u), 1\}, \min\{\sup u_A(u) + \sup u_B(u), 1\}], \\ & \quad [\min\{\inf v_A(u) + \inf v_B(u), 1\}, \min\{\sup v_A(u) + \sup v_B(u), 1\}], \\ & [\min\{\inf w_A(u) + \inf w_B(u), 1\}, \min\{\sup w_A(u) + \sup w_B(u), 1\}] > / u \in \mathbb{U}) \end{aligned} \right\}. \tag{4}$$

**Example 7** Reconsider example 4

$$\begin{aligned}
 F_A + G_B = & \{(x_1, \{\langle u_1, [1.0, 1.0], [1.0, 1.0], [0.5, 1.0] \rangle, \langle u_2, [0.7, 1.0], [0.6, 1.0], [0.6, 1.0] \rangle\}), \\
 & (x_2, \{\langle u_1, [0.7, 1.0], [0.7, 1.0], [0.7, 1.0] \rangle, \langle u_2, [0.6, 1.0], [1.0, 1.0], [0.7, 1.0] \rangle\}), \\
 & (x_3, \{\langle u_1, [0.5, 1.0], [0.3, 1.0], [0.7, 1.0] \rangle, \langle u_2, [0.7, 1.0], [0.3, 1.0], [0.8, 1.0] \rangle\}), \\
 & (x_4, \{\langle u_1, [1.0, 1.0], [1.0, 1.0], [1.0, 1.0] \rangle, \langle u_2, [0.9, 1.0], [0.9, 1.0], [0.3, 1.0] \rangle\}).
 \end{aligned}$$

**Definition 3.14**

Let  $F_A \in$  IVNHSS over  $\mathbb{U}$ , then its scalar multiplication is represented as  $F_A \cdot \check{\alpha}$ , where  $\check{\alpha} \in [0, 1]$  and defined as follows

$$F_A \cdot \check{\alpha} = \left\{ \begin{aligned} & (< u, [\min\{\inf u_A(u) \cdot \check{\alpha}, 1\}, \min\{\sup u_A(u) \cdot \check{\alpha}, 1\}], \\ & \quad [\min\{\inf v_A(u) \cdot \check{\alpha}, 1\}, \min\{\sup v_A(u) \cdot \check{\alpha}, 1\}], \\ & [\min\{\inf w_A(u) \cdot \check{\alpha}, 1\}, \min\{\sup w_A(u) \cdot \check{\alpha}, 1\}] > / u \in \mathbb{U}) \end{aligned} \right\}. \tag{5}$$

**Definition 3.15**

Let  $F_A \in$  IVNHSS over  $\mathbb{U}$ , then its scalar division is represented as  $F_A / \check{\alpha}$ , where  $\check{\alpha} \in [0, 1]$  and defined as follows

$$F_A / \check{\alpha} = \left\{ \begin{aligned} & (< u, [\min\{\inf u_A(u) / \check{\alpha}, 1\}, \min\{\sup u_A(u) / \check{\alpha}, 1\}], \\ & \quad [\min\{\inf v_A(u) / \check{\alpha}, 1\}, \min\{\sup v_A(u) / \check{\alpha}, 1\}], \\ & [\min\{\inf w_A(u) / \check{\alpha}, 1\}, \min\{\sup w_A(u) / \check{\alpha}, 1\}] > / u \in \mathbb{U}) \end{aligned} \right\}. \tag{6}$$

**Definition 3.16**

Let  $F_A \in$  IVNHSS over  $\mathbb{U}$ , then Truth-Favorite operator on  $F_A$  is denoted by  $\tilde{\Delta}F_A$  and defined as follows

$$\tilde{\Delta}F_A = \left\{ \begin{aligned} & (< u, [\min\{\inf u_A(u) + \inf v_A(u), 1\}, \min\{\sup u_A(u) + \sup v_A(u), 1\}], [0, 0], \\ & \quad [\inf w_A(u), \sup w_A(u)] > / u \in \mathbb{U}) \end{aligned} \right\}. \tag{7}$$

**Example 8** Reconsider example 1

$$\begin{aligned}
 \tilde{\Delta}F_A = & \{(x_1, \{\langle u_1, [1, 1], [0, 0], [1, 4] \rangle, \langle u_2, [7, 1], [0, 0], [2, 6] \rangle\}), \\
 & (x_2, \{\langle u_1, [7, 1], [0, 0], [3, 5] \rangle, \langle u_2, [6, 1], [0, 0], [3, 7] \rangle\}), \\
 & (x_3, \{\langle u_1, [3, 1], [0, 0], [7, 8] \rangle, \langle u_2, [5, 1], [0, 0], [5, 7] \rangle\}), \\
 & (x_4, \{\langle u_1, [1, 1], [0, 0], [1, 1] \rangle, \langle u_2, [1, 1], [0, 0], [1, 8] \rangle\})
 \end{aligned}$$

**Proposition 3.17**

Let  $\mathcal{F}_{\check{A}}, \mathcal{G}_{\check{B}} \in$  IVNHSS over  $\mathbb{U}$ , then

1.  $\tilde{\Delta}\tilde{\Delta}\mathcal{F}_{\check{A}} = \tilde{\Delta}\mathcal{F}_{\check{A}}$
2.  $\tilde{\Delta}(\mathcal{F}_{\check{A}} \cup \mathcal{G}_{\check{B}}) \subseteq \tilde{\Delta}\mathcal{F}_{\check{A}} \cup \tilde{\Delta}\mathcal{G}_{\check{B}}$

$$3. \tilde{\Delta}(\mathcal{F}_{\tilde{A}} \cap \mathcal{G}_{\tilde{B}}) \subseteq \tilde{\Delta}\mathcal{F}_{\tilde{A}} \cap \tilde{\Delta}\mathcal{G}_{\tilde{B}}$$

$$4. \tilde{\Delta}(\mathcal{F}_{\tilde{A}} + \mathcal{G}_{\tilde{B}}) = \tilde{\Delta}\mathcal{F}_{\tilde{A}} + \tilde{\Delta}\mathcal{G}_{\tilde{B}}$$

Proof of the above proposition is easily obtained by using definitions 3.4, 3.6, 3.13, and 3.16.

**Definition 3.18**

Let  $F_A \in$  IVNHSS over  $\mathbb{U}$ , then False-Favorite operator on  $F_A$  is denoted by  $\tilde{v}F_A$  and defined as follows

$$\tilde{v}F_A = \left\{ \begin{array}{l} \langle u, [\inf u_A(u), \sup u_A(u)], [0, 0], \\ [\min\{\inf w_A(u) + \inf v_A(u), 1\}, \min\{\sup w_A(u) + \sup v_A(u), 1\}] \rangle / u \in \mathbb{U} \end{array} \right\} \tag{8}$$

**Example 9** Reconsider example 1

$$\begin{aligned} \tilde{v}F_A = & \{ (x_1, \{ \langle u_1, [ .6, .8], [0, 0], [ .6, 1] \rangle, \langle u_2, [ .4, .7], [0, 0], [ .5, 1] \rangle \}), \\ & (x_2, \{ \langle u_1, [ .4, .7], [0, 0], [ .6, 1] \rangle, \langle u_2, [0, .3], [0, 0], [ .9, 1] \rangle \}), \\ & (x_3, \{ \langle u_1, [ .2, .9], [0, 0], [ .8, 1] \rangle, \langle u_2, [ .4, .9], [0, 0], [ .6, 1] \rangle \}), \\ & (x_4, \{ \langle u_1, [ .6, .9], [0, 0], [1, 1] \rangle, \langle u_2, [ .5, .9], [0, 0], [ .7, 1] \rangle \}) \end{aligned}$$

**Proposition 3.19**

Let  $\mathcal{F}_{\tilde{A}}$  and  $\mathcal{G}_{\tilde{B}} \in$  IVNHSS over  $\mathbb{U}$ , then

1.  $\tilde{v}\tilde{v}\mathcal{F}_{\tilde{A}} = \tilde{v}\mathcal{F}_{\tilde{A}}$
2.  $\tilde{v}(\mathcal{F}_{\tilde{A}} \cup \mathcal{G}_{\tilde{B}}) \subseteq \tilde{v}\mathcal{F}_{\tilde{A}} \cup \tilde{v}\mathcal{G}_{\tilde{B}}$
3.  $\tilde{v}(\mathcal{F}_{\tilde{A}} \cap \mathcal{G}_{\tilde{B}}) \subseteq \tilde{v}\mathcal{F}_{\tilde{A}} \cap \tilde{v}\mathcal{G}_{\tilde{B}}$
4.  $\tilde{v}(\mathcal{F}_{\tilde{A}} + \mathcal{G}_{\tilde{B}}) = \tilde{v}\mathcal{F}_{\tilde{A}} + \tilde{v}\mathcal{G}_{\tilde{B}}$

Proof of the above proposition is easily obtained by using definitions 3.4, 3.6, 3.13, and 3.18.

**4. Necessity and Possibility Operations on IVNHSS**

In this section, some further operations on IVNHSS are developed such as OR-Operation, And-Operation, necessity, and possibility operations with some properties.

**Definition 4.1**

Let  $F_A$  and  $G_B \in$  IVNHSS over  $\mathbb{U}$ , then OR-Operator is represented by  $F_A \vee G_B$  and defined as follows

$$\begin{aligned} u (F_{A \times B}) &= [\max\{\inf u_A(u), \inf u_B(u)\}, \max\{\sup u_A(u), \sup u_B(u)\}], \\ v (F_{A \times B}) &= [\min\{\inf v_A(u), \inf v_B(u)\}, \min\{\sup v_A(u), \sup v_B(u)\}], \\ w (F_{A \times B}) &= [\min\{\inf w_A(u), \inf w_B(u)\}, \min\{\sup w_A(u), \sup w_B(u)\}]. \end{aligned}$$

**Definition 4.2**

Let  $F_A$  and  $G_B \in$  IVNHSS over  $\mathbb{U}$ , then And-Operator is represented by  $F_A \wedge G_B$  and defined as follows

$$\begin{aligned} u (F_{A \times B}) &= [\min\{\inf u_A(u), \inf u_B(u)\}, \min\{\sup u_A(u), \sup u_B(u)\}], \\ v (F_{A \times B}) &= [\max\{\inf v_A(u), \inf v_B(u)\}, \max\{\sup v_A(u), \sup v_B(u)\}], \\ w (F_{A \times B}) &= [\max\{\inf w_A(u), \inf w_B(u)\}, \max\{\sup w_A(u), \sup w_B(u)\}]. \end{aligned}$$

**Proposition 4.3**

Let  $\mathcal{F}_{\tilde{A}}, \mathcal{G}_{\tilde{B}}, \mathcal{H}_{\tilde{C}} \in$  IVNHSSs, then

1.  $\mathcal{F}_{\check{A}} \vee \mathcal{G}_{\check{B}} = \mathcal{G}_{\check{B}} \vee \mathcal{F}_{\check{A}}$
2.  $\mathcal{F}_{\check{A}} \wedge \mathcal{G}_{\check{B}} = \mathcal{G}_{\check{B}} \wedge \mathcal{F}_{\check{A}}$
3.  $\mathcal{F}_{\check{A}} \vee (\mathcal{G}_{\check{B}} \vee \mathcal{H}_{\check{C}}) = (\mathcal{F}_{\check{A}} \vee \mathcal{G}_{\check{B}}) \vee \mathcal{H}_{\check{C}}$
4.  $\mathcal{F}_{\check{A}} \wedge (\mathcal{G}_{\check{B}} \wedge \mathcal{H}_{\check{C}}) = (\mathcal{F}_{\check{A}} \wedge \mathcal{G}_{\check{B}}) \wedge \mathcal{H}_{\check{C}}$
5.  $(\mathcal{F}_{\check{A}} \vee \mathcal{G}_{\check{B}})^c = \mathcal{F}^c(\check{A}) \wedge \mathcal{G}^c(\check{B})$
6.  $(\mathcal{F}_{\check{A}} \wedge \mathcal{G}_{\check{B}})^c = \mathcal{F}^c(\check{A}) \vee \mathcal{G}^c(\check{B})$

**Proof** We can prove easily by using definitions 4.1 and 4.2.

**Definition 4.4**

Let  $F_A \in$  IVNHSS over  $\mathbb{U}$ , then necessity operator IVNHSS represented as  $\oplus F_A$  and defined as follows

$$\oplus F_A = \{ \langle u, [inf u_A(u), sup u_A(u)], [inf v_A(u), sup v_A(u)], [1 - sup u_A(u), 1 - inf u_A(u)] \rangle : u \in \mathbb{U} \}$$

**Example 10** Reconsider example 1

$$\begin{aligned} \oplus F_A = & \{ (x_1, \{ \langle u_1, [ .6, .8], [ .5, 0.9], [ .2, .4] \rangle, \langle u_2, [ .4, .7], [ .3, .9], [ .3, .6] \rangle \}), \\ & (x_2, \{ \langle u_1, [ .4, .7], [ .3, .9], [ .3, .6] \rangle, \langle u_2, [ 0, .3], [ .6, .8], [ .7, 1] \rangle \}), \\ & (x_3, \{ \langle u_1, [ .2, .9], [ .1, .5], [ .1, .8] \rangle, \langle u_2, [ .4, .9], [ .1, .6], [ .1, .6] \rangle \}), \\ & (x_4, \{ \langle u_1, [ .6, .9], [ .6, .9], [ .1, .4] \rangle, \langle u_2, [ .5, .9], [ .6, .8], [ .1, .5] \rangle \}) \end{aligned}$$

**Definition 4.5**

Let  $F_A \in$  IVNHSS over  $\mathbb{U}$ , then possibility operator on IVNHSS represented as  $\otimes F_A$  and defined as follows

$$\otimes F_A = \{ \langle u, [1 - sup w_A(u), 1 - inf w_A(u)], [inf v_A(u), sup v_A(u)], [inf w_A(u), sup w_A(u)] \rangle / u \in \mathbb{U} \}$$

**Example 11** Reconsider example 1

$$\begin{aligned} \otimes F_A = & \{ (x_1, \{ \langle u_1, [ .6, .9], [ .5, 0.9], [ .1, .4] \rangle, \langle u_2, [ .4, .8], [ .3, .9], [ .2, .6] \rangle \}), \\ & (x_2, \{ \langle u_1, [ .5, .7], [ .3, .9], [ .3, .5] \rangle, \langle u_2, [ .3, .7], [ .6, .8], [ .3, .7] \rangle \}), \\ & (x_3, \{ \langle u_1, [ .2, .3], [ .1, .5], [ .7, .8] \rangle, \langle u_2, [ .3, .5], [ .1, .6], [ .5, .7] \rangle \}), \\ & (x_4, \{ \langle u_1, [ 0, 0], [ .6, .9], [ 1, 1] \rangle, \langle u_2, [ .2, .9], [ .6, .8], [ .1, .8] \rangle \}) \end{aligned}$$

**Proposition 4.6**

Let  $\mathcal{F}_{\check{A}}$  and  $\mathcal{G}_{\check{B}} \in$  IVNHSS over  $\mathbb{U}$ , then

1.  $\oplus (\mathcal{F}_{\check{A}} \cup \mathcal{G}_{\check{B}}) = \oplus \mathcal{F}_{\check{A}} \cup \oplus \mathcal{G}_{\check{B}}$
2.  $\oplus (\mathcal{F}_{\check{A}} \cap \mathcal{G}_{\check{B}}) = \oplus \mathcal{F}_{\check{A}} \cap \oplus \mathcal{G}_{\check{B}}$

**Proof 1.** As we know that

$$\mathcal{F}_{\check{A}} = \{ \langle u, [inf u_A(u), sup u_A(u)], [inf v_A(u), sup v_A(u)], [inf w_A(u), sup w_A(u)] \rangle / u \in \mathbb{U} \}$$

$$\mathcal{G}_{\check{B}} = \{ \langle u, [inf u_B(u), sup u_B(u)], [inf v_B(u), sup v_B(u)], [inf w_B(u), sup w_B(u)] \rangle / u \in \mathbb{U} \}$$

Then by using definition 3.5, we get

$$\mathcal{F}_{\check{A}} \cup \mathcal{G}_{\check{B}} = \left\{ \left( \langle u, [\max\{inf u_A(u), inf u_B(u)\}, \max\{sup u_A(u), sup u_B(u)\}], \right. \right.$$

$$\left. \left. \begin{aligned} & [\min\{inf v_A(u), inf v_B(u)\}, \min\{sup v_A(u), sup v_B(u)\}], \\ & [\min\{inf w_A(u), inf w_B(u)\}, \min\{sup w_A(u), sup w_B(u)\}] \end{aligned} \right\rangle / u \in \mathbb{U} \right\}$$

By using the necessity operator, we get

$$\oplus (\mathcal{F}_{\tilde{A}} \cup \mathcal{G}_{\tilde{B}}) = \left\{ \begin{array}{l} (< u, [\max\{\inf u_A(u), \inf u_B(u)\}, \max\{\sup u_A(u), \sup u_B(u)\}], \\ \quad [\min\{\inf \nu_A(u), \inf \nu_B(u)\}, \min\{\sup \nu_A(u), \sup \nu_B(u)\}], \\ [1 - \max\{\sup u_A(u), \sup u_B(u)\}, 1 - \max\{\inf u_A(u), \inf u_B(u)\}] >/ u \in \mathbb{U}) \end{array} \right\}.$$

$$\oplus (\mathcal{F}_{\tilde{A}} \cup \mathcal{G}_{\tilde{B}}) =$$

$$\left\{ \begin{array}{l} (< u, [\max\{\inf u_A(u), \inf u_B(u)\}, \max\{\sup u_A(u), \sup u_B(u)\}], \\ \quad [\min\{\inf \nu_A(u), \inf \nu_B(u)\}, \min\{\sup \nu_A(u), \sup \nu_B(u)\}], \\ [\min\{1 - \sup u_A(u), 1 - \sup u_B(u)\}, \min\{1 - \inf u_A(u), 1 - \inf u_B(u)\}] >/ u \in \mathbb{U}) \end{array} \right\}$$

$$\oplus \mathcal{F}_{\tilde{A}} = \{(< u, [\inf u_A(u), \sup u_A(u)], [\inf \nu_A(u), \sup \nu_A(u)], [1 - \sup u_A(u), 1 - \inf u_A(u)] >/ u \in \mathbb{U})\}$$

$$\text{and } \oplus \mathcal{G}_{\tilde{B}} = \{(< u, [\inf u_B(u), \sup u_B(u)], [\inf \nu_B(u), \sup \nu_B(u)], [1 - \sup u_B(u), 1 - \inf u_B(u)] >/ u \in \mathbb{U})\}$$

Again, by using definition 3.5 we get

$$\oplus \mathcal{F}_{\tilde{A}} \cup \oplus \mathcal{G}_{\tilde{B}} =$$

$$\left\{ \begin{array}{l} (< u, [\max\{\inf u_A(u), \inf u_B(u)\}, \max\{\sup u_A(u), \sup u_B(u)\}], \\ \quad [\min\{\inf \nu_A(u), \inf \nu_B(u)\}, \min\{\sup \nu_A(u), \sup \nu_B(u)\}], \\ [\min\{1 - \sup u_A(u), 1 - \sup u_B(u)\}, \min\{1 - \inf u_A(u), 1 - \inf u_B(u)\}] >/ u \in \mathbb{U}) \end{array} \right\}$$

Hence

$$\oplus (\mathcal{F}_{\tilde{A}} \cup \mathcal{G}_{\tilde{B}}) = \oplus \mathcal{F}_{\tilde{A}} \cup \oplus \mathcal{G}_{\tilde{B}}$$

Similarly, we can prove assertion 2.

**Proposition 4.7**

Let  $\mathcal{F}_{\tilde{A}}$  and  $\mathcal{G}_{\tilde{B}} \in$  IVNHSS, then we have the following

1.  $\oplus(\mathcal{F}_{\tilde{A}} \wedge \mathcal{G}_{\tilde{B}}) = \oplus \mathcal{F}_{\tilde{A}} \wedge \oplus \mathcal{G}_{\tilde{B}}$
2.  $\oplus(\mathcal{F}_{\tilde{A}} \vee \mathcal{G}_{\tilde{B}}) = \oplus \mathcal{F}_{\tilde{A}} \vee \oplus \mathcal{G}_{\tilde{B}}$
3.  $\otimes(\mathcal{F}_{\tilde{A}} \wedge \mathcal{G}_{\tilde{B}}) = \otimes \mathcal{F}_{\tilde{A}} \wedge \otimes \mathcal{G}_{\tilde{B}}$
4.  $\otimes(\mathcal{F}_{\tilde{A}} \vee \mathcal{G}_{\tilde{B}}) = \otimes \mathcal{F}_{\tilde{A}} \vee \otimes \mathcal{G}_{\tilde{B}}$

**Proof** By using definitions 4.1, 4.2, 4.4, and 4.5 the proof of the above proposition can be obtained easily.

**5. Conclusion**

In this paper, we study NHSS and IVNHSS with some basic definitions and examples. We extend the work on IVNHSS and proposed some fundamental operations on IVNHSS such as union, intersection, extended union, extended intersection, addition, and difference, etc. are developed with their properties and proved the De Morgan laws by using union, intersection, OR-operation, and And-Operation. We also developed the addition, difference, scalar multiplication, Truth-Favorite, and False-Favorite operators on IVNHSS. Finally, the concept of necessity and possibility operations on IVNHSS with properties are presented. For future trends, we can develop the interval-valued neutrosophic hypersoft matrices by using proposed operations and use them for decision making. Furthermore, several other operators such as weighted average, weighted geometric, interaction weighted average, interaction weighted geometric, etc. can be developed with their decision-making approaches to solve MCDM problems.

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